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STUDIES OF HEAVY ION REACTIONS
AND TRANSURANIC NUCLEI

JOHN R. HUIZENGA
PRINCIPAL INVESTIGATOR

SEPTEMBER 1, 1978 - AUGUST 31, 1979

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Progress Report For
Period September 1, 1978 - August 31, 1979

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I. ABSTRACT

Muon capture in nuclei produced by prompt neutron emission and fission induced by radiationless transitions in the atomic cascade of muons bound to ^{237}Np , ^{239}Pu and ^{242}Pu is reported. Lifetimes of muons bound to the targets, the (A-1) nuclei and the neutron-rich fission fragments were measured. The lifetimes of the fission fragment muonic atoms are consistent with the muon of the fissioning muonic atom going predominantly to the heavy-fragment atom.

Neutron emission has been measured for the damped-collisions and fusion-fission components of the $^{165}\text{Ho} + ^{56}\text{Fe}$ reaction at a bombarding energy of 8.5 MeV/u. The c.m. neutron energy spectra for damped collisions indicate equal temperatures for the light and heavy fragments, a result consistent with equilibration of the excitation energy during the interaction time.

Structure in the energy spectra of specific final fragments produced in symmetric heavy-ion damped collisions is shown to correlate with statistical evaporation processes following the prediction of the primary fragments by the well-known nucleon exchange process which occurs on a very short time scale.

Yields measured with discrete Z and A resolution for Fe-like fragments produced in damped collisions for Fe-induced reactions show that both the fragment A/Z ratio and the relationship between the variances of the mass and charge distributions depend upon the degree of energy damping. The results are consistent with similar time scales for the evolution of the corre-

lations in the nucleon exchange process, A/Z equilibration and energy dissipation.

Currently available data on fusion excitation functions for heavy-ion induced reactions are well reproduced with a classical dynamical model based on the proximity nuclear potential and one-body nuclear dissipation. Extensive calculations have been performed to test the sensitivity of the calculated fusion cross sections to a number of parameters.

II. INTRODUCTION

The research activity of the group supported under United States Department of Energy Contract EY-76-S-02-3496.A004 is focussed on two outside-user programs. The principal program is a study of heavy-ion reaction mechanisms at the SuperHILAC accelerator of the Lawrence Berkeley Laboratory. The other program is concerned with a study of muon-induced reactions on heavy targets at the Los Alamos Meson Physics Facility - LAMPF.

In Section III, preprints of the five articles are included. These describe some of the experimental and theoretical results of the above research programs. Publications and Activities of the group are summarized in Section IV while the Personnel are listed in Section V.

The principal investigator has devoted 40% of his time during the academic year and 100% of his time during the summer to this contract and expects to continue at these rates during the remainder of the current term. Research Associates, J.R. Birkelund, W.U. Schröder and W.W. Wilcke have devoted 100% of their time to this contract and will continue at this level for the remainder of the term. Visiting Scientist and Research Associate, D. Hilscher devoted 100% of his time to this contract until March 31, 1979 when he terminated and returned to Berlin. M.W. Johnson has devoted 100% of his time to the muon project (in residence at LAMPF). He was a graduate student up to December 31, 1978 and then remained in the group as a postdoctoral fellow until August 31, 1979. Graduate students, A.D. Hoover

and L.E. Tubbs have devoted 100% of their time to this contract and will continue at this level for the remainder of the contract period. Graduate student, R.A. Atcher devotes 100% of his time to research; however, he is supported by an Argonne National Laboratory Fellowship.

To the best of our knowledge we have complied with the contract requirements. It is a pleasure to acknowledge the direct support of our reserach programs by the United States Department of Energy. In addition, we are grateful to the National Science Foundation for providing indirect support through the use of the facilities at the Nuclear Structure Research Laboratory.

III. RESEARCH PROGRAM

Evidence for Atomic Muon Capture by Fragments
from Prompt Fission of Muonic ^{237}Np , ^{239}Pu and ^{242}Pu

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Muon capture in nuclei produced by prompt neutron emission and fission induced by radiationless transitions in the atomic cascade of muons bound to ^{237}Np , ^{239}Pu and ^{242}Pu is reported. Lifetimes of muons bound to the targets, the (A-1) nuclei and the neutron-rich fission fragments were measured. The lifetimes of the fission fragment muonic atoms are consistent with the muon of the fissioning muonic atom going predominantly to the heavy-fragment atom.

There has been a long-standing puzzle of how to interpret the systematically higher disappearance rates of muons in actinide muonic atoms obtained from studies of the time distributions of muon-induced fission fragments as compared to measurements¹ involving muon-decay electrons and muon-capture neutrons and γ -rays. Bloom² suggested an excitation of shape-isomeric nuclear states during the muonic cascade, introducing an additional fission channel to be responsible for the apparent discrepancies in the data. In contrast, Hadermann³ emphasized the importance of sequential atomic muon capture by fragments resulting from prompt muon-induced fission. As is experimentally well established, high-energy muonic E1 or E2 cascade transitions to the 1s ground state may proceed by radiationless excitation of the nucleus leading to prompt fission or neutron emission in the presence of the muon. The muon then remains on the daughter isotope or, in a fission event, on one of the fragments, if not ejected during the fission or subsequent de-excitation processes. From the muonic 1s level of such a fragment, the muon then disappears with a characteristic rate lower than that of an actinide, an effect enhanced by the large A/Z ratio of fission fragments.

Evidence in support of Hadermann's proposal has been obtained by Ganzorig et al.⁴ by measuring muon-decay electrons in coincidence with prompt fission fragments and by Wilcke et al.¹ by observing long-lived components in the time spectra

of neutrons emitted from the highly fissile muonic ^{237}Np , ^{239}Pu and ^{242}Pu . The present experiments, employing the latter targets and a more sensitive fission fragment-neutron coincidence technique, were undertaken in order to demonstrate that the evidence for the recently discovered mechanism is independent of both the experimental technique and nuclide. Such studies have also the interesting potential of extending the systematics of muon capture into regions of short-lived nuclei and may provide information on the coupling between muonic and collective degrees of freedom.

The experiments were performed with the stopped muon channel of the Los Alamos Meson Physics Facility (LAMPF). As indicated in Fig. 1 showing a schematic of the apparatus, muons entered a counter telescope (1-4) and stopped in a fission chamber containing Ti foils on which ^{237}Np , ^{239}Pu and ^{242}Pu targets were deposited as oxides. Six NE 213 neutron detectors⁵, surrounding the fission chamber at a mean distance of 10 cm., were used to detect neutrons in coincidence with fission fragments.

Typical results of an experimental run for ^{239}Pu are displayed in Fig. 2, showing the fission fragment-neutron coincidence rate plotted in a two-dimensional diagram versus the times t_{fission} and t_{neutron} between a μ -stop in the target located in the fission chamber and the detection of the resulting coincident fission fragment and neutron, respectively. Three ridges are clearly observable, extending from the domi-

nant prompt peak towards longer times. Ridge A is due to delayed muon-capture induced fission of ^{239}Pu , where a fission fragment is detected in prompt coincidence with one of its de-excitation neutrons. Ridge B results from prompt emission of a neutron in a radiationless process followed by muon-capture induced fission of the daughter isotope ^{238}Pu . Ridge C is due to prompt muon-induced fission of the target followed by muon-capture in one of the fission fragments as identified by the appearance of a delayed neutron. This latter ridge is of primary interest to the present study.

Projecting the different ridges in Fig. 2 onto the respective time axes, one obtains the time distributions of the three classes of events such as shown in Fig. 3 for a sample run on ^{239}Pu , where a constant background has been subtracted. The distributions reflect the lifetimes of muonic ^{239}Pu (A), of muonic ^{238}Pu (B), and of the muonic fission fragments (C). The experimental lifetimes of the different types of events as obtained from exponential fits to the data of several runs are displayed in Table I. The data in ridge (B) illustrate for the first time a new technique for the measurement of the lifetime of actinide muonic atoms with one neutron removed from the target (although statistics are presently inadequate for a precise lifetime determination).

Whereas the experimental slopes of the time distributions are very similar for the target and its daughter produced by prompt neutron emission, the distribution corresponding to the

fission fragments (C) indicates a much lower rate of disappearance of the muon. The later distributions were analyzed with a theoretical fit function including a random-time background and two exponentials corresponding to muon capture in the light and the heavy fission fragments. From the data, however, one exponential component was found with statistical significance, restricting a possible contribution from a second component to less than 10%. The muonic fission fragment lifetimes are also collected in Table I.

Given the above results it is appropriate to determine the identity of the "average fission fragment" which captures the muon, by a comparison with predicted lifetimes of muons bound to typical fission fragments. Such predictions may be made with the Goulard-Primakoff formula⁶ which describes on the average the muon-capture rates for a wide range of elements. This formula is given by

$$\lambda_{GP} = KZ_{eff}^4 \left(1 - \frac{\epsilon_{\mu}}{m_{\mu}}\right) \left(1 - \frac{m_{\mu}}{m_N} - \frac{\epsilon_{\mu}}{m_N}\right) \times$$

$$\left[1 - 0.03 \frac{A}{2Z} + 0.25 \frac{A-2Z}{2Z} + 3.24 \left(\frac{A-Z}{2A} + \left|\frac{A-2Z}{8ZA}\right|\right)\right] \quad (1)$$

where the notation is that of Ref. 1. The theoretical mean muonic lifetime τ is given by $\tau = (\lambda_{GP} + \lambda_0)^{-1}$, where $\lambda_0 \approx 4.55 \times 10^6 \text{ sec}^{-1}$ is the muon-decay rate. Application of Eq. (1) to representative nuclei produced in low-energy fission (after n emission) reveals that heavy fragments have characteristic lifetimes $\tau_H \approx 120 \text{ nsec}$, while the corresponding lifetimes for typical light fragments are $\tau_L \approx 200 \text{ nsec}$. The

experimental muonic lifetimes $\tau(\text{ff})$ for fission fragments displayed in Table I agrees well with the estimated value of τ_H indicating that, after a prompt fission process, the muon is dominantly captured by the heavy fragment. This result for the prompt fission of muonic ^{237}Np , ^{239}Pu and ^{242}Pu is in agreement with measurements of Ganzorig et al.⁴ for ^{232}Th and ^{238}U obtained with a different technique. An additional piece of information supporting this interpretation is seen in the fact that the muonic lifetimes for fission fragments are equal within the errors for targets ranging from ^{232}Th to ^{242}Pu , whereas significant variations should have been observable in the case of dominant muon-capture by the light fragment.

The absence of sequential muon-capture by the light fission fragments suggested by these experiments bears some implications on the dynamics of the coupled system comprised of the muon and fissioning nucleus. This subject is presently being studied by various theoretical groups. If the nuclear fission mode is highly damped, the nuclear collective velocity from saddle to scission is expected to be much smaller than typical velocities of the muon in its ground state ($v_\mu \approx 0.36c$). Hence, the primary fission process itself is thought to be adiabatic in the rearrangement in the muon configuration resulting in the muon being captured predominantly into the 1s state of the heavy fragment. However, calculations of electron inner-shell ionization of fission products during nuclear fission⁷ and an experiment providing some evidence for muon ejec-

tion⁸ during radiationless-transition fission, indicate that the above view based on consideration of velocities alone may be oversimplified⁹. Information about this subject, as well as the time scale associated with the fission process, may be obtained by a search for muonic X-rays (Doppler shifted) from muonic fission fragments.

In summary, evidence is presented in this work for the dominance of sequential muon-capture by the heavy fragments produced in prompt muon-induced fission of muonic actinides. The resulting probabilities and muonic lifetimes associated with this process are consistent with the observations of long-lived components in the time distributions of neutrons from these muonic atoms. Excitation of isomeric states during the muon cascade have not been observed in the present experiment. It is obvious from the data such as illustrated in Fig. 2, that true muon-capture rates of actinides are only obtained from experiments eliminating contributions from muon-capture in products of prompt radiationless processes. The present experiments are clearly consistent with the predictions of the Goulard-Primakoff theory for muon-capture rates for nuclei far off the β -stability valley. However, a more definitive test of the model would require a coincidence experiment, in which the prompt fission fragments are identified with respect to their A and Z .

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Table I. Mean lifetimes in nsec of muons in the target muonic atoms, $\tau(A)$, in the muonic daughter atom after emission of a neutron, $\tau(A-1)$, and in muonic fission fragments, $\tau(ff)$.

Target	$\tau(A)$	$\tau(A-1)$	$\tau(ff)$
^{237}Np	71.3 ± 0.9	71 ± 10	126 ± 7
^{239}Pu	70.1 ± 0.7	70 ± 10	133 ± 7
^{242}Pu	75.4 ± 0.9	75 ± 10	133 ± 9

Figure Captions

- Fig. 1. Schematic diagram of experimental set up. For explanation, see text.
- Fig. 2. Fission fragment-neutron coincidence rate plotted in a two-dimensional diagram versus the times t_{fission} and t_{neutron} between the arrival of a muon in the fission chamber and the detection of the fission fragment and neutron, respectively (see text).
- Fig. 3. Decay curves of the three classes of events obtained by projecting the different ridges of Fig. 2 onto the respective time axes (see text).

LEAD

TARGET

SCINTILLATOR

POLYETHYLENE

FISSION CHAMBER

- μ^- Capture by
- (A) ^{239}Pu
  - (B) ^{238}Pu
  - (C) Fission Fragments

Fig. 2

NEUTRON EMISSION IN THE REACTION

$^{165}\text{Ho} + ^{56}\text{Fe}$ at $E_{\text{lab}} = 8.5 \text{ MeV/u}$

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ABSTRACT:

Neutron emission and α -particle emission in the $^{165}\text{Ho} + ^{56}\text{Fe}$ reaction have been measured for the damped-collision and fusion-fission components of this reaction at a bombarding energy of 8.5 MeV/u. The c.m. neutron energy spectra for damped collisions indicate equal temperatures for the light and heavy fragments, a result consistent with equilibration of the excitation energy during the interaction time. Pre-equilibrium neutron emission is found to contribute less than 5%. An out-of-plane neutron anisotropy $A_2 = 0.2 \pm 0.1$ is found. The measured ratios of the neutron multiplicities for the light and heavy fragments suggest a rapid approach of the fragment N/Z ratios during the collision to that corresponding to minimum potential energy of the composite system.

[NUCLEAR REACTIONS: $^{165}\text{Ho}(^{56}\text{Fe}, \text{H} \times \text{n}) E_{\text{lab}} = 476 \text{ MeV}$; measured $\sigma_n(E_n, \theta_n, \phi_n)$; deduced nuclear temperatures, neutron multiplicities, neutron-to-proton ratio N/Z, neutron out-of-plane anisotropy A_2 .]

I. INTRODUCTION

The extensive study of heavy-ion reactions at a few MeV/u above the Coulomb barrier has led to important advances in understanding these reactions.¹ The experimental results suggest that a large fraction of the kinetic energy available in the entrance channel is converted within the short interaction time of 10^{-22} to 10^{-21} sec into intrinsic excitation and collective rotational and deformation energy of the fragments. After the break-up of the intermediate complex into two final fragments the communication between the ions is terminated, and the excitation energy acquired by the two fragments leads to the evaporation of light particles such as neutrons, protons and α -particles, as well as γ -ray emission during the final stages of the de-excitation cascade.

Although emitted only in a secondary process, de-excitation particles from the final fragments carry important information on the damped-reaction mechanism, in particular on the energy-dissipation processes involved. Of considerable interest in this respect is the key question of the present investigation: whether or not statistical equilibration of the excitation-energy degree of freedom is achieved during the short interaction time encountered in a damped collision. Only if such an equilibration has occurred does one expect the final fragments to acquire the same nuclear temperature. Since the particle-evaporation spectra reflect the fragment nuclear temperature, it is possible to investigate the energy-

equilibration processes pertaining to damped collisions by measuring the secondary particles. The study of neutron emission is advantageous because of the absence of Coulomb effects which complicate the interpretation of charged-particle spectra in terms of a uniform nuclear temperature. A test of the degree of excitation-energy equilibration achieved in a damped collision is expected to be more conclusive for a system where the projectile $[A_p]$ and target $[A_T]$ masses are significantly different, since the reaction fragments from a symmetric system are more likely to exhibit similar excitation energies and temperatures in the exit channel. As a compromise between large projectile-target asymmetry and low fissility of the target-like fragment, the system $^{165}\text{Ho} + ^{56}\text{Fe}$ with $A_T/A_p = 2.95$ was chosen in this work.

In addition to the particle spectra reflecting the nuclear temperature, the particle multiplicities provide an independent measure of the excitation energy E_v^* of fragment v :

$$E_v^* = \sum_{i=n,p,\alpha,\gamma} \langle E_{iv} + B_{iv} \rangle M_{iv} \quad (1)$$

where M_{iv} , E_{iv} , and B_{iv} are the multiplicity, kinetic energy and binding energy of particle i from fragment v . A complete experiment would require a measurement of all light particles including the γ -rays for both fragments, separately. However, since neutron emission is the dominant de-excitation channel, the neutron multiplicity represents an independent measure of the excitation energy requiring only approximate estimates for the corrections due to charged-particle and γ -ray emission.

Depending on the time scale on which light particles are emitted, there are three possible sources of these particles and thus, three different frames of reference to consider. These sources are the intermediate double-nucleus complex and the two final fragments, fully accelerated by their mutual Coulomb field. It is the goal and also the main complication of the data analysis to identify the contributions from these sources. This is possible, in principle, by the use of the distinct features of particle emission in different processes. De-excitation particles from the final, fully accelerated fragments are likely to exhibit evaporation-like energy spectra and angular correlations determined by the velocities and emission angles of the fragments. Pre-equilibrium particles, on the other hand, are emitted on a short time scale and will reflect the kinematics of the double-nucleus complex. Depending on the physical processes producing these pre-equilibrium neutrons, different angular correlations and energy spectra will result. Fermi-jet or PEP² (promptly emitted particles) neutrons will be highly energetic with a maximum energy of ≈ 20 MeV for the reaction under investigation and will have a maximum intensity around the beam direction. However, highly energetic neutrons originating from a reaction mechanism described by the piston model³ will preferentially be emitted in the direction of the linear-momentum transfer. For the strongly damped part of the reaction, this corresponds approximately to the recoil-direction of the

heavy fragment, whereas it is close to the beam direction for fusion-fission events.

There exists already some experimental evidence^{1,4} that the neutron-to-proton ratio (N/Z) of the final fragments from damped collisions equilibrates towards the N/Z of the composite system (1.38 in the present case). These observations are in accord with theoretical attempts⁵ to describe the relaxation of the N/Z degree of freedom which suggest a very short time constant ($\sim 10^{-22}$ sec) for this process. A study of neutron emission adds to the information on this relaxation mechanism because the neutron multiplicities depend on the binding energies of the neutron and competing other light particles, and thus on the N/Z ratio of the emitting fragment. This dependence is particularly strong for the asymmetric system $^{165}\text{Ho} + ^{56}\text{Fe}$ studied in this work. The N/Z ratios are very different for the nuclei in the entrance channel: 1.15 for ^{56}Fe and 1.46 for ^{165}Ho . In addition, the binding energies in the Fe region of the atomic mass table⁶ are very sensitive to the N/Z ratio. Hence, one expects a quite accurate determination of the pre-evaporation N/Z value from the measured neutron multiplicities to be possible for the Fe-like fragments produced at various stages of the reaction.

II. EXPERIMENTAL PROCEDURE AND DATA ANALYSIS

A. Experimental Setup

The experiments were performed at the Lawrence Berkeley Laboratory SuperHILAC accelerator. Neutron and charged-particle emission was studied in the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$ at 8.5 MeV/u bombarding energy, corresponding to 4 MeV/u of relative motion above the Coulomb barrier. Self-supporting ^{165}Ho targets were used with thicknesses of $435 \mu\text{g}/\text{cm}^2$ and $244 \mu\text{g}/\text{cm}^2$, in an initial overview experiment. A schematic setup of the experiment is shown in Fig. 1. The beam was dumped 4 m downstream from the target into a well-shielded graphite faraday cup. Even though the beam was carefully kept off the collimator slits ahead of the scattering chamber, these slits were also shielded by a combination of iron bricks and water.

Six liquid-scintillator (NE213) detectors of dimensions 2.5 cm x 12.7 cm, 5.1 cm x 12.7 cm and 3.8 cm x 10.2 cm (thickness x diameter) were used to detect the neutrons. In connection with these detectors highly efficient γ -n pulse-shape discrimination circuits⁷ were employed. In order to suppress low-energy γ -rays and atomic X rays ($K_{\alpha}(\text{Ho}) = 47 \text{ keV}$), all scintillators were covered with 2 mm-thick Pb-caps. A ΔE -E solid-state detector telescope subtending a solid angle of 22 msr was employed to measure the α -particles. Another ΔE -E solid-state detector telescope subtending a solid angle

of 4 msr was placed at selected angles to detect the projectile-like fragments in coincidence with neutrons and α -particles.

For the final experiment the heavy-ion telescope was positioned at laboratory angles $\theta_T = 25^\circ, 20^\circ, \text{ and } 16^\circ$. These angles are all forward of the grazing angle, $\theta_{gr} = 32.5^\circ$.⁸ The neutron detector angles θ_n were the same for all settings of θ_T and chosen as $25^\circ, 10^\circ, -10^\circ, -45^\circ, -70^\circ$ in the reaction plane ($\phi_n = 0$). One single neutron detector was placed out of the reaction plane at $\theta_n = 0^\circ$ and $\phi_n = 85^\circ$.^{Insert} The α -telescope was positioned at $\theta_{LI} = 53^\circ$ and -57.5° .

The angles $\theta_n = 25^\circ, 10^\circ$ and $\theta_n = -45^\circ$ and -70° were used as master detector angles (cf. Section II.c) for the detected light and the correlated heavy fragments, respectively. The neutron detectors at $\theta_n = 10^\circ$ and -10° were located as close as possible to the beam direction in order to detect possible Fermi-jet neutrons. Although the fragment rest frames assumed in the lab.-to-c.m. transformation (cf. Section II.c) do not apply to neutrons emitted from the composite complex, those should show up as highly energetic neutrons. However, the c.m. energies of Fermi-jet neutrons calculated by this procedure are somewhat over- or underestimated in the rest frame of the heavy or light fragment, respectively.

The neutron energy was determined with a time-of-flight method whereby the time difference between an event in the

Insert: The angle ϕ_n is defined as the angle between the reaction plane, determined by the heavy-ion telescope and the beam direction, and the neutron detector. Hence, the angle $\phi_n = 0$ corresponds to any direction in the reaction plane. (see Fig. 1).

ion telescope and one in a neutron detector was measured. This time was corrected, event by event, for the fragment-time of flight from the target to the ion telescope as deduced from the measured fragment Z and energy. The overall time resolution achieved for γ -rays was typically 1 to 1.5 ns resulting in an energy resolution of 0.5 to 2 MeV for neutron energies between 5 and 10 MeV, for the flight path of 70 cm used in the experiment.

A typical time-of-flight spectrum of events identified as neutrons by the pulse-shape discrimination circuitry is shown in Fig. 2. The spectrum was recorded with a neutron detector placed at an angle of $\theta_n = 10^\circ$ in coincidence with the heavy-ion telescope at $\theta_T = 20^\circ$. The small peak on the left of the broad neutron time distribution is due to γ -ray events not rejected by the pulse-shape discrimination system. Favorable background conditions are demonstrated by the small number of events in the random-time range below 90 and above 170 ns.

For each event, 12 parameters were written on magnetic tape including the energy loss ΔE and the residual energy E of both the heavy-ion and the α -particle telescopes, the time difference between an event in the neutron detector or in the heavy-ion telescope, the proton recoil energy signal from each neutron detector, and an analog route signal which distinguished between the various neutron detectors and between neutrons and γ -rays. This made it possible

to mix the output signals of several neutron detectors into one parameter. An event was only recorded if the heavy-ion telescope E-detector had fired.

The neutron detection efficiency was calculated with a computer code developed by Textor and Verbinski⁹ and checked against measured values of Drosge¹⁰. Calculated and experimental¹⁰ efficiencies agreed with each other within 5% for neutron energies between 2 and 10 MeV. The detection efficiency depends strongly on the threshold set on the proton-recoil energy. This threshold was, therefore, carefully adjusted to 1.08 or 0.5 MeV equivalent neutron energies^{11,12} using radioactive γ -sources. This calibration was checked continuously during the experiment by comparing the neutron energies as determined by the time of flight and the proton recoil energy spectrum. In addition, tests were performed before and after the experiment in which neutrons from ^{252}Cf fission fragments were measured in a geometry similar to that of the data-taking runs. Including the uncertainty of the neutron threshold, the accuracy of the absolute neutron efficiency was estimated to be approximately 10%.

Calculations, as well as test measurements with ^{252}Cf fission neutrons¹³ indicated that neutrons produced in the target and scattered off the walls of the scattering chamber in the initial setup posed serious background problems, and very large corrections were necessary due to neutron in- and out scattering. In order to reduce this background, a thin

(3.2 mm) stainless-steel shell was designed for the scattering chamber (76 cm diameter). The resulting attenuation of the neutron flux was calculated to amount to 8% and 4% for neutron energies of 4 and 14 MeV, respectively. Furthermore, all mounts of the solid-state detector telescopes were made as light as possible. The massive base of the chamber gave rise to delayed γ -rays from (n,γ) processes in aluminum, which could be clearly identified in the γ -time distribution, emphasizing the importance of an efficient γ -n pulse-shape discrimination in such experiments. These events, if not identified as γ -rays, would have been mistaken as highly energetic neutrons.

B. Complete Angular Distributions

In order to identify the major neutron emitters, complete in-plane and out-of-plane angular distributions were measured for neutrons in coincidence with the reaction fragments detected at a laboratory angle $\theta_T = 23^\circ$. The detection threshold was set at 0.5 MeV neutron energy in this case. Figures 3 and 4 show the angular dependence of the neutron cross section and of the mean neutron energy $\langle E_n \rangle$, respectively, for damped reaction fragments with $10 \leq Z \leq 29$, integrated over total kinetic-energy loss. As can be seen from Fig. 3, there is a clear concentration of the neutron intensity at the scattering angle of the projectile-like fragment indicated by the vertical arrow ($\theta_T = 23^\circ$). At this angle the neutrons

have also the highest mean kinetic energies as shown in Fig. 4. However, tails of the neutron angular distribution corresponding to significantly lower neutron energies (i.e. Fig. 4) extend to large angles on both sides of the recoil direction. They are due to neutrons from the corresponding heavy fragment moving with considerably smaller velocities (0.2 MeV/u) than the Fe-like fragment (5-7 MeV/u). The dashed and dashed-dotted curves shown in Figures 3 and 4 are the results of an evaporation calculation using Eqs. (A4) and (A6) of the appendix, respectively. The assumption made in this calculation implies an essentially isotropic emission of neutrons from the fully accelerated fragments in the fragments' rest frames. Obviously the data are in reasonable agreement with the solid curve accounting for the contributions from both the heavy and light fragments. The inserts in Figures 3 and 4 show the variation of the respective quantities with the out-of-plane angle ϕ which was defined previously. In

Fig. 5, the respective angular distributions are shown for the fusion-fission part of the reaction, i.e. $30 \leq Z \leq 55$.

From these results one concludes that the neutrons are, indeed, predominantly emitted from the fully accelerated fragments. Supported by this observation, laboratory neutron energies and intensities were transformed into the rest frames

of the fully accelerated light and heavy fragments. Any deviation from this assumption is then detectable in a comparison of the center-of-mass neutron intensities and energies at different neutron detection angles, each of these angles having a different sensitivity to neutrons from either of the two separated fragments or from the composite complex.

C. Data Analysis

In the lower part of Fig 6, the double-differential neutron cross sections for the detected light (solid lines) and the correlated heavy fragments (dashed lines) are shown for a total kinetic energy loss E_{Loss} of 200 MeV in a contour diagram plotted versus the neutron detection angle and the neutron laboratory energy. The respective scattering angles of the two fragments are indicated by the vertical arrows. This two-dimensional plot demonstrates very clearly the separation in energy and emission angle of neutrons emitted from the two sources. In fact, for total kinetic energy losses smaller than 200 MeV corresponding to partially damped events, the separation is considerably better than indicated in Fig 6. Because of this good separation it is possible to use not only the emission angle but also the neutron lab. energy as indications of the origin of the detected neutrons. In the following, the neutron detector angles in the direction of the light- and heavy-fragment scattering angles are denoted as the master angles for neutrons from the light and heavy fragments, respectively.

At these angles, the intensity of neutrons from one fragment is the dominant part of the measured spectrum, whereas the contribution from its reaction partner is small (See Fig. 6).

