ANNUAL PROGRESS REPORT

Contract No.: EY-76-02-3497

Principal Investigator: Albert Simon, Prof. of Mechanical
and Aerospace Sciences and
Prof. of Physics

Date of Report: August 31, 1979

Report Number: C00-3497-36

ABSTRACT

The research topics covered include a study of the nonlinear behavior of the drift cyclotron loss-cone instability in mirror machines and a study of the longer time behavior of the nonlinear O'Neil trapped particle saturation. The saturation amplitude for a single dominant mode near the stability onset boundary was determined for the drift cyclotron loss-cone instability for a variety of mirror-like distributions. The results are in good agreement with computer simulation studies of Cohen and Maron.

MASTER *[Handwritten signature]*

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

INTRODUCTION

This past year's research (September 1978 - August 1979), under this contract, was concentrated in two topics. One was the nonlinear behavior of the drift cyclotron loss-cone instability (DCLC) in mirror machines, while the other was a study of the longer time behavior of the nonlinear O'Neil trapped particle saturation.

The major effort related to the DCLC. The entire nonlinear analyses, using what we call Model I, was completed and a detailed account has been submitted for publication. Model I has no mirror losses or particle sources and thus is dominated by radial transport. The nonlinear D.C. potential is negative and increases negatively as the saturation level of oscillation increases. This is opposite to the way a laboratory experiment behaves, since electron losses through the mirror dominate over radial transport. The magnitude of the Model I potential is also much lower than in laboratory observations. Nevertheless, comparison is possible with "experiments" carried out on a computer. Simulation studies by Cohen and Maron show good agreement with the saturation levels predicted by the Model I analysis, for the single mode situation.

Calculations using Model II, which does incorporate mirror losses and particle sources, are well under way. The D.C. potential is now positive in equilibrium and increases positively with the saturation level of oscillations. The magnitude of the D.C. potential is now in general agreement with laboratory observations. Detailed comparison with PR-6 data is just beginning.

The second topic is motivated by the appreciable difference between the early-time saturation behavior of the bump-on-tail instability (or damping of a single finite-amplitude wave in a Maxwellian plasma) predicted by O'Neil, and the long-time behavior predicted by Simon and Rosenbluth. We are attempting to determine the time-scale on which the O'Neil solution might cease to be valid. It is known to be accurate for times of order τ , where τ is the "trapping time." We have obtained a more convenient expression for the time behavior of the damping coefficient and nonlinear frequency shift in the $t \approx \tau$ range, and we are now extending the time range by including nonlinear behavior of the non-resonant electrons.

1. Nonlinear electrostatic oscillations and D.C. potentials in mirror machines.

The onset of drift cyclotron loss-cone instability¹ (DCLC) is believed to be the factor limiting the density in modern mirror machines, e.g. 2XIIB,² PR-6³ and PR-7⁴. The onset of this instability can be delayed by various techniques such as axial injection of cool plasma.⁵ Even when the instability does occur, it is observed to stabilize at some steady level of oscillation. The nonlinear evolution of this instability has been described by a quasilinear model⁶ which has had considerable success in fitting the observations on 2XIIB.⁷

Nevertheless, some observations of the nonlinear saturated state in mirror machines have shown that the fluctuation spectrum is limited to a very narrow band near the central ion-cyclotron frequency⁸⁻⁹. Indeed the observed bandwidths (2% of the ion-cyclotron frequency) are instrumental.

Measurements of the azimuthal mode structure are often characterized by a single value of the perpendicular wave number k . These observations have motivated studies of ion motion in the presence of a single, azimuthally propagating wave.¹⁰

These considerations led us to study the time-asymptotic nonlinear saturation level, near the stability boundary in parameter space, of a single dominating unstable wave. We used a variation of the Bogoliubov procedure and one which has been applied in the past to a variety of plasma instabilities.¹¹ [For a list of these applications, see Ref. 11, or the list of "previous publications under this contract," included below.]

During this year, we completed an analysis of the DCLC instability using a highly simplified model of a mirror machine denoted as Model I. In this model, the ions and electrons are described as collisionless and governed by the Vlasov equation. The oscillations are electrostatic and the Poisson equation completes the set. No mirror sinks or particle losses are included, the local approximation is used, and there is no variation in the magnetic field direction. A loss-cone equilibrium with a simple exponential density gradient is assumed. In many respects, the calculation is the nonlinear extension of the model used by Post and Rosenbluth.

