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OPTIONS FOR AXISYMMETRIC OPERATION OF MFTF-B

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ABSTRACT

The flexibility of MFTF-B for axisymmetric experiments has been investigated. Interchanging the axicell coils and increasing their separation results in an axisymmetric plug cell with 12:1 and 6:1 inner and outer mirror ratios, respectively. For axisymmetric operation, the sloshing-ion neutral beams, ECRH gyrotrons, and the pumping system would be moved to the axicell. Stabilization by E-rings could be explored in this configuration. With the addition of octopole magnets, off-axis multipole stabilization could also be tested. Operating points for octopole and E-ring-stabilized configurations with properties similar to those of the quadrupole MFTF-B, namely $T_{ic} = 10 - 15$ keV and $n_c = 3 \times 10^{13} \text{ cm}^{-3}$, have been obtained. Because of the negligible radial transport of central-cell ions, the required neutral-beam power in the central cell has been dramatically reduced. In addition, because MHD stabilization is achieved by off-axis hot electrons in both cases, much lower barrier beta is possible, which aids in reducing the barrier ECRH power. Total ECRH power in the end cell is projected to be ≈ 1 MW. Possible operating points for both octopole and E-ring configurations are described along with the stability considerations involved.

I. INTRODUCTION

Recent studies of tandem mirror reactor configurations have shown significant advantages in devices with end cells shortened over those required for quadrupole stabilization.^{1,2} A switch from the quadrupole end cell allows several transition coils, required to maintain low radial transport in the quadrupole configuration, to be eliminated. Several axisymmetric end cells have been proposed, including simple mirror fields with hot electron ring or disc stabilization,^{3,4} and wall stabilization of both the central and end cells.⁵ Configurations which are nearly axisymmetric in the core plasma include multipole fields higher than quadrupole in the end cell,¹ ponderomotive stabilization with rf fields near the ion cyclotron frequency,⁶ and the TARA arrangement^{7,8} with the quadrupole stabilization anchor cell outboard of the axisymmetric plug/barrier cell.

Many of the proposed modifications to the tandem mirror will be tested in near term experiments such as TARA, Phaedrus, and STM. Should any of the methods prove successful the next step would be to test the configuration at the plasma operating parameters of MFTF-B. This report focusses on the flexibility of the MFTF-B for axisymmetric experiments. Figure 1 shows the magnet geometry of MFTF-B, and the field, density, and potential profiles for the choke coil mode. In this mode the 6T coil is turned down to eliminate the axicell. An axisymmetric end cell with inner and outer mirror ratios of 12:1 and 6:1, respectively, can be obtained by reversing the position of the present axicell magnets and lengthening the cell. The magnetic geometry for both the E-ring and octopole upgrades presented in this report is shown in Fig. 2. The central cell length, between the 12T coils, is shortened from the 20.2 meter length of the choke coil mode to 15.8 meters in these upgrades. One can see the location of the magnets in relation to the MFTF-B vessel in Fig. 3, which shows the yin yang coils in place. To operate in these modes the yin yang magnets would be removed. Finally, if the flexibility to operate in either the TARA mode or with a single axisymmetric end cell was desirable, the 6 T and 12 T coils would be located far enough in toward the center of the machine to allow operation of the transition coils and the yin-yang. For these experiments the barrier and plug gyrotrons, sloshing ion neutral beams, and the pumping system would all be moved to the axicell. With the MFTF-B transition magnets and yin-yang included, TARA mode experiments could be done.

Figure 1. Profiles of the MFTF-B magnetic field, plasma density, and electrostatic potential.

Figure 2. Magnet arrangements for the octupole [MFTF-B(0)] and E-ring [MFTF-B(E)] upgrades of MFTF-B.

Figure 3. Position of the octopole end cell coils in the MFTF-B vacuum vessel.

E-ring stabilization could also be tested by not energizing the quadrupole coils and forming the hot electron rings in the axicell. With the addition of modest octopole magnets, off axis minimum-B stabilization experiments could also be tried.

In this report we will discuss possible octopole and E-ring stabilized operation of MFTF-B. We first address stability issues which set windows for possible operating points. These issues also impact design considerations such as the strength of the octopole field and the use of a bucking coil in the E-ring configuration. Particle and power balance for possible operating points are computed. A comparison of operating points with an improved model of quadrupole operation of MFTF-B is also given.

II. ELMO RING STABILIZED CONFIGURATION

There are many factors which must be considered in putting together an axisymmetric, E-ring stabilized configuration for MFTF. Many of these are driven by stability issues specific to E-ring stabilization and will be discussed in some detail below. Generic tandem mirror design considerations incorporated in this study will be reviewed in Sec. IV. Calculations specific to the parameters of an E-ring stabilized MFTF-B [MFTF-B(E)] will be presented in Sec. II.4.

II.1. END CELL STABILITY ISSUES

An effort has been made in this study to address as many of the current stability issues for hot-electron stabilized symmetric tandem mirrors as possible. Many of the issues identified through 1981 have been summarized by Campbell and Logan.⁴ These include the so-called Nelson-Lee-VanDam (NLV) beta criterion,⁹⁻¹² hot electron diamagnetic well formation,^{13,14} hot electron interchange,^{15,16} compressional Alfvén modes,^{10,17} temperature anisotropy modes, the mirror instability, and Whistler instabilities.¹⁸ Recent improvements to the theories for these modes, such as finite Larmor radius (FLR) effects, leading to changes in the stability criterion associated with the mode have also been included in this study. In addition, stability criteria for the precessional mode identified by Baldwin and Berk¹⁹ have been

examined. Modifications to these criteria^{20,21} when axial shear in the $E \times B$ frequencies exists in the device have also been considered. Finally, results from very recent literature on the experimental verification of electron ring formation in the SM-1¹³ and EBT¹⁴ devices have been used. The assumptions involved in evaluating each of these issues for MFTF are given below.

II.1.1. NLV Beta Limit

To operate an axisymmetric plug/barrier cell for MFTF with E-ring stabilization in the same cell necessitates finite core beta at the ring location. This is because hot electron beta in the core is required to form the thermal barrier potential depression and sloshing ion beta is required to stabilize microinstabilities in the core and to form the confining potential peak. The sloshing ion, passing and trapped ion, and passing electron pressures account for beta of about 3% in the barrier region. The hot electron beta in the barrier varies as the total barrier beta (input to the power balance calculation) is changed. With the ring located at the axial position of the barrier, on field lines just radially outside those that map to the core plasma in the central cell, the barrier beta determines the core beta at the ring location in the Nelson-Lee-VanDam stability criterion. Without FLR effects the criterion takes the form

$$\beta_c(r_A) [1 + \beta_h(r_A)] < \frac{2 C_{c1} \Delta}{|R_c(r_A)|} \quad (1)$$

where Δ is the full width at half maximum of the electron ring, $\beta_c(r_A)$ and $\beta_h(r_A)$ are the core and hot electron beta values at the ring location and $R_c(r_A)$ is the vacuum radius of curvature at the ring. The coefficient C_{c1} can vary between 1 and 2 depending on details of the hot electron distribution function. To minimize ring thickness and power requirements, $C_{c1} = 2$ has been used in this study. With FLR effects inequality (1) takes the form^{22,23}

$$\beta_c(r_A) [1 + \beta_h(r_A)] + k_{\perp}^2 \rho_h^2 \beta_h < \frac{4\Delta}{|R_c(r_A)|} \quad (2)$$

where ρ_h is the hot electron relativistic Larmor radius, and the perpendicular wavelength is taken to be

$$k_1^2 = k_r^2 + k_\theta^2 = \frac{16}{(2\Delta)^2} + \frac{m^2}{r_A^2}$$

where m is the azimuthal mode number. Inequality (2) is valid if the core beta at the ring $\beta_c(r_A)$ does not exceed

$$\beta_c^{\text{crit}} = \frac{4\Delta}{|R_c|} - k_1^2 \rho_h^2 \beta_h \quad (3)$$

and if the hot beta $\beta_h(r_A)$ does not exceed

$$\beta_h^{\text{crit}} = \frac{4\Delta}{|R_c| k_1^2 \rho_h^2} \quad (4)$$

Inequality (2) subject to the constraints of Eqs. (3) and (4) determines the ring thickness Δ for given $\beta_c(r_A)$, $\beta_h(r_A)$, and $R_c(r_A)$. To minimize ring thickness and power requirements each of these quantities must be reduced as much as possible. The lower limit on $\beta_c(r_A)$ is determined by the required hot electron pressure to form the thermal barrier and the lower limit on $\beta_h(r_A)$ is given by the criterion for formation of a diamagnetic well at the ring (described below). The radius of curvature $R_c(r_A)$ can be reduced by adjusting the magnet coil placement and by using a bucking coil at the midplane of the ring/barrier/plug cell. The effects of this coil will be described in Sec. V.

Note also that $\beta_c(r_A)$ corresponds only to pressure at the ring location which responds as an MHD fluid. Since a large fraction of the barrier pressure is from hot electrons it is expected that $\beta_c(r_A)$ should be reduced from the total barrier beta by an amount corresponding to the fraction of the hot barrier electrons distribution function which is above an MHD cutoff energy. Estimates of this fraction (see Appendix A) show that up to 50% the barrier beta may not be responding as an MHD fluid for typical MFTF-B(E) parameters. The calculations made from the NLV criterion using $\beta_c(r_A) \approx \beta_b$ therefore tend to overestimate the required ring thickness and the required ring power.

II.1.2. Diamagnetic Well Formation

The critical value of $\beta_h(r_A)$ required to just flatten the radial gradient in $B_0 \oint \frac{dl}{B(z)}$ is given approximately by¹⁴

$$\beta^{\text{crit}} = 2\Delta \left\{ \frac{1}{R_c} + \frac{2\pi \alpha^2}{\Delta z} [1 - \alpha I_0(x)] \right. \\ \left. + [I_0(x) I_1(x) + \frac{1}{4} x I_0(x) - \frac{1}{4} I_1(x)] \right\} \quad (5)$$

where Δz is the full length at half maximum of the ring, $x = kr_A$, $k = 2\pi/L$, L is the distance between mirrors, and $(1 + \alpha)/(1 - \alpha)$ is the vacuum mirror ratio. To dig a diamagnetic well the ring beta $\beta_h(r_A)$ must be greater than β^{crit} . The precise value of $\beta_h(r_A)$ required to stabilize the device against both interchange and ballooning instabilities must be determined from a detailed stability calculation which has not been done. Typically $\beta_h(r_A) > 1.5 \beta^{\text{crit}}$ has been used. In addition, the radial gradient of $B_0 \oint dl/B(l)$ on the outer edge of the ring can impact the maximum stable central cell length predicted for the precessional mode so that higher ring beta values may be linked to longer stable central cells (see Sec. II.1.5).

II.1.3. Hot Electron Interchange and Compressional Alfvén Modes

The criteria for stability of hot electron interchange and compressional Alfvén modes are compactly written in the form¹⁰

$$\frac{2\alpha \left(\frac{k_\theta}{k_r}\right)^2 \frac{R_c}{\Delta} \beta_h}{\left[1 - \alpha \left(\frac{k_\theta}{k_r}\right)^2 \frac{v_k}{\omega_{ci} \Delta}\right]} < \frac{v_A^2}{v_k^2} < \frac{\alpha^2 k_r^2 \Delta R_c}{2\beta_h \left[1 + \alpha \left(\frac{k_\theta}{k_r}\right)^2 \frac{v_k}{\omega_{ci} \Delta}\right]} \quad (6)$$

where k_r and k_θ are the radial and poloidal wave vectors given by $k_r \approx 2/\Delta$ and $k_\theta \approx m/r_A$, ω_{ci} is the ion cyclotron frequency, v_A is the plug Alfvén speed and

$$v_k = 2.0 \times 10^7 \bar{E}_h / [B_V(r_A) |R_c(r_A)|] \quad (7)$$

In Eq. (7), $\bar{E}_h$ is the average ring electron energy in (keV), $B_V(r_A)$ is the vacuum magnetic field at the ring in (T) and $R_c(r_A)$ is in (cm). The parameter

α is thought to have a reasonable range of 0.2 to 1.0. In this study, $\alpha = 1$ has been used; this reduces the estimate of the required power for the ring and may be somewhat optimistic near the NLV limit.

Inequality (6) sets upper and lower limits on the ratio n_c/n_h (cold electron density/hot electron density) at the ring. The left hand inequality is the compressional Alfvén criterion while the right hand side gives the hot electron interchange limit. The hot electron interchange criterion given here is a parameterization for the results of a complicated theory¹⁵ which is useful for scaling purposes. When a final set of parameters for MFTF-B(E) were chosen, the detailed graphical results in Ref. 15 were checked to assure that the hot electron interchange criterion was satisfied.