For each neutron event in any of the neutron detectors located at angles θ_n and ϕ_n , the factors $f_v(E_n, \theta_n, E_v, \theta_v, m_v)$ applying to the transformation from the lab. to the c.m. system were calculated for both fragments. Here $m_v = (A/Z) \cdot Z + \delta m_v$ is the pre-evaporation mass of fragment v . The quantities E_v , θ_v and δm_v are the measured energy, angle and calculated evaporated mass, respectively, of the light fragment ($v = L$) or the correlated heavy fragment ($v = H$). In order to determine the mass of the detected fragment, it was assumed that its A/Z ratio was that of the valley of beta stability. With these transformation factors, the double differential cross section for neutrons from fragment v becomes

$$\left(\frac{d^2\sigma_v}{d\Omega dE}\right)_{cm} = f_v \cdot C_v \cdot \left(\frac{d^2\sigma}{d\Omega dE}\right)_{lab} \quad (2)$$

where C_v is a correction factor accounting for the contribution from the respective correlated fragment. For events with total kinetic energy losses of 200 MeV and 50 MeV this correction amounts to 30% and 7%, respectively, at the light-fragment master angle, and 5% to 1%, respectively, at the heavy-fragment master angle. Hence, at these angles the final results are largely independent of the assumption used to calculate C_v .

The correction factors C_v were calculated assuming evaporative neutron spectra and isotropic emission in the rest frame of each fragment. For a given angle, this results in a ratio R for the yields of neutrons of energy E_n from the heavy and light fragments given by

$$R = (M_H/M_L) \cdot (E_{cm}^H/E_{cm}^L)^{1/2} \cdot \exp\{(-E_{cm}^H/T_H) + (E_{cm}^L/T_L)\} \quad (3)$$

where M_v , E_{cm}^v and T_v are the neutron multiplicity, neutron c.m. energy and nuclear temperature of fragment v , respectively. The value of E_{cm}^v is calculated using the laboratory neutron energy E_n and the respective fragment velocity. With this ratio R one obtains $C_L = 1/(1+R)$ and $C_H = R/(1+R)$. Relation (3) results in $\langle R \rangle \ll 1$ or $\langle R \rangle \gg 1$ for the master angle of the light or heavy fragment, respectively, yielding $\langle C_v \rangle \approx 1$ in both cases. This indicates again that the results obtained for the master angles are largely independent of the details used to calculate R .

Values of the ratio M_H/M_L and of the temperatures T_H and T_L have to be specified as starting parameters for an iterative analysis of the data. In the procedure applied here, M_H/M_L was initially set equal to the fragment mass ratio m_H/m_L calculated event by event as described above, and the total excitation energy was distributed in proportion to the fragment masses. This implies that $T_H = T_L = (E_{loss})^{1/2}$ was assumed as an initial condition in the iterative procedure. One can

then use the results of the first event tape replay for M_H/M_L and T_H/T_L as input parameters for the second iteration. However, the final results were largely independent of these assumptions and very close to the situation described above. It was, therefore, not necessary in most of the analyses to make a second iteration.

In order to test quantitatively the event analysis described above with respect to the sensitivity of the final result to the nuclear temperatures and neutron multiplicities, a test program was used to generate synthetic neutron spectra. For instance, various spectra were analyzed corresponding to ratios T_H/T_L and $(M_H/M_L) \cdot (m_L/m_H)$ ranging from 1/3 to 3. In each case the analysis gave results within a few percent of the correct temperatures and multiplicities of the test spectrum.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A considerable amount ($\approx 25\%$) of the total reaction cross section of the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$ at the bombarding energy of 476 MeV is associated with fusion-fission leading to an average fragment Z of about 46 corresponding to symmetric fission of the composite system. In comparison, the strongly damped part of the reaction cross section is concentrated around $Z = 26$ (Fe) with respect to the detected light fragment. No contribution from sequential fission of the target-like nucleus was observed. In the following, the strongly damped

part of the reaction is defined by the requirement $20 \leq Z \leq 30$ and fusion-fission corresponds to $35 \leq Z \leq 55$.

A. Nuclear Temperatures of the Strongly Damped Reaction Fragments

In Fig. 7 the c.m. neutron energy spectra in the rest frames of the light (open circles) and heavy (filled squares) fragments are shown for all 6 neutron detector angles. The data are weighted averages over all three telescope angles $\Theta_T = +16^\circ, 20^\circ$ and 25° and total kinetic energy losses larger than 36 MeV. The error bars depicted include only statistical errors and do not account for the uncertainty of the neutron detection efficiency and background subtraction which become appreciable at high neutron energies. This makes the results above $E_{cm} \approx 10$ -15 MeV considerably more uncertain than indicated by the error bars. The interpretation of high-energy parts of the neutron spectra is further complicated by the deterioration of the neutron energy resolution at high energies (≈ 4 MeV at $E_n = 20$ MeV). The solid curves are calculated evaporation spectra with a temperature of 2 MeV (See Eq. (A1) with $n=0.5$) normalized to the spectra at the master angles for the light fragment at 25° and 10° and for the heavy fragment at -45° and -70° (heavy solid lines).

In addition to the neutron energy cut-off due to the detection threshold (1.08 MeV) there is also a kinematical cut-off, if a neutron detector is not aligned with the recoil

direction of the respective fragment, due to the strong focusing of low-energy neutrons into the recoil direction. This effect causes the low-energy fall-off of the $\theta_n = -45^\circ$ neutron spectrum for $E_{cm} \leq 5$ MeV in the case of neutrons associated with light fragments (See Fig. 7).

As has been discussed previously (cf. Section II.c), the results obtained for the master angles are largely independent of the specific assumptions made in performing the analysis. Inspecting these energy spectra closely, one observes that, with the exception of very high E_{cm} , the spectra corresponding to light and heavy fragments have approximately the same exponential decrease. Since the slope of the neutron spectrum reflects the nuclear temperature of the emitting fragment, one arrives at the important conclusion that the temperatures of the light and heavy fragment must be similar. Furthermore, the contribution of high-energy pre-equilibrium neutrons appears to be small, as can be inferred from the high-energy parts of these spectra. Integrating the neutron intensity in excess of the evaporation spectrum results in an upper limit of 5% of a possible contribution of pre-equilibrium neutrons. Assuming furthermore that a pre-equilibrium contribution is associated only with the first neutron of each cascade, one has to multiply the above result with the average neutron multiplicity. This results in an upper limit of 25% for the probability for the first neutron to be emitted by the heavy fragment in a pre-equilibrium process. However, including

a reasonable estimate for the systematic errors mentioned above the high energy points are almost consistent with the evaporation spectra given in Fig. 7. This assigns a low confidence level to the value deduced for the upper limit of the pre-equilibrium neutron emission probability.

The most forward neutron detectors at $\theta_n = \pm 10^\circ$ should be specifically sensitive to highly energetic neutrons from the Fermi-jet mechanism. However, the spectra show almost no contribution from highly energetic neutrons at these angles, thus providing no support for this emission mechanism.

The energy spectra of Fig. 7 have been averaged over all values of E_{Loss} larger than 36 MeV and, hence yield information only about the average degree of energy equilibrium. However, most significant for an understanding of the equilibrium mechanism appears to be its evolution as the interaction time proceeds. Adopting the assumption¹⁴ that the interaction time depends monotonically on the E_{Loss} , it is possible to infer the progress of energy equilibration with time by measuring the fragment temperatures as a function of E_{Loss} . The experimental dependence of the temperature on energy loss is shown in Fig. 8. According to the evolution of interaction times mentioned above, an E_{Loss} of 100 MeV corresponds^{1,8} to an interaction time of either ≈ 4 or $\approx 8 \cdot 10^{-22}$ sec. assuming the non-sticking or sticking situation, respectively. One observes that the temperatures deduced from the c.m. neutron energy spectra of the master angles are, within the experimental uncertainty, the same for both fragments already for interaction times as short as $\approx 5 \cdot 10^{-22}$ sec. Furthermore, the points follow closely

the theoretical curve representing the Fermi-gas relation between temperature and excitation energy assuming that the entire loss is converted into intrinsic excitation and the level density parameter to be given by $a = (A/8) \text{ MeV}^{-1}$. LeCouteur and Lang¹⁵ have derived an effective temperature $T = (11/12) \cdot \sqrt{E^*/a}$ for a neutron cascade where E^* is the initial excitation energy. This relation is shown by the curve in Fig. 8. Although the limited accuracy of the data does not allow firm conclusions in this respect, the results are consistent with the assumption that the intrinsic-excitation energy E^* is given by the measured E_{Loss} of the asymptotic fragment kinetic energies, a quantity that may, conceptually, be different from the amount of kinetic energy that is irreversibly dissipated into intrinsic degrees of freedom.

In concluding this section, it is emphasized again that the observation of equal temperatures for light and heavy fragments suggests strongly that nearly complete equilibration of the excitation energy is reached in a damped heavy-ion collision already for interaction times as short as $5 \cdot 10^{-22} \text{ sec.}$

B. In-Plane and Out-of-Plane Correlation

The fact that the experimental neutron spectra obtained for both fragments at various angles in the reaction plane agree so well in shape and magnitude with the spectra measured at the master angles is consistent with a process where neutrons are emitted isotropically in the rest frames of the fully accelerated fragments. However, so far this has only been

established for neutrons emitted in the reaction plane. In fact, the assumption of isotropic emission leads to disagreement with the out-of-plane data as shown in Fig. 7 for $\theta_n = 0^\circ$ and $\phi_n = 85^\circ$, corresponding to an emission angle of approximately 90° in the rest frame of the heavy fragment. The experimental points lie systematically below the evaporation spectra (curves) deduced from the spectra measured at the master angles. From this one obtains an out-of-plane anisotropy $A_2 = [\frac{d\sigma}{d\Omega}(\phi = 0^\circ) / \frac{d\sigma}{d\Omega}(\phi = 90^\circ)] - 1$ of 0.2 ± 0.1 for neutrons from the heavy fragment. The intensity of neutrons from the light fragment measured at this angle cannot be used to extract an independent value of A_2 for the light fragment because it is strongly dependent on the specific assumptions made in the analysis.

According to the theory of Ericson and Strutinsky¹⁶, the out-of-plane angular correlation W of particles evaporated from a nucleus with temperature T and moment of inertia j is a function of the spin I of the nucleus and the angular momentum l carried away by a particle:

$$W(\phi) \approx 1 + (1/2) \cdot \left(\frac{\hbar^2}{2jI}\right)^2 \langle I^2 \rangle \cdot \langle l^2 \rangle \cdot \cos^2 \phi \quad (4)$$

where $\langle I^2 \rangle$ and $\langle l^2 \rangle$ are the mean-squared values corresponding to the normal of the scattering plane. Assuming a rolling or sticking situation in the collision, typical values of the asymmetry are estimated to amount to 4% or 12%, respectively, in qualitative agreement with the experiment. However, since

the angular momentum ℓ carried away by a nucleon evaporated from a spherical nucleus is small (1 to $2\hbar$ in the present case), the measurement of the out-of-plane angular correlation does not provide as sensitive a test of the fragment alignment as would be possible with heavier particles.

C. Neutron Multiplicities

As has been pointed out in Sec. I (cf. Eq. 1), the neutron multiplicity also measures the excitation energy, although somewhat indirectly. The experimental neutron multiplicities for the light (open circles) and heavy (filled squares) fragments are plotted in the lower part of Fig. 9 versus the measured atomic number Z_L of the detected fragment and that of the correlated heavy fragment assumed to be given by $Z_H = Z_T + Z_P - Z_L$. The measured fragment Z values have not been corrected for possible charged-particle evaporation since no accurate experimental data are available. As shown in Fig. 9, the multiplicities increase approximately linearly with increasing atomic number or mass of the fragment. At $Z \approx 46$ (dashed vertical arrow) a discontinuity appears between the multiplicities deduced for the detected (open circles) and the correlated (filled squares) fragments, although they should be equal for the symmetric-fission fragments. This discrepancy can be understood in terms of charged-particle emission from these fragments. Using the evaporation code of Beckerman et al.¹⁷, one obtains a charged-particle (protons

and α -particles) multiplicity of ≈ 0.75 (indicated by the crosses in Fig. 9) or an average evaporated charge of 1.25 per fragment, for fragments with $Z \approx 46$. This is in qualitative agreement with approximate α -particle measurements performed in this work yielding an estimate for $M_\alpha \approx 0.5 - 1.0$ for fusion-fission fragments. If then a correction is applied accounting for charged-particle emission, the circles and squares have to be moved to the right and left on the Z axis, respectively, by 1.25 units each, resulting in a smooth transition between the two sets of data. The mean neutron multiplicity obtained for the fusion-fission fragments is $M_n = 5.7 \pm 0.7$ (See Table 1).

Also shown in Fig. 9 are the results of an extensive evaporation calculation using the code MBII¹⁷ (filled triangles and crosses). The dashed and dashed-dotted curves are drawn through the calculated points. The basic assumption for the parameters used in the calculation were: (1) the excitation energy was taken to be apportioned according to the mass ratio, (2) the binding energy of the first evaporated particle was assumed corresponding to a completely equilibrated ratio N/Z ($= 1.38$), and (3) the angular momentum used was estimated from the sticking limit for two touching spherical nuclei. The excitation energy was calculated from the average value of E_{Loss} and the ground-state Q -value Q_{gg} . Within the experimental errors and the uncertainty of the evaporation calculation (estimated to be 10-20%), one obtains very good agreement with the

measurements, except around $Z = 26$ and $Z = 67$ (solid vertical arrows) corresponding to the projectile and target Z , respectively. Here, however, the assumption of the sticking limit is not very realistic. By decreasing the angular momentum of the light fragment from $17 \hbar$ to $11 \hbar$, more thermal energy (≈ 5 MeV) is available for neutron evaporation and less for γ -ray emission. This results in an increased multiplicity represented by the upper triangle for $Z = 26$, as is shown in Fig 9. Although there are no γ -ray multiplicity measurements available for the system $^{165}\text{Ho} + ^{56}\text{Fe}$, a similar effect has been observed by Aleonard et al.¹⁸ for the dependence of the γ -ray multiplicity on the fragment atomic number in the reaction $^{165}\text{Ho} + ^{86}\text{Kr}$ at 618 MeV, which has a pronounced minimum at the projectile Z . Physically, this implies a close relation between the number of transferred particles which deposit their relative linear momentum and the transferred angular momentum.

In Fig. 9, the crosses connected by a dashed-dotted line represent the results of the calculation for the charged-particle multiplicities $M_{p+\alpha}$. The fact that charged-particle emission from fragments around $Z \approx 26$ is quite improbable is directly tied to the binding-energy assumption (2) listed above. This assumption is discussed in more detail below.

In the upper part of Fig. 9, the ratio M_H/M_L of the neutron multiplicities for the light detected fragment (M_L) and the heavy correlated fragment (M_H) are plotted as a function of Z_L . The solid curve represents the mass ratio m_H/m_L deduced

from the measured Z_L assuming $N/Z = 1.38$, i.e., N/Z of the composite system. Within the experimental error bars, good agreement with the relation $M_H/M_L = m_H/m_L$ is obtained.

Neglecting γ -ray emission this ratio is given to first approximation by

$$M_H/M_L = \frac{E_H^*}{E_L^*} \cdot \frac{\langle E_n + B_n \rangle_L}{\langle E_n + B_n \rangle_H} \cdot \frac{\{(\Gamma_p/\Gamma_n) + (\Gamma_\alpha/\Gamma_n) + 1\}_L}{\{(\Gamma_p/\Gamma_n) + (\Gamma_\alpha/\Gamma_n) + 1\}_H} \quad (5)$$

where E_v^* , E_n , B_n and Γ_i stand for the excitation energy of fragment v , the mean kinetic energy carried away per neutron ($E_n = 1.5 \cdot T$, see Eq. (A2), $n = 0.5$), the neutron binding energy and the evaporation width for particle i , respectively. Since (Γ_p/Γ_n) and (Γ_α/Γ_n) are sensitive to the difference of binding energies $(B_{p,(a)} - B_n)$ and thus the N/Z ratio, the third term also depends sensitively on the pre-evaporation N/Z ratio. The use of assumption (2) given above, results in a very small amount of charged-particle emission from either fragment and approximately equal binding energies B_{nL} and B_{nH} for the system under investigation. With the above statement and the equal-temperature relation, Eq. (5) can be written as:

$$M_H/M_L \approx E_H^*/E_L^* \approx a_H T_H^2 / a_L T_L^2 \approx m_H/m_L \quad (6)$$

where a_v is the level density parameter $a_v = A_v/8$. Consequently, observing a relation $M_H/M_L = m_H/m_L$ is consistent with a full equilibration of the excitation energy degree of freedom during the interaction time.

The neutron multiplicities shown in Fig. 9 are averages over all values of E_{Loss} above 86 MeV with an average E_{Loss} of ≈ 160 MeV corresponding to an average interaction time of $(8 \text{ or } 17) \cdot 10^{-22}$ sec assuming a nonsticking or sticking situation, respectively. It is again possible to obtain information about the time scale on which the excitation-energy degree of freedom equilibrates using the relation between energy loss and interaction time discussed previously.¹ For this purpose the neutron multiplicities for the light fragment ($20 \leq Z_L \leq 30$) and the correlated heavy fragment ($73 \geq Z_H \geq 63$) are plotted versus E_{Loss} in Fig. 10a. As shown in this figure, the neutron multiplicities increase almost linearly with E_{Loss} increasing up to 120 and 180 MeV for the light and heavy fragment, respectively. Furthermore, up to these energies the experimental results agree very well with the evaporation calculations (solid triangles) connected by the dashed line in Fig. 10a assuming N/Z equilibration. That is, provided that this assumption is true, one can conclude from the observed dependence of neutron multiplicities on E_{Loss} , that the excitation energy is equilibrated for values of E_{Loss} as small as 60 MeV corresponding to very short interaction times. This is consistent with the results given above for the nuclear temperatures.

However, there is a considerable discrepancy between the evaporation calculations and the experimental results for large energy losses, especially for the light fragment with

values of E_{Loss} above 120 MeV. This may be due to a considerable transfer of angular momentum in collisions damped to the Coulomb energy of spherical fragments or below. In the calculations it was assumed that the maximum possible fraction of the angular momentum corresponding to the rolling limit had been dissipated for an E_{Loss} of 130 MeV, decreasing linearly towards an E_{Loss} of zero and remaining constant for larger values of E_{Loss} . Such a dependence has been derived from γ -ray multiplicity measurements by Olmi *et al.*¹⁹ although for a different system, and is expected from the angular-momentum dissipation theory of Wolschin and Nörenberg²⁰. Possible larger amounts of angular momentum transferred would lead to a decreasing neutron multiplicity, since collective rotational energy is not available for particle emission.

The discrepancy discussed above may also be due to an N/Z equilibration process which is retarded or damped at high temperatures, a possibility which will be discussed below. However, in order to attribute the deviation of the neutron multiplicities from the results of the evaporation calculations at high values of E_{Loss} uniquely to either of the above effects, a more complete experiment is necessary in which all emitted particles, including protons and α -particles as well as γ -rays are measured.

In Table 1, the measured average neutron multiplicities, neutron energies and E_{Loss} values given for strongly damped collisions (SDC) and the fusion-fission (FF) part of the

reaction cross-section. Column 8 of Table 1 lists the average residual energy E_R , which is not carried away by neutrons but must go into p, α , and γ -ray emission.

Finally, a quantity of practical interest is the total neutron production cross section $\sigma_{n, \text{tot}}$. This quantity has been derived from the total kinetic energy loss spectrum⁸ of the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$ and the dependence of the total neutron multiplicity on the value of E_{Loss} . For the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$ at 476 MeV bombarding energy a value of $\sigma_{n, \text{tot}} = (26.7 \pm 3) \text{ b}$ is obtained. Similarly large neutron production cross-sections have resulted from the pioneering experiment of Broek²¹ who reported neutron production cross sections of 2.8 and 25 b in the reaction of 160-MeV oxygen on targets between aluminum and gold, respectively.

D. N/Z Equilibration

Since the largest fraction of the energy carried away per emitted neutron is given by the neutron binding energy B_n , the total number of emitted neutrons for a given excitation energy is almost directly proportional to B_n . By increasing the neutron-to-proton ratio of a given nucleus, the neutron binding energy decreases whereas the proton binding energy increases. Hence, the probability of charged-particle emission is drastically reduced for an increasing N/Z ratio, since $(\Gamma_n/\Gamma_{p,(\alpha)}) \propto \exp\{B_{p,(\alpha)} - B_n\}/T$ where $B_{p,(\alpha)}$ is the binding energy of protons and α -particles in-

cluding their respective Coulomb barriers. The α -binding energy has a weak N/Z dependence. Both effects amplify the sensitivity of the neutron multiplicity as a probe for the N/Z ratio. Since an increase in the N/Z ratio of the projectile-like fragment implies a decrease of N/Z of the target-like fragment, the sensitivity of the ratio M_H/M_L to the neutron-to-proton ratio is further enhanced, as can be seen from Eq. (5). Furthermore, in this ratio the dependence of M_ν on the total transferred angular momentum cancels, to first approximation.

In Fig. 11 the results of the evaporation calculation for the ratio M_H/M_L are plotted versus the neutron-to-proton ratio of the projectile-like fragments ($20 \leq Z_L \leq 30$). The error bars given represent uncertainties of the calculation. The excitation energy has been assumed to be apportioned according to the mass ratio, as discussed earlier. The hatched area represents the experimental value M_H/M_L at an E_{Loss} of 100 MeV. Obviously, the data are consistent with values of N/Z larger than 1.26 up to the N/Z equilibration value of the composite system ($N/Z = 1.38$).

Until now the equilibrium N/Z ratio has implicitly been identified with the N/Z ratio of the composite system. This is a priori a model-independent definition. The equilibrium neutron-to-proton ratio for a given charge division (Z_1, Z_2) and an angular momentum ℓ of the interacting system with a given shape is defined by the minimum potential-

energy surface $V_L(Z_1, N_1, Z_2, N_2)$ with respect to a variation in (N_1/N_2) . This potential surface has been approximated using the liquid-drop model,²² Coulomb point charges and rotational energies^{are} given by those of two nuclei at a separation distance equal to the strong-absorption radius or the sum of the two nuclear radii. These calculations accounting for shell corrections result in an equilibrium N/Z ratio of 1.30 for $L = 0$ and $20 \leq Z_L \leq 30$, in agreement with a value of 1.31 obtained using the maximum ground-state Q_{gg} values as deduced from the mass tables⁶. For a value of E_{Loss} of 100 MeV, the average initial angular momentum⁸ is $L = 193 \hbar$. In this case, one obtains 1.39 or 1.49 for the equilibrium N/Z value, depending on whether the strong absorption radius or the sum of the nuclear radii is used for the separation distance of the ions, respectively. Hence, for $L = 193 \hbar$ the model predicts the equilibrium N/Z value to be slightly larger than that of the composite system (1.38). Both predictions for the equilibrium N/Z values are consistent with the present experimental result at an E_{Loss} of 100 MeV.

In Fig. 10a,b the neutron multiplicities M_H and M_L and their ratios are compared with the evaporation calculations, assuming either $N/Z = 1.15$ (open triangles) or $N/Z = 1.38$ (filled triangles) for the projectile-like fragment. Here too, it is clear that the data are consistent with N/Z equilibration towards N/Z of the composite system for energy losses between 80 and 120 MeV. The evaporation calculations

become very uncertain for excitation energies close to the binding energy of one neutron. Hence, it is not feasible to perform an unambiguous comparison with the data at smaller TKE-losses. Thus, with this method it is not possible to deduce the N/Z ratio for fragments with values of E_{Loss} smaller than 80 MeV.

For the light fragment with energy losses above 120 MeV, the experimental multiplicities assume values in between the predictions based on either assumption made on the N/Z ratio. This could be interpreted as a different mechanism of N/Z equilibration at low angular momenta corresponding to large values of E_{Loss} or high temperatures resulting in a damping of this equilibration. However, since the present data are not complete, it is possible that this deviation is due to a transfer of amounts of angular momentum larger than assumed to the light fragment, at large values of E_{Loss} .

Summarizing the results discussed in this section, one may conclude that the neutron-to-proton ratio is almost equilibrated at interaction times as short as $(4 \text{ or } 8) \cdot 10^{-22}$ sec, depending on a non-sticking or sticking situation, respectively. Brosa and Krappe⁵ have predicted N/Z equilibration to be achieved within times of $\approx 2 \cdot 10^{-22}$ sec for the system $^{165}\text{Ho} + ^{56}\text{Fe}$ assuming the mass-to-charge equilibration mode to be not overdamped. This is consistent with the present results.

IV. CONCLUSIONS

Summarizing the results of this investigation, it is found that the excitation-energy degree of freedom and the neutron-to-proton ratio are equilibrated on the very short time scale of $(5-10) \cdot 10^{-22}$ sec. Neutrons are emitted from the fully accelerated fragments, that is after $\approx 5 \cdot 10^{-21}$ sec. Very few, if any, events are observed with a signature of pre-equilibrium processes. The data provide no evidence for Fermi-jet neutrons.

Very similar results, with respect to equilibration of the excitation energy, have been obtained in other investigations²³⁻²⁹ using a variety of different methods. All those investigations using neutron emission directly or indirectly as an indication for pre-equilibrium effects are summarized in Table 2. Except for the results of Hillis et al.²⁸, it is seen that pre-equilibrium effects do not become appreciable until the relative kinetic energy above the Coulomb barrier in the entrance channel is approximately 6-8 MeV/u.

The results for the $^{165}\text{Ho} + ^{56}\text{Fe}$ reaction at 476 MeV can be qualitatively understood by inspecting the relevant time scales for the evaporation time τ_p , nuclear relaxation time τ_R and interaction time τ_{INT} applicable to this experiment as given for different nuclear temperatures in Fig. 12. It appears that for nuclear temperatures reached at a bombarding energy of 4 MeV/u above the Coulomb barrier, even if a hot spot ($T \approx 5$ MeV) were formed on contact of the two ions, it

would decay within a typical relaxation time τ_R which is short compared to the interaction time ($\tau_R \ll \tau_{INT}$). That is, the system would equilibrate very rapidly to temperatures around 2 MeV where evaporation times are $\approx 10^{-20}$ sec, whereas the corresponding interaction time is of the order $\tau_{INT} \approx 10^{-21}$ sec. Adding to this interaction time the time the two fragments are accelerated within their mutual Coulomb field one obtains $(5-6) \cdot 10^{-21}$ sec which is still smaller than the evaporation time τ_p . From this estimate one expects particles to be evaporated only from the fully accelerated and completely equilibrated fragments.

For incident energies 7-8 MeV/u above the Coulomb barrier it may be possible to produce local temperatures of 8 MeV if the energy dissipation mechanism would allow an almost instantaneous loss of the available kinetic energy. In this case, the relaxation time τ_R is no longer small compared to the particle-evaporation time τ_p and one could, indeed, expect a considerable contribution of pre-equilibrium particles.

The essential quantity marking the boundary between the equilibrium and pre-equilibrium regimes appears to be the kinetic energy per nucleon above the Coulomb barrier in the entrance channel rather than the total excitation energy. Using, then, the kinetic energy per nucleon above the Coulomb barrier as a characteristic parameter, one can qualitatively put the (LI,xn) and (HI,xn) reactions on a comparable footing. For both types of reactions pre-equilibrium neutrons are observed for incident

energies higher than 5 to 6 MeV/u above the Coulomb barrier³² independent of whether tens or hundreds of MeV of relative kinetic energy have finally been dissipated into intrinsic excitation energy.

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APPENDIX

Assuming isotropic emission in the fragment rest frame, the double-differential cross section for neutrons from an evaporation cascade in this frame can be written as¹⁵

$$\frac{d^2\sigma}{d\Omega dE} = \frac{M}{4\pi} \cdot \left(\frac{1}{[\Gamma(n+1)] \cdot T^{n+1}} \right) \cdot E^n \exp(-E/T) \quad (A1)$$

where $\Gamma(n+1)$ is the gamma function, T is an effective nuclear temperature and M is the neutron multiplicity. The first energy moment $\langle E \rangle$ of this spectrum is given by

$$\langle E \rangle = (n+1) \cdot T \quad (A2)$$

where $\langle E \rangle$ corresponds to the mean kinetic energy carried away per neutron.

If on the average only one particle is evaporated, one has a Maxwellian³³ spectrum with $n=1$ ($\Gamma(2) = 1$), whereas for a neutron cascade, LeCouteur and Lang¹⁵ have shown that $n=0.45$. It has experimentally been demonstrated that evaporation spectra of fission neutrons³⁴ can be described very well by using $n=0.5$ ($\Gamma(3/2) = \sqrt{\pi}/2$), which will be used in the following,

$$\frac{d\sigma^2}{dE_n d\Omega} = \frac{M}{2(\pi \cdot T)^{3/2}} \cdot \sqrt{E_n} \exp\{-(E_n - 2\sqrt{\epsilon} E_n \cos \theta + \epsilon)/T\} \quad (A3)$$

where E_n is the laboratory neutron energy.

