1615

179
12/18/79

DECEMBER 1979

16. 419
PPPL-1615

UC-20g

TRANSPORT CODE SIMULATIONS OF
LOWER HYBRID HEATING IN TOKAMAKS

BY

J. OGDEN AND S. BERNABEI

PLASMA PHYSICS
LABORATORY

1615

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

Transport Code Simulations of
Lower Hybrid Heating in Tokamaks

J. Oden and S. Bernabei

Princeton University, Plasma Physics Laboratory
Princeton, New Jersey 08544

A simple model of RF heating near the lower hybrid frequency has been added to the BALDUR 1-D tokamak transport code. The code has been used to simulate lower hybrid heating in ATC and PLT. A comparison was made with ATC data, in which the code gave fair agreement. Simulations of PLT were carried out for various input RF power spectra. It was found that the RF waves effectively heated the plasma ions over a certain density range, ($2.5 - 3.5 \cdot 10^{13} \text{ cm}^{-3}$ for proposed experimental parameters), yielding considerably less heating at higher and lower densities. The code predicted a rise in the volume averaged ion temperature of about 1.8 eV per kilowatt of RF input power in the optimum density range.

UNCLASSIFIED

12R

12R

1. INTRODUCTION

There is considerable interest in using RF waves near the lower hybrid frequency to heat tokamak plasmas.^[1-4] The way in which the wave energy is absorbed by the plasma depends upon the plasma density and temperature profiles. The plasma temperature profile in turn depends upon the wave energy deposition profile. In an experiment with significant plasma heating, one expects that the deposition profile and plasma temperature profile evolve together in time in a complicated way. The question immediately arises as how to maximize heating of the central plasma. The effect of heating of plasma transport is also of interest. We have approached this problem by adding a model of lower hybrid heating to the BALDUR 1-D tokamak transport code.^[5]

We have used a simple theoretical model of the wave-plasma interaction. The wave travels radially inward from the plasma edge, losing energy to the electrons via Landau damping until it reaches the linear mode conversion (LMC) point. The wave then couples to an ion electrostatic mode, which is heavily damped via ion Landau or ion cyclotron damping. Thus, the wave loses energy to the electrons until it reaches the LMC point, where it gives up its remaining energy to the ions. Several other effects may be important in lower hybrid heating, including scattering of waves by density fluctuations at the plasma edge, variation of the parallel wave number $k_{\parallel}$ with magnetic field, parametric instabilities, and stochastic heating.

These effects have not been put into the simulation, because the physics is largely unknown. We have chosen a simple model to provide a reference point for experimental results. Comparison between the data and the code should help indicate which of the above effects are important. We have used empirical transport coefficients, which give good agreement with existing data for ohmically heated discharges. The wave energy deposition and transport models are described in detail in Section II.

We have used the code to simulate lower hybrid heating in ATF and PLT. The code gave fair agreement with ATF data. For PLT we have run several cases where most of the energy was deposited in the ions via linear mode conversion and other cases where linear electron Landau damping was the dominant conversion mechanism. The sensitivity of the final state to the initial target plasma and the input spectrum was explored.

II. PHYSICAL MODEL

A. Interaction of the lower hybrid wave with the plasma

We briefly describe the models of wave propagation and absorption which have been included in the code. For derivations of the results stated here the reader is referred to the literature.

1. Wave propagation

It is known that an electromagnetic wave launched from an array of wave guides at the plasma edge can couple efficiently with a lower hybrid wave with frequency $\omega_{UH} \approx \omega_{UH0}$, and a component of k parallel to the magnetic field.¹

Cold plasma theory with kinetic corrections gives the following dispersion relation for electrostatic waves with $\omega_{ci} \ll \omega \ll \omega_{ce}$:

$$(1) \quad \omega^2 = \omega_{LH}^2 \left[1 + \frac{m_i}{m_e} \cos^2 \theta + \frac{3}{2} \left(\frac{k_{\perp} v_i}{\omega} \right)^2 + \frac{3}{8} \frac{\omega^2}{\omega_{ci} \omega_{ce}} \left(\frac{k_{\perp} v_e}{\omega_{ce}} \right)^2 \right]$$

where

$$v_i = \left(\frac{2T_i}{m_i} \right)^{1/2}$$

$$v_e = \left(\frac{2T_e}{m_e} \right)^{1/2}$$

$$\omega_{ce} = \frac{eB}{m_e c}$$

$$\omega_{LH}^2 = \frac{\omega_{pi}^2}{1 + \frac{\omega_{pi}^2}{\omega_{pe}^2}}$$

