Verification of Theoretically Computed Spectra for a Point Rotating in a Vertical Plane

**D. C. Powell
J. R. Connell
R. L. George**

March 1985

**Prepared for the U.S. Department of Energy
under Contract DE-AC06-76RLO 1830**

**Pacific Northwest Laboratory
Operated for the U.S. Department of Energy
by Battelle Memorial Institute**

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

PACIFIC NORTHWEST LABORATORY
operated by
BATTELLE
for the
UNITED STATES DEPARTMENT OF ENERGY
under Contract DE-AC06-76RLO 1830

Printed in the United States of America
Available from
National Technical Information Service
United States Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161

NTIS Price Codes
Microfiche A01

Printed Copy

Pages	Price Codes
001-025	A02
026-050	A03
051-075	A04
076-100	A05
101-125	A06
126-150	A07
151-175	A08
176-200	A09
201-225	A010
226-250	A011
251-275	A012
276-300	A013

VERIFICATION OF THEORETICALLY COMPUTED
SPECTRA FOR A POINT ROTATING IN A
VERTICAL PLANE

David C. Powell
James R. Connell
Raymond L. George

March 1985

Prepared for
the U.S. Department of Energy
under Contract DE-AC06-76RLO 1830

Pacific Northwest Laboratory
Richland, Washington 99352

SUMMARY

A theoretical model is modified and tested that produces the power spectrum of the alongwind component of turbulence as experienced by a point rotating in a vertical plane perpendicular to the mean wind direction. The ability to generate such a power spectrum, independent of measurement, is important in wind turbine design and testing. The radius of the circle of rotation, its height above the ground, and the rate of rotation are typical for those for a MOD-OA wind turbine.

Verification of this model is attempted by comparing two sets of variances that correspond to individual harmonic bands of spectra of turbulence in the rotational frame. One set of variances is calculated by integrating the theoretically generated rotational spectra; the other is calculated by integrating rotational spectra from real data analysis. The theoretical spectrum is generated by Fourier transformation of an autocorrelation function taken from von Karman and modified for the rotational frame. The autocorrelation is based on dimensionless parameters, each of which incorporates both atmospheric and wind turbine parameters. The real data time series are formed by sampling around the circle of anemometers of the Vertical Plane Array at the former MOD-OA site at Clayton, New Mexico.

Three time series were selected from data taken in a previous study; the time series represent slightly unstable, definitely unstable, and stable atmospheric conditions. All series were analyzed for 8.5-min periods of data. The first series was also analyzed for a 17-min data period.

The theoretical model verifies at all frequencies from 1P to 5P with the 8.5-min runs except for the stable case. In all cases the ratios of variance in a harmonic band from the model to the corresponding variance from real data analysis decrease as the harmonic number increases. Since both sets of variances show decreasing magnitude with increasing harmonic number (from 1P, 2P,...5P), variances from the model show a more rapid decrease of energy with harmonic than do the variances from real data analysis. For the slightly unstable case these ratios ran from 1.38 at 1P to 0.83 at 5P. However, the

decrease of this ratio with frequency could result from noise in the observed data as well as from known and unknown limitations of the theoretical model.

The necessary input parameters for the model include the variance as well as both the longitudinal and lateral length scales of u-component turbulence. (The empirical specification of two length scales departs from the theoretical specification of a single length scale.) Evaluating these from real data analysis involves recognition that stationarity may not have existed. If the data are nonstationary, the measured variance and length scales may be larger. Therefore, the validity of the comparison of results based on specific magnitudes of these parameters is debatable, and sensitivity to a range of parameter magnitudes should be explored. Fortunately, band variances found from different sensitivity tests were within a factor of two of observed results. It was found that sensitivity to the lateral length scale is greater than sensitivity to the longitudinal length scale. It was also found that the effects of increasing variance and increasing length scales are roughly compensating but not exactly so, particularly in low frequencies. Therefore, the sensitivity of the model to an increase of these parameters caused by nonstationarity is much less than would be the case if the model sensitivity to concomitant changes were not compensating.

The spectrum of the shear of the mean wind is also modeled at the 1P harmonic at all stabilities and at the 2P harmonic for the slightly unstable and definitely unstable cases. The shear at higher harmonics is negligible.

The model is economical to operate, requiring typically about 4 min running time on the VAX 11/780 computer.

CONTENTS

SUMMARY	iii
1.0 INTRODUCTION	1
2.0 THEORY AND MATHEMATICS	3
3.0 COMPUTER PROGRAMMING OF THE CORRELATION AND SPECTRAL FUNCTIONS	7
4.0 DESCRIPTION OF VERIFICATION DATA	9
4.1 MEASURED MEAN SHEAR ANALYSIS	10
4.1.1 Assessment of the Accuracy of the Measured Mean Wind Speeds for the Purpose of Rotational Analysis	10
4.1.2 Assessment of the Prevailing Stability	10
4.1.3 Assessment of Accuracy of Measured Eulerian Variances for Purposes of Rotational Sampling	15
4.1.4 Adopted Values of Mean Wind Speed and Variance at Hub Height	15
4.2 MEASURED ENERGY IN HARMONIC BANDS	16
4.3 COMPARISON OF EULERIAN SPECTRUM WITH MODEL	17
5.0 VERIFICATION RESULTS - TURBULENCE WITHOUT MEAN SHEAR	19
5.1 VERIFICATION WITH OBSERVED VARIANCE AND LENGTH SCALES	19
5.1.1 Adoption of Length Scale Values	19
5.1.2 Variances in the Harmonic Bands and Spectra	20
5.2 VERIFICATION AND SENSITIVITY STUDY USING CALCULATED AND ARBITRARY VARIANCES AND LENGTH SCALES	24
5.3 VERIFICATION FOR THE 17-MIN CASE	29
5.4 RUNNING TIME FOR THE MODEL	29
5.5 CONCLUSIONS	30

6.0	MODELING THE MEAN WIND SHEAR	31
6.1	SPECTRAL ANALYSIS OF THE MODELED PROFILES	31
7.0	CONCLUSIONS	35
8.0	REFERENCES	37
APPENDIX -	RELATIONSHIP OF ROTATIONAL TO EULERIAN SPECTRA	A-1

FIGURES

2.1	Geometry of r in Equation (2.2)	4
4.1	Modeled and Measured Wind Speeds for Test 1	14
4.2	Modeled and Measured Wind Speeds for Test 2 and Test 3	14
4.3	Eulerian Spectra for Test 1; 17-min Analysis	18
5.1	Measured and Modeled Variances for Five Harmonics	21
5.2	Rotationally Sampled Spectra	22
5.3	Rotationally Sampled Spectra for Test 1	23
5.4	Measured and Modeled Variances for Five Harmonics Using Four Length Scale Scenarios	25
5.5	Rotationally Sampled Spectra for Four Length Scale Scenarios	25
5.6	Comparison of Rotational Spectra for Two Length Scale Scenarios	28
5.7	Comparison of Measured and Modeled Variances for Five Harmonics, Test 1, 17 min	29
6.1	Rotationally Sampled Spectrum Produced by the Model	33
A.1	Comparison of "Rotational Spectrum" at Zero Rotation Rate with von Karman Variant Spectrum	A-3
A.2	Comparison of von Karman Spectrum and Variant Spectrum	A-4
A.3	Comparison of Kaimal u Spectrum and the von Karman Variant	A-5

TABLES

4.1	Measured Mean Wind Speeds, Test 1	10
4.2	Measured and Modeled Mean Wind Speed Profiles, Test 1, Assuming Neutral Stability.	12
4.3	Measured Eulerian Variances, Test 1	15
4.4	Average Eulerian Variances at Different Height	16
4.5	Hub-Height Mean Wind Speed and Eulerian Variance Values for Each Test.	16
4.6	Measured Variances in the Harmonic Bands, Test 1	17
4.7	Measured Variances in the Harmonic Bands, Tests 2 and 3.	17
6.1	Variance in the Shear Contribution to the Measured Harmonic Band.	31
6.2	Corresponding Analysis Using Time Series Derived from Profile Equations	32
6.3	Ratios Between Modeled and Measured Values in the Above Tables	32

1.0 INTRODUCTION

Spectral analysis of turbulence sampled at a rotating point, such as a point near the tip of a wind turbine blade, has been shown to be relevant to the spectra of stresses on rotating wind turbine blades (Connell and George 1983). Techniques have been devised to calculate spectra in the rotating frame of reference using data from 12 positions on a circle at the Vertical Plane Array (VPA) at Clayton, New Mexico. Spectra so calculated have been compared to the spectra of bending moments, with favorable result, by Connell and George (1983).

