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**RAYLEIGH-TAYLOR GROWTH OF BUMPS ON
INERTIAL CONFINEMENT FUSION TARGETS**

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RAYLEIGH-TAYLOR GROWTH OF BUMPS ON INERTIAL CONFINEMENT FUSION TARGETS

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I. INTRODUCTION

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For the successful implosion of an ablatively driven inertial confinement fusion (ICF) target, it is essential that there be good implosion symmetry so that the shell of the target remains intact during the implosion process. This cannot occur if the target suffers from the effects of the Rayleigh-Taylor instability during the accelerative phase of the implosion. Since perturbations on the target's surface either initially present or produced by nonuniform illumination may be amplified in amplitude by the instability, it is important to be able to calculate and predict this growth. For as long as the amplitude of the perturbation remains small as compared to its wavelength, then the instability growth can be described by linear theory. Previous studies¹⁻⁵ have treated this linear instability growth for ICF targets and have produced dispersion relationships which give growth rates as functions of eigenmode wavenumber.

The purpose of this paper is to demonstrate a technique for determining the spatial and temporal evolution of surface imperfections which are composed of a spectrum of wavelengths using results from these linear studies. These surface imperfections, or bumps, may result from target fabrication techniques or may be intentionally inscribed on the target's surface for use in instability experiments. For the purposes of this paper we assume a particular spatial shape for the bump although the technique is applicable to other bump shapes. While the results are only valid during the linear phase of the instability, estimates can be obtained for predicting when the transition to nonlinear behavior might occur.

II. DISCUSSION

We consider one-dimensional small amplitude bumps on a flat surface where the amplitude of the bump at time t and at spatial position x is given by $a(x,t)$. Physically,

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this could approximate the seam produced by joining two hemispherical sections of a target where the width of the seam is much less than the radius of the resulting sphere, and the x direction is perpendicular to the seam with $x=0$ at the center of the bump. We choose the shape of the bump to be Gaussian with an initial amplitude given by

$$a(x, t = 0) = a_0 \exp \left[- \left(\frac{x}{L} \right)^2 \right] \quad (1)$$

where L is a measure of the width of the bump, a_0 is the initial amplitude of the bump's center and t is measured from the onset of the instability. As the instability develops, its effect on the eigenmodes which make-up the spectral content of the bump is determined by the growth rate, $\gamma(k)$. Here k is the wavenumber of the mode and $\gamma(k)$ gives the growth rate, i.e. $\gamma(k) > 0$, or damping rate, i.e. $\gamma(k) < 0$, of the mode. The initial spectral composition of the bump is found by the Fourier decomposition of Eq. (1) which gives

$$g(k, t = 0) = \frac{a_0 L}{\sqrt{2}} \exp \left(- \frac{k^2 L^2}{4} \right). \quad (2)$$

The amplification of these initial modes during the linear phase of the instability is given by

$$g(k, t) = g(k, t = 0) \exp [\gamma(k)t]. \quad (3)$$

The spatial form of the bump is obtained by the Fourier inversion of Eq. (3) which becomes

$$a(x, t) = \frac{a_0 L}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \cos(kx) \exp \left[- \frac{k^2 L^2}{4} + \gamma(k)t \right] dk \quad (4)$$

where Eq. (2) has been substituted for $g(k, t=0)$.

This same analysis can also be applied to two-dimensional point-like bumps. For this case Eq. (1) is replaced by

$$a(r, t = 0) = a_0 \exp \left[- \left(\frac{r}{L} \right)^2 \right] \quad (5)$$

where r is measured from the center of the isotropic Gaussian bump. Physically, this could represent a bump produced by the closure of a fill tube where L is a measure of the

bump's width which is assumed to be much less than the target's radius. Correspondingly the Fourier decomposition gives

$$g(k, t=0) = \frac{a_0 L^2}{2} \exp\left(-\frac{k^2 L^2}{4}\right) \quad (6)$$

and the subsequent inversion is

$$a(r, t) = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} g(k, t=0) \exp[-ikr \cos \theta + \gamma(k)t] k d\theta dk$$

where the inner integral can be evaluated in closed form to give

$$a(r, t) = \frac{a_0 L^2}{2} \int_0^\infty k J_0(kr) \exp\left[-\frac{k^2 L^2}{4} + \gamma(k)t\right] dk. \quad (7)$$

II. RESULTS

Once a form for the growth rate, $\gamma(k)$, is specified the development of the bump is found by the evaluation of Eq. (4) and Eq. (7) for the 1-d and 2-d cases, respectively. We choose three different forms for the growth rate $\gamma(k)$ and discuss the results. The first choice is an idealized form which allows the integral of Eq. (4) to be evaluated analytically. The next choice is the classical RT dispersion relationship⁶, while the last choice is of this same form but includes a stabilizing term due to convection¹⁻⁶.

