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AUTOMATIC GENERATION OF SYNCHRONIZATION INSTRUCTIONS FOR PARALLEL PROCESSORS

Samuel Pratt Midkiff

May 1986

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MASTER

AUTOMATIC GENERATION OF SYNCHRONIZATION
INSTRUCTIONS FOR PARALLEL PROCESSORS

BY

SAMUEL PRATT MIDKIFF

B.S., University of Kentucky, 1982

THESIS

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AUTOMATIC GENERATION OF SYNCHRONIZATION INSTRUCTIONS FOR PARALLEL PROCESSORS

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The development of high speed parallel multi-processors, capable of parallel execution of *doacross* and *forall* loops, has stimulated the development of compilers to transform serial FORTRAN programs to parallel forms. One of the duties of such a compiler must be to place synchronization instructions in the parallel version of the program to insure the legal execution order of *doacross* and *forall* loops.

This thesis gives strategies usable by a compiler to generate these synchronization instructions. It presents algorithms for reducing the parallelism in FORTRAN programs to match a target architecture, recovering some of the parallelism so discarded, and reducing the number of synchronization instructions that must be added to a FORTRAN program, as well as basic strategies for placing synchronization instructions. These algorithms are developed for two synchronization instruction sets.

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CHAPTER 1

INTRODUCTION

The need for faster computing has brought about projects to develop computers with up to several thousand processing units capable of being brought to bear on a single program. These computers seek to exploit two different types of parallelism: extra-loop and intra-loop. Extra-loop parallelism assigns program segments containing one or more loops to a processor and runs different segments in parallel. The University of Illinois Cedar project [GKLS83] and the Cray XMP series [Chen83] are the most notable examples of multi-processors exploiting this type of parallelism. Intra-loop parallelism assigns program segments containing one or more iterations, or part of an iteration, to a processor, and thereby runs loop iterations in parallel. Intra-loop parallelism is exploited by the Cedar project computer, the Alliant FX series [Alli85] computers, the NYU Ultra Computer [GGKM83], the Cray XMP using micro-tasking and the Denelcorp HEP-1 [Smit81] among others.

This thesis concentrates on automatic generation of synchronization for intra-loop parallelism. When the body of a loop is executed on several processors, it is often necessary to restrict the order of execution of some statements within the body of the loop so that the results produced by the parallel version of the program are identical to those produced by the serial version of the same program. It is the job of synchronization to enforce these restrictions on the execution order of statements. This thesis develops algorithms for the automatic generation of synchronization instructions. We concern ourselves with FORTRAN DO loop nests that have at most one loop whose parallelism is realized by spreading the loop across multiple processors.

We also assume that parallel loops have already been marked before the invocation of the algorithms in this thesis. The Parafrase compiler [KKLW81], [KLPL81], [Wolf82] that automati-

cally detects parallel loops, and performs code transformations to enhance the detection of parallel loops, is one example of software that is capable of doing this. Many of the algorithms in this thesis have been implemented in a pass of the Parafrase compiler, but their usefulness is in no way restricted to this compiler.

This thesis contains 5 chapters. The first introduces the architecture of the computer for which architecturally dependent algorithms are targeted. An algorithm is given for determining which loops should be executed as parallel loops on this architecture, to give an optimal running time, when multiple parallel loops exist in a loop nest. It also describes the notation used in this thesis, and provides some definitions of basic terms. Finally, the fundamental data structure used in this thesis, the *dependence graph* is explained in detail.

In the second chapter we modify the dependence graph to take into account the presence of certain types of control branches present in loops. These include branches outside the scope of the parallel loop being synchronized, and serial loops nested within the parallel loop being synchronized. These transformations are largely independent of the target architecture.

The third chapter defines two synchronization instruction sets and shows how synchronization is generated using these. Both the restricted case of adding synchronization instructions when no control branches are present in the parallel loop, and the general case of adding synchronization instructions to loops with arbitrary acyclic flow of control graph are covered.

The algorithms of the first three chapters are sufficient to determine where synchronization instructions are needed in a loop, and to place those synchronization instructions in the program. The fourth chapter presents algorithms for minimizing the number of dependences that must be synchronized, a process we refer to as *dependence arc elimination*. Dependence arc elimination is shown to be NP-hard, and a pseudo-polynomial time algorithm [GaJo79] is given. The transformations of this chapter have architecturally dependent and independent components.

The fifth chapter gives our conclusions, and areas for further study. It also presents a global algorithm that ties together the use of the other algorithms presented in the earlier chapters.

1.1. Architecture

A block diagram of the target machine is shown in Figure 1. Memory access is assumed to be deterministic: either memory accesses from processor units take a bounded amount of time, or a processor can suspend itself until all pending stores and fetches are complete.

Synchronization registers are contained within the global memory modules. By reading or writing these registers through the interconnection network, processors can signal and detect the completion of events.

The architecture is capable of supporting two levels of parallelism simultaneously. The first level, achieved by spreading iterations across processors, is referred to as *spread* parallelism in this thesis, and is designated in programs by the *dospread* loop construct. It will be con-

M_i is Global Memory Module i . P_j is Processor j .

Figure 1. System Architecture

sidered illegal for a *dospread* loop to contain a nested *dospread* loop. The second level is achieved by executing certain statements in vector mode. Statements to be so executed are designated in programs by being enclosed in a *dovector* loop. Each processor is assumed to have a functional unit capable of executing such statements in vector mode. Section 1.3.2, which discusses FORTRAN extensions, defines these loops in terms of other types of parallel loops found in the literature. An important point to note about the two types of parallelism is that spread parallelism may require synchronization statements be added to the program, whereas vector parallelism will never cause synchronization statements to be introduced. Within a processor, and therefore an iteration, all fetches and stores execute serially.

Dospread loops can be scheduled in different ways. Figure 2a shows how sixteen iterations of a loop would be horizontally spread [Jack85] over four processors, and Figure 2b shows how these same iterations would be vertically spread over four processors. This can be accomplished by self scheduling, or by loop blocking. With self scheduling, a processor will query a register or memory location for the value of the next iteration to be run. This it will do by locking, reading, incrementing and unlocking the register or memory location. The value thus read will be the number of the next iteration to be run. This process suffers from potentially serializing the execution of the program.

The other method involves *loop blocking*. In loop blocking, the *dospread* loop is made serial and a new *dospread* loop is placed around the just serialized loop, L_s . This *dospread* loop runs from one to the number of processors available, P , to execute the loop. For horizontal spreading, the loop bounds of L_s are adjusted to:

$$\text{do } i := p \text{ to } N \text{ by } P$$

where $p, 1 \leq p \leq P$, is the number of the processor executing L_s . If vertical spreading was used, then a processor would execute iterations $i, i + 1, i + 2, \dots, i + k$. It is often the case that the processor executing iteration $i + k + 1$ would have to wait until some statements of all

processor =	1	2	3	4
i =	1	2	3	4
	5	6	7	8
	9	10	11	12
	13	14	15	16

(a) Horizontal Spreading

processor =	1	2	3	4
i =	1	5	9	13
	2	6	10	14
	3	7	11	15
	4	8	12	16

(b) Vertical Spreading

Figure 2. Horizontal and Vertical Spreading of Iterations

iterations less than iteration $i + k + 1$ finished before beginning execution. Therefore iterations of loops with cross iteration dependences should not be vertically scheduled since this would serialize the loop.

1.2. Usable Parallelism Selection

Often, more parallelism will be found in a loop nest than can be utilized by the architecture of this thesis. For example, if there are three nested parallel loops in a program, it will never be possible to execute more than two of them as parallel loops. The inner loop may be executed as a *dovector* loop, and one outer loop as a *dospread* loop. The general problem of optimally assigning processors to various parallel loops in a loop nest is not solvable if certain assumptions cannot be made at compile time about the running times of loops. [Poly86] shows that with some slight restrictions this can be done in polynomial time. The task we give ourselves is much simpler. Because nested *dospread* loops are not allowed, we need only examine the parallel running time of each loop in the nest. If the loop nest executes faster by having this loop run in parallel and all surrounding and enclosed loops run serially, then the optimal

schedule for the loop nest will mark this loop as parallel and the others as serial. Since only one loop is running in parallel at a time, all available processors can be given to this loop. We make no assumptions about the number of processors available to run a loop, nor do we make any attempt at determining the optimal number of processors to execute a loop nest. We also do not attempt to do *loop interchanging* [Wolf82]. We simply decide, given a loop nest and some number of processors, which loops in the nest to run in parallel.

As will be seen, to do this requires visiting each loop only once. Therefore, scheduling for our architecture can be done in $O(n)$ time, where n is the number of loops in the loop nest.

The loop nest, which serves as input to this algorithm, is assumed to already have all parallel loops marked as such. Loops that can be executed in vector mode are marked as *dovector*, and all other parallel loops are marked as *dospread*.

A loop nest, such as is found in Figure 3a, can be represented as a tree, such as the one found in Figure 3b. Attached to each node are three times. The first, SS , gives the time it takes for the loop to execute serially, and for all loops nested within it to execute serially. The second, SP , gives the time for the loop to execute serially, and for all loops nested within it to execute in the fastest possible manner. That is, the time for the optimal schedule of the subnest is used. The third, PS , gives the time for the node to execute in parallel, and all loops nested within it to execute serially. Each node is also marked *spread* or *serial*, depending on whether the optimal schedule at this point requires the loop to be run as a *dospread* loop or not.

Figure 3b shows the times computed for the nodes b , c , d , e and f . These have already been computed by applying the bottom up algorithm of Figure 4. We compute the times for a loop by adding the appropriate times for nested loops to the time for *in-line* code in the loop. (*In-line* code is code that is nested within the loop being timed, but is not contained in any other more deeply nested loops.) This is then multiplied by the number of iterations in the loop, and for *dospread* loops, divided by the number of processors assigned to execute the loop. If the

```

DO 60 a = 1, 40
  Z(a) = X(a) + Y(a)
  DO 10 b = 1, 25
    ...
10  CONTINUE
  DO 50 c = 1, 15
    DO 30 d = 1, 10
      DO 20 e = 1, 20
        ...
20  CONTINUE
30  CONTINUE
  DO 40 f = 1, 20
    ...
40  CONTINUE
50  CONTINUE
60 CONTINUE

```

(a) A Loop Nest to be Scheduled

Figure 3. Loop Scheduling Example

dospread has lexically backward dependences, or has an exit-if, then the execution of iterations cannot totally overlap, and running time of the loop must be increased. It will be assumed that a value, *delay* [Cytr84] can be found that can be used to determine the true running time of the loop. For our example we will assume that no lexically backward dependences exist, i.e. that

$delay = 0$. Other functions, [Poly86] and [Cytr84], can be used to calculate the running time of a loop.

We now compute the time at node a . The SS time is the sum of the SS times for all immediately nested loops, i.e. b and c , plus the time for the in-line code. We will assume in-line code takes one time unit per statement to execute, times the number of iterations in the loop.

F_p - a timing function such as found in [Poly86] and [Cytr84]
 $\#processors$ - the number of processors assigned to execute the loop nest

All nodes are initially marked as *do* or *dovector*

Visit each node n using a post-order traversal

$SS[n]$ = execution time for the serial execution of all code, including nested loops
 $SP[n]$ = execution time for in-line code to execute serially, and nested loops in parallel if possible
 $PS[n]$ = execution time for inline code to execute in parallel, and nested loops to execute serially

```
foreach child  $c$  of  $n$  do
     $SS[n] := SS[n] + SS[c]$ 
    if  $c$  is marked as spread then
         $SP[n] = SP[n] + PS[c]$ 
    else
         $SP[n] = SP[n] + SP[c]$ 
```

```
 $PS[n] = F_p(SS[n], \#processors, delay)$ 
if  $PS[n] \leq SP[n]$  then
    mark loop as spread
```

```
Visit each node  $n$  using a preorder traversal of the tree
    if an ancestor of  $n$  is marked spread then
        mark  $n$  as serial.
    if  $n$  is a leaf then
        if  $n$  was a dovector loop then
            if  $n$  is marked spread then
                run  $n$  as a dospread loop with enclosing
                    statements executed as vector operations.
            else
                run  $n$  as a dovector loop
```

Figure 4. Algorithm for Scheduling Loops

Thus the *SS* time is $(6,600 + 75 + 1) \cdot 40$, or 267,040. The *SP* time is computed as above, except now we use the *PS* times for each of the immediately nested loops that is marked as *dospread*, and the *SP* time for each of the loops that is not marked as *dospread*. This leads to an *SP* time of 33,440. And the *PS* time is computed like the *SS* time, except that we divide the final result by the number of processors assigned to the loop. The *PS* time is 33,380.

Since the time for running *a* in parallel and its children serially is less than or equal to the time for running *a* serially and its children in parallel, we should run *a* as a *dospread* loop, and its children serially. This would be shown by marking *a* as *dospread*, and its children as *serial*.

1.3. Dependences and FORTRAN Extensions

This section seeks to both define those terms that are peculiar to this thesis, and to define and give some background on those terms that are more widely used. The discussion of the more general terms will of necessity be more abbreviated than that contained within the mentioned references.

1.3.1. Dependences and Dependence Graphs

This section defines what various types of dependences are, and shows how the dependence structure of a program can be represented by a directed graph called a *dependence graph*. We first define dependences and their properties, and give examples. We then show how to construct a dependence graph.