Using this model, we first determined a linear stability boundary in n, ϵ space, where n is the central number density and ϵ is the spatial inverse e-folding length of the density. This was done for a variety of loss-cone-like distributions including a delta-function

$[\delta(v_1 - v_0)]$, a "broadened delta," and subtracted Maxwellians. In every case, the boundary for onset of the instability was determined by a mini-max analysis of the dispersion relation, just as in Ref. 1. We then assumed a slightly unstable system obtained by increase of the parameter ϵ above its critical value ϵ_c by the amount $\Delta\epsilon_c$, where $\Delta \ll 1$. Expansion of the final oscillating state in powers of $\Delta^{1/2}$ was carried out, assuming a single dominating mode at the critical wave number and frequency. In second order, we obtained the D.C. modified distribution functions and nonlinear D.C. potential. In third order, we obtained an equation which can yield a secular solution. The requirement that a secular solution be absent determined the amplitude of the saturated oscillating potential. The nonlinear frequency shift remains indeterminate in this simple model.

Analytic formulas were obtained for the ratio of the square of the final amplitude $|\Gamma|^2$ to Δ . Numerical evaluation involved carrying out some sums over Bessel functions and a velocity integral. This was done on a computer. The resulting values of $|\Gamma|^2/\Delta$ were determined for a variety of points on the linear stability boundaries. In every case, $|\Gamma|^2/\Delta$ was positive and small (the potential being in units of KT/e). The stability of each solution was checked and found to be stable. It was noted that $|\Gamma|^2/\Delta$ decreases with increasing density.

The mechanism of saturation was also determined. The dominant mechanism was a slight filling-in of the loss-cone. The D.C. nonlinear potential which developed had a negligible effect. Incidentally, in all cases the nonlinear D.C. potential was negative, and the radial electric field was also negative. All this can be understood on the basis of simple

radial transport models.

Of course, this Model I cannot be compared with laboratory experiments which have mirror losses and particle sources which are far more effective than radial transport. In these situations, electron end-loss is dominant and the D.C. potential and radial electric fields are both positive and increase in the presence of instability, whereas the potential in Model I is negative in the presence of instability. Furthermore, the magnitude of the D.C. potential in Model I is much smaller than that observed in laboratory experiments. Model II, which we discuss below, is more realistic and should be capable of comparison with experiment.

However, it is fortunate that there is a way to compare this Model I theory with "experiment." A computer experiment, that is! Bruce Cohen and Neil Maron² have used a particle-fluid hybrid simulation code to study the nonlinear properties of the DCLC instability. Simulation of the DCLC in a uniform magnetic field, with conditions similar to those of Model I, have shown good agreement with our theory for the single mode situation. It is interesting that this saturation amplitude was reached in only about 20 ion-cyclotron periods. For further information, see Ref. 12.

Further details on our Model I calculation are contained in a manuscript submitted to Phys. Fluids.¹³ A preprint is attached to this report.

We are now working on the nonlinear saturation of the DCLC using a Model II. In this model, the electrons are lost by collisional scattering out the loss cone, using the Pastukhov¹⁴ loss-rate expression, while the ions

degrade in energy into the loss-cone by slowing down on the electrons. There are also particle sources. We have determined that the equilibrium potential is positive, and that it increases positively when an instability is present. The magnitude of the potential is now in rough agreement with observations on PR-6.³ We have also established that the nonlinear frequency shift is now determinate.

Further work on Model II is under way. We expect to evaluate the saturation levels and to compare both these and the D.C. potential to measurements on PR-6 and PR-7. A detailed analysis of the time-variation observed in these experiments, in the stable regime, should enable us to estimate the time-variation of the experimental ion distribution-function, and hence the location of the stability boundary. This, in turn, should allow prediction of the coupled variation of the nonlinear D.C. potential and oscillation amplitude.

2. Extended Time Behavior of Nonlinear Plasma Oscillations

We have been studying the relation between the early time behavior of a weakly unstable single mode near saturation (or equivalently, the behavior of a weakly damped single wave) and the late-time prediction of Simon and Rosenbluth.¹⁵ The early time behavior is correctly predicted in the pioneering papers of O'Neil¹⁶ and Morales and O'Neil¹⁷ for times of order τ , where τ is the trapping time. This has been verified by experiment¹⁸ and by some careful computer simulations done by Denavit.¹⁹ Since the saturation amplitude is quite different from that predicted in

Ref. 15, it is of interest to inquire as to the time-scale on which the trapped O'Neil solution might break up.

We are attempting to extend the Morales-O'Neil calculation to times well beyond τ , by including nonlinear behavior of the non-resonant electron. We exclude "sideband" behavior by restrictive boundary conditions. In checking the time τ analysis of O'Neil, we found that a much simplified expression can be obtained for the damping coefficient and for the nonlinear frequency shift. These expressions may constitute a more useful analytic description of the time-behavior than has been available previously.