II.1.4. Temperature Anisotropy, Mirror, and Whistler Modes

Other instabilities which can exist in an axisymmetric plug/barrier cell with an E-ring include those driven by temperature anisotropy and loss cone anisotropy, the mirror mode, and Whistler instabilities.^{4,18} The anisotropy modes set upper and lower bounds on the cold electron density in the cell given by

$$\frac{1}{\eta} \leq \frac{(\omega)_{pe,cold}^2}{(\Omega)_{ce}^2} \leq 1.34 \left(\frac{T_{\parallel}}{T_{\perp}} \right) + 0.45 \left(\frac{T_{\parallel}}{T_{\perp}} \right)^2 \quad (8)$$

where $\omega_{pe,cold}$ is the cold electron plasma frequency, Ω_{ce} is the electron cyclotron frequency and the temperature anisotropy is given by

$$\frac{T_{\parallel}}{T_{\perp}} < \frac{B_{\mu}}{B(r_A)} - 1 \quad (9)$$

where B_{μ} is the resonant plasma magnetic field and $B(r_A)$ is the midplane plasma field at the ring locations. In the long thin approximation $B(r_A) = B_V(r_A)(1 - \beta_h)^{1/2}$ where $B_V(r_A)$ is the midplane vacuum field at the ring and β_h is the ring beta. The inequality (8) is satisfied in the MFTF-B(E) cases considered.

The maximum ring beta is constrained by the more restrictive of the mirror and Whistler instability criteria. The constraint takes the form

$$\beta_h(r_A) < \min [0.8, 2 \left(\frac{T_{||}}{T_{\perp}} \right)^2] - \beta_c(r_A) \quad (10)$$

where the first factor in the brackets corresponds to the mirror mode, the second to Whistler modes. Typically, the Whistler constraint cannot be satisfied with the available gyrotron frequencies which also must form the thermal barrier in the same mirror cell. This constrains the available mirror ratio for second harmonic resonance at the ring which keeps $T_{||}/T_{\perp}$ low. The mirror mode constraint is satisfied.

II.1.5. Precessional and Shear ExB Modes

Perhaps the most serious instability predicted for E-ring stabilized tandem mirror configurations is the precessional mode of Baldwin and Berk.¹⁹ This instability results when the negative-energy precessional wave of the hot electron ring is destabilized by coupling to positive-energy shear-Alfven waves which can exist in the central cell. Coupling is avoided if the hot-electron precessional frequency exceeds the core-plasma ion gyrofrequency but for the parameters of MFTF-B(E) this cannot be achieved without going to very high ring electron energies (>20 MeV). Recently Tsang²⁰ has re-examined these modes when the axial potential profile of a tandem mirror is included. The criterion for stability of the precessional mode including axial shear in the ExB drift frequency is given in terms of an upper limit on the central cell length by

$$L_c < \frac{\pi V_{Ac}}{\max [|\Omega|, |\Omega + (1 + \beta_w) \omega_k|]} \quad (11)$$

where V_{Ac} is the central cell Alfven velocity. In inequality (11)

$$\beta_w = \beta_w (1 + 1/\beta_h)/(2\epsilon) \quad (12)$$

$$\epsilon = 1/(R_c \frac{\partial \ln B}{\partial r}) \quad (13)$$

$$\omega_k = 2\omega_{kh}/(1 + \beta_h/2) \quad (14)$$

$$\omega_{kh} = \bar{E}_h L_1 \Omega / (e \Phi_p R_c) \quad (15)$$

$$\Omega = \omega_{ec}^{ExB} - \omega_{cc}^{ExB} \quad (16)$$

In Eqs. (12) - (16), R_c is the vacuum radius of curvature at the ring (negative for simple mirror fields), $\partial \ln B / \partial r$ is evaluated locally on the outside edge of the ring where the gradient has been reversed (i.e., positive), $\bar{E}_h$ is the hot electron average energy, L_1 is the radial scale length of the plug potential $e\phi_p$ and ω_{ec}^{ExB} , ω_{cc}^{ExB} are the end cell and central cell ExB drift frequencies respectively.

The calculations in this study use $\beta_w = \beta_c(r_A)$, and $L = r_p / \gamma_{pro}$ where r_p is the plug plasma radius and γ_{pro} is the plug radial density profile exponent. In addition, it has been proposed²⁴ that for thermal barrier potential profiles the plug potential be averaged as

$$e\phi_p = \frac{L_b}{L_{tot}} (e\phi_b) + \frac{L_p}{L_{tot}} (e\phi_p) \quad (17)$$

where L_b , L_p are lengths over which the barrier potential ($e\phi_b$) and plug potential ($e\phi_p$) form and $L_{tot} = L_b + L_p$.^{*} This leads to the definition of the ExB frequency in the end cell as

$$\omega_{ec}^{ExB} = \frac{1.0 \times 10^7 (e\phi_p)}{(r_c)^2 B_c (1 - \bar{\beta}_c)^{1/2}} \text{ (rad/s)} \quad (18)$$

where $e\phi_p$ is in (keV), r_c is the central cell plasma radius in (cm), B_c is the central cell vacuum field in (T) and $\bar{\beta}_c$ is the average central cell beta. Finally, evaluation of ϵ from Eq. (13) requires a consistent calculation of the gradient reversal due to diamagnetic currents in the ring. Estimates of $\partial \ln B / \partial r$ on the outside edge of the ring have been made from results of previous experiments^{13,14} but the detailed calculation has not been done.

Note that when there is no axial shear in the ExB frequencies, $\Omega = 0$, inequality (21) gives the precessional mode condition. The important condition for thermal barrier tandem operation occurs when the ExB fluid

*A better treatment has recently been given by Tsang in Ref. 21. These results have not yet been incorporated in evaluating MFTF-B(E).

precession is in the opposite direction from the hot electron precession [Ω and $(1 + \beta_w) \omega_k$ opposite signs]. In this case the electron precessional mode is more stable.

II.2. HOT ELECTRON FORMATION

Recently there have been extensive studies of hot electron ring formation in the SM-1 device at TRW¹³ and the EBT-I at ORNL.¹⁴ These results point out several aspects of ring formation which, apart from stability analyses, must be considered when putting together a tandem mirror end cell design.

One point is that the hot-electron energy in the ring is not necessarily strongly dependent on the applied microwave power or neutral gas pressure in the mirror cell. Instead, as found in many experiments,^{13,25} the electron energy appears to be limited to values such that the ratio $\rho/|R_c| < 0.05$ where ρ is the relativistic electron gyroradius given by

$$\rho = 1.7 \times 10^{-1} (\gamma^2 - 1)^{1/2} / B(\text{tesla}) \quad \text{cm} \quad (19)$$

with $\gamma = 1 + \bar{E}_h(\text{keV})/511$ and R_c is the radius of curvature of the magnetic field lines at the ring. It has been suggested¹³ that this scaling indicates that the breakdown of invariance of the electron magnetic moment²⁶ plays a critical role in determining the temperature of the hot-electron ring population. Typically for MFTF-B(E) parameters this limit on ring energy has not been approached for any realistic designs because the radius of curvature and the field strength are large ($B \approx 1.0$ T, $|R_c| \sim 300$ cm).

Other rules-of-thumb for ring formation have emerged in the experiments. Typically a pressure distribution for the ring with $p_{||}/p \sim 0.2-0.3$ models the observed diamagnetism and stored energy. With this distribution, the ring length at half maximum pressure is comparable to the radius, $\Delta z \sim r_A$. Also observed is the tendency for the ring to form near the second harmonic of the applied microwave power in the vacuum field. This occurs in spite of finite beta in the ring which reduces the effective magnetic field with plasma. Finally, the required stored energy in the ring to flatten $\oint \phi dz/B$ scales approximately as the ring full width at half maximum squared, inversely with the length and is relatively insensitive to the hot electron pressure

distribution. Again, this emphasizes the need to reduce the ring width as much as possible. The minimum ring width calculated for the EBT-S geometry is the relativistic electron gyrodiameter plus approximately 1 cm to account for dispersion in the electron guiding centers. This limit should not be a consideration for large devices like MFTF-B(E) due to large radius of curvature.

11.3 GLOBAL STABILIZATION

A detailed global stability calculation including interchange and low- m ballooning modes with consistent kinetic treatment of the hot-electron ring is not yet available. Instead, estimates have been made for the required ring and central cell parameters of a stable configuration. These are at best crude estimates; the next step would be an analysis with the finite Larmor radius stability code FLORA.²⁷ However, this would only be an intermediate step since the assumption of ring rigidity (ring and core parameters far from the NLV limit) is used in the code. Treatment of the global stability problem near the NLV limit without FLR effects is under development.

The conditions for global stability can be estimated by evaluating the MHD growth rate for interchange modes²⁷

$$\gamma_{\text{MHD}}^2 = \frac{\int \frac{dz}{B} \left[\frac{\kappa}{rB} \frac{\partial p}{\partial \psi} - \frac{1}{B^2} \frac{\partial p_{\perp}^{\text{e-ring}}}{\partial \psi} \frac{\partial p_{\perp}}{\partial \psi} \right]}{\int \frac{dz}{B} \frac{\rho_m}{r^2 B^2}} \quad (20)$$

and requiring $\gamma_{\text{MHD}}^2 < 0$ for stability. In Eq. (20), κ is the field line curvature (R_c^{-1}), $p = p_{\perp} + p_{\parallel}$, $p_{\perp}^{\text{e-ring}}$ is the hot electron ring pressure, ρ_m is the core plasma mass density, $\psi = r^2 B/2$ and the integration is over the entire length of the machine. Since the denominator is always positive, the stability conditions are determined by

$$\int \frac{dz}{r R_c B^2} \frac{\partial p}{\partial \psi} < \int \frac{dz}{B^3} \frac{\partial p_{\perp}^{\text{e-ring}}}{\partial \psi} \frac{\partial p_{\perp}}{\partial \psi} \quad (21)$$

To estimate the integrals in inequality (21) use

$$\begin{aligned} \psi &\equiv r^2 B/2 ; \quad \partial/\partial\psi \equiv (rB)^{-1} \partial/\partial r ; \quad \partial p/\partial r \approx -p/L \\ &\partial p^{e\text{-ring}}/\partial r \approx -p^{e\text{-ring}}/\Delta ; \quad 2p/B^2 = \beta ; \quad L \approx r_p/\gamma_{\text{pro}} \end{aligned} \quad (22)$$

where r_p is the plasma radius and γ_{pro} is the density profile exponent. Using Eqs. (22), inequality (21) takes the form

$$\beta_h > \frac{2\Delta}{\bar{B}_b l_h} \left[\frac{\bar{B}_t}{\bar{B}_b} \frac{l_t \bar{B}_t}{|\langle R_{c,t} \rangle|} + \frac{l_b \bar{B}_b}{|\langle R_{c,b} \rangle|} \right] \quad (23)$$

where l_t , l_b are the lengths of bad curvature ($R_c < 0$) at the end of the central cell and in the plug/barrier cell respectively, $\bar{B}_t$, $\langle R_{c,t} \rangle$ are the average field and radius of curvature in the bad curvature region of the central cell, $\bar{B}_t = \bar{B}_{cc}$, and l_h is the axial extent of the hot electron ring.

Since inequality (23) is only approximate, the hot electron beta used in the power balance calculations is typically taken to be about a factor of 1.5 higher than this estimate. In addition, nearly flat radial pressure profiles in the plug core plasma have been used to reduce the effects of radial averages on the global stability criterion and to reduce the drive for interchange instabilities on field lines in the core. To support these profiles a nearly reactor grade halo plasma will be required (see Sec. IV.3). The assumption of rising radial pressure profiles in the central cell plasma,²⁷ though more conservative from a stability viewpoint, may be difficult to achieve in near term experiments. Finally, compressibility effects²⁸ which have not been included in the MHD growth rate [Eq. (20)] reduce the beta requirements for global stability. This is an area for future study.

II.4 STABILITY RESULTS

This section will describe results of stability calculations for the baseline MFTF-B(E) configuration. Power balance modeling indicates that the parameters for this operating point can be achieved with the available magnets, sloshing ion and central cell neutral beams, and end cell ECRH

systems (see Sec. VI). Results will be presented essentially in the order used in Secs. II.1-II.3. The critical stability parameters are given in Table 1.

To begin the design process a magnet configuration was obtained using the MFTF-B axicell magnets by reversing the positions of the 6T and 12T coils and lengthening the cell. For E-ring scenarios the radius of curvature at the ring location (midplane of the cell just outside the core plasma radius) must be kept as small as possible to reduce the ring thickness and required power. To do this a bucking coil is used at the end cell midplane with current running in the opposite direction from the mirror coils. The current used in this coil is limited by the requirement that no separatrix form inside the outer edge of the ring. The EIG input deck for the coil configuration as well as results from magnetics calculations are given in Sec. V.1. For this configuration the plug mirror-mirror length is 3.6 m, the central cell mirror-mirror length is 15.7 m ($\int dl/B$ length = 14 m), the plug radius of curvature is $R_{c,p} = 189$ cm at a radius $r = 30$ cm, the radius of curvature for the bad curvature ($R_c < 0$) transition region at the end of the central cell is $R_{c,tr} = -1500$ cm. The bad curvature lengths in the transition and plug regions are $l_{b,tr} \approx l_{b,p} = 121$ cm. Assuming $rR_c = \text{constant}$, the radius of curvature near the center of the ring $r_{ring} = 40$ cm is $R_{c,ring} = -141$ cm.