1.3.1.1. Definitions

We often refer to statements within the loop being synchronized. S_i refers to statement *i* in a FORTRAN source program. If $i < j$, then S_i lexically precedes S_j . L_i refers to some loop nested within the *dospread* loop being synchronized. Sometimes a nested loop, L_i , will be broken into several loops. These will be called $L_i^1, L_i^2, \dots, L_i^n$, where L_i^k is the lexically *k*'th loop.

Dependencies that must be synchronized fall into four categories: *flow*, *anti*, *output* [Kuck78] [Wolf82] and *control* [Kuck78]. Figure 5a, Figure 6a, Figure 7a and Figure 8a, respectively, give examples of these. Flow, anti and output dependencies are collectively known as *data* dependencies because they express a constraint on the flow of data through the program. Each of the data dependencies is defined using the *IN* and *OUT* sets of the statements involved in the dependence.

$OUT(S_i)$ is the set of the output variables of S_i . In an assignment statement this would be the left hand side, or *LHS*, variable [AhU177]. $IN(S_i)$ is the set of the input variables of S_i . In an assignment statement these would be the right hand side, or *RHS*, variables [AhU177].

A *flow* data dependence from S_i to S_j , $S_i \delta S_j$, exists if $\alpha \in OUT(S_i)$, $\alpha \in IN(S_j)$, and the value of α used in S_j could come from S_i . S_i is the *source* of the dependence, and S_j is the *sink* of the dependence.

```

DO 10 I = 1, N
  A(I) = B(I) + C(I)      S1
  D(I) = A(I-1) + B(I)    S2
10 CONTINUE

```

(a) Flow Data Dependence

(b) DG for (a)

Figure 5. Flow Data Dependence Example.

```

DO 10 I = 1, N
  A(I) = B(I) + C(I)      S1
  B(I-1) = C(I) + D(I)    S2
10 CONTINUE

```

(a) Anti Data Dependence

(b) DG for (a)

Figure 6. Anti Data Dependence Example

```

DO 10 I = 1, N
  A(I) = B(I) + C(I)      S1
  A(I-1) = B(I) + D(I)    S2
10 CONTINUE

```

(a) Output Data Dependence

(b) DG for (a)

Figure 7. Output Data Dependence Example

```

DO 10 I = 1, N
  IF (A(I).GT.B(I)) GOTO 10      S1
  X(I) = C(I) + D(I)             S2
10 CONTINUE

```

(a) Control Dependence

(b) DG for (a)

Figure 8. Control Dependence Example

An *anti*-data dependence from S_i to S_j , $S_j \bar{\delta} S_i$, exists if $\alpha \in IN(S_i)$, $\alpha \in OUT(S_j)$, and the value of α is used in S_i before a new value of α is stored in S_j . S_i is the *source* of the dependence, and S_j is the *sink* of the dependence. S_i must be executed before S_j .

A *output* data dependence from S_i to S_j , $S_j \delta^o S_i$, exists if $\alpha \in OUT(S_i)$, $\alpha \in OUT(S_j)$, and the value of α in S_j could be stored after the value of α is stored in S_i . S_i is the *source* of the dependence, and S_j is the *sink* of the dependence.

A *control* dependence from S_i to S_j , $S_j \delta^c S_i$, exists if whether S_j executes depends the execution of S_i . S_i is the *source* of the dependence, and S_j is the *sink* of the dependence.

That some statement is the source of a dependence will often be denoted by substituting so for the subscript i in a statement specification. A sink is denoted by substituting si . Therefore, S_{so} refers to the statement that is the source of some dependence. If $si > so$, i.e. the sink of the dependence is lexically forward of the source, the dependence is a *lexically forward dependence*, or *LFD*. If $si \leq so$, i.e. the sink of the dependence is lexically prior to the dependence, the dependence is a *lexically backward dependence*, or *LBD*.

The dependence distance, Δ , is the number of iterations spanned by the dependence. In Figure 9, we see the *iteration space* [Wolf82] of Figure 5a, with the dependences drawn in. Here the dependence distance is one. In general, the dependence distance from one occurrence of the array variable A , $A(f_1(I))$, to another occurrence, $A(f_2(I))$, is $\Delta = f_1(I) - f_2(I)$, where I is the index variable of the loop. Depending on the functions, f_1 and f_2 , the distance may either be constant or changing. If Δ is the distance of some dependence, then Δ_{min}^L is the minimum value of all Δ s in a loop L , and Δ_{max}^L is the maximum value of all Δ s in loop L .

Often it is not necessary to know the distance of a dependence: its *direction* alone suffices. The direction can be determined from the relative values of the respective subscript functions f_{so} and f_{si} , for the source and sink of the dependence. If I is an index value for some loop, then the direction, γ , of the dependence on that loop is $f_{si}(I) \gamma f_{so}(I)$, where $\gamma \in \{<, =, >, <=, >=, <>, <=>\}$. In other words, if γ is ' $<$ ', the dependence is from an earlier iteration to a later iteration. If γ is ' $>$ ', the dependence is from a later iteration to an earlier one. And if γ is ' $=$ ', the dependence is within the same iteration.

Figure 9. Iteration Space of Figure 1a

1.3.1.2. The Importance of Dependences

As stated before, a dependence is a relation between the source and sink of a dependence. This relation says that the source of the dependence must execute before the sink of the dependence if the parallel execution of the program is to be the same as the serial execution of the program. Therefore, the sink of a dependence must wait until the source executes before executing. It is the purpose of synchronization to force this. Thus, regardless of the synchronization scheme used, some action must be taken before or during the sink of a dependence to insure that it does not execute until the source has. Likewise, the execution of the source of a dependence must allow the sink to eventually execute.

1.3.1.3. Building the Dependence Graph

A dependence δ is a relation between two statements, S_{so} and S_{si} . This relation can be expressed as an arc on a graph whose nodes are statements. The dependence $\delta = (S_{so}, S_{si})$ can be represented by a directed arc whose head is at S_{so} and whose tail is at S_{si} . Such a graph will be referred to as a *dependence graph*, or *DG*. The next section explains how to build a DG. Figure 5b, Figure 6b, Figure 7b and Figure 8b give examples of DGs.

As has been mentioned, the primary tool used to generate synchronization for programs is the DG. The DG used for synchronizing programs with the architecture assumed in this thesis need only be a statement level DG. Nested loops are treated as complex statements, and the entire nested loop is represented as a single node on the DG. As well, the DG need only include dependences that have a dependence direction component of ' $<$ ' for the loop being synchronized. The direction components of other loops may be ignored, since those loops are serial and therefore cannot cause synchronization instructions to be generated. The reasons for this, which are discussed shortly, are entirely dependent on the architecture we have assumed. Other architectures may require the inclusion of dependences with ' $=$ ' and ' $>$ ' direction components, or require an atom level DG. We note that if the dependence we leave out were included in the DG

they would be removed by the dependence arc elimination algorithms of Chapter 4.

Control dependences, with the exception of exit-ifs (a control branch whose target is outside the *dospread* loop), have an '=' dependence direction. Therefore control dependences are not added to the DG at this time. In Chapter 2, we augment the DG with control dependences for exit-ifs. Thus when the DG is built, only flow, anti and output dependences are included.

1.3.1.3.1. Justification for a Statement Level DG

The use of a statement level DG for determining the synchronization points implies that synchronizing a dependence from an atom in some statement to another atom in some statement synchronizes all dependences from the former statement to the latter statement, assuming the dependence distances are the same.

For example, consider the program in Figure 10. There exists a flow dependence from $A(I)$ in S_1 to $A(I-1)$ in S_2 , and an anti-dependence from $B(I)$ in S_1 to $B(I-1)$ in S_2 . Since statement S_2 cannot begin executing until all fetches and stores in statement S_1 have finished,

```

DOSPREAD 10 I = 1, N
      A(I) = B(I) + C(I)    S1
      B(I-1) = A(I-1) + C(I) S2
10    CONTINUE

```

(a) Loop with Multiple Dependences with equal Δ s between Two Statements

(b) DG for the Program in (a)

Figure 10. Multiple Dependences and Equal Δ s

S_2 , cannot distinguish between the store of $A(I)$ and the fetch of $B(I)$. Thus the statement can be viewed as a single indivisible entity, and represented by a single node on the DG. Figure 10b shows the DG of the loop of Figure 10a.

Complications arise, however, when the dependences have different distances, as in the program of Figure 11a. We again have an anti-dependence of distance one from $B(I)$ in S_1 to $B(I-1)$ in S_2 . The flow dependence from $A(I)$ in S_1 to $A(I-2)$ in S_2 has a distance of two. If the DG is to be a statement level DG can both dependences from S_1 to S_2 be merged? In general, the answer is no. [LiZh85] proves this for one synchronization instruction set. Figure 11b shows the DG for the program of Figure 11a.

This is particularly bad when the dependence distance between two statements is not a constant function of the loop index. If the dependence distance is not a constant function of the loop index, then arcs must be generated for every possible value that the function can take. If the upper bound of the loop is not known, then it might not be possible to generate all these

```

DO 10 I = 1, N
    A(I) = B(I) + C(I)    S1
    B(I-2) = A(I-1) + C(I) S2
10  CONTINUE

```

(a) Program with Multiple Dependences with non-equal Δ s Between Two Statements

(b) DG for the Program in (a)

Figure 11. Multiple Dependences and Non-equal Δ s

arcs, making synchronization for certain instruction sets impossible. As shall be seen in Chapter 3, when the *testset* instruction set is used, the distance can be assumed to be Δ_{min}^L , the shortest dependence from one statement to another. Because we are treating the more general case at this point, we shall assume that all dependence arcs are placed in the DG. Furthermore, if synchronization cannot be generated for a loop, we assume that the loop was marked as serial either by other passes or by a prescan pass with a knowledge of synchronization.

1.3.1.3.2. Justification for Including Only '<' and '<=' Dependences

If a dependence does not have a '<' direction component it must have either '>' or '=' direction components. If the dependence direction is '=' the dependence is completely within an iteration. Since we assume that a single iteration always runs on a single processor, and fetches and stores within a processor execute serially the dependence is forced to be honored by the hardware. If the hardware did not constrain the order of fetches and stores within a single processor, then '=' dependences would have to be included in the DG.

If a dependence has a '>' direction then the source of the dependence is in a later iteration of the *dospread* loop than the sink. The only way such a dependence can be violated is for multiple invocations of the *dospread* loop to be active simultaneously. This, however, requires that a loop surrounding the *dospread* loop be parallel and executing on multiple processors. The architecture, however, only allows one level of spread parallelism, so this cannot occur. If multiple levels of spread parallelism were allowed, '>' dependences would also have to be included in the DG. The synchronization instruction semantics would also have to be altered to allow a synchronization statement to be associated with a particular *dospread* loop within a loop nest, and the sign of Δ in the *test* instruction would have to be taken into account.

If we need to construct a DG for a loop nested within a *dospread* loop to perform the transformations of Section 2.3.2 then we should include all dependences regardless of direction.

1.3.2. FORTRAN Extensions

Programs that are the object of the automatic synchronization techniques are written in extended FORTRAN 66. Four new statements are needed to specify synchronization actions and objects. These statements are not present in the input language, but are found only in the output language from synchronization. The new statements are *test*, *set*, *testset* and *quit*. They are explained in greater detail in Chapter 2.

The input language includes two new loop types, *dospread* and *dovector*.

A *dospread* loop is one that is spread across several processors to take advantage of intra-loop parallelism. A *dospread* loop can be a *doall* [Davi81], [LuBa80], a *forall* [Padu79], or a *doacross* [Cytr84] loop. A *doall* loop is any loop that has no cross-iteration dependences, i.e. any dependences that exist have a direction of '='. A *forall* loop may have cross iteration dependences, but they must all be from a statement to a lexically forward statement, i.e. LFDs. A *doacross* loop is a loop whose iterations may not all start at the same time. Instead, each iteration must wait some amount of time after the last iteration has started before itself starting. This wait will be caused, in this thesis, by synchronization. The wait is necessitated either by a branch out of the body of the loop, or by a LBD. Figure 12a gives an examples of a *doall* loop, and Figure 12b gives the DG for the *doall* loop. Figure 13a, Figure 13b, Figure 14a and Figure 14b give the same for *forall* and *doacross* loops respectively.

The *dovector* loop is one that can potentially be run as a set of vector instructions. Any loop that has been determined to be a *doall* or a *forall* loop is a candidate to be run as a *dovector* loop.

```

DO 10 I = 1, N
  A(I) = B(I) + C(I) S1
  D(I) = A(I) + B(I) S2
10 CONTINUE

```

(a) *Doall* Loop

S₁

S₂

(b) DG for (a)

Figure 12. Doall Loop Example

```

DO 10 I = 1, N
  A(I) = B(I) + C(I) S1
  D(I) = A(I-1) + C(I) S2
10 CONTINUE

```

(a) *Forall* Loop

(b) DG for (a)

Figure 13. Forall Loop Example

```
DO 10 I = 1, N
  A(I) = B(I) + C(I-1)
  C(I) = A(I-1) + C(I)
  E(I) = D(I) + C(I-2)
10 CONTINUE
```

 S_1 S_2 S_3

(a) *Doacross* loop

(b) DG for (a)

Figure 14. Doacross Loop Example

CHAPTER 2

MODIFICATIONS TO THE DG

In the previous chapter the loops to be synchronized were marked as *dospread*, and a DG was constructed for those loops. In this chapter this DG is modified to take into account certain control structures in the DG, namely branches whose targets are outside of the loop and nested loops. The DG so modified is then ready to be used to place synchronization instructions in the source loop using the algorithms of the Chapter 3.

