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BY

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**PLASMA PHYSICS
LABORATORY**

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Energy Confinement Comparison of Ohmically
Heated Stellarators to Tokamaks

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An empirical scaling prescribes that the energy confinement time in ohmically heated stellarators and tokamaks is proportional to the internal energy of the plasma and the minor radius, and inversely proportional to the current density. A thermal-conduction energy transport model, based on a heuristic assumption that the effective momentum transfer in the radial direction is proportional to the classical parallel momentum transfer which results in ohmic heating, is used to explain this scaling.

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In the late 1960's, ohmically heated Model C Stellarator discharges [1] persistently yielded electron temperatures ~ 100 eV and particle confinement times ~ 1 msec, lower than the then results [2] of T-3 Tokamak: $T_e \sim 1$ keV and $\tau_E \sim 5$ msec. The apparently superior tokamak results, especially the high electron temperature, led to the strong research effort in tokamaks in the current decade. Recently, ohmically heated stellarator discharges [3-7] have also yielded significant plasma parameters. However, a useful comparison of the stellarator results with those of tokamaks is difficult, since the electron energy transport, while poorly understood in tokamaks, is even less clear in stellarators because of the additional variable of external rotational transforms. The present work first proposes an empirical scaling for energy confinement in ohmically heated tokamaks. The stellarator results are then compared to those of tokamaks using the same scaling. Based on this comparison we observe that ohmically heated stellarator plasmas follow qualitatively the same scaling for tokamaks. This scaling proposes that in ohmically heated toroidal discharges the energy confinement time is proportional to the plasma internal energy and minor radius and inversely proportional to the ohmic heating current density. To explain this scaling, we offer an energy-transport model, based on a heuristic assumption that the anomalous cross-

field momentum transfer is proportional to the classical parallel momentum transfer which results in ohmic heating, and we relate further this momentum transfer to the driving voltage and current of the ohmic heating.

When an ohmically heated tokamak discharge is near a steady state, its energy confinement time is given by

$$\tau_E = (3/2) (1+\alpha) \langle T_e \rangle \langle n_e \rangle V/V_R I_t, \quad (1)$$

where $\langle T_e \rangle \equiv \int n_e T_e dV / \int n_e dV$, $\langle n_e \rangle \equiv \int n_e dV / V$, $\alpha \equiv \int n_i T_i dV / \int n_e T_e dV$, V is the total plasma volume, and $(3/2) (1+\alpha) \langle T_e \rangle \langle n_e \rangle$ is the internal energy of the plasma. The resistive loop voltage $V_R = 2\pi R E(R) \approx 2\pi R_0 E$ where R_0 is the major radius. The total plasma current can be written in terms of the safety factor q_a and device parameters:

$$I_t = 2\pi a^2 B / \mu_0 q_a R_0 \quad (2)$$

where B is the toroidal field strength at R_0 . With the aid of Eq. (2), Eq. (1) can be written as

$$\tau_E = (3\mu_0/4) (1+\alpha) \langle T_e \rangle \langle n_e \rangle q_a R_0 / BE. \quad (3)$$

The electric field E is still to be identified with device and/or plasma parameters. We note that discharges in contemporary tokamaks usually result in $q_0 \approx 1$, and $q_a \approx 4$.

In Fig. 1 we plot $\tau_E / (1+\alpha) \langle T_e \rangle$ vs. $\langle n_e \rangle q_a$ from recent PLT [8] and Alcator A [9,10] (Fig. 1a) and ISX [11,12] (Fig. 1b) experiments.

(The unit of density scale in the abscissa of Fig. 1a is 10^{13} cm^{-3} for PLT data, and 10^{14} cm^{-3} for the Alcator A data) The parameter range of these experiments is given in Table 1. The PLT results in Fig. 1a are limited to gettered experiments, including both carbon and stainless steel limiters, and all Z_{eff} . (For deuterium plasma, the majority of the experimental points near the origin of the plot have high Z_{eff} .) The pattern of the data points appears adequately linear and can be extrapolated to the origin.