The ring half thickness and critical beta are determined by iterating the NLV criterion and the requirement for gradient reversal, inequality (2) and Eq. (5). Taking $\beta_c(r_A) = \beta_b = 0.20$, $\beta_h = 0.40$ and the ring average energy $\bar{E}_h = 1.2$ MeV, the NLV criterion gives $\Delta = 9.4$ cm. To be conservative we have taken the ring thickness to be slightly larger, $\Delta = 10.4$ cm. The critical beta for flattening of the gradient in $\int dl/B$ is $\beta_h^{crit} = 0.23$ so that $\beta_h = 0.4$ should be sufficient to establish positive gradients in $\int dl/B$ on the outside of the ring.

The hot electron interchange and compressional Alfvén criteria given by inequality (6) are both satisfied at a ring energy of 1.2 MeV. The ratio $v_A^2/v_k^2 = 17$ for these parameters; the right hand side (hot electron interchange) is R.H.S. = 66 while the left hand side (compressional Alfvén mode) is L.H.S. = 0.76. Here, $B_v(r_A) = 0.96$ T has been used.

The criteria for stability of temperature anisotropy modes, inequality (8) is satisfied for cold electron density $n_{ec,r} = 4.0 \times 10^{12} \text{ cm}^{-3}$ at the ring. The ratio of cold electron plasma frequency to electron cyclotron

Table 1. MFTF-B(E) critical stability parameters.

Parameter	Description	Value
$\bar{E}_{\text{hot}}$	Ring hot electron energy	1.2 MeV
n_{hot}	Ring hot electron density	$1.0 \times 10^{12} \text{ cm}^{-3}$
$n_{\text{ec},r}$	Cold electron density at the ring	$4.0 \times 10^{12} \text{ cm}^{-3}$
β_{hot}	Ring beta	0.4
Δ	Ring full width half maximum	10.4 cm
r_A	Ring inner radius	30 cm
r_{ring}	Ring radius (center)	35 cm
l_{ring}	Ring effective length	66 cm
$ R_c(r_A) $	Radius of curvature at ring	141 cm
ρ_h	Hot electron Larmor radius	0.57 cm
$\beta_c(r_A)$	Core beta at the ring	0.2
$\bar{E}_{e,h}$	Barrier hot electron energy	195 keV
$n_{eh,b}$	Barrier hot electron density	$2.8 \times 10^{12} \text{ cm}^{-3}$
$n_{ec,b}$	Barrier cold electron density	$3.1 \times 10^{11} \text{ cm}^{-3}$

frequency squared is 0.32 while the right hand side of inequality (8) is R.H.S. = 0.52 for second harmonic resonance using the available 56 GHz ECRH power. The criteria for the mirror and Whistler modes, inequality (10) indicates that the Whistler mode may be unstable due to large temperature anisotropy, $T_{\parallel}/T_{\perp} = 0.34$. However, the Whistler instability has not been observed in the TMX-U end cell even at similarly high anisotropy.²⁹

The criteria for stability of precessional modes when axial shear is present in the radial electric field, inequality (21), cannot be satisfied for the baseline parameters of MFTF-B(E). For this configuration the Alfvén velocity in the central cell is $V_{Ac} = 2.4 \times 10^8$ cm/sec, the shear in ExB frequencies between the central cell and plug is $\Omega_{ExB} = -5.6 \times 10^4$ rad/sec, the coefficient $\hat{\beta}_w = -0.58$, and the frequency $\omega_k = 2.8 \times 10^6$ rad/sec. Under these conditions the maximum stable central cell length is $L_{c,max} = 3$ m. However, this calculation uses the most pessimistic assumption for the curvature drift frequency ω_k , namely the curvature drift frequency of the hot electrons in the ring, without considering radial potential profiles in the plug. Following Tsang,²⁰ with parabolic radial potential profiles gives $\omega_k = 2.3 \times 10^5$ rad/s implying a maximum central cell length of $L_{c,max} = 79$ m if the stabilizing effects of axial shear in the potential are neglected. Including the effects of axial shear in the potential results in $L_{c,max} = 136$ m. The only conclusion that can be reached from these calculations is that the baseline MFTF-B(E) specified in this report ($L_c = 14$ m) should operate in a parameter regime which will test the theories on this instability.

Finally, the global stability criterion, inequality (23), is satisfied with ring beta, $\beta_h = 0.40$. To evaluate the right hand side of inequality (23), axial shape factors have been obtained for the ring density profile (see Section IV.2) and averaging the contributions from scattering and drag processes yields a ring effective length $l_h = 68.6$ cm. For MFTF-B(E), the average field in the bad curvature region at the end of the central cell is $B_t = 2.8$ T while the average beta there is the same as the central cell value. $\bar{\beta}_t = \bar{\beta}_{cc} = 0.19$. When these values are combined with the geometry parameters given above, inequality (23) is satisfied provided that $\beta_h > 0.33$. Taking $\beta_h = 0.40$ provides some margin against inaccuracies in these estimates.

III. OCTOPOLE STABILIZATION

The octopole end cell was introduced¹ to allow shorter end cells in reactor designs and thereby reduce the central-cell length required for ignition. This type of end cell resembles in some ways the E-ring-stabilized end cell described in the previous section. The magnet field is formed by adding (large) main and (small) transition octopole coils in the axicell. A hot electron "mantle" plasma is created outside the core plasma just as in the E-ring stabilization scheme. However, there are two very important differences. Instead of a fast-drifting "rigid" electron ring, the mantle electrons are required to drift slowly enough to retain an MHD response to instabilities in the central cell. Furthermore, since these electrons are located in the minimum- $|B|$ well formed by the octopole field, many of the stability restrictions found for hot E-rings are absent. Because the minimum $|B|$ stabilizes interchange modes, the cold electron density in the mantle may be much less than the hot electron density. Since the cold-electron density is much lower, the power drain by escaping cold electrons is less and the electron heating efficiency is much higher than for an E-ring in a maximum- $|B|$ field. Experiments in the Ioffe-bar-stabilized mirrors, INTEREM and IMP^{30,31} and, more recently, in TMX-U,²⁹ have verified this picture of the cold-to-hot density ratio and the improved heating efficiency in minimum- $|B|$ wells. In addition, a peak β approaching unity was observed for the hot electrons in INTEREM.³⁰

Although the operating window for an octopole is not so restricted as for the E-ring end cell, several constraints must be satisfied by the magnetic field and plasma. These include requirements from MHD equilibrium and stability, discussed in the next section, and limits on the magnet design governed by considerations of the radial transport induced by the octopole field, which is covered in the subsequent section. In addition, the mantle and barrier electrons can be subject to the anisotropy-driven instabilities discussed in Sec. II.1.4. We provide for a mirror ratio of at least two on the field lines containing mantle electrons to minimize the possibility of large electron anisotropy. Electron-cyclotron-heating (ECH) at second harmonic should also reduce the anisotropy.³¹

III.1 MHD EQUILIBRIUM AND STABILITY

The radial magnetic field and pressure profiles in the central and end cells are shown schematically in Fig. 4. In the near axisymmetric core of the end cell, the magnitude of the field is decreasing with radius and the plasma is unstable without the mantle plasma. Although a detailed theory has not yet been worked out, it is expected that the core will bridge to the minimum- $|B|$ region through the pressure of the mantle electrons and thus be stable. In the only theoretical work specifically directed to multipole end cells,³² it has been found that the following criterion must be satisfied in the core,

$$\int_{-L/2}^{+L/2} \frac{\kappa}{rB^2} \frac{\partial p}{\partial \psi} ds < 0 \quad (24)$$

which is the same as Eq. (20) and inequality (21) without the hot electrons. If this criterion is not satisfied, deviation from axisymmetry of the equilibrium field (with plasma present) is predicted to occur in the core. We recognize inequality (24) as the familiar low-beta interchange criterion for MHD stability.

In the essentially axisymmetric core plasma, the average curvature is negative and the above criterion could not be satisfied if pressure decreases with flux (or radius) in both the central and end cells. To satisfy inequality (24) we add sufficient pressure from hot electrons in the end cell to form a positive pressure gradient out to the edge of the core, as indicated in Fig. 4. If the gradient is large enough, it will satisfy inequality (24). This region of positive pressure gradient extends axially over a limited length of the end cell; the pressure gradient must be large enough to balance out the negative gradients on other portions of each field line in the core. From the edge of the core to the minimum $|B|$ region, we maintain a zero or very small positive pressure gradient, which should allow stability. Once in the region of increasing $|B|$, the pressure gradient can become negative and satisfy inequality (24).

The stability of these multipole configurations to ballooning modes has not been checked in detail. Because of finite-Larmor radius (FLR) effects, we expect that the core will be stable to all but $m = 1$ modes, and that these will be rigid. The absence of high-energy ions in the mantle suggests that

Figure 4. Magnitude of the magnetic field, B , and the plasma pressure, p , vs. radial coordinate $= r^2 B_0 / 2$ at the midplane of the central cell (top) and end cell (bottom).

this region will not be rigid but that a radial structure to the mode will develop. Tools to analyze this problem have not yet been developed. We assume, therefore, that satisfaction of the above interchange criterion is sufficient to guarantee stability.

In order to apply inequality (24) to determine the required mantle beta for a given central cell and end cell betas, we must furnish pressure functions. We choose p to be separable in ψ and R ,

$$p = \omega(\psi) \hat{p}(R) \quad (25)$$

For the core plasma we use a parabolic profile in radius,

$$\omega = 1 - (\psi/\psi_p)^2 \quad (26)$$

and for the extra hot electrons in the end-cell core which are needed to stabilize the core,

$$\omega = (\psi/\psi_p)^2 \quad (27)$$

where ψ_p is the magnetic flux at the core plasma surface.

For the pressure function, $\hat{p} = \hat{p}_\perp + \hat{p}_\parallel$, in the core, we use the model functions given in Appendix B.

The "extra" stabilizing hot electrons in the core are expected to be narrower in axial extent than the combination of hot electrons and sloshing ions represented by Eqs. (B-6)-(B-8). We use the same pressure functions as in Eqs. (B-6)-(B-8) but with the replacement of β_b by β_m and B_b by B_m , where the mirror ratio B_{m2}/B_m is chosen to be two (see Fig. 5). Numerical evaluation of the interchange criterion for the magnet design described in Section V shows that the required mantle beta is related to the barrier and central-cell peak beta by

$$\beta_m = 1.2 \beta_b + 0.35 \beta_{cc} \quad (28)$$

One additional point to consider is the maximum energy of the hot mantle electrons. As described above, they must not drift faster than the growth rate of MHD instabilities in the central cell so as not to become rigid. In the

Figure 5. Magnetic field and potential vs distance z showing the cardinal points used in the modeling of the MFTF upgrades (not to scale).

octopole, the direction of electron drift reverses as one proceeds outward from the axis because of the minimum in B. Both sufficiently inside and sufficiently outside the minimum $|B|$ region, electron orbits will be axis encircling. Because islands of constant $|B|$ form in the minimum $|B|$ region, some electrons in this region will drift on surfaces which do not enclose the axis. Since it is not clear just how to predict rigidity of these electrons, it seems preferable to evaluate the drift frequency of axis encircling electrons just outside the minimum $|B|$ region, i.e. those electrons at the edge of the mantle plasma. Although we have not carried out the calculations for the MFTF-B octopole magnet, results for similar magnets proposed for a small reactor¹ and for TMX-U³³ show that limiting the mantle electron energy to less than about 1 MeV ensures MHD response. In fact, for all calculations presented in this report, the mantle energy was chosen to be 800 keV, well below this limit.

III.2 RADIAL TRANSPORT OF CENTRAL-CELL IONS

Replacement of the quadrupole magnets in the end cell by octopole magnets causes the field near the axis to be much more axisymmetric. The residual asymmetries still bring about neoclassical radial transport and we need to verify that the transport of central-cell ions is negligible compared to other loss mechanisms. The other principal loss is via collisional trapping into the end cell. For MFTF-B the trapping time averaged over the plasma radius is about 4 s.

