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MICROWAVE UNDULATOR

Kenneth Batchelor

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MICROWAVE UNDULATOR*

At the present time the use of a static magnetic field has been the dominant method by which undulator synchrotron radiation has been produced. A microwave undulator utilizing a plane rectangular waveguide operating in the TE_{10n} mode presents a possible alternate means of producing this synchrotron radiation.^{1,2} Also, since a typical resonator may be many guide wavelengths in length it is possible to make the undulator source tunable over a broad spectral range.

Theory

Consider a rectangular waveguide operating in the TE_{10} mode and having the dimensions shown in Figure 1. It will operate with field patterns as shown in Figure 2, where for guides operating with air or vacuum dielectric the guide wavelength λ_g is given by the relation:

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/\lambda_c)^2}} \quad (1)$$

λ_o is the free space wavelength and in general for a TE_{lmn} mode we have the relationship

$$\left(\frac{1}{\lambda_c}\right)^2 = \left(\frac{l}{2a}\right)^2 + \left(\frac{m}{2b}\right)^2 \quad (2)$$

For the TE_{10n} mode we see that the cut off wavelength, $\lambda_c = 2a$ and hence

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/2a)^2}} \quad (3)$$

The electric and magnetic fields are given by the relations

$$E_y = E_o \sin \frac{2\pi z}{\lambda_g} \sin \omega t \quad (4)$$

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$$B_x = B_0 \cos \frac{2\pi z}{\lambda_g} \cos \omega t \quad (5)$$

where E_0 and B_0 are the peak electric and magnetic field strengths of the microwave field, respectively.

The equation of motion of a relativistic electron is given by

$$\frac{dP}{dt} = -e(E + v \times B) \quad (6)$$

substituting equations (4) and (5) in (6) gives the transverse motion of electrons in the microwave undulator as:

$$\begin{aligned} \frac{d\beta_y}{dt} &= \frac{-e}{m_0 c \gamma} (E_y - v_z B_x) \\ &= \frac{eE_0}{m_0 c \gamma} \left(\frac{Z_0}{Z_w} \cos \frac{2\pi z}{\lambda_g} \cos \omega t - \sin \frac{2\pi z}{\lambda_g} \sin \omega t \right) \end{aligned}$$

(where $B_0 = \mu_0 H_0 = \sqrt{\mu_0 K_0} E_0$ from Maxwell's equations; the group velocity $v_z = \lambda_0 / \lambda_g$; Z_w the transverse wave impedance to is given by $Z_w = (\lambda_g / \lambda_0) Z_0$ and β_y is the electron velocity in the transverse direction divided by the velocity of light.)

Simplifying the above expression gives

$$\frac{d\beta_y}{dt} = \frac{eE_0}{2m_0 c \gamma} \left(\left(1 + \frac{Z_0}{Z_w}\right) \cos \left(\omega t + \frac{2\pi z}{\lambda_g}\right) - \left(1 - \frac{Z_0}{Z_w}\right) \cos \left(\omega t - \frac{2\pi z}{\lambda_g}\right) \right) \quad (7)$$

This shows that the electron is undulated by both a forward and backward wave component of the standing microwave fields.

Each of these waves will give rise to coherent radiation but as long as the wavelength of the forward wave component is much greater than the total undulator length the effect of this wave may be neglected. This implies operating the undulator with a low cut-off frequency; that is with a large broad dimension 'a' as compared to the operating wavelength λ_0 , where λ_g is very nearly equal to λ_0 . We are interested in obtaining short wavelength radiation so we will first investigate the backward wave contribution to the radiation spectrum as

$$\frac{d\beta_y}{dt} = \frac{eE_o}{2m_o c \gamma} \left(\frac{Z_o}{Z_w} + 1 \right) \cos \left(\frac{1}{\lambda_o} + \frac{1}{\lambda_g} \right) 2\pi z \quad (8)$$

where $z = ct$ and $\omega = 2\pi c/\lambda_o$, λ_o being the free space microwave wavelength.

