Energy Imparted, Energy Transferred,
and Net Energy Transferred*

by

F. H. Attix
Departments of Medical Physics
and Human Oncology
Room 1530 MSC
University of Wisconsin
Madison, WI 53706

Abstract

The ICRU-defined non-stochastic quantity absorbed dose is related to the stochastic quantity energy imparted. In the present paper the corresponding stochastic quantities energy transferred and net energy transferred are defined as precursors for kerma and collision kerma, respectively. This forms a rational fundamental framework for radiation dosimetry which facilitates its teaching and understanding. For neutrons collision kerma coincides with kerma, because the heavy secondaries do not lose significant energy by radiative processes (e.g., bremsstrahlung).

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Introduction

The International Commission on Radiation Units and Measurements (ICRU, 1971, 1980) defines the non-stochastic quantity absorbed dose, D, in terms of a stochastic quantity energy imparted, ϵ . The absorbed dose to matter at a point P is given by

$$D = \frac{d\bar{\epsilon}}{dm} \quad (1)$$

where $\bar{\epsilon}$ is the expectation value of ϵ , the energy imparted by ionizing radiation to matter of mass m in a finite volume V , and

$$\epsilon = R_{in} - R_{out} + \Sigma Q \quad (2)$$

in which

R_{in} = the radiant energy incident on the volume V ,

R_{out} = the radiant energy emerging from V ,

and

ΣQ = the net energy derived from rest mass within V (negative for the reverse process).

Radiant energy R is defined as the energy of particles (excluding rest energy) emitted, transferred, or received. (ICRU, 1980)

The point P where the absorbed dose is defined is internal to V ; dm is the mass in the infinitesimal volume dv at P. Defining the specific energy imparted to matter in V as $z = \epsilon/m$, D can be alternatively written (ICRU, 1980) as

$$D = \lim_{m \rightarrow 0} \bar{z} \quad (3)$$

which further clarifies the significance of Eq. (1). As volume V shrinks

to an infinitesimal about P, the expectation value of z becomes the dose at P.

The present author pointed out in earlier papers (1979 a,b) that some useful conceptual clarifications accrue from partitioning the kerma K for photons into two components, according to the fate of the energy involved:

Collision kerma, $K_c = \frac{\mu_{en}}{\mu_{tr}} K$, represents the portion of K that is spent in collision interactions, which contribute to local absorbed dose, while

Radiative kerma, $K_r = \frac{(\mu_{tr} - \mu_{en})}{\mu_{tr}} K$, represents the part of K that is spent in radiative interactions, producing energetic photons at the expense of charged particle kinetic energy.^a

Of course $K = K_c + K_r$. If the energy is transferred to heavy charged particles (e.g., by fast neutrons), $K_r = 0$. μ_{tr} and μ_{en} are the energy transfer coefficient and energy absorption coefficient, respectively (ICRU, 1980).

Since D is defined in terms of a stochastic quantity (ϵ), it is comparably important to define K and K_c in a similar manner. Doing so allows one to show that $D = K_c$ under charged-particle equilibrium (CPE) conditions, even when $K_r \neq 0$ so that $D \neq K$. Equations (12) and (15) of Attix (1979a) defined the quantities energy transferred, ϵ_{tr} , and net energy transferred, ϵ_{tr}^n as stochastic precursors for the quantities K and K_c , respectively. Unfortunately the form chosen for ϵ_{tr}^n in Eq. (15) of that reference does not account for radiative losses by electrons after

^aBremsstrahlung x-rays, and the kinetic-energy-derived part of in-flight positron annihilation.

they have escaped from the volume of interest.^b The present paper corrects that deficiency, while also restating the definitions in terms of the more recently defined radiant energy.

Definitions of Energy Transferred and Net Energy Transferred.

We may define the energy transferred in a specified volume V as

$$\epsilon_{tr} = (R_{in})_u - (R_{out})_u^{non-r} + \Sigma Q \quad (4)$$

where

$(R_{in})_u$ = the radiant energy of uncharged particles entering V,

$(R_{out})_u^{non-r}$ = the radiant energy of uncharged particles leaving V, except that which originated from radiative losses of kinetic energy by charged particles while in V, and

ΣQ was already defined in Eq. (2).