It would be desirable to demonstrate that the rotational spectrum (Connell 1981) that applies at a point on a wind turbine blade, under specified conditions of mean wind speed, stability, and surface roughness, can be calculated by formula, at least as well as can be done for the Eulerian, or non-moving point, spectrum. This should hold for the normal, radial and tangential components of wind relative to the disk of rotation. (Only the normal component is addressed in this report.)

In one respect modeling of the rotational spectrum should be easier than modeling the Eulerian spectrum. This is because interest is chiefly in the frequency band beginning at half the rotational frequency of the blade and extending upward into the upper harmonic frequencies. Therefore, modeling of energy content at low frequencies, regarding which uncertainty is considerable, is less critical. However, some concern in the low frequency modeling still remains because part of the energy at low frequencies is translated up into the rotational frequencies by the rotational process.

The purpose of this report is to compare rotational spectra computed with a theoretical model with spectra calculated from real data from the Clayton VPA, which have been formed into rotational sequence. Three samples of such data are chosen, taken under contrasting conditions of atmospheric stability. Because the model that produces the rotational spectrum has no diabatic terms (i.e., terms parameterizing the stable or the unstable atmosphere, as opposed to the neutral atmosphere), the analysis is weighted principally on the test during which atmospheric stability departed least from neutrality.

This report is organized as follows: Section 2 restates and slightly modifies the theory used in the rotational spectrum model. This spectrum applies to the wind component normal to the disc of rotation. Section 3 describes the analytical features of the computer program that makes the model calculations. Section 4 describes and qualifies the Clayton VPA data from which the measured rotational spectra were calculated.

The VPA data were analyzed in two modes. First, rotational spectra were calculated using the full rotational data. Second, the mean wind speed at each anemometer was removed, thus removing the effect of mean wind shear. Spectra were then calculated for these rotational data, containing only the turbulence effect. The theoretical model for the rotational spectrum models the turbulence only. The verification results for the turbulence-only rotational spectrum are given in Section 5. The spectrum of the shear of the mean wind, as observed and as calculated using boundary-layer profile theory, is evaluated in Section 6.

2.0 THEORY AND MATHEMATICS

The basic statistical theory for turbulence as experienced by a point moving in a vertical plane perpendicular to the mean wind direction in the wind turbine layer is given by (Connell 1981). The autocorrelation function of u-component turbulence in the path of the rotating point is given by

$$R_{11} = \frac{2}{\Gamma(1/3)} \sigma^2 \left(\frac{r}{2L} \right)^{1/3} \left\{ K_{1/3} \left(\frac{r}{L} \right) - \frac{r}{2L} \left[1 - \frac{x^2}{r^2} \right] K_{2/3} \left(\frac{r}{L} \right) \right\}. \quad (2.1)$$

where:

Γ is the gamma function

σ^2 is the turbulence variance

r is the separation distance

L is a length characteristic of the turbulence

$K_{1/3}$ is the modified Bessel function of the second kind of fractional order 1/3

x is the alongwind component of the separation distance.

To physically understand the r one must visualize the two ends of the segment defining r as follows. The one end is fixed to an arbitrary point on the blade and thus describes a circular path in the disc of rotation, perpendicular to the mean wind direction, as the blade rotates. The other end is downstream of its original position on the blade at a distance x , or Ut where U is mean wind speed and t is travel time. Mathematically this is expressed by (see Connell 1981)

$$r^2 = \left[2\rho \sin \left(\frac{\Omega t}{2} \right) \right]^2 + (Ut)^2 \quad (2.2)$$

where ρ is the radius of rotation of the one end and Ω is the rotation rate. The two terms express the rotational and translational effects, respectively (see Figure 2.1). Obviously the second term dominates the first as t becomes large.

P_0 is position of P at initial time.

$\Omega t = \theta + 2n\pi$ where n is number of revolutions in time t .

FIGURE 2.1. Geometry of r in Equation (2.2)

The development of a theoretical model of rotational spectra is facilitated by defining the following dimensionless variables. The first two relate turbulence and machine properties; the last is time related to the blade rotation rate:

$$\alpha = \rho/L_a \quad (2.3)$$

$$\beta = U/\Omega L_b \quad (2.4)$$

$$\bar{\tau} = \Omega \tau / 2. \quad (2.5)$$

L_a and L_b are two length scales, in place of the one in Equation (2.1). This is actually a modification of the theory of Equation (2.1), motivated by the fact that the length scale in α is associated with the radius, which extends in all lateral directions, while the length scale in β is associated with the longitudinal mean wind speed. Thus it is hypothesized that more than one length scale may be physically relevant.

L_b is the usual length scale of turbulence. Theoretically, it is the product of the mean wind speed and the infinite integral of the autocorrelation function,

$$L_b = (L_u)_x = U \int_0^{\infty} \rho(\tau) d(\tau). \quad (2.6)$$

This presumes that the following limit exists:

$$\lim_{T \rightarrow \infty} \int_0^T \rho(\tau) d(\tau). \quad (2.7)$$

Alpha (α) is a radial distance from the center of rotation divided by the turbulence length scale, L_a . U is the mean wind speed. Ω is blade rotation in radians per second. β is the period of rotation $2\pi/\Omega$ made dimensionless by multiplying by $U/2\pi L_b$. $\bar{\tau}$ is the correlation time made dimensionless by multiplying by half the rotation frequency in radians.

We note that

$$x/L_b = 2\beta\bar{\tau}. \quad (2.8)$$

We define the autocorrelation function

$$\rho(\bar{\tau}) = R_{11}/\sigma^2. \quad (2.9)$$

Substitution of Equations (2.2), (2.3), and (2.4) into (2.1) and then into (2.9) yields:

$$\rho(\bar{\tau}) = [2/\Gamma(1/3)] (\alpha^2 \sin^2 \bar{\tau} + \beta^2 \bar{\tau}^2)^{1/6} (A-B) \quad (2.10)$$

$$A = K_{1/3} [2 (\alpha^2 \sin^2 \bar{\tau} + \beta^2 \bar{\tau}^2)^{1/2}]$$

where

$$B = \frac{(\alpha^2 \sin^2 \bar{\tau})}{(\alpha^2 \sin^2 \bar{\tau} + \bar{\tau}^2 \beta^2)^{1/2}} K_{2/3} [2(\alpha^2 \sin^2 \bar{\tau} + \beta^2 \bar{\tau}^2)^{1/2}]$$

The function $\rho(\bar{\tau})$ is a correlation of the dimensionless time variable $\bar{\tau}$ in terms of the dimensionless parameters α and β .

The power spectrum is the Fourier transform of the autocorrelation function. Since we want the spectrum with a dimensional frequency argument, we use the autocorrelation function of dimensional time,

$$\rho(t) = \rho(\bar{\tau}), \quad \bar{\tau} = 2\tau/\Omega, \quad (2.11)$$

and write the spectral transform as follows:

$$S(n) = 4\sigma^2 \int_0^\infty \rho(t) \cos(2\pi n t) dt \quad (2.12)$$

where

$$\sigma^2 = \int_0^\infty S(n) dn. \quad (2.13)$$

In the rest of this section we use the logarithmic form of the spectrum, which is the principal form encountered in the meteorological literature. This is $nS(n)$, which integrates to yield the variance according to

$$\sigma^2 = \int_0^\infty nS(n) d[\ln(n)]. \quad (2.14)$$

The programming for the VAX 11/780 computer required to generate the rotational spectra according to Equations (2.1) through (2.10) involved significant problems not explicit in those equations. Therefore, the programming is described in some detail in the following section.