In the case of a conventional magnetic undulator the equation of motion is

$$\frac{d\beta_y}{dt} = \frac{eB_u}{m_o \gamma} \cos \frac{2\pi z}{\lambda_u} \quad (9)$$

So comparing equations (8) and (9) we see that the equivalent magnetic field and wavelength of the microwave undulator are given by the following relationships:

$$B_u = \frac{E_o}{2c} \left(\frac{Z_o}{Z_w} + 1 \right) \quad (10)$$

$$\lambda_u = \frac{\lambda_o \lambda_g}{\lambda_o + \lambda_g} \quad (11)$$

and for the case where λ_o is approximately equal to λ_g we have

$$B_u = \frac{E_o}{c} \quad (12)$$

$$\lambda_u = \frac{\lambda_o}{2} \quad (13)$$

Note that the electron is undulated with the microwave half wavelength and that the maximum value of B_u depends on the maximum electric field strength that the waveguide will allow or the microwave power source can provide.

The characteristics of the radiation from the microwave undulator are exactly the same as those from a static magnetic undulator³ in which the magnetic field and period are given by equations (12) and (13) respectively. According to Hofmann⁴ the wavelength and intensity of the undulator radiation are given by

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right) \quad (14)$$

$$\frac{dP}{d\Omega} = \frac{e^2 c \gamma^4 K^2}{\epsilon_0 \lambda_u^2 (1 + \frac{K^2}{2})^3} \cdot \frac{1 + 2\gamma^2 \theta^2 (1 - 2 \cos^2 \phi) + \gamma^4 \phi^4}{(1 + \gamma^2 \theta^2)^5} \quad (15)$$

$$\text{where } K = \frac{e B_u \lambda_u}{2 \pi m_0 c} = 0.093 B_u (\text{KG}) \cdot \lambda_u (\text{cm}) \quad (16)$$

$$\gamma^2 = \frac{\gamma^2}{1 + \frac{K^2}{2}} \quad (17)$$

The angles θ and ϕ are observation angles as defined in Figure 3 and ϵ_0 is the permittivity of a vacuum. Integrating equation (15) over ϕ and substituting from (14) we get

$$\frac{dP}{d\lambda} = \frac{3P_0}{\lambda} \left(\frac{\lambda_1^2}{\lambda} \right)^2 \left(1 - \frac{2\lambda_1}{\lambda} + \frac{2\lambda_1^2}{\lambda^2} \right) \quad (18)$$

where P_0 is the total radiated power from an electron given by

$$P_0 = \frac{1}{4\pi\epsilon_0} \frac{e^4 \gamma^2 B_u^2}{3m_0^2 c (1 + \frac{K^2}{2})^2} \quad (19)$$

and λ_1 is the wavelength of the on-axis radiation, i.e.

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} \right) \quad (20)$$

Since the particles in the beam have a statistical longitudinal distribution of a length much greater than the wavelength of the radiation, the radiation is not coherent and we can add up the powers radiated by each particle. So for a current I and an undulator length d we obtain

$$P_{0I} = \frac{Id}{4\pi\epsilon_0} \frac{e^4 \gamma^2 B_u^2}{3m_0 c (1 + \frac{K^2}{2})^2} \quad (21)$$

We now will consider the effect of the forward wave on the undulator spectrum.

Although the forward wave contribution to the undulator radiation spectrum may be made small by operating in a region where λ_g is very nearly equal to λ_o there will always be a component in the radiation spectrum resulting from the interaction of this wave component. Let us investigate the nature of this radiation. If we start with the equation of motion given by expression (7) we see that by taking the second term and comparing it with the equivalent expression for the conventional magnetic undulator we may define an equivalent undulator field B_{uf} as

$$B_{uf} = \frac{E_o}{2c} \left(1 - \frac{Z_o}{Z_w} \right) = \frac{E_o}{2c} \left(\frac{\lambda_g - \lambda_o}{\lambda_g} \right) \quad (22)$$

compared with the expression previously determined for the backward wave.

$$B_{ub} = \frac{E_o}{2c} \left(1 + \frac{Z_o}{Z_w} \right) = \frac{E_o}{2c} \left(\frac{\lambda_g + \lambda_o}{\lambda_g} \right) \quad (23)$$

we also may define the equivalent forward and backward wave wavelengths as

$$\frac{1}{\lambda_{uf}} = \frac{1}{\lambda_o} - \frac{1}{\lambda_g} \quad \text{or} \quad \lambda_{uf} = \frac{\lambda_o \lambda_g}{\lambda_g - \lambda_o} \quad (24)$$

$$\frac{1}{\lambda_{ub}} = \frac{1}{\lambda_o} + \frac{1}{\lambda_g} \quad \text{or} \quad \lambda_{ub} = \frac{\lambda_o \lambda_g}{\lambda_g + \lambda_o} \quad (25)$$