The angular distribution in the lab system is obtained by integrating Eq. (A3) over all neutron energies, yielding

$$\frac{d\sigma}{d\Omega} = \frac{M}{2\pi^{3/2}} \cdot \exp\left(\alpha^2 - \frac{\epsilon}{T}\right) \cdot \left\{ \sqrt{\pi}(\alpha^2 + 1/2) (1 - \text{erf}(c)) + e^{-c^2} (2\alpha + c) \right\} \quad (A4)$$

The energy spectrum in the lab system is deduced by integrating Eq. (A3) over the angles, yielding:

$$\frac{d\sigma}{dE_n} = \frac{M}{2\sqrt{\pi \cdot \epsilon \cdot T}} \cdot \exp\left(-\frac{(E_n + \epsilon)}{T}\right) \sinh\left(\frac{2\sqrt{\epsilon \cdot E_n}}{T}\right) \quad (A5)$$

Finally, the dependence of the first moment of the energy spectrum in the lab system on the neutron laboratory angle θ relative to the fragment scattering angle is given by:

$$\langle E_n \rangle = \frac{T \cdot \{ \sqrt{\pi} (3/4 + 3\alpha^2 + \alpha^4) \cdot (1 - \operatorname{erf}(c)) + (c^3 + 3/2c + 4\alpha c^2 + 4\alpha + 6\alpha^2 c + 4\alpha^3) e^{-c^2} \}}{\{ \sqrt{\pi} (\alpha^2 + 1/2) (1 - \operatorname{erf}(c)) + (2\alpha + c) e^{-c^2} \}} \quad (A6)$$

where,

$$c = \sqrt{\frac{E_s}{T}} - \alpha, \quad \alpha = \sqrt{\frac{\epsilon}{T}} \cos \theta,$$

and E_s and ϵ are the neutron energy threshold and the fragment energy/amu, respectively.

Particle evaporation leads to a distribution of recoil directions θ of the fragments, which is, however, rather narrow as has been shown by Lide³⁵:

$$\frac{dN(\theta)}{d\Omega} = \frac{2N_0}{2\pi k \chi_k} \cos^2 \theta \exp\left\{-\frac{\sin^2 \theta}{k \cdot \chi_k}\right\} \quad (A7)$$

and

$$\chi_k = 2 \cdot \left(\frac{A \cdot \sqrt{m \cdot T}}{(A - k \cdot m) P_i} \right) \quad (A8)$$

Here A is the fragment mass, k is the number of evaporated neutrons, m is the neutron mass and P_i is the initial fragment momentum. For the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$, this effects leads to a broadening of the fragment angular distribution by only a few degrees.

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The results of Hillis et al. are somewhat difficult to apply to the problem under consideration. These authors deduced the mean energy $\langle E \rangle$ carried away per neutron as a function of the number of neutrons emitted from the completely fused system by measuring the mean γ -ray multiplicity and energy. At 2.4 MeV/u (see Table 2) above the Coulomb barrier they deduce mean neutron energies $\langle E \rangle$ above and below the expected thermal value for the (Hf,6n) and (Hf,9n) events. However, by selecting only the 6n and 9n channels, one samples predominantly the high- and low-energy ends of an evaporation spectrum, respectively. Hence, it is not permissible to use only the mean neutron energies from the

(HI,6n) reaction as an indication for non-thermal, pre-equilibrium events. If one measures simultaneously, however, also the neutron energy spectra, as has been done by Westerberg et al.²⁶, this method can be a powerful tool to look specifically at pre-equilibrium neutrons. A conclusion similar to that above has been reported recently by M. Blann and R.L. Ferguson, Phys. Rev. C19 298 (1979).

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Table 1: Mean neutron multiplicities and neutron energies

θ_T	$\langle E^* \rangle^a)$ [MeV]	$\langle M_L \rangle^b)$ $\pm 10\%$	$\langle E_L \rangle^c)$ [MeV]	$\langle M_H \rangle^b)$ $\pm 10\%$	$\langle E_H \rangle^c)$ [MeV]	$\langle M_{TOT} \rangle$ $\pm 10\%$	$\langle E_R \rangle^d)$ [MeV] $\pm 15\%$	Reaction ^{e)}
16°	113	1.93	3.1	5.77	3.1	7.69	27.5	SDC
20°	100	2.02	3.0	5.59	3.1	7.65	15.2	SDC
25°	83	1.56	2.4	4.01	2.5	5.57	20.7	SDC
$\langle 16, 20, 25 \rangle$	224	5.7	3.6	5.7	3.6	11.4	88	FF

a) $\langle E^* \rangle = \langle E_{Loss} \rangle + \langle Q_{gg} \rangle$ weighted mean with $E_{Loss} > 36$ MeV.

b) Weighted-mean neutron multiplicity with $E_{Loss} > 36$ MeV.

c) Weighted-mean first moment of c.m. neutron energy spectrum.

d) Residual excitation energy that is not carried away by neutron emission.

e) SDC : $20 \leq Z_L \leq 30$.

FF : $35 \leq Z_L \leq 55$.

Table 2: Emission of preequilibrium neutrons

Reaction	E_{LAB} [MeV]	$(E_{\text{cm}} - V_{\text{CB}})/u$ [MeV/u]	Measurement	Preequilibrium	Ref.
			a)	b)	
$^{12}\text{C} + ^{158}\text{Gd}$	152	8.3	CF	+	26)
$^{16}\text{O} + ^{154}\text{Sm}$	112-169	2.6 - 6.3	CF	- to +	27)
$^{20}\text{Ne} + ^{150}\text{Nd}$	175	4.6	CF	-	26)
$^{56}\text{Fe} + ^{165}\text{Ho}$	476	4.0	SDC/FF	-	present work
$^{86}\text{Kr} + ^{166}\text{Er}$	602	2.5	SDC	-	24)
$^{40}\text{Ar} + ^{124}\text{Sn}$	161-236	0.5 - 2.4	CF	- to +	28)
$^{132}\text{Xe} + ^{197}\text{Au}$	990	2.3	SDC	(+) ^{c)}	29)
$^{63}\text{Cu} + ^{197}\text{Au}$	365	.8	SDC/FF	-	25)

a) CF: Complete fusion; FF: Fusion fission; SDC: Strongly damped collisions

b) +: evidence, -: no evidence for pre-equilibrium neutrons.

c) See also reference 24.

The results of Broek²¹⁾ are not included since this was an inclusive measurement. He did not observe any pre-equilibrium neutrons though the energies were 8.3-11.8 MeV/u above the Coulomb barrier.

- Fig. 1. Schematic diagram of the experimental setup.
- Fig. 2. Neutron time-of-flight spectrum. The small narrow peak at $t \approx 100$ ns corresponds to γ -rays not rejected by the pulse-shape discrimination system.
- Fig. 3. Angular distributions of the neutron intensity in the laboratory system for strongly damped collisions (SDC) where Z of the detected fragment is between 10 and 29. The heavy-ion telescope was located at an angle of 23° (vertical arrow). The dashed and dashed-dotted curves are the results of an evaporation calculation (Eq. (A4)) for the light and heavy fragment, respectively, averaged over all values of E_{Loss} . The solid line is the sum of the calculated intensities of the two fragments. The insert shows the out-of-plane distribution measured at $\theta_n = 25^\circ$.
- Fig. 4. Angular distribution of the mean neutron energy (Eq. (A6)) in the lab system for strongly damped collisions (SDC). The dashed, dashed-dotted and solid curves are the results for the light fragment, heavy fragment and the weighted average of both, respectively.
- Fig. 5. Angular distribution of neutrons emitted from fusion-fission (FF) fragments F_1 and F_2 ($30 \leq Z \leq 55$). See also figure caption of Fig. 3.
- Fig. 6. In the lower part of the figure, the double-differential cross-section ($d^2\sigma/dE d\Omega$) in the lab system

(Eq. (A3) is plotted in a contour diagram versus neutron detection angle and energy. The light (solid line) and heavy fragments (dashed line) are scattered to 20° and -40° , respectively. In the upper part of the figure, respective cross-sections integrated over the neutron energy are depicted. A large E_{Loss} value of 200 MeV has been used to generate the distributions.

Fig. 7. Neutron energy spectra in the rest frames of the light (open circles) and the heavy (filled squares) fragments for all neutron detector angles. The spectra were averaged over 2 between 20 and 30, E_{Loss} values larger than 36 MeV and all three settings of the telescope angle $\theta_T = 16^\circ, 20^\circ$ and 25° . The solid lines represent evaporation spectra $dN/dE = \sqrt{E} \cdot e^{-E/T}$ and are normalized to the experimental points at 25° and 10° for the light fragment (open circles) and at -45° and -70° for the heavy fragment (filled squares).

Fig. 8 Nuclear temperature as a function of the E_{Loss} for the light (open circles) and heavy (filled squares) fragments. The solid curve represents the calculated nuclear temperature $T = (11/12) \cdot (E_{\text{Loss}}/a_{\text{CN}})^{1/2}$ with $a_{\text{CN}} = [(A_P + A_T)/8] \text{MeV}^{-1}$ being the level density parameter of the composite system.

Fig. 9 Neutron multiplicities M_L (open circles) and M_H (filled squares) are shown as functions of Z_L and $Z_H = Z_P + Z_T - Z_L$.

respectively (see text). The filled triangles and crosses are the results of evaporation calculations¹⁷ for neutron and charged-particle (p+α) emission, respectively. In the upper part of this figure the ratio M_H/M_L is plotted and compared to the mass ratio m_H/m_L (solid line).

Fig. 10. (a) Neutron multiplicities M_L (open circles), M_H (filled squares) and (b) the ratio M_H/M_L (filled circles) are plotted versus E_{Loss} for $20 \leq Z \leq 30$. The open and filled triangles represent exaporation calculations¹⁷ for M_L , M_H and M_H/M_L assuming an N/Z value corresponding to the valley of beta-stability and a value equilibrated towards N/Z of the composite system, respectively. The solid and dashed lines in (a) and (b) are drawn through the calculated points to guide the eye.

Fig. 11. The calculated ratio M_H/M_L of the neutron multiplicities M_L and M_H of the light and heavy fragments, respectively, is plotted versus N/Z of light fragment with $20 \leq Z_L \leq 30$ and an $E_{Loss} \approx 100$ MeV. The hatched area represents the experimental value of M_H/M_L . The vertical dashed arrow indicates the N/Z value of the composite system, whereas the solid arrows show those of ^{56}Fe and ^{165}Ho .

Fig. 12. Relevant time scales in the reaction $^{165}\text{Ho} + ^{56}\text{Fe}$.

The curves labeled τ_p , τ_R and τ_{INT} represent the time required to evaporate a particle³⁰ from a nucleus of temperature T , that for the decay of a hot spot in a nucleus of temperature T corresponding to the nuclear relaxation time³¹, and the nuclear interaction time⁸ in a non-sticking situation for the given reaction at 476 MeV as a function of equilibrium temperature T , respectively. Only the interaction times that do not lead to fusion ($\ell > \ell_{crit}$) are shown. The smallest interaction time corresponds to the maximum angular momentum (ℓ_{max}).

007-4020

Fig. 2

Fig. 7

007-4139

Fig. 9

Fig. 10

007-4184

Fig. 12

Structure in the Energy Spectra of the Fragments
Produced in Damped Heavy-Ion Reactions

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ABSTRACT:

Structure in the energy spectra of specific final fragments produced in symmetric heavy-ion damped collisions is shown to correlate with statistical evaporation processes from primary fragments. The calculations are applied and described for ^{41}Ca produced by the $^{40}\text{Ca} + ^{40}\text{Ca}$ reaction at a bombarding energy of 400 MeV. The calculated energy spectrum has structure at the same energies as the experimental spectrum.

[NUCLEAR REACTIONS Calculated isotopic energy spectra
from heavy-ion damped collisions. Origin of structure.]

It appears by now that A and Z distributions of fragments observed from damped heavy-ion reactions can be reasonably well understood in terms of a statistical mechanism of nucleon exchange operating in these reactions. Although it is evident that an exchange of mass between the two constituents of the intermediate complex formed in the collision must be accompanied by the dissipation of a certain amount of the kinetic energy of relative motion, it is presently not at all clear to what extent such a dissipation mechanism is responsible for the high degrees of damping observed in heavy-ion reactions. In fact, vastly different energy-loss mechanisms have been offered as explanations of the conspicuous dissipative features of damped heavy-ion collisions. Experimental information on the dissipation mechanism is obtained from a study of the probability distribution of final fragment-kinetic energies and its relation to mass and atomic number of the final fragment, reaction angle and other reaction variables. The particular appearance of this distribution may then allow an identification of the dominant dissipation mechanisms.

Structure has been reported¹⁻⁴ in the double-differential cross section $d^2\sigma/d\Omega dE$ as a function of the energy of final fragments with a given Z and N produced in heavy-ion reactions at bombarding energies of 7 to 10 MeV/u. To date, this structure has been observed for nuclides produced by symmetric systems, namely $^{40}\text{Ca} + ^{40}\text{Ca}$ ^{1,2}, $^{56}\text{Fe} + ^{56}\text{Fe}$ ³,

$^{63}\text{Cu} + ^{63}\text{Cu}^1$, and possibly in $^{93}\text{Nb} + ^{93}\text{Nb}^4$. The magnitude of the structure is most pronounced for product fragments near the target (or projectile), although even for these nuclei the peak-to-valley ratio of the cross section bumps is small and sometimes the structure manifests itself only as a shoulder in the energy spectrum.

• Broglia, et al.^{5,6} have discussed the possible importance of giant-resonance excitation in damped collisions between heavy ions. They have suggested that this mechanism of excitation of collective surface vibrations accounts for a damping of the relative kinetic energy comparable to that produced by the particle-exchange mechanism. It has been suggested on numerous occasions (see, for example, refs. 1 and 2) that the observed structure discussed above may be evidence for the excitation of these high-energy giant resonances. However, one characteristic feature of the observed structure is the marked change in this structure when the (Z,N) value of the final nucleus changes, even when the change is as small as one unit in Z or N. Another aspect of the structure is that it tends to disappear when the energy spectra of isotopes of the same element are combined. This unsystematic change in the energy of the experimental structure for different product nuclei argues against the explanation of the structure as being due to high-energy giant resonances. Furthermore, it is not at all clear that the excitation of collective surface vibrations^{5,6} in the damping process should produce the observed structure.

Hence, in this article it is shown that structure of the type found experimentally can be produced by statistical evaporation processes following the production of the primary fragments by the well-known nucleon exchange process⁷.

It is important to emphasize that inclusive spectra of fragments measured with unit Z and A resolution from heavy-ion damped reactions are a result of a two-step process. In the first stage a distribution of primary fragments is produced during the short interaction time of the damped heavy-ion collision (of the order of 10^{-21} to 10^{-22} sec). The second stage, which occurs on a much longer time scale, accounts for the deexcitation of these primary fragments by light particle and gamma ray emission. In order to calculate the energy spectrum of a particular nucleus (Z,N) observed in the laboratory system it is necessary to make several simplifying assumptions. These assumptions deal with the following aspects of the first and second stages of the reaction, namely: (1) the yield distribution of the primary fragments (Z_i, N_i) as a function of excitation energy, (2) the partition of the excitation energy between the two primary fragments, and (3) the details of the light-particle emission. Each of these subjects are discussed briefly in the following paragraphs.

Yield Distribution of Primary Fragments: The probability P for the production of a given primary fragment (Z_3, N_3) at a total excitation energy E^* of both primary fragments, where $E^* = \text{TKE} \cdot \text{loss} + Q_{\text{gg}}$ and Q_{gg} is the ground-state Q value, is

assumed to be given by

$$\frac{d^3P}{dZ_3 dN_3 dE^*} = \frac{1}{2\pi\sigma_Z^2\sigma_N^2} \exp\left(-\frac{(Z_3-Z_1)^2}{2\sigma_Z^2} - \frac{(N_3-N_1)^2}{2\sigma_N^2}\right) \left(\frac{E^*}{B^2}\right) \exp\left(-\frac{E^*}{B}\right) \quad [1]$$

where

$$\sigma_Z^2 \approx \sigma_N^2 \approx -\frac{Z_1}{A_1} \cdot \frac{\mu}{m \cdot \alpha} \cdot \ln\left(1 - \frac{E^*}{E_{cm} - V_C}\right) \quad [2]$$

The quantities Z_1 and N_1 are the number of protons and neutrons, respectively, of the projectile of mass A_1 (or the identical target). The center-of-mass energy, reduced mass and Coulomb barrier in the entrance channel are given by E_{cm} , μ and V_C , respectively. The value of the parameter α , which relates the variance σ_Z^2 and TKE-loss, is taken to be 2.0 from other studies⁸. The parameter B is used to estimate the primary yield distribution as a function of the total energy and, for example, is chosen to be 10 MeV for the $^{40}\text{Ca} + ^{40}\text{Ca}$ reaction.

Although the primary yield distribution given by Eq. [1] may be qualitatively correct for the more damped events, individual yields of particular primary fragments may deviate considerably from this prediction, especially for small damping. Therefore, we reiterate that the actual primary yields for these reactions are not known. Ultimately one hopes to be able to correct for the second-stage evaporation processes in order to gain information on the excitation energy dependence of these primary yields.

Partition of Excitation Energy: There is a considerable amount of evidence^{9,10} that the total excitation energy E^* is apportioned to the final fragments from damped reactions according to their mass ratio, $E_3^* = [A_3/(A_1+A_2)](E^*)$. However, this division of excitation energy is expected to hold only on the average. Hence, it is assumed here that the excitation energy of primary fragment (Z_3, N_3) is a Gaussian distribution centered at E_3^* with a dispersion σ given by $[A_3/(A_1+A_2)](E^*)^{1/2}$. The positions of the structure in an energy spectrum are insensitive to the assumed dispersion. However, there may be deviations from this simple division of excitation energy, especially for small energy damping. Again it is hoped that experiments of the type discussed here will give information on this question.

Light-Particle Emission: If a primary fragment has sufficient excitation energy to decay by particle emission, it is assumed that this process definitely occurs (of course, prior to the detection of the final fragment). De-excitation by γ -ray emission is assumed to occur only when particle emission is energetically forbidden. The competition between different particle decay channels (n, p, and α particles) is calculated with the approximate relation¹¹

$$\frac{r_2}{r_1} = \frac{(2s_2+1)m_2\sigma_2^0}{(2s_1+1)m_1\sigma_1^0} \exp \{[(S_1-S_2) + (V_1'-V_2')]/T\} \quad [3]$$

The quantities r_i , s_i , m_i , σ_i^0 , S_i , V_i' and T are the decay

width, spin, mass, inverse capture cross-section, separation energy¹², effective Coulomb barrier and average temperature of the two residual nuclei, respectively. The partial evaporation spectra in the rest frame of the emitting nucleus have been assumed to be Maxwellian in accordance with experimental observations^{9,10}.

If the primary fragment A_3 with laboratory kinetic energy E_3 emits a particle of mass m and kinetic energy ϵ_{cm} in the rest frame of A_3 , the observed kinetic energy E'_3 of this fragment after particle emission is given by¹¹

$$E'_3 = E_3 \left(1 - \frac{m}{A_3}\right) \left[1 + \frac{m\epsilon_{cm}}{A_3 E_3} + 2 \left(\frac{m\epsilon_{cm}}{A_3 E_3}\right)^{1/2} \cos\theta\right] \quad [4]$$

The angle θ is the light-particle emission angle in the rest frame of heavy fragment A_3 with respect to the direction of its laboratory velocity. On the average, particle emission decreases the kinetic energy of fragment A_3 and causes an energy broadening even if A_3 is monoenergetic. For example, the maximum widths [calculated with $\theta = 0$ and 180° in Eq. (4)] of distributions following the emission of 1-MeV γ -rays and 1-MeV neutrons by a 10 MeV/u ^{40}Ca fragment are 0.3 and 12.3 MeV, respectively. The broadening produced by charged-particle emission is even larger since the minimum emission energy as given by the Coulomb barrier can be rather large.

The line shape due to this recoil broadening depends on the angular distribution of the emitted particles in the rest frame and to some extent also on the angular distribution of

the primary fragments. This problem has already been discussed for γ -ray emission from heavy ions^{13,14}. In principle, one can investigate the fragment alignment by studying the spectral line shapes. For example, a $\sin^{-1}\theta$ -type angular distribution of the emitted light particles causes a spectral peak to split into two peaks. In calculations to be shown here, either maximum recoil broadening corresponding to isotropic particle emission in the rest frame¹⁴ or no broadening at all was assumed. The latter assumption corresponds to preferential emission at $\theta = 90^\circ$, a possibility which might arise from strong focussing of the primary fragments so that only light-particle emission to 90° will deflect the heavy final fragment into the detector aperture. The positions of the structure in the energy spectrum are rather insensitive to the assumed magnitude of the recoil broadening (see Fig. 1).

All primary nuclei which can energetically decay by one- or two-particle chains (and a few selected three-particle chains) into the observed product nucleus have been included in the evaporation calculations. Shown in Fig. 1 is the calculated laboratory energy spectrum of ^{41}Ca produced in the $^{40}\text{Ca} + ^{40}\text{Ca}$ reaction at a bombarding energy of 400 MeV and a detection angle of 10° . The vertical arrows indicate the positions of the bumps or shoulders in the experimental energy spectrum². The solid and dashed curves are the results

based on the assumption of maximum and no recoil broadening, respectively. Fig. 2 shows the same calculations for maximum energy broadening and includes the individual distributions for the different decay channels. In Figs. 1 and 2, the peak around 370 MeV is due to direct primary production of ^{41}Ca . When the excitation energy of ^{41}Ca is sufficiently high ($E_{\text{lab}} \approx 363$ MeV), this mode of direct production will disappear due to particle emission. The width of the peak at 370 MeV is determined by the assumption on the distribution of the excitation energy between the primary fragments. The second and third peaks at 345 and 320 MeV in Fig. 1 and 2 are predominantly due to the 1p and 2p decay of ^{42}Sc and ^{43}Ti , respectively. In general it should be emphasized that, independent of the specific primary yield distribution assumed, the second peak is due to one-nucleon decay and the third peak is due to two-nucleon and/or α -particle decay. The exact position of the bump or shoulder can shift somewhat in energy depending on the primary yields and which decay channels predominate as can be seen in Fig. 2.

In conclusion, the observed structure in the energy spectrum of a particular product nucleus produced in a heavy-ion damped collision process for symmetric systems can be qualitatively understood as due to statistical evaporation processes from primary fragments. The bumps or shoulders in the calculated energy spectra coincide in energy with experiment. No attempt was made, however, to fit simultaneously

the differential cross sections of all product nuclei as a function of energy by an iterative procedure. To do this, one would require better information on the yields of all primary fragments and experimental data on the angular distributions of the emitted light-particles both in plane and out of plane. We wish to emphasize that before one interprets the observed structure as due to high-energy giant resonances, one must eliminate the expected structure due to the statistical processes discussed in this article. Even if the symmetric systems would have narrow energy peaks, this structure will be broadened considerably in inclusive spectra due to the recoil process. If the primary fragment is produced at low enough excitation energies for particle emission to be energetically forbidden, the observed energy spectra will still be broadened by γ -ray recoil, although the magnitude will be considerably less. Further elucidation of these processes will require coincidence techniques.

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FIGURE CAPTIONS

- Fig. 1: Calculated laboratory energy spectrum of ^{41}Ca at a detection angle of $\theta_{\text{lab}} = 10^\circ$. The dashed and solid curves represent the results of calculations using no broadening and maximum recoil broadening, respectively, as described in the text. The vertical arrows indicate the positions of the bumps or shoulders in the experimental energy spectrum².
- Fig. 2: The heavy line is the calculated laboratory energy spectrum of ^{41}Ca using maximum recoil broadening (see Fig. 1). The remaining curves represent the individual contributions to the production of ^{41}Ca from various primary fragments. The curve labeled ^{41}Ca corresponds to the yield of primary fragment ^{41}Ca that has survived particle decay. The other curves are labeled by the particles emitted from the primary fragments to produce the final fragment ^{41}Ca . The vertical arrows indicate the positions of the bumps or shoulders in the experimental energy spectrum².

Fig. 1

Fig. 2

Nucleon Exchange and A/Z Equilibration
in Interactions of 8.3 MeV/u ^{56}Fe Ions with ^{56}Fe , ^{165}Ho and ^{209}Bi

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Abstract

Yields have been measured with discrete Z and A resolution for Fe-like fragments produced in damped collisions between 8.3 MeV/u ^{56}Fe ions and ^{56}Fe , ^{165}Ho and ^{209}Bi targets. Both the fragment A/Z ratio and the relationship between the variances σ_A^2 and σ_Z^2 of the mass and charge distributions, respectively, are found to depend upon the degree of energy damping. These observations are consistent with similar time scales for evolution of the correlations in the nucleon exchange process, A/Z equilibration, and energy dissipation.

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There is considerable evidence that the major part of the energy loss in damped collisions between complex nuclei is due to dissipation mechanisms occurring on the same time scale as nucleon exchange processes.¹ Under this condition, the variances of the product mass distributions at any value of the dissipated energy is related to the number of nucleons exchanged during the interaction time. Most previous measurements involving projectiles heavier than ⁴⁰Ca have determined only the charge distributions separately. However, the relation between the variances σ_N^2 , σ_Z^2 and σ_A^2 of the respective neutron, charge and mass distributions is of considerable interest since it depends on the degree of correlation² between neutron and proton exchange. The correlation coefficient ρ is defined by

$$\sigma_A^2 = \sigma_Z^2 + \sigma_N^2 + 2\rho\sigma_Z\sigma_N$$

with $-1 \leq \rho \leq 1$. In a classical model² for fully correlated neutron and proton exchange, $\rho = 1$ and $\sigma_A^2 = (A/Z)^2 \sigma_Z^2$; for uncorrelated exchange, $\rho = 0$ and $\sigma_A^2 = (A/Z) \sigma_Z^2$ (where A and Z represent the combined system).

Another important feature of damped collisions is the degree of mass-to-charge equilibration of the fragments as a function of energy damping and target-projectile mass asymmetry. From inspection of the potential energy

surfaces for reactions between heavy complex nuclei, as derived from the Q_{gg} systematics, one in general finds a gradient toward an equilibrium A/Z ratio which minimizes the potential energy of the composite system. Although several experiments have provided evidence in support of A/Z equilibration^{1,3-7}, no data presently exist which demonstrate the detailed dependence of this equilibration process on energy loss. In order to investigate the above questions, yields of projectile-like fragments have been measured with discrete A and Z resolution as a function of energy loss for both symmetric and asymmetric systems with varying A/Z ratios.

The measurements were performed with 465-MeV ^{56}Fe ions from the Lawrence Berkely Laboratory SuperHILAC accelerator. Self-supporting targets of monoisotopic ^{56}Fe , ^{165}Ho and ^{209}Bi with thicknesses of approximately 500 $\mu\text{g}/\text{cm}^2$ were used. The projectile-like fragments were detected with a time-of-flight ΔE -E telescope consisting of 17 μm ΔE and 100 μm E silicon surface-barrier detectors. Data were taken at selected angles forward of the grazing angles. The separation distance between the ΔE and E detectors was 100cm and timing signals were derived from Sherman-Roddick fast-timing electronics.⁸ For elastically scattered ^{56}Fe ions a timing resolution of 100psec and an energy resolution of approximately 3 MeV were obtained,

resulting in a mass resolution of $\approx 0.7u$. The $\Delta E-E$ signals provided Z resolution of ≤ 0.8 charge units, thus permitting discrete Z and A identification for all Fe-like products. In Fig. 1 a two-dimensional plot of experimental yields for the $^{209}\text{Bi} + ^{56}\text{Fe}$ bombardment is shown.

Since the data represent the yields of residual Fe-like fragments after de-excitation, it is necessary to correct for particle evaporation in order to obtain primary yields. The available excitation energy is assumed to be shared between the two primary products according to their mass ratio. Neutron multiplicity and energy measurements support this assumption for the $\text{Ho} + \text{Fe}$ system.⁷ Using nucleon binding energies from the compilation of Wapstra and Bos⁹ and treating the energy spectra of the emitted particles as described in Ref. 7, the measured yields and energies were transformed by an iterative, event-by-event procedure into primary fragment yields and center-of-mass energies. These calculations indicate that neutron evaporation predominates over charged-particle emission for the $\text{Ho} + \text{Fe}$ and $\text{Bi} + \text{Fe}$ cases due to the neutron excess and relatively low excitation energies of the primary Fe-like fragments. Hence, the measured charge distributions for these two heavier systems can be considered to be essentially primary

yields. However, in the Fe + Fe system greater proton richness and higher excitation energies increase the relative importance of charged-particle emission. These conclusions are corroborated by measurements of charged-particle emission performed simultaneously in these experiments. While particle emission gives measured average masses significantly lower than the average mass of the primary fragments, the effect on the variances of the mass distributions is small. Neutron evaporation leads to measured mass distributions which are narrower than the primary distributions, whereas proton evaporation produces the opposite effect. Thus the primary variances of the mass distributions for the Fe + Fe reaction fragments are essentially the same as those of the measured distributions.