3.0 COMPUTER PROGRAMMING OF THE CORRELATION AND SPECTRAL FUNCTIONS

A computer program has been written that reads in a list of turbulence and wind turbine parameter values and then calculates the correlation and spectral functions for a point rotating in a vertical plane perpendicular to the mean wind direction in the homogeneous isotropic turbulence. One Eulerian spectrum by Kaimal and one following von Karman are also computed for comparison. These spectra are given in Equations (A.3) and (A.4) in the Appendix.

The program first reads in a required set of Eulerian and rotational parameters:

- Eulerian parameters (required)
 - Z - hub height
 - U - mean wind speed at height z
 - z_0 - surface roughness length
- rotational parameters
 - ENO - rotation rate
 - R - rotation radius.

Other parameters handled optionally by read statements or by calculation in the code are σ_u and the two length scales L_a and L_b .

The program reads two other input parameters that govern the length and time step of the autocorrelation function:

- KMAX - the upper bound for the number of values to be calculated for the autocorrelation function
- TFAC - ratio of the time interval in the correlation function to half the reciprocal of the Nyquist frequency of the calculated spectrum.

KMAX prevents the program from calculating and storing more than a given number of values in the autocorrelation vector. If KMAX is too low, the truncated autocorrelation function may be transformed into a spectrum with negative values. If there is no restriction (KMAX very large), the program may store correlation values beyond the dimensioned capacity.

Intuitively one would assume that the product of the time interval of the correlation function and Nyquist frequency should be one-half. Therefore, TFAC

has a trivial value of one. If this is done, the spectrum is badly aliased on the high frequency end. Making TFAC smaller reduces aliasing and also increases the cost by requiring more values in the correlation function. The value 0.25 was considered optimum.

The required value of KMAX depends on the value of TFAC used. The adopted value of KMAX was 12000.

The program calculates the dimensionless parameters α and β and then calculates the correlation function from Equation (2.10). Next the following spectra for the u component are computed:

- nS(n) - rotational
- S(n) - rotational
- nS(n) - Eulerian Kaimal
- S(n) - Eulerian Kaimal
- nS(n) - Eulerian von Karman.

The nS(n) spectra are computed at frequencies that are evenly spaced on a logarithmic scale, at 40 per frequency decade. Thus the ratio of successive frequencies for nS(n) is $10^{0.025}$. The S(n) spectra are computed at evenly spaced frequencies, the difference between successive frequencies being one-tenth of the rotation rate. The rotation rate is called 1P and is the tenth frequency at which S(n) is calculated.

In order to calculate nS(n) at the exact harmonic frequencies of the blade rotation rate, it is necessary to find the nearest frequency in the set to each harmonic (1P, 2P...NP) and arbitrarily alter the frequency to match the harmonic. This is a small change made by multiplying or dividing the original value of n by a number not exceeding $10^{0.0125}$, which is 1.0292. If this is not done, a plot of an nS(n) spectrum may show obvious chopping off at the tops of some of the harmonic spikes.

This section concludes the theoretical and mathematical background of the report. In the succeeding sections the results of data analysis are given.

4.0 DESCRIPTION OF VERIFICATION DATA

The description of the verification data must first of all include the fitness of the data for the intended verification exercise. The sensitivity of the verification results to deficiencies in the data depends on the particular parameters used in the verification. These are of two types: rotational spectra, finely resolved in frequency; and the variances calculated by integrating these spectra across individual frequency bands surrounding the harmonic spikes at frequencies $1P$, $2P$ $5P$. The energy expressed in these variances is the second parameter sought.

The data analyzed to produce the verification spectra and variances were taken with the VPA of fixed-propeller anemometers at Clayton, New Mexico. Results obtained from analysis of these data have been reported by Connell and George (1983).

The geometry of the VPA is that of a circle in the vertical plane that should be normal to the mean wind direction. The center of the circle is 30.5 m above surface, and the radius is 19 m.

According to Connell and George (1983), the data composing the neutral, stable, and unstable cases were taken as follows:

Test 1: 11/25/81 starting 11:49

Test 2: 12/18/81 starting 16:11

Test 3: 3/22/82 starting 09:22

where Test 1, Test 2, and Test 3 refer to the cases called, for simplicity, neutral, stable, and unstable, respectively. All three have 512 samples at one sample per second.

These data are available as portions of longer segments. In order to avoid making the verification results dependent upon the sample length of data chosen, Test 1 was also analyzed as a 17-min segment.

4.1 MEASURED MEAN SHEAR ANALYSIS

4.1.1 Assessment of the Accuracy of the Measured Mean Wind Speeds for the Purpose of Rotational Analysis

The mean wind speeds measured from the VPA for Test 1 are given in Table 4.1.

TABLE 4.1. Measured Mean Wind Speeds, Test 1

z(m)	8.5-min								17-min							
	Wind Speed (m/s)								Wind Speed (m/s)							
60	8.71								8.22							
49	8.46								7.94							
47	8.51				8.57				7.96				8.02			
41	8.47						8.30		7.92				7.78			
30.5	8.29	8.18	8.12	8.18	8.18	8.37	8.15	7.81	7.66	7.58	7.63	7.65	7.84	7.66		
20	7.88						7.94		7.41				7.43			
14	7.51				7.82				7.09				7.24			
12	7.74								7.20							

These mean values have several features that appear to be imperfect. One notices differences between two readings at the same level; for example, there is a difference of 0.31 m/s at the 14-m level in the 8.5-min analysis. This difference drops to 0.15 m/s in the 17-min analysis. Therefore, the differences between anemometers at the same level are not the same for the 8.5-min analysis as they are for the 17-min analysis. An unexpected feature is that the average values at the 47-m level are less than the value measured at 49 m.

In summary, there appears to be unevenness in the measured profile that is not characteristic of the steady atmosphere over uniform terrain. As a result, spectral analysis of data in the rotational frame, including mean wind speeds, will have some content in the harmonic bands that is probably not of atmospheric origin.

4.1.2 Assessment of the Prevailing Stability

One method of assessing meteorological stability is to examine the variation of the mean wind with height, $U(z)$. Values of $U(z)$ averaged over several minutes at several heights may be represented by a continuous function of z that is called a profile. The profile should ordinarily be a monotonic increasing

function of height. The increase of wind with height dU/dz (called the shear) depends on both surface roughness and stability.

Therefore, errors can be made in estimating stability from the profile if the surface roughness has been inaccurately estimated. This estimation may be made roughly using tables based on surface cover such as those given by Counihan (1975). However, refined estimation of surface roughness length for a particular site must be made only from wind profiles that are assumed to be representative of neutral stability.

Stability estimation for data taken at a particular site requires assuming a surface roughness value z_0 for that site and examining estimates for many samples taken under known conditions. If no bias is apparent, when analytically estimated stability is compared to stability estimated by rough human observation, the assumed surface roughness length may be assumed correct.

The surface roughness that applies at Clayton varies, as it does at many other sites, with wind direction because the topography is of different roughness at different directions from the tower. The data analyzed in this report were all taken when the wind direction was in that of least roughness, for which a value of 0.005 m has been estimated (see Chan et al. 1983).

All this discussion should warn the reader that stability estimation using wind shears at the typical wind turbine site is necessarily gross. The final assessment of the stability for the three tests is

Test 1 - slightly unstable
Test 2 - definitely stable
Test 3 - definitely unstable.

The formula for calculating the mean wind speed at one level from that at another level, assuming neutral stability, is

$$U(z_2) = U(z_1) \ln(z_2/z_0)/\ln(z_1/z_0). \quad (4.1)$$

Here z_1 is a reference level that should be in the surface boundary layer, which includes 30.5 m, the central height of the rotational disc used for z_1 in this report. Comparison of $U(z_2)$ calculated from Equation (4.1) may be made with the measured profile. If the measured profile has more shear than that calculated, a stable atmosphere is assumed. Conversely, if the measured profile has less shear than that calculated, an unstable atmosphere is assumed. The average wind speeds at the 30.5-m level for the two periods are 8.21 and 7.69 m/s, respectively. The average values at each height of measured values and values calculated using 0.005 m for the value of z_0 are given in Table 4.2.

