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NEOCLASSICAL DIFFUSION OF HEAVY IMPURITIES IN A ROTATING
TOKAMAK PLASMA

By

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DIST.

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ROTATING TOKAMAK PLASMA

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ABSTRACT

Particle orbits in a rotating tokamak plasma are calculated from the equation of motion in the frame that rotates with the plasma. It is found that heavy particles in a rotating plasma can drift away from magnetic surfaces significantly faster, resulting in a diffusion coefficient much larger than that for a stationary plasma. Particle simulation is carried out and the results offer a qualitative explanation for some experimental data from the Tokamak Fusion Test Reactor (TFTR).

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Existing theories^{1,2} predict that impurity transport can be altered by an external momentum source, like neutral beam injection. Co-injection produces outward impurity convection and counter injection produces inward convection. Although experimental evidence of such impurity behavior was reported in the ISX-B tokamak experiment,³ impurity injection experiments on the Princeton Large Torus (PLT)⁴ showed no significant difference in impurity transport with co- or counter-beams because the temporal evolutions of impurity line radiation were quite similar in both cases. The only noticeable difference was in the amount of impurity penetrating into the plasma interior. With the same injected amount, the impurity concentration in the core plasma with counter-beams was two to three times higher than that with co-beams. Similar results have been obtained recently from TFTR.⁵ It is found that diffusion plays an important role in impurity transport. Therefore, it is important to calculate the effect of plasma rotation on impurity diffusion. In this paper, we calculate the orbits of impurity ions in a rotating plasma and determine the neoclassical diffusion coefficient D_{neo} . We discover that D_{neo} for heavy impurities can be one to two order of magnitude higher in a rotating plasma than in a stationary one, and our calculations agree qualitatively with some TFTR experimental data.⁵

Let us assume concentric circular magnetic surfaces which rotate as rigid shells about the axis of symmetry with angular frequency $\omega(r)$. We choose a toroidal coordinate system which rotates with the plasma so that the magnetic field lines are stationary. The equation of motion for a charged particle with charge number Z in a rotating frame is:

$$m \frac{d\vec{v}}{dt} = Z e (\vec{E} + \vec{v} \times \vec{B}) - 2 m \vec{\omega} \times \vec{v} - m \vec{\omega} \times [\vec{\omega} \times (\vec{R}_0 + \vec{r})] . \quad (1)$$

R_0 is the plasma major radius and the last two terms on the right hand side of Eq. (1) are the Coriolis force and the centrifugal force. We assume that the bulk plasma consists of electrons and a single ion species of mass m_i and charge number Z_i . We put Z_i equal to the plasma effective ion charge number Z_{eff} and $m_i = 2 Z_{eff} m_p$ where m_p is the proton mass. The impurity ions with mass m_I and charge number Z_I are assumed to be in the "trace limit," such that their presence causes negligible changes in Z_{eff} and other plasma parameters. We know that the magnetic surface is approximately an equipotential surface, so that $\phi = \phi_0(r)$. In a rotating plasma, the ions are pushed towards the large-major-radius side by the centrifugal force. An electrostatic potential ϕ_1 is set up to maintain charge neutrality:⁶

$$\phi_1 = \frac{\omega^2}{2} \frac{m_i}{e} \frac{T_e}{T_i + Z_i T_e} (R^2 - \langle R^2 \rangle), \quad (2)$$

where $\langle \rangle$ denotes averaging over the magnetic surface. It has been shown⁷ that the magnetic moment $\mu = 1/2 m_i v_\perp^2 / B$ and the particle energy are adiabatic invariants in the rotating frame. Let R_1 and R_2 be the maximum and minimum of R on a flux surface. Impurity ions at $R = R_1$ are trapped if

$$\frac{1}{2} m_I [v_\parallel^2 - (\frac{R_1}{R_2} - 1) v_\perp^2] \leq \frac{1}{2} m_I \omega^2 (R_1^2 - R_2^2) + Z_I e [\phi(R_2) - \phi(R_1)] . \quad (3)$$

The boundary between trapped and untrapped particles in velocity space is a hyperbolic surface defined by Eq. (3) with the equal sign. Nearly all the heavy impurity ions are trapped when a plasma is rotating at high velocity like the energetic ion mode plasma in TFTR,⁸ which is characterized by low density and high ion temperature with high co-injecting neutral beam power.