Note that $K_{uf} = 0.093 B_{uf} \lambda_{uf} = \lambda_o E_o / 2c = 0.093 B_{ub} \lambda_{ub} = K_{ub}$ i.e. the K factor for both forward and backward wave components is the same. For a single pass device there will be components in the spectrum of wavelengths λ_{uf} and λ_{ub} of amplitudes defined by the ratio $(\lambda_g - \lambda_o) / (\lambda_g + \lambda_o)$. This will give a long wavelength noise superimposed as the desired short wavelength radiation spectrum. For an undulator in a storage ring this component could result in a net deflection of the electron beam which may ultimately destroy the beam.

In order that forward wave has no net effect on beam we require that $m\lambda_u$ backward = λ_u forward where m is a positive integer, i.e.,

$$\frac{m\lambda_o \lambda_g}{\lambda_o + \lambda_g} = \frac{\lambda_o \lambda_g}{\lambda_g - \lambda_o}$$

$$m(\lambda_g - \lambda_o) = (\lambda_o + \lambda_g)$$

$$\lambda_o(m + 1) = \lambda_g(m - 1)$$

$$\lambda_g = \frac{\lambda_o(m+1)}{m-1} \quad (26)$$

$$\text{But } \lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/\lambda_c)^2}} = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/2a)^2}} \text{ for TE}_{10} \text{ mode.}$$

$$\text{so we require that } \frac{m+1}{m-1} = \frac{1}{\sqrt{1 - (\lambda_o/2a)^2}}$$

$$1 - \left(\frac{\lambda_o}{2a}\right)^2 = \left(\frac{m-1}{m+1}\right)^2$$

$$\lambda_o = 2a \sqrt{1 - \left(\frac{m-1}{m+1}\right)^2} \quad (27)$$

We may choose λ_o so that equation (7) is satisfied and then, outside of the undulator region itself, there is no net deflection of the electron beam by either the forward or backward waves. There is, however, a contribution to the undulator output spectrum from the forward wave interaction. Note that K is the same for both forward and backward wave contributions so that the wavelength and power of the undulator radiation from the forward and backward waves depends

only on the undulator wavelengths λ_{ub} and λ_{uf} and equivalent undulator field strengths B_{ub} and B_{uf} . The allowable operating wavelengths and equivalent undulator wavelengths and magnetic field strengths are given in Table I for a waveguide of broad dimension (a) cms.

In order to eliminate the effect of the forward wave we would need to operate with λ_0 very nearly equal to λ_g or $\lambda_0 \ll 2a$.

Let us consider further the effect of both forward and backward waves on the undulator spectrum. Returning to equation (7) we may integrate this equation of motion to obtain.

$$y = \frac{eE_0 \lambda_0^2}{8\pi^2 m_0 c^2 \gamma} \left(\frac{-\lambda_g}{\lambda_0 + \lambda_g} \cos\left(\frac{1}{\lambda_0} + \frac{1}{\lambda_g}\right) 2\pi z + \frac{\lambda_g}{\lambda_g - \lambda_0} \cos\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_g}\right) 2\pi z \right) \quad (28)$$

and

$$\frac{dy}{dz} = \frac{eE_0 \lambda_0}{4\pi^2 m_0 c^2 \gamma} \left(\sin\left(\frac{1}{\lambda_0} + \frac{1}{\lambda_g}\right) 2\pi z - \sin\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_g}\right) 2\pi z \right) \quad (29)$$

From Equation (28) we see that the maximum amplitude of the backward wave oscillation is given by:

$$y_b^{MAX} = \frac{eE_0 \lambda_0^2}{8\pi^2 m_0 c^2 \gamma} \left(\frac{\lambda_g}{\lambda_0 + \lambda_g} \right) = \frac{K\lambda_0}{2\pi\gamma} \left(\frac{\lambda_g}{\lambda_0 + \lambda_g} \right) \quad (30)$$

and the maximum amplitude of the forward wave oscillation is

$$y_f^{MAX} = \frac{eE_0 \lambda_0^2}{8\pi^2 m_0 c^2 \gamma} \left(\frac{\lambda_g}{\lambda_g - \lambda_0} \right) = \frac{K\lambda_0}{2\pi\gamma} \left(\frac{\lambda_g}{\lambda_g - \lambda_0} \right) \quad (31)$$

so that the ratio of the maximum amplitude of the backward to the forward wave is given by

$$\frac{y_b^{MAX}}{y_f^{MAX}} = \frac{\lambda_g - \lambda_0}{\lambda_g + \lambda_0} \quad (32)$$