For the first case we assume a form for the growth rate which allows for the analytic evaluation of Eq. (4). We choose the following idealized dispersion relationship

$$\gamma(k) = \gamma_m \left[1 - \left(\frac{k - k_m}{k_m} \right)^2 \right] \quad (8)$$

where γ_m is the maximum value of γ which occurs at $k=k_m$, the most unstable mode. This form has the desired qualitative features of a more realistic dispersion relationship which includes some sort of stabilizing effect. Evaluation of Eq. (4) gives

$$a(x, t) = \frac{a_0}{\left(1 + \frac{4\gamma_m t}{k_m^2 L^2}\right)^{1/2}} \exp\left[\frac{4\left(\frac{\gamma_m t}{k_m L}\right)^2 - \left(\frac{x}{L}\right)^2}{1 + \frac{4\gamma_m t}{k_m^2 L^2}}\right] \cos\left[\frac{4\gamma_m t\left(\frac{x}{L}\right)}{k_m L \left(1 + \frac{4\gamma_m t}{k_m^2 L^2}\right)}\right] \quad (9a)$$

which at late times, i.e. $\gamma_m t \gg k_m^2 L^2$, is

$$a(x, t \gg k_m^2 L^2 / \gamma_m) = \frac{a_0 k_m L}{2\sqrt{\gamma_m t}} \cos(k_m x) \exp(\gamma_m t) \exp \left[- \left(\frac{x}{L} \right)^2 \left(\frac{k_m^2 L^2}{4\gamma_m t} \right) \right] \quad (9b)$$

This shows the expected result that the eigenmode corresponding to the most unstable mode survives from the initial gaussian bump and exponentially grows with growth rate γ_m , but unexpectedly shows the $t^{-1/2}$ dependence from spectral narrowing near k_m .

We now consider classical RT growth as applied to incompressible fluids. This serves as a limiting case since it includes no stabilizing phenomena or other complexities. The growth rate is $\gamma(k) = (\alpha g k)^{1/2}$ where g and α are the acceleration and Atwood number, respectively. These quantities are assumed constant over the time span of the linear instability growth. The development of the 1-d line bump is found from Eq. (4) to be

$$a(x, t) = \frac{a_0 L}{\sqrt{\pi}} \int_0^\infty \cos(kx) \exp \left[-\frac{k^2 L^2}{4} + (\alpha \gamma k)^{1/2} t \right] dk$$

and for the 2-d point bump from Eq. (7) to be

$$a(r, t) = \frac{a_0 L^2}{2} \int_0^\infty k J_0(kr) \exp \left[-\frac{k^2 L^2}{4} + (\alpha \gamma k)^{1/2} t \right] dk.$$

These expressions are now solved numerically using standard integration techniques.

The scaled spatial profiles for the 1-d line bump are shown at various times in Fig. 1a. The solid line (at the upper left corner of the plot) is the initial bump at $t=0$, while the other lines are at later times separated in time intervals, Δt , of $(\alpha g/L)^{1/2} \Delta t = 1$. (The first nonzero time is the dashed line.) The amplitude of the bump is scaled to its maximum value which occurs at the bump's center, and the distance from the bump's center, x , is scaled to L , i.e. the plot presents $a(x, t)/a(x=0, t)$ vs. x/L . At early times the bump rapidly becomes more narrow, with material redistributing from the outer to the central portions of the bump. A second peak of smaller amplitude begins forming at about $(\alpha g/L)^{1/2} t = 5$. At this time the $\cos(k_m x)$ -like modulation seen in Eq. (9b) is becoming apparent, but with increasing time the wavenumber at which the growth occurs shifts to

larger values of k . This is consistent with the maximum growth rate occurring at $k \rightarrow \infty$, and the second peak moves towards the bump's center corresponding to this increasingly shorter wavelength. The amplitude of the bump's center scaled to its initial value, i.e. $a(x=0,t)/a_0$, is plotted in Fig. 1b as a function of the dimensionless time $(\alpha g/L)^{1/2}t$. The quantity, $\exp[(\alpha g/L)^{1/2}t]$, which gives the amplification of an eigenmode with $k=1/L$ is included on this figure for comparison. At early times this growth rate is essentially the same as that of the bump's center, whereas at later times the bump grows at a faster rate. Similar results for the 2-d point bump are given in Figs. 1c and 1d. The scaled spatial profiles seen in Fig. 1c are somewhat similar to the 1-d profiles but show less modulation. The growth of the 2-d bump's center is plotted in Fig. 1d and shows a greater growth rate than for the 1-d bump.