TABLE 4.2. Measured and Modeled Mean Wind Speed Profiles,
Test 1, Assuming Neutral Stability

z(m)	8.5-min Wind Speed (m/s)		17-min Wind Speed (m/s)	
	Log Law	Average Measured	Log Law	Average Measured
60	8.85	8.71	8.29	8.22
49	8.66	8.46	8.11	7.94
47	8.62	8.54	8.07	7.99
41	8.49	8.39	7.95	7.85
30.5	8.21	8.21	7.69	7.69
20	7.81	7.91	7.32	7.42
14	7.48	7.66	7.00	7.17
12	7.33	7.74	6.87	7.20

In spite of the distortion in the measured shear profile, it is evident that this log-law model predicts too much shear. Less shear will be predicted if either a lower value of z_0 is assumed, or if the atmosphere is assumed to be unstable. Corroborating evidence for the latter assumption will be presented later in the Eulerian variance analysis.

To assess the degree of instability, it is necessary to estimate a value for the Monin-Obukhov stability length, ($-L$; see Businger et al. 1971). For the unstable atmosphere $-L$ is conventionally positive and approaches infinity as the instability approaches neutrality. Representative values are 10 to 30 m under the afternoon sun. Because Test 1 was thought to have taken place under neutral conditions, we estimate a larger value for $-L$, namely 100 m, and use it in the version of Equation (4.1) for the unstable atmosphere (see Paulson 1970). This equation is:

$$U(z_2) = U(z_1) \frac{\ln(z_2/z_0) - \psi(x_2)}{\ln(z_1/z_0) - \psi(x_1)} \quad (4.2)$$

where

$$\psi(x) = 2 \ln \frac{1+x}{2} + \ln \frac{1+x^2}{2} - 2 \tan^{-1} x + \pi/2$$

$$x = (1 - 15 z/L)^{1/4}.$$

Note that $x = 1$ when $-L$ is infinite (neutral atmosphere) and $\psi = 0$.

To assess the stability of Test 3, the log-linear law for stable conditions is needed. This is given by (see Dyer 1974).

$$U(z_2) = U(z_1) \frac{\ln(z_2/z_0) + 5 z_2/L}{\ln(z_1/z_0) + 5 z_1/L} \quad (4.3)$$

Note that $+L$ is used to model the stable case.

For the slightly unstable case, the profiles of wind speed measured and wind speed modeled after Equation (4.2) are shown in Figure 4.1. Since the model fits well at all except the highest and lowest anemometers, the $-L$ value of 100 m is assumed appropriate, and the mean wind speed readings are assumed correct except at the highest and lowest anemometers, where they appear to be slightly high.

In a similar way, the stabilities prevailing at the time of Test 2 and Test 3 were assessed. These turned out to be definitely stable and unstable, agreeing with previous description. The measured and modeled profiles are shown in Figure 4.2.

The modeled wind profiles obviously disagree with the measured profiles in curvature at certain heights. However, the curvature cannot be changed without changing also the mean slope, which appears to be reasonably good for the unstable case, in general, and also for the upper half of the rotational disc in the stable case. Therefore, we assume that the definitely stable character prevailed during Test 2 and that the definitely unstable character prevailed

FIGURE 4.1. Modeled and Measured Wind Speeds for Test 1

FIGURE 4.2. Modeled and Measured Wind Speeds for Test 2 and Test 3.

during Test 3. Estimated L values used in modeling these profiles were +35 m and -25 m, respectively.

4.1.3 Assessment of Accuracy of Measured Eulerian Variances for Purposes of Rotational Sampling

The Eulerian variances from the Test 1 analysis are as follows given in Table 4.3.

TABLE 4.3. Measured Eulerian Variances, Test 1

<u>z(m)</u>	<u>8.5-min Variance (m/s)²</u>								<u>17-min Variance (m/s)²</u>							
49	0.52								0.83							
47	0.52				0.67				0.86				0.92			
41	0.48				0.64				0.86				1.00			
30.5	0.62	0.61	0.59	0.48	0.58	0.67	0.62	0.88	0.88	0.92	0.86	1.02	1.17	1.04		
20	0.83				0.72				1.02				1.17			
14	0.96				0.72				1.10				1.17			
12	0.79								1.12							

These variances obviously differ at the same level but not in the same way that the mean wind speeds differ at the same level. The difference is that the higher mean wind speed value of two recorded at the same level is at the same anemometer in both the 8.5- and 17-minute segments, while this is not so in the Eulerian variance analysis. This is shown below in Table 4.4, where the letters E and W indicate that the higher variance was recorded on the east or the west anemometer. Below 30.5 m, this pattern reverses.

From the above we shall assume that the variances are not distorted by problems peculiar to the individual anemometers. There are two reasons:

1. The ratios in Table 4.4 decrease when the 17-min analysis is compared with the 8.5-min analysis.
2. The greater variance of two at the same level may change sides when the 8.5-min result is compared with the 17-min result. This is hard to explain as an instrumental aberration.

TABLE 4.4. Average Eulerian Variances at Different Heights

z(m)	8.5 min		17 min	
	Avg.	Ratio at Same Height(a)	Avg.	Ratio at Same Height(a)
49	0.52		0.83	
47	0.59	1.29 E	0.89	1.07 E
41	0.56	1.33 E	0.93	1.16 E
30.5	0.62	1.00	0.96	1.18 E
20	0.77	1.15 W	1.09	1.15 E
14	0.84	1.33 W	1.13	1.06 E
12	0.79		1.12	

- (a) At hub height the average and the ratios are taken from the two outer anemometers on the rotational path only. The ratios are the ratio of the higher of the two readings to the lower, regardless of position.

4.1.4 Adopted Values of Mean Wind Speed and Variance at Hub-Height

The values given in Table 4.5 were adopted for hub-height mean wind speed and variance for the three tests.

TABLE 4.5. Hub-Height Mean Wind Speed and Eulerian Variance Values for Each Test.

	<u>Test 1 - 8.5 min</u>	<u>Test 1 - 17 min</u>	<u>Test 2</u>	<u>Test 3</u>
Mean (m/s)	8.21	7.69	9.28	5.64
Variance	0.62	0.96	0.65	0.19

4.2 MEASURED ENERGY IN HARMONIC BANDS

The principal concern of this document is the modeling of energy in the harmonic bands of the first five harmonics of the rotational frequency. Modeled versions of the kind of spectrum that was integrated to obtain these results are shown later in Figures 5.2, 5.5, and 5.6.

From the Test 1 these values given in Table 4.6 were obtained.

We note that variances in the bands are greater for the 17-min period than for the 8.5-min period although the mean wind speed was greater for the 8.5-min

TABLE 4.6. Measured Variances in the Harmonic Bands, Test 1

Band	Variances (m/s) ²			
	Means Included		Means Removed	
	8.5-min	17-min	8.5 min	17-min
1P	0.2840	0.2849	0.1794	0.1992
2P	0.0692	0.0847	0.0644	0.0834
3P	0.0407	0.0430	0.0355	0.0408
4P	0.0250	0.0278	0.0244	0.0272
5P	0.0209	0.0238	0.0209	0.0222
lowest 0.5P	0.3614	0.6373	0.3550	0.6275

period. Also the overall variances for the 17-min period are greater than those for the 8.5-min period, but the ratios of the overall variances of the 17-min period to those of the 8.5-min period are greater than the corresponding ratios of the band variances. Thus, exact modeling of the band variances for short time segments will not be a simple task.

In a similar way we arrive at average mean wind speeds at each height and the average hub-height variance for Test 2 and Test 3 as shown in Table 4.5.

The mean wind speeds and variances with the mean shear removed are given in Table 4.7.

