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INVESTIGATION OF THE ELASTIC TRANSFER PROCESS

P. J. Griffin
Physics Department, Ohio University, Athens, Ohio 54701 U.S.A.

and

R. D. Koshel
Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

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DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

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A FINITE-RANGE COUPLED REACTION CHANNEL
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P. J. Griffin
Physics Department, Ohio University, Athens, Ohio 45701 U.S.A.

R. D. Koshel
Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.
and
Physics Department, Ohio University, Athens, Ohio 45701 U.S.A.*

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ABSTRACT

Results of calculations for the elastic transfer reaction
 $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ at laboratory energies of 15 and 87 MeV are presented.
The exchange amplitude is calculated using finite range CRC and DWBA
methods. Recoil was included in all calculations. The results of
the calculations indicate that CRC effects are important at the lower
of the two energies while recoil effects are not. At the higher energy
for the reaction studied the results of the calculation indicate that
CRC effects are negligible while recoil effects are important.

KEYWORD ABSTRACT

[Nuclear Reactions $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$, E = 15 and 87 MeV; CRC and DWBA
calculations of exchange elastic scattering.]

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I. INTRODUCTION

The study of the elastic and inelastic transfer process has been suggested to be a powerful tool in the study of nuclear properties, in particular, it is felt that the analysis of such processes could be a good test of proposed nucleus-nucleus potential models.¹ Up to now the analysis of these type of processes have been made by means of two methods; the linear combination of nuclear orbitals (LCNO) method² and a direct one-step treatment³, where the exchange amplitude is obtained from a DWBA calculation. Both of these approaches suffer from deficiencies. The LCNO method has made use of a low energy approximation which is essentially similar to the neglect of recoil effects⁴, while the DWBA approach is a one-step method which neglects higher order contributions to the amplitude. Also, DWBA calculations performed up to now have also neglected some recoil effects.

This paper will deal with the elastic transfer process by means of a finite-range coupled reaction channel (CRC) analysis which includes recoil effects. We will use the computer code PHOEBUS⁵ for this purpose. We will study the elastic scattering of $^{12}\text{C} + ^{13}\text{C}$ at two incident laboratory energies, 15 and 87 MeV, respectively. Thus, we should be able to learn something about recoil and multiple-step effects at two different energies.

In section II, we shall present the method by which we obtain the total elastic amplitude, which is a coherent sum of the direct and exchange amplitudes. The results of calculations are then presented in section III and finally in section IV we present a summary and a discussion of the main results of the paper.

II. DERIVATION OF THE TOTAL SCATTERING AMPLITUDE

Let us consider the general elastic scattering reaction $A(a,a)A$, where $A = a + b$. Pictorially we can represent this reaction as consisting of two terms in the center of mass system, as shown in Fig. 1a. The first term is the direct scattering, where particle a scatters directly from A , which is composed of b and a_1 . We make use of a subscript on a_1 in order to indicate that even though a and a_1 are identical particles we wish to differentiate them from each other in the present discussion. The second term is the exchange term, where we now have a_1 emerging at the same center of mass angle and with the same momentum $\vec{k}_f$ as does a in the direct term. Since a and a_1 are indistinguishable these two processes contribute to the observed cross section. The total scattering amplitude will be a coherent sum of the two individual amplitude. Thus, if we denote $f_d(\theta)$ as the direct amplitude and $f_{ex}(\theta)$ as the exchange amplitude, the total amplitude $f(\theta)$ is given by

$$f(\theta) = f_d(\theta) + P f_{ex}(\theta), \quad (1)$$

where $P = \pm 1$, depending upon the exchange properties of the particles involved. It is obvious on inspection of Fig. 1a. that the quantity P is determined by the statistical properties of the particle a , which is the particle exchanged. Thus, if a is a boson $P = +1$ and if a is a fermion $P = -1$.

In this paper we calculate the two individual amplitudes in a slightly different manner. We directly couple the direct process to the transfer process as shown in Fig. 1b, so that we calculate the direct and transfer amplitudes. Once we have the transfer amplitude $f_{tr}(\theta)$, it is obvious

upon comparison of the exchange and transfer diagrams in Fig. 1, that if we let $\vec{k}'_f = -\vec{k}_f$ in the transfer process, then we have the exchange process, i.e.,

$$f_{\text{ex}}(\theta) = f_{\text{tr}}(\pi - \theta). \quad (2)$$

Thus, our total amplitude is given by

$$f(\theta) = f_d(\theta) + P f_{\text{tr}}(\pi - \theta). \quad (3)$$

The value of P is determined as before. There has been, in the past, some question concerning any additional phase that f_{tr} may have. This is due to the fact that f_d and f_{tr} may be separately calculated. This is not true here, because both amplitudes are calculated from a set of coupled equations, so that the relative phase is given by the calculation and all we need to determine is P .