We next choose the form of $\gamma(k)$ which includes the effects of convective stabilization, i.e. $\gamma(k) = c_1 k^{1/2} - c_2 k$, which can be rewritten in the form

$$\gamma(k) = 4\gamma_m \left[\left(\frac{k}{k_0} \right)^{1/2} - \frac{k}{k_0} \right] \quad (10)$$

where $k_0 = (c_1/c_2)^2$, $\gamma_m = c_1^2/(4c_2)$ (γ_m is the maximum growth rate which occurs at $k=k_0/4$) and k_0 is the value of k such that $\gamma(k=k_0)=0$. With this choice for $\gamma(k)$ the time and spatial dependence of the 1-d bump is found from Eq. (4) to be

$$a(x,t) = \frac{a_0 L}{\sqrt{\pi}} \int_0^\infty \cos(kx) \exp \left\{ -\frac{k^2 L^2}{4} + 4\gamma_m t \left[\left(\frac{k}{k_0} \right)^{1/2} - \frac{k}{k_0} \right] \right\} dk \quad (11a)$$

and that for the 2-d bump from Eq. (7) is

$$a(r,t) = \frac{a_0 L^2}{2} \int_0^\infty k J_0(kr) \exp \left\{ -\frac{k^2 L^2}{4} + 4\gamma_m t \left[\left(\frac{k}{k_0} \right)^{1/2} - \frac{k}{k_0} \right] \right\} dk \quad (11b)$$

These expressions can now be evaluated using standard numerical integration techniques and results are presented for a range of values of $k_0 L$. At late times an asymptotic form for the growth of the bump's center similar to Eq. (9b) for the idealized dispersion relationship can be obtained from Eq. (11a) for the 1-d bump to be

$$a(x=0, t \gg k_0^2 L^2 / \gamma_m) \approx \frac{a_0 k_0 L}{2\sqrt{\gamma_m t}} \exp(\gamma_m t)$$

and from Eq. (11b) for the 2-d bump to be

$$a(r=0, t \gg k_0^2 L^2 / \gamma_m) \approx \frac{a_0 k_0^2 L^2}{16} \sqrt{\frac{\pi}{\gamma_m t}} \exp(\gamma_m t).$$

Figure 2a presents the spatial profiles for the 2-d bump scaled to the maximum amplitude which occurs at $r=0$ as a function of r/L for $k_0 L=1$ at various times. The solid line is the initial bump and the dotted lines show the bump at evenly spaced time intervals of $\gamma_m \Delta t=1$. The bump is seen to spread rapidly at early times with the rate slowing considerably at later times. The time history of the maximum amplitude, $a(r=0, t)$ scaled to its initial value, a_0 , as a function of $\gamma_m t$ is shown in Fig. 2b along with the curve for $\exp(\gamma_m t)$, the growth which would occur for a pure eigenmode with wavenumber corresponding to that of γ_m , i.e. $k/k_0=1/4$. At early times the maximum amplitude of the bump is seen to initially decrease by approximately a factor of $1/2$ and then increase back to its original amplitude at $\gamma_m t \approx 2$. At this same time the full width half maximum, FWHM, of the bump is approximately 3 times greater than its initial value, see Fig. 2a. At later times the bump's maximum amplitude is seen to grow at roughly the same rate as $\exp(\gamma_m t)$. Figure 2c shows the scaled initial spectral content of the bump, $g(k, t=0)/(a_0 L^2)$ and the scaled growth rate γ/γ_m as functions of kL . As seen here eigenmodes with wavenumbers in the range $kL < 1$ are amplified while those modes with $kL > 1$ are damped (i.e. $\gamma < 0$ for $k > k_0$) with damping rates given by Eq. 10. Alternatively we also investigate the case where $\gamma=0$ for $k > k_0$, this cut-off does not allow the damping of any modes as in the previous case. Spatial profiles similar to those presented in Fig. 2a are shown in Fig. 2d for the cut-off case, while the dotted line in Fig. 2b gives the time evolution of the bump's maximum amplitude. Comparison of these figures show that at early times the bump spreads more slowly but at a more uniform rate, while the late time behaviors are identical. Similar results are shown in Figs. 3 for the 1-d line bump with $k_0 L = 1$. Spatial profiles, as seen in Fig. 3a, show similar early time broadening of the bump but with greater displacement of material away from the bump's center than for the 2-d bump. Figure 3b gives the growth