and θ is the angle between the wave vector $\vec{k}$ and the magnetic field. This dispersion relation can be used to trace the path of a lower hybrid wave into the plasma. It can be shown that the group velocity $v_g = \partial\omega/\partial k$ of the wave is perpendicular to the phase velocity $\vec{v}_p = \omega/k \hat{k}$. Near the plasma edge the density is low and $\omega \gg \omega_{LH}$. From Eq.(1) the angle θ between $\vec{k}$ and $\vec{B}$ is small. The wave vector and phase velocity are closely aligned with the magnetic field, and the group velocity is almost radially inward. As the wave moves into the plasma, θ increases, approaching 90° , when $\omega = \omega_{LH}$. The group velocity swings in direction for almost radially inward to almost along the magnetic field (Fig. 1). [6-9]

This picture is valid only if

$$(2) \quad n_{\perp}^2 = (k_{\perp} c / \omega)^2 \cdot 1 + \omega_{pe}^2 / \omega^2.$$

Otherwise the fast mode branch of the dispersion relation couples with the slow mode (Fig. 2) and there is a maximum density [7,10] which can be reached, which can be considerably less than that of the lower hybrid layer. The accessibility condition Eq. (2) is satisfied by the input power spectra in all of our simulations.

3. Linear Mode Conversion

As the wave approaches the lower hybrid layer, where $\omega = \omega_{UH}$, it can couple to a short wavelength ion wave which is heavily damped. This occurs when the density satisfies the condition [11]

$$(3) \quad \frac{\omega_{pe}^2}{\omega^2} = \frac{1}{1 - \frac{\omega_{pe}^2}{\omega^2} + \frac{v_{Te}^2}{v_{Ti}^2} \left[\frac{T_i}{T_e} + 3 \left(\frac{\omega_{pe}^2}{\omega^2} \right) \right]}. \quad 1/2$$

The density corresponding to this density is the linear mode conversion (LMC) point. Cold plasma theory predicts that a resonance occurs at $\omega = \omega_{UH}$. We see from Eq. (3), that linear mode conversion takes place at lower density than the lower hybrid resonance. Fig. 3 illustrates the density dependence of the perpendicular index of refraction $n_{\perp}$. The LMC radius depends on n_e , T_i , T_e , B , and $n_{\perp}$. The LMC point moves outward (toward lower density) as $n_{\perp} = ck_{\perp}/\omega$ and temperature increase.

3. Electron Landau damping

When the phase velocity of the wave along the magnetic field direction $v_{p\parallel} = \omega/k_{\parallel}$ is near the thermal velocity v_e of the electrons, the wave can lose energy to the electrons via Landau damping. For a wave propagating from the edge ($r = a$) to a point $r = x_0$, the power absorbed by the electrons between $r = a$ and $r = x_0$ is

$$(4) \quad P = A(k_{\parallel}) \left\{ 1 - \exp \left[- \int_{x_0}^a 2 k_x''(x) dx \right] \right\}$$

where

$$(5) \quad k_x'' = \left(\tau \frac{m_i}{m_e} \right)^{1/2} \frac{\omega}{v_e} \left(\frac{v_{p\parallel}}{v_e} \right)^2 \frac{\exp \left(- \left(\frac{v_{p\parallel}}{v_e} \right)^2 \right)}{\left(\frac{\omega^2}{\omega_{IH}^2} - 1 \right)^{1/2}}$$

and $A(k_{\parallel})$ is an amplitude function of $k_{\parallel}$, which depends on the input spectrum. [12]

4. Energy Absorption by the Plasma

A spectrum of waves of frequency ω is launched from an antenna array at the plasma edge. The power amplitude as a function of parallel index of refraction $n_{\parallel} = k_{\parallel}c/\omega$ is calculated from the boundary value theory developed by Brambilla. [13]

We assume that the waves propagate inward, losing energy to the electrons via linear Landau damping, until the linear mode conversion (LMC) point is reached. At this point the remaining wave energy goes to the ions. The LMC point is different for each wavenumber, and the energy transferred to the electrons via Landau damping also depends upon the wavenumber. The spread in wavenumber of the input RF spectrum causes a radial spread in energy deposited in the plasma.