Depending on the relative sizes of the collision, bounce, and azimuthal drift frequencies, the neoclassical diffusion can obey different relations. For conditions operative in MFTF-B, diffusion falls in the boundary-layer region if radial electric fields are controlled by potential-control plates in the end tank.³⁴ The diffusion coefficient follows the relation

$$D = (a^2/4) (\nu_i \nu_b/R)^{1/2} \quad (29)$$

where a is the maximum radial deflection of a central-cell ion on a single reflection from the end cell, R is the mirror ratio of the choke to the central cell, ν_b is the bounce frequency of central-cell ions, and ν_i is the collision frequency.

$$v_i = \frac{n_i \ln A}{3.5 \times 10^{17} m_i^{1/2} T_i^{3/2}} \quad (30)$$

All quantities are in SI units with T in keV and m in amu. The radial deflection, a , is found from an integral along the ion trajectory

$$\begin{aligned} a &= - \frac{2m_i v_{\parallel} r_0}{B_0 q} \int_0^{s_t} \frac{\mathcal{H} ds}{r_0^2} \\ &= - 9.1 \times 10^{-3} \frac{r_0 (\epsilon m_i)^{1/2}}{B_0} \int_0^{s_t} \frac{\mathcal{H} ds}{r_0^2} \end{aligned} \quad (31)$$

where r_0 is the starting radial position of the ion in the central cell, ϵ is the mean ion energy, B_0 is the central-cell magnetic field, s_t is the turning point of the ion, and $\mathcal{H}$ is the geodesic curvature. In an octopole field near the axis, the geodesic curvature depends on azimuthal angle as $\cos(4\theta)$. The amplitude of $\mathcal{H}$, found for the octopole coil set to be presented in Section V, is shown in Fig. 6. We see that there are regions of both positive and negative $\mathcal{H}$. If we plot the integral of Eq. (31) along the field line of maximum $\mathcal{H}$ as a function of the assumed turning point location, we get the result shown in Fig. 7. We note that the magnitude of the integral is quite sensitive to the choice of the turning point and could be zero if the ion turns at two specific locations between the minimum- $|B|$ point and the outer mirror. However, electric fields, curvature, and grad B forces cause the ion to drift azimuthally and thus sample the curvature on adjacent field lines and it is unlikely that ions will have zero deflection without very good control of the radial electric field. We have chosen to take the maximum value of the integral as a conservative estimate of the deflection.

From the diffusion coefficient at any radial location we can estimate the radial confinement time from

$$\tau_1 = \frac{\int_0^r n r' dr'}{rD \left| \frac{\partial n}{\partial r} \right|} \quad (32)$$

Figure 6. (a) Magnetic field B on axis vs z for the octopole configuration.
 (b) Geodesic curvature, $10/r_0^2$, vs z for a field line started at
 $r = 0.20$ m and $\theta = 22.5^\circ$ in the central cell.

Figure 7. Running integral of geodesic curvature, $\int_0^s ds/r_0^2$ vs z for the field line of Fig. 3.

For a parabolic radial density profile

$$n = n_0 [1 - (r/r_p)^2] \quad (33)$$

Eq. (41) becomes

$$\tau_1 = \frac{r^2}{8D} \left[2 \left(\frac{r_p}{r} \right)^2 - 1 \right] \quad (34)$$

The confinement time depends on radius, both directly through Eq. (34) and indirectly through the dependence of the quantity a on radius. For octopole fields, the quantity a scales as r^3 , so confinement time decreases strongly with radius.¹ Results for the coil set described in Sec. V are presented in Table 2. Even at the edge of the plasma, the confinement time of 9 s is considerably greater than the average trapping time into the end cell of 4 s. The result of 140 s at 20 cm is much improved over the time of 0.4 s predicted for the quadrupole version of MFTF-B.³⁴

IV. GENERIC ISSUES IN TANDEM MIRROR DESIGN

Several aspects of the design of MFTF-B with E-ring or octopole end cells are independent of the stabilization method. These are discussed in the following sections.

IV.1 SLOSHING ION ADIABATICITY

As the length of the end cell is reduced, the possibility that the sloshing ions will not behave adiabatically must be considered. When an E-ring is used for stabilization, the need to reduce the ring thickness in the NLV criterion by reducing the radius of curvature must be balanced against the need for sloshing ion adiabaticity. We can use an expression of Cohen, Rowlands, and Foote²⁶ to compute the minimum ion energy at the midplane of the end cell for a specified confinement time due to non-adiabatic effects. The losses due to non-adiabaticity are assumed to be stochastic in this model. The expression reads (SI units)

Table 2. Neoclassical diffusion coefficients and radial confinement time for ions starting at $r = 0.2$ and 0.3 m in the central cell.

r (m)	$\left \int_0^s ds/r^2 \right _{\max}$	a (m)	D (m ² /s)	τ (s)
0.2	0.074	9.0×10	7.2×10	140
0.3	0.20	3.7×10	1.2×10	9

$$W_{ad} = \frac{10^3 \kappa^2 B_{min}^2 L_{||} z_i^2}{m_i (1 - 0.036 \ln \Lambda)^2} \quad (35)$$

where B_{min} is the field at the minimum, $z_i e$ is the ion charge, m_i is its mass, $L_{||}$ is the scale length for a parabolic well at the minimum, Λ is defined below, and

$$\kappa = \frac{1}{2\lambda^2} \left[\frac{1 + \lambda^2}{2\lambda} \ln \left(\frac{1 + \lambda}{1 - \lambda} \right) - 1 \right] \quad (36)$$

with $\lambda = v_{||}/v$ at the minimum. A representative value of λ is 0.55 for which we find $\kappa = 0.77$. Inserting representative numbers into the full expression for Λ in Ref. 26 we obtain

$$\Lambda = 0.129 m_i (L_{a'a}/\tau)^2 \Delta^4 / W_{ad} \quad (37)$$

where $L_{a'a}$ is the distance between ion turning points in meters, τ is the collisional confinement time for the sloshing ions, 0.5 s for MFTF-B, and Δ is given by

$$\Delta = 1 - (B_{min}/\lambda B_{max}) \quad (38)$$

The expression for $L_{||}$ is

$$L_{||} = (2B)^{1/2} \left(\frac{d^2 B}{ds^2} \right)^{-1/2} \quad (39)$$

evaluated at the minimum field. To find the magnetic field with plasma present we use the long-thin-approximation,

$$B^2 = B_v^2 - 2\mu_0 p_l(B) \quad (40)$$

with the pressure function in Eq. (B-8). The plasma magnetic field is used in both Eqs. (35) and (39).

Results for the E-ring and octopole coils are given in Table 3. The latter has a somewhat higher energy limit for adiabatic behavior because of

Table 3. Maximum ion energy for adiabatic confinement of sloshing ions in the end cell.

Beta	E-ring (keV)	Octopole (keV)
0.0	120	180
0.10	100	150
0.15	90	135
0.20	80	120
0.25	70	105
0.30	--	95

its longer length. At the barrier beta of 0.20 used in the operating-point calculations of Section VI, an 80 keV sloshing ion energy is only marginally adiabatic in the E-ring end cell. We would expect that the losses of sloshing ions would therefore be somewhat higher than computed from collisional effects alone. No problems with non-adiabaticity are anticipated for the octopole end cell unless the barrier beta exceeds 0.30.

IV.2 Hot Electron Axial Shape Factors

The power required to sustain hot barrier electrons in the core and hot ring electrons at outer radii must balance losses due to synchrotron radiation, scattering, and drag. Because of axial variations in densities, we must average over the density profile to obtain axial profile factors, which multiply the densities at the cardinal point "b". The axial density distributions are the following: for the hot electrons in the core

$$n_{eh}(z) = \frac{\beta_b \beta_b^2}{1.6 E_{ehb}} \left(\frac{3\beta_{max} - \beta}{3\beta_{max} - \beta_b} \right) \left(\frac{\beta_{max} - \beta}{\beta_{max} - \beta_b} \right) \quad (41)$$

and for the cold density in the core

$$n_{ec}(z) = \begin{cases} n_{ecb} \exp [-(z_b - z)^2 / \ell^2] & z < z_b \\ n_{ecb} & z \geq z_b \end{cases} \quad (42)$$

with $\ell^2 = (z_b - z_{ml})^2 / \ln [n_{ecb} / n_{ec}(z_{ml})]$, z_{ml} the axial position of the end cell inner mirror and n_{ecb} the cold electron density at the barrier (see Fig. 5). For the hot electrons in the ring and mantle the axial profile is more peaked due to lower collision frequency,

$$n_h^{ring} = \frac{\beta_h \beta_b^4}{1.6 E_{eh}^{ring} \beta^2} \left[\frac{\beta_{max} - \beta}{\beta_{max} - \beta_b} \right]^{5/2} \quad (43)$$

Axial profile factors are then calculated from

$$A = \frac{1}{L_b} \int_c^{L_b} f(z) \frac{B_b}{B} dz \quad (44)$$

where the factor $B_b/B(z)$ accounts for the variation of the flux-tube area along the axis. For synchrotron radiation losses $f(z) = n_{eh}(z) B^2(z)$, for hot electron self scattering $f(z) = n_{eh}^2(z)$, for hot electron-cold electron scattering, and drag in the core $f(z) = n_{eh}(z) n_{ec}(z)$ while in the ring $f(z) = n_{ec}(z) n_{eh}(z)$.

IV.3 Halo Density and Thickness

To achieve the radial pressure profiles used in this study, a sufficient halo plasma must be maintained between the core plasma and the walls to shield out recycled gas in the form of neutrals, Frank-Condon and impurities. Detailed analysis for MARS indicated that this halo would form with electrons at about 50 eV and that an electron density of $2-5 \times 10^{12} \text{ cm}^{-3}$ would be required to shield out both impurity atoms and hydrogen neutrals. In MFTF with octopole or E-ring stabilization, the power required to sustain this electron plasma comes from scattering and drag of these cold electrons on the hot (mantle or ring) electrons through which the halo passes in both end cells.

V. END CELL MAGNETS

V.1 MAGNETS FOR THE E-RING CONFIGURATION

The end cell for the E-ring configuration uses only circular coils and is therefore completely axisymmetric. The MFTF-B axicell coils are repositioned with the 6T coil placed 3.6 m outboard of the 12 T coil. To increase the normal curvature at the midplane of the cell where the E-ring is to be formed, a bucking coil is used in each end cell with current running in the opposite direction from the mirror coils. These coils are very similar in size and construction to the MFTF-B central cell coils so that no costly development is required for them. The specifications for the end cell coils are given in Table 4. These coils are shown in Fig. 8. The EIG³⁵ input for this configuration is given in Appendix E. The magnetic field strength and

Table 4. Magnet parameters for the E-ring configuration.

Choke coil: Superconducting insert and outer coil		
Total magnetic field (on axis) 12T		
Axial position $z = 7.88$ m		
Insert coil:	Cross section	65.7 x 35.2 cm
	Radius	38.8 cm
	Current density	2.0 kA/cm ²
Outer coil:	Cross section	93.8 x 40.7 cm
	Radius	92.8 cm
	Current density	2.36 kA/cm ²
Outer mirror coil: Total magnetic field (on axis) 6T		
Axial position $z = 11.5$ m		
	Cross section	93.8 x 40.7 cm
	Radius	92.8 cm
	Current density	2.42 kA/cm ²
Bucking coil: Magnetic field bucks end cell midplane field		
Axial position $z = 9.75$ m		
	Cross section	16.3 x 33.6 cm
	Radius	250 cm
	Current density	-0.5 kA/cm ²

Figure 8. View of the end-cell coils for the E-ring stabilized design (central cell is to the left).

normal curvature on axis for this end cell configuration are shown in Fig. 9a and 9b. Finally, Fig. 10 shows a vertical cut through the coils, mod-B contours off axis and the position of the end cell core plasma and ELMO ring.

V.2 OCTOPOLE END CELL

A view of the coils in the octopole end cell is given in Fig. 11 with a listing of the EIG input in Appendix F. The inner (choke) and outer mirror coils are identical to those used in the E-ring-stabilized end cell while the two main and two transition octopole coils (each end) must be fabricated. The current in the main coil has been adjusted to furnish an off-axis minimum at all azimuthal angles. The current in the transition coil is adjusted to cause the circular plasma surface at the midplane of the central cell to map into a circle at the minimum $|B|$ plane (point b in Fig. 5) in the main octopole.

Figures 12 and 13 show contours of $|B|$ and the trajectories of several field lines in the r - z planes at 22.5° and 45° , respectively. One field line is started at the assumed plasma radius of 0.3 m in the central cell, while the others are lines on which the hot mantle electrons will reside. We see from the figures that the off-axis minimum $|B|$ is at about $r = 0.55$ m, $z = 10.75$ m and the mantle radius at this plane can be taken as 0.60 m. For the mantle field line just at the edge of the mantle in the $\theta = 45^\circ$ plane, the outer mirror ratio is about 1.85, less than the ratio of two desired for minimization of electron anisotropy. Further work with the design of the main coil could probably increase this mirror ratio. In Sec. III we saw that the neoclassical radial confinement of central-cell ions was much better than needed. Furthermore, the peak field at the conductor is only about 6.5 T, well below a nominal design value of 8 T for Nb-Ti superconductors. We expect that reductions of the size are possible and could lead to lower fabrication cost as well as lower power requirements for the mantle electrons.

Figure 9. (a) Magnetic field B on axis vs z for the E-ring configuration.
 (b) Normal curvature $R \equiv R_C^{-1}(r_A)$ on the ring flux tube vs z .

Figure 10. Elevation view of the E-ring end cell showing coil cross-sections, core plasma, and the ELMO ring plasma. Mod-B contours from 0.7 T to 2.4 T are also shown (dotted lines).

Figure 11. View of the end-cell coils for the octopole-stabilized design (central cell is to the left).

Figure 12. Contours of $|B|$ in the $\theta = 22.5^\circ$ plane in the octopole end cell. Projections of field lines are shown which start at (from innermost field line): $z = 0$, $r = 0.3$ m; $z = 10.8$, $r = 0.4, 0.5, 0.6$ m in the $\theta = 22.5^\circ$ plane.