The kerma at point P can now be expressed in the same form as for the absorbed dose:

$$K = \frac{\bar{\epsilon}_{tr}}{dm} \quad (5)$$

where $\bar{\epsilon}_{tr}$ is the expectation value of ϵ_{tr} .

^bMy attention was drawn to this omission through a question asked by Tim W. Holmes, a student in my radiological physics class.

The net energy transferred can likewise be defined for volume V as

$$\epsilon_{tr}^n = (R_{in})_u - (R_{out})_u^{non-r} - R_u^r + \Sigma Q \quad (6)$$

in which R_u^r is the radiant energy emitted as radiative losses by the charged particles which themselves originated in V, regardless of where the radiative loss events occur.

The corresponding definition of K_c then is

$$K_c = \frac{\overline{d\epsilon_{tr}^n}}{dm} \quad (7)$$

To see how these quantities can be applied, consider Fig. 1. Photon $h\nu_1$ enters volume V, undergoing a Compton interaction which produces scattered photon $h\nu_2$ and an electron with kinetic energy T. The electron is assumed to produce one bremsstrahlung x-ray ($h\nu_3$) before leaving V with remaining energy T'. It then produces another x ray, $h\nu_4$. In this example the energy imparted, energy transferred, and net energy transferred in V are, respectively,

$$\begin{aligned} \epsilon &= h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0 \\ \epsilon_{tr} &= h\nu_1 - h\nu_2 + 0 = T \\ \epsilon_{tr}^n &= h\nu_1 - h\nu_2 - (h\nu_3 + h\nu_4) + 0 \\ &= T - (h\nu_3 + h\nu_4) \end{aligned}$$

A second example, shown in Fig. 2, illustrates the significance of the ΣQ term in Eqs. (2), (4), and (6). A γ -ray $h\nu_1$ is emitted by a radioactive atom in V. The photon undergoes pair production, giving kinetic energy T_1 to the electron and T_2 to the positron. Both are assumed to run their course in V; the positron is then annihilated and the resulting photons (0.511 MeV each) are shown escaping from V. For this case the

quantities in Eqs. (2), (4) and (6) are all equal, being given in MeV by:

$$\epsilon = \epsilon_{tr} = \epsilon_{tr}^n = 0 - 1.022 + \Sigma Q$$

$$\text{where } \Sigma Q = h\nu_1 - 2m_0c^2 + 2m_0c^2 = h\nu_1,$$

$$\text{Hence } \epsilon = \epsilon_{tr} = \epsilon_{tr}^n = h\nu_1 - 1.022$$

$$= T_1 + T_2$$

Note that there are no radiative losses here, since the annihilation photons derive their entire energy from rest mass (the $+2m_0c^2$ term), not from kinetic energy.

If the positron had been annihilated in flight when its remaining kinetic energy was T_3 , then the total quantum energy of the annihilation photons would have been $(1.022 + T_3)$ MeV. Assuming they escaped from V, the quantities in question become

$$\epsilon = 0 - (1.022 + T_3) + h\nu_1 = T_1 + T_2 - T_3$$

$$\epsilon_{tr} = 0 - 1.022 + h\nu_1 = T_1 + T_2$$

$$\epsilon_{tr}^n = 0 - 1.022 - T_3 + h\nu_1 = T_1 + T_2 - T_3$$

In this case T_3 , which was derived from charged-particle kinetic energy, constitutes radiative loss. It is only a part of the energy carried out by the annihilation photons.

Charged-Particle and Radiation Equilibrium.

Equation (2) can be expanded to

$$\epsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \Sigma Q \quad (8)$$

in which $(R_{in})_u$ = the radiant energy of uncharged particles
incident on V,

$(R_{in})_c$ = the radiant energy of charged particles
incident on V, and so on.

Let us now define the condition called charged particle equilibrium
(CPE) in the following more or less conventional way:

CPE exists with respect to volume V if each charged particle of
a given type and energy leaving V is replaced by an identical
particle of the same energy entering, in terms of expectation values.

The ICRU (1980) has said (in footnote 13) that

"Charged particle equilibrium exists if the energies, number
and direction of the charged particles are constant throughout the
volume of interest. This is equivalent to saying that the
spectral distribution of charged particle radiance does not vary
within the volume. In particular, it follows that the sums of the
energies (excluding rest energies) of the charged particles
entering and leaving the volume are equal."