We find that the results from these different devices follow a single empirical scaling when the electric field strength E in Eq. (3) is made inversely proportional to the minor radius of the device, i.e.,

$$E = c_1/a, \quad (4)$$

where c_1 is a dimensional constant. The empirical scaling is then

$$\tau_E = (3\mu_0/4c_1) (1+\alpha) \langle T_e \rangle \langle n_e \rangle q_a a R_0 / B. \quad (5)$$

This scaling is shown in Fig. 2, in which we plot the data in Fig. 1 in the form of $\tau_E / (1+\alpha) \langle T_e \rangle$ vs. $\langle n_e \rangle q_a a R_0 / B$ on log-log scales.

For stellarator plasmas heated solely by ohmic-heating current, the total rotational transform ι_a at the boundary is the sum of the vacuum transform ι_{a0} and the transform due to

Table 1: Parameter Range of PLT, Alcator A,
and ISX A Experiments

Device	R_0/a (m/m)	$n_e(0)$ $(10^{13} \text{ cm}^{-3})$	$\langle n_e \rangle$ $(10^{13} \text{ cm}^{-3})$	$T_e(0)$ (keV)	$\langle T_e \rangle$ (keV)	B, Tesla	Z_{eff}	q_a	Plasma ion species
PLT	1.34/ .4	.81- 9.2	.62- 6.9	.54- 3.3	.22- 1.5	-3.2	-1- 8.4	3.5- 6.2	deuterium, hydrogen
Alcator A	.54/ .1	20- 100	3.3- 31	.7- 1.5	.37- .58	6.0	-1- 2.3	3.5- 4.5	deuterium
ISX A	~.93/~.24	1.5- 7.5	.54- 3.7	.53- 1.3	.24- .56	.81- 1.5	1.2- 5.6	2.6- 4.5	deuterium, hydrogen

ohmic-heating current. The total plasma current in a stellarator corresponding to Eq. (6) is $I_t = 2\pi\bar{a}^2 B(\tau_a - \tau_{a0}) / \mu_0 R_0$ where $\bar{a}$ is the radius of an equivalent circle of the outermost closed magnetic surface. Figure 3 shows Wendelstein VII-A [3] and Cleo [6] stellarator results, plotted in the form of $\tau_E / (1+\alpha) \langle T_e \rangle$ vs $\langle n_e \rangle \bar{a} R_0 / (\tau_a - \tau_{a0}) B$. The line is an "average" line in Fig. 2, $\tau_E = 25(1+\alpha) \langle T_e \rangle \langle n_e \rangle q_a \bar{a} R_0 / B$ msec. Based on this comparison, we observe that, qualitatively, the energy confinement in ohmically heated stellarators is comparable to that in tokamaks. The scaling prescribes that in toroidal discharges heated by ohmic-heating current, the energy confinement time is proportional to the internal energy and the minor radius, and inversely proportional to the current density. According to this scaling, the input ohmic-heating power limit is determined by the device parameters (a, R_0, B) and the safety factor q_a . In the following we will offer a thermal-conduction energy transport model as a plausible explanation of this scaling.

If the effective cross-field energy transport is due to the destruction of magnetic surfaces [13,14] by turbulence so that magnetic lines wander about stochastically, the resultant time-averaged, momentum transfer in the radial direction can be expected to be related to the classical parallel momentum transfer which results in ohmic heating. Specifically, the fluctuating component of a magnetic line, on the average, would have a radial component. The time-averaged momentum transfer in the radial direction is then proportional to the classical parallel momentum transfer. Accordingly, the effective radial thermal conductivity is inversely proportional to the classical parallel resistivity, multiplied by a function of T_e which characterizes the step

size and the energy content of the anomalous diffusion process. The parallel resistivity can be further related to the driving ohmic-heating voltage and current. Thus,

$$\chi_{\perp}(r) = \frac{1}{n_e} f(T_e) \approx \frac{1}{E} \frac{I(r)}{\pi r^2} f(T_e), \quad (6)$$

where $I(r) = 2\pi \int_0^r j(r) r dr$, $j(r)$ is the current density, E is the electric field assumed to be constant across the cross section. As we shall describe shortly, the precise dependence of $f(T_e)$ on T_e does not affect the basic feature of the scaling: different functional dependences result in different temperature and current density profile shapes, and, therefore, different numerical constants in the scaling.