TABLE 4.7. Measured Variances in the Harmonic Bands, Tests 2 and 3

Band	Variances (m/s) ²			
	Means Included		Means Removed	
	Test 2	Test 3	Test 2	Test 3
1P	1.1188	0.2364	0.0066	0.1632
2P	0.0292	0.0779	0.0061	0.0741
3P	0.0083	0.0499	0.0040	0.0458
4P	0.0038	0.0338	0.0031	0.0318
5P	0.0032	0.0276	0.0027	0.0244
lowest 0.5P	0.0081	0.3383	0.0070	0.3317

4.3 COMPARISON OF EULERIAN SPECTRUM WITH MODEL

The Eulerian *u* spectra for the two anemometers in the Test 1 17-min analysis are shown along with a Kaimal *u* spectrum for the neutral atmosphere (see Equation A.4) in Figure 4.3.

SPECTRA - CLAYTON 11/25 11:49

30-M HEIGHT

FIGURE 4.3. Eulerian Spectra for Test 1; 17-min Analysis

The spectrum for the neutral atmosphere was used rather than that for the unstable atmosphere because the former requires less parameterization and the sample spectrum is not defined back in the low-frequency domain where the difference between the two theoretical spectra is the greatest. The level of the Kaimal spectrum was established by computing u_* using the logarithmic wind law (see Equation A.5).

The measured spectra show high frequency damping of anemometer responsiveness, which is expected because of the distance constant of the measurement system. The measured spectra follow only the low frequency end of the isotropic (5/3 law) region. We shall not presume to "correct" the measured Eulerian variances.

5.0 VERIFICATION RESULTS - TURBULENCE WITHOUT MEAN SHEAR

Verification depends upon obtaining three sets of harmonic variances that match those given for the three cases, called Test 1, Test 2, and Test 3, in Section 4. In all analyses, a variance and two length scales must be specified. In this section we present two verification efforts:

1. using variance and length scale analyses from the same data analysis that produced the harmonic spikes
2. using variance and length scales of quasi-theoretical derivation.

Test 1 was analyzed in both the 8.5- and 17-min record lengths. Since verification is much better in the former case, it is presented in greater detail.

Observed variances were reported in Section 4. Adoption of an observed length scale value is a process requiring some description. Such a length scale is always in the along-wind direction. For the u-component, or horizontal, wind speed we designated this scale as $(L_u)_x$.

5.1 VERIFICATION WITH OBSERVED VARIANCE AND LENGTH SCALES

5.1.1 Adoption of Length Scale Values

Observed length scales are also available from the sample spectral analysis program after some interpretation of output. Referring back to Equations (2.6) and (2.7), we see that the integral of the autocorrelation function must converge in order for the length scale to be defined. Autocorrelation functions of samples of u or v component turbulence frequently do not converge. Therefore, some other criterion must be sought if a length scale value is to be used. Frequently, the value of the integral at its first maximum is taken as the value of the length scale. This is what we call an observed value. This is also the first point where the autocorrelation function becomes negative. If the data set contains data from several anemometers exposed at the same level so that the individual data records should be part of the same ensemble, there may be sufficient disagreement among observed values that an average value taken over all individual values showing relative agreement is the best interpretation.

A second qualifying aspect is data handling before analysis. Most analysts remove trends from data before spectral analysis. Linear trends were removed from the data analyzed for this report. Such trend removal also significantly reduces calculated variance and integral scale. In the analysis to be presented here, we use the results of analyzing linearly detrended data of 8.5- and 17-min periods, as were used by Connell and George (1983). The values of the length scale $(L_u)_x$ were estimated from this analysis. Values for $(L_u)_y$, the length scale of horizontal wind speed in the lateral direction, were assumed to be 0.4 $(L_u)_x$ (see Counihan 1975). For the stable case, a larger value of $(L_u)_y$, 0.5 $(L_u)_x$, was used. This was necessary because the theoretical spectrum produced by the component program described in Section 3, using $(L_u)_y = 0.4 (L_u)_x$, Test 2, had negative values in it. In other words the program produces an autocorrelation function that is not positive-definite when the length scale is too short.

The length scales selected were

	<u>$(L_u)_x$ (m)</u>	<u>$(L_u)_y$ (m)</u>
Test 1	112	45
Test 2	36	18
Test 3	90	36

5.1.2 Variances in the Harmonic Bands and Spectra

The variances in the harmonic bands, calculated and measured, are shown in Figure 5.1. One graph is presented for each of the three tests.

In all three cases the modeled values are found both above and below the measured ones. Therefore, the modeled estimates are of about the right magnitude. In all three cases the modeled values decrease more rapidly with increasing frequency than do the measured values. That is, the absolute value of the slope is less for the curve representing the measured variances than for the curves representing the modeled variances. Therefore, it will be instructive in the next section to use several derived length scales so that the sensitivity of both level and slope of the modeled values to different selection of the length scale may be shown. (Change of variances changes all modeled values by the same ratio as the ratio of variance change.)

The spectra that were integrated to obtain the modeled values are shown in Figure 5.2.

(a)

(b)

(c)

FIGURE 5.1. Measured and Modeled Variances for Five Harmonics: (a) Test 1, Slightly Unstable; (b) Test 2, Stable, (c) Test 3, Unstable.

(a)

U-8.21M/S, Z-30.5M, R-19.05M, NO-0.567HZ
SIGU-0.79M/S, ZO-0.005M, LA-45M, LB-112M

(b)
U-5.61M/S, Z-30.5M, R-19.05M, NO-0.667HZ
SIGU-0.19M/S, ZO-0.005M, LA-18M, LB-36M

(c)

U-9.28M/S, Z-30.5M, R-19.05M, NO-0.667HZ
SIGU-0.98M/S, ZO-0.005M, LA-36M, LB-90M

FIGURE 5.2. Rotationally Sampled Spectra: (a) Test 1 Neutral; (b) Test 2 Stable; (c) Test 3 Unstable

These spectra are quite similar, except that the stable case departs more from the von Karman/Eulerian spectrum than the other spectra.

The correspondence of the rotational spectra from data analysis and from the model for Test 1 is shown in Figure 5.3. This fit is also quite good, although the troughs in the modeled spectrum are noticeably deeper. The difference could be due to noise in the data. The spectrum from data terminates at the fifth harmonic because the higher harmonics are beyond the Nyquist frequency of the data.

One way to make the troughs of the spectra from the data deeper is to subject the data to low-pass filtering at each anemometer before doing the rotational sampling (see George 1984). This has interesting implications discussed in the next section.

FIGURE 5.3. Rotationally Sampled Spectra for Test 1

5.2 VERIFICATION AND SENSITIVITY STUDY USING CALCULATED AND ARBITRARY VARIANCES AND LENGTH SCALES

Rotational spectra applicable for neutral stability and small deviations therefrom have been produced using calculated values of turbulence rms and length scale. The stability limitation is necessary because there are no diabatic terms in the model. A turbulence rms value may be calculated using the mean wind speed of 8.21 m/s in Equations (A.6) and (A.7). This yields

$$\sigma_u = 0.904 \text{ m/s.} \quad (5.1)$$

Using Equation (A.7) to calculate a value of $(L_u)_x$, we obtain

$$(L_u)_x = 128 \text{ m.} \quad (5.2)$$

If we assume that $(L_u)_y$ is 40% of $(L_u)_x$, we obtain

$$(L_u)_y = 51 \text{ m.} \quad (5.3)$$

In the following demonstration, we use the two values of length scale to demonstrate the sensitivity of the model by running four scenarios with length scales as follows:

$$\begin{aligned} L_a &= 128 \text{ m, } L_b = 128 \text{ m} \\ L_a &= 128 \text{ m, } L_b = 51 \text{ m} \\ L_a &= 51 \text{ m, } L_b = 128 \text{ m} \\ L_a &= 51 \text{ m, } L_b = 51 \text{ m} \end{aligned}$$

where the values in the third scenario correspond to the assumptions above. Note that the third scenario is the only one approaching the measured values. The variances in the harmonic bands are plotted along with the observed variances in Figure 5.4. The corresponding spectra are given in Figure 5.5.

There are several interesting observations to make.