This statement is certainly a sufficient condition for CPE, but seems
needlessly restrictive if intended also as a necessary condition. In that
case it would be requiring CPE to exist in every elemental volume within V,
in order to say that CPE existed with respect to V as a whole. This would
represent a departure from past common usage of CPE, for example in
describing the operation of a free-air ionization chamber. For purposes of
the present paper we will assume that the definition of CPE in italics above
is both necessary and sufficient, while the ICRU statement will be taken as
being sufficient but not necessary.

Now returning to Eq. (8), if CPE exists with respect to volume V, the
equation can be simplified to

$$\bar{\epsilon} = \overline{(R_{in})_u} - \overline{(R_{out})_u} + \overline{\Sigma Q} \quad (9)$$

since $\overline{(R_{in})_c} = \overline{(R_{out})_c}$, where the bars indicate expectation values throughout.

Substituting Eq. (9) into Eq. (6) we see that

$$\overline{\epsilon_{tr}^n} = \bar{\epsilon} + \overline{(R_{out})_u} - \overline{(R_{out})_u^{non-r}} - \overline{R_u^r} \quad (10)$$

However, under CPE conditions any radiative interaction by a charged particle after it leaves V will be replaced by an identical interaction inside of V. As a result, and assuming that V is small enough to let the radiative-loss photons escape,

$$\overline{(R_{out})_u} = \overline{(R_{out})_u^{non-r}} + \overline{R_u^r} \quad (11)$$

and we are left with

$$\bar{\epsilon} = \overline{\epsilon_{tr}^n} \quad (12)$$

Making use of Eqs. (1) and (7), one sees that by continuing to require CPE conditions for volume V as it shrinks to dv about point P, one can write

$$D \stackrel{\text{CPE}}{=} K_c \quad (13)$$

where the "CPE" above the equality symbolizes its dependency upon that condition.

Note that Eq. (13) is itself a necessary, but not a sufficient, condition for the existence of CPE at P, since Eq. (13) only requires equilibrium for the energy carried in and out by the charged particles.

It can be seen in the examples discussed in relation to Fig. 2 that $\epsilon = \epsilon_{tr}^n$ even when radiative losses occur, while $\epsilon < \epsilon_{tr}$ in that case. Since no charged particles enter or leave V, CPE can be said to exist in the limit of expectation values, for which $\bar{\epsilon} = \overline{\epsilon_{tr}^n}$ as in Eq. (12). Equation (13) follows.

Radiation equilibrium (RE) can be defined by striking out the word "charged" from the above (italics) definition of CPE. The energy imparted to matter of mass m in a finite volume V under RE conditions is (from Eqs. 2, 6 and 11):

$$\bar{\epsilon} = \overline{\epsilon_{tr}^n} = \overline{\Sigma Q} \quad (14)$$

but, from Eqs. (4) and (11),

$$\begin{aligned} \bar{\epsilon}_{tr} &= (\overline{R_{in}})_u - (\overline{R_{out}})_u + \overline{R_u^r} + \overline{\Sigma Q} \\ &= \overline{R_u^r} + \overline{\Sigma Q} = \overline{R_u^r} + \overline{\epsilon_{tr}^n} \end{aligned} \quad (15)$$

In comparing Eq. (15) with $K = K_r + K_c$ and Eqs. (5) and (7), one sees that K_r is identifiable as

$$K_r = \frac{\overline{dR_u^r}}{dm} \quad (16)$$

Conclusions

The subdivision of kerma into K_c and K_r is important because the relationships

$$K = \Psi \left(\frac{\mu_{tr}}{\rho} \right)$$

and

$$K_c = \Psi \left(\frac{\mu_{en}}{\rho} \right)$$

help clarify the difference between the mass energy transfer coefficient and the mass energy absorption coefficient, in terms of the energy fluence Ψ .

Since ϵ is a stochastic precursor for the absorbed dose, there are benefits to be gained from also defining K and K_c in terms of corresponding

stochastic quantities. The logical underpinnings of radiation dosimetry are thereby strengthened, and its teaching becomes more straightforward and easier to understand.

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Fig. 1. Illustration of the concepts of energy imparted, energy transferred, and net energy transferred, for the case of a Compton interaction followed by bremsstrahlung emission. (See text.)

Fig. 2. Example involving γ -ray emission, pair production, and annihilation.