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One-Dimensional Model of Fluidized Bed Combustor Dynamics

R. B. Perez

**OPERATED BY
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FOR THE UNITED STATES
DEPARTMENT OF ENERGY**

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Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A04; Microfiche A01

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INSTRUMENTATION AND CONTROLS DIVISION

ONE-DIMENSIONAL MODEL OF FLUIDIZED BED COMBUSTOR DYNAMICS

R. B. Perez

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Date Published - January 1980

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HIGHLIGHTS

The I&C Division undertook an exploratory study of the physics of fluidized bed combustors (FBCs). Specific goals of this study were to: (1) compile and understand the basic multiphase equations describing the various processes which determine the behavior of an FBC, (2) develop a one-dimensional model for further dynamic studies of the system, (3) identify those areas of expertise in the I&C Division which could be applied to FBC systems, and (4) develop a systematic approach for using diagnostic techniques developed at I&C to aid the industry.

A detailed study of current literature treating the physics of multiphase systems was conducted. The resulting equations of continuity, chemical reactions, momentum transport, and energy balance were reduced to a more tractable one-dimensional system.

The present work has accomplished the first two goals cited above, thus providing the groundwork for development of tools for the surveillance and control of coal-fired power plants.

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I. INTRODUCTION

The burning of coal in a fluidized bed combustor (FBC) appears to be an interesting energy source alternative with low environmental risks.¹ A panoramic view of the utilization of the fluidized bed technique as applied to both coal burning power plants and to the chemical industry is given in Refs. 1 and 2. For thorough reviews of some of the fundamental theoretical aspects of fluidization, the reader is referred to Refs. 3, 4, and 5.

The primary purpose of this work is to develop a one-dimensional model of the time-dependent hydrodynamic and enthalpy equations for a fluidized bed combustor. This endeavor is motivated by the need to generate suitable diagnosis techniques for the surveillance and control of FBC systems. The physical ideas and the analytical formulation of the multiphase equations are those developed by Soo.⁶

In Section II we present a quick survey of the physical characterization of the fluidized process. The continuity and momentum equations are developed in Section III. Section IV is devoted to the derivation of the energy balance. The enthalpy equations are given in Section V, and the one-dimensional version of the multiphase equations is developed in Section VI. The reaction, breakup, and conglomeration rates are studied in Appendix A. The Appendices B and C act as an aid to the development of the one-dimensional, multiphase equations for a cylindrical fluidized bed combustor.

II. PHYSICAL CHARACTERIZATION OF THE FLUIDIZATION PROCESS*

Several interacting processes take place in a fluidized bed combustor. They are chemical reactions, momentum exchange, and heat exchange processes. These processes operate in a multicomponent, multiphase system, composed of a solid phase and a fluid phase. Eventually pockets of gas (bubbles) appear intermixed in the solid phase. The overall physical picture is that of a solid suspension in a fluid, with swarms of ascending bubbles, just like the physical situation associated with a boiling liquid.⁷ The solids component of the suspension contains N_e solid species. Within each species there is a distribution of particle sizes, $[P^{(e)}(a_s^e) da_s^e]$ characterized by an effective particle radius, $a_s^{(e)}$, for the e^{th} species of solid and the s^{th} size. We shall approximate the continuous size distribution by $N_s^{(e)}$ "groups" of sizes (i.e. a histogram description of a continuous distribution). Also to make the notation less cumbersome, we shall use the subindex, s , to indicate both the species and size of a solid particle. This means that we consider $[N_e \cdot N_s^{(e)}]$ "species" of particles. This leads to the convention that the summation sign $\sum_{|s|}$ indicates a sum over all species and sizes. For example, if there are 100 particle sizes of limestone and 500 particle sizes of coal, the subindex, s , from $s = 1$ to 100, indicates limestone, and from $s = 101$ to $s = 600$ indicates the various sizes of coal. The bulk density of the s^{th} solid species is ρ_{0s} . The fluid phase is made of N_f components of theoretical density, ρ_{0q} , ($q = 1, 2 \dots N_f$).

*A list of symbols follows the References in this document, page 33.

II.A. Chemical Processes

We include both chemical reactions and the process of particle coagulation and break-up. The following processes are assumed to occur:

(a) chemical reactions between molecules of the q^{th} species in the fluid phase and particles of the s^{th} species in the particulate phase. The generation (removal) rates in the fluid and particulate phases are Γ_q and Γ_s , respectively (in units of $\text{Kg/m}^3 \cdot \text{sec}$); (b) break-up and coalescence in the particulate phase, expressed in terms of the rate parameters, K_s ($\text{Kg/m}^3 \text{ sec}$); and (c) chemical reactions in the fluid phase, which include change of state, for condensable fluids, expressed in terms of the rate constant K_q ($q = 1 \dots N_f$), ($\text{Kg/m}^3 \text{ sec}$). The above rate constants are studied in detail in Appendix A.

II.B. Momentum Exchange

In order to write the momentum balance equations (equations of motion) one must account for various types of momentum exchange which lead to drag forces (newton/m^3). The "smear" densities for the s^{th} solid species and for the fluid are ρ_s and ρ , respectively. All fluid components have the same velocity $\vec{U}$. Due to the inertia of the solid particles each species, s , has the velocity $\vec{U}_s$.

We now identify the following drag forces: (a) drag force, $\vec{P}_s^{(\text{ex})}$, due to the momentum exchange rate between particle and fluid:

$$\vec{P}_s = \gamma_s \rho_s (\vec{U} - \vec{U}_s) \quad (\text{II.1})$$

where $\gamma_s (\text{sec}^{-1})$ is a momentum exchange rate constant; (b) drag force, $\vec{P}_{s's'}$, arising from momentum exchange rate between particles of different species:

$$\vec{P}_{s's} = \gamma_{s's} \rho_s (\vec{U}_{s'} - \vec{U}_s) \quad (\text{II.2})$$

where $\gamma_{s's} (\text{sec}^{-1})$ is the corresponding exchange rate constant;

(c) momentum exchange due to chemical reactions, $\Gamma_s (\vec{U} - \vec{U}_s)$, which arises because Γ_s ($\text{Kg/m}^3 \text{ sec}$) of solid particles have been produced (destroyed) by chemical reaction with the fluid, and appear in other phase with a different velocity; (d) momentum exchange rate due to coagulation and break-up of solid particles, $\delta \vec{H}_s$, (newtons/m^3) (described in Appendix A); (e) viscous drag forces in the solid cloud, fluid phase, and their interactions with the walls. These forces are described by the viscous tensors, $\hat{T}_s$ and $\hat{T}$ for the s^{th} solid component and for the fluid phase, respectively. The rate constants, γ_s , $\gamma_{s's}$, and the viscous tensors, $\hat{T}_s$ and $\hat{T}$, are described in Appendix B.

II.C. Heat Exchange Terms

In writing the dynamic heat balance equation (energy or enthalpy equations) there are several heat exchange rate terms between particles of different species and the fluid which must be accounted for:

(a) viscous heat exchange rate, $\gamma_s \rho_s (\vec{U} - \vec{U}_s)^2$ and $\gamma_{ss'} \rho_s (\vec{U}_{s'} - \vec{U}_s)^2$, arising from particle-fluid, and particle-particle momentum exchange; (b) fluid-particle cloud, heat convection, $c_s \rho_s g_s (T - T_s)$, where c_s is the specific heat for the bulk, s^{th} species, T_s its temperature, T the fluid temperature and g_s a rate constant (sec^{-1}); (c) particle-particle heat convection, $c_s \rho_s g_{s's} (T_{s'} - T_s)$, where $g_{s's} (\text{sec}^{-1})$ is a rate constant and $T_{s'}$ the temperature of the $(s')^{\text{th}}$ -species of particles; (d) kinetic energy exchange rate due to coagulation and break-up δW_s (derived in Appendix A). The rate constants, g_s and $g_{s's}$ are given in Appendix B.

II.D. The Bubble-Wake Phase

It is not clear why bubbles form in a fluidized bed. However, it has been shown by Jackson⁸ that uniform flow is unstable, a fact which leads to the surmise that bubble formation may be a direct consequence of flow instability, Rowe.⁹ Extensive experimental studies⁹ on bubble formation and dynamics have nevertheless confirmed that bubbles are associated with ordered fluid flow patterns and that the appearance of bubbling is determined by the ratio of solid to fluid density.

Flow of gas through a bubble varies between once-through flow to closed circulation inside the bubble. The gas flow pattern in the bubble controls the chemical and mass transfer rates between gas and particles.

The solid particles flow around the rising bubble. Consequently, there is an upward drift of solids around the vertical axis of the bubble path, with an associated sinking of the rest of the solids. The particles that form the innermost boundary of the bubble flow around it and meet at the rear stagnation point of the bubble. As a result of the impact, particles are tossed upwards creating a volume of particles in agitated conditions, which results into a "wake", with continuous exchange of particles between the wake and the solid phase. When the bubble arrives at the surface, the wake is left behind.

For remote bubbles, that is for isolated bubbles with sizes very small compared with the size of the fluidized bed, the shape of a bubble is essentially spherical, with an indented base where the wake is accommodated. To a large extent, the bubble can be defined by an effective radius, r_b , and by the wake fraction, f_b , which is the ratio of the volume occupied by the indentation to the volume of bubble sphere, V_b .

Isolated bubbles rise substantially at constant velocity, $U_{b\infty}$, which is proportional to their size according to the empirical⁹ correlation

$$U_{b\infty} = K(g r_b)^{1/2} \quad (\text{II.3})$$

where g is the acceleration of gravity, r_b the radius of the bubble, and K a parameter which depends on the physical properties and size of the solids.