1. The modeled harmonic band variances change more with change of L_a than with change of L_b except at the first harmonic, 1P.

FIGURE 5.4. Measured and Modeled Variances for Five Harmonics Using Four Length Scale Scenarios

(a)

FIGURE 5.5. Rotationally Sampled Spectra for Four Length Scale Scenarios

U=8.21M/S, Z=30.5M, R=19.05M, NO=0.667HZ
 SIGU=0.90M/S, ZO=0.005M, LA=128M, LB=51M

(b)

U=8.21M/S, Z=30.5M, R=19.05M, NO=0.667HZ
 SIGU=0.90M/S, ZO=0.005M, LA=51M, LB=128M

(c)

FIGURE 5.5. (Continued)

FIGURE 5.5. (Continued)

2. Reduction of L_x somewhat reduces the decrease of energy with harmonic number (slope of a line connecting points).
3. The best fit is obtained not with the third scenario, but with the fourth, which used the shorter length scale for both L_a and L_b .
4. The rotational spectrum from data analysis, Figure 5.3, obviously fits the theoretical spectrum from the observed values better than any of the theoretical spectra in Figure 5.5 except possibly the third, which is based on similar values of L_a and L_b .
5. Decreasing L_b while holding L_a constant decreases the contrast between peak and valley that defines the harmonic spikes. Fewer spikes are defined. Decreasing L_a while holding L_b constant increases the contrast between peak and valley. Spikes are generated out to greater multiples of the rotation rate.
6. When the data are low-pass-filtered before analysis, thus lengthening $(L_u)_x$, the contrast between peaks and valleys is increased. Thus, the behavior of $(L_u)_x$ is like that of L_b in the model. This fact seems to indicate the need for the scale in the longitudinal direction. Since

neither of the spectra using identical values for both L_a and L_b show sufficient contrast between peak and valley, the need for the second distinct length scale is also apparent.

7. The spectra for the two cases whose length scales are the same, first and fourth, appear similar, but there is more variance with the shorter length scales. This shows that increasing variance (which increases all spectral values in direct proportion) and decreasing both length scales (which increases all spectral values but not exactly by the same ratio) has a similar but not identical effect. This is very important. It means that selection of analysis period length and data detrending procedure, as determined by some criterion (probably debatable) of appropriateness to the total physical situation, is not extremely critical. The reason is that the increase of observed variance and increase of observed length scale are compensating features in the determination of the higher frequency portion of the rotational spectrum. However, a distinction in the low-frequency range is possible; see Figure 5.6.

FIGURE 5.6. Comparison of Rotational Spectra for Two Length Scale Scenarios

5.3 VERIFICATION FOR THE 17-MIN CASE

The harmonic band variances and the rotational spectrum for the 17-min case are shown in Figure 5.7. We note that the difference in slope between modeled and measured results is greater in this case than in the 8.5-min cases. This is expected because L_b has been increased. It appears that the 8.5-min time frame used in the analysis previous to this report was well chosen.

FIGURE 5.7. Comparison of Measured and Modeled Variances for Five Harmonics, Test 1, 17-min

5.4 RUNNING TIME FOR THE MODEL

The running time of this model on the VAX 11/70 computer is proportional to the length scales used. Typically the runs take about 4 min of execution time. Extreme times were about 2 min for the short length scales used in Test 2 and 12 min for the long length scales used in the 17-min version of Test 1.

5.5 CONCLUSIONS

We conclude that the theoretical model verifies well against measured winds at a VPA with the 8.5-min runs except for the stable case. If such verification can be extended to include measured data from one hot-film device mounted on a rotating blade, expensive measurement programs may be sometimes circumvented by the use of theory.

Modeled verification may be accomplished with physically inappropriate values since increasing variance and decreasing both length scales in the model have a similar but not identical effect.

Because the rotational period is very short compared to even the shortest sample (or data record) period, which was 8.5 min, it seems fair to conclude also that short sampling periods should be preferred for assessment of variance and length scale. The larger values of variance and length scale obtained by making the sample longer have a compensating effect in the rotational analysis, and a direct effect in escalating the cost of running the model.

6.0 MODELING THE MEAN WIND SHEAR

Mean wind shear in the boundary layer can be modeled by the logarithmic wind law for the neutral simple boundary layer, or by Paulson's extension [see Equation (4.2)] for the unstable boundary layer. There is also the log-linear law for simpler stable case [see Equation (4.3)]. Correct modeling depends on the data for surface roughness and the estimation of the Monin-Obukhov length.

Once a modeled profile has been obtained, this can be rotationally sampled and the resulting time series subjected to spectral analysis to obtain variances for the harmonic bands. Since one revolution of these values is sufficient, the variances of the bands are those of the individual harmonics of the spectral analysis.

6.1 SPECTRAL ANALYSIS OF THE MODELED PROFILES

We assume that the variance obtained for the total wind with the mean shear left in is the sum of the parts obtained by analyzing mean shear only and turbulence only. This assumes that the contribution of the mean shear to the low harmonics is independent of the turbulence with respect to phase.

The differences between the energy in the harmonic bands for the total wind and for the wind with the mean shear removed (from Section 4) are given in Table 6.1.

TABLE 6.1. Variance in the Shear Contribution to the Measured Harmonic Bands.

<u>Band</u>	<u>Variances (m/s)²</u>		
	<u>Test 1</u>	<u>Test 2</u>	<u>Test 3</u>
1F	0.1048	1.1122	0.0732
2P	0.0048	0.0231	0.0038
3P	0.0052	0.0043	0.0041
4P	0.0006	0.0007	0.0020
5P	0.0022	0.0005	0.0032

The corresponding analysis of modeled shear, derived from profile equations, is given in Table 6.2.

TABLE 6.2. Corresponding Analysis Using Time Series Derived from Profile Equations

Band	Variances (m/s) ²		
	Test 1	Test 2	Test 3
1P	0.1131	1.0727	0.0924
2P	0.0050	0.0014	0.0043
3P	0.0003	0.0001	0.0003
4P, 5P	0.0	0.0	0.0

The ratios (modeled value divided by measured value) from these tables are given in Table 6.3.

TABLE 6.3. Ratios Between Modeled and Measured Values in Tables 6.1 and 6.2.

	Ratio of Modeled to Measured Values		
	Test 1	Test 2	Test 3
1P	1.08	0.96	1.26
2P	1.04	0.06	1.13
3P	0.06	0.02	0.15

Care must be taken not to attach too much meaning to the more favorable ratios in Table 6.3 since the modeled profiles are all functions of estimated values of Monin-Obukhov length (see Section 4). If the length had been estimated at other values, these ratios would have been different. But the ratios are encouraging: they indicate that if the shear can be modeled correctly, the first, and maybe the first two, harmonic bands can also be modeled correctly. In the higher harmonic bands the shear contribution is unimportant compared to the turbulence contribution. (For the neutral and unstable cases, this is also true of the second harmonic band.)

The unfavorable ratios, showing much more variance in the higher frequency bands for the measured values than for those modeled, are believed to result from inaccuracies in the measured profile (see Table 4.2 and discussion).

In the rotational spectra for mean wind included and mean wind at each anemometer removed (and consequently mean shear deleted), the difference seen in

the tabulated variances must be in the single harmonic frequency of each spike rather than spread out over the band. This is shown in Figure 6.1, where both are presented on the same graph. Although the difference is scarcely visible in this plot, the contribution to the variance is, nevertheless, important.

FIGURE 6.1. Rotationally Sampled Spectrum Produced by the Model

7.0 CONCLUSIONS

We conclude that the energy of the first five harmonics of wind speed data (frequencies 1P, 2P....5P), rotationally sampled in the plane of rotation of the blade of a horizontal-axis wind turbine, can be well represented by the theoretical model described in this report. Qualifications to this conclusion are: 1) that the rotationally sampled data used for verification were constructed by sampling about a circle of anemometers in a vertical plane array rather than by collecting continuous data as could be done on a device attached to a rotating blade, and 2) that the verification was well demonstrated for tests conducted in the slightly unstable atmosphere, which may be taken to represent the neutral atmosphere, and in the definitely unstable atmosphere, but was less convincing in a test conducted for the stable atmosphere.