Figure 2 presents the experimental results as a function of total kinetic energy loss (E_{Loss}). The data points on the left-hand side of the plot represent the shifts of the average mass, $\Delta\bar{A} = \bar{A} - 56$, whereas on the right-hand side the variances of the mass distributions, σ_A^2 , are plotted. Triangles indicate measured data points and circles show values of $\Delta\bar{A}$ and σ_A^2 corrected for neutron emission. The curves represent the corresponding measured values of $\Delta\bar{Z} = \bar{Z} - 26$ and σ_Z^2 , multiplied by A/Z of the composite systems (dashed lines) and $(A/Z)^2$ (solid

lines), respectively, for comparison with the mass data. Values for $\bar{Z}$, $\bar{A}$, σ_A^2 and σ_Z^2 were obtained from Gaussian fits to the data.

First, the implications of these data on the charge equilibration mechanism are discussed. For all three systems the data exhibit a drift of $\bar{Z}$ toward smaller atomic numbers as the energy damping increases. For the Fe + Fe system this drift is attributed to charged-particle emission in the de-excitation of the primary fragments. Since for the Ho + Fe and Bi + Fe systems charged particle emission is strongly inhibited, their decrease in $\bar{Z}$ must be attributed to a charge equilibration process in which protons are preferentially transferred from the Fe-like fragments to the heavy partner.

From the measured values of Z and A , corrected for neutron evaporation, for the two heavier systems, it is possible to deduce the dependence of $\bar{A}/\bar{Z}$ on E_{Loss} , as shown in Fig. 3. Also shown as dashed lines are the A/Z ratios of the ^{56}Fe projectile (2.15) and of the composite systems (2.38 for Ho + Fe and 2.43 for Bi + Fe). The data support a nucleon exchange mechanism in which there is an initial increase in the $\bar{A}/\bar{Z}$ value of the primary Fe-like fragments as soon as the Q values permit. Thereafter a monotonic increase in $\bar{A}/\bar{Z}$ is observed which may

saturate at energy losses in the range of 100-150 MeV at $\bar{A}/\bar{Z} = 2.3$ for both asymmetric systems. These values are close to the equilibrium values defined by potential energy surfaces based on the liquid drop model including shell corrections, which are somewhat smaller than the $\bar{A}/\bar{Z}$ of the combined systems.

The measured variances of the mass and charge distributions are compared on the right hand side of Fig. 2. Neither assumption of correlated or uncorrelated diffusion (dashed lines) describe the variances of the mass distributions for any of the systems studied in this work. Instead, a behavior intermediate between uncorrelated and correlated exchange is observed for partially damped events, but with increasing energy loss all three systems evolve toward correlated exchange (consistent with the square of the observed $\bar{A}/\bar{Z}$ ratio). This aspect of the data is demonstrated more explicitly in Fig. 4 where the dependence of σ_A^2/σ_Z^2 on energy loss is plotted. For comparison the upper and lower dashed lines represent the A/Z and $(A/Z)^2$ values of the combined system, respectively. It is noted in Fig. 4 that the limit of correlated exchange is reached for the Fe + Fe system at about half the energy loss of the two asymmetric systems. Previous studies of the $^{168}\text{Er} + ^{86}\text{Kr}$ system⁶ at energy losses greater than 100 MeV have shown similar agreement with the relation $\sigma_A^2 = (A/Z)^2 \sigma_Z^2$. However, the present data stress the importance of the degree of energy damping in reaching the correlated limit.

In summary, charge and mass distributions have been measured for the products formed in damped collision processes in three heavy-ion systems of widely differing target-projectile mass asymmetries, and A/Z values. The excellent resolution for Z and A permits the determination of precise values for $\bar{A}$, $\bar{Z}$, σ_Z^2 , and σ_A^2 . The $\bar{Z}$ and $\bar{A}$ results show that charge equilibration develops gradually with increasing energy loss. Hence, for very asymmetric systems, an equilibrium in A/Z is not achieved until the reaction is almost totally damped, requiring comparatively long interaction times. The relationship between σ_Z^2 and σ_A^2 supports a correlated nucleon diffusion mechanism for interaction times sufficiently long that the limiting $\bar{A}/\bar{Z}$ value has been reached. This is consistent with similar time scales for evolution of the correlations in the nucleon exchange, the equilibration of A/Z , and the energy dissipation. The energy loss time scale leads to an interaction time of $(4-8) \times 10^{-22}$ sec at 100 MeV energy loss for the non-sticking and sticking conditions,^{1,7} at which time the correlations in nucleon exchange and A/Z ratio seem to be close to steady-state values.

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Figure Captions

Fig. 1:

Plot of charge versus mass for the Bi + Fe reaction at $60 \leq E_{\text{Loss}} \leq 160$ MeV. The sizes of the points correspond to counts on a quadratic scale. The region of currently known particle-stable nuclei falls between the limits indicated by the intensified solid lines.

Fig. 2:

Values of $\Delta\bar{A}$ (left sides) and variances of the mass distributions (right sides) as a function of energy loss for the reactions of ^{56}Fe ions with (a) ^{56}Fe at 8° ; (b) ^{165}Ho at 30° ; and (c) ^{209}Bi at 34° and 38° in the laboratory system. Note the change in scale for σ^2 for the Fe + Fe reaction. Arrows indicate the respective Coulomb barriers for touching spheres in the entrance channels. Triangles give measured points with statistical errors; circles represent the data corrected for neutron emission (errors are comparable to measured points). The dashed lines show the values of $\Delta\bar{Z} = 26 - \bar{Z}$ and σ_Z^2 multiplied by (A/Z) ; the solid line represents σ_Z^2 times $(A/Z)^2$.

Fig. 3:

Ratios of $\bar{A}/\bar{Z}$, corrected for neutron evaporation, as a function of energy loss for the (a) ^{165}Ho and (b) $^{209}\text{Bi} + ^{56}\text{Fe}$ systems. The dashed lines give A/Z of the projectile and of the composite systems.

Fig. 4:

Ratios of σ_A^2/σ_Z^2 as a function of energy loss for the
(a) ^{56}Fe , (b) ^{165}Ho , and (c) $^{209}\text{Bi} + ^{56}\text{Fe}$ systems.
Data points in (a) (triangles) represent the measured
values, in (b) and (c) (circles) values corrected for
neutron evaporation. The dashed lines represent
 A/Z and $(A/Z)^2$ of the composite systems.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

HEAVY-ION FUSION: COMPARISON OF EXPERIMENTAL DATA
WITH CLASSICAL TRAJECTORY MODELS

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Abstract:

Currently available data on fusion excitation functions for heavy-ion induced reactions over a wide mass range are compared to results calculated with a classical dynamical model based on the proximity nuclear potential of Blocki et al., the Coulomb potential of Bondorf et al., and one-body nuclear friction in the proximity formalism of Randrup. With these conservative and dissipative forces and the radial parameters of Myers, overall good agreement is obtained between the theoretical excitation functions and most of the available data. Extensive calculations have been performed to test the sensitivity of the calculated fusion cross sections to a number of parameters, including the radial dependence of the Coulomb and nuclear potentials, the radial and tangential friction form factors as well as the projectile and target radii. The theoretical excitation functions for the lighter heavy-ion systems are rather insensitive to changes in either the conservative or dissipative forces. The calculations show that tangential friction sufficient to produce the rolling condition is necessary to explain the magnitude of the fusion cross sections at high energies, which are also sensitive to the magnitude of the radial friction component. This is in contrast to the fusion cross sections at low energies which are determined by the nuclear potential at larger separations, and to a lesser extent by tangential friction. The low energy fusion data are most sensitive to the nuclear radii. The

calculations reveal the importance of more experimental measurements of fusion cross sections at high energies, especially for heavy systems where the magnitudes of the fusion cross section are the most sensitive to the assumed forces. However, even for these cases the effects of the conservative and dissipative forces are similar and difficult to separate. These studies indicate, however, that it is possible to construct a conservative potential that will give calculated fusion excitation functions which are in good agreement with all experimental data over the entire mass range. The maximum fusion cross sections as defined here exceed considerably the liquid-drop limiting value for very heavy systems.

1. Introduction

In this paper a detailed comparison is made of experimental and theoretical heavy-ion fusion excitation functions over a wide range of mass and energy. The theoretical fusion cross sections are calculated with a classical dynamical model which contains conservative and dissipative forces. Results based on this model have already been published [1] for some reactions. In the present calculation the trajectory for each value of the angular momentum is followed as a function of time in order to determine whether the ions are trapped behind a potential barrier (fusion) or escape over the barrier at a reduced energy (damped collision).

The fusion cross section is a measure of the range of impact parameters or number of angular momenta ℓ that are trapped behind a potential barrier. This number depends combinatively on the radial dependence of a) the Coulomb potential, b) the nuclear ion-ion potential and c) the dissipative forces. In addition, the sensitivity of the fusion cross section to a), b) and c) depends on the bombarding energy and the projectile and target masses. The present global comparison of theoretical calculations with fusion data was initiated to investigate the overall quality of the fits obtained with the proximity nuclear potential [2] and the proximity one-body nuclear friction [3].

Information about the ion-ion nuclear potential has been reported from analyses of experimental heavy-ion fusion

excitation functions based on a simple 'friction-free model' [4-10]. This model assumes zero friction up to a certain point and then an infinite short-range friction (see section 4.4). The fusion cross section in this model is given by

$$\sigma_{\text{fus}}(E) = \pi R_B^2 \{1 - [V_B(R_B)/E]\} \quad (1)$$

where R_B is the barrier radius, $V_B(R_B)$ is the conservative potential at the barrier radius and E is the center-of-mass energy. Values of R_B and $V_B(R_B)$ are determined from the slope and intercept of a plot of $\sigma_{\text{fus}}(E)$ vs $(1/E)$, as illustrated in fig. 1. Such an analysis of fusion data for a particular reaction gives values of both R_B and $V_B(R_B)$, where $V_N(R_B) = V_B(R_B) - V_C(R_B)$, that are in qualitative agreement with the proximity potential [8,9]. However, rather sizeable errors in R_B result from experimental uncertainties in even the best fusion data and this leads to a sizeable ambiguity in the determination of the nuclear potential [11]. This point is illustrated by figs. 1 and 2. In fig. 1 the data of Blann et al. [6,7] for the $\text{Sn} + {}^{35}\text{Cl}$ reaction are plotted along with four lines fitted to this data. Line (a) is derived from the proximity potential. Line (b) is fitted to the higher energy data and nearly parallel to line (a). Line (c) is that given by David et al. [7] and line (d) is fitted to only the lower-energy data. The corresponding values of R_B and $V_B(R_B)$ for lines (a) to (d) (see fig. 1) are transformed to ζ and ϕ values (see section 2.3) and compared [11] to the theoretical

proximity potential in fig. 2. The values of (ζ, Φ) derived from lines (b) to (d) are all very different. However, each set of values (ζ, Φ) is in approximate agreement with the theoretical proximity potential. This example illustrates the ambiguity in Φ and ζ that results from such an analysis of fusion data [11].

In addition to the major difficulty discussed above, there are further problems [11] associated with the simple analysis of the fusion data with eq. (1) giving independent values of R_B and $V_B(R_B)$. These include the neglect of the dependence of R_B on ℓ and the neglect of friction. Although friction has a negligible effect on the fusion cross sections at very low energies, it is just at these energies that barrier penetration effects may play an important role and obscure the importance of the nuclear potential.

In the present analysis of the fusion data, we assume the nuclear proximity function [1] to be correct but allow for small variations [1] in the nuclear radius and/or surface diffuseness from the systematic formulae developed by Myers [12]. The inclusion of dissipation makes it possible to compare theory and experiment over the entire energy range. This allows one to examine the radial dependence of the nuclear potential over a much larger variation in distance. In addition, even at energies where the friction does not play a dominant role, it is better to include a first-order correction for dissipation rather than to neglect

it completely.

In section 2 the dynamical theory is discussed in some detail. The forms of the Coulomb and nuclear potentials as well as the dissipative force are described. The definition of a fusion trajectory as used in this study is also discussed in some detail. In section 3 comparisons between theoretical and experimental fusion excitation functions for a large number of heavy-ion reactions covering a wide range of systems and bombarding energies are presented. Section 4 is concerned with the sensitivity of the calculated fusion cross sections to changes in various parameters. Hence, we explore in section 4 the changes in the fusion excitation functions as the Coulomb, nuclear and dissipative forces as well as the nuclear radius are altered. In section 5, fusion barrier parameters are discussed. In addition, the number of angular momenta making up the fusion cross section is compared with the liquid-drop limiting value for a number of reactions. Conclusions are presented in section 6 and acknowledgments in section 7.

2. Description of theoretical calculations

Classical calculations have been used in the past both for the study of fusion [13-15] and strongly damped collisions [16-19]. Some of the earliest dynamical calculations of fusion cross sections incorporated only the degrees of freedom [14-15] of the relative motion and ignored the intrinsic rotation of the ions. The first classical calculation which included the conservation of angular momentum was reported by Bondorf et al. [16]. However, at this time little was known about the radial dependence of the ion-ion potential and the frictional force. This lack of knowledge prompted the parameterization of the conservative and dissipative forces to fit the data.

In the present calculations we make use of the recently proposed proximity nuclear potential [2] and the proximity one-body nuclear friction [3] in order to calculate the fusion excitation functions from first principles. The initial success [1] of such a model for some heavy-ion systems has motivated us to examine both a wider variety of systems and to extend the excitation functions to higher energies [20].

2.1 Equations of motion

The system of two interacting heavy ions is described by four degrees of freedom as shown in fig. 3. These four dynamical variables are r , θ , θ_T and θ_P . The first two quantities, r and θ , define the radial separation and angu-

lar orientation of the system and the second two quantities θ_T and θ_P are the angles specifying the orientation of the target and projectile, respectively. In order to determine a particular trajectory, the following equations of motion are solved:

$$\mu \ddot{r} = - \frac{\partial V_C}{\partial r} - \frac{\partial V_N}{\partial r} + \mu r \dot{\theta}^2 - f_r(r) \dot{r} \quad (2a)$$

$$\mu r^2 \dot{\theta} = L = L_0 + L_T + L_P \quad (2b)$$

$$\frac{dL_T}{dt} = - C_T \left(\frac{r}{C_T + C_P} \right)^2 f_\theta(r) [C_T(\dot{\theta}_T - \dot{\theta}) + C_P(\dot{\theta}_P - \dot{\theta})] \quad (2c)$$

$$\frac{dL_P}{dt} = - C_P \left(\frac{r}{C_T + C_P} \right)^2 f_\theta(r) [C_T(\dot{\theta}_T - \dot{\theta}) + C_P(\dot{\theta}_P - \dot{\theta})] \quad (2d)$$

In these equations V_C is the Coulomb potential (see section 2.2) and V_N is the nuclear potential (see section 2.3). The reduced mass μ is defined by $M_T M_P / (M_T + M_P)$. The radial and tangential friction form factors, $f_r(r)$ and $f_\theta(r)$, respectively, are defined in section 2.4. The torques dL_T/dt and dL_P/dt are proportional to the product of the tangential friction form factor and the relative velocity v_t of the surfaces of the two heavy ions where v_t is calculated from the relation

$$v_t = \frac{r}{(C_T + C_P)} [C_T(\dot{\theta}_T - \dot{\theta}) + C_P(\dot{\theta}_P - \dot{\theta})] \quad (3)$$

and C_T and C_P are the half-density radii of the target and projectile, respectively, as defined by eq. (15). The instantaneous angular momentum L is given by eq. (2b) where L_0 is the initial angular momentum and L_T and L_P are the intrinsic

sic angular momenta of the target and projectile. The latter angular momenta are calculated by the relations

$$L_T = \frac{2}{5} M_T R_T^2 \dot{\theta}_T, \quad L_P = \frac{2}{5} M_P R_P^2 \dot{\theta}_P \quad (4)$$

where R_T and R_P are the equivalent sharp surface radii of the target and projectile, respectively, as defined by eq. (15).

Although degrees of freedom in the present calculation are similar to those introduced previously [16], the present calculation differs in that the torques on the two heavy ions are used rather than the forces.

2.2 Coulomb potential

The Coulomb potential plays an important role in the interaction between two very heavy ions. In this paper extensive use is made of a phenomenological Coulomb potential suggested by Bondorf et al. [16]. This potential is defined by the following equations:

$$V_C(r) = 1.438 Z_T Z_P / r \quad r > R_C \quad (5)$$

$$V_C(r) = V_0 - kr^n \quad r \leq R_C \quad (6)$$

where the nuclear charge radius is given by $R_C = R_{CT} + R_{CP}$, V_C is given in MeV and each radius is given in Fermi. The value of V_0 in MeV is given by

$$V_0 = 0.6 \left[\frac{(Z_T + Z_P)^2}{(R_{CT}^3 + R_{CP}^3)^{1/3}} - \frac{Z_T^2}{R_{CT}} - \frac{Z_P^2}{R_{CP}} \right] (1.438) \quad (7)$$

The quantities n and k are evaluated by matching the potentials and forces at the point $r = R_C$ and are given by

$$n = 1.438 Z_T Z_P / R_C (V_0 - 1.438 Z_T Z_P R_C^{-1}) \quad (8)$$

and

$$k = (V_0 - 1.438 Z_T Z_P R_C^{-1}) / R_C^n \quad (9)$$

The charge radius of each ion is calculated by the method of Myers [21] and is given by

$$R_{Ci} = r_0 [2Z_i / (1 - 3\bar{\epsilon})(1 - \bar{\delta})]^{1/3} \quad (10)$$

where $r_0 = 1.16$ fm. In order to calculate the charge radius R_{Ci} , $\bar{\epsilon}$ and $\bar{\delta}$ must be known. Relations for these quantities are as follows [21]:

$$\bar{\epsilon} = (-2a_2 A^{-1/3} + L \bar{\delta}^2 + C_1 Z^2 A^{-4/3}) / K \quad (11)$$

$$\bar{\delta} = [1 + (3C_1/8Q)Z^2 A^{-5/3}] / [1 + (9J/4Q)A^{-1/3}] \quad (12)$$

where

- $a_2 = 22.0$ MeV, surface energy coefficient
- $L = 99$ MeV, density-symmetry coefficient
- $C_1 = 0.745$ MeV, Coulomb energy coefficient
- $K = 300$ MeV, compressibility coefficient
- $I = (N-2)/2$
- $Q = 25$ MeV, effective surface stiffness
- $J = 35$ MeV, symmetry energy coefficient

The radial dependence of the above Coulomb potential is compared in figs. 4-6 with two point charges, two Fermi charge distributions and a point plus a spherical charge distribution for three heavy-ion reactions of different masses. The latter Coulomb potential is often used in optical model calculations. It is defined by eq. (5) for $r > R_C$ and by

$$V_C(r) = (1.438 Z_T Z_P / 2R_C) (3 - r^2 / R_C^2) \quad r < R_C \quad (13)$$

for $r < R_C$, where R_C is now given by $r_0(A_T^{1/3} + A_P^{1/3})$. In figs. 4-6 this potential is calculated with $r_0 = 1.3$ fm, a value frequently used in the literature.

The phenomenological Coulomb potential of Bondorf et al. [16] is correct in the limits of $r=0$ and $r > R_C$. In addition, this potential follows that of two Fermi charge distributions at distances where the tails of the two charge distributions begin to overlap. However, when the separation radius r becomes smaller than $(C_T + C_P)$, the Bondorf et al. potential is less than that calculated for two Fermi charge distributions where no account is taken of volume conservation. This is qualitatively correct. However, the present matching point at $R_C = R_{CT} + R_{CP}$ may not be the optimum value (see arrows in figs. 4-6 and eqs. (6) and (10)).

The Coulomb potential defined by eqs. (5) and (13), although a recipe often used in optical model calculations, is a poor approximation of the Coulomb potential of two heavy ions when R_C is calculated with r_0 values as large as 1.3 Fermi. Such a formalism puts a point charge inside a spherical charge distribution at much too large a value of R_C . This results in a Coulomb potential considerably less than that of two Fermi charge distributions even for values of $r > C_T + C_P$. Some improvement in this potential can be obtained by arbitrarily making r_0 much smaller.

In this paper the phenomenological Coulomb potential of Bondorf et al. [16] is used with the parameters described in

this section. Comparison of the results based on this Coulomb potential with that of two point charges and a point and spherical charges are made for a number of reactions.

2.3 Nuclear potential

The magnitude and radial dependence of the ion-ion nuclear potential is of utmost importance in the calculation of heavy ion trajectories. Hence, the determination of this potential acting between two complex nuclei from measurements of the elastic and inelastic processes is a goal of experimentalists. Even so, many different types of potentials have been proposed including phenomenological, folding, proximity, self consistent microscopic model and liquid-drop model potentials. These different potentials have been reviewed [22] recently and references to the original literature can be found there.

In this paper, calculations are performed with the nuclear proximity potential of Blocki et al. [2]. These authors have shown that for leptodermous (thin-skinned) matter distributions, the real part of the nuclear ion-ion potential V_N is given by

$$V_N(\zeta) = 4\pi\gamma b\bar{R}\phi(\zeta) \quad (14)$$

where $\bar{R} = C_T C_P / (C_T + C_P)$. The surface energy coefficient $\gamma = 0.9517(1 - 1.7826 I^2)$ where $I = (N - Z)/A$ and N , Z and A refer to the combined system of the two interacting nuclei. The radii C_i (C_P and C_T) are calculated from the equivalent sharp surface radii R_i by the relations

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3} \quad (15)$$

and

$$C_i = R_i [1 - (b/R_i)^2 \dots]$$

where b is a constant related to the surface diffuseness with a value of approximately 1 fm. The indefinite integral of the universal interaction potential $\phi(\zeta)$ depends on the separation distance ζ of the two surfaces where ζ is given by

$$\zeta = (r - C_T - C_P)/b \quad (16)$$

The nuclear potential of eq. (14) is a product of a universal function $\phi(\zeta)$ and a geometrical factor. Blocki et al. [2] have calculated and tabulated this universal function $\phi(\zeta)$ using the nuclear Thomas-Fermi approximation. We use these tabulated values of $\phi(\zeta)$ in our calculations. We refer to the nuclear potential calculated with eq. (14) as the standard proximity potential.

Recently Randrup [23] suggested a modified version of the proximity potential for distances where $\zeta < 0$. In the standard proximity treatment an excess density builds up for these negative values of ζ giving rise to a strong repulsion. However, such configurations are not expected to occur for nuclear collisions at low to moderate energies and suggest some modification of the standard proximity potential for these close collisions. Randrup proposed in this case that the nuclear potential is given by the standard proximity potential for $\zeta=0$ minus the surface energy associated with the two erased caps. The area of the

erased caps is $2\pi\rho^2 = 4\pi\bar{R}s$ and the modified nuclear potential energy for $\zeta < 0$ is

$$\begin{aligned} V_N(\zeta) &= 4\pi\gamma b\bar{R}\phi(\zeta=0) + \gamma(4\pi\bar{R}b\zeta) \\ V_N(\zeta) &= 4\pi\gamma b\bar{R}\hat{\phi}(\zeta) \end{aligned} \quad (17)$$

where

$$\hat{\phi}(\zeta) = \phi(\zeta = 0) + \zeta.$$

The nuclear potential calculated with eq. (17) is referred to as the modified proximity potential. Although Randrup [23] used the modified proximity potential in conjunction with a point-charge Coulomb potential, we have also utilized it with the less repulsive Bondorf Coulomb potential.

It has been shown [1] previously that small changes in the radial parameters from the systematic values given by eq. (15) improves the agreement between experimental fusion excitation functions and theory for some reactions. Examples of calculations where the radial parameters are slightly altered are discussed in section 4.5. Similar improvement in fits is obtained by a change in the surface diffuseness b .

Some calculations have been performed also with a unified nuclear potential suggested recently by Krappe et al. [24]. In fig. 7, the proximity, modified proximity, and unified nuclear potentials are compared for two heavy-ion reactions, $^{27}\text{Al} + ^{16}\text{O}$ and $^{165}\text{Ho} + ^{56}\text{Fe}$. Krappe et al. [24] suggest that their unified nuclear potential is valid only for distances larger than the sum of the half-density radii (illustrated by dashed arrow in fig. 7). Extension of this

potential into smaller distances is necessary before it is useful for fusion calculations (see section 4.3).

In figs. 8 and 9 the total conservative potential $V = V_C + V_N$ is plotted as a function of the radial distance r for two heavy-ion reactions, $^{58}\text{Ni} + ^{35}\text{Cl}$ and $^{165}\text{Ho} + ^{56}\text{Fe}$. The curves for the different angular momentum values ℓ are labelled in each figure. The magnitude of the conservative potential at small radial separations depends critically on the combination of nuclear and Coulomb potentials. A listing of these combinations starting with the most repulsive potential are as follows: (1) standard nuclear proximity potential and point-charge Coulomb potential (SP), (2) standard nuclear proximity potential and Bondorf Coulomb potential (SB), (3) modified nuclear proximity potential and point-charge Coulomb potential (MP), and (4) modified nuclear proximity potential and Bondorf Coulomb potential (MB). These different conservative potentials will be referred to by the notations in parentheses.

2.4 Dissipative forces

Heavy-ion reactions at energies several MeV/amu above the Coulomb energy are distinctly characterized by a large loss in kinetic energy of relative motion. Hence, in any classical dynamical calculation of heavy-ion trajectories it is important to account for the frictional or dissipative forces. Several authors including Gross and Swiatecki contributed to the early development of models for energy dis-

sipation or friction. This subject has been reviewed in a recent article [22].

In the present calculations we use the nuclear one-body proximity friction as proposed by Randrup [3]. This friction is based on the idea that at moderate energies, where the nucleon mean free path is long, the dissipation is expected to be predominantly produced by the inelastic collisions of individual nucleons with the time-dependent average single-particle potential. This model [3] is developed to the same degree of accuracy as the conservative nuclear force by virtue of the proximity treatment [2].

The Rayleigh dissipation function for the one-body proximity friction of Randrup is given by

$$F = \pi n_0 \bar{R} b \Psi(\zeta) (2v_r^2 + v_t^2) \quad (18)$$

where n_0 is the bulk flux of nucleons within the nucleus which is taken to be $n_0 = 0.264 \times 10^{-22}$ MeV sec fm⁻⁴ and $\Psi(\zeta)$ is the universal proximity flux function tabulated by Randrup [3]. The Rayleigh tensor is diagonal in radial and tangential velocities which were introduced in section 2.1 with eqs. (2) and (3).

The respective radial and tangential friction form factors derived from the Rayleigh dissipation function are

$$f_r(r) = 4\pi n_0 \bar{R} b \Psi(\zeta) \quad (19)$$

and

$$f_\theta(r) = 2\pi n_0 \bar{R} b \Psi(\zeta)$$

In the present calculations we use Randrup's tabulated

values of the universal proximity function $\Psi(\zeta)$ which represents the form factor for energy dissipation.

2.5 Definition of fusion

Within the context of the model described in section 2.1 it is possible to determine unambiguously whether a trajectory leads to fusion of the target and projectile. However, trajectories which lead to fusion within the context of the model may not produce fusion in the commonly understood sense of a system in statistical equilibrium in all its intrinsic degrees of freedom, with a long lifetime, and with deformations which are inside the saddle point for fission on the potential surface. The model described in this paper accounts approximately for deformation of the ions, by modification of the nuclear potential for ζ less than 0, while mass transfer or the equilibration of the intrinsic degrees of freedom are not explicitly considered. Thus the fusion predictions of the model refer only to whether the system will be caught behind a barrier in the internuclear potential, and there is no consideration of the question of the subsequent evolution of the trapped system.

Qualitative consideration of the evolution of a trapped di-nuclear system leads to the expectation that the final states may be divided into two categories. These categories include trajectories which lead to fully equilibrated fusion of the two nuclei, perhaps followed by fission for the heavier systems, and trajectories which lead to a composite system in which some of the intrinsic degrees of freedom are

only partially equilibrated before fission occurs. In addition, as discussed below, very long interaction times are exhibited by some of the trajectories with angular momenta ℓ slightly greater than ℓ_f , the highest angular momentum leading to fusion. These long-lived untrapped systems comprise a third category of trajectories which may also lead to final states difficult to distinguish from those contributed by trapped systems.

Practical distinctions between the first two categories of final states described above cannot be made by the model described in this paper, since distinctions are dependent on the evolution of the degrees of freedom unaccounted for by the model. However, trajectories in the third category, which have long interaction times but are not trapped, also have di-nuclear system lifetimes determined by the inter-nuclear potential and the friction. There is evidence that some of the degrees of freedom equilibrate very rapidly. These rapidly equilibrating modes include the charge-to-mass ratio and the excitation energy division degrees of freedom [22,25-27]. Other degrees of freedom equilibrate more slowly, especially the mass asymmetry [22,25]. Thus, fragments from untrapped trajectories but long interaction times may be distinguishable in mass asymmetry from fragments produced on trapped trajectories, where the still longer interaction times are determined by more detailed considerations of the internal

structure of the system.