We are now evaluating the contribution of higher order terms in the Taylor expansion of the electron distribution function about the resonant velocity. Some preliminary analysis has suggested that the breakup time may be of order $(\omega_p \tau)^2 \tau$, where ω_p is the plasma frequency. Some details will be presented in Boston (Nov. 1979).

3. Personnel

FACULTY -

Albert Simon (MAS and Physics) - Dr. Simon has devoted 20% of his time to this project during the academic year, and full time for two summer months. He supervised two graduate students, Richard L. Myer and Thierry DeWandre.

GRADUATE RESEARCH ASSISTANTS -

(a) Richard L. Myer (MAS Department) - Mr. Myer, who completed his undergraduate studies at Stony Brook, is now in his fifth year of graduate study

and has nearly completed his dissertation on the nonlinear saturation of the drift cyclotron loss-cone instability.

(b) Thierry DeWandre (MAS Department) - Mr. DeWandre, who completed his undergraduate and master's level studies at the Catholic University of Louvain, Belgium, is now beginning his third year at Rochester. He is working on the extended time behavior of nonlinear plasma oscillations.

Other Federal Support. None of the personnel working on this contract have other federal support.

REFERENCES

1. R.F. Post and M.N. Rosenbluth, Phys. Fluids 9, 731 (1966).
2. F.H. Coensgen, W.F. Cummins, B.G. Logan, A.W. Molvik, W.E. Nexsen, T.C. Simonen, B.W. Stallard, and W.C. Turner, Phys. Rev. Lett. 35, 1501 (1975).
3. B.I. Kanaev and E.E. Yushmanov, Sov. Physics JETP, 40, 290 (1975).
4. Y.T. Baiborodov, 6th Eur. Conf. Controlled Fusion and Plasma Physics (Proc. Conf. Moscow, 1973) 2, 122 (1973).
5. H.L. Berk, T.K. Fowler, L.D. Pearlstein, R.F. Post, J.D. Callen, W.C. Horton, and M.N. Rosenbluth, in Plasma Physics and Controlled Nuclear Fusion Research, Vol. II, p. 151 (International Atomic Energy Agency, Vienna, 1969).
6. D.E. Baldwin, H.L. Berk and L.D. Pearlstein, Phys. Rev. Lett. 36, 1501 (1976).
7. H.L. Berk and J.J. Stewart, Phys. Fluids 20, 1080 (1977).
8. W.C. Turner, J. Phys. (Paris) 38, C6-121 (1977).
9. W.C. Turner, E.J. Powers and T.C. Simonen, Phys. Rev. Lett. 39, 1087 (1977).
10. G.R. Smith, J.A. Byers and L.L. LoDestro, Phys. Fluids (to be published).
11. S. Migliuolo and A. Simon, Phys. Fluids 21, 2218 (1978).
12. B. Cohen and N. Maron, to be submitted to Phys. Fluids; see also Mirror Theory Monthly, Lawrence Livermore Laboratory, July 15, 1979.
13. R.C. Myer and Albert Simon, submitted to Phys. Fluids.
14. V.P. Pastukhov, Nuc. Fusion 14, 3 (1974).
15. A. Simon and M.N. Rosenbluth, Phys. Fluids 19, 1567 (1976).

16. T.M. O'Neil, Phys. Fluids 8, 2255 (1965).
17. G.J. Morales and T.M. O'Neil, Phys. Rev. Lett 28, 417 (1972).
18. T.M. O'Neil, J.H. Winfrey and J.H. Malmberg, Phys. Fluids 14, 1204 (1971).
19. J. Denavit, private communication.

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT

1. A. Simon, "Nonlinear Transport in Weakly Ionized Plasma," Phys. Fluids 11, 1181 (1968).
2. A. Simon, "Convection of Weakly Ionized Plasma in a Non-uniform Magnetic Field," Phys. Fluids 11, 1186 (1968).
3. J.J. Kim and A. Simon, "Nonlinear Theory of the ExB Instability," Phys. Fluids 12, 895 (1969).
4. A. Simon and J.N. Shiau, "Quasilinear Theory of the Positive Column Spiral Instability," Phys. Fluids 12, 2630 (1969).
5. J.N. Shiau and A. Simon, "Nonlinear Theory of the Positive Column Spiral Instability," Phys. Fluids 13, 2569 (1970).
6. A.M. Sleeper and A. Simon, "Nonlinear Theory of the Drift-Dissipative Instability," Phys. Fluids 13, 2757 (1970).
7. Albert Simon and C.I. Weng, "Quasilinear Theory of the Mirror Flute Instability," Phys. Fluids 15, 2341 (1972).
8. V.G. Abhyankar and A. Simon, "Linear Threshold of the Buneman-Farley Instability," Phys. Fluids 17, 261 (1974).
9. Donald A. Monticello and Albert Simon, "Nonlinear Theory of the Collisional Drift-Wave Instability," Phys. Fluids 17, 791 (1974).
10. V.G. Abhyankar and A. Simon, "Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines," Phys. Fluids 17, 2040 (1974).
11. A. Simon, L. Gross and P.J. Catto, "Shear Stability Boundaries for Collisionless Drift Modes in Finite Geometry," Phys. Fluids 18, 231 (1975).
12. M.N. Rosenbluth and P.J. Catto, "An Improved Calculation of Critical Magnetic Shear in an Inhomogeneous Plasma," Nucl. Fusion 15, 573 (1975).
13. W.M. Tang, C.S. Liu, M.N. Rosenbluth, P.J. Catto, and J.D. Callen, "Finite Beta and Resonant Electron Effects on Trapped Electron Instabilities," Nucl. Fusion 16, 191 (1976).
14. A. Simon and M.N. Rosenbluth, "Single-Mode Saturation of the Bump-on-Tail Instability: Immobile Ions," Phys. Fluids 19, 1567 (1976).