The steady-state radial-direction thermal conduction equation is

$$Ej = - \frac{1}{r} \frac{d}{dr} (r \chi_{\perp} \frac{dT_e}{dr}). \quad (7)$$

Integrating Eq. (7) once and making use of Eq. (6), we obtain

$$2 f(T_e) \frac{dT_e}{dr} = - E^2 r. \quad (8)$$

Equation (8) can be integrated once more to give the temperature profile if $f(T_e)$ is known. For $f(T_e) \propto 1/T_e$, the profile is gaussian, and for $f(T_e) = \text{const}$, the profile is parabolic.

For $f(T_e) \propto T_e^{-1/2}$, the profile is square of a parabola.

The effective diffusion coefficient in this case is proportional to the drift parameter,

$$D \propto j/n_e T_e^{1/2} \propto v_{\text{drift}}/v_{\text{thermal}}, \quad (9)$$

and the corresponding current distribution requires $q_a = 4q_0$. Regardless of the true dependence of f on T_e , the boundary conditions of temperature (for non-gaussian profiles $T_0 = 0$ at $r = a$

or at certain effective radius) requires the electric field to be inversely proportional to the minor radius a , i.e.,

$$E = \frac{c_1}{a}, \quad (4)$$

where c_1 is related to the dimensional constant factor in $f(T_e)$. We note that E in Eq. (4) should be regarded as an average field since it has been assumed to be a constant: the substitution of $n_w = E/(1/\pi r^2)$ rather than $n_w = E/j$ in Eq. (6) allows a direct integration of Eq. (7). The scaling so obtained is therefore a global average. The results as expressed in Eq. (4) can also be obtained by simply examining the scaling of the various quantities in Eq. (7), i.e., $r \sim a$, $j \sim I_t/a^2$, and $n_w \sim Ea^2/I_t$.

Several remarks on the scaling can now be made. 1) The fact that high- Z_{eff} data also follow the same pattern suggests that in these experiments the impurity ions in the hot interior might be nearly fully stripped and therefore affect the scaling only indirectly through higher $\langle T_e \rangle$, though undoubtedly radiation loss affects the total loss. The inclusion of non-gettered experimental results, however, would have produced more irregular data pattern and lower confinement time in general. Presumably in these cases, radiation from non-fully stripped impurities in both the interior and the edge may have contributed more substantially to the energy loss and its level becomes more irregular based on the present scaling. 2) Since we are examining a regime in which the electron-energy loss is dominant, the saturation of energy confinement time at high densities [11, 12, 9] due to neoclassical ion energy transport cannot be expected to emerge from the present scaling. 3) The absence of major

disruption in stellarator discharges allows an internal, finite, $q(r) = 1$ region to exist, inside of which density and temperature profiles are quite flat. This may result in a somewhat higher internal energy content and confinement time [15]. 4) In the Wendelstein VII-A experiments, saturation of energy confinement time at high densities has also been observed, and this saturation has been correlated with enhanced $m = 2$ mode activity concomitant with the more intense impurity influx. 5) The usual scaling of $\tau_E \propto n_c$ is consistent with Eq. (5) except it does not take into account of the profile effect, and the usual $\tau_E \propto a^2$ scaling also agrees with the present scaling if we rewrite aR_0 in Eq. (5) in the form of $a^2(\frac{R_0}{a})$. 6) τ_E is not necessarily proportional to $1/\chi_\perp$, as it is often assumed. 7) Relating the cross-field momentum transfer to the driving voltage and current is tantamount to relating the current-driven modes as the responsible mechanism for cross-field energy transfer.

Several useful features concerning ohmically heated plasmas in a torus are revealed by the present scaling.