of the bump's scaled amplitude and the maximum possible growth, $\exp(\gamma_m t)$. Again, as in the 2-d case the growth rate asymptotes to approximately that of γ_m but does so at an earlier time. The dashed line which is the growth from considering the cut-off in γ for $k > k_0$ is only different at early times, i.e. for $\gamma_m t < 2$. This is consistent with the growth rate approaching the maximum growth rate at an earlier time.

We now present results similar to those in Figs. 2 and 3 for values of $k_0 L = 10^{-1}$, 10^1 and 10^2 . Spatial profiles and the time history of the growth of the bump's amplitude for the 2-d bump are seen in Figs. 4a and 4b, while the 1-d results are shown in Figs. 4c and 4d for $k_0 L = 10$. For this value of $k_0 L$ the results from the cut-off case are identical to the results shown. The scaled spectral content and scaled growth rate are seen in Fig. 4e. The growth of the maximum amplitude for these cases is approximately the same as that predicted by γ_m , with the 2-d result showing larger amplification. The next case shown in Figs. 5 presents the same information but for $k_0 L = 10^2$. Comparison of these results shows that the spatial profiles are very similar, but the amplitude's amplification is much less than predicted by γ_m . This behavior is explained by comparing Figs. 4e and 5e which show that for $k_0 L = 10$ a greater portion of the spectral content of the bump corresponds to modes with growth rates nearest γ_m than for $k_0 L = 10^2$. Results for the final case are given in Figs. 6 for $k_0 L = 10^{-1}$ with no cut-off for γ . Figures 6a and 6c show that spatial profiles for the 2-d and 1-d cases are quite similar, while Figs. 6b and 6d show that the amplitude amplification is greatest for the 1-d bump. Included on these figures are results (dashed lines) for the γ cut-off case. The behavior for this case is much different than for the no cut-off case where the maximum amplitude first decreases for small times and then ultimately increases. This is consistent with Fig. 6e which shows that the bump is composed mainly of modes with $k > k_0$ which are damped if no cut-off is considered. The spatial profiles for the cut-off case are shown in Figs. 6f and 6g for the point and line bumps, respectively. At sufficiently large $\gamma_m t$ the profiles are very similar to those of Figs. 6a and 6c showing significant spreading, while the early time rate of spreading is not as

rapid.

III. APPLICATION OF RESULTS

An example of how these results may be utilized is now presented. Suppose a point (2-d) bump on an ICF target has an initial amplitude of $1\mu\text{m}$, a characteristic width of $40\mu\text{m}$ (i.e. $a_0 = 1\mu\text{m}$ and $L=40\mu\text{m}$) and is amplified by the Taylor instability. If during the implosion the target is uniformly accelerated such that the product of the Atwood number and acceleration is $ag=40\mu\text{m}/\text{ns}^2$, and if the classical RT theory is used to approximate the growth rate, then the results in Figs. 1c and 1d are used. For example, after 3ns of growth, $(ag/L)^{1/2}t = 3$, the amplification of the bump's center is found from Fig. 1d to be 81.4. Had the amplification been approximated by using the growth rate for a pure eigenmode with wavelength chosen to be twice the FWHM of the bump, then this gives an amplification of only 81.6 after 3ns of growth. The shape of the bump as shown in Fig. 1c is still somewhat gaussian at this time but with a narrower FWHM.

Now results for the same bump experiencing instability growth with the following dispersion relationship, $\gamma = (40)^{1/2}k^{1/2} - (160)^{1/2}k$, which includes stabilizing effects due to ablation are presented. These parameters are chosen so that $k_0 = .25\mu\text{m}^{-1}$, $\gamma_m = .79\text{ns}^{-1}$ and $k_0L=10$. Results for this case are found in Figs. 4. The scaled spatial profiles are at time intervals of 1.26ns in Fig. 4a, while Fig. 4b shows that after 3ns of instability growth the bump's center has been amplified by a factor of 9. If we assume that the linear growth will saturate and become nonlinear say when the bump's amplitude is approximately equal to its characteristic length, i.e. $a(r=0,t) \approx L$, then for this example this occurs after approximately 5ns of growth.