For bubbles in swarms, the bubble velocity, U_b , is given by the approximate expression²

$$\vec{U}_b = (\vec{U} - \vec{U}_{mf}) + \vec{U}_{b\infty} \quad (\text{II.4})$$

where U is the fluid velocity, $\vec{U}_{mf}$ the minimum fluidization velocity, and $\vec{U}_{b\infty}$ is a vector in the upward direction of motion, with modulus, $|\vec{U}_{b\infty}| = U_{b\infty}$ [Eq. (II.3)].

Isolated bubbles develop downward pointing cusps growing rapidly into "knives" which eventually lead to the phenomenon of bubble splitting.⁹ When bubbles are brought together coalescence may occur.^{6,9}

III. THE BASIC EQUATIONS FOR MULTIPHASE SYSTEMS

The multiphase system described in the previous section is represented analytically by the hydrodynamic equations for suspensions developed by Soo.⁶

We consider the following phases in the combustor: (a) particulate phase - this phase consists of $[v_s = N_s^{(e)} N_e]$ solid "species" of bulk density ρ_{0s} ($s = 1, 2, \dots, v_s$), volume fraction, α_s , and particle velocity, $\vec{U}_s$; (b) fluid phase - this phase consists of v_f , gas species, of theoretical density, ρ_{0q} , and volume fraction, α_q . All the species in the fluid phase have the same velocity, $\vec{U}$.⁶

The volume fractions defined above must satisfy the relation:

$$\sum_s \alpha_s(\vec{x}, t) + \sum_q \alpha_q(\vec{x}, t) = 1 \quad (\text{III.1})$$

III.A. Continuity Equations

The continuity equations are obtained from material balances accounting for the chemical reactions among solids, fluid phase, and between both phases. We have:

(a) Continuity equation for the q^{th} gas species,

$$\frac{\partial}{\partial t} \rho_q(\vec{x}, t) + \vec{\nabla} \cdot \vec{J}_q(\vec{x}, t) = -\Gamma_q(\vec{x}, t) + K_q(\vec{x}, t); \quad (\text{III.2})$$

(b) Continuity equation for the s^{th} solid species,

$$\frac{\partial}{\partial t} \rho_s(\vec{x}, t) + \vec{\nabla} \cdot \vec{J}_s(\vec{x}, t) = \Gamma_s(\vec{x}, t) + K_s(\vec{x}, t) \quad (\text{III.3})$$

where we introduced the following quantities:

$$\begin{aligned}\rho_q(\vec{x}, t) &= \text{"smear" density of the } q^{\text{th}} \text{ gas component (Kg/m}^3\text{)} \\ &= \alpha_q \rho_{0q}\end{aligned}\quad (\text{III.4})$$

$$\alpha_q = \text{volume fraction of the } q^{\text{th}} \text{ gas component} = V_q/V_T$$

$$V_q = \text{volume occupied by the } q^{\text{th}} \text{ gas component.}$$

$$\vec{J}_q(\vec{x}, t) = \text{mass flow vector} = \rho_q(\vec{x}, t) \vec{U}_q(\vec{x}, t) \text{ (Kg/m}^2\text{·sec)} \quad (\text{III.5})$$

$$\Gamma_q(\vec{x}, t) = \text{chemical reaction losses to the particulate phase (Kg/m}^3 \text{ sec)}$$

$$K_q(\vec{x}, t) = \text{chemical reactions, break-up and coalescence, and condensation processes in the fluid phase (Kg/m}^3 \text{ sec)}$$

$$\begin{aligned}\rho_s(\vec{x}, t) &= \text{"smear" density of the } s^{\text{th}} \text{ solid species (Kg/m}^3\text{)} \\ &= \alpha_s \rho_{0s}\end{aligned}\quad (\text{III.6})$$

$$\begin{aligned}\vec{J}_s(\vec{x}, t) &= \text{mass flow vector for the } s^{\text{th}} \text{ solid species (Kg/m}^3\text{·sec)} \\ &= \rho_s(\vec{x}, t) \vec{U}_s(\vec{x}, t)\end{aligned}\quad (\text{III.7})$$

$$\Gamma_s(\vec{x}, t) = \text{chemical reaction rate in the solid phase (Kg/m}^3\text{·sec)}$$

$$K_s(\vec{x}, t) = \text{rate of break-up and coalescence in the solid phase}^{10} \text{ (Kg/m}^3\text{·sec)}$$

Also (from Appendix A) we have the relations:

$$\sum_s \Gamma_s = \sum_q \Gamma_q = \Gamma \quad (\text{III.8})$$

$$\sum_s K_s = 0 \quad (\text{III.9})$$

$$\sum_q K_q = 0 . \quad (\text{III.10})$$

The overall continuity equation for the fluid phase is obtained by summation over all fluid components of Eq. (III.2), we have:

$$\frac{\partial}{\partial t} \rho(\vec{x}, t) + \vec{\nabla} \cdot \vec{J}(\vec{x}, t) = - \Gamma \quad (\text{III.11})$$

where we utilized Eqs. (III.8) and (III.10) and defined the following quantities:

$$\rho(\vec{x}, t) = \sum_q \rho_q(\vec{x}, t) = \sum_q \alpha_q \rho_{0q} \quad (\text{III.12})$$

$$\vec{J}(\vec{x}, t) = \sum_q \vec{J}_q(\vec{x}, t) = \rho(\vec{x}, t) \vec{U} . \quad (\text{III.13})$$

A similar procedure applied to Eq. (III.3) yields the following continuity equation for the overall solid phase:

$$\frac{\partial}{\partial t} \rho_m(\vec{x}, t) + \vec{\nabla} \cdot \left(\sum_s \vec{J}_s \right) = \Gamma(\vec{x}, t) \quad (\text{III.14})$$

where we utilized Eqs. (III.8) and (III.10) with the definitions

$$\rho_m(\vec{x}, t) = \text{total solids density} = \sum_s \rho_s(\vec{x}, t) = \sum_s \alpha_s(\vec{x}, t) \rho_{0s} . \quad (\text{III.15})$$

The overall continuity equation for the system is obtained by adding up Eqs. (III.11) and (III.14), i.e.:

$$\frac{\partial}{\partial t} [\rho_m(\vec{x}, t) + \rho(\vec{x}, t)] + \vec{\nabla} \cdot [\vec{J}(\vec{x}, t) + \sum_s \vec{J}_s(\vec{x}, t)] = 0 . \quad (\text{III.16})$$

It is now convenient to write down the various continuity equations derived above in dimensionless form. To this end we define the following characteristic parameters:

L_o = characteristic length (m)

t_o = characteristic time (sec)

U_o = characteristic speed (m sec⁻¹) .

On this basis the differential operators become

$$\frac{\partial}{\partial t} = \frac{1}{t_o} \frac{\partial}{\partial \tau} \quad (\text{III.17})$$

$$\vec{\nabla} = \frac{1}{L_o} \vec{\nabla}^* \quad (\text{III.18})$$

where we introduced the dimensionless time and spatial coordinates

$$\tau = t/t_o \quad (\text{III.19})$$

$$\vec{x}^* = \vec{x}/L_o . \quad (\text{III.20})$$

With the above definitions the continuity Eqs. (III.2) and (III.3) become:

$$\frac{\partial}{\partial \tau} \alpha_q + \vec{\nabla}^* \cdot \vec{J}_q^* = -\Gamma_q^* + K_q^* \quad (\text{III.21})$$

and

$$\frac{\partial}{\partial \tau} \alpha_s + \vec{\nabla}^* \cdot \vec{J}_s^* = \Gamma_s^* + K_s^* \quad (\text{III.22})$$

where

$$\vec{J}_q^* = \alpha_q \vec{U}^* \quad (\text{III.23})$$

$$\vec{J}_s^* = \alpha_s \vec{U}_s^* \quad (\text{III.24})$$

$$\vec{U}^* = \text{reduced velocity} = \vec{U}/U_o \quad (\text{III.25})$$

$$\vec{U}_s^* = \text{reduced velocity for the } s^{\text{th}} \text{ solid species} = \vec{U}_s/U_o \quad (\text{III.26})$$

$$\Gamma_q^* = \Gamma_q \left(\frac{t_o}{\rho_{0q}} \right) \text{ (dimensionless)} \quad (\text{III.27})$$

$$\Gamma_s^* = \Gamma_s \left(\frac{t_o}{\rho_{0s}} \right) \text{ (dimensionless)} \quad (\text{III.28})$$

$$K_q^* = K_q \left(\frac{t_o}{\rho_{0q}} \right) \text{ (dimensionless)} \quad (\text{III.29})$$

$$K_s^* = K_s \left(\frac{t_o}{\rho_{0s}} \right) \text{ (dimensionless)} \quad (\text{III.30})$$

Consequently, the overall continuity equations for the fluid phase, the solid phase, and the mixture become, respectively:

$$\frac{\partial}{\partial \tau} \alpha + \vec{\nabla}^* \cdot \vec{J}^* = - \Gamma^* \quad (\text{III.31})$$

$$\frac{\partial}{\partial \tau} \alpha + \vec{\nabla}^* \cdot \sum_s \vec{J}_s^* = \Gamma^* \quad (\text{III.32})$$

$$\vec{\nabla}^* \cdot (\vec{J}^* + \sum_s \vec{J}_s^*) = 0 \quad (\text{III.33})$$

where

$$\alpha = \sum_q \alpha_q \quad (\text{III.34})$$

and we utilized the relation $\sum \alpha_s = \alpha_m = 1 - \sum_q \alpha_q = 1 - \alpha$ [see Eq. (III.1)] with

$$\vec{J}^* = \sum_q J_q^* = \alpha \vec{U}^* . \quad (\text{III.35})$$

III.B. Equations of Motion

We now write down the momentum balance equations for the particulate and fluid phases:

(a) Particulate phase

$$\rho_s \left(\frac{\partial}{\partial t} + \vec{U}_s \cdot \vec{\nabla} \right) \vec{U}_s = \vec{\nabla} \cdot \vec{T}_s + \rho_s \vec{g} + \vec{P}_{0s} \quad (\text{III.36})$$

where $\vec{P}_{0s}$ is the following rate of momentum exchange per unit volume

$$\vec{P}_{0s} = \vec{P}_s^{(ex)} + \sum_{s'} \vec{P}_{s's}^{(ex)} + \vec{P}_{0s}^{(ch)} + \vec{P}_{0s}^{(cg)} \quad (\text{III.37})$$

with (see Section II.B)

$$\vec{P}_s^{(ex)} = \gamma_s \rho_s (\vec{U} - \vec{U}_s) \quad (\text{III.38})$$

$$\vec{P}_{s's}^{(ex)} = \rho_s \gamma_{s's} (\vec{U}_s - \vec{U}_{s'}) \quad (\text{III.39})$$

$$\vec{P}_{0s}^{(ch)} = \Gamma_s (\vec{U} - \vec{U}_s) \quad (\text{III.40})$$

$$\vec{P}_{0s}^{(cg)} = \delta \vec{\Theta}_s \quad (\text{III.41})$$

$$\sum_s \vec{P}_{s's}^{(ex)} = 0 \quad (\text{III.42})$$

and where following Soo's formalism we have neglected the term in the gradient of the pressure in Eq. (III.36).

(b) Fluid phase - The overall momentum balance for the fluid phase is:

$$\rho \left(\frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla} \right) \vec{U} = - \vec{\nabla} P + \vec{\nabla} \cdot \vec{T} + \rho \vec{g} - \sum_s \vec{P}_s^{(ex)} - \sum_s \vec{P}_{0s}^{(ch)} \quad (\text{III.43})$$

where P is the pressure (pascals).

We also shall find it convenient to rewrite Eqs. (III.36) and (III.43) in dimensionless form. We then obtain for the particulate and fluid phases, respectively:

$$\alpha_s \left(\frac{\partial}{\partial \tau} + \vec{U}_s^* \cdot \vec{\nabla}^* \right) \vec{U}_s^* = \vec{\nabla}^* \cdot (\vec{T}_s^*) + \alpha_s \vec{g}^* + \vec{P}_{0s}^* \quad (\text{III.44})$$

$$\alpha \left(\frac{\partial}{\partial \tau} + \vec{U}^* \cdot \vec{\nabla}^* \right) \vec{U}^* = - \vec{\nabla}^* P^* + \vec{\nabla}^* \cdot (\vec{T}^*) + \alpha \vec{g}^* - \left\{ \sum_s \frac{\rho_{0s}}{\rho_0} \left[\vec{P}_s^{*(ex)} + \vec{P}_{0s}^{*(ch)} \right] \right\} \quad (\text{III.45})$$

with

$$\vec{T}_s^* = \vec{T}_s / (\rho_{0s} U_o^2) \quad (\text{III.46})$$

$$\vec{g}^* = \vec{g} / g_o \quad (\text{III.47})$$

$$g_o = \text{characteristic acceleration} = U_o / t_o \quad (\text{III.48})$$

$$\vec{p}_{0s}^* = \vec{p}_s^{*(ex)} + \sum_s \vec{p}_{ss}^{*(ex)} + \vec{p}_{0s}^{*(ch)} + \vec{p}_{0s}^{*(cg)} \quad (\text{III.49})$$

where all the dimensionless momentum exchange rates in Eq. (III.49) are equal to their corresponding values [Eqs. (III.38) up to (III.41)] divided by the characteristic momentum exchange rate per unit volume, Π_{0s} :

$$\Pi_{0s} = \rho_{0s} g_o . \quad (\text{III.50})$$

We also have

$$P^* = \text{dimensionless pressure} = P/(\rho_o U_o^2) \quad (\text{III.51})$$

and

$$T^* = T/(\rho_o U_o^2) . \quad (\text{III.52})$$

Finally to obtain the equations of motion in terms of dimensionless mass flow vectors and momentum flow tensors, we utilize the relations

$$\alpha_s \frac{\partial}{\partial \tau} \vec{U}_s^* = \frac{\partial}{\partial \tau} (\alpha_s \vec{U}_s^*) - \vec{U}_s^* \frac{\partial}{\partial \tau} \alpha_s \quad (\text{III.53})$$

$$\alpha \frac{\partial}{\partial \tau} \vec{U}^* = \frac{\partial}{\partial \tau} (\alpha \vec{U}^*) - \vec{U} \frac{\partial}{\partial \tau} \alpha \quad (\text{III.54})$$

together with the continuity Eqs. (III.21) and (III.22), to obtain

$$\alpha_s \frac{\partial}{\partial \tau} \vec{U}_s^* = \frac{\partial}{\partial \tau} \vec{J}_s^* + \vec{U}_s^* \vec{\nabla} \cdot \vec{J}_s^* - \vec{U}_s^* (\Gamma_s^* + K_s^*) \quad (\text{III.55})$$

$$\alpha \frac{\partial}{\partial \tau} \vec{U}^* = \frac{\partial}{\partial \tau} \vec{J}^* + \vec{\nabla} \cdot \vec{J}^* + \vec{U}^* \Gamma^* \quad (\text{III.56})$$

where we utilized the definitions (III.24) and (III.35). Insertion of Eq. (III.53) into (III.44) and Eq. (III.54) into (III.45) yields the desired results

$$\frac{\partial}{\partial \tau} \vec{J}_s^* + \vec{\nabla}^* \hat{M}_s^* = \vec{\nabla}^* \hat{T}_s^* + \alpha_s \vec{g}^* + \vec{P}_s^* \quad (\text{III.57})$$

$$\frac{\partial}{\partial \tau} \vec{J}^* + \vec{\nabla}^* \hat{M}^* = - \vec{\nabla}^* P^* + \vec{\nabla}^* \hat{T}^* + \alpha \vec{g}^* - \vec{P}_f^* \quad (\text{III.58})$$

where we defined the dimensionless momentum flow tensors

$$\hat{M}_s^* = \alpha_s \vec{U}_s^* \vec{U}_s^* \quad (\text{III.59})$$

$$\hat{M}^* = \alpha \vec{U}^* \vec{U}^* \quad (\text{III.60})$$

and where

$$\vec{P}_s^* = \vec{P}_{0s}^* + \vec{U}_s^* (\Gamma_s^* + K_s^*) \quad (\text{III.61})$$

$$\vec{P}_f^* = \sum_s \left(\frac{\rho_{0s}}{\rho_0} \right) \left[\vec{P}_s^{*(ex)} + \vec{P}_{0s}^{*(ch)} \right] + \vec{U}^* \Gamma^* \quad (\text{III.62})$$

In what follows we shall use Eqs. (III.56) and (III.57) as the equations of motion for the combustor.

IV. THE ENERGY BALANCE EQUATIONS

On the basis of our discussion in Section II.C we can now write the following energy balances:

(a) Solid phase

$$\rho_s \left(\frac{\partial}{\partial t} + \vec{U}_s \cdot \vec{\nabla} \right) \left(\frac{1}{2} \vec{U}_s^2 \right) + \rho_s \left(\frac{\partial}{\partial t} + \vec{U}_s \cdot \vec{\nabla} \right) H_s = \vec{\nabla} \cdot \left[\vec{Q}_s^{(v)} + \vec{Q}_s^{(h)} \right] + Q_{0s} \quad (\text{IV.1})$$

where we defined the following quantities

$$H_s = \text{enthalpy of the } s^{\text{th}} \text{ particulate component} = c_s T_s \text{ (joules/Kg)} \quad (\text{IV.2})$$

$$c_s = \text{specific heat for the } s^{\text{th}} \text{ component in solid phase (joules/Kg-}^\circ\text{C)}$$

$$T_s = \text{temperature of the } s^{\text{th}} \text{ solid component (}^\circ\text{C)}$$

$$\vec{Q}_s^{(v)} = \text{viscous energy vector} = \vec{U}_s \cdot \vec{\nabla} T_s \text{ [joules(sec)m}^2\text{]} \quad (\text{IV.3})$$

$$\vec{Q}_s^{(h)} = \text{heat flux vector} = \kappa_s \vec{\nabla} \cdot T_s \text{ (joules/sec x m}^2\text{)} \quad (\text{IV.4})$$

$$\kappa_s = \text{thermal conductivity (joules/-}^\circ\text{C x sec x meter)}$$

and

$$Q_{0s} = Q_s^{(\text{ex})} + Q_s^{(\text{c})} + Q_s^{(\text{cg})} + Q_s^{(\text{ext})} + Q^{(\text{ch})} \quad (\text{IV.5})$$

where

$$Q_{0s}^{(\text{ex})} = \text{viscous heat exchange} = \gamma_s \rho_s (\vec{U} - \vec{U}_s)^2 \text{ (joules/m}^3 \text{ x sec)} \quad (\text{IV.6})$$

$$Q_{ss}^{(\text{ex})} = \text{interparticle clouds viscous heat exchange}$$

$$= \gamma_{ss} \rho_s (\vec{U}_s - \vec{U}_s)^2 \text{ (joules/m}^3 \text{ x sec)} \quad (\text{IV.7})$$