For the two categories of trapped trajectories, the lifetimes of the di-nuclear system may be so long that, even if fission occurs before full equilibration, it may be difficult to distinguish the resultant fragments from those produced by fission after fully equilibrated fusion. The formation of stable evaporation residues is dependent on factors such as the rotational stability of the nucleus against fission [28], and the shape of the nucleus at the fission saddle point [29]. The influence of these factors may prevent the formation of evaporation residues, or even fully equilibrated fusion, especially for the heavier systems and higher energies. However, the interaction times for the trapped systems may be so long that the resultant fragments will be indistinguishable from fully equilibrated fusion followed by fission. In this connection it may turn out that the width of the resulting mass distribution from fusion is a parameter that characterizes the degree of equilibration of such composite systems. Experimental measurements of the width as a function of energy are desired.

Fig. 10 shows the interaction times as a function of the asymptotic entrance channel ℓ value for some trajectories near ℓ_f at various incident energies for the system $^{165}\text{Ho} + ^{56}\text{Fe}$. The interaction time is defined as the time spent by the system inside the strong absorption radius determined from the systematics of elastic scattering [30,

31]. This figure shows that at higher energies there are a considerable number of trajectories which lead to long interaction times and a corresponding large number of rotations of the di-nuclear system during interaction.

There are two possible conditions which, the model shows, may lead to fusion. The simplest case arises in calculations made with the standard proximity potential with a hard core. In this case the effective potential, consisting of the sum of nuclear, Coulomb and centrifugal potentials, is repulsive at small separations of the two nuclei for all partial waves. Thus, all trajectories will have an apsidal distance and will lead to at least the onset of separation of the target and projectile in the latter stages of the interaction. However, for all but the heaviest systems, the trajectories of lowest ℓ will show a 'pocket' and a barrier in the effective potential at some radius. If during the separation in the exit channel, the system should reach the inside of the barrier with insufficient energy to surmount the barrier, then the separation of the two nuclei will cease, and the system will fuse.

The second condition which may lead to fusion arises in calculations with the modified proximity potential combined with the Coulomb potential of Bondorf et al. In this case the modified proximity potential is still attractive even at low values of the radial separation and the Coulomb potential is not singular at $r = 0$. Thus, for

values of ℓ sufficiently low, the effective potential is not repulsive at any value of r . High energy trajectories with ℓ below ℓ_f in this second case, may not show an apsidal distance because of the absence of a repulsive core in the effective potential. Such trajectories approach arbitrarily small separations and are taken to lead to fusion. For the purpose of the calculations any trajectories which reach a separation of 0.2 fm are taken to lead to fusion under the second condition.

It is most important to note that whether a trajectory will lead to fusion cannot be determined from inspection of the effective potential alone, as in the 'friction free' models. This situation arises primarily because of the effect of tangential friction which leads to a continuous transfer of orbital to intrinsic angular momentum during the interaction. Thus, a trajectory which shows no barrier and pocket in the effective potential for the asymptotic entrance channel value of ℓ may experience such a barrier in the potential at some stage during the interaction because the initial value of ℓ is reduced during the interaction and the repulsive centrifugal potential is also reduced. Alternatively, for fusion under the second condition described above, a trajectory which has a repulsive core in the effective potential because of the centrifugal potential for the asymptotic value of ℓ , may not experience a repulsive potential at small values of r because of the

reduction in ℓ caused by the tangential friction. It is the reduction in ℓ due to the tangential friction which is necessary to reproduce the magnitude of the fusion cross section at high energies. Where simple inspection of the effective potential would predict much lower cross sections. Energy loss alone cannot produce the enhancement of the fusion cross section at high energies over the expectations of "friction free" models (see section 4). It is necessary that pockets appear in the effective potentials and this can only be done for the larger impact parameters at high energy by the reduction in the centrifugal barrier produced by the transfer of orbital angular momentum to intrinsic spin. Of course the radial friction also influences the trajectories of the nuclei as discussed below, but the expected reduction of the fusion cross section due to the radial friction which could occur at near barrier energies because of the lowering of the kinetic energy below the barrier, does not occur since the reduction in ℓ due to tangential friction lowers the barrier height [1].

The production of the kinetic energy loss, however, is more evenly divided between the tangential and radial friction. Whether radial or tangential friction is dominant in the production of the energy loss is dependent both on the target-projectile system and on the incident projectile energy. The energy losses due to tangential and radial friction are shown separately as a function of ℓ in

figs. 11 and 12 for the two systems $^{165}\text{Ho} + ^{56}\text{Fe}$ and $^{27}\text{Al} + ^{32}\text{S}$, respectively. Included also in figs. 11 and 12 is the energy in intrinsic spin of the fragments as a function of angular momentum ℓ . The intrinsic spin energy is not dissipated energy in this model, however, it is energy removed from the translational kinetic energy of the fragments. For the trajectories of highest ℓ , which are grazing trajectories with predominantly tangential velocity, the energy loss is predominantly produced by the tangential friction. This is true for all non-fusing trajectories of the $^{27}\text{Al} + ^{32}\text{S}$ system, since most of the reaction cross section leads to fusion and the non-fusing trajectories are all of a grazing type. For the heavier systems with the standard proximity potential, trajectories of lower ℓ are more deeply penetrating and, as fig. 11 shows, such trajectories have energy losses predominantly due to the radial friction. However, at lower energies, even for the heavy systems, no trajectories lead to sufficiently deep penetration for radial friction to produce larger energy losses than tangential friction for non-fusing trajectories. In addition, as shown in fig. 11, for heavy systems with the modified proximity friction, the increased number of 'pockets' in the effective potential produces fusion at sufficiently high ℓ that non-fusing trajectories have energy losses predominantly caused by tangential friction, unless the bombarding energy is raised to very high values. Whether

the energy loss is primarily due to radial or tangential friction is determined by the depth of interpenetration of the nuclei on a trajectory. Deeply penetrating trajectories have energy losses primarily due to radial friction, whereas grazing trajectories are dominated by energy loss due to tangential friction. This situation arises since the tangential friction can, at most, reduce the nuclei to a rolling condition and thereafter no further energy loss or angular momentum transfer can be produced by the tangential friction. Hence, the more deeply penetrating trajectories will be brought to rolling early in the interaction and all subsequent energy loss must be due to radial friction.

3. Results

3.1 Description of models

In this section comparisons of fusion excitation function data to calculations are discussed. The classical trajectory program described in section 2 is run to determine l_f , the maximum angular momentum that fuses. The fusion cross section is then calculated with the relation

$$\sigma_{\text{fusion}} = \pi \lambda^2 l_f(l_f + 1) \quad (20)$$

This formulation of the fusion cross section is not valid at bombarding energies near the fusion barrier where barrier penetration effects may be important. Hence, in this article no attempt is made to fit the very low energy data.

The experimental measurements are due to many different experimental groups. These excitation functions can for purposes of discussion be divided into three groups of heavy-ion systems.

a. Light systems where it is generally assumed that the entire fusion cross section is contained in heavy evaporation residues. These residues result from de-excitation of the fused system by light particle emission. Although this definition of fusion is very straightforward, it is not simple in practice to decide which final products to include as part of the fusion evaporation residues. This is particularly true for high bombarding energies when the masses of the evaporation residues may overlap with the masses of the target and projectile and are difficult to identify unambiguously. Some information on the relative importance of fusion and damped collisions for producing these intermediate fragments may be obtained by making

Hauser-Feshbach statistical calculations [32]. At high energies, it is possible that fission makes some contribution to the fusion cross section. However, no experimental information is available and the identification of fission fragments for these light (nearly symmetric) systems is most difficult.

b. Intermediate systems where the fusion cross section is composed of both evaporation residues and fission fragments. For these systems there is usually no ambiguity in the assignment of products to the evaporation residues. However, difficulty arises in the identification of the fission fragments when the target and projectile masses are nearly equal. An example of an intermediate system is the $^{118}\text{Sn} + ^{35}\text{Cl}$ reaction. At the higher bombarding energies reaction products are expected in four general mass regions. First of all, evaporation residues are expected in the mass region just below A of 150. Fission fragments are expected in a broad mass region centered at $A \approx 75$. In addition, two peaks associated with the deep-inelastic or damped collision process are expected at masses centered around the target and projectile masses. At a bombarding energy of $E_{\text{c.m.}} = 130$ MeV, for example, the ratio of the fission to evaporation residue cross section is 24%. At higher energies this ratio increases sharply.

c. Heavy systems where the fusion cross section is given almost entirely by the fission cross section. Many of these systems have a very large deep-inelastic or

damped collision cross section, especially at the higher energies. Hence, fusion-fission cross sections are measured with least ambiguity when the target and projectile are well separated in mass. In section 2.5, our definition of fusion is discussed. Operationally, fusion is distinguished from the damped collision process by the additional requirement that the mass asymmetry degree of freedom is equilibrated.

The data and calculated excitation functions are compared in figs. 13-68. In general, the experimental data are compared with three different theoretical calculations where the radial dependence of the nuclear potential V_N and the Coulomb potential V_C are varied. The three calculations will be referred to as models (MB), (SB) and (SP).

1. Model (MB). The Coulomb potential of Bondorf et al. [16] (see eqs. (5) and (6)) is combined with the modified nuclear proximity potential of Randrup [23] (see eqs. (14) and (17)). This model is expected to give the largest values of the theoretical fusion cross section and is depicted by a dashed line in figs. 33-68. Randrup [23] suggested this modified nuclear potential be used with point-charges for the Coulomb potential. Hence, the fusion cross sections calculated here with the less repulsive Bondorf Coulomb potential should be viewed as upper limits to the cross sections.

2. Model (SB). The Coulomb potential of Bondorf is

combined with the standard nuclear proximity potential of Blocki et al., [2] (see eq. (14)). The results from this model give intermediate values of the fusion cross section and these theoretical excitation functions are shown in figs. 54-68 as dotted lines.

3. Model (SP). The Coulomb potential for point-charges is combined with the standard nuclear proximity potential. This potential is the most repulsive of the three potentials. Although the Coulomb potential of point-charges is much too large for close heavy-ion collisions and, therefore, unrealistic, these calculations were performed in order to give a lower limit to the fusion cross section and help establish the sensitivity of the results to the choice of Coulomb potential. The theoretical results based on model SP are shown as short-long dashed lines in figs. 33-68. In a later section (section 4.1) calculations using a Coulomb potential which is less repulsive than the Bondorf potential will be discussed.

In conjunction with all three of the above models, we employ the nuclear one-body proximity friction as proposed by Randrup [3]. At low energies the three above models lead to identical excitation functions. When all three models give a common excitation function, this function is illustrated in figs. 13-68 as a solid line. For light systems the excitation functions for the three different models are close to identical at high energies and, hence, only a

solid line is shown for such systems at high energy in figs. 13-32. As the mass of the target-projectile system is increased the high energy fusion cross sections are largest for model (MB), least for model (SP) and of intermediate magnitude for model (SB). For some systems, where the differences at high energy between the three model calculations, are not large, only the excitation functions based on models (MB) and (SP) are shown in figs. 33-53 as they establish the upper and lower limits of the calculated fusion cross sections. It should be emphasized that all the calculations in section 3 were performed with the radii defined by eq. (15). Examples of results based on small changes in these radii are reported in section 4.5.

3.2 Comparison of experimental and calculated fusion excitation functions for systems where all three models give the same result ($Z_T Z_P < 150$).

The calculated fusion excitation functions for the heavy ion systems discussed in this section are the same for models (MB), (SB), and (SP) as shown in figs. 13-32. The insensitivity of these systems to changes in the nuclear and Coulomb potentials at small distances (i.e., radius values less than the sum of $(C_T + C_P)$) is associated with the fact that the fusion barrier distances are much larger than the distance $C_T + C_P$. In fact, for these light systems the fusion barrier distance is only slightly smaller (0.6 to 0.8 fm) than the corresponding interaction radius (see table 1).

3.2.1. $^{162}\text{Dy} + ^4\text{He}$

The experimental data [33] are compared to the calculations in fig. 13. The total experimental fusion cross section is obtained by summing the (α, xn) cross sections. The agreement between theory and experiment is excellent.

3.2.2. $(^{12}\text{C} \text{ and } ^{27}\text{Al}) + ^{12}\text{C}$

The data for the $^{12}\text{C}+^{12}\text{C}$ system are compared to the calculations in fig. 14. The data shown as solid dots are from Sperr et al., [34] and the data shown as triangles are from Namboodiri et al. [35,36]. Both sets of data have been measured by Z identification of evaporation residues with ΔE - E counter telescopes. Angular distribution measurements have been made. The definition of evaporation residues is complicated for light systems by the effects of particle evaporation, which may leave the Z of the residue close to or even below Z of the target or projectile. Thus, uncertainties arise in the distinction of evaporation residues from the fragments resulting from reaction mechanisms other than complete fusion. At high incident energies for light systems there are also problems of cross section normalization which arise because only a very small angular range near the beam direction exhibits pure Coulomb elastic scattering [37]. Hence, as can be seen from fig. 14, there is some disagreement between the measurements of ref. [34] and [36]. This disagreement seems to arise since Sperr et al. [34] define all residues with $Z > 6$ as evaporation residues, where Namboodiri et al. [36]

consider fragments with Z as low as 3 to be evaporation residues if they satisfy appropriate energy and angular distribution criteria. It is shown by Namboodiri et al. that the yield of low Z evaporation residues increases as the incident energy is raised. Thus, the fusion cross sections of Sperr et al. are likely to be increasingly below the true fusion cross section as the projectile energy increases. The theoretical calculations predict fusion cross sections too high at low energies and too low at high energies. The calculations are not expected to reproduce the structure in the excitation functions, which is sometimes observed for light systems, since this structure may be due to specific nuclear structure effects for some systems [38]. Recent measurements [39] of the total fusion cross section are in qualitative agreement with earlier results.

The comparison of data and experiment for $^{27}\text{Al} + ^{12}\text{C}$ reactions is shown in fig. 15. The data are from Betts et al. [40,41] and Natowitz [42]. This system is largely free of difficulties produced by particle evaporation as is demonstrated in ref. [40]. The high energy points shown in fig. 15 by squares and triangles are measurements made by detection of evaporation residues with ΔE - E telescopes [40] and mica track detectors [42]. The solid dots are measurements derived from γ -ray yields of evaporation residues [41]. For this system the Z identification measurements of the evaporation residues are unambiguous except for a small con-

tamination from low-energy projectile-like fragments [40]. Mica track detector measurements are somewhat more difficult to make because of the problem of separating evaporation residues from other light fragments which may also produce particle tracks. Evaporation residue yields may be determined by measuring the entirety of known γ -ray transitions in the nuclei. Such γ -ray measurements are difficult because of the large amount of nuclear structure information necessary to obtain absolute evaporation residue yields. In addition, the γ -ray measurements are insensitive to production of evaporation residues in reaction channels which do not lead to γ -ray emission. In some cases this de-excitation without γ -ray production has been found to be non-negligible [43]. However, in the high energy region the $^{27}\text{Al} + ^{12}\text{C}$ fusion cross sections, measured by the three techniques, are in good agreement. The agreement between data and the theory may be improved by increasing both target and projectile radii by 0.24 fm (see section 4.5).

3.2.3 $^{12}\text{C} + ^{13}\text{C}$

Data are shown in fig. 16 from γ -ray yield deduced evaporation residue cross sections of Dayras et al. [44] and Z identification telescope measurements of Kovar et al. [38,45] at higher energies. The γ -ray yield measurements are shown as dots and the Z measurements as triangles. The data of Dayras et al. are measurements of evaporation residues which have emitted one α -particle, proton or neutron only. A detailed discussion of the γ -ray measurement technique

for determination of fusion cross sections is given by Dayras et al. [44]. The Z identification measurements of Kovar et al. have the same difficulties as those of ref. [34] discussed in section 3.2.2 since only fragments with $Z > 6$ are taken to be evaporation residues. Thus, the triangles in fig. 16 at high energies are probably lower limits to the $^{12}\text{C}+^{13}\text{C}$ fusion cross section. The agreement between the data and calculations is good except at the highest energies.

3.2.4. (^{12}C and ^{16}O) + ^{14}N

The data for $^{12}\text{C}+^{14}\text{N}$ are compared to the calculated excitation function in fig. 17. Evaporation residue measurements were made with energy loss techniques for Z identification by Stokstad et al. [32] and are shown as diamonds. Total reaction cross section measurements made by Kuehner and Almquist [46] are shown as dots. Evaporation residue measurements made with $\Delta E-E$ Z identification techniques by Kovar et al. [38,47] are shown as squares. Fusion cross sections inferred from γ -ray yields by Switkowski et al. [48] are shown as triangles. The major difficulties in the identification of evaporation residues occur for $^{12}\text{C}+^{14}\text{N}$ at the higher energies. Stokstad et al. [32] define the evaporation residues down to $Z=3$ by use of angular distribution and energy criteria guided by the expectations of Hauser-Feshbach calculations. The agreement between data and calculation is within the error bars.

Only lower energy data are available for the $^{16}\text{O}+^{14}\text{N}$ system. The data are evaporation residue cross sections deduced from γ -ray yields by Switkowski et al. [48]. The calculated cross sections are in reasonable agreement with the data within the error bars as shown in fig. 18. A better fit would be obtained with a small increase in the nuclear radii. The low-energy tail in the excitation function clearly seen for $^{16}\text{O}+^{14}\text{N}$ is not expected to be reproduced by the calculations, which take no account of barrier penetration effects.

3.2.5 $^{12}\text{C}+^{15}\text{N}$

The calculated fusion excitation function for the $^{12}\text{C} + ^{15}\text{N}$ reaction and the available experimental data are compared in fig. 19. The data of Conjeaud et al. [39] were taken with a E- Δ E counter telescope. The experimental cross sections are in reasonable agreement with the calculations.

3.2.6 (^{10}B , ^{12}C , ^{16}O , ^{24}Mg , ^{26}Mg , ^{27}Al and ^{28}Si) + ^{16}O

The comparison between the calculated fusion excitation function and the experimental data [49] for the $^{10}\text{B}+^{16}\text{O}$ reaction is shown in fig. 20. The agreement between the data and the calculations is worse for this reaction than any of the other light systems.

Calculations and data for the $^{12}\text{C}+^{16}\text{O}$ reaction are shown in fig. 21. Measurements by Sperr et al. [50] using a Z identification telescope are shown as dots. Time-of-flight evaporation residue measurements of Fernandez et al. [51] are shown as triangles. Time-of-flight combined with

Z identification measurements at one incident energy by Weidinger et al. [43] are shown also. Sperr et al. exclude fragments with $Z < 10$ from the category of evaporation residues on the ground of a low yield of fragments with $Z = 9$ and the large negative Q value for 3 α -particle evaporation necessary to produce residues with $Z = 8$. The time-of-flight data of Fernandez et al. includes all fragments with $A > 19$ and is therefore roughly equivalent to the measurements of Sperr et al. in the definition of evaporation residues. However, Fernandez et al. suggest on the basis of observed evaporation residue mass distributions, that the 3 α -particle reaction may not be negligible at laboratory projectile energies above 60 MeV. If such an observation is correct, the four higher energy points in fig. 21 represent a lower limit to the fusion cross section. Weidinger et al. [43] with both Z and A identification of fragments include evaporation residues with $A > 16$, after some separation of direct reaction contributions on the basis of energy spectra. The data point so derived still excludes the 3 α -particle evaporation reaction and is, therefore, not greatly different from the other two measurements in definition of the fusion cross section. The calculations are in rough agreement with the data although only a limited range of energy has been measured. It may be that the higher energy cross section data is too low and measurements nearer the barrier would determine whether a nuclear

radius enhancement would improve the agreement between data and calculations.

The $^{16}\text{O}+^{16}\text{O}$ fusion data and calculations are shown in fig. 22. The low-energy data of Spinka and Winkla [52] shown as diamonds, are in considerable disagreement with the data of Kolata et al. [53] shown as dots. The fusion cross sections of Spinka and Winkla are derived from measurements of yields of light particles (p, α , n, d) and some decay measurements of radioactive species produced in the reaction. These measurements are somewhat more indirect than the data of Kolata et al. which are derived from γ -ray yield measurements from evaporation residues. Kolata et al. identify 12 isotopes with $Z > 10$ as evaporation residues. Unidentified evaporation residues may be important at the higher energies, thus explaining the apparent underestimate of the fusion cross section made by Kolata et al. when compared to the data of Fernandez et al. [51] which are shown as triangles. The time-of-flight technique used by Fernandez et al. has already been considered in this section. In the case of $^{16}\text{O}+^{16}\text{O}$ reaction, Fernandez et al. have additional difficulties associated with a TiO_2 target, namely the difficulty of obtaining cross section normalization from elastic scattering due to interference from the titanium and the effects of carbon build-up on the target. Cross section normalization is obtained by Fernandez et al. from optical model analysis

of elastic scattering. Fernandez et al. define as residue fragments nuclei with $A > 20$. Other measurements contributing data to fig. 22 are those of Weidinger et al.

[43] and Conjeaud et al. [54]. The Z and A identification measurements of Weidinger have been considered earlier in this section. In the case of $^{16}\text{O}+^{16}\text{O}$, Weidinger et al. define evaporation residues to be fragments with $A > 16$, with separation of direct reaction contributions on the basis of energy spectra. An $E-\Delta E$ telescope was used by Conjeaud et al. in their measurement of the evaporation residues from the $^{16}\text{O}+^{16}\text{O}$ system. Recent measurements of Tserruya et al. [55] are shown on fig. 22 as hexagons. Evaporation residues were detected using either the $\Delta E-E$ telescope or time-of-flight technique. All products with $Z > 10$ or $A > 20$ were considered as evaporation residues and defined the fusion yield. With the exception of the low-energy data of Spinka and Winkla and considering the spread in the data from different experiments the calculations agree well with the data.

The systems $^{24}\text{Mg}+^{16}\text{O}$ and $^{26}\text{Mg}+^{16}\text{O}$ are shown in figs. 23 and 24, respectively. The data are from Tabor et al. [56]. The measurements are from $\Delta E-E$ particle identification techniques and all fragments with $Z \geq 14$ were taken to be evaporation residues. With these somewhat heavier systems, the confusion between target-like, projectile-like and evaporation residues is largely removed and the

ΔE - E spectra are generally easily separated into evaporation residue and other components, except at very low energies where the Z identification properties of the energy loss curves are no longer present. The fusion excitation functions are derived from a combination of complete evaporation residue angular distributions and deduction of the total cross section from the differential cross section at $6^\circ(\text{lab})$. As is true of all reactions in section 3.2, calculations with the three models defined in section 3.1 give the same fusion excitation function represented in figs. 23 and 24 by the solid line. The agreement between the data and the model calculations is reasonable, although the data show a slightly steeper slope than the calculations at lower energies.

Calculations and data for $^{27}\text{Al}+^{16}\text{O}$ are shown in fig. 25. The data are from Eisen et al. [57], Back et al. [58], Dauk et al. [59], Kozub et al. [60] and Rascher et al. [61]. The dots in fig. 25 represent evaporation residue cross sections determined from γ -ray measurements [59]. These data show what appears to be an underestimate of the fusion cross section at the higher energies when compared to the Z identification telescope measurements of Eisen et al. [57] shown in diamonds. This behavior of γ -ray evaporation residue identification measurements has been discussed previously for $^{16}\text{O}+^{16}\text{O}$ reaction. The γ -ray residues include 12 isotopes with $Z > 17$. The Z

identification technique of Eisen et al. works well for this system, since particle evaporation does not confuse the identification. Full evaporation residue angular distributions have been measured at all measured energies by Eisen et al. except for very forward angles where the cross sections were determined by extrapolation. This extrapolation introduced corrections to the measured cross sections of up to 25%. There was also a small correction of ~2% for the cross section at unmeasured backward angles. For the heavier systems this problem of measurement of evaporation residues at very forward angles is a disadvantage of the Z or A identification techniques which rely on ΔE -E telescope or time-of-flight measurements since a considerable amount of the cross section is within a few degrees of the beam direction. Similar Z identification methods were used by Kozub et al. [60], to give the data shown as squares in fig. 25. A time-of-flight technique has been used for $^{27}\text{Al}+^{16}\text{O}$ by Back et al. [58]. Angular distributions for the evaporation residues were measured at all measured energies and the resultant fusion cross sections are shown as triangles in fig. 25. Rascher et al. [61] also use a time-of-flight technique to identify the evaporation residues shown as inverted triangles. The calculations agree well with the data over the entire energy range.

Measurements of the fusion cross section for the

$^{28}\text{Si}+^{16}\text{O}$ reaction have been made by two groups [59,61]. The data of Rascher et al. [61] shown in fig. 26 as dots, were measured by time-of-flight techniques and agree very well with the calculated excitation function. The data of Dauk et al. [59], shown as triangles in fig. 26, have sizeable uncertainties and are in poor agreement with the calculations.

3.2.7. $^{27}\text{Al}+^{17}\text{O}$

The calculated fusion excitation function and the experimental fusion data of Eisen et al. [57] are shown in fig. 27. The fusion data were obtained from Z identification telescope measurements as discussed in section 3.2.6. for the $^{27}\text{Al}+^{16}\text{O}$ reaction. The agreement between data and calculations may be improved by increasing the radii of the ions slightly.

3.2.8. (^{12}C , ^{24}Mg , ^{27}Al and ^{28}Si) + ^{16}O

The data for $^{12}\text{C}+^{16}\text{O}$ are taken from Sperr et al. [34] and shown in fig. 28. The measurements are taken from telescope Z identification of all evaporation residues with $Z > 8$. This measurement has already been considered in section 3.2.2., and may underestimate the fusion cross section at high energies. The results of calculations based on the three models of section 3.1 give very similar excitation functions and are shown as a single solid line. The agreement between data and calculations may be improved at lower energies by enhancement of the nuclear radii.

The data and calculations for the $^{24}\text{Mg}+^{16}\text{O}$ reaction

are shown in fig. 29. The measurements are taken from telescope Z identification and angular distribution measurements of evaporation residues by Tabor et al. [56]. These measurements have been discussed in section 3.2.6. The calculations agree well with the data.

The experimental data [57,61] and calculations for the $^{27}\text{Al}+^{18}\text{O}$ reaction are shown in fig. 30. The measurements of Eisen et al. [57], shown as triangles, are based on the Z identification telescope technique as discussed in section 3.2.6 for the $^{16}\text{O}+^{16}\text{O}$ reaction. The data of Rascher et al. [61], shown as dots, were obtained by detecting the evaporation residues in a time-of-flight spectrometer. The agreement between data and calculations is satisfactory although it may be improved by a small increase in the nuclear radii.

The experimental fusion cross sections of Rascher et al. [61] for the $^{28}\text{Si}+^{18}\text{O}$ reaction are compared with calculations in fig. 31. The evaporation residues were measured by a time-of-flight technique. Agreement between data and theory is satisfactory.

3.2.9. $^{12}\text{C} + ^{19}\text{F}$

The experimental fusion cross sections of Sperr et al. [34] for the $^{12}\text{C}+^{19}\text{F}$ reaction are compared with theory in fig. 32. The data are based on Z identification by ΔE -E telescope measurements of evaporation residues with $Z > 9$. Inherent difficulties associated with this

measurement technique have been discussed in section 3.2.2. The experimental fusion cross sections are larger than the calculated values. An improved fit is obtained by increasing the nuclear radii.

3.3. Comparison of experimental and calculated fusion excitation functions for systems where the three models give slightly different results at high energies ($150 < Z_T Z_P < 850$).

The calculated fusion excitation functions for the heavy-ion systems discussed in this section are slightly different for models (MB), (SB), and (SP). At center-of-mass energies up to the maximum in the excitation function, all models give the same fusion cross sections. However, as the energy increases above the cross section maximum, the results from the different models begin to deviate slightly as shown in fig. 33-53. Curves are shown only for the two models giving the most extreme results, i.e., the modified nuclear proximity potential with the Bondorf Coulomb potential (MB) and the standard nuclear proximity potential with a point-charge Coulomb potential (SP). Results with model (MB) are shown as dashed lines. The standard nuclear proximity potential with the Bondorf Coulomb potential (SB) give results intermediate to models (MB) and (SP), however these results are not shown in the figures.

3.3.1 $^{152}\text{Sm} + ^{12}\text{C}$

The available experimental fusion data [33] for the

$^{152}\text{Sm}+^{12}\text{C}$ reaction are compared to calculations in fig. 33. The experimental total fusion cross section is obtained by adding together the $(^{12}\text{C},\text{xn})$ cross sections. The agreement between experiment and theory is very good. It is assumed that the $(^{12}\text{C},\alpha\text{xn})$ and $(^{12}\text{C},\text{pxn})$ reactions are not part of the fusion cross section.