III. CALCULATIONS

In order to see the effect of multi-step processes and recoil on elastic transfer cross sections calculations were performed for the $^{12}\text{C} + ^{13}\text{C}$ elastic reaction at incident laboratory energies of 15 and 87 MeV. This reaction was chosen because an LCNO² and DWBA³ analyses of the elastic transfer cross section has been performed for the smaller of the two energies and a finite-range DWBA with recoil study⁶ of the transfer cross section at the higher energy has been made. The optical model parameters used in the present study were taken from these previous studies.

The direct scattering processes were calculated using an optical interaction between the incident projectile and the target. The interaction coupling the direct channel and the transfer channel was one which is used in direct nuclear reaction studies. Thus, if we were to calculate a DWBA matrix element for the transfer process, our transition matrix element would be given by

$$T_{fi} \sim \langle \chi_f^{(-)} | V_{ab} | \chi_i^{(+)} \rangle, \quad (4)$$

where $\chi_f^{(-)}$ and $\chi_i^{(+)}$ are, respectively, the distorted wave states for the final and initial states and V_{ab} is the interaction between particles a and b. Thus, if we were to calculate a (p,d) reaction, $V_{ab} = V_{pn}$; the interaction between the incident proton and the neutron picked up from the target nucleus.

The finite-range code PHOEBUS⁵ used in the calculations is based on a convergent iterative process⁷ so that it is also possible to calculate the DWBA elastic transfer process. In this way we can compare CRC and DWBA results in order to determine higher order effects. PHOEBUS also allows one to perform a coherent superposition of the two amplitudes, as given by Eq. (3). For the reaction being studied here, the exchanged particle is

^{12}C so that $P = +1$.

A. The $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ Reaction at $E_L = 15$ MeV.

A Woods-Saxon potential with real and imaginary volume terms was used to describe the $^{12}\text{C} + ^{13}\text{C}$ interaction. Potential (1) of Ref. 3 was used at this energy. The relevant parameters describing this potential are listed in Table I. A spectroscopic factor $S = 0.61^8$ for the neutron in ^{13}C was used in the calculations at both energies. This corresponds nicely with the value $S = 0.6$ obtained in the work of Ref. 3.

The results of our calculations are shown in Fig. 2. The dashed line shows the results of a standard optical model calculation, the dotted line corresponds to a calculation where the transfer amplitude was calculated using the DWBA, and the solid curve to a calculation using a CRC amplitude. It is obvious that the inclusion of an exchange amplitude definitely alters the calculated cross section and that there are differences between the DWBA and CRC calculations.

Since the calculations presented here did take into account recoil effects, the transfer amplitudes have $\ell = 1$ transfer. In order to test the importance of the $\ell = 1$ transfer at this energy, both the DWBA and CRC calculations were repeated without the $\ell = 1$ transfer. The resulting calculations were indistinguishable from those shown in Fig. 2 which included both the $\ell = 0$ and $\ell = 1$ transfers. This does not mean that nonrecoil calculations were performed, as full recoil was included in the calculations which only included the $\ell = 0$ transfer, but only that the $\ell = 1$ transfer was omitted.

If one compares the DWBA calculation shown in Fig. 2 with a similar one in Ref. 3 one finds good agreement between the two calculations except for a minor difference in peak and valley values in the oscillating parts of the calculated cross sections and also a small difference in the

calculated cross sections for angles greater than 150° . In the calculations presented here the separation energy procedure was used wherein we adjusted the strength of a Wood-Saxon potential with a radial parameter of 1.25 fm and a diffuseness of 0.65 fm to give the proper separation energy. A different procedure was used in Ref. 3.

The CRC calculation does show differences from the DWBA calculation, particularly at the larger angles. To see why this occurs we show the results of the transfer calculation in Fig. 3. In this figure the dashed curve is the DWBA calculation and the solid curve the CRC one. One sees a significant difference in the cross section at the forward angles, particularly at 15° . It should be pointed out that here 0° corresponds to 180° in the elastic transfer cross section. The fact that the transfer cross sections are almost identical at 0° but yield different results for the elastic transfer indicates that higher order effects can not only alter the magnitude of calculated amplitudes but can also alter their phase.

B. The $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ Reaction at $E_L = 87$ MeV.

The optical potential used in this calculation was again one with real and imaginary volume terms. The potential parameters used were obtained from Ref. 9 and are given in Table I.