$$Q_s^{(ex)} = Q_{0s}^{(ex)} + \sum_s Q_{ss'}^{(ex)} \quad (IV.8)$$

$$Q_{0s}^{(c)} = \text{fluid-cloud heat convection} = g_s \rho_s c_s (T - T_s) \text{ (joules/m}^3 \text{ sec)} \quad (IV.9)$$

$$Q_{ss'}^{(c)} = \text{interparticle clouds heat convection} = g_{ss'} \rho_s c_s (T_{s'} - T_s) \text{ (joules/m}^3 \text{ sec)} \quad (IV.10)$$

$$Q_s^{(c)} = Q_{0s}^{(c)} + \sum_s Q_{ss'}^{(c)} \quad (IV.11)$$

$$Q_s^{(cg)} = \text{kinetic energy change from coagulation and break-up} \text{ (joules/cm}^3 \text{ sec)} \text{ (see Appendix A)} = \delta W_s \quad (IV.12)$$

$$Q_s^{(ext)} = \text{external work on the system (for example by pumps)} \\ \text{or output work (turbines, as well as heat} \\ \text{exchange to walls)} \text{ (joules/m}^3 \text{ sec)}$$

$$Q_s^{(ch)} = \text{chemical heat} = \beta_s \Gamma_s \text{ (joules/m}^3 \text{ sec)} \quad (IV.13)$$

$$\beta_s = \text{heat yield (joules/Kg)} .$$

(b) Fluid phase

$$\rho \left(\frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla} \right) \left[\frac{1}{2} (\vec{U})^2 + H_f \right] = \frac{\partial}{\partial t} P + \vec{\nabla} \cdot \left[\vec{Q}^{(v)} + \vec{Q}^{(h)} \right] + Q_0 \quad (IV.14)$$

with

$$H_f = \text{enthalpy in the fluid phase} = cT \text{ (joules/Kg)} \quad (IV.15)$$

$$c = \text{specific heat in the fluid phase (joules/Kg-}^\circ\text{C)} .$$

T = temperature of the fluid ($^{\circ}\text{C}$)

$$\vec{Q}^{(v)} = \text{viscous energy vector} = \vec{U} \cdot \vec{\nabla} \quad (\text{joules/sec} \times \text{m}^2) \quad (\text{IV.16})$$

$$\vec{Q}^{(h)} = \text{heat flux vector} = \kappa \vec{\nabla} T \quad (\text{joules/m}^2 \times \text{sec}) \quad (\text{IV.17})$$

κ = thermal conductivity ($\text{joules/m} \times \text{sec} \times ^{\circ}\text{C}$)

and

$$Q_0 = Q_0^{(ex)} + Q^{(c)} + Q_0^{(ch)} + Q^{(ext)} \quad (\text{IV.18})$$

with

$$Q_0^{(ex)} = - \sum_s Q_{0s}^{(ex)} \quad (\text{IV.19})$$

$$Q^{(c)} = - \sum_s Q_{0s}^{(c)} \quad (\text{IV.20})$$

$$Q_0^{(ch)} = - \sum_s Q_s^{(ch)} \quad (\text{IV.21})$$

$Q^{(ext)}$ = input (output) work on the fluid system.

Following the ideas and procedures developed in Section III, we now rewrite the energy balance Eqs. (IV.1) and (IV.14) in dimensionless form. We have for the solid phase

$$\alpha_s \left(\frac{\partial}{\partial \tau} + \vec{U}_s^* \cdot \vec{\nabla}^* \right) \left[\frac{1}{2} (\vec{U}_s^*)^2 + H_s^* \right] = \vec{\nabla}^* \cdot \left[\vec{Q}_s^{*(v)} + \vec{Q}_s^{*(h)} \right] + Q_{0s}^* \quad (\text{IV.22})$$

with

$$H_s^* = \text{dimensionless enthalpy} = c_s^* T_s^* \quad (\text{IV.23})$$

$$T_s^* = T_s / T_o \quad (\text{IV.24})$$

$$c_s^* = c_s T_o / U_o^2 \quad (\text{IV.25})$$

$$\vec{Q}_s^*(v) = \vec{U}_s^* \gamma_s^* T_s^* \quad (\text{IV.26})$$

$$\vec{Q}_s^*(h) = \kappa_s^* \vec{\nabla}^* T^* \quad (\text{IV.27})$$

$$\kappa_s^* = \kappa_s \left(\frac{t_o T_o}{L_o^2 \rho_{0s} U_o^2} \right) \quad (\text{IV.28})$$

and

$$\begin{aligned} Q_{0s}^* = \alpha_s \left[q_s^{(ex)} + \sum_{s'} q_{ss'}^{(ex)} + q_s^{(c)} + \sum_{s'} q_{ss'}^{(c)} \right] + Q_s^{*(cg)} \\ + Q_s^{*(ch)} + Q_s^{*(ext)} \end{aligned} \quad (\text{IV.29})$$

with

$$q_s^{(ex)} = \gamma_s^* (\vec{U}^* - \vec{U}_s^*)^2 \quad (\text{IV.30})$$

$$q_{ss'}^{(ex)} = \gamma_{ss'}^* (\vec{U}_{s'}^* - \vec{U}_s^*)^2 \quad (\text{IV.31})$$

$$q_s^{(c)} = g_s^* c_s^* (T^* - T_s^*) \quad (\text{IV.32})$$

$$q_{ss'}^{(c)} = g_{ss'}^* c_{ss'}^* (T_{s'}^* - T_s^*) \quad (\text{IV.33})$$

$$Q_s^{*(cg)} = \frac{t_o}{\rho_{0s} U_o^2} \delta W \quad (\text{see Appendix A}) \quad (\text{IV.34})$$

$$Q_s^{*(ch)} = \left(\frac{t_o}{\rho_{0s} U_o^2} \right) \beta_s \Gamma_s \quad (\text{IV.35})$$

$$Q_s^{*(ext)} = \left(\frac{t_o}{\rho_{0s} U_o^2} \right) Q_s^{(ext)} \quad (\text{IV.36})$$

$$\gamma_s^* = \gamma_s t_o \quad (\text{IV.37})$$

$$\gamma_{s'}^* = \gamma_{s'} t_o \quad (\text{IV.38})$$

$$g_{s'}^* = g_{s'} t_o \quad (\text{IV.39})$$

$$g_s^* = g_s t_o \quad (\text{IV.40})$$

In a similar fashion we obtain for the fluid phase

$$\alpha \left(\frac{\partial}{\partial \tau} + \vec{U}^* \cdot \nabla^* \right) \left[\frac{1}{2} (\vec{U}^*)^2 + H_f^* \right] = \frac{\partial}{\partial \tau} P^* + \vec{\nabla}^* \cdot \left[\vec{Q}^{*(v)} + \vec{Q}^{*(h)} \right] + Q_0^* \quad (\text{IV.41})$$

with

$$H_f^* = \text{dimensionless enthalpy} = c^* T^* \quad (\text{IV.42})$$

$$T^* = T/T_o \quad (\text{IV.43})$$

$$c^* = c T_o / U_o^2 \quad (\text{IV.44})$$

$$\vec{Q}^{*(v)} = \vec{U}^* \cdot \vec{T}^* \quad (\text{IV.45})$$

$$\vec{Q}^{*(h)} = \kappa^* \vec{\nabla}^* T^* \quad (\text{IV.46})$$

$$\kappa^* = \kappa \left(\frac{t_o T_o}{L_o^2 \rho_o U_o^2} \right) \quad (\text{IV.47})$$

$$P^* = \text{dimensionless pressure} = P / (\rho_o U_o^2) \quad (\text{IV.48})$$

and

$$Q_0^* = - \sum_s \left\{ \alpha_s \left(\frac{\rho_{0s}}{\rho_o} \right) \left[q_s^{(ex)} + q_s^{(c)} \right] + \left(\frac{\rho_{0s}}{\rho_o} \right) Q_s^{*(ch)} \right\} + Q^{*(ext)} \quad (\text{IV.49})$$

with

$$Q^{*(ext)} = \frac{t_o}{\rho_o U_o^2} Q^{(ext)} \quad (\text{IV.50})$$

V. THE ENTHALPY EQUATIONS

We shall find it convenient to rewrite the energy balance equations by eliminating the kinetic energy terms. The result, known as the "Enthalpy Equations" is obtained as follows: For the particulate phase we form the scalar product between the vector $\vec{U}_s^*$ and the equation of motion (III.44), we obtain

$$\alpha_s \left(\frac{\partial}{\partial \tau} + \vec{U}_s^* \cdot \vec{\nabla}^* \right) \left[\frac{1}{2} (U_s^*)^2 \right] = \vec{U}_s^* \cdot \vec{\nabla}^* (\tilde{T}_s^*) + \alpha_s \vec{U}_s^* \cdot \vec{g}_s^* + \vec{U}_s^* \cdot \vec{P}_{0s}^* . \quad (V.1)$$

Next we subtract the result (V.1) from Eq. (IV.22) to obtain

$$\alpha_s \left(\frac{\partial}{\partial \tau} + \vec{U}_s^* \cdot \vec{\nabla}^* \right) H_s^* = \vec{\nabla}^* \cdot \left[\vec{Q}_s^{*(v)} + \vec{Q}_s^{*(h)} \right] - \vec{U}_s^* \cdot \vec{\nabla}^* (\tilde{T}_s^*) - \alpha_s \vec{U}_s^* \cdot \vec{g}_s^* + Q_s^* \quad (V.2)$$

with

$$Q_s^* = Q_{0s}^* - \vec{U}_s^* \cdot \vec{P}_{0s}^* . \quad (V.3)$$

Also from Eq. (IV.26)

$$\vec{\nabla}^* \cdot Q_s^{*(v)} - \vec{U}_s^* \cdot \vec{\nabla}^* \tilde{T}_s^* = \vec{\nabla}^* \cdot (\vec{U}_s^* \tilde{T}_s^*) - \vec{U}_s^* \cdot \vec{\nabla}^* \tilde{T}_s^* = \vec{\nabla}^* \cdot \vec{U}_s^* \tilde{T}_s^* = \phi_s^* \quad (V.4)$$

Where ϕ_s^* is the dimensionless rate of energy dissipation (see Appendix C).