3.3.2. (^{40}Ca , ^{63}Cu , ^{140}Nd , ^{150}Nd and ^{208}Pb) + ^{16}O

The data and calculations for the $^{40}\text{Ca}+^{16}\text{O}$ system are shown in fig. 34. The data are from ΔE -E measurements of angular distributions by Vigdor [62] and Geesaman et al. [37]. Some γ -ray identification measurements were also performed by Geesman et al. [37]. The Z identification measurements are unambiguous except for very low energy evaporation residues and provide the primary data for the whole $^{40}\text{Ca}+^{16}\text{O}$ excitation function. There is possibly some loss of evaporation residues at the higher energies because of the probability of fission which has not been measured. As pointed out earlier, the results from model (MB) are shown as a dashed line and those from model (SP) are shown as a long-short dashed line. The results from model (SB) lie between the other two, and all calculations lead to identical results at lower energies and are shown as a solid curve. These data are not well fitted by the calculation. An enhancement of the nuclear radii improves the overall agreement between theory and experiment (see section 4.5).

The experimental fusion cross sections [63] for the $^{63}\text{Cu}+^{16}\text{O}$ reaction are compared to theory in fig. 35. The experimental fusion cross sections are obtained by summing excitation functions of individual product nuclei identified by characteristic γ -rays. An uncertainty of 12% in the fusion cross sections is reported as due to uncertainties in the absolute efficiency of the Ge(Li) detector and in target thickness. Agreement between theory and experiment is good at the lower energies, especially since the cross section of four reactions were estimated because the residual nuclei were stable or long-lived. In addition, some direct reaction cross section may be included, for example, part of the $(^{12}\text{C},\alpha n)$ reaction cross section. The poor agreement for the two highest energy points may be due to missing channels including fission.

The data and calculations for the $^{146}\text{Nd}+^{16}\text{O}$ and $^{150}\text{Nd}+^{16}\text{O}$ reactions are shown in figs. 36 and 37, respectively. The experimental fusion cross sections [33] are obtained by summing the $(^{16}\text{O},xn)$ reaction cross sections. As mentioned in section 3.3.1, it is assumed that the $(^{16}\text{O},\alpha xn)$ and $(^{16}\text{O},pxn)$ reactions are not part of the fusion cross section. Agreement between theory and data is good for both reactions.

The data for $^{208}\text{Pb}+^{16}\text{O}$ are taken from Videbaek et al. [64], and are shown together with the calculations in fig. 38. The data are taken by measurement of fission

fragment angular distributions. Since the Z values of the target and projectile are widely different in the $^{208}\text{Pb}+^{16}\text{O}$ system, the fission fragments are easily distinguished from target or projectile-like fragments, because fission produces a roughly symmetric splitting of the composite system. By comparison of the fusion cross section derived from fission fragment measurements with the total reaction cross section derived from elastic scattering, Videbaek et al. conclude that a negligible fraction of the evaporation residues do not fission. The data require a small increase in radius. At high energies, the model (MB) calculations shown as dashes and the model (SP) calculations shown as short-long dashes, yield slightly different excitation functions. The fusion excitation function calculated with model (SB) lies between the other two. However, no data are currently available to test the calculation for $^{208}\text{Pb}+^{16}\text{O}$ at high energies.

3.3.3 $^{144}\text{Nd}+^{16}\text{O}$

The data and calculations for the $^{144}\text{Nd}+^{16}\text{O}$ reaction are shown in fig. 39. The experimental fusion cross sections [33] are obtained by summing the $(^{16}\text{O},\text{xn})$ reaction cross sections. The $(^{16}\text{O},\alpha\text{xn})$ and $(^{16}\text{O},\text{pxn})$ reaction cross sections are not included as part of the fusion cross section (these are about 10% of the magnitude of the fusion cross section). The overall agreement is good.

3.3.4 $^{28}\text{Si}+^{28}\text{Si}$

Betts et al. [41] have measured the fusion excitation function for $^{28}\text{Si}+^{28}\text{Si}$ shown in fig. 40. The measurements were made from γ -ray yields of evaporation residues and it is possible that unmeasured channels have produced the low cross section at the highest energy. A small nuclear radius enhancement is necessary to improve the agreement between the data and the calculations.

3.3.5 (^{24}Mg , ^{27}Al , ^{40}Ca and ^{58}Ni) + ^{32}S

Fusion measurements with sulphur projectiles have been made by Gutbrod et al. [4] and for $^{24}\text{Mg}+^{32}\text{S}$ by Kovar et al. [65]. The measurements have been made by Z identification of evaporation residues with ΔE - E techniques and angular distribution measurements of the evaporation residues. These techniques work well for systems of intermediate mass where the evaporation residues are well separated in Z from target and projectile fragments the particle evaporation does not confuse the identification of the residues, and fission of the evaporation residues is negligible. The results from model (MB) (dashes) and model (SP) (long-short dashes) calculations are compared with the data in fig. 41-44. Agreement between data and calculations is good for all reactions except perhaps for the $^{40}\text{Ca}+^{32}\text{S}$ reaction where nuclear radii enhancements are necessary. Such an enhancement was also required for the $^{40}\text{Ca}+^{16}\text{O}$ reaction.

3.3.6 (^{27}Al , ^{48}Ti , ^{56}Fe , ^{58}Ni , ^{60}Ni , ^{62}Ni , ^{64}Ni , and ^{90}Zr) + ^{35}Cl

Fusion measurements with ^{35}Cl projectiles have been made by Scobel et al. [6] for ^{27}Al , ^{48}Ti , ^{56}Fe , ^{58}Ni , ^{60}Ni , ^{62}Ni , ^{64}Ni and ^{90}Zr targets. Additional measurements at higher energies for the $^{62}\text{Ni} + ^{35}\text{Cl}$ reaction have been reported by Sikora et al. [66]. All the ^{35}Cl projectile measurements have been made by Z identification with ΔE -E telescope techniques. Measurements of evaporation residue angular distributions have been made also. At the higher bombarding energies with the heavier targets the evaporation residue measurements have been supplemented by measurements of fission fragment angular distributions, since the fusion products have increasing probability for fission as the projectile energy rises. For $^{48}\text{Ti} + ^{35}\text{Cl}$ at a c.m. energy of 92.2 MeV, the fusion-fission events contribute 6% to the fusion cross section. Fusion-fission contributions to the total fusion cross section vary between 5%-10% for $^{62}\text{Ni} + ^{35}\text{Cl}$ between c.m. energies of 102 and 108 MeV, and increase sharply as the energy is increased. The agreement between theory and experiment is excellent for all targets except the nickel isotopes where slight increases in the nuclear radii improve the agreement as discussed in section 4.5. Experiment and theory are compared in figs. 45-52.

3.3.7 $^{40}\text{Ca} + ^{40}\text{Ca}$

Data for this system are reported by Doubre et al. [67] and shown in fig. 53. The fusion cross sections con-

tain only evaporation residues and these were measured with a position sensitive E- ΔE telescope. The calculations and data are in very poor agreement. Improvement is obtained by increasing the nuclear radii. This was found to be necessary also for the $^{40}\text{Ca}+^{32}\text{S}$ and $^{40}\text{Ca}+^{16}\text{O}$ reactions, the only other ^{40}Ca target measurements in this compilation. The highest energy point is considered a lower limit insofar that no measurement has been made of the fission cross section.

Extensive efforts to fit the fusion excitation function for this reaction by both two- and three-dimensional time-dependent Hartree-Fock calculations have been reported [68,69]. The results of the best theoretical fits by this method are of no better quality than the results of the present model.

3.4 Comparison of experimental and calculated fusion excitation functions for systems where the three models give quite different results at high energies ($Z_T Z_P > 850$).

The calculated fusion cross sections discussed in this section are limited to reactions between very heavy systems where the theoretical excitation functions calculated with the three models are quite different at high energy. For some of these systems the largest angular momentum ℓ_f that fuses may be considerably in excess of the liquid-drop critical angular momentum. It is impor-

tent to have additional experimental measurements of fusion cross sections at high energies for these systems, especially for the very heaviest systems where the sensitivity to the conservative and dissipative forces are the largest. For some of the systems with the largest values of $Z_T Z_P$ a fourth model calculation is shown. In this model the conservative potential is calculated with the modified nuclear proximity potential and the point-charge Coulomb potential (MP). At high energy, the magnitude of the calculated fusion cross section increases in the order of the listed models, (SP), (SB), (MP), and (MB). As mentioned previously the (MB) model probably underestimates the conservative potential at small radial separation distances leading to fusion cross sections that are too large.

3.4.1. $^{235}\text{U} + ^{20}\text{Ne}$

Data for the system $^{235}\text{U} + ^{20}\text{Ne}$ are shown in fig. 54. The data are measurements of Viola et al. [70]. With a highly fissionable target such as ^{235}U , the major problem in measurements of the fusion cross section arises because fission is expected from both the fusion products and target nuclei excited in inelastic collisions and transfer reactions. Hence, angular correlation measurements of fission fragments must be made to ensure that a complete transfer of momentum from projectile to the composite system occurred before fission. Usually the separation of complete fusion from incomplete momentum transfer events must

be made through separation of two peaks in a double humped correlation function. Because of these difficulties large error bars are shown on the data in fig. 54. The calculations are shown as dashes for model (MB), dots for model (SB), and short-long dashes for model (SP). The calculations agree with the data to within the error bars. The calculations for this system also illustrate the increasing separation of the calculated excitation functions at high energies, which occurs as the mass of the system increases.

3.4.2 (^{112}Sn , ^{116}Sn , ^{120}Sn , ^{124}Sn and ^{134}Pm) + ^{35}Cl

Experimental measurements [6, 7, 66] of the fusion cross sections for ^{35}Cl induced reactions on targets of ^{112}Sn , ^{116}Sn , ^{120}Sn , ^{124}Sn and ^{134}Pm are in very good agreement with theoretical calculations as shown in figs. 55-59. The experimental fusion cross sections are obtained by summing the evaporation residue and fission cross sections. All of the measurements were made by Z identification with ΔE -E telescope techniques. The present experimental data are at energies where all models give the same cross sections. The two experimental points [66] at the highest energies for the $^{116}\text{Sn} + ^{35}\text{Cl}$ reaction are approaching the region where the model results begin to diverge. Again, in this region the fission cross section is becoming more important as the energy is increased. The fission cross sections for the tin isotopes in the energy range $E_{\text{C.M.}} = 122$ to 132 MeV increases from 8 to 23% of the fusion cross section, depending on the isotope, and is

considerably larger for the two highest energy points for the target ^{116}Sn . The fission cross section for the $^{141}\text{Pr} + ^{35}\text{Cl}$ reaction in the energy range $E_{\text{c.m.}} = 124$ to 136 MeV increases from 16 to 40% of the total fusion cross section.

3.4.3 (^{109}Ag , ^{121}Sb , ^{163}Ho , ^{197}Au and ^{238}U) + ^{40}Ar

Some fusion measurements have been made for the $^{109}\text{Ag} + ^{40}\text{Ar}$ system by Britt et al. [71]. Measurements were made by Z identification of evaporation residues and fission fragments with ΔE -E techniques. The measurements for this reaction are complicated by the presence of a large strongly damped reaction component in the reaction cross section. The damped collisions reduce the counting rate in the fusion channel and require a separation on the basis of Z between damped fragments and fission fragments. Since the Z distributions at large energy losses are broad, this separation is not always unambiguous. The data are compared to the three calculations in fig. 60. Agreement is well within the errors. However, the excitation function data at high energies are not sufficiently accurate to allow a choice of the best potentials for the calculation. The fission cross section for the $^{109}\text{Ag} + ^{40}\text{Ar}$ reaction in the energy range $E_{\text{c.m.}} = 124$ to 247 MeV increases from 4 to 35% of the total fusion cross section.

Fusion data and calculations for the $^{121}\text{Sb} + ^{40}\text{Ar}$ reaction are compared in fig. 61. The fusion data repre-

sent a sum of the cross sections for evaporation residues [71,72] and fission fragments [73]. The evaporation residues were identified with ΔE -E telescope techniques. Cross sections for argon-induced fission included only those events following full momentum transfer. This method eliminated fission events following transfer reactions. Angular distributions of the fission fragments have a shape close to $1/\sin \theta$, and the total fission cross section is obtained by integrating the measured distribution over angle. Fusion data for the $^{165}\text{Ho}+^{40}\text{Ar}$ [73] $^{197}\text{Au}+^{40}\text{Ar}$ [74] and $^{235}\text{U}+^{40}\text{Ar}$ [73] reactions are shown in figs. 62-64, respectively. Mass and energy distributions of the reaction products were measured at different angles for these reactions. One of the special features of the heavier targets is that ℓ_f , the largest ℓ wave leading to fusion, exceeds considerably the liquid-drop limiting value.

3.4.4 $^{165}\text{Ho} + ^{56}\text{Fe}$

A single experimental point for the fusion cross section of the $^{165}\text{Ho}+^{56}\text{Fe}$ reaction has been measured [75] at 8.5 MeV/amu. A fourth theoretical fusion excitation function is included in fig. 65. This curve (two dots and a dash) is based on model (MP) for the conservative potential (see section 2.3 and figs. 8 and 9). The present data point falls midway between the calculations of the conservative potential based on models (MB) and (MP). The in-

dication is that the conservative potential has to be more repulsive than that of model (MB) and less repulsive than that of model (MP).

3.4.5 ($^{165}\text{Ho} + ^{209}\text{Bi}$) + ^{84}Kr

The available experimental fusion data [76, 77] for the $^{165}\text{Ho} + ^{84}\text{Kr}$ and $^{209}\text{Bi} + ^{84}\text{Kr}$ reactions are compared with calculations in figs. 66 and 67, respectively. The fusion cross section is difficult to measure for these heavy systems and hence, the experimental data are either limits or very approximate values. However, it appears that the conservative potential must be more repulsive than that of model (MB). Qualitative agreement with experimental data is obtained with model (MP). It is interesting to note that the difference in the fusion excitation functions calculated with models (MB) and (MP) increases markedly as one increases the mass of the very heavy systems (compare, e.g., models (MB) and (MP) in figs. 65, 66 and 67). Another interesting feature shows up for the $^{209}\text{Bi} + ^{84}\text{Kr}$ reaction when model (SB) is employed. An angular momentum window begins to appear just above the fusion threshold energy and moves to higher l values as the energy is increased.

3.4.6. $^{56}\text{Fe} + ^{132}\text{Xe}$

The single experimental fusion cross section [78] for this system is compared with theory in fig. 68. This measurement was made by E- ΔE telescope identification of the fusion-fission products. The present point is at an energy where all models

give essentially the same result. Higher energy data are required for gaining insight into the approximate nuclear potential at small radii.

4. Sensitivity of calculated fusion cross sections to parameters

In this section we discuss the effects on the theoretical fusion excitation functions produced by varying some of the parameters in the calculations.

4.1 Coulomb potential

In section 3 comparisons of fusion cross sections calculated with models (SB) and (SP) are shown for a large number of reactions. Model (SB) employs the standard nuclear proximity potential and the Bondorf Coulomb potential whereas model (SP) uses the standard nuclear proximity potential and point charges for the Coulomb potential. The fusion cross sections at the higher energies for the heavier systems are always smaller for the more repulsive point charges.

The Coulomb potential defined by eq. (13) and discussed in section 2.2 has often been used in optical model calculations, and hence it is interesting to compare fusion results based on this potential with those obtained with the Bondorf potential. In both cases the standard proximity potential is used. Figs. 69-70 compare such results of calculations with the Bondorf Coulomb potential to those with the optical model Coulomb potential for the systems

$^{58}\text{Ni} + ^{35}\text{Cl}$ and $^{109}\text{Ag} + ^{40}\text{Ar}$. The optical model Coulomb calculations were made with $r_0 = 1.3$ fm, a value often used in the literature (see for example ref. [79]). At low energies the

fusion cross sections are insensitive to the Coulomb potential and all calculations give the same result.

However, at higher energies the optical model Coulomb potential calculations give larger fusion cross sections. The degree of the discrepancy increases markedly as the masses of the target and projectile increase. Better agreement in the fusion cross sections for the heavier systems is obtained by arbitrarily reducing the value of r_0 used in optical model Coulomb calculations. If one generates the Coulomb potential from a point charge and a homogeneously charged sphere, the radius of the sphere should be characterized by $R = (R_p^3 + R_t^3)^{1/3}$. For example, if one chooses to represent R by $r_0(A_t^{1/3} + A_p^{1/3})$, then the value of r_0 is approximately given by $1.3(A_t + A_p)^{1/3} / (A_t^{1/3} + A_p^{1/3})$.

4.2 Moments of inertia

The choices of the moments-of-inertia of the target and projectile arise from different possible choices for the approximate nuclear radius, and the way the nuclear surface diffuseness is accounted for. For most applications of the model, the moments-of-inertia of both the target and projectile have been taken to be that of a sharp sphere of uniform density (see eq. (4)) and a radius equal to the equivalent sharp radius. In fig. 71 the effects of two possible variations in the above assumption are shown for the $^{16}\text{O} + ^{56}\text{Fe}$ reaction. The dotted line shows the effect of using the half-density

matter radius, C , for the radius of the sharp sphere. This has the effect of slightly reducing the fusion cross section at high energies since the smaller moments-of-inertia thus produced allow higher rotational velocities for the nuclei at a given spin angular momenta. This in turn implies that the relative surface velocities of the nuclei may be reduced to zero with a smaller transfer of orbital angular momentum to intrinsic spin. Since the removal of some orbital angular momentum is a necessary condition for fusion at high bombarding energies, and a relative surface velocity of zero prevents tangential friction from transferring orbital to spin angular momentum, the lower moments-of-inertia reduce the fusion cross section. Conversely, the dashed line shown in fig. 71 illustrates the effect of increased moment-of-inertia obtained by assuming the nuclear matter has a Fermi distribution with a half-density radius of C and diffuseness of 0.551 fm. In this case, the larger moments-of-inertia enhance the fusion cross section at high energies. At lower energies, where friction effects are less and the potential barrier positions most important, the effects of moments-of-inertia variations are slight.

The third curve, shown in fig. 71 as a short-dash-long-dash line, illustrates the standard calculation with R defining the moments-of-inertia, and the effect of replacing C by R in eq. (3). The use of R in eq. (3) shifts

slightly the effective point of application of the tangential force on the nuclei. As fig. 71 shows such a shift has a negligible effect on the excitation function. At lower energies, radius changes in the parameters shown in fig. 71 have a negligible effect on the excitation function.

4.3 Nuclear potential

In section 3 results of calculations performed with both the standard nuclear proximity potential (see eq. (14)) and the modified nuclear proximity potential (see eq. (17)) are shown for a large number of systems. For light systems the fusion cross sections are the same for both nuclear potentials at all energies. For heavy systems the fusion cross sections are the same for both nuclear potentials at low energies, however, the modified nuclear potential gives larger fusion cross sections at high energies. As mentioned previously, Randrup [23] used point charges for the Coulomb potential when employing the modified nuclear proximity potential. The fusion cross section resulting from this conservative potential is illustrated in figs. 65-67 as a two dot-dash line for these three reactions. As expected for higher energies, the cross sections are smaller than those calculated with a modified nuclear potential and the Bondorf Coulomb potential (dashed line).

Some calculations have also been performed with the Krappe-Nix-Sierk [24] nuclear potential. These authors

point out that their potential is not applicable for small separations where the nuclei overlap. If, however, one uses their formalism to calculate the potential into smaller radii as shown in fig. 7, one obtains fusion cross sections which are much smaller than those shown in section 3 and in serious disagreement with the existing experimental data. However, good agreement with experimental fusion excitation functions is obtained with the Krappe-Nix-Sierk nuclear potential if for distances inside the contact point the potential is represented by a parabolic function [80]. Several of the experimental fusion excitation functions have been fitted [81] with this extended form of the Krappe-Nix-Sierk potential, where for distances outside the contact point the original potential function [24] is employed. For the light-ion reactions, $^{27}\text{Al}+^{12}\text{C}$ and $^{27}\text{Al}+^{16}\text{O}$, this potential gives a better fit to the fusion data, especially in the energy region of the maximum fusion cross section. For the very heavy ion reactions, the fusion excitation functions calculated with this potential gives a better representation of the small amount of available data than do the other potentials used in figs. 65-68 [81].

4.4 Nuclear dissipative force

In all the calculations reported in section 3, the nuclear one-body proximity friction [3] has been used (see section 2.4). The importance of including the dissipative force in the trajectory calculations is illustrated in figs. 72-74 for the $^{56}\text{Ni}+^{35}\text{Cl}$, $^{115}\text{Sn}+^{35}\text{Cl}$ and $^{163}\text{Ho}+^{56}\text{Fe}$ reactions.

Shown in these figures are results of fusion cross section calculations based on eq. (20) where l_f is determined by either the bombarding energy and/or the presence of a 'pocket' in the conservative potential for the asymptotic angular momentum wave in question. Although this model is sometimes referred to as a 'friction-free' model it actually assumes an infinite short-range friction such that the trajectories of all l waves proceeding beyond the barrier into a pocket lead to fusion. At bombarding energies where the number of asymptotic l waves exceeds the number of potential pockets, this model predicts that the fusion cross section decreases in proportion to k^2 .

Inspection of figs. 72-74 reveals that the maximum value of the fusion cross section is reached in the above model at a lower bombarding energy for each system than that observed for our classical trajectory model with the nuclear one-body proximity friction. In addition, the magnitude of the maximum value of the fusion cross section is much less without the longer-range friction force. Experimental measurements of fusion cross sections exceed the calculated values from the 'friction-free' or short-range friction models (see, for example, figs. 72-74). As can be seen in figs. 50, 56 and 65, the inclusion of a longer-range friction increases the fusion cross sections to values consistent with experimental data. It is the reduction of l due to the transfer of orbital angular momentum into intrinsic spin

by the tangential friction that enables a larger number of asymptotic ℓ waves to be trapped in the pockets of the effective potentials. This results in the fusion cross section increasing in magnitude with energy beyond the limit of the so-called 'friction-free' model leading to results in accord with experiments. This same effect can be approximated in short-range friction models by assuming that at some close distance the two nuclei stick and rotate as a rigid body and in this way convert part of their relative angular momentum into intrinsic rotation of the fragments [81].

Critical distance models [5, 42, 82-85] make the assumption also that no friction occurs until a critical distance is reached. At this critical distance R_{CR} (usually taken to be approximately $1.1 (A_T^{1/3} + A_P^{1/3})$ fm) an infinitely strong friction sets in and traps all nuclei reaching this distance. In this model the fusion cross section has two branches, a low-energy branch following eq. (1) and a high energy branch where, $\sigma_{fus} = \pi R_{CR}^2 (1 - V(R_{CR})/E)$. By the appropriate choice of $V(R_{CR})$, it is possible to obtain a downward trend in the fusion cross section as a function of increasing excitation energy as observed experimentally (see section 3). For the very light systems this model gives reasonable fits to the available data, however, for intermediate and heavy systems the predicted limiting value of the fusion cross section of πR_{CR}^2 far exceeds existing experimental data. Another difference of the critical distance model,

even for light nuclei, is that all l waves reaching R_{CR} are predicted to fuse and, hence, there is no upper limit to l_f except as dictated by the energy requirement.

The energy loss due to radial friction is an important factor controlling the suppression of fusion for trajectories with $l \approx 0$. There is some experimental evidence for this ' l -window'[86] and some time-dependent Hartree Fock calculations have also shown such effects [69, 87, 88]. For all except the very heaviest systems the friction proposed by Randrup does not show the ' l -window' when combined with the proximity potential in either of its two forms. However, if the radial component of the friction is reduced an ' l -window' may appear at the higher energies. Fig. 75 shows the limiting values of l predicted to fuse as a function of incident laboratory energy for the $^{28}\text{Si}+^{28}\text{Si}$ reaction with different reduction factors for the radial friction. For the $^{16}\text{O}+^{56}\text{Fe}$ system a change in the magnitude of the radial friction form factor by ≈ 0.5 is sufficient to produce an ' l -window'. Heavier systems like the $^{209}\text{Bi}+^{86}\text{Kr}$ reaction show an ' l -window' with the model (SB) potential. For lighter systems large reductions in the radial friction by a factor of ≈ 0.2 are necessary before the ' l -window' appears. For the $^{28}\text{Si}+^{28}\text{Si}$ reaction, it can be seen in fig. 75 that an ' l -window' appears at a bombarding energy of 150 MeV when the radial friction form factor is $0.2 f_r(r)$. This effect occurs because at higher energies both a reduction in l to form a potential barrier, and a reduction in energy

due to radial friction are necessary for fusion to occur. When the energy loss due to radial friction is reduced, then the trajectories of lower ℓ will be the first to fail to fuse as the radial friction is decreased, since such trajectories are dominated by radial friction energy losses. Variations in the tangential friction are of lesser importance in determining the presence of an ' ℓ -window', provided that the tangential friction is sufficiently large to produce the rolling condition.

4.5 Nuclear radius

The effects of a variation in the nuclear radius of the interacting ions are illustrated in figs. 76-78 for the $^{27}\text{Al}+^{12}\text{C}$, $^{40}\text{Ca}+^{16}\text{O}$ and $^{58}\text{Ni}+^{35}\text{Cl}$ reactions. In these figures, the excitation functions are plotted for calculations performed with model (MB). In each figure, the solid curve is calculated with the nuclear radii from Myers' systematics (see eq. 15)). The dashed curves in these figures are calculated with the nuclear radius of both target and projectile increased by 0.24, 0.18 and 0.16 fm, respectively, for the reactions $^{27}\text{Al}+^{12}\text{C}$, $^{40}\text{Ca}+^{16}\text{O}$ and $^{58}\text{Ni}+^{35}\text{Cl}$. These small changes in the nuclear radius may be within the accuracy of the Myers' systematics of nuclear radii. The shift in each of these excitation functions so produced by a radius change improves somewhat the agreement between the calculations and the experimental data [1, 10].

Theoretically the radius changes alter the excitation

functions through the geometrical factors in the proximity potential and friction (eqs. (14) and (19)) and through the equations which determine the torques (eqs. (2c) and (2d)). The important factor in these effects on excitation functions comes from the radius variations in the total sum of the radii, and how these variations are distributed between the target and projectile is of secondary importance.

The approximate changes in radius needed to improve the agreement between theory and experiment are listed in table 1, where $\Delta R = \Delta R_T + \Delta R_P$. When no value of ΔR is given, the agreement between theory and experiment is within the uncertainties of the data. The magnitude of these radius changes are somewhat dependent on the model used to calculate the fusion cross sections. As discussed previously, the present classical calculations utilize eq. (20) with the assumption that the transmission coefficients are unity for $l \leq l_f$ and zero for $l > l_f$.

5. Fusion barrier parameters and angular momenta

5.1 Fusion Barrier radii and heights

Fusion barrier characteristics are tabulated in table 1. The calculated fusion barrier radii, $R_B(l=0)$, and potentials $V_B(R_B, l=0)$ are listed in table 1. The barriers, calculated for $l=0$, are independent of the model choices described in the paper, since the barriers all lie at radii greater than the sum of the charge and matter radii of the target and projectile. In cases where improved agreement between data and calculations may be obtained by nuclear radius changes, these changes are shown in table 1 as $\Delta R = \Delta R_T + \Delta R_P$, together with the recalculated barrier radii and heights (R_B' and V_B'). ΔR was distributed equally between the target and projectile radii for the calculation of R_B' and V_B' . As mentioned in section 4.5, the actual distribution of ΔR between the target and projectile radii is not an important factor. Also tabulated in table 1 are the interaction or strong absorption radii obtained from the systematics of elastic scattering [30,89]. In general radius changes necessary to improve the agreement between data and calculations are ≤ 0.4 fm. Only the reactions $^{40}\text{Ca} + ^{40}\text{Ca}$ and $^{27}\text{Al} + ^{12}\text{C}$ require changes in target and projectile radii which are larger than 0.4 fm. In addition, except for $^{12}\text{C} + ^{12}\text{C}$ the required changes in radii are all increases in the nuclear radius.

As $Z_T Z_P$ increases, the decreasing importance of fusion may be seen by a comparison of the fusion barrier radius to the strong absorption radius. The difference $\Delta = R_{SA} - R_B$ increases from

Table 1

Barrier radius $R_B(l=0)$ and barrier height $V_B(R_B, l=0)$ for angular momentum $l=0$. Energy E_{cm}^M and cross section σ_{fus}^M for the excitation function maximum calculated from model (SB). Nuclear radius changes $\Delta R = \Delta R_T + \Delta R_P$ necessary to improve the agreement between data and calculations, and the consequent new barrier parameters R'_B and V'_B . The strong absorption radius R_{SA} is calculated from systematics [30, 89].