2.1. Control Arcs and Exit-ifs

The targets of GOTO statements within a *dospread* loop can either be to other statements within the loop or to statements outside the loop. If the former, then the GOTO will either be lexically forward or lexically backward. If it is lexically forward nothing needs to be done to the DG; the actions described in Chapter 3 will insure that the execution of the loop proceeds safely. If the GOTO is lexically backward, we will assume that all enclosing loops are serial. This is a consequence of dependence testing being extremely difficult when backward GOTOs are present in a loop. If the conditions governing the backward GOTO are simple enough the backward GOTO can be converted into a DO loop.

GOTOs that are to targets outside the loop, and their associated IF statement, are referred to as *exit-ifs*. When exit-ifs are present the DG must be modified to reflect this.

Figure 15a shows a *dospread* loop that contains an exit-if statement. The exit-if causes two problems. The first problem is that the first statement of some iteration $i + 1$ of the *dospread* loop should not begin executing until the exit-if of iteration i has executed and taken the false path. The order of execution that must be maintained is shown in Figure 15b.

```

      DOSPREAD 10 I = 1, 4
      A(I) = B(I) + C(I)      S1
      IF (A(I-1).GT.4.0) GOTO 20 S2
      D(I) = A(I-1) + F(I)    S3
10    CONTINUE
      ...
20    X(I) = Y(I) + Z(I)

```

(a) Loop Containing an Exit-if

i =	1	2	3	4
t = 1	S ₁			
= 2	S ₂			
= 3	S ₃	S ₁		
= 4		S ₂		
= 5		S ₃	S ₁	
= 6			S ₂	
= 7			S ₃	S ₁
= 8				S ₂
= 9				S ₃

(b) Execution Order of the Loop of (a)

(c) Modified DG of the Loop of (a)

Figure 15. Exit-If Example: DG Modification

The second problem arises when the true path of an exit-if in iteration i is taken, and the loop should terminate. When this happens the processor running the exiting iteration should wait until all iterations less than i have completed, and then terminate the loop.

The first problem can be solved by modifying the DG and is dealt with here. The second problem requires a change in the source code of the program and some knowledge about the synchronization instruction set being used, and is dealt with in Chapter 3.

We now consider the first problem. A LBD with distance 1 is inserted into the DG from the exit-if statement to the first statement of the loop body that changes the value of any variable used outside that iteration. A control dependence actually exists from the exit-if statement to every statement in the loop. With the architecture of this thesis, all dependence arcs but the one we add will be eliminated by the algorithms of Chapter 4. Therefore we do not need to include these dependence arcs in the DG. Because this dependence will be satisfied by synchronization, iteration $i + 1$ cannot execute any statements that will alter any data used outside the loop until iteration i has signaled that it is not going to exit the loop. Since it is the execution of the next iteration that is affected, the distance of the control dependence is one. Figure 15c shows a modified DG where the dashed line is the added LBD.

Figure 16 gives the algorithm for this.

2.2. Nested Loops

As we mentioned, only one level of spread parallelism is allowed in the loop nest to be synchronized. This *dospread* loop can, however, have other loops nested within it. As explained previously, these nested loops are represented as a single node in the DG. If a dependence is lexically forward and its source and sink are not contained within the same nested loop, then the

```

foreach exit-if branch  $S_E$  do
    place a dependence from  $S_E$  to  $S_1$  in the DG
  
```

Figure 16. Exit-if DG Modification Algorithm

DG can be left unchanged. Consider, however, the program of Figure 17a, and its DG in Figure 17b. The *dospread* loop is effectively serialized by the presence of the dependence cycle on L_1 . The next two sections show how the DG can be modified to recover some or all of the parallelism inherent in a loop nest.

2.2.1. Synchronization within Nested Vector Loops

If the complete inner loop nest consists of vector loops then we can fully distribute [Wolf82] the inner loops over their statements. The synchronization instructions can then be placed between the resulting fully distributed loops. Such a loop nest is shown in Figure 17a, and its DG in Figure 17b.

By fully distributing the inner loops, we get the loop nest of Figure 17c, and its DG, shown in Figure 17d. This is the approach to take if loops are to operate on vectors, since they will execute as an atomic vector operation. Depending on the overheads associated with vector operations, and the length of vectors, it might be desirable to use loop splitting, discussed in the next section, along with loop distribution.

2.2.2. Splitting Non-Distributable Loops

The second method of dealing with nested loops involves splitting the offending loop into sub-loops, and placing synchronization instructions between the sub-loops. This splitting up of the iteration space of the loop and placing of the synchronization instructions will be referred to as *loop splitting*. A *sub-loop* is a loop that executes a set of adjacent iterations of the original loop. For example, a loop that has iterations from 1 to 1000 might be split into two sub-loops, the first of which executes iterations 1 to 500, the second iterations 501 to 1000. Loop splitting differs from loop distribution in that in loop splitting each sub-loop contains all the statements of the original loop, but only part of the iterations. In loop distribution, each distributed loop contains all the iterations of the previous loop, but only part of the statements.

```

      DOSPREAD 20 I = 1, L
      DOVECTOR 10 J = 1, N
      A(I,J) = B(I,J) + C(I,J)
      D(I,J) = A(I-1,J-1) + B(I,J)
10      CONTINUE
20      CONTINUE

```

(a) Loop Nest With Inner Vector Loops

(b) DG for the Loop Nest of (a)

```

      DOSPREAD 30 I = 1, L
      DOVECTOR 10 J = 1, M
      A(I,J) = B(I,J) + C(I,J)
10      CONTINUE
      DOVECTOR 20 J = 1, N
      D(I,J) = A(I-1,J-1) + B(I,J)
20      CONTINUE
30      CONTINUE

```

(c) Loop Nest with Inner Loops Fully Distributed and Synchronization Added

(d) Final DG for (c)

Figure 17. Dependence within a Nested Vector Loops

Loop splitting will allow execution of iterations of the inner loop to be overlapped. Thus the inner loop is executed in parallel. Since the inner loop is executed in parallel, dependences on the inner loop may be violated if not properly synchronized. [Bane79] and [Wolf82] discuss dependence testing and the construction of DGs for multiply nested parallel loops in detail. For

our purposes, we can assume that a DG for the inner loop can be built in the same manner as for the the outer *dospread* loop, except that directions of all dependences must be included since there is an outer parallel loop.

When constructing a DG for a *dospread* loop, only '<' direction components are allowed. This means that all dependence distances are positive. When constructing a DG for a nested loop, it is possible that '=' and '>' dependence directions exist. This means that dependence distances may be zero and negative. When determining the value of Δ_{max}^{nested} for a nested loop the sign of the distance will be ignored. Therefore, if $\Delta_1^{nested} = -4$, and $\Delta_2^{nested} = 3$, then $\Delta_{max}^{nested} = -4$

The goal of loop splitting is to arrange the DG in such a way that synchronizing the dependences will cause both the dependences on the outer *dospread* and inner serial loop to be honored. Dependences on the outer loop will be honored by the distance attached to the dependences added to the DG during loop-splitting. Dependences on the inner loop will be honored because of the location of the dependences in the DG, which will not allow a sub-loop L_k , in some iteration i of the outer *dospread* loop, to begin executing until all sub-loops that generate data for sub loops L_k have completed.

Figure 18a shows a loop nest whose nested inner loop is a candidate for loop splitting. Figure 18b gives the DG for the *dospread I* loop, and Figure 18c gives the DG for the inner nested *J* loop. Figure 18d shows the loop nest of Figure 18a with the inner loop split. Dependence arcs have been drawn that represent dependences between the nested loops that must be satisfied. Figure 18e shows a DG for the loop of Figure 18d with the sub-loops represented as a single node for compactness. If L_k is a sub-loop, and the sink of such a dependence, then it can be easily determined what sub-loop $L_k + offset$ is the source. *Offset* is a function of the minimum dependence distance, Δ_{max}^J , on the nested *J* loop, and the maximum number of iterations, ι , in any sub-loop. The formula is:

$$offset := \left\lceil \frac{l - \Delta_{max}^J}{l} \right\rceil - 1$$

A proof is given in Figure 19.

Figure 20 shows values for *offset* obtained for different ranges of Δ_{max}^J with different dependence directions. The values have been derived from the formula for *offset*. A more intuitive explanation of these values follows.

An *iteration dependence graph* (IDG) will be used in the examples that follow. An IDG is like a DG except that nodes on the graph are iterations, not statements, and arcs represent dependences between iterations and not between statements. An IDG can be created by merging

```

DOSPREAD 20 I = 1, N
DO 10 J = 1, N
    A(I,J) = B(I,J) + C(I,J)      L1
    B(I+1,J+1) = A(I-1,J) + E(I,J) S1
    B(I+1,J+1) = A(I-1,J) + E(I,J) S2
10  CONTINUE
20  CONTINUE

```

(a) Loop Nest with Non-vector Inner Loop

(b) DG for DOSPREAD Loop of (a)

(c) DG for Nested Loop of (a)

Figure 18. Loop Splitting a Nested Loop

```

DOSPREAD 40 I = 1, N
      DO 10 J = 1,  $\left\lceil \frac{M}{L} \right\rceil$                                  $L_1^1$ 
           $S_1$ 
           $S_2$ 
10      CONTINUE
      ...
      DO  $L_k$  J =  $\left\lceil \frac{M}{L} \right\rceil + 1, 2 * \left\lceil \frac{M}{L} \right\rceil$                  $L_1^k$ 
           $S_1$ 
           $S_2$ 
      CONTINUE
      ...
      DO  $L_{l-1}$  J =  $(l-1) * \left\lceil \frac{M}{L} \right\rceil + 1, M$                  $L_1^{l-1}$ 
           $S_1$ 
           $S_2$ 
40      CONTINUE

```

(d) Loop Nest of (a) after Loop Splitting

(e) The DG for the Split Loop of (d)

Theorem 1 If *offset* sub-loops execute between the source and sink of a dependence, the dependence will be honored.

Proof By definition, a dependence will be honored if Δ_{max}^L iterations execute between its source and sink, where Δ_{max}^L is the maximum distance of any dependence on some loop L. Let I be the minimum number of iterations that executes between the source and sink for a given offset. Then

$$\begin{aligned}
 I &= |\text{offset} \cdot \iota| \\
 &= \left| \iota \left(\left\lceil \frac{\iota - \Delta_{max}^L}{\iota} \right\rceil - 1 \right) \right| \\
 &\geq |(\iota - \Delta_{max}^L) - \iota| \\
 &\geq |\iota - \Delta_{max}^L - \iota| \\
 &\geq |-\Delta_{max}^L|
 \end{aligned}$$

Figure 19. Proof of Validity of OFFSET values

direction	ι	offset
$< (\Delta_{max}^J > 0)$	$\iota \leq \Delta_{max}^J$	$\Delta_{max}^J \leq \text{offset} < 0$
	$\iota > \Delta_{max}^J$	$\text{offset} = 0$
$= (\Delta_{max}^J = 0)$	all values	$\text{offset} = 0$
$> (\Delta_{max}^J < 0)$	$\iota \leq \Delta_{max}^J $	$1 < \text{offset} \leq - \Delta_{max}^J $
	$\iota > \Delta_{max}^J $	$\text{offset} = 1$

Figure 20. Values of Offset for Different Dependence Distances and ι

the rows of an iteration space diagram into a single row. Figure 21a gives a loop nest with '<'

dependences on both the I and J loops, and $\Delta_{max}^J = 3$. Figure 21b gives the DG for the nested loop, and Figure 21c gives the iteration space diagram. Finally Figure 21d gives the IDG.

In Figure 22a the IDG of Figure 21d has been marked to show the range of sub-loops where $\iota = 1$. All dependences to a particular sub-loop begin at an earlier sub-loop L_{k-2} . That all

```

DOSPREAD 20 I = 1, N
DO J = 10 1, 20
    A(I+1,J+3) = B(I-1,J) + C(I,J) S1
    B(I,J+3) = D(I,J) + E(I,J) S2
10    CONTINUE
20    CONTINUE

```

(a) A Loop Nest

(b) DG for the J Loop of (a)

(c) Iteration Space for (a)

Figure 21. Continued

(d) IDG for (c)

Figure 21. Loop Nest Appropriate for Loop Splitting and its Associated IDG

dependences begin at an earlier sub-loop will always be true if $\iota < |\Delta_{max}^J|$, since the dependence must be from an earlier iteration and the sub-loop is too small to contain both the source and the sink of the dependence.

In Figure 22b we can see the IDG with sub-loops marked where $\iota = 4$. In this case, where $\iota > |\Delta_{max}^J|$, a dependence whose sink is in some L_k may also have its source in L_k . Therefore sub-loop L_k cannot execute until sub-loop L_k has finished in a previous iteration of the *dospread* loop. Synchronization accomplishing this occurs when *offset* = 0. This remains true no matter how much larger ι becomes. Figure 22c shows the loop split with $\iota = 4$. Figure 22d shows the DG for the loop of Figure 22c, and Figure 22e shows the execution order of the iterations in the program.

In Figure 23a we have a loop nest where $\Delta_{max}^J = 0$. Figure 23b gives the IDG for the nested loop. Sub-loops with $\iota = 2$ are shown on the IDG for the nested loop. Regardless of the size of ι , sub-loops must always have dependences whose source and sink are within the sub-loop. Thus, sub-loop L_k in some iteration of the *dospread* loop must always wait for sub-loop L_k in some previous iteration of the *dospread* loop to finish. Again, *offset* = 0 accomplishes this.