Introduction of the dimensionless gravitational potential

$$\phi^* = (\vec{r}^* \cdot \vec{g}^*) \quad (V.6)$$

with

$$\vec{r}^* = \vec{r} / L_0 \quad (V.7)$$

allows us to write Eq. (V.2) in the form

$$\alpha_s \left(\frac{\partial}{\partial \tau} + \vec{U}_s^* \cdot \vec{\nabla}^* \right) H_s^* = \vec{\nabla} \cdot \vec{Q}_s^{*(h)} + \Phi_s^* + Q_s^* + \alpha_s \vec{U}_s^* \cdot \vec{\nabla}^* \Phi_s^* . \quad (V.8)$$

In a similar fashion we obtain for the fluid phase the following result:

$$\alpha \left(\frac{\partial}{\partial \tau} + \vec{U}^* \cdot \vec{\nabla}^* \right) H_f^* = \vec{\nabla} \cdot \vec{Q}^{*(h)} + \Phi^* + Q^* + \alpha \vec{U}^* \cdot \vec{\nabla}^* \Phi^* + \left(\frac{\partial}{\partial \tau} + \vec{U}^* \cdot \vec{\nabla}^* \right) P^* \quad (V.9)$$

with

$$\Phi^* = (\vec{\nabla}^* \cdot \vec{U}^*) : \vec{T}^* \quad (V.10)$$

and

$$Q^* = Q_0^* + \sum_s \left(\frac{\rho_{0s}}{\rho_0} \right) \vec{U}_s^* \left[\vec{P}_s^{*(ex)} + \vec{P}_{0s}^{*(ch)} \right] . \quad (V.11)$$

The enthalpy Eqs. (V.8) and (V.9) are conveniently rewritten through the use of the relations

$$\alpha_s \frac{\partial H_s^*}{\partial \tau} = \frac{\partial}{\partial \tau} (\alpha_s H_s^*) - H_s^* \left(\frac{\partial}{\partial \tau} \alpha_s \right) \quad (V.12)$$

$$\alpha \frac{\partial H_f^*}{\partial \tau} = \frac{\partial}{\partial \tau} (\alpha H_f^*) - H_f^* \left(\frac{\partial}{\partial \tau} \alpha \right) \quad (V.13)$$

and the continuity Eqs. (III.22) and (III.31). We obtain for the particulate and fluid phase, respectively

$$\frac{\partial}{\partial \tau} (\alpha_s H_s^*) + \vec{\nabla} \cdot (H_s^* \vec{J}_s^*) = \vec{\nabla} \cdot \vec{Q}_s^{*(h)} + \Phi_s^* + \alpha_s \vec{U}_s^* \cdot \vec{\nabla}^* \Phi_s^* + Q_{1s}^* \quad (V.14)$$

$$\frac{\partial}{\partial \tau} (\alpha H_f^*) + \vec{\nabla} \cdot (H_f^* \vec{J}^*) = \vec{\nabla} \cdot \vec{Q}^{*(h)} + \Phi^* + \alpha \vec{U}^* \cdot \vec{\nabla}^* \Phi^* + \left(\frac{\partial}{\partial \tau} + \vec{U}^* \cdot \vec{\nabla}^* \right) P^* + Q_1 \quad (V.15)$$

with

$$Q_{1s}^* = Q_s^* + (\Gamma_s^* + K_s^*) H_s^* \quad (V.16)$$

$$Q_1^* = Q^* - H_f \Gamma^* . \quad (V.17)$$

The results of (V.14) and (V.15) express the balance of enthalpy in the form which will be utilized in the present work.

VI. THE ONE-DIMENSIONAL VERSION OF THE MULTIPHASE EQUATIONS

In this section we shall derive the one-dimensional version of the multiphase equations by an averaging procedure over the cross-sectional area of the combustor.

The following assumptions will be made:

- (1) The material properties, κ , κ_s , c , c_s , μ_o , μ_{os} , μ , and μ_s will be taken as constants.
- (2) The remaining quantities, such as the volume fractions α_1 and α_s , are assumed to be function of the radial and axial space variables and of the time variables.
- (3) The velocity vectors are

$$\vec{U} = \vec{k} U_z(r, z, t) \quad (\text{VI.1})$$

$$\vec{U}_s = \vec{k} U_{sz}(r, z, t) . \quad (\text{VI.2})$$

VI.A. The Continuity Equations

In view of the above assumptions and utilizing the nomenclature of Appendix C, we get from Eqs. (III.22) and (III.31) the following continuity equations:

$$\frac{\partial}{\partial \tau} \alpha_s + \frac{\partial}{\partial \gamma} (J_{s,z}^*) = \Gamma_s^* + K_s^* \quad (\text{VI.3})$$

$$\frac{\partial}{\partial \tau} \alpha + \frac{\partial}{\partial \gamma} (J_z^*) = - \Gamma^* . \quad (\text{IV.4})$$

We now operate on Eqs. (VI.3) and (VI.4) with $\frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi$, where A^* is the dimensionless transverse area

$$A^* = \pi R_0^2 / L_0^2 = \pi R^{*2} . \quad (\text{VI.5})$$

In this manner we obtain

$$\frac{\partial}{\partial \tau} \bar{\alpha}_s + \frac{\partial}{\partial \gamma} (\bar{J}_{s,z}^*) = \bar{\Gamma}_s^* + \bar{U}_s^* \quad (\text{VI.6})$$

$$\frac{\partial}{\partial \tau} \bar{\alpha} + \frac{\partial}{\partial \gamma} (\bar{J}_z^*) = - \bar{\Gamma}^* \quad (\text{VI.7})$$

where calling $X(\xi, \gamma, \tau)$, any of the quantities involved in the above equations, we have

$$\bar{X}(\gamma, \tau) = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi X(\xi, \gamma, \tau) . \quad (\text{VI.8})$$

VI.B. The Equations of Motion

In this instance let us define the unit vectors

$$\vec{\lambda} = (0, 0, \vec{k}) \quad (\text{VI.9})$$

$$\lambda_s = (0, 0, \vec{k}_s) \quad (\text{VI.10})$$

and operate on Eqs. (III.57) and (III.58) with the operators $\frac{2\pi}{A^*} \vec{\lambda}_s \int_0^{R^*} \xi d\xi$, and $\frac{2\pi}{A^*} \vec{\lambda} \int_0^{R^*} \xi d\xi$, respectively, one obtains in view of the relations in Appendix C

$$\frac{\partial}{\partial \tau} \bar{J}_{s,\gamma}^* + \frac{\partial}{\partial \gamma} (\bar{\alpha}_s U_{s,\gamma}^{*z}) = \bar{D}_{s,\gamma}^* + g^* \bar{\alpha}_s + \bar{P}_{s\gamma}^* \quad (\text{VI.11})$$

with

$$\left(\overline{\alpha_s U_{s,\gamma}^{*2}} \right) = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi \alpha_s (U_{s,\gamma}^*)^2 \quad (\text{VI.12})$$

$$g^* = g/g_0 \quad (\text{VI.13})$$

$$\overline{D_{s,\gamma}^*} = \frac{\ell^*}{A^*} T_{s,\xi,\gamma}^* + \frac{\partial}{\partial \gamma} \overline{T_{s,\gamma\gamma}^*} \quad (\text{VI.14})$$

where $\ell^* = 2\pi R^*$ is the dimensionless perimeter of the combustor, and

$$T_{s,\xi,\gamma}^* = \frac{1}{R_e(s)} \left(\frac{\partial U_{s,\gamma}^*}{\partial \xi} \right)_{\xi=R^*} \quad (\text{VI.15})$$

$$\overline{T_{s,\gamma\gamma}^*} = \frac{2\pi}{A^*} \left[\frac{1}{R_{le}(s)} \right] \int_0^{R^*} \xi d\xi \left(\frac{\partial^2 U_{s,\gamma}^*}{\partial \gamma^2} \right) \quad (\text{IV.16})$$

with the following Reynolds numbers:

$$R_e(s) = \frac{L_o \rho_{0s} U_o}{\mu_s} \quad (\text{VI.17})$$

$$R_{le}(s) = \frac{L_o \rho_{0s} U_o}{2 \mu_s + \mu_{2s}} \quad (\text{VI.18})$$

The term $\overline{P_{s,\gamma}^*}$ in Eq. (VI.11) is given from Eqs. (III.61), (III.49), (III.50), and (III.38) up to (III.42) by

$$\overline{P_{s,\gamma}^*} = \overline{P_{0s\gamma}^*} + \left[\overline{U_{s,\gamma}^* (T_s^* + K_s^*)} \right] \quad (\text{VI.19})$$

with

$$\overline{p_{0s,\gamma}^*} = \overline{p_{s,\gamma}^*(ex)} + \sum_s \overline{p_{s,s,\gamma}^*(ex)} + \overline{p_{0s,\gamma}^*(cg)} + \overline{p_{0s,\gamma}^*(ch)} \quad (VI.20)$$

$$\overline{p_{s,\gamma}^*(ex)} = [\alpha_s \gamma_s^* (U_\gamma^* - U_{s,\gamma}^*)] \quad (VI.21)$$

$$\overline{p_{s,s,\gamma}^*(ex)} = [\alpha_s \gamma_{ss}^* (U_s^* - U_{s\gamma}^*)] \quad (VI.22)$$

$$\overline{p_{0s,\gamma}^*(cg)} = \delta \theta_{s\gamma}^* / (\rho_{0s} g_o) \quad (VI.23)$$

$$\overline{p_{0s,\gamma}^*(ch)} = [\Gamma_s^* (U_\gamma^* - U_{s\gamma}^*)] \quad (VI.24)$$

with

$$\delta \theta_{s\gamma}^* = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi \delta \theta_{s\gamma}^* . \quad (VI.25)$$