TABLE 1

Reaction	$R_B(\ell=0)$ fm	$V_B(R_B, \ell=0)$ MeV	$E_{cm}^M (\sigma_{fus}^M)$ MeV	σ_{fus}^M mb	ΔR fm	$R_B'(\ell=0)$ fm	$V_B'(\ell=0)$ MeV	R_{SA} fm
$^{162}\text{Dy} + ^4\text{He}$	10.01	17.63	69.0	1870				10.68
$^{12}\text{C} + ^{12}\text{C}$	7.41	6.32	26.3	1000	-.20	7.13	6.54	8.04
$^{27}\text{Al} + ^{12}\text{C}$	8.01	12.76	46.5	1120	+.48	8.67	11.88	8.81
$^{152}\text{Sm} + ^{12}\text{C}$	10.39	47.94	122.0	1680				11.32
$^{12}\text{C} + ^{13}\text{C}$	7.53	6.23	25.0	1040				8.11
$^{12}\text{C} + ^{14}\text{N}$	7.50	7.29	29.4	1020				8.17
$^{16}\text{O} + ^{14}\text{N}$	7.68	9.52	33.9	1040	+.20	7.96	9.22	8.42
$^{12}\text{C} + ^{15}\text{N}$	7.61	7.20	22.0	1060	+.10	7.75	7.08	8.23
$^{10}\text{B} + ^{16}\text{O}$	7.49	6.95	24.4	1020	+.40	8.06	6.51	8.14
$^{12}\text{C} + ^{16}\text{O}$	7.59	8.25	32.8	1040				8.29
$^{16}\text{O} + ^{16}\text{O}$	7.76	10.77	36.4	1060				8.54
$^{24}\text{Mg} + ^{16}\text{O}$	8.06	15.62	62.5	1110				8.93
$^{26}\text{Mg} + ^{16}\text{O}$	8.20	15.39	54.1	1160				9.02
$^{27}\text{Al} + ^{16}\text{O}$	8.19	16.68	54.8	1150				9.06
$^{28}\text{Si} + ^{16}\text{O}$	8.19	17.96	58.8	1130				9.10
$^{40}\text{Ca} + ^{16}\text{O}$	8.54	24.72	71.4	1190	+.36	9.02	23.50	9.51
$^{63}\text{Cu} + ^{16}\text{O}$	9.13	33.70	53.3	1320				10.11
$^{148}\text{Nd} + ^{16}\text{O}$	10.52	61.08	135.1	1580				11.52

$^{150}\text{Nd} + ^{16}\text{O}$	10.55	60.92	131.6	1580				11.54
$^{208}\text{Pb} + ^{16}\text{O}$	11.19	78.70	163.9	1640	+.20	11.73	75.50	12.20
$^{27}\text{Al} + ^{17}\text{O}$	8.28	16.51	55.6	1190	+.23	8.60	15.95	9.12
$^{12}\text{C} + ^{18}\text{O}$	7.76	8.08	32.3	1100	+.28	8.16	7.72	8.40
$^{24}\text{Mg} + ^{18}\text{O}$	8.24	15.31	54.8	1180				9.04
$^{27}\text{Al} + ^{18}\text{O}$	8.37	16.36	57.1	1220	+.18	8.62	15.93	9.17
$^{28}\text{Si} + ^{18}\text{O}$	8.38	17.61	54.1	1180	+.10	8.59	17.10	9.21
$^{148}\text{Nd} + ^{18}\text{O}$	10.71	60.14	142.9	1680				11.63
$^{12}\text{C} + ^{19}\text{F}$	7.75	9.10	35.1	1070	+.41	8.33	8.53	8.45
$^{235}\text{U} + ^{20}\text{Ne}$	11.59	106.57	186.9	1520				12.67
$^{28}\text{Si} + ^{28}\text{Si}$	8.63	29.98	85.1	1160	+.23	8.93	29.05	9.66
$^{24}\text{Mg} + ^{32}\text{S}$	8.62	29.40	85.1	1170	+.20	8.89	28.58	9.64
$^{27}\text{Al} + ^{32}\text{S}$	8.75	31.41	93.5	1180				9.77
$^{40}\text{Ca} + ^{32}\text{S}$	9.09	46.63	111.1	1180	+.41	9.64	44.22	10.22
$^{58}\text{Ni} + ^{32}\text{S}$	9.54	62.41	137.0	1200				10.71
$^{27}\text{Al} + ^{35}\text{Cl}$	8.86	32.99	87.0	1210				9.87
$^{48}\text{Ti} + ^{35}\text{Cl}$	9.47	52.47	120.5	1300				10.56
$^{56}\text{Fe} + ^{35}\text{Cl}$	9.64	60.99	135.1	1260				10.76
$^{58}\text{Ni} + ^{35}\text{Cl}$	9.65	65.60	138.9	1230	+.32	10.07	63.13	10.81
$^{60}\text{Ni} + ^{35}\text{Cl}$	9.72	65.17	138.9	1250	+.32	10.14	62.73	10.86
$^{62}\text{Ni} + ^{35}\text{Cl}$	9.79	64.77	138.9	1290	+.32	10.21	62.37	10.90
$^{64}\text{Ni} + ^{35}\text{Cl}$	9.85	64.38	138.9	1320	+.32	10.27	62.01	10.95

$^{90}\text{Zr} + ^{35}\text{Cl}$	10.29	88.14	161.3	1270	+.21	10.54	86.11	11.47
$^{112}\text{Sn} + ^{35}\text{Cl}$	10.61	106.92	181.8	1230				11.83
$^{116}\text{Sn} + ^{35}\text{Cl}$	10.69	106.16	181.8	1260				11.89
$^{120}\text{Sn} + ^{35}\text{Cl}$	10.77	105.45	185.2	1300				11.95
$^{124}\text{Sn} + ^{35}\text{Cl}$	10.85	104.78	185.2	1320				12.01
$^{141}\text{Pr} + ^{35}\text{Cl}$	11.03	121.54	196.1	1230				12.24
$^{109}\text{Ag} + ^{40}\text{Ar}$	10.80	104.81	181.2	1320				11.94
$^{121}\text{Sb} + ^{40}\text{Ar}$	10.98	111.93	192.3	1330				12.13
$^{165}\text{Ho} + ^{40}\text{Ar}$	11.52	140.20	214.3	1240				12.70
$^{197}\text{Au} + ^{40}\text{Ar}$	11.85	160.70	224.4	1140				13.06
$^{238}\text{U} + ^{40}\text{Ar}$	12.23	181.23	245.4	1050				13.46
$^{40}\text{Ca} + ^{40}\text{Ca}$	9.31	57.00	122.0	1170	+.64	10.15	52.70	10.49
$^{165}\text{Ho} + ^{56}\text{Fe}$	11.77	197.11	261.0	788				13.14
$^{165}\text{Ho} + ^{84}\text{Kr}$	12.24	261.12	299.4	540				13.73
$^{209}\text{Bi} + ^{84}\text{Kr}$	12.51	312.76	452.0	133				14.21
$^{56}\text{Fe} + ^{132}\text{Xe}$	11.42	164.05	224.72	980				12.72

≈ 0.6 fm for $Z_p Z_T \approx 30$ to ≈ 1.7 fm at $Z_p Z_T \approx 3000$. The value of Δ increases slowly as $Z_p Z_T$ increases, and there seems to be no simple relation between Δ and $Z_p Z_T$. However, the increasing nuclear penetration necessary to reach the fusion barrier as the mass of the system increases, leads to an increase in the relative importance of non-fusion reaction channels for the heavier systems.

Generally, all fusion excitation functions, calculated from the models described in this paper, show a maximum σ_{fus}^M at some bombarding energy which is designated as E_{cm}^M . These values of cross section and energy are tabulated also in table 1. For light systems, very similar values of σ_{fus}^M and E_{cm}^M are determined from calculations with all the different conservative potentials described in this paper. However, as the mass of the system increases the values of σ_{fus}^M , from the different models, change as the potentials are altered. In general model (MB) predicts larger values of σ_{fus}^M than model (SB), which in turn predicts larger values than model (SP). This trend may be seen by comparison of fig. 50 for the comparatively light system $^{62}\text{Ni} + ^{35}\text{Cl}$, with fig. 65 for the heavier system $^{165}\text{Ho} + ^{56}\text{Fe}$. The single experimental fusion cross section for the $^{165}\text{Ho} + ^{56}\text{Fe}$ reaction suggests that an appropriate conservative potential is somewhat more repulsive than that of model (MB) but less repulsive than model (MP).

5.2 Maximum angular momentum for fusion determined by different model calculations

The variation of l_f^M with the mass of the system has been considered by Wilczyński [90] and Bass [82]. The method of Wilczyński relies on the assumption that the barrier for the trajectory of angular momentum l_f^M lies at the touching radius $z=0$, where the inter-nuclear force is assumed to be at a maximum. Then, by equating the attractive nuclear force to the sum of the repulsive Coulomb and centrifugal forces, an equation for l_f^M is obtained

$$(l_f^M)^2 = - \frac{\mu(C_T + C_P + \delta)^3}{\hbar^2} \left[4\pi\gamma R\phi_M + \frac{Z_T Z_P e^2}{(C_T + C_P + \delta)^2} \right] \quad (21)$$

where in the formalism of Wilczyński $\delta=0$ and $\phi_M=-1$. Equation (21) is derived without consideration of the effects of tangential friction. Bass [82] has suggested that the target and projectile will reach the rolling condition at the barrier and, hence, the value of l_f^M found from eq. (21) should be increased by a factor of 7/5. For comparison to the values of l_f^M found from model (SB), eq. (21) needs different values of δ and ϕ_M . The nuclear force determined from the proximity potential reaches a maximum at $z \approx 0.3$ fm, thus requiring values of $\delta = 0.3$ fm and $\phi_M = -0.96$ in eq. (21). Even with these modifications eq. (21) will still lead to an underestimate of l_f^M since the Wilczyński method does not select the barrier precisely, but rather selects the potential minimum of the pocket in the effective potential. This situation arises since the radial derivative of the sum of

the Coulomb and centrifugal forces is negative at the point where the nuclear force is a maximum. This in turn implies that the second radial derivative of the potential is positive which is a condition for a local minimum in the potential. The true position of the barrier for the trajectory with angular momentum ℓ_f^M is at a value of ζ slightly larger than 0.3 fm.

The Wilczyński condition may be applied also when the Bon-dorf Coulomb potential is used for cases where

$R_{CT} + R_{CP} > C_T + C_P + 0.3$. This leads to the following equation for ℓ_f^M

$$(\ell_f^M)^2 = - \frac{\mu(C_T + C_P + \delta)^3}{\hbar^2} \left[4\pi\gamma R\phi_M + kn(C_T + C_P + \delta)^{n-1} \right] \quad (22)$$

where n and k are defined as eqs. (8) and (9). In table 2, a comparison is given of the values of ℓ_f^M found from eq. (21), or eq. (22) where appropriate, and the values found with the additional requirement of the rolling condition of Bass. These values are compared in table 2 with the highest value of ℓ to show a potential pocket in model (SB) and the value of ℓ_f^M determined from the complete trajectory calculation. As can be seen from table 2, the criterion of Bass gives a good estimate of ℓ_f^M , with the small discrepancy simply due to the fact that the Wilczyński criterion does not exactly select the barrier potential maximum.

For most cases shown in table 2, the value of ℓ_f^M is larger than the value of ℓ at the maximum of the excitation function, $\ell_f(\sigma_{fus}^M)$. For the lighter systems, model (SB) shows values of

ℓ_f approaching ℓ_f^M asymptotically as the energy increases. For the heavier systems, ℓ_f decreases for energies much higher than the energy at the peak of the excitation function, and ℓ_f^M reaches values only slightly larger than $\ell_f(\sigma_{fus}^M)$. However, in all cases, model (SB) gives $\ell_f^M \geq \ell_f(\sigma_{fus}^M)$.

Table 2. Maximum fusion values of angular momentum l_f^M calculated from eq. (21) or eq. (22). These values increased by 7/5 to account for rolling are included also. The largest value of angular momentum having a pocket in the effective potential for model (SB), l_{pocket}^{SB} , is tabulated for each reaction. The largest fusing value of angular momentum obtained with the calculation for model (SB) is tabulated as $l_f^M(SB)$. The liquid-drop model limiting value of angular momentum at which the fusion barrier is zero is listed in the last column.

Reaction	$l_f^M(\text{eq. 21})$	l_{pocket}^{SB}	$(7/5)l_f^M(\text{eq. 21})$	$l_f^M(SB)$	l_{LDM}
$^{162}\text{Dy} + ^4\text{He}$	18.5	19	25.9	33.4	90.7
$^{12}\text{C} + ^{12}\text{C}$	11.9	13	16.6	18.8	23.7
$^{27}\text{Al} + ^{12}\text{C}$	19.2	20	27.0	28.7	39.0
$^{152}\text{Sm} + ^{12}\text{C}$	39.7	41	55.6	60.0	88.8
$^{12}\text{C} + ^{13}\text{C}$	12.5	14	17.5	19.3	24.9
$^{12}\text{C} + ^{14}\text{N}$	13.1	14	18.3	21.8	25.7
$^{16}\text{O} + ^{14}\text{N}$	15.7	17	22.0	26.3	29.7
$^{12}\text{C} + ^{15}\text{N}$	13.7	15	19.2	20.8	27.0
$^{10}\text{B} + ^{16}\text{O}$	12.6	13	17.6	20.7	25.7
$^{12}\text{C} + ^{16}\text{O}$	14.2	15	19.9	22.3	27.7
$^{16}\text{O} + ^{16}\text{O}$	17.1	18	24.0	26.5	31.7
$^{24}\text{Mg} + ^{16}\text{O}$	21.8	23	30.5	33.3	39.6
$^{26}\text{Mg} + ^{16}\text{O}$	22.9	24	32.1	34.9	42.2
$^{27}\text{Al} + ^{16}\text{O}$	23.3	25	32.7	35.5	42.9
$^{28}\text{Si} + ^{16}\text{O}$	23.7	25	33.2	36.0	43.4
$^{40}\text{Ca} + ^{16}\text{O}$	28.4	29	39.8	41.0	53.2
$^{63}\text{Cu} + ^{16}\text{O}$	35.1	36	49.1	51.0	72.0

Table 2 continued:

Reaction	$\epsilon_f^M(\text{eq. 21})$	$\epsilon_{\text{pocket}}^{\text{SB}}$	$(7/5)\epsilon_f^M(\text{eq. 21})$	$\epsilon_f^M(\text{SB})$	ϵ_{LDM}
$^{148}\text{Nd}+^{16}\text{O}$	47.2	49	66.1	71.0	88.8
$^{150}\text{Nd}+^{16}\text{O}$	47.4	50	66.3	71.0	90.7
$^{208}\text{Pb}+^{16}\text{O}$	51.9	54	72.7	79.0	76.9
$^{27}\text{Al}+^{17}\text{O}$	24.4	26	34.2	37.0	44.1
$^{12}\text{C}+^{18}\text{O}$	15.3	16	21.4	23.8	30.2
$^{24}\text{Mg}+^{18}\text{O}$	23.7	25	33.2	38.0	42.2
$^{27}\text{Al}+^{18}\text{O}$	25.4	27	35.6	38.5	45.4
$^{28}\text{Si}+^{18}\text{O}$	25.9	27	36.2	41.0	46.0
$^{148}\text{Nd}+^{18}\text{O}$	53.1	55	74.3	80.0	90.7
$^{12}\text{C}+^{19}\text{F}$	15.8	17	22.1	24.7	31.0
$^{235}\text{U}+^{20}\text{Ne}$	59.6	62	83.4	89.0	59.0
$^{28}\text{Si}+^{28}\text{Si}$	33.2	34	46.4	49.0	53.2
$^{24}\text{Mg}+^{32}\text{S}$	32.7	33	45.7	48.0	53.2
$^{27}\text{Al}+^{32}\text{S}$	35.1	36	49.2	52.0	56.1
$^{40}\text{Ca}+^{32}\text{S}$	42.6	44	59.7	61.0	63.8
$^{58}\text{Ni}+^{32}\text{S}$	50.3	52	70.4	70.4	72.4
$^{27}\text{Al}+^{35}\text{Cl}$	36.9	38	51.7	55.0	58.9
$^{48}\text{Ti}+^{35}\text{Cl}$	49.6	52	69.5	72.0	74.0
$^{56}\text{Fe}+^{35}\text{Cl}$	52.8	55	73.8	75.0	76.6
$^{58}\text{Ni}+^{35}\text{Cl}$	53.0	55	74.2	76.0	74.2
$^{60}\text{Ni}+^{35}\text{Cl}$	54.1	56	75.7	78.0	77.3
$^{62}\text{Ni}+^{35}\text{Cl}$	55.1	58	77.2	79.0	80.2
$^{64}\text{Ni}+^{35}\text{Cl}$	56.1	59	78.5	81.0	83.0
$^{90}\text{Zr}+^{35}\text{Cl}$	62.0	65	86.8	89.0	82.1
$^{112}\text{Sn}+^{35}\text{Cl}$	65.1	69	91.1	96.0	73.7
$^{116}\text{Sn}+^{35}\text{Cl}$	66.7	71	93.4	98.0	78.3

Table 2 continued:

Reaction	$\ell_f^M(\text{eq. 21})$	$\ell_{\text{pocket}}^{\text{SB}}$	$(7/5)\ell_f^M(\text{eq. 21})$	$\ell_f^M(\text{SB})$	ℓ_{LDM}
$^{120}\text{Sn} + ^{35}\text{Cl}$	68.1	72	95.3	100.0	82.8
$^{124}\text{Sn} + ^{35}\text{Cl}$	69.3	73	97.0	100.3	87.2
$^{141}\text{Pr} + ^{35}\text{Cl}$	69.8	74	97.7	101.0	78.2
$^{109}\text{Ag} + ^{40}\text{Ar}$	73.0	77	102.1	106.0	83.3
$^{121}\text{Sb} + ^{40}\text{Ar}$	75.3	79	105.5	107.0	82.4
$^{165}\text{Ho} + ^{40}\text{Ar}$	79.9	84	111.9	116.0	78.6
$^{197}\text{Au} + ^{40}\text{Ar}$	80.7	85	112.9	116.7	65.0
$^{238}\text{U} + ^{40}\text{Ar}$	80.4	85	112.5	119.0	40.1
$^{40}\text{Ca} + ^{40}\text{Ca}$	47.5	49	66.5	65.1	67.2
$^{165}\text{Ho} + ^{56}\text{Fe}$	79.3	86	111.0	114.0	66.4
$^{165}\text{Ho} + ^{84}\text{Kr}$	74.2	84	103.8	116.0	50.5
$^{209}\text{Bi} + ^{84}\text{Kr}$	29.1	49	40.7	64.0 ^(a)	0
$^{56}\text{Fe} + ^{132}\text{Xe}$	82.4	89	115.3	114.0	79.5

(a) For this heavy system, model (SB) has an angular momentum window, $7 \leq \ell \leq 64$, at E_{cm}^M . The maximum ℓ increases for $E_{\text{cm}} > E_{\text{cm}}^M$; however, the lower angular momentum window becomes larger as the energy increases.

5.3 Systematics of maximum fusion cross sections from model (SB)

The maximum value of the fusion cross section, σ_{fus}^M , and the energy at which the maximum fusion cross section occurs, E_{cm}^M , can be obtained for model (SB) by a relationship using the systematics of strong absorption radii determined from elastic scattering [30]. The angular momentum of the grazing trajectory ℓ_g is given by:

$$\ell_g = (R_{\text{SA}}^2 \kappa^{-2} + 2n\kappa^{-1} R_{\text{SA}})^{1/2} \quad (23)$$

where n and κ are the Coulomb parameter and wave number for the system. The strong absorption radius R_{SA} is given by [89]

$$R_{\text{SA}} = C_T + C_P + 4.49 - (C_T + C_P)/6.35 \text{ fm} \quad (24)$$

Values of R_{SA} are tabulated in table 1. The value of $\ell_f(\sigma_{\text{fus}}^M)$ is found empirically to be related to ℓ_g by

$$\ell_f(\sigma_{\text{fus}}^M) \approx 0.8 \ell_g \quad (25)$$

It is important to note that eq. (25) is only valid at the energy E_{cm}^M . The energy E_{cm}^M may be estimated for systems with $Z_P Z_T \lesssim 1700$ by the relation

$$E_{\text{cm}}^M \approx (51.92 Z_P Z_T + 963.6)^{1/2} - 25.51 \text{ MeV} \quad (26)$$

For most systems eq. (26) is accurate to about 10 MeV. Thus, the maximum fusion cross section σ_{fus}^M at energy E_{cm}^M is qualitatively given by

$$\sigma_{\text{fus}}^M(E_{\text{cm}}^M) \approx 0.64 \pi \hbar^2 (E_{\text{cm}}^M)^{-1} \ell_g^2 \quad (27)$$

Equation (27) implies that over a wide range of masses, the maximum fusion cross section at energy E_{cm}^M is the same fraction

of the reaction cross section at that energy. However, it is important to realize that the value of E_{cm}^M approaches the fusion threshold as the mass of the system increases, and the maximum fusion cross section decreases as a consequence. In the limit, for systems which have no pockets in the effective potential, the fusion cross section is zero at all energies.

5.4 Experimental fusion angular momenta and liquid-drop model limits

In table 3, a comparison is made of some experimentally measured values of angular momentum and the liquid-drop model limits. The experimentally measured values of l shown in table 3 are near to or larger than the l_f^M predicted by the trajectory calculations. It should be emphasized that models (MP) and (MB) give larger values of l_f^M which would be in better agreement with the data and that the limited data available indicate that the true potential will lie between the (MP) and (MB) models in this mass region. The currently measured values of l_f are in excess of the liquid-drop model limits for heavy systems over a considerable mass range. For lighter systems the liquid-drop model limits are not approached by the experimental fusion cross sections since an insufficient number of trajectories exhibit pockets in the effective potential.

For the heavy systems shown in table 3 the explanation for the enhancement of the number of fusion trajectories over the expectations of the liquid-drop model, may lie simply in the very long interaction times of the 'trapped' trajectories.

This subject has already been discussed in section 2.5. Although the system may have too much angular momentum to be stable as a compound nucleus, the friction and effective potential produces such long interaction times that the mass asymmetry degree of freedom has sufficient time to relax. Hence, several rotations of the system may occur before the final separation of the fragments. Under such conditions it is experimentally difficult to distinguish fission of long-lived systems above the liquid drop model angular momentum limit from fission of a system which has passed through a compound system with a non-zero fission barrier. Currently, neither fragment masses, angular distributions, or kinetic energies provide a distinguishing characteristic of long-lived di-nuclear systems above the liquid-drop model angular momentum. However, it is possible that more subtle criteria may be found, perhaps by detailed examination of mass distributions, for example.

Table 3. Comparison of experimentally measured values of ℓ_f with the liquid-drop model limiting angular momentum ℓ_{LDM} . Shown also are the measured fusion cross sections at particular energies.

Reaction	E_{cm} (MeV)	σ_{fus} (mb)	ℓ_f	ℓ_{LDM}
$^{116}\text{Sn} + ^{35}\text{Cl}$	130.1	695	89	78
$^{109}\text{Ag} + ^{40}\text{Ar}$	246.5	975	109	83
$^{121}\text{Sb} + ^{40}\text{Ar}$	222.5	1130	107	82
$^{165}\text{Ho} + ^{40}\text{Ar}$	241.5	1450	126	79
$^{238}\text{U} + ^{40}\text{Ar}$	256.8	1030	117	40
$^{165}\text{Ho} + ^{56}\text{Fe}$	344.0	763	129	66
$^{56}\text{Fe} + ^{136}\text{Xe}$	225.0	1118	123	80

6. Conclusions

In this article available data on fusion excitation functions for heavy-ion induced reactions over a wide mass range are compared with theoretical calculations based on a classical dynamical model. Most of the calculations utilize a conservative potential comprised of the proximity nuclear potential of Blocki et al. [2], the Coulomb potential of Bon-dorf et al. [16], and one-body nuclear friction in the proximity formalism of Randrup [3]. This combination of conservative and dissipative forces gives a good representation of the available fusion data except for the heaviest systems.

Calculations were performed for four quite different conservative potentials identified as models (MB), (MP), (SB), and (SP) (defined in section 2.3). The radial dependence of these different potentials are shown in figs. 8 and 9 for the $^{58}\text{Ni}+^{35}\text{Cl}$ and $^{165}\text{Ho}+^{56}\text{Fe}$ reactions. The calculated fusion excitation functions for the lighter heavy-ion reactions ($Z_T Z_P < 150$) are independent of these potentials. However, as the heavy ion system becomes heavier, the calculated fusion excitation functions begin to diverge more and more. The most repulsive conservative potential giving, of course, the smallest fusion cross section. The very heaviest systems require a conservative potential more repulsive than model (MB) but less repulsive than models (SP) and (SB) for close distances. A conservative potential that has a radial dependence near that of model (MP) will give a good fit to the present

fusion excitation function data over the whole mass range and also reproduce experimental damped heavy ion deflection functions. An example of such a nuclear potential is the recent Krappe-Nix-Sierk [80] potential which reproduces the experimental fusion excitation functions over a wide mass range [81].

The experimental fusion excitation functions require some form of energy and angular momentum dissipation in the present classical dynamical model. Without dissipation the maximum value of the fusion cross section is reached at too low an energy and the magnitude of the maximum value of the fusion cross section is much too small (see figs. 72-74). In these figures, the fusion cross sections are calculated with a so-called 'friction-free' model. This model actually assumes zero friction up to the barrier distance and infinite friction at smaller distances such that all ℓ waves which can energetically reach potential pockets will fuse. In contrast, when friction of finite range is included, it is not possible to predict whether a trajectory will fuse or not, based on the effective conservative potential alone. Tangential friction leads to a continuous transfer of orbital to intrinsic angular momentum during the interaction and the possibility of fusion for asymptotic entrance channel values of ℓ beyond that possible for the 'friction-free' model. Critical distance models are capable of giving reasonable fits to the fusion excitation functions of light heavy-ion reactions. However, for intermediate and heavy systems the predicted limiting value of the

fusion cross section with the simple critical distance model far exceeds known experimental data.

It is shown that the theoretical fusion cross sections of the very heavy systems depend most sensitively on the assumed conservative and dissipative forces. Although it is possible to gain considerable information on these forces, it is also shown here that conservative and dissipative forces lead to similar effects and for most reactions are difficult to separate.

The fusion cross sections at the lower energies are less sensitive to the conservative and dissipative forces than at higher energies. One observes, for example, that all models of the conservative potential give similar fusion cross sections. However, the magnitude of the fusion cross sections at these lower energies is quite sensitive to the value of the nuclear radius as pointed out previously [1]. The fusion data are mostly of insufficient accuracy to extract simultaneously information on both the barrier radius and the nuclear potential at this radius [11]. Hence, in the analysis of the fusion data presented here, the nuclear potential is assumed to be given by the proximity potential of Blocki et al. [2] for radial separations greater than $C_T + C_P$. The appropriateness of the proximity potential is judged on the quality of the fits to fusion data.

With the definition of fusion presented here, the maximum λ wave leading to fusion λ_f far exceeds the liquid-drop

limiting value for very heavy systems. A number of these trajectories may not produce fusion in the commonly understood sense of a system in statistical equilibrium in all its intrinsic degrees of freedom and with a deformation which is inside the saddle point for fusion. However, in the present context all trajectories which have sufficiently long lifetimes lead to equilibration of the mass asymmetry degree of freedom and a $1/\sin \theta$ fission fragment angular distribution and are identified as fusion. The important criteria for determining the l waves that fuse are the presence of potential pockets and the strength of the energy and angular momentum dissipation. These factors increase the lifetimes of trajectories with $l \leq l_f$ and lead to mass asymmetry equilibration. In future experiments it will be important to search for features that might characterize fusion trajectories of small and large angular momenta, e.g., the widths of the mass distributions.

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Figure captions

- Fig. 1. The fusion cross section vs $1/E_{c.m.}$ for the $\text{Sn} + {}^{35}\text{Cl}$ system [6,7]. The fitted curves are:
- Derived from the proximity potential with "friction-free" assumptions.
  - Parallel to (a) shifted to fit the data at higher energies.
  - Fit given by David et al., [7].
  - Fitted to the low energy data.
- Fig. 2. The indefinite integral of the universal interaction potential $\Phi(\zeta)$ and ζ , the dimensionless nuclear surface separation (solid line). The points (a) to (d) correspond to values of ϕ and ζ derived from a friction-free analysis of the corresponding excitation functions (a) to (d) from fig. 1.
- Fig. 3. Degrees of freedom for two interacting heavy ions.
- Fig. 4. Coulomb potentials for the ${}^{62}\text{Ni} + {}^{35}\text{Cl}$ reaction. V_C plotted versus r where r is the radial separation of the two ions. $C_T + C_P$, the sum of half-density matter radii and $R_{CT} + R_{CP}$, the Coulomb radii are indicated by the arrows (see section 2.2).
- Fig. 5. Similar to fig. 4 for the ${}^{116}\text{Sn} + {}^{35}\text{Cl}$ reaction.