1. Loop Voltage. Equation (4) states that the resistive loop voltage is a function of the aspect ratio only and is proportional to (R_0/a) . An "average" discharge in Fig. 2 corresponds to $V_R \approx 0.37 (R_0/a)$ V. For PLT, Alcator A, and ISX A devices, $V_R = 1.2, 2,$ and 1.3 V, respectively. As density increases, the loop voltage should increase slightly to sustain the increasingly higher ion losses. We note that a scaling of $V_R \propto (R_0/a)^2$ was proposed by Coppi and Mazzucato [16], and $V_R \propto (R_0/a)$ was proposed by Coppi [17].

2. Maximum Temperature Attainable. Since electron heating follows Ohm's law, the peak electron temperature at the center is given by

$$T_{e0} \approx 4.52 \times 10^{-2} \left(\frac{Z_{eff} \ln \Lambda}{\gamma_E q_0} \frac{B}{V_R} \right)^{2/3} \text{ keV.} \quad (8)$$

Since $V_R \propto (R_0/a)$, the peak electron temperature is proportional to $(\frac{aB}{R_0})^{2/3}$. In a given device, the electron temperature is therefore essentially constant (except for the monotonic decrease with increasing density because of increasing coupling to ions). The electron temperature in a torus with a larger minor radius is the same as that with a smaller minor radius unless the aspect ratio is reduced or the toroidal field is increased. The "best" discharge condition (upper bound) according to Fig. 2 corresponds to $V_R \approx 0.27 (R_0/a) V$. For $Z_{eff} = 1$ (and $\gamma_E = 0.58$), $\ln \Lambda \approx 16$, and $q_0 \approx 0.5$, the highest attainable electron temperature is $\max T_{e0} \approx 1.5 (\frac{aB}{R_0})^{2/3} \text{ keV}$. If ohmic-heating current alone is used for heating a plasma to $T_{e0} = 10 \text{ keV}$, at $R_0/a = 3$ the required magnetic field strength would be ~ 50 Teslas. We note that, based on the present scaling, the usual "Alcator scaling" $\tau_E \propto n_e a^2$ has an apparent validity. In clean discharges ($Z_{eff} = 1$), $\tau_E \propto \langle n_e \rangle a^2 [q_a (R_0/aB)^{1/3}]$ with the term inside the bracket having only very limited variations in past and current tokamaks. The Alcator scaling, therefore, reflects the energy confinement limit as imposed by the limit of electron temperature due to ohmic heating, and the scaling would result in an underestimating of the energy confinement time in future larger devices if an increasing fraction of the electron temperature is sustained by nonohmic heating means. The energy confinement limit in a given toroidal device by continuously increasing the electron temperature (with nonohmic heating means) is yet to be established.

3. Effects of Disruption. The first effect of disruption is setting an upper limit of energy confinement time in ohmically heated discharges. This is because (1) plasma disruption sets an upper operation limit of density [18] which is proportional to the current density, i.e., $\max(n_e) q_a R_0 / B$ in Eq. (5) is a constant [and approximately its value is $10 (10^{13} \text{ m/cm}^3\text{-Tesla})$], and (2) $\langle T_e \rangle$ scales as $(aB/R_0)^{2/3}$. For tokamaks of same aspect ratio and magnetic field, the highest attainable energy confinement time only increases, linearly, with the minor radius. Approximating $\langle T_e \rangle / (\max T_{e0}) \approx 0.35$ and $\alpha \approx 0.75$, we obtain $\max \tau_E \approx 350a(aB/R_0)^{2/3} \text{ msec}$. The second effect of disruption is to set a minimum magnetic field for attaining a desired density level. We note that in ohmically heated stellarators, moderate rotational transform reduces or eliminates major disruption and, therefore, a discharge of equal density to that of a tokamak of same R_0 can be sustained at a lower magnetic field. The possibility to sustain a discharge of given density at lower magnetic field may have practical usefulness in future larger devices in which plasma heating is primarily accomplished by nonohmic heating means such as the injection of energetic neutral beams, since it relaxes the stringent requirement on magnetic field strength needed to sustain a desired density to absorb the beam energy. (An alternate scheme to increase the density to a desired level is to inject successively a series of beams with increasingly higher energy.)