In a similar fashion one obtains the following radially weighted one-dimensional equation of motion for the fluid phase:

$$\frac{\partial}{\partial \tau} \overline{J_\gamma^*} + \frac{\partial}{\partial \gamma} [\alpha U_\gamma^{*2}] = \overline{D_\gamma^*} + g^* \overline{\alpha} - \frac{\partial}{\partial \gamma} \overline{p^*} - \overline{p_{f,\gamma}^*} \quad (VI.26)$$

with

$$\overline{\alpha U_\gamma^{*2}} = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi \alpha U_\gamma^{*2} \quad (VI.27)$$

$$\overline{D_Y^*} = \frac{\ell^*}{A^*} T_{\xi, \gamma}^* + \frac{\partial}{\partial \gamma} \overline{T_{\gamma\gamma}^*} \quad (\text{VI.28})$$

$$T_{\gamma, \xi}^* = \frac{1}{R_e^{(f)}} \left(\frac{\partial}{\partial \xi} U_{\gamma}^* \right)_{\xi=R^*} \quad (\text{VI.29})$$

$$T_{\gamma\gamma}^* = \frac{2\pi}{A^*} \left[\frac{1}{R_{1e}^{(f)}} \right] \int_0^{R^*} \xi d\xi \frac{\partial^2}{\partial \gamma^2} U_{\gamma}^* \quad (\text{VI.30})$$

with the following Reynolds numbers

$$R_e^{(f)} = \left(\frac{L_o \rho_o U_o}{\mu} \right) \quad (\text{VI.31})$$

$$R_{1e}^{(f)} = \frac{L_o \rho_o U_o}{2 \mu + \mu^2} . \quad (\text{VI.32})$$

We also have:

$$\overline{P^*} = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi P^*(\xi, \gamma, z) \quad (\text{VI.33})$$

$$\overline{P_{f, \gamma}^*} = \sum_s \left(\frac{\rho_{0s}}{\rho_0} \right) \left[\overline{P_{s, \gamma}^{*(ex)}} + \overline{P_{0s\gamma}^{*(ch)}} \right] + (\overline{U_{\gamma}^*} \overline{T^*}) . \quad (\text{VI.34})$$

VI.C. The Weighted One-Dimensional Enthalpy Equations

To derive the one-dimensional version of the enthalpy equations, we operate on Eqs. (V.14) and (V.15) with $\frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi$, to obtain:

$$\begin{aligned} \frac{\partial}{\partial \tau} (\alpha_s c_s^* T_s^*) + \frac{\partial}{\partial \gamma} (c_s^* T_s^* J_{s, \gamma}^*) &= \kappa_s \left[\frac{\partial^2 \overline{T_s^*}}{\partial \gamma^2} + \frac{\ell^*}{A^*} \left(\frac{\partial}{\partial \xi} T_s^* \right)_{R^*} \right] \\ &- (\alpha_s \overline{U_{s, \gamma}^*}) g^* + \overline{Q_{1s}^*} + \overline{\Phi_s^*} \end{aligned} \quad (\text{VI.35})$$

with

$$\overline{Q_{1s}^*} = \overline{Q_s^*} + \overline{[c_s^* T_s^* (\Gamma_s^* + K_s^*)]} \quad (\text{VI.36})$$

$$\overline{Q_s^*} = \overline{Q_{0s}^*} - \overline{(U_{s,\gamma}^* P_{0s,\gamma}^*)} \quad (\text{VI.37})$$

$$\overline{Q_{0s}^*} = \frac{2\pi}{A^*} \int_0^{R^*} \xi d\xi \overline{Q_{0s}^*} \quad (\text{VI.38})$$

where Q_{0s}^* is given by Eq. (IV.29), and the weighted dimensionless dissipation function $\overline{\phi_s^*}$ is given by Eq. (C.16) in Appendix C.

Similarly, we obtain for the fluid phase:

$$\begin{aligned} \frac{\partial}{\partial \tau} (\alpha c^* T^*) + \frac{\partial}{\partial \gamma} (c^* T^* J_\gamma^*) &= \kappa \left[\frac{\partial^2}{\partial \gamma^2} T^* + \frac{\ell^*}{A^*} \left(\frac{\partial}{\partial \xi} T^* \right)_{R^*} \right] - (\alpha U_\gamma^*) g^* \\ &+ \overline{Q_1} + \frac{\partial}{\partial \tau} \overline{P^*} + \left(\overline{U_f^* \frac{\partial P^*}{\partial \gamma}} \right) + \overline{\phi^*} \end{aligned} \quad (\text{VI.39})$$

with

$$\overline{Q_1^*} = \overline{Q^*} - \overline{(c^* T^* \Gamma^*)} \quad (\text{VI.40})$$

$$\overline{Q^*} = \overline{Q_0^*} + \sum_s \left(\frac{\rho_{0s}}{\rho_0} \right) \left\{ \overline{U_\gamma^* [P_{s,\gamma}^{*(ex)} + P_{0s,\gamma}^{*(ch)}]} \right\} \quad (\text{VI.41})$$

where Q_0^* is given by Eq. (IV.49) and ϕ^* is the radially weighted dimensionless dissipation function for the fluid phase given by Eq. (C.17) in Appendix C.

VII. SUMMARY AND DISCUSSION

Starting from Soo's basic multiphase equations we have developed a set of one-dimensional time-dependent hydrodynamic and enthalpy equations for a fluidized bed reactor by averaging over its cross sectional area.

For the time being we have not considered the following effects in our derivation of the FBC equations:

- (a) Forces to accelerate the apparent mass of the solid particle.¹¹
- (b) Basset force.¹¹
- (c) Heat exchange by radiation between solids and fluid or within each phase.
- (d) Electrodynamic effects.

Inclusion of the above effects will have carried this report beyond its scope. Within these restrictions, the material developed here forms the basis for a sequel to this report devoted to the development of stability studies and to the application of stochastic methods for FBC surveillance.

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LIST OF SYMBOLS

N_e = Number of solid species in the fluidized bed

$a_s^{(e)}$ = Radius of the e^{th} species of solid material of size s

$N_s^{(e)}$ = Number of different sizes of particles of the e^{th} species

ρ_{0s} = Bulk density of the s^{th} solid species (see Section II for the convention regarding the definition of a solid species)
 $[s = 1, \dots, N_e \cdot N_s^{(e)}]$

N_f = Number of fluid components in the fluid phase

ρ_{0q} = Theoretical density of the q^{th} fluid component ($q = 1 \dots N_f$)

Γ_q = Removal rate of the q^{th} fluid species ($\text{Kg/m}^3 \times \text{sec}$)

Γ_s = Generation rate of the s^{th} species ($\text{Kg/m}^3 \times \text{sec}$)

K_s = Generation rate of the s^{th} species by break down and conglomeration
 $(\text{Kg/m}^3 \times \text{sec})$

K_q = Generation rate of the q^{th} fluid component from reactions in the fluid phase ($\text{Kg/m}^3 \times \text{sec}$)

ρ_s = Density of the s^{th} "cloud" of solid particles (Kg/m^3)

ρ = Density of the fluid phase (Kg/m^3)

$\vec{U}$ = Velocity of the fluid phase (m/sec)

$\vec{U}_s$ = Velocity of the particles in the s^{th} solid species (m/sec)

γ_s = Momentum exchange rate parameter (sec^{-1})

$\gamma_{s,s}$ = Interparticle clouds momentum exchange rate parameter (sec^{-1})

τ = Viscous tensor for the fluid phase (pascals)

τ_s = Viscous tensor for the s^{th} solid component (pascals)

g_s = Fluid-particle cloud heat convection rate parameter (sec^{-1})

g_{ss} = Intercloud heat convection rate parameter (sec^{-1})

r_b = Bubble effective radius (m)

V_b = Bubble volume (m^3)

$U_{b\infty}$ = Speed of a rising isolated bubble (m/sec)

$\vec{U}_{mf}$ = Minimum fluidization velocity

α = Fluid phase volume fraction

α_s = Volume fraction of the s^{th} solid component

α_q = Volume fraction of the q^{th} fluid component

L_o = Characteristic length (m)

t_o = Characteristic time (sec)

U_o = Characteristic speed (m/sec)

$\tau = t/t_o$ dimensionless time variable

$\vec{x}^* = \vec{x}/L_o$ dimensionless coordinate vector

$\vec{\nabla}^*$ = Dimensionless gradient operator = $L_o \vec{\nabla}$

$\vec{U}^*$ = $\vec{U}/U_o$, dimensionless fluid phase velocity

$\vec{U}_s^*$ = $\vec{U}_s/U_o$, dimensionless velocity for the s^{th} solid species

$\vec{g}$ = Acceleration of gravity vector

g_o = Characteristic acceleration = U_o/t_o ($m \text{ sec}^{-2}$)

$\vec{g}^*$ = Dimensionless acceleration of gravity vector = $\vec{g}/g_o$

P = Pressure (pascals)