The energy added to the electrons and ions as a function of radius is calculated in the BALDUR 1-D transport code as follows. At the beginning of each timestep, BALDUR supplies density and temperature profiles. The power spectrum of the RF source is input to the code as a series of magnitudes corresponding to the range of parallel wavenumbers excited. Typically, the spectrum is broken up into about 20 modes. Each mode is followed separately as it moves radially into the plasma. In each radial zone, the attenuation of the wave due to electron Landau damping is calculated as a function of n_e , n_i , n , and $T_e(r)$, and that amount of energy is added into a source term in the electron energy equation in BALDUR. This process continues the wave moving inward radially, until the linear mode conversion point is encountered. The energy left in the mode is then added to the ion energy equation. It is assumed that the wave is damped in a small region near the LMC point, so that only small errors are introduced by adding all the wave energy into one radial zone. This process is illustrated in Fig. 4 for an input spectrum of four modes.

If a given mode never encounters its LMC point the calculation is continued until the wave has passed through the plasma once. The calculation of electron Landau damping is then terminated. Each mode is followed in this way and the energy deposited in each radial zone is summed over $n_{\perp}$ to give ion and electron energy source terms. BALDUR uses these electron and ion energy source terms (the total power to electrons or ions curves in Fig. 4) in solving the equations for electron and ion energy as a function of time, producing new temperature profiles, for the next time step.

B. Transport Model

A semi-empirical transport model has been used in our simulations. The values of the transport coefficients were determined by matching the code results with existing data for Ohmically heated discharges on ATC and PLT, in order to set up reasonable target plasmas.

We have assumed that the density diffusion coefficient D_1 and the electron heat conductivity X_e were proportional to $1/(n_e T_e^{\frac{1}{2}})$, where α lies between 0 and 1, depending on the simulation. The ion heat conductivity X_i was assumed to be the neoclassical value. When $q(r) < 1$, we have modelled enhanced transport due to MHD turbulence by increasing D_1 , X_i , and X_e to 1% of the Bohm values. For some ATC and PLT cases, we have included impurities. Any effects of impurities upon wave absorption have been neglected. The impurity transport was assumed to be neoclassical.

IV. SIMULATION RESULTS

A. ATC Simulations

Lower hybrid heating experiments were carried out on ATC.¹¹ Electron heating was observed in the central plasma. Ion heating also occurred. In addition a high energy component of ions may have formed in response to the waves. It was uncertain whether ion heating occurred in the central plasma or near the edge. The ion temperature decayed and the tail disappeared in a small fraction of an energy confinement time after the RF was turned off, suggesting that the heating took place near the plasma edge. Since our model doesn't include the creation of tails, we have concentrated on matching the electron data.

In the experiment about 120 kW of RF power at 340 MHz was injected for 5 milliseconds into a D^+ plasma with central density $n_p(0) = 2 \times 10^{13} \text{ cm}^{-3}$, $Z_{\text{eff}} = 4$, $B_0 = 19 \text{ kG}$ and $T_e(0) = 100 \text{ eV}$. The power input spectrum of the RF is shown in Fig. 5. Experimental electron temperature profiles before and during the RF are plotted vs. radius at the bottom of Fig. 5. The central electron temperature increases by $\sim 150 \text{ eV}$. Figure 6 illustrates the code results for the same input power and approximately the same target plasma. About 24 kW of power is absorbed by the electrons, and $T_e(0)$ increases by $\sim 100 \text{ eV}$. The simulation shows less central heating than the experiment, but a broader temperature profile. The simulation and electron data match fairly well. The ion energy absorption via linear

mode conversion is negligible for this case; therefore, the code predicts no ion heating. Clearly, the codes' repertoire of ion energy absorption mechanisms needs to be larger to explain the ATC results. The energy deposition profile was approximately constant in time, with a broad maximum 5-10 cm from the plasma center.

B. PLT Simulations

We have simulated lower hybrid heating on PLT for two types of target plasmas. In the first set of runs, we have chosen the initial plasma and RF power spectrum so that the waves undergo linear mode conversion in the plasma and are predominantly damped by the ions. In the second set of runs, we have tailored the input spectrum and plasma parameters so that most of the RF energy is absorbed by the electrons via Landau damping. These two spectra can be produced experimentally by proper phasing of the waveguides constituting the array.

1. PLT Ion Heating Cases

Figure 7 shows the input RF power in arbitrary units vs. $n_{||} = ck_{||}/\omega$. The total power injected is 0.5 MW at frequency $f = 8 \times 10^8$ Hz, with pulse duration of 0.1 seconds. The initial D^+ plasma has $Z_{eff} = 1$, $B_T = 30$ kG, and $I = 400$ kA. In Fig. 8, we show a 3-D computer plot of the ion temperature profile $T_i(r)$ as a function of time for initial $n_e(0) = 3.0 \times 10^{13} \text{ cm}^{-3}$. The RF is turned on at $t = 0.4$ seconds and continues until $t = 0.5$ seconds. The maximum in the power deposition profile occurs at the same place as the ion temperature maximum. We see that T_{imax} moves radially outward in time.