Figure 13. Contours of $|B|$ in the $\theta = 45^\circ$ plane in the octopole end cell. Same field lines as in Fig. 7 but these field lines remain in the 45° plane because of the symmetry of the field.

VI. POWER AND PARTICLE BALANCE

Parameters for steady state operating points of MFTF-B(O) and MFTF-B(E) have been obtained using the TMRBAR particle and power balance code³⁶ modified to include central cell neutral beams.³⁷ Further modifications have been made for this study including modeling of axisymmetric end cells (both E-ring and octopole) and evaluation of E-ring parameters and stability criteria. In addition, models of MFTF-B (quadrupole) configurations have been included so that some degree of comparison against previous power and particle balance calculations can be made. The following sections describe briefly the MFTF-B calculations done for comparison (see Appendix C), the parameters obtained for the "most reasonable cases" of MFTF-B(E) and MFTF-B(O), and discussion of the fundamental differences between operation of the quadrupole stabilized MFTF-B and the axisymmetric E-ring or octopole stabilized upgrades. Note that "most reasonable case" is taken to mean that scenario which satisfies as many of the local stability criteria as possible, satisfies the global stability criteria given the approximations used, and achieves the calculated operating parameters using the available MFTF-B magnet and heating systems. Design of the most attractive axisymmetric E-ring or octopole stabilized tandem mirror of MFTF size, building from the ground up, was not within the scope of this study.

VI.1 POWER BALANCE RESULTS FOR MFTF UPGRADES

This section will describe the power balance results for the octopole and E-ring stabilized axisymmetric upgrades of MFTF. All of these results were obtained using the modified TMRBAR power balance code. The performance of the upgrades will also be compared with results for MFTF-B (quadrupole) with similar physics assumptions.

The parameters for the axisymmetric cases and the quadrupole case are given in Table 5. In addition to the parameters defined in Appendix D, this table includes the equivalent plasma Q (fusion power/input power) for a 50-50 D-T mixture at the density and temperature listed. The value for $Q_{DT,cc}$ takes into account only the power input to the central cell (neutral beam power) while $Q_{DT,TOT}$ includes input power to the central cell, end cell core, and in

Table 5. Comparison of MFTF-Upgrade and MFTF-B performance.

Parameter (see Appendix D)	MFTF-B(Q)	MFTF-B(O)	MFTF-B(E)
n_c (10^{13} cm^{-3})	4.54	3.37	3.47
n_H (10^{13} cm^{-3})	1.29	0.58	0.46
T_{ic} (keV)	9.5	15.2	14.9
E_H (keV)	30.8	41.0	40.6
T_{ec} (keV)	3.4	6.3	7.2
ϕ_e (kV)	21.3	43.1	47.6
$\eta_e = \phi_e / T_{ec}$	6.2	6.84	6.61
ϕ_b (kV)	16.1	30.7	33.8
$\eta_b = \phi_b / T_{ec}$	4.7	4.87	4.69
ϕ_c (kV)	33.1	44.3	45.9
$\eta_c = \phi_c / T_{ic}$	3.5	2.91	3.08
$(n\tau)_{CA}$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	2.1	2.0	2.4
$(n\tau)_P$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.24	1.53	2.17
$(n\tau)_r$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.24	∞	∞
$\tau_{ }^c$ (s)	4.6	5.9	6.9
$\tau_{ }^H$ (s)	0.67	2.1	2.79
$\tau_{ }^{\text{tot}}$ (s)	2.0	4.7	5.9
$\tau_{\perp}$ (s)	0.53	∞	∞
τ_{pump} (s)	0.53	4.54	6.26
τ_E (s)	0.11	0.52	0.36
β_D	2.75	2.78	2.75
H_D	2.3	2.0	2.0
β_D	0.55	0.17	0.20

Table 5. (Continued.)

Parameter (see Appendix D)	MFTF-B(Q)	MFTF-B(O)	MFTF-B(E)
$n_{eh,b}$ (10^{12} cm^{-3})	3.8	2.3	2.8
$n_{ec,b}$ (10^{12} cm^{-3})	0.43	0.26	0.31
$n_{eh,a}$ (10^{12} cm^{-3})	2.5	1.3	1.5
$n_{ew,a}$ (10^{12} cm^{-3})	4.2	1.9	3.5
$n_{pass,b}$ (10^{12} cm^{-3})	0.66	0.54	0.54
T_{ew} (keV)	55	43	58
E_{eh} (keV)	416	204	195
SLQSH	1.4	1.4	1.4
R_{effs}	1.20	1.14	1.10
S_{den}	2.7	3.0	3.1
$P_{NB,cc}$ (MW)	2.71	0.33	0.27
$I_{NB,cc}$ (A)	24.3	3.7	2.9
$I_{gas,cc}$ (A)	115	-0	-0
I_{pump} (A)	31.2	1.8	1.3
I_{past} (A)	3.6	1.4	1.2
I_{hot} (A)	4.0	0.5	0.4
I_{radial} (A)	31.3	0	0
$P_{ECRH,a}$ (kW)	866	55	152
$P_{ECRH,b}$ (kW)	1496	203	267
$P_{ECRH,hal}$ (kW)	--	641 (302) ^a	--
$P_{ECRH,man}$ (kW)	--	507	670 (503) ^b
$P_{NB,a'}$	206	98	235
$Q_{DT,cc}$	0.22	1.05	1.45
$Q_{DT,TOT}$	0.094	0.19 (0.24) ^a	0.24 (0.27) ^b

^aWith flat radial pressure profile in the end cell core plasma.^bWith ring thickness taken to be 1.5 times FWHM.

the axisymmetric cases, end cell hot electron plasmas required for stabilization. Note that $Q_{DT,TOT}$ is normalized to a central cell length of 14 m for comparison.

Before comparing the performance parameters several fundamental differences between axisymmetric and quadrupole operation of MFTF must be identified. First, the outer mirror ratio of the end cell is higher in the axisymmetric cases (6:1 compared with 3:1 in MFTF-B) because the cell uses circular mirror coils instead of the quadrupole yin-yang coil. This allows higher confining potentials and deeper barrier potential depressions for the same sloshing ion energy in the axisymmetric configurations. Second, since radial transport is negligible in the octopole and E-ring cases, the total particle throughput of these devices is significantly reduced over MFTF-B. This allows the central cell particle balance to be maintained by ionization of the neutral beam current without additional cold gas fueling. Charge exchange on the cold gas used to fuel the central cell of MFTF-B has a large effect on the power balance. Third, the transition region between the central cell mirror and the yin-yang anchor in MFTF-B has been eliminated in the axisymmetric configurations so that the volume for trapping of passing ions is significantly reduced. This reduces the requirements on the pumping system as well as contributing to the reduction in particle throughput for the axisymmetric cases. Finally, in both the axisymmetric configurations the stabilization mechanism relies on off-axis pressure using hot electrons. Compared with MFTF-B, which requires core pressure in the quadrupole min- $|B|$ fields, this allows operation at much lower barrier beta in the axisymmetric cases. This aids in reducing the barrier ECRH power if the hot electron energy does not decrease so far that scattering and drag power losses become large. On the other hand, ECRH power must be provided and precisely absorbed in the off axis regions to form the stabilizing mantle pressure in the octopole and the rigid electron ring for the E-ring. The required beta (and ECRH power) in these regions for stabilization tends to scale with the barrier beta (and other quantities) so that there is an advantage to using the lowest possible barrier beta. Taking a realistic lower limit on the maintainable barrier electron energy of 200 keV sets the minimum barrier beta for the axisymmetric cases at $\beta_b = 0.17$ for the octopole and $\beta_b = 0.20$ for the E-ring.

Other considerations in the selection of cases for Table 5 are the following. The cold electron fraction at the barrier minimum was set to

$f_{ec} = 0.10$ [higher than the value of $f_{ec} = 0.05$ in previous MFTF-B(Q) calculations (see Appendix C)] based on present experimental evidence that hot electron fractions greater than 90% are difficult to achieve. In addition, an attempt was made to provide ample distance between the barrier minimum and the confining potential peak to keep the axial electric field in this region as small as possible. Typical values for the axisymmetric configurations in Table 5 are in the range 1.5-2.5 kV/cm. Finally, the sloshing ion injection energy was taken to be 80 keV while the central cell beam average energy was 60 keV, assuming that the performance would be optimized if the available pure 80 keV beams were used to form the potential peak.

As discussed in Appendix B, the modeling options used for this comparison include: (1) zero secondary electron current entering the central cell from the end walls, (2) trapped-ion pumping by a drift pumping system so that no pump beams are used and no cold electron source is included at the potential peak, (3) the weak rf model for calculating required ECRH power at the barrier and potential peak, and, finally (4) for MFTF-B, the ratio of charge exchange to ionization cross sections for cold gas $R_{cx} = 1.5$. The central cell neutral beam power listed is the sum of the ionization power P_i and the power delivered to the central cell plasma from charge exchange events on beam neutrals, P_{cx} . The calculation takes the form

$$P_{NB,cc} = P_i^{cc} + P_{cx}^{cc} \quad (45)$$

where

$$P_i^{cc} = \frac{q}{1000} \frac{n_H n_{ec}}{(n\tau)_{pc}} v_{cc} \bar{E}_{inj} \quad (46)$$

$$P_{cx}^{cc} = \frac{\langle \sigma v \rangle_{cx}^B}{\langle \sigma v \rangle_i} \frac{P_i^{cc}}{\bar{E}_{inj}} (\bar{E}_{inj} - \bar{E}_c) \quad (47)$$

$$\bar{E}_c = \left(\frac{3}{2} n_c T_{ic} + n_H \bar{E}_H \right) / (n_c + n_H) \quad (48)$$

(see Appendix D for notation). Detailed modeling of cold gas charge exchange off the central cell warm population and the mirror trapped hot ions has not been done.

Finally, it should be emphasized that the purpose of Table 5 is to draw some comparison between quadrupole and axisymmetric operation of MFTF. To that end, an attempt has been made wherever possible to present a fair comparison. Common parameters for all of the cases are given in Table 6. Table 5 does not therefore present a baseline case for MFTF-B for either 0.5 s or 30 s operation, nor does it necessarily show the optimum operating point for either the octopole or E-ring configuration. It does show the difference in performance that can be achieved between quadrupole and axisymmetric stabilization of an MFTF sized tandem mirror.

The performance of MFTF-U is significantly improved over quadrupole MFTF-B due primarily to the elimination of radial transport. The effective $Q_{DT,TOT}$ is improved by a factor of 2-3 in the axisymmetric cases in spite of the high ECRH power required to sustain the off-axis hot electron pressure for stabilization. The central cell $Q_{DT,cc}$ increases by a factor of 5-7 due to a reduction in neutral beam power by nearly an order of magnitude. The equivalent fusion power per unit length is slightly lower in the axisymmetric cases because they operate at significantly higher temperature (and lower density at fixed beta). The higher temperature operation is linked to the 6:1 vacuum mirror ratio in the end cells (3:1 in quadrupole MFTF-B) which allows the sloshing ions to be confined at higher confining potential for fixed 80 keV injection energy in the axisymmetric cases. Particle and energy confinement times are significantly higher in the axisymmetric cases due primarily to an increase in electron temperature T_{ec} by about a factor of 2.

The explanation for many of these differences can be found by comparing the particle balances for the two stabilization schemes. In quadrupole MFTF-B radial transport accounts for 31.3 A while the ion current which traps in the long transition and yin-yang end cell (which must be pumped out) adds another 31.2 A. The sum of these two currents in the axisymmetric configurations ($I_{RADIAL} = 0$) is less than 2 A. This allows the operating point to be adjusted so that at balance the sum of all the particle losses can be supplied by the central cell ionization current. In quadrupole MFTF-B, if an operating point could be found which satisfied this criterion (and this is not assured

Table 6. Common parameters in the quadrupole and axisymmetric cases.

Parameter	Description	Value
$\hat{\beta}_{cc}$	Central cell peak beta	0.383
γ	Density profile exponent	2.0 (parabolic)
r_c	Plasma radius (central cell)	30 cm
$B_{c,v}$	Vacuum field (central cell)	1.0 T
f_{ec}	Cold electron fraction at barrier	0.1
g_b	(Passing density fraction at barrier) ⁻¹	2.75
$E_{inj,a'}$	Sloshing ion injection energy at a'	80 keV
$\bar{E}_{inj}$	Central cell injection energy (average)	60 keV
B_b	Barrier minimum field	1.0 T
B_{m1}	Central cell mirror field	12.0 T

because energy balance must also be satisfied), the resulting neutral beam requirement would be prohibitively large. Instead, the fueling is provided with cold gas and the penalty is taken for charge exchange events between a cold gas neutral and a hot central cell plasma ion. Note that the results here model the average energy lost per charge exchange event as a density weighted average of the central cell Maxwellian and hot mirror trapped ion energies. This energy loss results in lower central cell ion and electron temperatures.