(a) IDG of Figure 21d Marked for $\iota = 1$

(b) IDG of Figure 21d Marked for $\iota = 4$

Figure 22. Loop Splitting with ' $<$ ' Dependences

```

DOSPREAD 50 I = 1, N
DO 10 J = 1, 5 L11
DO 20 J = 6, 10 L12
DO 30 J = 11, 15 L13
DO 40 J = 16, 20 L14
CONTINUE
40 CONTINUE
50 CONTINUE

```

(c) Loop Nest after Splitting, with $\iota = 4$

(d) The DG for the Split Loop of (c)

Figure 22. Continued

i=	1	2	3	4
t = 1	L_1^1			
= 2	L_1^2	L_1^1		
= 3	L_1^3	L_1^2	L_1^1	
= 4	L_1^4	L_1^3	L_1^2	L_1^1
= 5		L_1^4	L_1^3	L_1^2
= 6			L_1^4	L_1^3
= 7				L_1^4

(e) Execution Order of the Sub-loops.

Figure 22. Continued

Figure 23c shows the transformed program, Figure 23d its DG, and Figure 23e the execution order of the iterations.

Figure 24a shows a loop nest where the nested loop has $\Delta_{maz}^J = 3$. Figure 24b gives the IDG for the nested loop, showing sub-loops where $\iota = 4$. When $\Delta_{maz}^J > 0$, some sources for dependences whose sinks are in the sub-loop will always be in a later sub-loop than the sink. Thus it is always true that $offset > 0$. If $\iota \leq \Delta_{maz}^J$, then no dependence can extend further than some sub-loop to the previous sub-loop. Therefore $offset$ will equal 1. If $\iota < \Delta_{maz}^J$, then a dependence will extend through several sub-loops, and $offset$ will always be greater than 1. Figure 24c shows the transformed loop nest, Figure 24d the DG for the loop, and Figure 24e shows the execution order of the iterations.

Figure 25 summarizes the rules for handling nested loops.

2.2.3. Splitting Loops with '<' and '=' Direction Components

If a loop contains only dependences with '<' and '=' direction components, the software necessary to split these loops is simpler than in the more general case. For reason for this can be seen by examining the table of Figure 20. If the nested loop contains only '<' and '=' direction

```

      DOSPREAD 20 I = 1, N
        DO 10 J = 1, 8
          A(I,J) = B(I,J) + C(I,J)      S1
          D(I,J) = A(I-2,J) + E(I,J)    S2
10      CONTINUE
20      CONTINUE

```

(a) Program with '=' Dependence on the Nested Loop

```

j = 1  ○ → 2
      = 2  ○ → 2
      = 3  ○ → 2
      = 4  ○ → 2

```

(b) IDG for (a)

```

      DOSPREAD 50 I = 1, N
        DO 10 J = 1, 2 L11
          ...
        DO 20 J = 3, 4 L12
          ...
        DO 30 J = 5, 6 L13
          ...
        DO 40 J = 7, 8 L14
          ...
50      CONTINUE

```

(c) Loop Nest of (a) after Loop-splitting

Figure 23. Loop Splitting with '=' Dependences

(d) The DG for the Split Loop of (c)

i=	1	2	3	4	5	6
t = 1	L_1^1	L_1^1				
t = 3	L_1^2	L_1^2	L_1^1	L_1^1		
t = 4	L_1^3	L_1^3	L_1^2	L_1^2	L_1^1	L_1^1
t = 5	L_1^4	L_1^4	L_1^3	L_1^3	L_1^2	L_1^2
t = 6			L_1^4	L_1^4	L_1^3	L_1^3
t = 7					L_1^4	L_1^4

(e) Execution Order of Split Loops

Figure 23. Continued

```

      DOSPREAD 20 I = 1, N
        DO 10 J = 1, 7
          A(I,J) = B(I,J) + C(I,J)
          D(I,J) = A(I-1,J+3) + E(I,J)
10      CONTINUE
20      CONTINUE

```

(a) Loop Nest with '>' Dependences

(b) IDG for (a)

```

      DOSPREAD 50 I = 1, N
        DO 10 J = 1, 2 L11
        ...
        DO 20 J = 2, 4 L12
        ...
        DO 30 J = 5, 6 L13
        ...
        DO 40 J = 7, 7 L14
        ...
50      CONTINUE

```

(c) Loop Nest of (a) after Loop Splitting

Figure 24. Loop Splitting with '>' Dependences

(d) The DG for the Split Loop of (d)

i=	1	2	3	4
t = 1	L_1^1			
= 2	L_1^2			
= 3	L_1^3	L_1^1		
= 4	L_1^4	L_1^2		
= 5		L_1^3	L_1^1	
= 6		L_1^4	L_1^2	
= 7			L_1^3	L_1^1
= 8			L_1^4	L_1^2
= 9				L_1^3
= 10				L_1^4

(e) Execution Order of the Sub-loops

Figure 24. Continued

D - a set of all dependences in the DG.
 L - the *dospread* loop being synchronized.
 L_{ub} - the upper bound of the loop for which the DG built.
 R - the number of synchronization registers.
 k - the minimum number of iterations in a loop that can be efficiently executed in light of startup overheads, etc.

$Distance(\delta)$ returns the distance of a dependence δ , and is defined as:
 return the actual maximum dependence distance, if determinable. Otherwise,
 if the direction of δ is ' $<$ ' then
 return -1
 if the direction of δ is '=' or '<=' then
 return 0
 if the direction of δ is '>', '>=', or '<=>' then
 return L_{ub} .

foreach dependence $\delta \in D$ **do**
 if δ is immediately nested in a *vector* loop then
 fully distribute the vector loop.

foreach dependence $\delta \in D$ **do**
 L_{so} is the outermost loop containing the source that is nested within the *spread* loop, or is undefined otherwise.
 L_{si} is the outermost loop containing the source that is nested within the *spread* loop, or is undefined otherwise.
 if $L_{so} = L_{si} \neq \text{undefined}$ then
 find Δ_{min}^{spread} , the minimum dependence distance of any dependence completely within L .
 remove δ and all other δ' completely within L from D
 build a DG for L_{so} , including all dependences regardless of direction.
 find Δ_{max}^L , the maximum distance of a dependence in L_{so} .
 $\iota := \max\left(k, \left\lceil \frac{L_{ub}}{R} \right\rceil\right)$
 compute *offset* as given in the text
 split the loop into $\left\lceil \frac{L_{ub}}{\iota} \right\rceil$ sub-loops, numbered lexically from 1 to n .
 foreach dependence $\delta'' \in D$ **do**
 if $source(\delta'') = \text{DO of } L$ or
 $sink(\delta'') = \text{CONTINUE of } L$ then
 replicate the dependence to or from each L_i , $1 \leq i \leq n$
 foreach sub-loop l **do**
 place a dependence from the end of the sub-loop to the beginning of sub-loop j , $j = l + \text{offset}$. If $j < 1$ do not add the dependence.
 Mark the dependence as having a distance of Δ_{min}^{spread} .

Figure 25. Algorithm for Nested Loops

components, then we can assume that the value of ι is always greater than the value of Δ_{max}^J . This can be done because this assumption always leads to the most conservative synchronization for this case. Therefore, for any ι , $offset = 0$. If ' $>$ ' direction components are present, then the most conservative synchronization requires assuming Δ_{max}^J equals the upper bound of the nested J loop. This in turn leads to the nested J loop being run completely serially, with synchronization to insure the dependence is filled before the loop, and synchronization to show that the dependence is filled after the loop. This means that to synchronize nested loops with ' $>$ ' direction components, we need an upper bound on the value of Δ_{max}^J less than the upper bound of the J loop in order to salvage some parallelism, whereas in the latter case we need only know the direction.

2.2.4. Removing Extraneous Dependences Introduced in Loop Splitting

It is possible that loop splitting will cause extraneous dependence to be introduced into the DG. These dependences result from a dependence source or sink being moved out of a loop, as in Sections 2.2.1 and 2.2.2, and the loop being subsequently split. As the loop is split, the dependences attached to the DO and CONTINUE statements of the loop are replicated, and attached to the DO and CONTINUE statements of each sub-loop. Figure 26a gives a program, Figure 26b its DG, and Figure 26c the DG after splitting the inner loop into four sub-loops. Leaving these dependences in the DG will not cause incorrect synchronization to be generated, but it may cause unnecessary synchronization to be generated. And in the case of the extra dependences being LBDs, the parallelism gained by loop splitting can be eliminated by the extra dependences. Therefore the extra dependences should be removed if possible.

To do this, it is necessary only to retrieve the source and sinks from the dependence arcs, and repeat dependence testing on these statements. If a dependence is not present, then the corresponding arc can be eliminated. Figure 26d shows the DG of Figure 26c after performing additional data dependence testing.

```

DO 20 I = 1, N
  B(I) = B(I) + A(I-1,I)
  DO 10 J = 1, N
    A(I,J) = C(I,J) + D(I,J)
    E(I,J) = A(I-1,J) + F(I,J)
  10    CONTINUE
20    CONTINUE

```

S_1
 L_1

(a) A Loop Nest

(b) The DG for (a)

Figure 26. Extraneous Dependence Removal Example

(c) The DG after Splitting the Inner Loop into Four Sub-loops

(d) The DG after Removing Extraneous Dependences

CHAPTER 3

GENERATION OF SYNCHRONIZATION INSTRUCTIONS

At this point a DG has been constructed in the previous chapters that accurately reflects the dependence relations among statements in the program that must be honored. This chapter uses this information to add synchronization statements to the source program. We cover two major topics in doing this. The first is the definition of two sets of synchronization instructions, and how they can be used to synchronize loops in the absence of any control branches in the source loop. The second is the generation of synchronization in the presence of control branches in the source loop.

3.1. Synchronization Instructions

We use two different sets of synchronization instructions in this thesis. The first set consists of the *test* and *set* instructions, the second set of the *test* and *testset* instructions. The *test* instructions in the two sets are different, but this will not cause any problems since the two instruction sets will never be intermixed in a program. Both instruction sets contain the *quit* instruction. The semantics of synchronization instructions are defined only within a *dospread* loop. The relative advantages of the two instruction sets are covered in section 3.3.

3.1.1. SET and TEST Instructions

The test and set instructions operate on synchronization registers that are accessed as bit vectors. The registers are long enough so that there is one bit per iteration of the loop. If the execution order of the iterations of a loop is constrained so that the lowest running iteration number is never k more than the highest running iteration number, then the register needs to only have k bits. This is not too difficult to enforce if loops are scheduled at run time, i.e are

not scheduled by blocking. When the *dospread* loop begins running, all bits are assumed to be initialized to zero. We will refer to bit i of register r as $r[i]$.

The *set* instruction has the syntax $set(r)$, where r is a synchronization register. If a *set* instruction executes in some iteration i , it performs the assignment $r[i] := 1$.

The *test* instruction has the syntax $test(r) \Delta$, where r is a synchronization register and Δ is a dependence distance. If $test(r) \Delta$ executes on iteration i , then it busy waits for $r[i - \Delta]$ to be equal to one. Figure 27 summarizes the actions of these instructions.

The *set* instruction signals the completion of an action, in particular the execution of a dependence source. The *test* waits until the completion of that action before starting another action such as execution of a dependence sink.

3.1.2. TEST and TESTSET Instructions

The *test* and *testset* instructions operation on a synchronization register that can hold an integer. The register should be able to hold an integer as large as the loop upper bound. The register is set to zero when the concurrent loop begins execution.

The *testset* instruction has the syntax $testset(r)$. It first checks to see if a *testset*(r) instruction has been executed in the previous iteration. This it does by checking if the value of

$$r[i] := 1$$

(a) Semantics of $set(r)$ executing in iteration i .

if $i - \Delta > 0$ then
 l : if $r[i - \Delta] = 0$ then goto l

(b) Semantics of $test(r) \Delta$ executing in iteration i .

Figure 27. Semantics of the *set* and *test* Instructions

$r = i - 1$, where i is the iteration number (not the value of the loop index) of the *testset* executing. If so, it increments r making it equal to i , otherwise it waits. Figure 28a summarizes this.

The *test* instruction has the syntax *test*(r) Δ , where Δ is the dependence distance. A *test* instruction in iteration i busy waits on r until $r = i - \Delta$, i.e. until a *testset*(r) is executed in iteration $i - \Delta$. Figure 28b summarizes the semantics of the *test* instruction.

3.1.3. The QUIT Instruction

The syntax of the *quit* instructions is simply *quit*. Its purpose is to halt execution of a loop before all iterations of the loop have executed, and is used in this thesis only to terminate *dospread* loops when exited by an *exit-if*. When a *quit* statement is executed the parallel loop is terminated immediately on all processors executing the loop.

3.2. Synchronizing Within Blocks of Assignment Statements (BAS)

Synchronizing a dependence is easiest if no control branches occur between the source and sink of the dependence. In this section we deal with loops whose bodies are blocks of assignment

```

l:    if  $r = i - 1$  then
         $r := i$ 
    else
        goto l

```

(a) Semantics of *testset*(r) executing in iteration i .

```

    if  $i - \Delta > 0$  then
l:    if  $r < i - \Delta$  then goto l

```

(b) Semantics of *test*(r) Δ in iteration i .