Equation (1) can be solved by using the guiding center approximation.

The guiding center drift velocity is:

$$\vec{v}_1 = \frac{v}{Z_1 e B^2} \left(1 + \frac{2 v_1^2}{v_1^2} \right) \vec{B} \times \nabla \vec{B} - \frac{2 m_I (\vec{\omega} \times \vec{v}) \times \vec{B}}{Z_1 e B^2} - \frac{m_I [\vec{\omega} \times (\vec{\omega} \times (\vec{R}_0 + \vec{r}))]}{Z_1 e B^2} \times \vec{B} + \frac{\vec{E} \times \vec{B}}{B^2}, \quad (4)$$

where $\vec{E} = -\nabla(\phi_0 + \phi_1)$. Guiding center orbits can be obtained by integrating Eq. (4) together with the parallel component of Eq. (1). For typical energetic-ion mode plasmas in TFTR, the banana widths of impurity ion orbits are about a few centimeters, which is only a small fraction of the minor radius (82 cm). Therefore, it is a reasonably good approximation to neglect the variation of $\omega(r)$ along the particle orbit. For a more accurate treatment, variation of $\omega(r)$ can be accounted for by adding a radial electric field which can be absorbed into $\phi_0(r)$. The orbit characteristics are sensitive to the direction of rotation. It offers a qualitative explanation for the observation of higher impurity concentration in rotating plasmas with counter-beams.⁹

The neoclassical diffusion coefficient of the impurity ions can be calculated from the radial component of the drift velocity as follows:

$$D_{\text{neo}} = \lim_{\tau \rightarrow \infty} \frac{(\Delta r)^2}{2\tau} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_0^\tau dt \int_0^\tau dt' W_r(t) W_r(t'). \quad (5)$$

Consider particles doing a random walk¹⁰ along magnetic field lines. If the collisional mean-free-path is much less than the plasma major radius, the impurity distribution function is locally Maxwellian with a temperature T_I , the same as the bulk ion temperature. Equation (5) can then be evaluated to yield

$$D_{\text{neo}} = \frac{1}{4} \left(\frac{m_i}{Z_i e B} \right)^2 \left[\omega^4 R^2 \left(1 - \frac{m_i}{m_I} \frac{Z_i T_e}{T_i + Z_i T_e} \right)^2 + \omega^2 \left(1 - \frac{m_i}{m_I} \frac{Z_i T_e}{T_i + Z_i T_e} \right) \frac{4 T_i}{m_I} + \frac{4 T_i^2}{m_I^2 R^2} \right] \frac{m_i \nu}{T_i} R^2 q^2, \quad (6)$$

where ν is the impurity ion collision frequency and q is the safety factor. In a stationary plasma, $\omega = 0$ and Eq. (6) is reduced to the same result as obtained by Galeev and Sagdeev.¹¹ When $m_i \omega^2 R^2 \gg T_i$, the diffusion coefficient becomes much larger. It is interesting to note that Eq. (6) depends on even powers of ω which means that in the highly collisional regime, the diffusion coefficient is independent of the direction of plasma rotation.

The above result applies only to impurity ions in the Pfirsch-Schlüter regime. In the TFTR energetic-ion mode plasma, impurity ions are collisional only at the edge; they are mildly collisionless near the plasma core. A 3-D particle simulation code¹² is used to calculate the neoclassical diffusion coefficient. The code follows the impurity guiding center positions with drift velocities given by Eq. (4). Pitch angle scattering is modeled by a Monte-Carlo collision operator.¹³ Thermal equilibration between impurity ions and the bulk plasma is also included. At $t = 0$, 1024 impurity particles are distributed uniformly on a flux surface with $T_i = T_e$. The diffusion coefficient is calculated from $D_{\text{neo}} = \langle (\Delta r)^2 \rangle_p / 2t$ where Δr is the excursion of a particle from the original flux surface and $\langle \rangle_p$ denotes averaging over all the particles. In order to compare with experiments in TFTR, we choose germanium as the impurity species. The charge number Z_i is determined by the local electron temperature and the ionization potential $\phi_i(Z)$: $Z_i = Z$ if $\phi_i(Z) < T_e \leq \phi_i(Z + 1)$. Lorentzian plasma parameter profiles are used for computational convenience: $n_e(r) = n_e(0)/(1 + 4 r^2/a^2)$, $T_e(r) = T_e(0)/(1 +$