Analytically, the energy in the harmonics comes from two phenomena: the vertical shear of the mean wind and the turbulence. Except possibly in the stable case, the important contribution of vertical shear is entirely to the first harmonic, 1P. This energy was well modeled for the neutral and unstable cases; ratios of energy calculated from the model to energy calculated from data analysis were well within the 0.90 to 1.10 bracket.

From measured turbulence analysis (mean wind shear excluded), we found that all of the first 5 harmonics have significant turbulent energy levels. The ratios of energy calculated by the model to energy calculated by analysis of the data of the neutral test decreased monotonically from 1.38 at 1P to 0.74 at 5P. Comparable ratios were obtained by applying the same verification exercise to the data of the unstable test. The attempt to similarly model rotational turbulence in the stable atmosphere was less successful.

The measured energy in a harmonic decreased in all cases as harmonic number increased. The modeled decrease was always steeper than the measured decrease, as shown by the decreasing set of ratios in the previous paragraph. This difference could result in part from noise in the rotationally sampled data, caused by any persistent anomaly of the data level at a single anemometer in the rotationally sampled ring. However, comparison of mean wind measurements for a 17-min period and an included 8.5-min period indicated that such an anomaly could be natural.

Among the input parameters to the model are the turbulence variance (σ_u^2) and the longitudinal and lateral length scales of the u component of turbulence. The length scale in the longitudinal direction $(L_u)_x$ may be calculated from data analysis. The length scale in the lateral direction $(L_u)_y$ is estimated at 40% of $(L_u)_x$, if it cannot be estimated from specific measurements. Both σ_u^2 and $(L_u)_x$ may be too large if the data analysis period is too long and nonstationarity of the wind grossly affects the calculations. A measurement period of about 8.5 min for normal boundary layer conditions seems to work best. Fortunately, the mathematical effects on model calculations from excess in σ_u on the one hand and excess in the two length scales on the other hand are to a large extent compensating. Therefore, the sensitivity of the model to an increase of these parameters caused by nonstationarity is much less than would be the case if model sensitivity to concomitant changes was not compensating.

A reasonably good verification of theory was also obtained using a theoretically calculated variance and a value of $(L_u)_x$ that was about four times the measuring height. Sensitivity tests performed by varying the two length scales indicate that the model is more sensitive to variation of $(L_u)_y$ than of $(L_u)_x$. Several other conclusions from this exercise are listed at the end of Section 5.2.

The model is economical to run. Execution time depends on the length of autocorrelation function needed to achieve convergence, and this depends on the value of $(L_u)_x$ chosen -- another reason for avoiding an excessive value that results from too long a data sampling time. One to two minutes is a representative execution time on the VAX 11/70.

8.0 REFERENCES

- Businger, J. A., J. C. Wyngaard, Y. Izumi, and E. F. Bradley. 1971. "Flux-profile Relationships in the Atmospheric Surface Layer." Jour. of the Atmospheric Sciences. 28:181-189.
- Chan, S. M., D. C. Powell, M. Yoshimura, and D. H. Curtice. 1983. "Operations Requirements of Utilities with Wind Turbine Generation." IEEE Transactions on Power Apparatus and Systems. Volume PAS-102 No. 9. ISSN 0018-9510. IEEE Power Engineering Society, New York, New York.
- Connell, J. R. 1981. "The Spectrum of Wind Speed Fluctuations Encountered by a Rotating Blade of a Wind Energy Conversion System." Solar Energy, 29:363-375.
- Connell, J. R., and R. L. George. 1983. "A New Look at Turbulence Experienced by a Rotating Wind Turbine." Paper presented at Energy Sources Technology Conference and Exhibition, January 30-February 3, 1983, Houston, Texas.
- Counihan, J. 1975. "Review Paper - Adiabatic Atmospheric Boundary Layers: A Review and Analysis of Data from the Period 1880-1972." Atmospheric Environment. 9:871-905.
- Dyer, A. J. 1974. "A Review of Flux-Profile Relations." Boundary-Layer Meteorology. 7:363-372.
- George, R. L. 1984. Simulation of Winds as Seen by a Rotating Vertical Axis Wind Turbine Blade. PNL-4914, Pacific Northwest Laboratory, Richland, Washington.
- Paulson, C. A. 1970. "The Mathematical Representation of Wind Speed and Temperature in the Unstable Surface Layer." Journal of Applied Meteorology. 9:115-130.

APPENDIX

RELATIONSHIP OF ROTATIONAL TO EULERIAN SPECTRA

APPENDIX

RELATIONSHIP OF ROTATIONAL TO EULERIAN SPECTRA

In this appendix, we demonstrate the correspondence of the rotational spectrum to the corresponding Eulerian spectrum, the von Karman u-component spectrum, when the rotation rate is reduced to zero. Also, the von Karman spectrum is compared with the more state-of-the-art Kaimal u-component spectrum for neutral stability. There are some problems here, as will be shown below. Therefore, the principal point we can make is that we advocate the method given in this report because, in spite of the theoretical problems, it may be successfully used to model blade-bending moments.

The first step is made by plotting the spectrum that results from Fourier transformation of the covariance function of Equation (2.1) when the rotational effort is omitted. In this case, Ω is zero, x and r are identical, and the second term in the summation vanishes, leaving

$$(R_{11})_x = \frac{2}{\Gamma(1/3)} \sigma_u^2 \left(\frac{x}{2L} \right)^{1/3} \left\{ K_{1/3} \left(\frac{x}{L} \right) \right\} \quad (A.1)$$

where $(R_{11})_x$ is the correlation of the u component in the x direction. This correlation function was expected to transform into the von Karman spectrum for the u component, which is given by

$$\frac{nS_u(n)}{\sigma_u^2} = \frac{4F}{[1.0 + (1.339 \times 2\pi F)^2]^{0.8333}} \quad (A.2)$$

where

$$F = nL_b/U.$$

Several points about this spectrum deserve discussion. First, since there are no diabatic terms, the spectrum models the neutral atmosphere rather than the unstable or the stable atmosphere. Neutral stability has widely been regarded as sufficient for wind energy applications because atmospheric stability approaches neutrality during high operational wind speeds (Frost and Shieh 1981). However, during some operational conditions, the atmosphere is unstable and turbulence is more intense (see Powell 1984). For these periods, such a spectrum may underpredict the turbulence in the rotational spikes by a factor of approximately 25% to 50% depending on the degree of instability. Second, the spectrum correctly models the high-frequency range of isotropic turbulence where $nS_u(n)$ decreases with the $2/3$ power of n . A third feature is that the high-frequency range may not properly model vertical variation of the spectrum unless the length scale L_b is proportional to height. Panofsky and Dutton (1984) point out that the length scale of the u component in the alongwind direction $(L_u)_x$ increases more slowly with height than the first power in neutral conditions in simple terrain. Therefore, they do not recommend von Karman spectra for atmospheric modeling.

However, the same objection applies to the Kaimal spectrum, as well as to any other state of the art spectrum for neutral stability. Therefore, regardless of which spectrum we use, we will have to live with these objections.

When the zero-rate rotational spectrum was compared to Equation (A.2), it was found to be too low at low frequencies and too high at high frequencies. However, it was found to coincide very well with the following:

$$\frac{nS_u(n)}{\sigma_u^2} = \frac{4F}{1.339 [1.0 + (2\pi F)^2]^{0.833333}} \quad (A.3)$$

This equation is like (A.2) except that one coefficient, the 1.339, has been relocated. We shall call this the von Karman variant spectrum. The plot is shown in Figure A.1. Why the correlation function by von Karman transforms into the variant spectrum is not known.

FIGURE A.1. Comparison of "Rotational Spectrum" at Zero Rotation Rate with von Karman Variant Spectrum

The distinction between is important insofar as the plots of the von Karman spectrum and the variant spectrum differ. This comparison is shown in Figure A.2.