Table I. Possible Operating Wavelengths for a Standing Wave Undulator

m	λ_o	λ_g	λ_{ub}	λ_{uf}	B_{ub}	B_{uf}	K
2	1.8856a	5.6568a	1.4142a	2.8284a	$1.3333 \frac{E_o}{2c}$	$0.6667 \frac{E_o}{2c}$	$0.0877 \frac{E_{o,a}}{c}$
3	1.7321a	3.4642a	1.1547a	3.4642a	$1.5000 \frac{E_o}{2c}$	$0.5000 \frac{E_o}{2c}$	$0.0805 \frac{E_{o,a}}{c}$
4	1.6000a	2.6667a	1.0000a	4.0000a	$1.6000 \frac{E_o}{2c}$	$0.4000 \frac{E_o}{2c}$	$0.0744 \frac{E_{o,a}}{c}$
5	1.4907a	2.2361a	0.8944a	4.4718a	$1.6667 \frac{E_o}{2c}$	$0.3333 \frac{E_o}{2c}$	$0.0693 \frac{E_{o,a}}{c}$
6	1.3997a	1.9595a	0.8165a	4.8997a	$1.7143 \frac{E_o}{2c}$	$0.2857 \frac{E_o}{2c}$	$0.0651 \frac{E_{o,a}}{c}$
7	1.3229a	1.7639a	0.7559a	5.2913a	$1.7500 \frac{E_o}{2c}$	$0.2500 \frac{E_o}{2c}$	$0.0615 \frac{E_{o,a}}{c}$
8	1.2571a	1.6163a	0.7071a	5.6565a	$1.7778 \frac{E_o}{2c}$	$0.2222 \frac{E_o}{2c}$	$0.0585 \frac{E_{o,a}}{c}$
9	1.2000a	1.5000a	1.6667a	6.000a	$1.8000 \frac{E_o}{2c}$	$0.2000 \frac{E_o}{2c}$	$0.0558 \frac{E_{o,a}}{c}$
10	1.1499a	1.4054a	0.6325a	6.3252a	$1.8182 \frac{E_o}{2c}$	$0.1818 \frac{E_o}{2c}$	$0.0535 \frac{E_{o,a}}{c}$
11	1.1055a	1.3266a	0.6030a	6.6330a	$1.8333 \frac{E_o}{2c}$	$0.1667 \frac{E_o}{2c}$	$0.0514 \frac{E_{o,a}}{c}$
12	1.0659a	1.2597a	0.5774a	6.9284a	$1.8462 \frac{E_o}{2c}$	$0.1538 \frac{E_o}{2c}$	$0.0496 \frac{E_{o,a}}{c}$
13	1.0302a	1.2019a	0.5547a	7.2111a	$1.8571 \frac{E_o}{2c}$	$0.1429 \frac{E_o}{2c}$	$0.0479 \frac{E_{o,a}}{c}$
14	0.9978a	1.513a	0.5345a	7.4837a	$1.8667 \frac{E_o}{2c}$	$0.1333 \frac{E_o}{2c}$	$0.0464 \frac{E_{o,a}}{c}$
15	0.9682a	1.1066a	0.5164a	7.7412a	$1.8750 \frac{E_o}{2c}$	$0.1250 \frac{E_o}{2c}$	$0.0450 \frac{E_{o,a}}{c}$

We may also determine the maximum angular deflection of both forward and backward waves from Equation (29) as

$$\left(\frac{dy}{dz}\right)_{\text{MAX}} = \frac{eE_0 \lambda_g}{4\pi m_0 c^2 \gamma} = \frac{K}{\gamma} \quad (33)$$

Expression (33) is exactly that obtained for the static magnetic field undulator in reference (3). It is also possible to operate the undulator in a travelling wave mode and thus avoid the effects of the forward wave. Alternatively an external magnetic field could be applied in order to suppress the effect of the forward wave.