This is because the linear mode conversion point for a given mode occurs at a lower density (nearer the plasma edge) when the temperature increases. This effect is also seen in Fig. 9, which is a similar case with a slightly lower central density $n_0(0) = 2.6 \times 10^{13} \text{ cm}^{-3}$. We have summarized the results in Figs. 8 and 9 in Table I. We see that slightly higher ion temperatures are achieved and that the central heating is somewhat better at lower density. The power deposition maximum moves from 10 cm to 18 cm during the run for $n_0(0) = 2.6 \times 10^{13}$ and from 15 cm to 22 cm for $n_0(0) = 3 \times 10^{13}$. In both of these cases the power is deposited within the main plasma throughout the run. However, if we take a case with $n_0(0) = 4 \times 10^{13}$, we find edge heating, with T_{max} occurring at radius $r = 26$ cm, and for $n_0(0) = 2 \times 10^{13}$, most of the power spectrum "misses" the plasma and there is little heating. We find that there is a density range for which ion heating of the central plasma is quite effective.

2. BLT Electron Heating Cases

We have carried out simulations of RF heating in BLT where electron Landau damping is the principal energy absorption mechanism. The target plasma has $I = 450 \text{ kA}$, $B_T = 30 \text{ kG}$, $T_i(0) = 800 \text{ eV}$, $T_e(0) = 150 \text{ eV}$, $n_0(0) = 1.5 \times 10^{13} \text{ cm}^{-3}$, and $Z_{\text{eff}} = 3$, where the impurity is assumed to be carbon. The frequency is 800 MHz and the pulse length 0.2 seconds. Two cases with powers of 0.5 and 1.0 MW were simulated. The power input spectrum is shown in Fig. 10. The

electron heating for the 0.5 MW case is shown in Fig. 11. The electron temperature profile $T_e(r)$ is plotted vs. time. The RF is turned on at 0.1 seconds and off at 0.3 seconds. Time evolution of the central electron temperature and volume averaged electron temperature are shown in Fig. 12. Figures 13 and 14 contain similar computer plots for 1.0 MW of injected RF power.

The results are summarized in Table 2.

V. SUMMARY AND CONCLUSIONS

We have included a simple model of lower hybrid heating in the BALDUR 1-D tokamak transport code, in which wave energy is absorbed by the ions via linear mode conversion and by the electrons via linear Landau damping. A comparison was made between ATC data and the code results. Fair agreement was obtained for electron heating. The code results confirm that linear mode conversion was not the ion heating mechanism in ATC, and that some more sophisticated model is needed to match the data. Simulations of LH were carried out for two types of target plasmas; one in which the input spectrum and plasma density were designed to give mostly ion heating, the other mostly electron heating. Assuming that linear mode conversion is the primary ion absorption mechanism, we find that the ions near the plasma center are effectively heated over a fairly wide range of plasma densities. Electron heating cases for 0.5 and 1.0 MW of power were run. Considerable increase in electron temperature was obtained.

As better experimental data becomes available from the planned lower hybrid experiments on PLT and ALCATOR, the code can be used as a basis for comparison and modified to include more physics as necessary.

ACKNOWLEDGMENTS

We would like to thank W. Booze for helpful discussions and suggestions.

This work was supported by the United States Department of Energy Contract No. EY-76-C-02-3073.

PLASMA PARAMETERS

$$D^+, z_{eff} = 1, I = 400 \text{ kA}, B_T = 30 \text{ kG}$$

RF POWER PARAMETERS

$$f = 800 \text{ MHz}, P = 500 \text{ kW}, \Delta t = 0.1 \text{ sec.}$$

$n_e(t)$ (cm^{-3})	BEFORE RF				AFTER RF				
	T_i (eV)	$T_{i\text{max}}$ (eV)	T_e (eV)	OH (kW)	T_i (eV)	$T_{i\text{max}}$ (eV)	T_e (eV)	OH (kW)	RF (kW)
$2.1 \cdot 10^{13}$	360	722	507	495	363	775	504	484	5
$2.5 \cdot 10^{13}$	369	725	508	498	1136	2661	552	344	462
$3.0 \cdot 10^{13}$	382	728	507	495	1124	2108	565	335	590
$3.0 \cdot 10^{13}$	396	709	506	496	1116	1165	549	367	500

TABLE 1. PLT Low Heating Simulations

PLASMA PARAMETERS

D^+ , $Z_{\text{eff}} = 3$ (Carbon), $I = 450$ kA, $B_T = 30$ kG
 $n_e(0) = 1.6 \cdot 10^{13} \text{ cm}^{-3}$

RF POWER PARAMETERS

$f = 300$ MHz, $P = 0.5$ MW, 1.0 MW, $t = 0.2$ sec.