VII. ENGINEERING AND COST CONSIDERATIONS

A variety of upgrades^{38,39} to MFTF-B have been studied, the most extensive being one that allows DT operation for 100 hour runs to do fusion blanket testing. That upgrade requires the complete replacement of the end cell magnets, a new shielded vacuum vessel, a new halo pumping scheme, and modification of the beamlines for tritium operation. At a cost of approximately \$400 M, about equal to the original construction cost of MFTF-B, that upgrade is an upper bound to the costs that might be acceptable for machine upgrades.

At the other end of the scale on cost, one could rather easily modify MFTF-B to operate in the TARA configuration.⁶ Referring to the magnetic geometry of MFTF-B shown in Fig. 1, the TARA mode can be run by exchanging the 6T and 12T coils, then lengthening the axicell from 2.22 meters to 4.6 meters. (See Fig. 14 for the TARA mode geometry.) Sloshing beams, ECRH, and drift pumping would all be moved to the axicells. Since plasma leaving the 6T coil must map through the transition to the anchors, its position relative to the transition coils is predetermined. The 12T coil would therefore be moved into the central cell by 2.43 meters, shortening the central cell by 4.86 meters.

Modifications to operate in the E-ring stabilized mode are similar to those for the TARA mode but with some differences of major importance. Primary among these is that the central cell solenoid length has been chosen to be the same as in MFTF-B. Specifically, the distance between the 6T coils in MFTF-B is 15.76 m, as is the distance between 12T coils in the E-ring and octopole designs. Thus, in these upgrades the 6T coil must be moved towards the end cell into what was the transition region in MFTF-B. Now, the transitions must be removed, as must the yin yang anchors since the flux will

Figure 14. Magnet geometry for TARA-mode operation of MFTF-B.

not map through them. Also, the end region cryopanel was designed for the elliptical flux bundle that exits the yin yang, so the end wall and cryopanel geometry must be changed. It is these changes to the transition, anchors, and end regions which make this a more expensive upgrade than the TARA upgrade.

An alternative is to shorten the central cell, as would be done for the TARA mode, map the flux through the transition and anchors, and therefore leave as is the magnet geometry of the ends. While this alternative is less expensive than the one envisioned for the E-ring operating mode of this paper it would result in lower overall performance owing to the shorter central cell. One might indeed choose this scenario for its lower cost, but for purposes of comparison with the current MFTF-B configuration and its performance we chose a common central cell solenoid length. It is important to note that the central cell length for MFTF-B is different in the choke coil mode than in the axicell mode. In the choke coil mode used for comparison here the 6T coils are run at low current to eliminate the 6T mirror, making a mirror-to-mirror length (between 12T coils) of 20.20 m.

Modifications to incorporate the octopole are essentially just those to construct the E-ring configuration, with the addition of the octopole coils. Also, the separation between the field peaks is 3.6 m in the E-ring mode and 5.2 m in the octopole mode. To configure the machine in this mode would take approximately the same amount of time (about 12-18 months) as for the E-ring. However, the octopole magnets would require 2-3 years to construct, so they would be started while MFTF-B was still operational so as not to increase the downtime for building the upgrade.

While the costs of these upgrades were not estimated one can make educated guesses of the cost by noting that, except for the octopole coils and the bucking coils for the E-ring cell, there is essentially no new fabrication. What new fabrication there is, such as structural support for magnets, modifications for relocating existing hardware, new cryogenic pipe runs, etc, involves relatively low cost compared to labor costs. From the photo of the completed MFTF-B in Fig. 15, showing extensive cryogenic plumbing and dewars on top of the machine, one can begin to appreciate the effort needed to get inside the vessel to remove magnets. Annual construction crew costs on MFTF-B ranged from \$20-30 M (including both the 150-man LLNL crew and contractor crews), so if one assumes that the full construction crew is

Figure 15. The MFTF-B at the end of the construction phase 1986.

required then the estimated 12-18 month modification time gives an upper bound on the costs.

For the octopole, the costs are roughly the same for construction but the costs for building the octopole coils must be added. Since the yin yang pair for MFTF-B cost \$32 M, the smaller octopoles could probably be built for less than that. Based on this comparison, the octopole upgrade might cost \$20-25 M more than the E-ring upgrade. In total, its cost is perhaps in the \$40-70 M range.

Another possibility for implementing both the E-ring and octopole plugs is to build the axicell in the position of the present yin yang coils, and then lengthen the central cell for increased performance. Since the central coils were originally designed for a 2 meter spacing, rather than the 1.25 meter spacing called for in the axicell design, it would not require many more coils to do this. Their cost was approximately \$0.75 M each, so the cost would not be too significant compared to the total.

VIII. DISCUSSION

This section will summarize results from the stability criteria calculations for both the E-ring and octopole configurations, the calculation of generic features for MFTF-like tandem mirrors and the performance comparison between MFTF quadrupole and axisymmetric operation.

VIII.1 E-RING STABILITY

The parameters used to evaluate the performance of MFTF-B(E) are, for the most part, consistent with both local and global stability criteria for hot electron ring stabilization of a tandem mirror. Two exceptions to this are the mirror mode microinstability criterion and the conditions for stability against precessional modes. High anisotropy conditions similar to those specified for MFTF-B(E) have been observed in TMX-U but the detrimental effects of strong mirror mode activity have not been observed. For this reason, the mirror mode is not anticipated to be a major problem for the MFTF-B(E) configuration.

The stability criteria for the precessional mode in a thermal barrier tandem mirror are still evolving as more calculations are done. The only

conclusion that can be made for MFTF-B(E) at this time is that, with a 14 m central cell this device should be in the correct parameter range to experimentally test these theories. This is an important positive step since the specifics of this instability must be verified before realistic designs of axisymmetric E-ring stabilized tandem mirror reactors can proceed.

Finally, though the estimates made here for the global stability of MFTF-B(E) indicate that the configuration operated with hot electron ring beta $\beta_h = 0.40$ is stable, detailed calculations of the interaction of the ring with a consistent MHD calculation of the core plasma are certainly required. It should be noted, however, that in addition to some margin which has been included in the ring beta used for the power balance calculations, compressibility effects which have not been included reduce the beta requirements for global stability. Also, further optimization of the bad curvature region at the end of the central cell should be done to reduce the requirements for global stability.

VIII.2 OCTOPOLE STABILIZATION

In the preceeding sections it has been shown that use of octopole end-cell magnets permits a considerable reduction in radial losses over quadrupole magnets. Partly offsetting this advantage is the necessity of forming a hot-electron mantle with ECRH. It could be argued that one should go to completely axisymmetric operation with E-rings for stabilization, but the operating window for stable operation has been found to be small and some question remains as to whether it exists at all. The octopole magnets allow more confidence in a low power MHD-stable operation because the stabilization mechanism relies on plasma behavior in a minimum of the vacuum magnetic field for which there is a larger data base.

VIII.3 GENERIC ISSUES

Independent of the mode of stabilization, tandem mirror operation requires that several specific aspects of the device be checked for consistency. For this study we have considered the question of sloshing ion adiabaticity, calculated axial shape factors for use in the calculation of power requirements in the end cells, and specified minimum anticipated halo

parameters required to shield the core plasma from neutrals. For the end cell parameters used, the sloshing ions in MFTF-B(E) marginally satisfy the adiabaticity conditions at barrier beta $\beta_b = 0.20$ while adiabaticity would not be a problem for the octopole configuration unless β_b exceeded 0.30. Axial shape factors for the core and hot electron regions (E-ring or mantle) have been calculated consistent with the magnetics calculations and used in determining ECRH power requirements for the barrier and off-axis hot electrons. Finally, based on calculations from previous reactor studies, the halo parameters, which affect power requirements for the off-axis hot electrons in both axisymmetric configurations, have been chosen to shield out both impurity atoms and hydrogen neutrals. Power to sustain the halo plasma comes from scattering and drag on the hot off-axis electrons.

VIII.4 PERFORMANCE COMPARISON

Key performance improvements which have been obtained for the octopole and E-ring configurations are summarized in Table 7. The values in the table were all obtained with the modified TMRBAR^{36,37} power balance code. The required central cell neutral beam power has been reduced by almost an order of magnitude due primarily to elimination of radial transport in the axisymmetric configurations. This allows central cell operation fueled entirely by a small neutral beam current. In MFTF-B large radial transport due to the quadrupole fields requires fueling of the central cell by a large quantity of cold gas. The charge exchange on this gas leads to an operating point at lower temperature and higher required neutral beam power. Finally, the equivalent Q for a device with a 14 m central cell (equivalent DT fusion power/input power) has been improved by a factor of 3 for the axisymmetric configurations due to the fact that the reduction in the central cell power requirement is much larger than the additional power required to sustain the off-axis hot electrons for stabilization.

Table 7. Key performance improvements.

	MFTF-B(Q)	MFTF-B(O)	MFTF-B(E)
P_{NB}^{cc} (MW)	2.71	0.33	0.27
I_{NB}^{cc} (A)	24.3	3.7	2.9
I_{gas}^{cc} (A)	115	~0	~0
I_{pump}^{ec} (A)	31.2	1.8	1.3
P_{ECRH}^{ec} (MW)	2.36	1.41 (1.07) ^a	1.09 (0.92) ^b
$Q_{DT} _{L_c=14\text{ m}}$	0.10	0.19 (0.24) ^a	0.24 (0.27) ^b

^aWith flat radial pressure profile in the end cell core plasma.

^bWith ring thickness taken to be 1.5 times FWHM.

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APPENDIX A. FRACTION OF BARRIER HOT ELECTRON BETA IN $B_c(r_A)$

To calculate the fraction of the hot electron pressure in the barrier which is responding as an MHD fluid in the NLV sense begin with a relativistic Maxwellian distribution in momentum space of the form

$$f(p) = \frac{1}{4\pi m^2 c T K_2(z)} \exp \left[- (m^2 c^4 + p^2 c^2)^{1/2} / T \right] \quad (A-1)$$

where $T = 0.8 E_{eh}$ is the hot electron temperature ($\bar{E}_{eh}$ is the average energy), $z = mc^2/T$, m is the electron mass, c is the speed of light, $p = (\bar{E}_{eh}^2 - m^2 c^4)^{1/2}/c$ is the momentum, and K_2 is the modified Bessel function of order two. Assuming a nearly isotropic distribution (this may not be a very good approximation for the mirror trapped barrier electrons) the distribution in speed takes the form

$$f(p) = \frac{p^2}{m^2 c T K_2(z)} \exp \left[- (m^2 c^4 + p^2 c^2)^{1/2} / T \right] \quad (A-2)$$

which in terms of the average energy $E \equiv \bar{E}_{eh}$ is

$$f(E) = \frac{E(E^2 - m^2 c^4)^{1/2}}{m^2 c T K_2(z)} \exp (-E/T) \quad (A-3)$$

where $\int_{E_0}^{\infty} f(E) dE = 1$ with $E_0 = mc^2$. The fraction of this distribution which is taken to be MHD is obtained by integrating Eq. (A-3) from the electron rest energy to a cutoff energy E_{MHD} above which the electrons are rigid. Using $x = E/mc^2$ the MHD fraction is

$$f_{\text{MHD}} = \int_1^a dx \frac{z}{K_2(z)} \times (x^2 - 1)^{1/2} e^{-zx} \quad (\text{A-4})$$

where $a = 1 + E_{\text{MHD}} \text{ (keV)}/511$. The integral in Eq. (A-4) is done numerically. The cutoff energy E_{MHD} is calculated by setting the curvature drift frequency of the barrier electrons equal to the MHD growth rate for the central cell and solving for the required hot electron energy.

APPENDIX B. PRESSURE FUNCTIONS USED IN EVALUATING INEQUALITY (24)

Central-Cell Ions ($0 < z < z_{m1}$)

$$\hat{p}_{\parallel} = \hat{p}_{\perp} = \frac{1}{2} \beta_c B_c^2 \quad (\text{B-1})$$

Passing Ions ($z_{m1} < z < z_b$)

$$\hat{p}_{\parallel} = \frac{1}{2} \beta_c B_c^2 \frac{1}{4} \left[5 \frac{B}{B_{m1}} - \left(\frac{B}{B_{m1}} \right)^5 \right] \quad (\text{B-2})$$

$$\hat{p}_{\perp} = \frac{1}{2} \beta_c B_c \left(\frac{B}{B_{m1}} \right)^5 \quad (\text{B-3})$$

Passing Ions ($z_b < z < z_{m2}$)

$$\hat{p}_{\parallel} = \frac{1}{2} \beta_c B_c^2 \left[\frac{5}{4} \frac{B_b}{B_{m1}} - \frac{1}{4} \left(\frac{B_b}{B_{m1}} \right)^5 \left(\frac{B}{B_b} \right)^{1+\delta} \left(\frac{B_{m2} - B}{B_{m2} - B_b} \right)^2 \right] \quad (\text{B-4})$$

$$\hat{p}_{\perp} = \left(\frac{2B}{B_{m2} - B} - \delta \right) \hat{p}_{\parallel} \quad (\text{B-5})$$

$$\delta = \frac{2B_b}{B_{m2} - B_b} + \frac{4B_b^4}{B_b^4 - 5B_{m1}^4} \quad (\text{B-6})$$

Sloshing Ions and Hot Electrons ($z_{m1} < z < z_{m2}$)

$$\hat{p}_{\parallel} = \frac{1}{2} \beta_b B_b^2 \frac{(B_{m2} - B)^2}{B_{m2}^2 - B_b^2} \quad (\text{B-7})$$

$$\hat{p}_\perp = \frac{1}{2} \beta_b E_b^2 \frac{B_{m2}^2 - B^2}{B_{m2}^2 - B_b^2} \quad (B-8)$$

$$\hat{p}_\parallel = \hat{p}_\perp = 0 \quad , \quad B > B_{m2} \quad (B-9)$$

β_{cc} and β_b are input values for the plasma properties, and B_c , B_{m1} , B_b , and B_{m2} are magnetic fields at key locations (see Fig. 5).