Figure 28. Semantics of the *testset* and *test* Instructions

statements. The principles outlined in this section also apply if control branches occur, but additional actions need to be taken, as discussed later in this chapter. We show how to synchronize programs using both *test* and *set*, and *test* and *testset* instructions.

3.2.1. Synchronizing LFDs and LBDs Using TEST and SET

Consider the program of Figure 29a, and the DG of Figure 29b. There exists a LFD of distance 2 from S_1 to S_2 on A . To insure that the dependence is honored, S_2 , on any iteration $i > 2$, must execute after S_1 , on iteration $i - 2$, executes. This means a *set* instruction should be placed after S_1 , and a *test* instruction, with $\Delta = 2$, before S_2 . The synchronized program is shown in Figure 29c.

```

        DOSPREAD 10 I = 1, N
            A(I) = B(I) + C(I)      S1
            X(I) = A(I-2) + B(I)    S2
10      CONTINUE

```

(a) Program with LFD

(b) DG for the Program of (a)

```

        DOSPREAD 10 I = 1, N
            A(I) = B(I) + C(I)      S1
            SET (1)
            TEST (1) 2
            X(I) = A(I-2) + B(I)    S2
10      CONTINUE

```

(c) Program with Test and Set Instructions Added

Figure 29. Program and DG with LFD

The program and DG of Figure 30a and Figure 30b, respectively, show a LBD. The goal is the same as with a LFD, namely to notify the dependence sink at S_1 when the source at S_2 has executed. Again, this is done by placing a *set* after the source and a *test* before the sink of the dependence. The synchronized program is shown in Figure 30c.

In general, with a dependence having distance Δ from S_i to S_j , $set(r_s)$ placed after the source of the dependence and $test(r_s) \Delta$ placed before the sink of the dependence will force the dependence to be honored.

```

DOSPREAD 10 I = 1, N
...
    B(I) = A(I) + C(I)    S1
    A(I+2) = B(I) + C(I) S2
...
10    CONTINUE

```

(a) Program with LBD

(b) DG for the Program of (a)

```

DOSPREAD 10 I = 1, N
...
    TEST(1) 2
    B(I) = A(I) + C(I)    S1
    A(I+2) = B(I) + C(I) S2
    SET(1)
...
10    CONTINUE

```

(c) Program with Test and Set Instructions Added

Figure 30. Program and DG with LBD

Figure 31 summarizes these rules.

3.2.2. Synchronizing LFDs and LBDs Using TEST and TESTSET

The goals of synchronizing using *test* and *testset* are the same as with *test* and *set*. With LFDs, fulfilling this goal is easier using the *testset* instruction. Again consider the program of Figure 29a. To synchronize this program we need only to place a *testset* between S_1 and S_2 . This is shown in Figure 32. S_2 in iteration i cannot execute before the *testset* in iteration i , since the *testset* precedes S_2 . But the *testset* of iteration i cannot execute before the *testset* of iteration $i - 1$, which cannot execute before the *testset* of iteration of $i - 2$. But the *testset* of $i - 2$ cannot execute until S_1 in $i - 2$ has finished, since S_1 precedes the *testset*. Thus S_1 in iteration $i - 2$ must execute before S_2 in iteration i , and the dependence is satisfied. Therefore LFDs can be synchronized by putting a *testset* between the source and the sink of the LFD.

If Δ for a dependence being synchronized is not constant, then the *test* instruction should be given the minimum value of Δ , Δ_{min} . As can be seen from the algorithms of Chapter 4, this

```

foreach dependence  $\delta = (S_{so}, S_{si})$ , with distance  $\Delta$  do
    after  $S_{so}$  place the statement set( $r_i$ )
    before  $S_{si}$  place the statement test( $r_i$ )  $\Delta$ 
  
```

Figure 31. Placing Test and Set Instructions in BAS

```

        DOSPREAD 10 I = 1, N
            A(I) = D(I) + C(I)      S1
            TESTSET(1)
            X(I) = A(I-2) + B(I)    S2
10      CONTINUE
  
```

Figure 32. Program with LFD Synchronized using TEST

is the same as synchronizing for each value of Δ and then eliminating the redundant dependence arcs.

LBDs are synchronized as before with *test* and *set*, except now *testset* is used instead of *set* to signal *test*. Figure 33 shows the program of Figure 30a synchronized using *test* and *testset*.

Figure 34 summarizes these rules.

3.3. Advantages and Disadvantages of the Instruction Sets

Now that we know how the instructions are used to synchronize dependences, it is possible to examine the relative advantages of the two instruction sets. The most obvious difference is

```

DOSPREAD I = 1, N
...
TEST(1) 2
B(I) = A(I) + C(I)    S1
A(I+2) = B(I) + C(I)  S2
TESTSET(1)
...
10    CONTINUE

```

Figure 33. Program with LBD Synchronized using TEST and TESTSET

```

foreach dependence  $\delta = (S_{so}, S_{si})$  with distance  $\Delta$  do
  if  $so < si$  then
    place the instruction testset( $r_i$ ) after  $S_{so}$  and before  $S_{si}$ .
  else
    place the instruction testset( $r_i$ ) after  $S_{so}$ 
    place the instruction test( $r_i$ )  $\Delta$  before  $S_{si}$ 

```

Figure 34. Placement of *test* and *testset* Instructions in BAS

the amount of storage needed for a synchronization register. With a bit needed for each iteration, *test* and *set* instructions will need 4096 bytes of storage to synchronize a loop with an upper bound of 32767, whereas the *test* and *testset* instruction will need only two bytes. As has been stated, though, by restraining the execution of iterations, the storage requirements for *test* and *set* can be eased.

More importantly, using *test* and *set*, it is possible to execute a *forall* loop completely in parallel. Using *test* and *testset* requires that any loop requiring synchronization have a serial component. Figure 35a gives a loop that has been synchronized using *test* and *set* instructions. Figure 35b gives an optimal execution order of the statements. At time $t = 2$ every iteration i will set bit i of r . At time $t = 3$ every iteration i will read bit $i - 1$ of r . Since each processor is accessing a different element of r at each time step, the loop can execute completely in parallel. Of course, if some processors are delayed, the loop may not execute completely in parallel, but it is still theoretically possible for it to do so.

Using the *testset* instruction, this is not possible. Figure 36a gives the same loop as before, except now synchronized using a *testset* instruction. Figure 36b gives the optimal execution order of the statements in the loop. Note that now six time steps are needed to execute the four iterations of the loop. This is because the *testset* of iteration two must wait for the *testset* of iteration 1 to finish before it executes, and cannot execute in parallel with the synchronization instructions of other iterations as in the previous example. Therefore the savings of space in the synchronization registers is paid for in execution time.

Another difference in the two instructions will become apparent when we discuss dependence arc elimination in Chapter 4. The *testset* instruction is more powerful than what is needed to simply synchronize many dependences. The additional power allows us to potentially use fewer synchronization instructions with *test* and *testset* instructions than with *test* and *set* instructions.

```

      DOSPREAD 10 I = 1, 4
      A(I) = B(I) + C(I)      S1
      SET(r)                  Ss
      TEST(r) 1               St
      D(I) = E(I) + A(I-1)    S4
10    CONTINUE

```

(a) A Loop Synchronized Using TEST and SET Instructions

i =	1	2	3	4
t = 1	S ₁	S ₁	S ₁	S ₁
= 2	S _s	S _s	S _s	S _s
= 3	S _t	S _t	S _t	S _t
= 4	S ₂	S ₂	S ₂	S ₂

(b) Optimal Execution of (a)

Figure 35. Optimal Execution of a Loop using TEST and SET Instructions

```

      DOSPREAD 10 I = 1, 4
      A(I) = B(I) + C(I)      S1
      TESTSET(r)              TS
      D(I) = E(I) + A(I-1)    S2
10    CONTINUE

```

(a) A Loop Synchronized Using TEST and TESTSET Instructions

i =	1	2	3	4
t = 1	S ₁	S ₁	S ₁	S ₁
= 2	TS			
= 3	S ₂	TS		
= 4		S ₂	TS	
= 5			S ₂	TS
= 6				S ₂

(b) Optimal Execution of (a)

Figure 36. Optimal Execution Using TEST and TESTSET Instructions

3.4. Loop Modifications for Exit-ifs

In Chapter 2 the DG was modified to reflect the presence of exit-ifs. This modification insured that a loop would only begin executing if the previous loop did not take the exit branch of an IF statement. If the exit branch of an IF statement was taken, processors executing later iterations of the loop will deadlock while waiting for a signal that they can begin their iterations unless a *quit* instruction is executed. The *quit* cannot be immediately executed, however, as this might halt the execution of an iteration previous to the exiting iteration. This section shows how the loop can be modified so that these problems are overcome.

The solution to the above problem requires both adding synchronization instructions and additional FORTRAN statements. To insure all iterations less than the terminating iteration have completed we will place *set(0)* or *testset(0)* at the end of the loop body. All branches within the loop to the CONTINUE statement at the end of the loop will instead branch to the *set* or *testset* instruction. Therefore, each time a loop iteration ends, synchronization register 0 will be set.

To terminate the *dospread* loop the processor exiting the loop needs to execute a *quit* instruction when all previous iterations have finished. This is done by adding a block of code to the end of the loop for each exit-if target in the loop. If an exit-if target is *T*, then at the bottom of the loop the following code sequence is added:

```
T' TEST(0) 1
    QUIT
    GOTO T
```

The *test* instruction makes sure that all iterations previous to this iteration have finished. If they have, the *quit* instruction is executed, which releases all processors from executing this loop, and allows them to continue to execute other code. Finally, the GOTO instruction causes the terminating processor to branch to the target of the exit-if.

To signal the termination of a loop a statement *set(0)* is added immediately before the blocks just added that terminates and exits the loop. To prevent the loop from falling into the just added blocks a GOTO to the CONTINUE statement is placed immediately after the *set(0)* statement. If *L* is the label of the CONTINUE statement, then this appears as:

```
SET(0)
GOTO L
```

Figure 37a shows the program of Figure 15a, and Figure 37b the program with all the changes necessary to synchronize the exit-if.

```

        DOSPREAD I = 1, 4
            IF(A(I).GT.4.0) GOTO 20
            D(I) = A(I) + B(I)
10      CONTINUE
        ...
20      X(I) = Y(I) + Z(I)
```

(a) The Loop of Figure 15a

```

        DOSPREAD I = 1, 4
            TEST(1) 1
            IF(A(I).GT.4.0) GOTO 201
            SET(1)
            D(I) = A(I) + B(I)
            SET(0)
            GOTO 10
201      TEST(0) 1
            QUIT
            GOTO 20
10      CONTINUE
        ...
20      X(I) = Y(I) + Z(I)
```

(b) The Loop of (a) With Synchronization for the Exit-if Added

Figure 37. Exit-if Example: Completely Synchronized Exit-if

Figure 38 summarizes the rules for synchronizing exit-ifs.

3.5. Adding Instructions in Loops with Complex Flow of Control

The previous discussion covered the simple case of adding instructions to loops whose bodies are BASes. In the more general case the loop body is not a BAS, but instead contains arbitrary forward branches within the loop. We now show how to handle this more complex case. A graph is first constructed that represents the flow of control within the loop body. Then, using this graph and the DG constructed previously, synchronization instructions are added to the loop.

```

 $T$  is the set of targets of exit-ifs
 $S_c$  is the CONTINUE statement at the end of the spread loop.
 $S_b$  is a GOTO to  $S_c$ .
 $S_1$  is the first statement in the loop.

if exit-ifs exist then
    place  $S_b$  immediately before  $S_c$ .
    place the statement set(0) (or testset(0)) immediately before  $S_b$ .

foreach exit-if  $E$  do
    add the target of  $E$  to  $T$ .

foreach  $t \in T$  do
    after  $S_b$  place the statements

         $t'$       test(0) 1
                quit
                GOTO  $t$ 

foreach exit-if with target  $t$  do
    change  $t$  to  $t'$ .
  
```

Figure 38. Exit-if Algorithm

3.5.1. The Flow of Control Graph (FCG)

This section gives the algorithms for constructing the two *flow of control graphs* (FCG) needed to add synchronization instructions to the FORTRAN program and for eliminating dependences.

The FCG is represented as a boolean matrix, $reach[i, j]$, where $reach[i, j]$ is true if and only if block i can reach block j without going through any other blocks. A block is similar to a basic block in [AhU177] except that branches to targets outside the loop, nested DO statements, statements whose labels are the targets only of nested DO statements, and statements that terminate execution of the program unit (RETURN, STOP) are ignored. The first three are ignored because they have been dealt with in Chapter 2, and the DG already reflects their presence. The last are ignored because synchronization is moot if the program unit is halted. Figure 39 gives the algorithm for building the FCG for the body of a loop. It is again noted that the FCG must be acyclic since backward branches are not allowed.

The first and last statements in a block are held by *block_limits*. $Block_limits[b].first$ gives the first statement in a block, and $block_limits[b].last$ gives the last statement in a block. The block that a statement is in is contained in *stmt_to_block*. $Stmt_to_block[s]$ gives the block that encloses statement s . Both of these data structures are computed at the same time the FCG is. A second FCG, $reach^*$, is also needed. $Reach^*$ is the transitive closure of $reach$. $Reach^*(i, j)$ is true if block i can reach block j .