Power (MW)	BEFORE RF $t = 0.1$ s				AFTER RF $t = 0.1$ s				
	T_e (eV)	$T_{e\text{max}}$ (eV)	T_i (eV)	OH (kW)	T_e (eV)	$T_{e\text{max}}$ (eV)	T_i (eV)	OH (kW)	RF (kW)
0.5	798	1429	342	708	1072	2054	371	411	464
1.0	900	1625	348	592	1638	2510	289	250	994

TABLE 2- PLT Electron Heating Simulations

REFERENCES

- ¹S. Bernabei et al., Third Symposium on Plasma Heating in Toroidal Devices, Varenna, Sept. 6-17 (1976), 68.
- ²S. Bernabei, M. A. Heald, W. M. Hooke, and F. J. Paoloni, Phys. Rev. Lett., 34, 866 (1975).
- ³G. Tonon et al., "Lower Hybrid Frequency Heating in Toroidal Devices with Emphasis on WEGA Results," Third Int'l Conf. on Waves and Instabilities in Plasmas, Palaiseau, France, June 1977.
- ⁴J. Schuss et al., Phys. Rev. Lett., 43, 278 (1979).
- ⁵A. McKenney, Baldur 1-D Tokamak Transport Code Manual, PPPL, (1978).
- ⁶R. Troyon and F. W. Perkins, Proc. Lubbock Conf. on RF Heating, Lubbock, Texas, Paper B4, (1974).
- ⁷T. H. Stix, The Theory of Plasma Waves, McGraw-Hill, New York (1962).
- ⁸R. K. Fisher and R. W. Gould, Phys. Fluids, 14 857 (1971).
- ⁹R. J. Briggs and R. R. Parker, Phys. Rev. Lett. 29 852 (1972).
- ¹⁰V. E. Golant, Sov. Phys. Tech.-JEP, 16, 1980 (1972).
- ¹¹T. H. Stix, Phys. Rev. Lett. 15, 878 (1965).
- ¹²S. Bernabei and I. Fidone, Third Symposium on Plasma Heating in Toroidal Devices, Varenna, Sept. 6-17 (1976) 92.
- ¹³M. Brambilla, Nuclear Fusion, 16, 47, (1976).

Fig. 1. Wave path in the tokamak.

79.2599

792586
Fig. 2. Structure of the dispersion relation for lower hybrid waves when accessibility condition is (a) violated, $n_{\perp}^2 < 1 + \omega_{pe}^2 / \omega_{ce}^2$, (b) satisfied, $n_{\perp}^2 > 1 + \omega_{pe}^2 / \omega_{ce}^2$.

Fig. 3. $n_{\perp}^2$ vs density. Dashed line shows cold plasma result.

Input Spectrum

Power Deposition
One Mode

Summed Over All Modes

792591

Fig. 4. Power deposition model.

792590
Fig. 5. AEC data.

792587
Fig. 6. ATC simulation with same parameters as Fig. 5.

792581
Fig. 7. RF power spectrum for PLT ion heating simulations.

Fig. 8. PLT ion heating simulation. $T_i(r,t)$ vs r and t ,
 $n_e(0) = 3 \times 10^{13} \text{cm}^{-3}$, 0.5 MW RF power.

792582

Fig. 1. IIS ion heating simulation. T_i (keV) vs r and t ,
 $n_0(t) = 2.5 \times 10^{19}$, 0.7 MW RF power.

Fig. 10. RF power spectrum for PLT electron heating simulations. 792583

Fig. 11. PFT electron heating simulation. $T_e(r, t)$ vs. r and t , 0.5 MW RF power.

792592

Fig. 12. PLT electron heating simulation. $T_e(0)$, $T_i(0)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs t , 0.5 MW RF power.

792586

792194
 FIG. 15. IIT electron heating simulation. $T_e(r, t)$ vs r and t , 1.0 MW RF power.

792593
Fig. 14. PLT electron heating simulation. $T_e(t)$, $T_i(t)$, $\langle T_e \rangle$, $\langle T_i \rangle$ vs. t , 1.0 MW RF power.