Obviously, the variant is simply a shift of the original to higher frequency. Inasmuch as the y-axis difference varies from 33% on the left side to 22% on the right side, the difference is of some significance.

Another interesting comparison is between the von Karman variant spectrum and the Kaimal u-component spectrum (Kaimal et al. 1972), which is considered state of the art for neutral stability by meteorologists. This is given by

$$\frac{nS(n)}{u_*^2} = \frac{105f}{[1.0 + 33f]^{1.666667}} \quad (A.4)$$

FIGURE A.2. Comparison of von Karman Spectrum and Variant Spectrum.

where

$$f = nz/U$$

$$z = \text{height}$$

u_* = is the friction velocity given by the logarithmic wind law:

$$u_* = 0.4 U / \ln (z/z_0) \quad (\text{A.5})$$

z_0 = is the surface roughness length.

Note that height serves directly as the length scale here; therefore the length scale is defined as the first power of height.

The Kaimal spectrum and the von Karman variant spectrum are compared in Figure A.3. This plot was facilitated by setting

$$\sigma_u = 2.4 u_* \quad (\text{A.6})$$

$$(L_u)_x = 4.2 z. \quad (\text{A.7})$$

FIGURE A.3. Comparison of Kaimal u Spectrum and the von Karman Variant.

Neither of these are arbitrary fabrications. The first is in agreement with Panofsky and Dutton (1984). The second may be obtained using Kaimal's methods (Kaimal 1973). Although the subject of this paper is spectral description for the stable atmosphere, the result applies reasonably well to the neutral atmosphere.

APPENDIX REFERENCES

- Frost, W. and C. F. Shieh. 1981. Guidelines for Siting WECS Relative to Small-Scale Terrain Features. DOE/ET/20242-78-1, Available from the National Technical Information Service, Springfield, Virginia.
- Kaimal, J. D., J. C. Wyngaard, Y. Izumi, and O. R. Cote. 1972. "Spectral Characteristics of Surface-Layer Turbulence." Quarterly Journal of the Royal Meteorological Society. 98:563-589.
- Kaimal, J. D. 1973. "Turbulence Spectra, Length Scales and Structure Parameters in the Stable Surface Layer." Boundary Layer Meteorology. 4:289-309.
- Panofsky, H. A., and J. Dutton. 1984. Atmospheric Turbulence. Wiley-Interscience, New York.
- Powell, D. C. 1984. Meteorological Analysis for Clusters of Wind Turbines: Interim Report. PNL-5024, Pacific Northwest Laboratory, Richland, Washington.

DISTRIBUTION LIST

No. of
Copies

No. of
Copies

OFFSITE

Carl Aspliden
Battelle Memorial Institute
Washington Office
2030 M Street NW
Washington, DC 20036

S. J. Hightower
Bureau of Reclamation
Denver Federal Center
Building 67, Code 254
Denver, CO 80225

J. Cadogan
Department of Energy
Wind Energy Technology Division
1000 Independence Avenue
Forrestal Building, Room 5H048
Washington, DC 20585

Willi Sadeh
Department of Civil Engineering
Colorado State University
Fort Collins, CO 80523

D. F. Ancona
Department of Energy
Wind Energy Technology Division
1000 Independence Avenue
Forrestal Building, Room 5H048
Washington, DC 20585

V. A. Sandborn
Department of Civil Engineering
Colorado State University
Fort Collins, CO 80523

2 Dr. Edgar DeMeo/Frank Goodman, Jr.
Electric Power Research Institute
3412 Hillview Avenue
Palo Alto, CA 94303

G. P. Tennyson
Department of Energy
Albuquerque Operations Office
P.O. Box 5400
Albuquerque, NM 87110

R. L. George
TERRA Sciences, Inc.
7555 W. 10th Avenue
Lakewood, CO 80214

30 DOE Technical Information Center

Peter Lissaman
Aerovironment, Inc.
145 Vista Avenue
Pasadena, CA 91107

Tom Hiester
FloWind Corporation
1183 Quarry Lane
Pleasanton, CA 94566

Tom Gray
American Wind Energy Association
1516 King Street
Alexandria, VA 22314

2 J. M. Kos/Eugene Di Valentin
Hamilton Standard
Bradley Field Road
Windsor Locks, CT 06096

W. A. Vachon
P.O. Box 149
Manchester, MA 01944

G. L. Park
Department of Electrical
Engineering
Michigan State University
East Lansing, MI 48824

No. of
Copies

David Spera
NASA/Lewis Research Center
21000 Brookpark Road
Cleveland, OH 44135

Phillip French
NASA Scientific and Technical
Information Facility
P.O. Box 8757
Baltimore/Washington International
Airport
Baltimore, MD 21240

J. C. Kaimal
National Oceanic and Atmospheric
Administration/Environmental
Research Laboratories/Wave
Propagation Laboratory
3000 Marine Street
Boulder, CO 80302

G. M. Gregorek
Aeronautics and Astronautics
Research Lab
Ohio State University
2300 W. Case Road
Columbus, OH 43220

Peter M. Moretti
Oklahoma State University
Mechanical and Aerospace
Engineering
Engineering North 218
Stillwater, OK 74074

Dr. H. A. Panofsky
6222 Agee Street #5
San Diego, CA 92122

J. Dutton
Department of Meteorology
Pennsylvania State University
University Park, PA 16902

Andrew Trenka
Solar Energy Research Institute
Rocky Flats
P.O. Box 464
Golden, CO 80401

No. of
Copies

E. Kadlec
Sandia Laboratories
Division 5443, P.O. Box 5800
Albuquerque, NM 87115

Robert Noun
Solar Energy Research Institute
1617 Cole Boulevard
Golden, CO 80401

Sherman M. Chan
Systems Control, Inc.
1801 Page Mill Road
P.O. Box 10025
Palo Alto, CA 94303

R. H. Kirchhoff
Department of Mechanical
Engineering
University of Massachusetts
Amherst, MA 01003

Earl L. Davis
U.S. Windpower, Inc.
2305 S. Vasco Road
Livermore, CA 94550

H. Tieleman
Department of Engineering Science
& Mechanics
Virginia Polytechnic Institute and
State University
Blacksburg, VA 24061

Susan Hosch
Washington State Energy Office
400 E. Union Avenue, 1st Floor
Olympia, WA 98504

Farrell Smith Seiler
Wind Energy Report
Box 14 - 104 S. Village Avenue
Rockville Centre, NY 11571

No. of Copies
Copies

No. of
Copies

FOREIGN

Leif Kristensen
Department of Physics
Risø National Laboratory
4000 Roskilde
DENMARK

Dr. Neil Cherry
Lincoln College
Canterbury
NEW ZEALAND

ONSITE

Per Lundsager
Risø National Laboratory
P.O. Box 49
DK-4000 Roskilde
DENMARK

DOE Richland Operations Office

H. E. Ransom/D. R. Segna

31 Pacific Northwest Laboratory

Dr. M. B. Anderson
Sir Robert McAlpine & Sons, Ltd.
40 Bernard Street
London WC1N 1LG
ENGLAND

J. W. Buck
J. R. Connell
D. W. Dragnich
C. E. Elderkin
D. L. Elliott
P. C. Hays
S. K. Heflick
B. D. Holst
A. H. Miller
V. R. Morris
E. L. Owczarski
D. C. Powell (5)
D. S. Renne
H. L. Wegley
L. L. Wendell
Publishing Coordination (2)
Technical Information (5)
WCPE Program Office (5)

Dr. D. Wilson
Cavendish Laboratory
Madingley Road
Cambridge CB3 0HE
ENGLAND

H. H. Rosenbrock
The University of Manchester
Institute of Science and Technology
P.O. Box No. 88
Manchester M60 1QD
ENGLAND

D. Lindley
Taylor Woodrow Construction, Ltd.
Taywood House
345 Ruislip Road
Southall
Middlesex UBI 2QX
ENGLAND

Dr. Werner Dub
Forschungsprojekt Windenergie
Universitätsstrasse 31
D-8400 Regensburg
GERMANY, FRG