A second FCG, built at the statement level, is used in Chapter 4. This graph, represented by the boolean matrix $reach_stmt$, is constructed using the data structures built during the construction of $reach$. Figure 40 gives the algorithm for building this.

3.5.2. Adding Statements using the Flow of Control Graph (FCG)

In this section we use the DG built in the previous chapters, and the FCG built in the previous section to add synchronization statements to a loop. *Set* and *testset* instructions are

first is the first statement in the loop
last is the last statement in the loop
branch(s) = true if statement *s* is a branch not handled in Chapter 2.
label(s) = true if statement *s* is labeled and is the target of a branch

```

/* build block_limits */
b := 1
block_limits[b].first := first
for s := first to last - 1 do
  if branch(s) or label(s + 1) then
    block_limits[b].last := s
    block_limits[b + 1].first := s + 1
    b := b + 1

block_limits[b].last := last

/* build stmt_to_block */
for i := 1 to b do
  for j := block_limits[i].first to block_limits[i].last do
    stmt_to_block[j] := i

/* build reach */
reach := false
for i := 1 to b - 1 do
  if block_limits[i].last is a branch then
    foreach non exit-if target t in the branch do
      reach[i, stmt_to_block[t]] := true
  
```

Figure 39. Algorithm for Building a FCG, Block_limits and Stmt_to_block

```

#blocks = number of blocks in reach
Nf := block_limits(1).first
Nt := block_limits(#blocks).last

/* build a statement level flow graph */
stmt_reach := false
for i := 1 to #blocks do
    /* add arcs from non-branching statements */
    for j := block_limits[i].first to block_limits[i].last - 1 do
        stmt_reach[j, j + 1] := true
    /* add arcs from possible branching statements */
    for j := 1 to #blocks - 1 do
        if reach[i, j] = true then
            stmt_reach[block_limits[i].last, block_limits[j].first] := true

```

Figure 40. Building a Statement Level Control Flow Graph

identical for the purposes of this section. Therefore we will only refer to the *set* instruction.

Both *test* instructions are also equivalent for the purposes of this section.

3.5.2.1. Adding TEST Instructions

Adding *test* instructions is trivial. The *test* instruction is placed immediately before the source of the dependence it is synchronizing. The only flow of control consideration is if the dependence source has a label. If so, the label must be moved to the *test* instruction so that it is not jumped over by the branch. The rules for this are given in Figure 41.

```

foreach synchronization arc  $\alpha$  do
     $\delta_{\alpha_i} := head(\alpha)$ 
    add test( $r_\alpha$ )  $\Delta$  immediately before  $\delta_{\alpha_i}$ 
    if  $\delta_{\alpha_i}$  has label  $l$  then
        remove  $l$  from  $\delta_{\alpha_i}$ 
        add  $l$  to test( $r_\alpha$ )  $\Delta$ 

```

Figure 41. Algorithm for Adding Test Instructions

3.5.2.2. Adding SET Instructions

Adding *set* instructions is more difficult than adding *test* instructions. The reason for this is that when adding a set instruction, deadlock must be prevented. Therefore, it is essential that a *set* be executed for every synchronization register in the loop regardless of the path taken. If it is possible to execute an iteration of the loop without executing a set instruction, then a deadlock will occur when a *test* in a later iteration of the loop waits indefinitely for the *set* to be executed.

Each dependence in a loop is treated in turn. A *set*(*r*) is first placed immediately after the source of the dependence, which is in BAS B_{so} . Each branch in the loop is then examined. If one of its targets is B_{so} , then we must insure that if any of the other branches is taken a *set*(*r*) is also executed. Each target block, B_T is examined. If B_T can reach B_{so} then a *set* cannot be placed in B_T because the *set* may execute before the source executes. If a *set* is placed in all the other B_T then regardless of the path taken either the *set* in B_{so} or a *set* in some B_T will execute. Figure 42 gives the algorithm.

```

foreach dependence  $\delta$  in the DG do
   $B_{so} :=$  the BAS containing  $\delta_{so}$ 
  place set( $r_\delta$ ) after  $\delta_{so}$  in  $B_{so}$ 
  foreach branch in the loop with one target  $T = B_{so}$  do
    foreach target  $T'$  of the branch,  $T' \neq B_{so}$  do
      if reach( $T', B_{so}$ ) = false then
        add set( $r_\delta$ ) to  $T$ 

```

Figure 42. Algorithm for Adding *set* Instructions

CHAPTER 4

DEPENDENCE ARC ELIMINATION AND DEPENDENCE FOLDING

4.1. Introduction

In this chapter we discuss an optimization for reducing the amount of synchronization needed in a parallel loop. Dependence arc elimination is not necessary to produce correct code. If desired it should be done immediately before the code generation algorithms of Chapter 3. This chapter uses the modified DG constructed in Chapter 2, and produces a DG used by Chapter 3.

We first tell how to perform dependence arc elimination in BAS, and using this prove several properties about it. It is then shown how to eliminate dependences within loops which have flow of control branches.

Dependence arc elimination involves determining what dependences can be safely ignored when generating synchronization instructions. Dependence arcs which can be ignored are called *redundant*, and are said to be *covered* by zero or more other dependences in the DG. A dependence arc is redundant if it is forced to be honored by either the covering dependences or a combination of the covering dependences and the control structure of the target architecture.

Dependence arc elimination is important for two reasons. The most important reason is that every instruction that is executed in a loop body uses some finite resource. Consequently, if synchronization instructions are generated needlessly for redundant dependences, resources are consumed needlessly. In particular, since some types of synchronization imply a serialization of the execution of a loop body, this needless synchronization will increase the running time of the loop. Of less practical importance is the restriction placed on the fixed number of dependences that can be synchronized by the number of synchronization registers. Since each dependence in

the DG to be synchronized requires one synchronization register, loops with more dependences than synchronization registers cause problems. It is possible that after dependence arc elimination the number of dependences to be synchronized falls within acceptable limits.

The need to synchronize dependences arises from having to force some data accesses to follow the same order they would on a serial architecture. As some dependences are synchronized, access to data is restricted and becomes closer to a serial order. As data accesses tend more and more towards a serial access, it becomes likely that some dependences that have not yet been synchronized will be forced to be honored by the synchronization of other data dependences along with the control structure of the architecture. These dependences are redundant and should be eliminated. It should be noted that the elimination of a redundant dependence arc does not make a loop less parallel than it would be otherwise.

Other work on this problem has been done by Li [LiZh85]. Our solution differs from theirs in three major ways. First, we separate the architecturally dependent portion of dependence arc elimination from the architecturally independent portion. Second, our algorithm, with the algorithms in Section 3.1, works on code segments with arbitrary flow of control graphs. Third, our algorithms allow several dependences to be used to eliminate another dependence arc.

Later in this chapter, a related subject, *dependence folding*, is covered. Dependence folding involves introducing new dependences to eliminate other dependence arcs. The goal of dependence folding is to reduce the number of synchronization registers needed. Dependence folding can cause loops to be more serial than necessary.

4.2. Definitions and Basic Strategies

A dependence arc with distance Δ gives a constraint on the legal execution of a program segment: Δ iterations of the loop must execute between the source and sink of the dependence. When the dependence has been synchronized, there now exists a *controlled path* from one statement to another across Δ iterations. A controlled path arc, labeled with distance Δ from S_i to

S_j asserts that all dependences from S_i to S_j of distance Δ will be enforced. Thus, while a dependence represents a condition that must be satisfied, a controlled path represents a condition that is satisfied, thereby forcing the dependence to be honored.

When a loop is synchronized two things happen. The first is that synchronization statements are placed in the loop. The second is that dependences in the loop are replaced by controlled paths from synchronization statement to synchronization statement in the loop. The loop can now be represented by a *Controlled Path Graph*, or CPG. Nodes in the CPG will either be *synchronization nodes*, which represent synchronization statements, or *source nodes*, which represent the original program source statements. Arcs, or *controlled path arcs* in the CPG will either be *synchronization arcs*, which represent controlled paths introduced by synchronization statements, or *machine arcs*, which represent controlled paths introduced by the architecture of the target machine. In examples, where CPGs are depicted pictorially, synchronization arcs in the CPG will be shown as solid lines and machine arcs will be shown as dashed lines.

In the algorithms presented in this section, a CPG will be represented as a *connectivity matrix*. For singly nested parallel loops, a connectivity matrix C will be a two dimensional matrix indexed by ordered pairs of the form $\langle i, S \rangle$. $C[\langle i, S \rangle, \langle i', S' \rangle]$ has the value true if there is either a machine or synchronization arc from S in iteration i to S' in iteration i' . The transitive closure of C is C^* . $C^*[\langle i, S \rangle, \langle i', S' \rangle]$ is true if there exists a controlled path from statement S in iteration i to statement S' in iteration i' . As will be seen, C needs to only be large enough to contain the first $\Delta_{max}^L + 1$ iterations of the loop.

As before, Δ_{max}^L stands for the maximum dependence distance in the loop being synchronized.

4.3. Building a CPG in a BAS

Building a CPG is straightforward, and the algorithm for doing so is given in Figure 43. The process can be broken down into three distinct phases. Initially a *synchronization dependence graph*, or *SDG*, is built for the DG by inserting nodes for the synchronization statements that must be added to the loop to synchronize it according to the basic rules of Section 3.1. Dependence arcs are removed from the SDG and replaced with controlled path arcs corresponding to the synchronization arcs that will be added to the final CPG. Figure 44a shows a loop,

```

 $S_f, S_l$  = nodes corresponding to the first and last nodes in the loop
/* Build an SDG */
replicate the DG as the SDG
foreach statement  $S_{so}$  in the SDG do
    foreach dependence  $\delta = (S_{so}, S_{st})$  do
         $\Delta_\delta$  is the distance of  $\delta$ 
        place synchronization instructions on the SDG using the
            appropriate algorithms of Chapter 3
        place an arc from each  $set(r_\delta)$  or
             $testset(r_\delta)$  to the corresponding
             $test(r_\delta)$   $\Delta_\delta$  labeling
            the arc  $\Delta_\delta$ 

/* build the connectivity graph */
 $C := \text{false}$ 

/* add synchronization arcs */
access each node  $n$  of the SDG in lexical order
foreach arc  $\alpha$  out of  $n$  do
     $\Delta_\alpha :=$  label of  $\alpha$ 
     $h :=$  head of  $\alpha$ 
    for  $i := 1$  to  $1 + \Delta_{maz}^L - \Delta_\alpha + 1$  do
         $C[ \langle i, n \rangle, \langle i + \Delta_\alpha, h \rangle ] := \text{true}$ 

/* add machine arcs */
for  $i := 1$  to  $\Delta_{maz}^L + 1$  do
    for  $n := S_f$  to  $S_l - 1$  do
         $C[ \langle i, n \rangle, \langle i, n + 1 \rangle ] := \text{true}$ 

```

Figure 43. Algorithm for Building a CPG

Figure 44b shows the DG for the loop, and Figure 44c shows the SDG for the loop. The first part of the algorithm in Figure 43 gives the details of how this is done. The primary purpose of building the SDG is to be able to determine the location of potential synchronization instructions in the final loop, and to be able to see the positions of controlled path arcs relative to the sources and sinks of all dependences in the loop.

If it is only desired to determine how many dependences are inherently redundant, then the step of the algorithm where a SDG is built for the DG can be skipped. This means that dependence arcs are to be treated as controlled path arcs, and the DG is to be used as the SDG. This would also be a valid SDG for a synchronization instruction set in which the store or fetch of a variable would be done by the synchronization instruction itself. In any case, the dependence arc to be removed should not be placed in the SDG.

```

        DOSPREAD 10 I = 1, N
            A(I) = B(I) + C(I)
            C(I) = A(I - 1) + D(I)
            D(I + 1) = E(I) + C(I - 2)
10      CONTINUE

```

(a) A Loop

(b) The DG for (a)

Figure 44. A SDG Example

(c) The SDG for (a)

DOSPREAD 10 I = 1, N	S_1
A(I) = B(I) + C(I)	TS_1
TESTSET(1)	T_2
TEST(2) 1	S_2
C(I) = A(I - 1) + D(I)	TS_3
TESTSET(3)	S_3
D(I + 1) = E(I) + C(I - 2)	TS_2
TESTSET(2)	
10 CONTINUE	

(d) Loop Corresponding to the SDG of (c)

Figure 44. Continued

Once the SDG has been built it is simple to construct the CPG. First we observe that only iterations 1 to Δ_{max}^L need be included in the CPG. This will be shown later, after we prove the condition necessary to allow a controlled path arc to be eliminated.

Each node in the SDG is examined in turn. Each arc out of a node represents a controlled path, since all dependences were replaced with the synchronization necessary to enforce the dependence. Thus, for each arc in the SDG from S to S' of length Δ , we add an arc from every $\langle i, S \rangle$ to every $\langle i + \Delta, S' \rangle$. This arc is labeled with the number of iterations over which it extends. With arcs going from a *set* to a *test*, or from a *testset* to a *test*, this distance is the dependence distance, and corresponds to the value of Δ which will be appended to the *test* instruction. When going from a *testset* to itself, the value is 1, corresponding to $\Delta = 1$ in the implied *test* instruction. This takes care of all of the synchronization arcs. Figure 45a shows the partially constructed CPG for the SDG of Figure 44c with synchronization arcs added.