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT (Con't)

15. P.J. Catto, K.T. Tsang, J.D. Callen and W.M. Tang, "Resonant Electron Effects on Trapped Electron Instabilities," *Phys. Fluids* 19, 1596 (1976).
16. A. Simon, "Single-Mode Saturation of the Bump-on-Tail Instability: II. Mobile Ions," *Phys. Fluids* 20, 79 (1977).
17. P.J. Catto and K.T. Tsang, "Linearized Gyro-Kinetic Equation with Collisions," *Phys. Fluids* 20, 396 (1977).
18. A. Simon and L.S. Gross, "Nonlinear Saturation of the Collisionless Drift Instability," *Phys. Fluids* 20, 946 (1977).
19. A. Simon and S. Migluelo, "Nonlinear Saturation of Two Unstable Modes: Survival Competition," *Phys. Fluids* 20, 1855 (1977).
20. K.T. Tsang, J.D. Callen and P.J. Catto, "Collisional Effects on Trapped Electron Instabilities," *Phys. Fluids* 20, 2113 (1977).
21. K.T. Tsang and P.J. Catto, "Analytic Solutions of the Two-dimensional Eigenvalue Problem for the Trapped-Electron Instability in Tokamaks," *Phys. Rev. Lett.* 39, 1664 (1977).
22. P.J. Catto, "Adiabatic Modifications to Plasma Turbulence Theories," *Phys. Fluids* 21, 147 (1978).
23. P.J. Catto and K.T. Tsang, "Trapped Electron Instability in Tokamaks: Analytic Solution of the Two-dimensional Eigenvalue Problem," *Phys. Fluids* 21, 1381 (1978).
24. K.T. Tsang, P.J. Catto, J.D. Whitson and J. Smith, "Absolute Universal Instability is Not Universal," *Phys. Rev. Lett.* 40, 327 (1978).
25. P.J. Catto, "Linearized Gyrokinetics," *Plasma Physics* 20, 719 (1978).

LIST OF PUBLICATIONS (Sept. 1978 - Aug. 1979)

A. WORK COMPLETED PREVIOUSLY, BUT PUBLISHED THIS YEAR

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percentage DOE Support</u>
Nonlinear Sturation of the Dissipative Trapped Electron Instability	S. Migliuolo and A. Simon	Phys. Fluids <u>21</u> , 2218 (1978)	100%

B. WORK COMPLETED THIS YEAR, OR CONTINUING

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percentage DOE Support</u>
Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability	R.C. Myer and A. Simon	Phys. Fluids (submitted)	100%

THESES COMPLETED UNDER THIS CONTRACT

1. John J. Kim, "The Second Order Shifts in Frequency and Amplitude in the Non-Linear Theory of ExB Instability," Physics Dept., 1969.
2. Jeng-Nan Shiau, "Small Amplitude Nonlinear Theory of the Positive Column Spiral Instability," MAS Department, 1969.
3. Arthur M. Sleeper, "The Drift Dissipative Instability," Physics Dept., 1970.
4. Cheng-I Weng, "Nonlinear Theory of the Mirror Flute Instability," MAS Dept., 1972.
5. Donald A. Monticello, "Nonlinear Theory of the Collisional Drift Wave Instability," Physics Dept., 1973.
6. Vishwas Govind Abhyankar, "Density and Temperature Gradient Flute-Mode Instabilities," MAS Dept., 1974.
7. Leonard Gross, "Nonlinear Theory of the Collisionless Drift Instability," Applied Physics Program, 1975.
8. Stefano Migliuolo, "Nonlinear Saturation of the Dissipative Trapped Electron Instability," Physics Dept., 1977.