APPENDIX C. COMPARISONS WITH PREVIOUS MFTF-B QUADRUPOLE PROJECTIONS

Previous calculations of MFTF-B operating parameters have been done using the MFTF code.⁴⁰ Modeling of axisymmetric tandem configurations has been done in the upgraded TMRBAR code.^{36,37} Since these models were now to be applied to MFTF size devices, a comparison of results from the two codes for a quadrupole configuration was done. Complete agreement has not been achieved; the two codes are so fundamentally different in their modeling philosophy that this has not been possible in the time allotted to this study. In addition, this calibration effort has identified several improvements which are required in the MFTF code models. Until these improvements are completed no direct calibration is possible. The differences and their impact on comparisons between MFTF-B and the MFTF upgrades results are described below.

The MFTF code has been designed specifically for MFTF-B configurations and operating parameters. The results are accurate if key parameters are consistent with the input used in detailed multi-region Fokker Planck⁴¹ calculations of specific aspects of the device, e.g., sloshing ion behavior, hot barrier electron interaction with the warm electrons and applied ECRH etc. This design philosophy requires several aspects of the power and particle balance to be fixed to the Fokker-Planck results even as other parameters are varied for particular scenarios. The modified TMRBAR code, on the other hand, has been designed to search wide ranges of parameter space for consistent operating parameters for a variety of tandem mirror devices from test cell configurations to demonstration and full scale reactors. Of necessity, parameterization of Fokker Planck results for typical configurations is used rather than specific results for each configuration. However, the operating point can then be obtained by treating the device as a whole, including interactions between different regions and scaling of the Fokker-Planck results with plasma parameters which are varying; the MFTF operating point requires that a consistent set of parameters be found which also adheres to the separate conditions set by input in each Fokker-Planck calculation. For example, the axial potential profile input to the Fokker-Planck runs may not be consistent with the final profile obtained by the MFTF code but the results for particular aspects of the device are fixed to the Fokker-Planck results.

Major differences between the MFTF and the upgraded TMRBAR code calculations found during this calibration effort are:

1. Electron Power Balance - MFTF fixes the electron temperature T_e by input and calculates the required secondary electron current emitted from the end wall to achieve power balance of the electrons. This can result in secondary electron emission coefficients much larger than the space charge limited value. TMRBAR allows the secondary electron emission coefficient to be set by input or calculated based on a parameterization of space charge limited calculations⁴² and the electron temperature is calculated consistently.
2. Central Cell Neutral Beams - MFTF fixes central cell ion temperature T_c , hot ion average energy $\bar{E}_H$, and hot-cold density fraction n_H/n_c to the values obtained from a Fokker-Planck run and solves for the required trapped neutral beam current and cold gas current for central cell ion power and particle balance. The axial confinement of the hot mirror trapped ions is taken to be the same as the Pastukhov confinement. TMRBAR calculates the consistent T_c and $\bar{E}_H$ based on a Logan-Rensink plug model⁴³ for the beam injection. Confinement of hot particles is calculated consistent with a mirror trapped distribution including scattering and drag on a warm Maxwellian background population. Fixed central cell beta β_{cc} and n_H/n_c determine the particle balance.
3. Warm Electrons in the Plug - Particle and power balance for the warm electrons in the ion confining potential region is obtained in MFTF using the "strong rf" model* of Cohen⁴⁴ including a source of cold electron current from neutral beams in the end cell. TMRBAR uses the weak rf model⁴⁵ and does not model a cold electron source because the sloshing ions are assumed to be injected at the conjugate point "a'" and barrier pumping is assumed to be done

*The "strong rf" model referenced here is that described by Logan⁴⁶ in 1980. Further improvements to this model have been made in the interim.

entirely by a drift pumping system. The option to use the "strong rf" model including a cold electron source has been added to TMRBAR-U as part of this calibration effort.

4. Pastukhov Confinement - MFTF contains several options for scaling the Pastukhov confinement of central cell ions. In the runs used for comparison with TMRBAR, the Pastukhov confinement parameter $(n\tau)_{CA}$ was calculated from an analytical fit based on TMX-U results. TMRBAR uses the formula given by Pastukhov for $(n\tau)_{CA}$ increased by a leading coefficient $R \approx 1.5$ to account for the effective enhancement of axial confinement due to radial transport.
5. Central Cell ECRH Power - MFTF assumes that all of the ECRH power used to form the thermal barrier at "b" and plug potential at "a" ($P_{ECRH,a} + P_{ECRH,b}$) eventually ends up in the central cell as a source to the electron power balance. In TMRBAR only the drag power from the hot electrons at the barrier is input to the central cell. Synchrotron radiation is assumed to escape the low density plasma in the end cell and, at least for reactor parameters, the scattering power is lost from the end cell on a much shorter time scale than it is input to the central cell. However, the option to use all of the end cell ECRH power as a source to the central cell electron power balance has been added to TMRBAR as part of this calibration effort.
6. Passing Density Formulas - The formulas for the passing density in the end cells used in MFTF do not take into account the energy cutoff attributable to finite ion confining potential. TMRBAR includes this cutoff which can result in lower passing density than the MFTF models if ϕ_c/T_{ic} gets too small.
7. Mirror Ratios - The mirror ratios in MFTF confinement calculations are obtained using the average beta reduced fields assuming a parabolic radial pressure profile. TMRBAR uses the centerline peak beta reduced fields in all cases. This can lead to non-negligible differences especially in the quadrupole end cell of MFTF-B with peak barrier beta $\hat{\beta}_b \approx 0.55$.

Even though several of the differences between the MFTF modeling and the TMRBAR models have been eliminated, (primarily by adding options in the upgraded TMRBAR to use the MFTF models) a satisfactory calibration of the two codes has not been completed. The effects of secondary electron emission and charge exchange on the cold gas used to fuel the central cell have been investigated and will be presented below. At this writing, no converged operating points for an MFTF-B configuration using the "strong-rf" model with significant cold electron source at the potential peak have been obtained using TMRBAR.

It should be emphasized here that the continuing development of TMRBAR-U models for MFTF-B configurations does not affect our results for the axisymmetric configurations. The models under development do not pertain to the axisymmetric configurations for the following reasons: (1) fueling of the central cell is done entirely with neutral beams so that charge exchange on cold gas is negligible (assuming a high quality halo), (2) end cell pumping is assumed to be done entirely with a drift pumping system so that no cold electrons are introduced at the potential peak by pump beams, (3) the available ECRH power is not enough to put the end cell plasma in the strong rf limit, and, finally, (4) analysis⁴² of the experimental observations near the end walls of the TMX-U experiment indicate that secondary electron emission has a small effect on the central cell axial particle flow. For these reasons, a comparison has been made between TMRBAR generated operating points for the MFTF-B (quadrupole), MFTF-B(O) and MFTF-B(E) configurations assuming in each case the weak rf model, zero cold electron source at the potential peak and zero secondary electron current into the central cell. These cases are described in detail in Sec. VI.1.

VI.1.1 Results for MFTF-B from the Upgraded TMRBAR

The effect of secondary electron emission from the end walls of MFTF-B has been modeled in TMRBAR. The results are shown in Table C-1 along with the corresponding case from the MFTF code.⁴⁷ All of the TMRBAR cases use the weak rf model with no cold electron source at the potential peak. In addition, charge exchange on the cold gas used to fuel the central cell is not included. For case MFTF-1, all of the ECRH power in the end cells is assumed to be a power source for the central cell electrons while in cases MFTF-2,3 only the

Table C-1. Comparison of MFTF and TMRBAR-U code results.

Case:	MFTF-B	MFTF-1	MFTF-2	MFTF-3
Code:	(MFTF)	(TMRBAR)	(TMRBAR)	(TMRBAR)
Secondary e^- current (A)	217	218	95 (SCL)	0
P_{ECRH} to central cell	Yes	Yes	No	No
<hr/>				
<u>Parameter</u>				
n_c (10^{13} cm^{-3})	3.20	3.60	3.84	3.49
n_H (10^{13} cm^{-3})	0.91	1.02	1.09	0.99
T_{1c} (keV)	8.1	12.9	12.1	13.2
E_H (keV)	49.8	36.7	35.1	37.4
T_{ec} (keV)	6.0	4.1	3.6	4.31
ϕ_e (kV)	19.9	18.6	19.2	26.3
$\eta_e = \phi_e/T_{ec}$	3.32	4.54	5.33	6.10
ϕ_b (kV)	19.0	17.4	17.2	22.3
$\eta_b = \phi_b/T_{ec}$	3.17	4.24	4.78	5.17
ϕ_c (kV)	30.0	40.7	39.1	41.6
$\eta_c = \phi_c/T_{1c}$	3.70	3.16	3.23	3.15
$(n\tau)_{CA}$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	2.1	2.1	2.1	2.1
$(n\tau)_p$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.18	0.38	0.34	0.42
$(n\tau)_r$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.24	0.24	0.24	0.24
$\tau_{ }^c$ (s)	5.1	5.8	5.5	6.0
$\tau_{ }^H$ (s)	∞	0.98	0.85	1.0
$\tau_{ }^{\text{tot}}$ (s)	5.1	2.8	2.5	2.9
τ_l (s)	0.75	0.67	0.62	0.69
τ_{pump} (s)	0.56	1.07	0.89	1.2
τ_e (s)	0.21	0.23	0.20	0.21

Table C-1. (Continued.)

Case: Code:	MFTF-B (MFTF)	MFTF-1 (TMRBAR)	MFTF-2 (TMRBAR)	MFTF-3 (TMRBAR)
λ_{sec}	0.97	0.85	0.68	0.0
$n_{\text{eh},b}$ (10^{12} cm^{-3})	4.3	3.3	3.5	3.0
$n_{\text{ec},b}$ (10^{12} cm^{-3})	0.20	0.17	0.18	0.15
$n_{\text{eh},a}$ (10^{12} cm^{-3})	3.0	2.34	2.48	2.22
$n_{\text{ew},a}$ (10^{12} cm^{-3})	3.4	2.27	2.29	1.66
$n_{\text{pass},b}$ (10^{12} cm^{-3})	0.69	0.53	0.56	0.48
T_{ew} (keV)	49	40.3	43.9	33.2
E_{eh} (keV)	353	498	465	547
SLOSH	1.18	1.18	1.18	1.18
$R_{\text{eff}s}$	1.44	1.23	1.29	1.21
S_{den}	2.51	2.33	2.27	2.16
g_{tran}	1.7	1.7	1.7	1.7
g_b	2.75	2.75	2.75	2.75
H_{tran}	2.6	1.52	1.52	1.52
H_b	4.4	2.30	2.30	2.30
$P_{\text{NB},\text{cc}}$ (MW)	2.88	1.03	1.09	0.80
$I_{\text{NB},\text{cc}}$ (A)	29.4	13.1	16.3	12.0
$I_{\text{gas},\text{cc}}$ (A)	24.8	22.8	26.9	20.9
I_{pump} (A)	22.5	12.2	15.8	10.6
I_{past} (A)	5.0	2.2	2.6	2.1
I_{hot} (A)	0.0	1.9	2.4	1.8
I_{radial} (A)	30.0	19.6	22.4	1.8
$P_{\text{ECRH},a}$ (kW)	1026	150	189	514
$P_{\text{ECRH},b}$ (kW)	1374	801	881	707
$P_{\text{NB},a}$ (kW)	123	198	170	214

drag power at the barrier location is assumed to heat the central cell electrons.

Clearly the codes yield significantly different predictions of the operating parameters. TMRBAR tends to predict better Pastukhov confinement of the central cell electrons than MFTF ($n_e \equiv \phi_e/T_{ec} = 4.54$ compared with 3.32) and this may be the fundamental difference which drives the rest of the parameters. Preliminary work using the "strong rf" model and a source of cold electrons at the potential peak indicate that this has the effect of pushing n_e down significantly. In addition, TMRBAR tends to predict lower electron temperature and hot ion average energy which result in higher hot and cold ion densities at fixed beta.