The results of the calculation are shown in Fig. 4. The optical model calculation is again given by the dashed line and disappears at the bottom of the graph by the time a scattering angle of 100° is reached. The solid line depicts the CRC calculation and the dotted line that of the DWBA calculation. There is very little difference between these two calculations.

In these calculations both the $\ell = 0$ and $\ell = 1$ angular momentum transfers

are definitely needed. This was dramatically demonstrated by DeVries⁶ in a full finite-range DWBA calculation with recoil for the reaction $^{13}\text{C}(^{12}\text{C}, ^{13}\text{C})^{12}\text{C}$ at this energy. He showed that if one retained only the $\ell = 0$ transfer one obtained an oscillating angular distribution in the forward direction of his reaction. This forward direction corresponds to our backward direction, so that if we retained why the $\ell = 0$ transfer part we would observe an oscillating angular distribution at large angles.

The exchange contribution to this reaction is quite large. The interference region at this energy approximation lies between the center of mass angles 90° and 110° . The peak in the cross section at an angle slightly larger than 100° is due to a constructive interference between the direct and exchange amplitudes.

IV. SUMMARY AND DISCUSSION OF THE RESULTS

The calculations for the reaction $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ definitely have shown that recoil effects are not important at the lower of the two energies considered but are definitely needed at the high energy. The effect of including higher order terms, i.e., performing a CRC calculation, is definitely more important at the lower energy. A change of the order of seventy percent was obtained at the back angles when these higher order effects were included in the calculation. However, at the higher energy these higher order effects were not significant.

The fact that recoil effects for the CRC results are not as important at the lower of the two energies is not surprising. Similar effects have been found for DWBA calculations which included recoil effects.^{6,10}

It is also apparent that CRC effects should be more important at lower energies. For if we look at a reaction in a classical sense the time the two nuclei overlap each other, the transit time, increases as the energy decreases. Thus, it would be more likely that multiple transfers can occur. However, it is very surprising, even with the shorter transit time, that there was no alteration in the CRC result for the elastic scattering from the optical model calculation. One would expect a change because the transfer cross is relatively large so that one would expect that a strong coupling would induce some change in the incident channel. This effect has been observed in the analysis¹¹ of the $^3\text{He} + ^4\text{He}$ elastic scattering reaction when the neutron transfer channel has been coupled to the direct elastic scattering.

The results presented here indicate that the elastic transfer reaction

may be a sensitive way to test various models for the nucleus-nucleus optical model. However, it is also apparent that further calculations are necessary to more fully understand the physical processes that are occurring. For example what is the effect of the coupling to inelastic states? Also, is it always true that at higher energies that CRC effects do not alter the calculated elastic scattering at forward angles? If this is the case one can then perhaps obtain an optical potential by fitting only at the forward angles. We hope to explore and report on these and other questions of coupling effects in nuclear reactions in future publications.

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Table I. OPTICAL MODEL PARAMETERS

Energy	V_0	W_0	r_0	r_I	a_0	a_I	Ref.
(MeV)	(MeV)	(MeV)	(fm)	(fm)	(fm)	(fm)	
15	17.0	4.0	1.35	1.45	0.49	0.35	3
87	50.0	25.0	0.95	1.18	0.70	0.53	9

FIGURE CAPTIONS

- Fig. 1. (a) The two processes which contribute to the elastic scattering reaction $a + A \rightarrow a + A$, where $A = a + b$;
 (b) The two processes which are included in the CRC calculations.
- Fig. 2. The calculated elastic scattering cross section in ratio to Rutherford for the reaction $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ at $E_L = 15$ MeV. The dashed curve is an optical model calculation; the solid curve is a result where the direct and exchange amplitudes are obtained from a CRC calculation; and the dotted curve is a result where the direct amplitude is the optical model calculation and the exchange amplitude is obtained from a DWBA calculation.
- Fig. 3. The calculated transfer cross section for the reaction $^{13}\text{C}(^{12}\text{C}, ^{13}\text{C})^{12}\text{C}$ at $E_L = 15$ MeV. The dashed curve is the result of a DWBA calculation and the solid curve of a CRC calculation.
- Fig. 4. The calculated elastic scattering cross section in ratio to Rutherford for the reaction $^{13}\text{C}(^{12}\text{C}, ^{12}\text{C})^{13}\text{C}$ at $E_L = 87$ MeV. The key to the curve is the same as in Fig. 2.

THE REACTION $A(a, a)A$

$$A = a + b$$

(a)

(b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4