T = Temperature in the fluid phase ($^{\circ}C$)

T_s = Temperature in the s^{th} solid species ($^{\circ}C$)

T_o = Characteristic temperature ($^{\circ}C$)

$\hat{M}$ = Momentum flow tensor in the fluid phase

$\hat{M}_s$ = Momentum flow tensor in the s^{th} particulate phase

c = Specific heat (fluid phase) (joules/Kg $^{\circ}C$)

c_s = Specific heat (s^{th} solid species) (joules/Kg $^{\circ}C$)

c^* = Dimensionless specific heat (fluid phase)

c_s^* = Dimensionless specific heat (s^{th} solid species)

κ = Thermal conductivity (fluid phase) (joules/ $^{\circ}C \times \text{sec} \times m$)

κ_s = Thermal conductivity (s^{th} solid species) (joules/ $^{\circ}C \times \text{sec} \times m$)

κ^* = Dimensionless thermal conductivity (fluid phase)

κ_s^* = Dimensionless thermal conductivity (s^{th} solid species)

Φ = Gravitational potential (joules/Kg)

ϕ^* = Dimensionless gravitational potential

r = radial coordinate (m)

H = Azimuthal coordinate

z = Axial coordinate

ξ = Dimensionless radial coordinate = r/L_0

γ = Dimensionless axial coordinate = z/L_0

R_0 = Radius of the combustor (m)

R^* = Dimensionless radius = R_0/L_0

A = Transverse area of the combustor

A^* = Dimensionless transverse area of the combustor

$\vec{\lambda}$ = Unit vector in the direction of the fluid velocity

$\vec{\lambda}_s$ = Unit vector in the direction of the s^{th} solid species velocity vector

ℓ^* = Dimensionless perimeter of the combustor

m_s = Mass of the s^{th} solid species particles (Kg)

μ_0 = Fluid bulk viscosity (poise)

μ = Fluid viscosity (poise)

μ_{os} = Particle cloud bulk viscosity (poise)

μ_s = Particle cloud viscosity (poise)

$\eta(s, s')$ = Fraction of impactation (dimensionless)

$\vec{u}_r$ = Unit vector along the radial coordinate (dimensionless)

$\vec{u}_\theta$ = Unit vector along the azimuthal coordinate (dimensionless)

$\vec{k}$ = Unit axial vector (dimensionless)

$\vec{k}_s$ = Unit axial vector (for the s^{th} species velocity vector, $\vec{V}_s$)

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APPENDIX A. CHEMICAL REACTIONS, BREAK-UP, AND COALESCENCE

The following processes occur in a fluidized bed:

- (a) Chemical reactions between molecules of the (q) species in the fluid phase and particles of the (s) species in the particulate phase. The generation (removal) rates in the fluid and particulate phases ($\text{Kg/m}^3 \text{ sec}$) are Γ_q and Γ_s , respectively.
- (b) Break-up and coalescence in the particulate phase, expressed in terms of the rate parameter, K_s ($\text{Kg/m}^3 \text{ sec}$).
- (c) Chemical reactions in the fluid phase, which include change of state, for condensable fluids expressed in terms of the rate parameter K_q ($\text{Kg/m}^3 \text{ sec}$).

Assuming binary collisions only, the rates defined above can be written in terms of combination (C_{jk}^i) and splitting (S_{jk}^i) coefficients, as well as mass conservation.¹⁰ Let X_i be any of the above defined rates ($\Gamma_q, \Gamma_s, K_q, K_s$) for the (i) species of molecule (particle), then we have

$$X_{i/m_i} = \sum_{jk} \left[(C_{jk}^i - S_{jk}^i) \delta_i(j+k) + (S_{ik}^j - C_{ik}^j) \delta_j(i+k) \right] \quad (\text{A.1})$$

where

C_{jk}^i = number of effective collisions ($\text{m}^{-3} \text{ sec}^{-1}$) between particles (i) and (k) to form the particle of the (i) species.

S_{jk}^i = number of (i) species splitting into the (j) and (k) species ($\text{m}^{-3} \text{ sec}^{-1}$),

and where mass conservation for each elementary interaction is introduced

by the Kronecker symbols [m_i = mass of the (i) species of particles]

$$\delta_{i(j+k)} = \begin{cases} 0 & \text{if } m_i \neq m_j + m_k \\ 1 & \text{if } m_i = m_j + m_k \end{cases} \quad (\text{A.2})$$

For instance, from the general expression (A.1) one obtains

$$\frac{\Gamma_s}{m_s} = \sum_q \sum_{q'} \left[(C_{qq'}^s - S_{qq'}^s) \delta_{s(q+q')} + (S_{qq'}^q - C_{qq'}^q) \delta_{q(q+s)} \right] \quad (\text{A.3})$$

where (q) and (q') are molecular species in the fluid phase, and the index (s) indicates the (s) particle species. Similarly we have for K_s the break-up and coalescence rate, the following expression:

$$\frac{K_s}{m_s} = \sum_s \sum_{s'} \left[(C_{ss'}^s - S_{ss'}^s) \delta_{s(s+s')} + (S_{ss'}^{s'} - C_{ss'}^{s'}) \delta_{s'(s+s')} \right]. \quad (\text{A.4})$$

In view of the general Eq. (A.1), one easily verifies the relations (III.9) and (III.10).

The momentum and kinetic energy changes due to break-up and conglomeration, $\delta \vec{\theta}_s$ and δW_s , respectively, are given by

$$\delta \vec{\theta}_s = \sum_s \sum_{s'} \left\{ \left[C_{ss'}^s \vec{p}_{ss'}^{(s)} - S_{ss'}^s \vec{p}_s \right] \delta_{s(s+s')} + \left[S_{ss'}^{s'} \vec{p}_s - C_{ss'}^{s'} \vec{p}_{ss'}^{(s')} \right] \delta_{s'(s+s')} \right\} \quad (\text{A.5})$$

and

$$\delta W_s = \sum_s \sum_{s'} \left[(C_{ss'}^s W_{ss'}^s - S_{ss'}^s W_s) \delta_{s(s+s')} + (S_{ss'}^{s'} W_s - C_{ss'}^{s'} W_{ss'}^{s'}) \delta_{s'(s+s')} \right] \quad (\text{A.6})$$

where

$$\vec{P}_s = m_s \vec{U}_s \quad (A.7)$$

$$\vec{P}_{s's''}^{(s)} = \vec{P}_{s'} + \vec{P}_{s''} - \vec{P}_s \quad (A.8)$$

$$W_s = (\vec{P}_s)^2 / 2m_s \quad (A.9)$$

$$W_{s's''}^s = W_{s'} + W_{s''} - W_s . \quad (A.10)$$

From the above definitions one can verify the following relations:

$$\sum_s \delta \vec{\theta}_s = 0 \quad (A.11)$$

$$\sum_s \delta W_s = 0 . \quad (A.12)$$

The momentum $\delta \vec{\theta}_s$ and kinetic energy δW_s changes due to break-up and coagulation can be put in dimensionless form as follows. From Eqs. (A.5), (A.7), and (A.8) we have

$$\begin{aligned} \delta \vec{\theta}^* = \frac{\delta \vec{\theta}}{(\rho_{0s} U_o / t_o)} = \sum_s \sum_{s''} & \left[\left(C_{s's''}^{s*} \vec{\pi}_{s's''}^s - S_{s's''}^{s*} \vec{U}_s^* \right) \delta_{s(s+s'')} \right. \\ & \left. + \left(S_{ss''}^{s'*} \frac{m_{s'}}{m_s} \vec{U}_{s'}^* - C_{ss''}^{s'} \vec{\pi}_{ss''}^{s'} \right) \delta_{s'(s+s'')} \right] \end{aligned} \quad (A.13)$$

with

$$C_{s's''}^{s*} = C_{s's''}^s \left(\frac{t_o m_s}{\rho_{0s}} \right) \quad (A.14)$$

$$S_{s's'}^{s*} = S_{s's'}^s \left(\frac{t_o m_s}{\rho_{0s}} \right) \quad (\text{A.15})$$

$$S_{ss'}^{s*} = S_{ss'}^s \left(\frac{t_o m_s}{\rho_{0s}} \right) \quad (\text{A.16})$$

$$C_{ss'}^{s*} = C_{ss'}^s \left(\frac{t_o m_s}{\rho_{0s}} \right) \quad (\text{A.17})$$

$$\vec{\pi}_{s's'}^{s*} = \vec{p}_{s's'}^s / (m_s U_o) \quad (\text{A.18})$$

$$\vec{\pi}_{ss'}^{s*} = \vec{p}_{ss'}^s / (m_s U_o) \quad (\text{A.19})$$

We also have from Eqs. (A.6), (A.9), and (A.10)

$$\begin{aligned} \delta W_s^* = & \sum_s \sum_{s'} \left\{ \left[C_{s's'}^{s*} W_{s's'}^{*s} - S_{s's'}^{s*} (\vec{U}_s^*)^2 \right] \delta_{s(s+s')} \right. \\ & \left. + \left[S_{ss'}^{s*} (\vec{U}_s^*)^2 - C_{ss'}^{s*} W_{ss'}^{*s} \right] \delta_{s'(s+s')} \right\} \end{aligned} \quad (\text{A.20})$$

with

$$W_{s's'}^{*s} = W_{s's'}^s / m_s U_o^2 \quad (\text{A.21})$$

$$W_{ss'}^{*s} = W_{ss'}^s / m_s U_o^2 \quad (\text{A.22})$$

APPENDIX B. THE VISCOUS TENSOR AND THE RATE CONSTANTS,

$$\gamma_s, \gamma_{ss}, g_s, \text{ and } g_{ss}$$

We summarize in this appendix the expressions for the viscous tensors $\tilde{T}$ and $\tilde{T}_s$, as well as the rate constants $\gamma_s, \gamma_{ss}, g_s$, and g_{ss} , as given by Soo.⁶