$9 r^2/a^2$), $T_i(r) = T_i(0)/(1 + 9 r^2/a^2)$, $\omega(r) = \omega(0)/(1 + 4 r^2/a^2)$, $B_T = 40$ kG. We put the zeroth order radial electric field to be $E_r = 400/(1 + 17 r^2/a^2)$ volts/m, and the safety factor to be $q(r) = 1.1(1 + 6.5455 r^2/a^2)$ which corresponds to $I_p = 700$ kA. The central parameters are chosen to be similar to the experimental values: $n_e(0) = 2 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = 4 \text{ keV}$, $T_i(0) = 10 \text{ keV}$, $\omega(0) = 2 \times 10^5 \text{ rad/sec}$. Figure 1 shows the evolution of the spatial distribution of the particle guiding centers. As the particles diffuse from the original flux surface, they tend to move towards the large-major-radius side and become trapped after a few collisions. In a stationary plasma, we find that $\langle \Delta r^2 \rangle_p$ increases linearly with time after many collisions. D_{neo} converges to a fixed value at $t \geq 8 \text{ ms}$ at which the excursion from the original flux surface is still small ($\Delta r \ll a$) for most of the particles, and the solid points in Fig. 2a were taken at 8.2 ms. In the rotating plasma, neoclassical diffusion is much faster so that $\Delta r \sim a/2$ at $t \sim 8 \text{ ms}$, and the values of D_{neo} were taken at $t = 1.6 \text{ ms}$ at which $\Delta r \leq 0.1 a$ for the majority of the particles. It is apparent that neoclassical diffusion is enhanced by one to two orders of magnitude due to plasma rotation. In a stationary plasma, D_{neo} is very small and impurity transport is governed by some anomalous processes. In a rotating plasma, D_{neo} can be large enough to compete with anomalous transport and causes enhanced diffusion in impurity injection experiments. In fact, the values shown in Fig. 2a are close to what is needed ($3 \times 10^4 \text{ cm}^2/\text{sec}$) to explain the experimental data from the TFTR tokamak.⁵ Quantitative comparison with experiment is not possible at this time due to the lack of profile data on $\omega(r)$, $T_i(r)$ and $\phi_0(r)$. As an independent check for the particle simulation results, we assume that all the impurity ions are trapped and then estimate the diffusion coefficient from $D = v_{\text{eff}}^2 \Delta_b^2$; Δ_b is the banana width of a "typical" particle with $v_i = \sqrt{T_i/m_i}$, v_i

$= \sqrt{2} T_i / m_i$ at $\theta = 0$, and $v_{\text{eff}} = 1.27 v_{90^\circ}$, is the collision frequency for $v_{\parallel}$ to change sign. It should be noted that the trapping fraction is significantly less than unity near $r = 0$, and the impurity ions are not in the banana regime near the plasma edge. Therefore, the triangular points in Fig. 2a can only be treated as an order-of-magnitude estimate; it is an overestimate at $r \sim a$, as well as at $r \sim 0$. Figure 2b shows the variation of D_{neo} with the plasma current at two different rotation velocities. The shape of current density profile and other plasma parameters remain the same. D_{neo} is obtained by particle simulation at $r = a/2$. The results show that D_{neo} drops to a low value at high plasma current and low rotation velocity, at which we expect that impurity transport would be dominated by other anomalous processes. This also qualitatively agrees with the observations reported previously.⁵

In summary, we have shown that neoclassical diffusion of heavy impurity ions can be much larger in a rotating plasma because of the enhanced drift velocity. Particle simulation results are in qualitative agreement with some experimental data from TFTR.

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Figure Captions

Fig. 1 Temporal evolution of impurity guiding center distributions in a rotating plasma at (a) $t = 0$, (b) $t = 0.66$ ms, and (c) $t = 3.28$ ms.

Fig. 2 (a) Calculated diffusion coefficients at various radial locations with $\omega(o) = 2 \times 10^5$ rad/sec and $\omega(o) = 0$.
(b) Variation of diffusion coefficient (at $r = a/2$) with plasma current at two rotation velocities.

Fig. 1

Fig. 2

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