The machine arcs are more straightforward. Since the loop is a BAS, S_{i+1} always executes after statement S_i . Since stores and fetches execute serially on a single processor, data from within an iteration for S_{i+1} must be ready when S_i has completed. Therefore a machine arc can be placed from S_i to S_{i+1} . Figure 45b shows the completed CPG.

If our architecture was different, or loop iterations were executed differently, we would have to place machine arcs differently. For example, if the execution of a single iteration was spread across different processors [Veid85] then it would no longer be true that stores and fetches within an iteration occurred in a serial fashion. Stores and fetches for that part of an iteration executing on a single processor would occur in a serial order, but no assumptions could be made about order of stores and fetches in relation to stores and fetches on different processors.

An even more extreme example is given by Zhu and Yew in [ZhYe84]. In their model, store and fetch requests occur in order, but the order in which stores and fetches actually occur depends upon the conflicts encountered in the interconnection network. No assumptions can be made about fetches and stores within a single processor, and therefore no machine arcs can be placed in the CPG.

(a) Partially Constructed CPG for the DG of Figure 44

(b) The Completed DG

Figure 45 A CPG

The difficult part of determining the placement of synchronization and machine arcs for a new architecture is mapping the semantics of the instructions and control structure onto control arcs in the graph. Once this is done, building the CPG automatically is fairly easy. The conservative approach is to leave out arcs when in doubt. This underestimates the constraints placed on the program execution by the architecture and synchronization, and potentially leads to unnecessary synchronization being added. This leads to slower running times, but the running

times are no slower than if dependence arc elimination had not been tried, and the execution of the program is correct. In any case, this sort of analysis is necessary to determine what dependences need synchronizing and how they might be legally synchronized.

4.4. Dependence Arc Elimination in a BAS

To eliminate some synchronization arc α , and the synchronization instructions that form it, it is only necessary to show that there exist at least one path in the CPG from the source to the sink of α , that does not include α . In other words, a controlled path arc can be eliminated if a controlled path not containing it performs the same function. To see if this condition holds requires only that we find an alternate path from the source to the sink of the dependence in the CPG. This can be done very easily by using the transitive closure of the connectivity matrix CPG, C^* .

Having computed C^* , determining whether an alternate path exists from the source to the sink of the dependence arc being eliminated is straightforward. If S_{α} is the source of a dependence in the controlled path arc to be eliminated, $S_{\alpha'}$ is the sink of the same dependence, and Δ_{α} is the distance of the dependence, then an alternate path exists if $C^*[\langle 1, \delta_{S_{\alpha}} \rangle, \langle \Delta_{\alpha} + 1, \delta_{S_{\alpha'}} \rangle] = \text{true}$.

We now prove that finding an alternate controlled path is necessary and sufficient for eliminating a dependence arc.

Lemma 1: The CPG is acyclic.

Proof: All arcs in the SDG are either LBDs or LFDs. All dependence distances attached to an arc will either be < 0 , $= 0$, or > 0 . From this, we get the following four cases:

case 1: An LBD with $\Delta \leq 0$. Arcs of this type cannot be added to the graph as this would require the flow of data from a statement to a lexically prior

statement in either the same or an earlier iteration. Note that if backwards branches were allowed, this would not be true.

case 2: An LFD with $\Delta < 0$. Arcs of this type cannot be added to the graph, as it would require the flow of data from a statement to a lexically forward statement in an earlier iteration.

case 3: An LBD with $\Delta > 0$. Such an arc can only form a cycle if we have at least one arc of the type in case 1 or case 2. Since such arcs cannot appear in the CPG, we cannot have a cycle from arcs of this type.

case 4: An LFD with $\Delta \geq 0$. Such an arc can only form a cycle if we have at least one arc of the type in case 1. Since such arcs cannot be in the CPG, arcs of this type cannot cause a cycle.

Since no other arcs can exist, the CPG must be acyclic.

Lemma 2: All CPG of length Δ constructed using the Algorithm of Figure 43 for some SDG will be isomorphic with respect to one another.

Proof: Assume two graphs have been constructed using the Algorithm. All arcs into and out of a node depend on three things: the structure of the SDG, the distance of the node from the first iteration in the graph and the statement the node represents. Clearly, if a node is γ iterations from the first node in the graph in both graphs, then the nodes coming into and out of the node will be identical. If the first iteration of the first graph is ι , and the first iteration of the second graph is ι' , then we can shift all iterations in both graphs by ι . Therefore the node in both new graphs will be at γ , and will be indistinguishable. A similar argument can be constructed for each node.

Theorem 1: A controlled path arc α can be removed from an CPG if and only if for each dependence δ covered by α there exists a controlled path arc from the source of δ in

iteration 1 of the CPG to the sink of δ in iteration $1 + \Delta_\delta$.

Proof: We first show that the condition is sufficient. Let δ_{so} be the source of some dependence covered by α , and δ_{si} be the sink of some dependence covered by α . Now assume that there is a path over Δ_δ iterations from δ_{so} to δ_{si} . This path consists of arcs, in the order traversed, $A_1, A_2, \dots, A_n$, none of which is α . By definition, no A_i can be traversed until A_{i-1} has been traversed. But A_1 cannot be traversed until δ_{so} has executed, thus A_n cannot execute until δ_{so} has executed. Since δ_{si} cannot execute until after A_n has been traversed, δ_{si} cannot execute until after δ_{so} . Therefore the dependence is satisfied. By Lemma 1, the graph is acyclic. Therefore we need not worry about any iterations less than iteration 1 or greater than $\Delta_\alpha + 1$. By Lemma 2, the iterations can be renamed, thus the dependence is satisfied for all iterations.

The condition must be necessary, for if there is no controlled path from the source to the sink of the dependence, then nothing exists to force the sink to execute after the source.

4.4.1. Dependence Arc Elimination is NP-Hard

We now show that dependence arc elimination in a SDG is NP-Hard. The proof proceeds by first showing that elimination of a dependence arc in an acyclic DG is NP-hard. This result is then used to show that dependence arc elimination in an acyclic SDG is also NP-hard. Both of these are shown by mapping the elimination of a single dependence arc onto the Partition problem. [CaJo79].

The Partition problem can be stated as follows: Let $A = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$, where each α_i is an integer. The problem is solved if the question:

Do there exist two subsets of A , A' and $A - A'$, such that $\sum_{\alpha \in A'} \alpha = \sum_{\alpha \in A - A'} \alpha$

is answered. Figure 46 gives an example of some A for which the answer would be "yes".

We map this problem onto dependence arc elimination in the following manner. Let A be as above. The SDG of Figure 47 is built for A . The lengths of the dependence labeled α_i is α_i . The arc to be eliminated is the one of length $M/2$, where $M = \sum_{\alpha \in A} \alpha$. Next a CPG and CPG* are built for the SDG. If a path exists from $\langle 1, N \rangle$ to $\langle M/2, N_{n+1} \rangle$ then there exists a valid partition of A . A' contains those non-zero arcs used in getting from N to N_{n+1} , and $A - A'$ contains those non-zero arcs not used. Since the total number of iterations that can be covered is M , and we covered $M/2$ of them, the arcs not traversed must also cover $M/2$. Two things should be noted. The first is that taking a zero arc is the same as leaving the corresponding α out of A' . The second is that each arc can only be used once, as required in the Partition problem. This is because the absence of any LBDs in the SDG prevents a path existing from an occurrence of some α_i in some iteration to another occurrence of α_i in a later iteration. Therefore α_i can only be used once.

We now consider the case of cyclic SDGs. Since we never know in advance if a back arc is useful in eliminating a dependence arc, we can make the length of the back arc any value. Figure 47 is modified by adding a back arc of length $(M/2) + 1$. Since we only wish to traverse

$$A = \{2, 3, 5, 6\}$$

$$M = 16$$

(a) The Set A and M for A

$$A' = \{2, 6\}$$

$$A - A' = \{3, 5\}$$

(b) A Partition of A

Figure 46. An Example of a Partitionable Set

Figure 47. A SDG Equivalent A of Figure 46

$M/2$ iterations, the back arc will never be used. Therefore eliminating the dependence arc $M/2$ is the same as before. Thus dependence arc elimination is NP-Hard in general for both cyclic and acyclic SDGs.

4.5. Dependence Arc Elimination with Flow of Control

When the loop body over which dependence arc elimination takes place contains flow of control branches, we must account for this flow of control in eliminating dependence arcs. The problem that arises with flow of control branches is that the controlled paths that can possibly be taken during an iteration of the *dospread* loop can depend on what statements are executed in that path. Consider, for example, the program of Figure 48a. Figure 48b gives the CPG for the loop assuming that the IF statement always takes the true path. Since statements S_3 through S_8 are never taken, there cannot exist controlled paths between those statements. Figure 48c gives the CPG for the loop assuming that the IF statement always takes the false branch. In this case statements S_9 through S_{13} are

never taken. In both cases, however, we can eliminate the dependence.

In Figure 48d we see a CPG for the loop when the true path is taken for the first four iterations, and then the false path is taken for the rest. Note in this case we cannot eliminate the dependence, because from iteration four we cannot get to iteration seven using controlled paths of length two. Thus we see that the existence of a single controlled path through the loop is not sufficient, nor is the existence of controlled path through every flow of control path in the loop. Instead, what must be shown is that there is a controlled path through the loop for every combination of flow of control paths possible when executing the loop for iterations one through Δ_{max}^L . Figure 47 gives an algorithm for determining all the possible flow of control paths through a loop.

	DOSPREAD 30 I = 1, N	
	A(I) = B(I) + C(I)	S_1
	SET(1)	S_2
	IF (A(I).GT.0) GOTO 10	S_3
	TEST(2) 2	S_4
	SET(3)	S_5
	D(I) = A(I) + F(I)	S_6
	F(I+2) = D(I) + A(I)	S_7
	SET(2)	S_8
	GOTO 20	S_9
10	CONTINUE	
	TEST(3) 3	S_{10}
	SET(2)	S_{11}
	D(I) = A(I) + G(I)	S_{12}
	G(I+3) = D(I) + A(I)	S_{13}
	SET(3)	S_{14}
20	CONTINUE	
	TEST(1) 6	S_{15}
	H(I) = A(I-6) + D(I)	S_{16}
30	CONTINUE	

(a) A Synchronized Loop

Figure 48. Problems Arising from Dependence Arc Elimination with Branches.

(b) CPG for (a) when only the True Path is taken.

Figure 48. Continued

(c) CPG for (a) when only the False Path is Taken

Figure 48. Continued

(d) A CPG for (a) when both the True and False Path are taken

Figure 48. Continued

max_paths - maximum number of flow of control paths through a loop
max_length - maximum number of statements in any flow of control path
num_stmts - number of statements in the loop.
paths[*max_paths*] of *paths* - *paths*[*i*] is a list of statements in the *i*'th path
num_paths - number of flow of control paths through the loop body found so far
this_path[*max_length*] of *statements* - path currently being found

find_all_paths(*this_path*, *length*, *S*)
length - length of path currently being found
leaf - true if no statements are executed after *S* in the loop
S - current statement in *reach_s*

```

    length := length + 1
    this_path[length] := S
    leaf := true
    for i := 1 to num_stmts do
        if reachs(S, i) then
            leaf := false
            find_all_paths(this_path, length, i)
    if leaf then
        num_paths := num_paths + 1
        paths[num_paths] := this_path
end find_all_paths

```

```

num_paths := 0
find_all_paths(this_path, 0, S1)

```

Figure 47. Algorithm for Finding all Flow of Control Paths through a Loop Body

When all the possible flow of control paths through the loop have been found, a CPG is built for each combination of flow of control paths over the iteration range being considered for dependence arc elimination. If P is the number of flow of control paths being constructed, then $\Delta_{max}^{L(I')}$ CPGs must be generated and checked. Fortunately, most dependence distances are two or less, so this is not as bad as it might initially seem. Figure 49 gives an algorithm for building a CPG using flow of control information. Note that when building the CPG when flow of control is being considered that only nodes which are executed in a particular iteration are at the head or tail of a controlled path.

```

 $\delta_e$  is the dependence arc being eliminated
SDG, paths, path_list are assumed already built

C := false

/* add synchronization arcs */
access each node n of the SDG in lexical order
for each arc  $\alpha$  out of n do
     $\Delta_\alpha :=$  label of  $\alpha$ 
    h := head of  $\alpha$ 
    for i := k to k +  $\Delta_{max}^L - \Delta_\alpha + 1$  do
        /* i is the iteration of the tail of the synchronization arc */
        /* i sub h is the iteration of the head of the synchronization arc */
         $i_h := i + \Delta_\alpha$ 
        if n in paths[path_list[i]] and  $i_h$  in paths[path_list[i_h]] then
            C[ < i, n >, <  $i_h$ , h > ] := true

/* add machine arcs */
for i := k to k +  $\Delta_{max}^L + 1$  do
    for n :=  $N_l$  to  $N_i$  do
        for t := n + 1 to  $N_i$  do
            if stmt_reach(n, t) and n in paths[path_list[i]]
               and t in paths[path_list[i]] then
                C[ < i, n >, < i, t > ] := true

```

Figure 49. Algorithm for Building a CPG with Flow of Control Information

If the dependence arc is redundant over all possible combinations of flow of control paths, then the arc may be eliminated. The algorithm for dependence arc elimination taking flow of control into account is given in Figure 50. The degenerate case of this algorithm, where no branches exist in the loop body, is the same as the algorithm for dependence arc elimination in a BAS.