The viscous tensor for the fluid phase (considered to be a Newton fluid) is given by

$$\tilde{T} = 2 \mu \tilde{d} + \mu_2 \tilde{z} \quad (\text{B.1})$$

where the deformation, $\tilde{d}$, and dilatation tensors, $\tilde{z}$, are given by (in cylindrical coordinates),

$$d_{rr} = \frac{\partial U_r}{\partial r} \quad (\text{B.2})$$

$$d_{r\theta} = d_{\theta r} = \frac{1}{2} \left[r \frac{\partial}{\partial r} \left(\frac{U_\theta}{r} \right) + \frac{1}{r} \frac{\partial U_r}{\partial \theta} \right] \quad (\text{B.3})$$

$$d_{\theta\theta} = \frac{1}{r} \frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r} \quad (\text{B.4})$$

$$d_{\theta z} = d_{z\theta} = \frac{1}{2} \left(\frac{\partial U_\theta}{\partial z} + \frac{1}{r} \frac{\partial U_z}{\partial \theta} \right) \quad (\text{B.5})$$

$$d_{zz} = \frac{\partial U_z}{\partial z} \quad (\text{B.6})$$

$$d_{zr} = d_{rz} = \frac{1}{2} \left(\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} \right) \quad (\text{B.7})$$

$$\gamma_z = \vec{\gamma} \cdot \vec{\nabla} \cdot \vec{U} \quad (\text{B.8})$$

where $\vec{\gamma}$ is the unit tensor and

$$\vec{\nabla} \cdot \vec{U} = \frac{1}{r} \frac{\partial}{\partial r} (r U_r) + \frac{1}{r} \frac{\partial U_\theta}{\partial \theta} + \frac{\partial U_z}{\partial z} \quad (\text{B.9})$$

The parameter, μ , is the fluid viscosity and μ_2 is given by

$$\mu_2 = \mu_0 - \frac{2}{3} \mu \quad (\text{D.10})$$

where μ_0 is the fluid "bulk" viscosity.

For the s^{th} component of the particulate phase one defines the deformation and dilatation tensors of the cloud $\gamma^{(s)}$ and $\gamma_z^{(s)}$, respectively by the same expressions for the fluid phase with the fluid velocity $\vec{U}$ replaced by $\vec{U}_s$ and

$$\vec{T}_s = \mu_s \gamma^{(s)} + \mu_{2,s} \gamma_z^{(s)} \quad (\text{B.11})$$

Following Soo,⁶ the viscous parameters are given by

$$\mu_s = A \mu \left(\frac{\alpha_{s,T}}{\alpha_s - \alpha_{s,T}} \right) \quad (\text{B.12})$$

$$\mu_2 = B \mu \left(\frac{\alpha_{s,T}}{\alpha_s - \alpha_{s,T}} \right)^3 - \frac{2}{3} \mu_s \quad (\text{B.13})$$

where A and B are constants of order unity, μ is the fluid viscosity, α_s the solid fraction of the s^{th} component and

$$\alpha_{s,T} = \sum_s \alpha_s . \quad (B.14)$$

Corrections for the non-Newtonian nature of the particulate phase have been considered by Soo.⁶

The rate parameter, γ_s , for fluid-particle momentum exchange is given by

$$\gamma_s = \frac{3}{8} C_D (\rho_0/\rho_{0s}) \left(\frac{|\vec{U} - \vec{U}_s|}{a_s} \right) \quad (B.15)$$

where C_D is the drag coefficient at relative speed $|\vec{U} - \vec{U}_s|$ and a_s the radius of the solid particle.

The rate parameter, $\gamma_{ss'}$, for momentum exchange between the s^{th} and s'^{th} kind of solid particles is given by

$$\gamma_{ss'} = \frac{3}{4} \frac{\eta^{(s,s')}(a_s + a_{s'})^2 |\vec{U}_{s'} - \vec{U}_s| \rho_s}{a_s^3 \rho_s \left(1 + \frac{m_{s'}}{m_s}\right)} \quad (B.16)$$

where $\eta^{(s,s')}$ is the fraction of impaction between particles s and s' .

The rate constant, g_s , for the convective heat transfer between fluid and particle is given by

$$g_s = \left[4\pi a_s^2 \kappa N_{\text{Nu}}^{(s)} / 2 a_s \right] / (m_s c_s) \quad (B.17)$$

where κ is the thermal conductivity of the fluid, c_s the specific heat of the solid and $N_{\text{Nu}}^{(s)}$ the Nusselt number

$$N_{\text{u}}^{(s)} = \frac{2 a_s h_s}{\kappa} \quad (B.18)$$

with h_s equal to the heat transfer coefficient between fluid and particle.

The rate constant $g_{ss'}$ for the convective heat transfer between the s^{th} and s'^{th} particle clouds is

$$g_{ss'} = C_{ss'} \left(\frac{\rho_{s'}}{c_s} \right) \frac{\sqrt{\kappa_s \kappa_{s'}}}{\rho_s \rho_{s'}} (a_s a_{s'})^{-1} \quad (\text{B.19})$$

where $C_{ss'} = C_{ss'}$ is a collision coefficient.

APPENDIX C. SOME USEFUL RELATIONS IN CYLINDRICAL COORDINATES

In this appendix we work out some of the relations needed in Section VI. We assume azimuthal symmetry. Also the only nonzero components of the velocities are the axial components U_z and $U_{s,z}$. We then have

$$\vec{\nabla}^* \cdot \vec{J}^* = \frac{\partial J^*}{\partial \gamma} = \frac{\partial}{\partial \gamma} (\alpha U_\gamma^*) \quad (C.1)$$

$$\vec{\nabla}^* \cdot \vec{J}_s^* = \frac{\partial J_{s,\gamma}}{\partial \gamma} = \frac{\partial}{\partial \gamma} (\alpha_s U_{s,\gamma}) \quad (C.2)$$

with

$$\gamma = z/L_0. \quad (C.3)$$

From Appendix B the viscous tensor becomes

$$T_{rr} = \mu_2 \frac{\partial U_z}{\partial z} \quad (C.4)$$

$$T_{\theta\theta} = \mu_2 \frac{\partial U_z}{\partial z} \quad (C.5)$$

$$T_{zz} = 2 \mu \frac{\partial U_z}{\partial z} + \mu_2 \frac{\partial U_z}{\partial z} \quad (C.6)$$

$$T_{zr} = \mu \frac{\partial U_z}{\partial r} = T_{rz} \quad (C.7)$$

with

$$T_{\theta r} = T_{r\theta} = T_{\theta z} = T_{z\theta} = 0 . \quad (C.8)$$

Next from the expression for the divergence of a tensor

$$\vec{\nabla} \cdot \vec{T} = \vec{u}_r \vec{\nabla} \cdot \vec{T}_r + \vec{u}_\theta \vec{\nabla} \cdot \vec{T}_\theta + \vec{k} \vec{\nabla} \cdot \vec{T}_z \quad (C.9)$$

with

$$\vec{T}_r = \vec{u}_r T_{rr} + \vec{k} T_{zr} \quad (C.10)$$

$$\vec{T}_\theta = \vec{u}_\theta T_{\theta\theta} \quad (C.11)$$

$$\vec{T}_z = \vec{u}_r T_{zr} + \vec{k} T_{zz} \quad (C.12)$$

we get

$$\vec{\nabla} \cdot \vec{T} = \vec{u}_r \left[\frac{1}{r} \frac{\partial}{\partial r} (r T_{rr}) + \frac{\partial}{\partial z} T_{zr} \right] + \vec{k} \left[\frac{1}{r} \frac{\partial}{\partial r} (r T_{zr}) + \frac{\partial}{\partial z} T_{zz} \right] \quad (C.13)$$

in view of the azimuthal symmetry of the system.

The momentum flow tensor becomes:

$$\vec{M}^* = \alpha (U_Y^*)^2 \vec{k} \quad (C.14)$$

with

$$\vec{\nabla} \cdot \vec{M}^* = \vec{k} \cdot \frac{\partial}{\partial Y} (\alpha U_Y^{*2}) . \quad (C.15)$$

From the definition (V.4) of the dissipation function, we get:

$$\overline{\phi_s^*} = \frac{t_o}{\rho_{0s} U_0^2} \overline{(\vec{\nabla} \vec{U}_s : \vec{T}_s)} = \frac{1}{R_{1e}^{(s)}} \left(\overline{\frac{\partial U_{s,Y}}{\partial \gamma}} \right)^2 + \frac{1}{R_e^{(s)}} \left(\overline{\frac{\partial U_{s,Y}}{\partial \xi}} \right)^2 \quad (C.16)$$

for the s^{th} solid species. Similarly one obtains for the fluid phase [see Eq. (V.10)] the result

$$\overline{\phi^*} = \frac{t_o}{\rho_0 U_0^2} \overline{(\vec{\nabla} \vec{U} : \vec{T})} = \frac{1}{R_{1e}^{(f)}} \left(\overline{\frac{\partial U_Y}{\partial \gamma}} \right)^2 + \frac{1}{R_e^{(f)}} \left(\overline{\frac{\partial U_Y}{\partial \xi}} \right)^2 \quad (C.17)$$

where the Reynolds numbers, $R_e^{(s)}$, $R_{1e}^{(s)}$, $R_e^{(f)}$, and $R_{1e}^{(f)}$, have been defined in Section VI.

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