- Fig. 6. Similar to fig. 4 for the $^{165}\text{Ho}+^{84}\text{Kr}$ reaction.
- Fig. 7. Nuclear potentials V_N for the $^{27}\text{Al}+^{16}\text{O}$ and $^{165}\text{Ho}+^{56}\text{Fe}$ reactions plotted vs r where r is the radial separation of the two ions (see section 2.3). The arrows indicate the radius where ζ goes to zero for each model. The models are discussed in refs. [24, 2, 23], respectively. The K-N-S model is applicable only for $\zeta > 0$.
- Fig. 8. Conservative potential (V_N+V_C) plus the centrifugal potential for selected ℓ values for the $^{62}\text{Ni}+^{35}\text{Cl}$ reaction. The models for the conservative potential are discussed in section 3.1.
- Fig. 9. Same as fig. 8 except for the $^{165}\text{Ho}+^{56}\text{Fe}$ reaction.
- Fig. 10. Interaction times vs asymptotic entrance channel ℓ -values for trajectories with ℓ values exceeding the last ℓ wave that fuses, ℓ_f .
- Fig. 11. Energy loss due to tangential friction radial friction and spin energy vs angular momentum with the standard proximity nuclear friction for the $^{165}\text{Ho}+^{56}\text{Fe}$ reaction. Energy loss due to tangential friction and spin energy with the modified proximity potential are also shown. The arrows indicate the highest ℓ -wave that fuses for the proximity and modified proximity cases.

Fig. 12. Same as fig. 11 except for the $^{27}\text{Al}+^{32}\text{S}$ reaction.

Fig. 13. Fusion cross section plotted vs $E_{\text{C.M.}}^{-1}$ for the $^{162}\text{Dy}+^4\text{He}$ reaction. Calculations with models (MB), (SB) and (SP) give the same results at all energies as shown by the solid line (see section 3.1). The data are from ref. [33].

Fig. 14. Same as fig. 13 except for the $^{12}\text{C}+^{12}\text{C}$ reaction. Data are from refs. [34-36, 39].

Fig. 15. Same as fig. 13 except for the $^{27}\text{Al}+^{12}\text{C}$ reaction. Data are from refs. [40-42].

Fig. 16. Same as fig. 13 except for the $^{12}\text{C}+^{13}\text{C}$ reaction. Data are from refs. [38, 44, 45].

Fig. 17. Same as fig. 13 except for the $^{12}\text{C}+^{14}\text{N}$ reaction. Data are from refs. [32, 38, 46-48].

Fig. 18. Same as fig. 13 except for the $^{16}\text{O}+^{14}\text{N}$ reaction. Data are from ref. [48].

Fig. 19. Same as fig. 13 except for the $^{15}\text{N}+^{12}\text{C}$ reaction. Data are from ref. [39].

Fig. 20. Same as fig. 13 except for the $^{10}\text{B}+^{16}\text{O}$ reaction. Data are from ref. [49].

Fig. 21. Same as fig. 13 except for the $^{12}\text{C}+^{16}\text{O}$ reaction. Data are from refs. [43, 50, 51].

- Fig. 22. Same as fig. 13 except for the $^{16}\text{O}+^{16}\text{O}$ reaction.
Data are from refs. [43, 51-55].
- Fig. 23. Same as fig. 13 except for the $^{24}\text{Mg}+^{16}\text{O}$ reaction.
Data are from ref. [56].
- Fig. 24. Same as fig. 13 except for the $^{26}\text{Mg}+^{16}\text{O}$ reaction.
Data are from ref. [56].
- Fig. 25. Same as fig. 13 except for the $^{27}\text{Al}+^{16}\text{O}$ reaction.
Data are from refs. [57-61].
- Fig. 26. Same as fig. 13 except for the $^{28}\text{Si}+^{16}\text{O}$ reaction.
Data are from refs. [59, 61].
- Fig. 27. Same as fig. 13 except for the $^{27}\text{Al}+^{17}\text{O}$ reaction.
Data are from ref. [57].
- Fig. 28. Same as fig. 13 except for the $^{12}\text{C}+^{18}\text{O}$ reaction.
Data are from ref. [34].
- Fig. 29. Same as fig. 13 except for the $^{24}\text{Mg}+^{18}\text{O}$ reaction.
Data are from ref. [56].
- Fig. 30. Same as fig. 13 except for the $^{27}\text{Al}+^{18}\text{O}$ reaction.
Data are from refs. [57, 61].
- Fig. 31. Same as fig. 13 except for the $^{28}\text{Si}+^{18}\text{O}$ reaction.
Data are from ref. [61].
- Fig. 32. Same as fig. 13 except for the $^{12}\text{C}+^{19}\text{F}$ reaction.
Data are from ref. [34].

Fig. 33. Fusion cross section plotted vs $E_{c.m.}^{-1}$ for the $^{152}\text{Sm}+^{12}\text{C}$ reaction. Calculations with models (MB), (SB) and (SP) give the same results at lower energies as shown by the solid line. At higher energies the models give slightly different results. The extreme results are given by the (MB) model (dashed line) and the (SP) model (long short dashed line). The experimental data are from ref. [33].

Fig. 34. Same as fig. 33 except for the $^{40}\text{Ca}+^{16}\text{O}$ reaction. Data are from refs. [37, 62].

Fig. 35. Same as fig. 33 except for the $^{63}\text{Cu}+^{16}\text{O}$ reaction. Data are from ref. [63].

Fig. 36. Same as fig. 33 except for the $^{148}\text{Nd}+^{16}\text{O}$ reaction. Data are from ref. [33].

Fig. 37. Same as fig. 33 except for the $^{150}\text{Nd}+^{16}\text{O}$ reaction. Data are from ref. [33].

Fig. 38. Same as fig. 33 except for the $^{208}\text{Pb}+^{16}\text{O}$ reaction. Data are from ref. [64].

Fig. 39. Same as fig. 33 except for the $^{148}\text{Nd}+^{18}\text{O}$ reaction. Data are from ref. [33].

Fig. 40. Same as fig. 33 except for the $^{28}\text{Si}+^{28}\text{Si}$ reaction. Data are from ref. [41].

- Fig. 41. Same as fig. 33 except for the $^{24}\text{Mg}+^{32}\text{S}$ reaction.
Data are from ref. [4, 65].
- Fig. 42. Same as fig. 33 except for the $^{27}\text{Al}+^{32}\text{S}$ reaction.
Data are from ref. [4].
- Fig. 43. Same as fig. 33 except for the $^{40}\text{Ca}+^{32}\text{S}$ reaction.
Data are from ref. [4].
- Fig. 44. Same as fig. 33 except for the $^{58}\text{Ni}+^{32}\text{S}$ reaction.
Data are from ref. [4].
- Fig. 45. Same as fig. 33 except for the $^{27}\text{Al}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].
- Fig. 46. Same as fig. 33 except for the $^{48}\text{Ti}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].
- Fig. 47. Same as fig. 33 except for the $^{56}\text{Fe}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].
- Fig. 48. Same as fig. 33 except for the $^{58}\text{Ni}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].
- Fig. 49. Same as fig. 33 except for the $^{60}\text{Ni}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].
- Fig. 50. Same as fig. 33 except for the $^{62}\text{Ni}+^{35}\text{Cl}$ reaction.
Data are from refs. [6, 66].
- Fig. 51. Same as fig. 33 except for the $^{64}\text{Ni}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].

Fig. 52. Same as fig. 33 except for the $^{90}\text{Zr}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].

Fig. 53. Same as fig. 33 except for the $^{40}\text{Ca}+^{40}\text{Ca}$ reaction.
Data are from ref. [67].

Fig. 54. Fusion cross section plotted vs $E_{\text{c.m.}}^{-1}$ for the $^{235}\text{U}+^{20}\text{Ne}$ reaction. Calculations with models (MB), (SB) and (SP) give the same results at lower energies as shown by the solid line. At higher energies the models give different results. The calculated fusion cross sections decrease in the order model (MB), model (SB) and model (SP). The experimental data are from ref. [70].

Fig. 55. Same as fig. 54 except for the $^{112}\text{Sn}+^{35}\text{Cl}$ reaction.
Data are from ref. [7].

Fig. 56. Same as fig. 54 except for the $^{115}\text{Sn}+^{35}\text{Cl}$ reaction
Data are from refs. [6, 66].

Fig. 57. Same as fig. 54 except for the $^{120}\text{Sn}+^{35}\text{Cl}$ reaction.
Data are from ref. [7].

Fig. 58. Same as fig. 54 except for the $^{124}\text{Sn}+^{35}\text{Cl}$ reaction.
Data are from ref. [6].

Fig. 59. Same as fig. 54 except for the $^{141}\text{Pr}+^{35}\text{Cl}$ reaction.
Data are from ref. [7].

- Fig. 60. Same as fig. 54 except for the $^{109}\text{Ag}+^{40}\text{Ar}$ reaction.
Data are from ref. [71].
- Fig. 61. Same as fig. 54 except for the $^{121}\text{Sb}+^{40}\text{Ar}$ reaction.
Data are from ref. [71-73].
- Fig. 62. Same as fig. 54 except for the $^{165}\text{Ho}+^{40}\text{Ar}$ reaction.
Data are from ref. [73].
- Fig. 63. Same as fig. 54 except for the $^{197}\text{Au}+^{40}\text{Ar}$ reaction.
Data are from ref. [74].
- Fig. 64. Same as fig. 54 except for the $^{235}\text{U}+^{40}\text{Ar}$ reaction.
Data are from ref. [73].
- Fig. 65. Same as fig. 54 except for the $^{165}\text{Ho}+^{56}\text{Fe}$ reaction.
Data are from [75]. An additional theoretical
curve is included that utilizes the modified nuclear
proximity potential and point Coulomb charges (MP)
for the conservative potential.
- Fig. 66. Same as fig. 65 except for the $^{165}\text{Ho}+^{84}\text{Kr}$ reaction.
Data are from refs. [76, 77].
- Fig. 67. Same as fig. 65 except for the $^{209}\text{Bi}+^{84}\text{Kr}$ reaction.
Data are from ref. [76, 77].
- Fig. 68. Same as fig. 54 except for the $^{56}\text{Fe} + ^{132}\text{Xe}$ reaction.
Data are from ref. [78].

- Fig. 69. Fusion cross section plotted vs $E_{c.m.}^{-1}$ for the $^{62}\text{Ni}+^{35}\text{Cl}$ reaction. The solid curve is calculated with model (SB). The dotted curve is the result of calculations with a Coulomb potential based on a point plus a spherical charge distribution. (see eq. (13) and $r_0 = 1.3$ fm).
- Fig. 70. Same as fig. 69 except for the $^{109}\text{Ag}+^{40}\text{Ar}$ reaction.
- Fig. 71. Dependence of the calculated fusion cross section on the moments of inertia. Model (MB) has been used in the calculation (see sections 4.2 and 2.1).
- Fig. 72. Comparison of the experimental fusion cross sections for the $^{62}\text{Ni}+^{35}\text{Cl}$ reaction with a calculation based on a simple 'friction-free' model. This model actually assumes an infinite short-range friction such that the trajectories of all l waves penetrating beyond the barrier into a pocket lead to fusion. Compare with the calculations shown in fig. 50.
- Fig. 73. Same as fig. 72 except for the $^{116}\text{Sn}+^{35}\text{Cl}$ reaction. Compare with the calculations shown in fig. 56.
- Fig. 74. Same as fig. 72 except for the $^{165}\text{Ho}+^{56}\text{Fe}$ reaction. Compare with the calculations shown in fig. 65.

Fig. 75. Highest and lowest ℓ values that fuse as a function of laboratory energy for the $^{28}\text{Si}+^{28}\text{Si}$ reaction. Various reductions in the magnitude of the radial friction are assumed. For example, with a radial friction of $0.2 f_r(r)$, fusion occurs at $E_{\text{lab}} = 175$ MeV for ℓ waves between 15 and 45 λ .

Fig. 76. Illustration of the effect of a variation in the nuclear radius of the interacting ions for the $^{27}\text{Al}+^{12}\text{C}$ reaction (see section 4.5). The dashed line represents a radius increase of each ion by 0.24 fm. Both lines are calculated with the MB potential.

Fig. 77. Same as fig. 76 except for the $^{40}\text{Ca}+^{16}\text{O}$ reaction. The radius increase was 0.18 fm.

Fig. 78. Same as fig. 76 except for the $^{62}\text{Ni}+^{35}\text{Cl}$ reaction. The radius increase was 0.16 fm.

Fig. 1

SCATTERING MODEL

FOUR DEGREES OF FREEDOM (r, θ, θ_p AND θ_T)

Fig. 4

Fig. 5

Fig. 6

007-4141

Fig. 7

Fig. 8

Fig. 9

007-4158

Fig. 11

Fig. 13

Fig. 14

Fig. 15

007-4150

Fig. 17

007-4147

Fig. 18

007-4211

007-4161

Fig. 21

Fig. 22

007-4155

Fig. 23

007-4145

Fig. 24

007-4166

Fig. 25

007-223

Fig. 26

007-4215

Fig. 27

007-4148

Fig. 28

007-4152

Fig. 29

007-4218

Fig. 30

007-4235

Fig. 31

007-4146

Fig. 32

007-4224

Fig. 33

007-4160

Fig. 34

007-4210

Fig. 35

007-4225

Fig. 36

Fig. 37

007-4154

Fig. 38

007-4213

Fig. 39

007-4163

Fig. 40

007-4153

Fig. 41

Fig. 42

007-4127

Fig. 43

Fig. 44

007-4143

Fig. 45

Fig. 46

Fig. 47

Fig. 48

Fig. 49

Fig. 50

Fig. 51

Fig. 52

Fig. 53

Fig. 54

007-4122

Fig. 55

Fig. 56

Fig. 57

Fig. 58

007-4117

Fig. 59

Fig. 60

Fig. 61

007-4204°

Fig. 62

007-4236

Fig. 63

007-4237

Fig. 64

007-4165

Fig. 65

007-4246

Fig. 66

007-4245

Fig. 67

007-4137

Fig. 68

007-4244

Fig. 69

Fig. 70

Fig. 72

Fig. 73

007-4219

Fig. 74

007-4140

Fig. 75

Fig. 76

Fig. 77

Fig. 78

IV. PUBLICATIONS AND ACTIVITIES

A. Articles

R. D. Griffioen, R. C. Thompson and J. R. Huizenga
LEVELS OF ^{235}Np EXCITED BY THE $^{234}(\text{}^3\text{He},\text{d})$ AND $^{234}(\alpha,\text{t})$ RE-
ACTIONS

Phys. Rev. C18, 671-678 (1978).

W. U. Schröder, J. R. Birkelund, J. R. Huizenga, K. L. Wolf,
and V. E. Viola, Jr.

MECHANISMS OF VERY HEAVY ION COLLISIONS: THE $^{209}\text{Bi} + ^{136}\text{Xe}$
REACTION AT $E_{\text{Lab}} = 1130 \text{ MeV}$

Physics Reports 45, No. 5, 301-343 (1978).

W. W. Wilcke, M. W. Johnson, W. U. Schröder, J. R. Huizenga,
and D. G. Perry

NEUTRON EMISSION FROM ACTINIDE MUONIC ATOMS

Phys. Rev. C18, 1452-1462 (1978).

S. L. Tabor, D. A. Goldberg and J. R. Huizenga

RELEVANCE OF THE PROXIMITY POTENTIAL TO LIGHT-ION SCATTERING

Phys. Rev. Lett. 41, 1285-1288 (1978).

J. R. Huizenga, J. R. Birkelund, L. E. Tubbs, J. N. De and
D. Sperber

ROLE OF CONSERVATIVE POTENTIAL AND DISSIPATION IN HEAVY-ION
FUSION

Proceedings of the International Workshop on Gross Properties
of Nuclei and Nuclear Excitations VII, Hirschegg, Austria,
p. 45, January (1979) [INKA-Conf.-79-001-010].

D. Hilscher, W. W. Wilcke, W. U. Schröder, A. D. Hoover,
J. R. Birkelund, J. R. Huizenga, A. Mignerey, K. L. Wolfe,
H. Breuer and V. E. Viola

NEUTRON EMISSION FROM STRONGLY DAMPED REACTION $^{165}\text{Ho} + ^{56}\text{Fe}$
AT 8.5 MeV/u

Proceedings of the International Workshop on Gross Properties
of Nuclei and Nuclear Excitations VII, Hirschegg, Austria,
p. 94, January (1979) [INKA-Conf-79-001-017].

H. Breuer, B. G. Glagola, V. E. Viola, K. L. Wolf, A. C.
Mignerey, J. R. Birkelund, D. Hilscher, A. D. Hoover,

J. R. Huizenga, W. U. Schröder and W. W. Wilcke,
NUCLEON EXCHANGE AND A/Z EQUILIBRATION IN INTERACTIONS OF
8.3 MeV/u ^{56}Fe IONS WITH ^{56}Fe , ^{165}Ho and ^{209}Bi

Phys. Rev. Lett., in press.

D. Hilscher, J. R. Birkelund, A. D. Hoover, W. U. Schröder,
W. W. Wilcke, J. R. Huizenga, A. Mignerey, K. L. Wolf,
H. Breuer and V. E. Viola, Jr.
NEUTRON EMISSION FROM THE $^{165}\text{Ho} + ^{56}\text{Fe}$ REACTION AT 8.5 MeV/u
Phys. Rev., in press.

D. Hilscher, J. R. Birkelund, A. D. Hoover, W. U. Schröder,
W. W. Wilcke, J. R. Huizenga, A. Mignerey, K. L. Wolf,
H. F. Breuer and V. E. Viola
STRUCTURE IN THE ENERGY SPECTRA OF THE FRAGMENTS PRODUCED
IN DAMPED HEAVY-ION REACTIONS
Phys. Rev., in press.

W. U. Schröder, W. W. Wilcke, M. W. Johnson, D. Hilscher,
J. R. Huizenga, J. C. Browne and D. G. Perry
EVIDENCE FOR ATOMIC MUON CAPTURE BY FRAGMENTS FROM PROMPT
FISSION OF MUONIC ^{237}Np , ^{239}Pu AND ^{242}Pu
Phys. Rev. Lett., in press.

J. R. Birkelund, L. E. Tubbs, J. R. Huizenga, J. N. De AND
D. Sperber
HEAVY-ION FUSION: COMPARISON OF EXPERIMENTAL DATA WITH
CLASSICAL TRAJECTORY MODELS
Physics Reports, in press.

R. W. Atcher, A. M. Friedman and J. R. Huizenga
PRODUCTION OF ^{52}Fe FOR USE IN A RADIONUCLIDE GENERATOR SYSTEM
International Journal of Applied Radiation and Isotopes, in
press.

R. W. Atcher, A. M. Friedman, J. R. Huizenga, G.V.S. Rayudu,
E. A. Silverstein and D. A. Turner
 $^{52\text{m}}\text{Mn}$: A NEW SHORT-LIVED, GENERATOR PRODUCED RADIONUCLIDE:
A POTENTIAL GENERATOR FOR POSITRON TOMOGRAPHY
Journal of Nuclear Medicine, in press.

B. Ph.D. Thesis

"MUON CAPTURE AND FISSION PHENOMENA IN ACTINIDE MUONIC ATOMS"
by M. William Johnson, Department of Chemistry, University of
Rochester, Jan. 5, 1979.

C. Contributed Papers at Professional Meetings

J. N. De, A. Sherman, D. Sperber, J. R. Birkelund and J. R.
Huizenga
FUSION EXCITATION FUNCTIONS AS A TEST OF THE RADIAL DEPENDENCE
OF THE PROXIMITY POTENTIAL
International Conference on the "Dynamical Properties of
Heavy Ions," Johannesburg, South Africa, August, 1978.

S. L. Tabor, D. A. Goldberg and J. R. Huizenga
APPLICATION OF THE PROXIMITY POTENTIAL TO LIGHT-ION
SCATTERING

Bull. Am. Phys. Soc. 23, 931 (1978).

H. F. Breuer, B. G. Glagola, V. E. Viola, A. Mignerey, D. G.
Raich, K. L. Wolf, J. R. Birkelund, D. Hilscher, W. W. Wilcke,
J. R. Huizenga and W. U. Schröder

STUDY OF THE $^{56}\text{Fe} + ^{209}\text{Bi}$ REACTION AT 385 AND 480 MeV

Bull. Am. Phys. Soc. 23, 958 (1978).

W. W. Wilcke, M. W. Johnson, W. U. Schröder, D. Hilscher,
J. R. Huizenga, J. C. Browne and D. G. Perry
MUON CAPTURE BY FISSION FRAGMENTS PRODUCED FOLLOWING RADIA-
TIONLESS MUON CASCADE PROCESSES

Bull. Am. Phys. Soc. 24, 573 (1979).

M. W. Johnson, W. W. Wilcke, W. U. Schröder, J. R. Huizenga,
D. Hilscher, J. C. Browne and D. G. Perry

EFFECTS OF BOUND MUONS ON FISSION BARRIERS IN ACTINIDE NUCLEI

Bull. Am. Phys. Soc. 24, 573 (1979).

L. E. Tubbs, J. R. Birkelund, D. Hilscher, A. D. Hoover, W. U.
Schröder, W. W. Wilcke and J. R. Huizenga

FISSION INDUCED WITH VERY HEAVY IONS AND THE LIQUID DROP MODEL

Bull. Am. Phys. Soc. 24, 574 (1979).

A. C. Mignerey, K. L. Wolf, J. R. Birkelund, D. Hilscher, W. U.
Schröder, W. W. Wilcke, J. R. Huizenga, V. E. Viola and H. Breuer
STRUCTURE IN THE ENERGY SPECTRA FROM THE REACTION $^{56}\text{Fe} + ^{56}\text{Fe}$
AT 8.5 MeV/u

Bull. Am. Phys. Soc. 24, 629 (1979).

A. D. Hoover, D. Hilscher, J. R. Birkelund, W. U. Schröder,
W. W. Wilcke, J. R. Huizenga, A. Mignerey, K. L. Wolf, H. Breuer
and V. E. Viola, Jr.

STUDY OF THE REACTION $^{165}\text{Ho} + ^{56}\text{Fe}$ AT 8.5 MeV/u

Bull. Am. Phys. Soc. 24, 629 (1979).

W. U. Schröder, W. W. Wilcke, J. R. Birkelund, D. Hilscher,
J. R. Huizenga and J. Randrup

ONE-BODY TRANSPORT IN DAMPED HEAVY-ION REACTIONS

Bull. Am. Phys. Soc. 24, 629 (1979).

H. F. Breuer, B. G. Glagola, V. E. Viola, A. Mignerey, K. L.
Wolf, J. R. Birkelund, D. Hilscher, J. R. Huizenga, W. U.
Schröder and W. W. Wilcke

STUDY OF Z AND A DISTRIBUTIONS AS A FUNCTION OF ENERGY DAMPING
FOR REACTIONS OF ^{56}Fe WITH ^{165}Ho AND ^{209}Bi .

Bull. Am. Phys. Soc. 24, 629 (1979).

J. R. Huizenga

WHEN AND WHY DO HEAVY IONS FUSE?

177th Annual Meeting of the American Chemical Society,
Honolulu, April 1-6 (1979)

R. W. Atcher, A. M. Friedman, J. R. Huizenga and R. P. Spencer
PRODUCTION OF SHORT-LIVED ALPHA EMITTERS FROM RADIONUCLIDE
GENERATORS, $^{223}\text{Ra} - ^{211}\text{Pb}$

Society of Nuclear Medicine 26th Annual Meeting

D. Invited Lectures

"INTERACTIONS OF STOPPED MUONS WITH ACTINIDE NUCLEI," Miami Beach
ACS Meeting, September 10-15, 1978 (by M. W. Johnson).

"HEAVY-ION FUSION, FRICTION AND THE NUCLEAR POTENTIAL," South-
eastern Sectional Meeting of the American Physical Society,
Blacksburg, Virginia, October 26-28, 1978 (by J. R. Huizenga).
See Bull. Am. Phys. Soc. 24, 103 (1979).

"NEUTRON EMISSION FROM STRONGLY DAMPED HEAVY-ION REACTIONS,"
Physics Colloquium, SUNY, Stony Brook, September 12, 1978,
Nuclear Structure Seminar, University of Rochester, October 25,
1978, Asilomar Meeting of the American Physical Society, Novem-
ber 1-3, 1978, Physics Seminar, University of Wisconsin, Novem-
ber 22, 1978, and Physics Seminar, Michigan State University,
February 21, 1979 (by D. Hilscher).

"MUON CAPTURE AND FISSION PHENOMENA IN ACTINIDE MUONIC ATOMS,"
Chemistry Seminar, University of Rochester, January 5, 1979
(by M. W. Johnson).

"NEUTRON EMISSION FROM THE STRONGLY DAMPED REACTION $^{165}\text{Ho} + ^{56}\text{Fe}$
AT 8.5 MeV/u," Hahn-Meitner Institut, Berlin, January 8, 1979,
Gesellschaft für Schwerionenforschung (GSI), Darmstadt, January 10,
1979, Technical University of Munich, Munich, January 12, 1979,
International Workshop on Gross Properties of Nuclei and Nuclear
Excitations, Hirschegg, Austria, January 17, 1979, (by W. W. Wilcke).

"THE ROLE OF THE CONSERVATIVE POTENTIAL AND ONE-BODY DISSIPATION
IN NUCLEAR FUSION," University of Frankfurt, Frankfurt, January 11,
1979, Technical University of Darmstadt, Darmstadt, January 12,
1979, International Workshop on Gross Properties of Nuclei and
Nuclear Excitations, Hirschegg, Austria, January 16, 1979, Cyclo-
tron Institute, Texas A&M University, January 24, 1979 (by J. R.
Huizenga).

"CLASSICAL TRAJECTORY MODELS OF HEAVY-ION FUSION," Third National
Congress of the Australian Institute of Physics, Perth, Western
Australia, January 15-19, 1979 (by J. R. Birkelund).

"DO THE FRAGMENTS IN STRONGLY DAMPED COLLISIONS REACH THERMAL EQUILIBRIUM?" Physics Seminar, University of Maryland, March 26, 1979 (by D. Hilscher).

"NUCLEAR AND ATOMIC PHYSICS OF ACTINIDE MUONIC ATOMS," Seminar at Lawrence Berkeley Laboratory, March 27, 1979, Seminar at University of Maryland, April 17, 1979, Seminar at Oak Ridge National Laboratory, May 1, 1979 (by M. W. Johnson).

"HEAVY-ION FUSION EXCITATION FUNCTIONS," Nuclear Physics seminar, Hahn-Meitner Institute, West Berlin, June 19, 1979 (by J. R. Birkelund).

"ONE-BODY TRANSPORT IN HEAVY-ION REACTION," Nuclear Physics Colloquium, Hahn-Meitner Institute, West Berlin, June 25, 1979 (by W. U. Schröder).

"CURRENT STATUS OF HEAVY ION REACTION MECHANISMS," Gordon Conference on Nuclear Structure Physics, Tilton School, Tilton, New Hampshire, July 9-13, 1979 (by J. R. Huizenga).

"FUSION REACTIONS AT HIGH ENERGIES," Symposium on Heavy Ion Physics from 10 to 200 MeV/u, Brookhaven National Laboratory, July 16-20, 1979 (by J. R. Huizenga).

E. Professional Activities

Member of the Curriculum Review Committee, College of Arts and Science

Member of Faculty Recruiting Committee, Department of Chemistry

Member of Research Grants and Contracts Committee, Department of Chemistry

Member of Faculty Bridging Committee

Member of Los Alamos Meson Physics Facility Program Advisory Committee

Member of Nuclear Chemistry Division Review Committee, Lawrence Livermore Laboratory

Member of ACS Division of Nuclear Chemistry and Technology Committee on Professional Awards

Member of ACS Division of Nuclear Chemistry and Technology Committee on Funding

Member of Program Committee of the Division of Nuclear Physics, APS

Chaired session on Relativistic and Deep Inelastic Heavy Ion Reactions, Washington APS Meeting, April 24, 1979

Member of International Advisory Committee for Conference on the Theory and Applications of Moment Methods in Many Fermion Systems (to be held September 10-14, 1979 in Ames, Iowa)

Member of International Advisory Committee for Symposium on Continuum Spectra of Heavy Ion Reactions (to be held December 3-5, 1979 in San Antonio, Texas)

Member of Nuclear Science Advisory Committee to DOE and NSF (NUSAC); also member of Facility Committee and Long Range Planning Committee.

V. PERSONNEL

Dr. J. R. Huizenga, Tracy H. Harris Professor of Chemistry and Physics

Dr. J. R. Birkelund, Research Associate

Dr. D. Hilscher*, Visiting Scientist and Research Associate

Dr. W. U. Schröder, Senior Research Associate

Dr. W. W. Wilcke, Research Associate

Dr. M. W. Johnson**, graduate student and postdoctoral fellow

Mr. R. A. Atcher***, graduate student, Department of Chemistry

Ms. A. D. Hoover, graduate student, Department of Chemistry

Ms. L. E. Tubbs, graduate student, Department of Chemistry

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**Completed dissertation and final examination for PhD degree on January 5, 1979. Terminated on August 31, 1979.

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