4. Model C Stellarator Results. Model C Stellarator parameters are $R_0 = 1.1 \text{ m}$, $\bar{a} \approx 0.05 \text{ m}$, $\langle T_e \rangle \approx 0.1 \text{ keV}$, $\langle n_e \rangle = \frac{1}{2} n_e(0) = 10^{13} \text{ cm}^{-3}$, $B = 3.5 \text{ Tesla}$, and $q_a \approx 10$. For $\alpha \approx 0.3$, we obtain

$\tau_E \approx 0.5$ msec. Since the peak current density due to the $q_0 = 1$ condition in a stellarator is reduced from that of a corresponding tokamak by a factor of $(1 - \tau_{a0})$, the factor $1/q_0$ in Eq. (5) is replaced by $(1 - \tau_{a0})$ in calculating the peak electron temperature for a stellarator. The highest attainable electron temperature for the Model C stellarator is approximately 230 eV, also in approximate agreement with observation [19]. We emphasize that this temperature limit refers to the ohmic heating regime; in the Model C Stellarator $T_i = 600$ eV was observed by Rothman, et al. [20], when the plasma was heated by rf at the ion cyclotron frequency.

5. Effects of Non-Ohmic Heating. The present scaling states that τ_E is proportional to the plasma internal energy, although the form of temperature dependence of the thermal conductivity is not known. This scaling is consistent with the recent high-power, neutral beam-injection heating experiment in the PLT Tokamak [21,22]. The very recent results of neutral beam injection experiment in Wendelstein VII-A [23] shows that, in addition to the increased confinement time due to higher internal energy, additional confinement improvement is achieved by reducing ohmic-heating current during injection. It may be conjectured that non-ohmic heating of electrons in tokamaks and stellarators in general leads to better confinement if the heating does not induce additional losses; reducing the ohmic-heating current while sustaining the plasma internal energy by non-ohmic heating in a stellarator also improves the confinement.

The scaling suggested in the present work is necessarily a simplified representation of ohmically heated toroidal discharges. In particular, radiation loss has not been explicitly included, although the scaling is not inconsistent with a model

that thermal conduction transports heat from the interior to the edge and there it is lost by radiation and/or conduction to the limiter. The scaling describes several important features of ohmically heated toroidal discharges; these features are characterized by a single phenomenon that for given aspect ratio and toroidal field strength, the electron temperature is fixed except for the monotonic decrease with increasing density (for $Z_{\text{eff}} = 1$). The lower operation limit of density is presumably determined by runaway conditions, and the upper limit, in the case of tokamaks, by major disruptions. We conclude by pointing out that in tokamaks, ohmic-heating current is also needed for maintaining plasma equilibrium, and therefore cannot be arbitrarily reduced or removed. In stellarators, in which the equilibrium does not require an ohmic-heating current, a different regime of electron energy confinement for plasmas embedded in toroidal flux surfaces can be expected to reign if plasma heating is entirely sustained by other means than ohmic heating. If the heating is sustained by injecting energetic neutral beams, recent experiments [21-23] demonstrated that improved confinement can be expected.

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Fig. 1. (a) $\tau_E/(1+\alpha)\langle T_e \rangle$ vs. $\langle n_e \rangle q_a$ for PLT and Alcator A experiments. The units of $\langle n_e \rangle q_a$ for PLT data is 10^{13}cm^{-3} , for Alcator A data is 10^{14}cm^{-3} . 796446

Fig. 1. (b) Same plot for ISX A experiments. (Note: a , R_0 , and B in ISX A experiments were varied slightly, see Table 1).

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Fig. 2. The data in Fig. 1 are replotted as $\tau_E / (1+\alpha) \langle T_e \rangle$ vs. $\langle n_e \rangle q_a R_0 / B$. 796448

Fig. 3. $\tau_E / (1+Q) < T_e >$ vs. $\langle n_e \rangle \bar{a} R_0 / (t_a - t_{a0}) B$ for W VII-A and Cleo Stellarators. The line represents an average tokamak discharge based on Fig. 2.

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