The TMRBAR-U results tend also to predict much lower central cell neutral beam requirements than MFTF ($P_{NB,cc} = 1.03$ MW compared with 2.88 MW). As the following sections will show, this is due primarily to the fact that charge exchange losses from the cold central cell fueling gas have been neglected. In addition, however, the TMRBAR models typically predict lower radial transport current than in MFTF [for fixed radial transport (n_r)] because it is assumed that there is no radial transport of the hot mirror trapped particles since they are confined to the axisymmetric central cell. This leads to a lower central cell beam ionization current and, at fixed energy, lower beam power. Finally, the weak rf model used in TMRBAR predicts lower ECRH powers at the barrier and potential peak locations than the models in MFTF. This is due in part to the differences in the models, partly to the lack of cold electron source at the potential peak in the TMRBAR-U cases, and partly to the energy cut-off in the passing density formulas used in TMRBAR which yield lower passing density in the end cells for fixed central cell density than the MFTF code.

Comparison of the TMRBAR calculations with and without secondary electron current (MFTF-2 and 3) shows only slight differences. The most important of these is that the central cell electron temperature is higher without secondaries ($T_{ec} = 4.3$ keV compared with 3.6 keV). This is expected since each secondary electron which reaches the central cell represents a net energy loss of T_{ec} to the central cell energy balance since it is born with negligible energy at the end wall. Higher electron temperature results in better confinement and higher ion temperatures (cold ions $T_{ic} = 13.2$ keV compared with 12.1 keV and hot ions $\bar{E}_h = 37.4$ keV compared with 35.1 keV). At

fixed beta this results in lower density and, combined with higher confinement ($n\tau$) values, yields lower central cell beam power and current requirements.

The results in Table C-2 show the effect on the operating point of charge exchange on the cold gas fueling current in MFTF-B. Cases A-D all assume zero secondary electron current, the weak rf model with no cold electron source at the potential peak and all the ECRH power from the end cells reaches the central cell electrons. The cases are distinguished by the ratio of the charge exchange cross section to the ionization cross section for cold gas fueling of the central cell, R_{cx} . The appropriate value of this ratio depends on the electron temperature T_{ec} and on radial profile effects neither of which has been modeled to be consistent with the rest of the numerical solution of the balance equations. In the MFTF modeling, T_{ec} is fixed at 6.0 keV and R_{cx} is taken to be 2.8. At lower T_{ec} with radially decreasing temperature profiles the ratio will be lower.

The trend in Table C-2 is to much lower central cell temperatures and consequently higher density as R_{cx} increases. This means that the central cell neutral beam current and cold fueling current must increase with R_{cx} to satisfy particle balance. Comparing Case A with $R_{cx} = 1.2$ to Case D with $R_{cx} = 2.0$ the beam current increases by nearly a factor of 3 and the cold fueling current increases by nearly a factor of 5. The neutral beam power increases to 3.55 MW from 1.27 MW for this range of R_{cx} . Finally, since the passing density increases with R_{cx} , the ECRH power required at both the barrier and potential peak increases; from 0.1 MW and 0.7 MW respectively at $R_{cx} = 1.2$ to 0.3 MW and 1.65 MW respectively at $R_{cx} = 2$. From these results there is a clear advantage to operating the device with the lowest possible cold fueling current. In axisymmetric configurations with negligible radial transport, operating points with the central cell fueled only by neutral beams can be found. The corresponding improvements in plasma performance and equivalent fusion Q for the device are described in Sec. VI.

Table C-2. Effect of charge exchange on cold gas fueling current.

Case:	A	B	C	D
$R_{cx} = \frac{\langle \sigma v \rangle \text{ charge exchange}}{\langle \sigma v \rangle \text{ ionization}}$	1.2	1.5	1.8	2.0
<hr/>				
<u>Parameter</u>				
n_c (10^{13} cm^{-3})	3.55	3.97	4.61	5.28
n_H (10^{13} cm^{-3})	1.01	1.13	1.31	1.50
T_{ic} (keV)	12.4	10.7	8.9	7.5
E_H (keV)	34.6	31.4	27.8	25.0
T_{ec} (keV)	5.1	4.7	4.2	3.8
ϕ_e (kV)	29.8	28.1	25.2	22.6
$n_e = \phi_e / T_{ec}$	5.84	5.98	6.0	5.95
ϕ_b (kV)	25.9	24.2	21.6	19.3
$n_b = \phi_b / T_{ec}$	5.08	5.15	5.14	5.08
ϕ_c (kV)	39.8	36.1	31.8	28.2
$n_c = \phi_c / T_{ic}$	3.21	3.37	3.57	3.76
$(n\tau)_{CA}$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	2.1	2.1	2.1	
$(n\tau)_p$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.42	0.33	0.24	0.18
$(n\tau)_r$ ($10^{14} \text{ cm}^{-3} \text{ s}$)	0.24	0.24	0.24	0.24
$\tau_{ }^c$ (s)	5.9	5.3	4.6	4.0
$\tau_{ }^H$ (s)	0.95	0.80	0.63	0.50
$\tau_{ }^{tot}$ (s)	2.7	2.4	1.91	1.57
$\tau_{\perp}$ (s)	0.68	0.60	0.52	0.45
τ_{pump} (s)	1.18	0.83	0.52	0.34
τ_e (s)	0.23	0.20	0.15	0.10

Table C-2. (Continued.)

Case:	A	B	C	D
λ_{sec}	0.0	0.0	0.0	0.0
$n_{\text{eh},b}$ (10^{12} cm^{-3})	2.9	3.2	3.8	4.4
$n_{\text{ec},b}$ (10^{12} cm^{-3})	0.15	0.17	0.20	0.23
$n_{\text{eh},a}$ (10^{12} cm^{-3})	2.07	2.3	2.7	3.1
$n_{\text{ew},a}$ (10^{12} cm^{-3})	2.19	2.2	2.3	2.3
$n_{\text{pass},b}$ (10^{12} cm^{-3})	0.46	0.52	0.62	0.72
T_{ew} (keV)	40	40.6	42.2	45.4
E_{eh} (keV)	569	500	418	355
SLOSH	1.18	1.18	1.18	1.18
$R_{\text{eff}s}$	1.10	1.24	1.42	1.60
S_{den}	2.44	2.30	2.14	2.02
g_{tran}	1.7	1.7	1.7	1.7
g_b	2.75	2.75	2.752	2.75
H_{tran}	2.6	1.52	1.52	1.52
H_b	4.4	2.30	2.30	2.30
$P_{\text{NB},cc}$ (MW)	1.27	1.70	2.50	3.55
$I_{\text{NB},cc}$ (A)	13.4	17.8	26.3	37.8
$I_{\text{gas},cc}$ (A)	45.3	71.7	127	215
I_{pump} (A)	10.9	17.4	32.1	56.4
I_{past} (A)	2.2	2.7	3.7	4.8
I_{hot} (A)	1.8	2.5	3.8	5.9
I_{radial} (A)	19.1	23.9	32.2	42.4
$P_{\text{ECRH},a}$ (kW)	101	132	198	300
$P_{\text{ECRH},b}$ (kW)	700	862	1192	1650
$P_{\text{NB},a}$ (kW)	345	171	121	109

APPENDIX D. GLOSSARY OF SYMBOLS

The following symbols are used extensively in the tables of results.

n_c	- central cell peak warm ion density
n_H	- central cell peak hot (mirror trapped) ion density
T_{ic}	- central cell warm ion temperature
$\bar{E}_H$	- central cell hot ion average energy
T_{ec}	- central cell and passing electron temperature
ϕ_e	- central cell potential
ϕ_b	- barrier potential dip relative to ϕ_e
ϕ_c	- ion confining potential relative to ϕ_e
$(n\tau)_{CA}$	- axial (Pastukhov) confinement parameter for n_c
$(n\tau)_p$	- radial (pumping) confinement parameter for n_c
$(n\tau)_r$	- radial (transport) confinement parameter for n_c
$\tau_{ }^c$	- axial (Pastukhov) confinement time for n_c
$\tau_{ }^H$	- axial (mirror) confinement time for n_H
$\tau_{ }^T$	- density weighted total confinement time for central cell ions
$\tau_{\perp}$	- radial (transport) confinement time for n_c
τ_{pump}	- radial (pumping) confinement time for n_c
τ_E	- energy confinement time for central cell ions (stored energy/input power)
I_{sec}	- secondary electron current from end walls
$n_{eh,b}$	- hot electron peak density in the barrier
$n_{ec,b}$	- passing electron peak density in the barrier
$n_{eh,a}$	- hot electron peak density at the potential peak
$n_{ew,a}$	- warm electron peak density at the potential peak
$n_{\text{pass},b}$	- passing ion peak density in the barrier
T_{ew}	- warm electron temperature at the potential peak
$\bar{E}_{eh}$	- hot electron average energy in the barrier
SLOSH	- ratio of vacuum field at potential peak to that at barrier
R_{effs}	- sloshing ion effective mirror ratio
S_{den}	- sloshing ion peak to midplane density ratio

g_j	- ratio of (passing + trapped) to passing density j = tran in transition region j = b in barrier
H_j	- coefficient in Futch-LoDestro trapping formula for region j
$P_{NB,cc}$	- trapped central cell neutral beam power
$I_{NB,cc}$	- trapped central cell neutral beam current
$I_{gas,cc}$	- trapped central cell cold gas current (ionization current)
I_{pump}	- total pumped ion current (transition + barrier)
I_{PAST}	- total axial (Pastukhov) loss current
I_{HOT}	- total hot ion direct pitch angle scattering loss current
I_{radial}	- total radial (transport) loss current
$P_{ECRH,a}$	- total trapped ECRH power at potential peak
$P_{ECRH,b}$	- total trapped ECRH power in the barrier
$P_{NB,a'}$	- total sloshing ion neutral beam power
B_c	- central cell vacuum magnetic field
B_{CH}	- central cell mirror magnetic field
B_t	- transition vacuum field
B_{mp}	- plug inner mirror vacuum field
B_b	- plug midplane vacuum field
B_{max}	- plug outer mirror vacuum field
L_t, L_p	- transition and plug $\omega d/B$ effective length
β_c, β_b	- central cell and barrier peak beta values
n_{ec}	- central cell peak electron density
V_{ec}	- central cell volume
E_{inj}	- central cell neutral beam average injection energy
$(n\tau)_{pc}$	- central cell hot (mirror trapped) ion confinement parameter
q	- electronic charge

APPENDIX E. EIG INPUT DECK FOR E-RING-STABILIZED END CELL OF MFTF-B

MFTF-AX1 Bcc=1.0 Bm2=5.77

coil=wa2 \$

sol 11.50 0.9280 0.9380 0.4073 2.420e+07 \$

coil=wb2 \$

sol 9.75 1.70 .163 .336 -2.50e+07 \$

coil=wa1 \$

sol 7.88 0.9280 0.9380 0.4073 2.364e+07 \$

coil=wa2i \$

sol 7.88 0.3883 0.6572 0.3523 2.000e+07 \$

coil=ws6 \$

sol +6.875 2.50 .163 .336 3.140e+07 \$

coil=ws5 \$

sol +5.625 2.50 .163 .336 3.140e+07 \$

coil=ws4 \$

sol +4.375 2.50 .163 .336 1.808e+07 \$

coil=ws3 \$

sol +3.125 2.50 .163 .336 1.808e+07 \$

coil=ws2 \$

sol +1.875 2.50 .163 .336 1.783e+07 \$

coil=ws1 \$

sol +0.625 2.50 .163 .336 1.783e+07 \$

coil=es1 \$

sol -0.625 2.50 .163 .336 1.783e7 \$

coil=es2 \$

sol -1.875 2.50 .163 .336 1.783e7 \$

coil=es3 \$

sol -3.125 2.50 .163 .336 1.808e7 \$

coil=es4 \$

sol -4.375 2.50 .163 .336 1.808e7 \$

coil=es5 \$

sol -5.625 2.50 .163 .336 3.140e7 \$

coil=es6 \$

sol -6.875 2.50 .163 .336 3.140e7 \$

coil=ea2i \$

sol -7.88 0.3883 0.6572 0.3523 2.000e+07 \$

coil=ea1 \$

sol -7.88 0.9280 0.9380 0.4073 2.364e+07 \$

coil=eb2 \$

sol -9.75 1.70 .163 .336 -2.50e+07 \$

coil=ea2 \$

sol -11.5 0.9280 0.9380 0.4073 2.420e+07 \$

APPENDIX F. EIG INPUT DECK FOR OCTOPOLE END-CELL COILS IN MFTF-B
(Central-cell coils are the same as in Appendix E.)

```

MFTF-AX1 Bcc=1.0 Bm2=5.77
p1=0.
***
option = noprint $
coil=wa2 4 3 $
sol 13. 0.9280 0.9380 0.4073 2.420e+07 $
coil = big8 2 1 $
octo 1.3 .83 .5 .3 .22 0. 1. 0. 10.9 .25 .3 3.e7 0. $
coil = small8 2 1 $
cone8 9. .4 .5 .6 1. 0. .1 0. .13 .19 3.e7 $
coil=wa1 4 3 $
sol 7.88 0.9280 0.9380 0.4073 2.364e+07 $
coil=wa2i 3 2 $
sol 7.88 0.3383 0.6572 0.3523 2.000e+07 $
sol -6.875 2.50 .163 .336 3.140e7 $
***
zxy 10. .05 12. 0. 0. 0. 0. 0 1 $
***

```