III. CONCLUSIONS

We have demonstrated a method for determining the evolution of surface perturbations on ICF targets due to linear instability growth. For $k_0L < 1$ the growth of the bump is significantly reduced by spectral narrowing from that predicted by γ_m , and considerable

bump spreading is seen at late times. For $k_0 L = 10$ the bump's growth is approximately the same as that of γ_m , and the bump's shape is not significantly changed. For larger values of $k_0 L$, the growth of the bump is reduced from that of γ_m and the bump becomes more narrow at later times. For all values of $k_0 L$ the effect of the cut off, i.e. $\gamma = 0$ for $k > k_0$, has little effect for sufficiently large $\gamma_m t$.

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APPENDIX I

The evaluation of Eq. (4) using the assumed form for $\gamma(k)$ given by Eq. (8) proceeds as follows:

In order that the growth rate be symmetric in k , then Eq. (3) is replaced by

$$\gamma(k, t) = \frac{g(k, t \approx 0)}{2} [\exp(\gamma_+ t) + \exp(\gamma_- t)]$$

where

$$\gamma_+ = \gamma_m \left[1 - \left(\frac{k - k_m}{k_m} \right)^2 \right]$$

and

$$\gamma_- = \gamma_m \left[1 - \left(\frac{k + k_m}{k_m} \right)^2 \right]$$

and Eq. (4) becomes

$$a(x, t) = \frac{a_0 L}{4\sqrt{\pi}} \left[\int_{-\infty}^{+\infty} \cos(kx) \exp \left\{ -\frac{k^2 L^2}{4} + \gamma_m t \left[1 - \left(\frac{k - k_m}{k_m} \right)^2 \right] \right\} dk + \int_{-\infty}^{+\infty} \cos(kx) \exp \left\{ -\frac{k^2 L^2}{4} + \gamma_m t \left[1 - \left(\frac{k + k_m}{k_m} \right)^2 \right] \right\} dk \right]$$

Now these integrals can be combined by replacing k with $-k$ in the first integral to obtain

$$a(x, t) = \frac{a_0 L}{2\sqrt{\pi}} \exp \left(\frac{b^2}{4a} \right) \int_{-\infty}^{+\infty} \cos(kx) \exp \left[- \left(a^{1/2} k + \frac{b}{2a^{1/2}} \right)^2 \right] dk$$

where

$$a = \frac{L^2}{4} \left(1 + \frac{4\gamma_m t}{L^2 k_m^2} \right)$$

and

$$b = \frac{2\gamma_m t}{k_m}$$

Now the variable substitution $y = a^{1/2} k + b/(2a^{1/2})$ reduces the integral to

$$a(x, t) = \frac{a_0 L}{(\pi a)^{1/2}} \exp \left(\frac{b^2}{4a} \right) \cos \left(\frac{bx}{2a} \right) \int_0^{\infty} \exp(-y^2) \cos \left(\frac{yx}{a^{1/2}} \right) dy$$

which can be evaluated in closed form to give

$$a(x,t) = \frac{a_0 L}{2a^{1/2}} \exp\left(\frac{b^2 - x^2}{4a}\right) \cos\left(\frac{bx}{2a}\right).$$

Now substituting for a and b ultimately gives

$$a(x,t) = \frac{a_0}{\left(1 + \frac{4\gamma_m t}{L^2 k_m^2}\right)^{1/2}} \exp\left[\frac{4\left(\frac{\gamma_m t}{k_m L}\right)^2 - \left(\frac{x}{L}\right)^2}{\frac{4\gamma_m t}{(k_m L)^2} + 1}\right] \cos\left[\frac{4\gamma_m t\left(\frac{x}{L}\right)}{k_m L \left(1 + \frac{4\gamma_m t}{L^2 k_m^2}\right)}\right]$$

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1d

Fig. 2a

Fig. 2b

Fig. 2c

Fig. 2d

Fig. 3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 4c

Fig. 4d

Fig. 4a

Fig. 5a

Fig. 5b

Fig. 5c

Fig. 5d

Fig. 5a

Fig. 6a

Fig. 6b

Fig. 6c

Fig. 6d

Fig. 6a

Fig. 6f

Fig. 6g