4.6. Optimal Dependence Arc Elimination

Consider the loop of Figure 51a, and its associated CPG in Figure 51b. Neither of the dependences in the loop can be eliminated. If the code is rearranged, as in Figure 51c, yielding the CPG of Figure 51d, then one dependence can be eliminated. Therefore the algorithm we

max_paths - maximum number of paths through the loop
paths[*max_paths*] - *paths*[*i*] contains a list of all statements in the *i*'th path
path_list[Δ_{max}^L] - *path_list*[*i*] is the path taken on the *i*'th iteration
num_paths - number of entries in *paths*
redundant - **true** if dependence is redundant, **false** otherwise
reach_s is a statement level reachability matrix
 δ - the dependence arc being checked for redundancy
 δ_{so}, δ_{si} - the source and sink of δ
this_path[Δ_{max}^L] - dummy parameter for *find_all_paths*

check_dep(*path_list*, *iteration*)

C is a CPG, and *C*^{*} is its transitive closure

```

if iteration >  $\Delta_{max}^L$  then
    build a C using the algorithm of Figure 49
    C* := transitive_closure(C)
    if not C[ $\delta_{so}, \delta_{si}$ ] then
        redundant := false /
    else
        for i := 1 to  $\Delta_{max}^L$  do
            path_list[iteration] := i
            check_dep(path_list, iteration + 1)
end check_dep

```

find all flow of control paths using the algorithm of Figure 47

build an SDG by duplicating the DG and adding synchronization nodes

foreach synchronization arc δ in the SDG **do**

redundant := **true**

check_dep(*path_list*, 0)

if *redundant* = **true** **then**

 remove δ and its associated synchronization nodes from the SDG

Figure 50. Dependence Arc Elimination with Flow of Control

present is optimal in the number of dependences only for a given code arrangement, and is not optimal for all semantically correct executions of the loop. The rules for determining the optimal number of dependence arcs given the possibility of code motion are beyond the scope of this paper.

```

      DOSPREAD 10 I = 1, N
      A(I) = B(I) + C(I)      S1
      B(I) = A(I-1) + E(I)   S2
      F(I) = G(I) + H(I)     S3
      P(I) = F(I-1) + Q(I)   S4
10    CONTINUE

```

(a) A Loop

(b) The CFG for (a)

```

      DOSPREAD 10 I = 1, N
      A(I) = B(I) + C(I)      S1
      F(I) = G(I) + H(I)     S3
      P(10 = F(I-1) + Q(I)   S4
      D(I) = A(I-1) + E(I)   S2
10    CONTINUE

```

(c) The Loop of (a) with the Statements Rearranged

(d) The CFG for (c)

Figure 51. Optimal Dependence Arc Elimination Requiring Code Motion

4.7. Dependence Arc Elimination in Nested Parallel Loops

The same techniques used to eliminate dependences with only one level of parallel loops can be used to eliminate dependences with multiple levels of nested loops. It is beyond the scope of this thesis to give details on the placement of synchronization in multiply nested parallel loops. Therefore, we will discuss dependence arc elimination from a SDG which is identical to the DG for the loop.

With unnested parallel loops, CPG is a function $f: [\langle I, S \rangle, \langle I', S' \rangle] \rightarrow \text{boolean}$. With multiply nested parallel loops, the CPG has as its domain $(\langle I_1, I_2, I_3, \dots, I_n, S \rangle, \langle I'_1, I'_2, I'_3, \dots, I'_n, S' \rangle)$, where n is the level of nesting of the parallel loop. In a proof similar to that given for Theorem 1 it can be shown that the CPG must be acyclic; this is a result of dependence distances on the outermost parallel loop having to be ≥ 0 . It is also true that the CPG need only have iterations from 1 to Δ_{max}^L in it for each parallel loop L in the loop nest.

Figure 52a gives a loop nest with two parallel loops. Figure 52b gives the DG for the loop. The first number on each dependence arc is the dependence distance on the outer loop and the second number is the dependence distance on the inner loop. Figure 52c gives a pictorial version of the CPG for the loop.

Again, eliminating a dependence requires finding an alternate controlled path from the source to the sink of the dependence. By following the dependence with a distance of $\langle 1, -1 \rangle$ from I iteration to I iteration, and the zero length machine arcs from statement to statement within a J loop, we can go from the source to the sink of the dependence with a distance of $\langle 2, -2 \rangle$ without traveling on this arc. Therefore it can be eliminated.

4.8. Dependence Folding

It may be that after dependence arc elimination there are still too many dependences to be synchronized with the available number of synchronization registers. If so, it is possible to

```

      DOSPREAD 20 I = 1, N
        DOSPREAD 10 J = 1, N
          E(I,J) = F(I,J) + K(I-1,J+1)  S1
          K(I,J) = H(I,J) + L(I,J)      S2
          M(I,J) = E(I-2,J+2) + P(I,J)  S3
10      CONTINUE
20      CONTINUE

```

(a) A Loop Nest

(b) DG for (a)

Figure 52. Dependence Arc Elimination in Nested Parallel Loops

eliminate other dependences by adding dependences. This process is called *dependence folding*, and the dependences being eliminated are said to be *folded* into the added dependence.

In dependence folding, we replace two or more dependences with one new one. Rules for dependence folding vary from architecture to architecture, and are formulated by considering what types of dependences can be added that will cause the old dependences to be eliminated. We present here some ideas on dependence folding for the two synchronization instruction sets of this thesis.

Since LBDs cannot be eliminated by LFDs with the instructions and architectures discussed in this thesis, folding LBDs will require the addition of extra LBDs. LFDs can only be eliminated by a new LFD under one circumstance.

Figure 52. Continued

If all the LFDs have the same distance (or *testset* instructions are used), and if the dependences overlap, then folding is very easy. Two LFDs overlap if the source of the lexically last precedes the sink of the lexically first LFD. Let δ_1 be the first dependence, δ_2 be the second, and so_1, si_1, so_2, si_2 be their sources and sinks. Also let $so_1 \leq so_2 < si_1 \leq si_2$. A new dependence is added, of length Δ_{δ_1} from so_2 to si_1 , and δ_1 and δ_2 are removed from the DG. Figure 53a shows two LFDs with equal Δ s folded into a single LFD.

If this condition cannot be met, then the LFDs must be eliminated with an LBD.

```

      DOSPREAD 10 I = 1, N
      A(I) = B(I) + C(I)      S1
      B(I) = C(I) + D(I)      S2
      E(I) = A(I-2) + C(I)    S3
      F(I) = B(I-2) + C(I)    S4
10    CONTINUE

```

(a) A Loop

(b) The CPG for (a)

(c) The CPG of (b) after Folding

Figure 53. Folding Two LFDs with Equal Δ s into a single LFD

To fold n LBDs, or LFDs that do not meet the above conditions, into a single LBD, we place an LBD from the last source or sink to the first source or sink. The distance, Δ_{lbd} , attached to the LBD should be the greatest common divisor, or GCD, of all the dependences to be folded. The distance is the GCD since there exists some integer C_i for dependence i being eliminated, such that $C_i \cdot \Delta_{lbd}$ equals the distance of dependence i . Figure 54 shows two LBDs folded into a single LBD. Figure 55 shows two LFDs folded into a single LBD.

```

      DOSPREAD 10 I = 1, N
      A(I) = B(I) + C(I)      S1
      C(I-3) = D(I) + A(I)   S2
      E(I) = B(I) + F(I-2)   S3
      F(I) = E(I) + P(I)     S4
10    CONTINUE

```

(a) A Loop

(b) The CPG for (a)

(c) The CPG of (b) after Folding

Figure 54. Folding Two LBDs into a single LBD

In both cases, the dependences being folded can be removed from the graph.

In practice adjacent pairs of dependences should be folded, since this minimizes the lexical length of the serial portion of the loop. The loop should be timed for all such pairings being folded. Then the pair that gives the lowest time should be picked for folding. As with all compile-time timings, the results will only be approximate. This should be repeated until the number of dependences is equal to the available number of synchronization registers. As a general rule, as many registers as possible should be given to splitting inner loops, if it is not possi-

```

DOSPREAD 10 I = 1, N
    A(I) = B(I) + C(I)      S1
    B(I) = A(I-2) + E(I)    S2
    H(I) = P(I) + Q(I)      S3
    R(I) = H(I-4) + J(I)    S4
10    CONTINUE

```

(a) A Loop

(b) The CPG for (a)

(c) The CPG of (b) after Folding

Figure 55. Folding Two LFDs into a single LBD

CHAPTER 5

CONCLUSIONS

This chapter covers three major topics: meshing the algorithms of the last three chapters into an overall algorithm for synchronization, our conclusions about automatic synchronization, and areas for further study.

5.1. A Global Algorithm for Synchronization

Synchronization of programs proceeds in much the same order that the algorithms have been presented in this thesis. Each loop nest is first examined, and the most useful parallelism extracted. Once this is done, it is known which loops must be synchronized.

Next a DDG is constructed for the *dospread* loop. The loop is examined for exit-ifs, and if any are found the DDG is modified accordingly. Every dependence is then examined, and those that have a source or a sink in a nested loop are dealt with. This will require modifying the DDG, and if loop-splitting is performed, the source program. At this point the DDG is completed.

Redundant dependences can now be removed from the DDG. The DDG is passed on to the flow of control algorithms, which emit synchronization instructions in the proper places. This series of steps is then repeated for each *dospread* loop in the program.

These steps are summarized in Figure 56.

5.2. Conclusions

We have implemented many of the algorithms of this thesis in the SYNCH pass of the Parafrase compiler. The SYNCH pass runs as the last pass in the compiler; previous passes detect parallel loops and perform program transformations that aid in the detection of parallel

Schedule loops using the algorithm of Figure 6, Chapter 1

```

foreach loop  $L$  do
  Build a DDG

   $T := \phi$ 
  foreach statement  $s$  in the loop do
    if  $s$  is a branch with targets  $t$  outside of  $L$  then
      add  $t$  to  $T$ 
  if  $T \neq \phi$  then
    /* exit-ifs are present */
    perform the algorithm of Figure 16, Chapter 2
    perform the algorithm of Figure 43, Chapter 3

    /* take care of dependences within nested loops */
    perform the algorithm of Figure 28, Chapter 2

    /* begin dependence elimination */

    /* build a CPG */
    perform the algorithm of Figure 54, Chapter 4

    /* generate instructions */
    /* build the statement level flow of control graph */
    perform the algorithm of Figure 45, Chapter 3

    foreach synchronization  $\alpha$  in the DDG do
      /* add test instructions */
      perform the algorithm of Figure 46, Chapter 3
      /* add set or testset instructions */
      perform the algorithm of Figure 47, Chapter 3

```

Figure 56. A Global Algorithm for Synchronization

loops. The SYNCH pass generates *test* and *testset* instructions. The algorithms implemented form a subset of the algorithms discussed in this thesis that is sufficient to insure the legal execution of a program on a multiprocessor.

The SYNCH pass does the following to add synchronization: nested loops are handled correctly, although loop splitting is not done at this time; dependence elimination is done; dependence folding is done for overlapping LFDs; and flow of control is handled. The addition of

dependence elimination and loop splitting as discussed in this thesis is planned for the near future. Using eight synchronization registers, the need for general dependence folding has not been shown, and will probably not be implemented.

The running time of the pass is on the order of the running time needed to detect *doacross* loops, i.e. it is fairly long for a standard Parafrase pass, but not excessively so. As well, no effort was made in the writing of the software to optimize the running time of the pass.

Since the algorithms for generating synchronization instructions of the types mentioned in this thesis can be made to run as a small portion of the total execution time of the entire process of automatically detecting parallelism, it is reasonable that these instructions should be generated automatically. Synchronization, like many transformation to detect and control parallelism, essentially involves the analysis of a data dependence graph. Manual data dependence analysis seems to be an operation for which humans are ill-suited. Therefore, even compilers which accept programs in parallel form should have as an option the generation of synchronization for loops marked parallel. Errors resulting from incorrect synchronization can be difficult to detect since they are often not repeatable.

5.3. Areas for Future Research

Several areas for research remain. It is obvious from the algorithms of Chapter 4 that what dependences can be eliminated or folded into other dependences depends strongly on the control structure of the target architecture and the synchronization instruction set of that architecture. The synchronization overhead associated with a loop depends on the number of synchronization instructions in the loop, the execution time for each synchronization instruction to execute, and the degree of inherent parallelism allowed to remain in the loop after synchronization. All three of these factors depend on the synchronization instruction set as well as on the structure of the loop's DDG. The tradeoffs between the complexity of a synchronization instruction set, the execution time of the synchronization instructions, the effects of a synchroni-

zation instruction set on dependence elimination, and the amount of parallelism robbed from the loop by the synchronization instruction set is not well understood.

The topic of synchronizing recurrences is closely related to the amount of parallelism that is allowed to remain in a loop. It is impossible, given the synchronization instruction sets of this thesis, to adequately synchronize any recurrence. Given the more general synchronization of [ZhYe84] recurrences can be synchronized, but the overhead associated with synchronizing a general recurrence is enormous and overwhelms any savings in time gained by running the recurrence in parallel. It is desirable that strategies be developed to recognize special dependence patterns and that optimal synchronization strategies be developed for them.

Finally, the generation of synchronization instructions for multiply nested loops has only been barely touched on in this thesis. Although dependence elimination will not be much more difficult than for a single level of *spread* parallelism, the issues surrounding the placement, generation and types of synchronization instructions for multiple levels of *spread* parallelism are not so clear.

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