Higher Order Modes

For a resonator installed in a storage ring other possible resonant modes are of concern since the beam may excite them. For a plane guide of dimensions a, b, d as shown in Figure 1 there are many possible modes with resonant wavelength, λ_0 , given by the relation

$$\lambda_0 = \frac{2}{\sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{d}\right)^2}} \quad (34)$$

for either transverse electric TE_{lmn} or transverse magnetic TM_{lmn} modes. Those with the longest cut off wavelength are the TM_{11n} and TE_{11n} families of modes both of which have a longest cut off wavelength, λ_c , given by

$$\lambda_c = \frac{2a}{\sqrt{1 + \left(\frac{a}{b}\right)^2}} \quad (35)$$

For a standard guide with $a/b = 2$ this gives $\lambda_c = 0.894a$ so that the useful wavelength region for operation of a resonator in the TE_{10n} mode is given by $0.895a < \lambda_0 < 2a$. Note that for the TE_{20n} family the cut-off wavelength $\lambda_c = a$, so it may be desirable to restrict the operating wavelength region to a $\lambda_0 < 2a$. This clearly conflicts with the need to make λ_g very nearly equal to λ_0 or $\lambda_0 \ll 2a$.

It is possible to increase the available wavelength range by use of a ridged waveguide of cross section as shown in Figure 4. However, this has the effect of increasing the cut off wavelength of both the TE_{10n} and TE_{11n} or TM_{11n} modes so that the overall waveguide dimensions have to be reduced in order to operate at the desired wavelength region. Also, since the waveguide dimension in the high electric field region is reduced in size the voltage breakdown limit is also lowered. For storage ring operation the significant gains achieved by use of a ridged guide are the increased available wavelength range and the increased R/Q attained by the reduction in gap size. A detailed optimization of all of these factors may lead to a ridged waveguide configuration. For single pass, pulsed operation, higher peak electric fields are attainable and the aperture may be reduced so a ridged guide may then be preferable.

Undulator Cavity Design

The peak transverse electric field E_0 (V/m) is given by the shunt impedance per unit length R (Ω/m) and the input microwave power per unit length P (w/m) as follows

$$E_0 = \sqrt{2PR} \quad (36)$$

Note that equation (38) is not a function of the cavity length and the shunt impedance per unit length is determined only by the cross sectional dimensions. It is usual to define a quality factor Q per unit length as

$$Q = \omega \frac{W}{-\frac{dW}{dt}} \quad (37)$$

where ω is the microwave angular frequency and W is the stored energy per unit length. Q is approximately equal to the quality factor Q_0 defined for the total volume of the cavity. Generally for resonant cavities we measure the effectiveness of the cavity in terms of R/Q the transverse shunt impedance divided by the quality factor since this is a function of cavity geometry only.

The parameter R/Q as calculated in Appendix I for a cavity of length $\frac{\lambda_g}{2}$ is given by

$$\frac{R}{Q} = \frac{4Z_0 b}{\pi \sqrt{a^2 + \left(\frac{\lambda_g}{2}\right)^2}} \quad (38)$$

The Q value for a cavity of length $n\lambda_g/2$ where n is a positive integer is also calculated in Appendix I as

$$Q_n = \frac{\pi Z_0}{4R_g} \frac{[a^2 + (\frac{\lambda_g}{2})^2]^{3/2}}{\left\{ \left(\frac{\lambda_g}{2}\right)^3 + \frac{a^3}{n} + \frac{a\lambda_g}{4b} (a^2 + (\frac{\lambda_g}{2})^2) \right\}} \quad (39)$$

Possible Undulator Configuration

Let us consider a standard waveguide of internal dimensions $a = 2$ cm, $b = 1$ cm and a length of 100 cm. The lowest practical operating wavelength, λ_0 , is given by equation (27) which defines the cut-off wavelength for the TE_{11n} , TM_{11n} mode families at 1.95 cm. Let us choose $\lambda_0 = 2$ cm which is well below the cut-off wavelength of 4 cm for the TE_{100} mode. The guide wavelength is then given by equation (3) as

$$\lambda_g = \frac{2}{\sqrt{1 - (\frac{\lambda_0}{4})^2}} = \frac{4}{\sqrt{3}} = 2.31 \text{ cm}$$

The maximum practical electric field strength for a 1 cm gap at this frequency is about 15 MV/m for c.w. operation in air. This gives an equivalent magnetic undulator field from equation (10) as

$$B_u = \frac{E_0}{2c} \left(1 + \frac{Z_0}{Z_w}\right) = \frac{15 \times 10^6 \times 10^4}{2 \times 3 \times 10^8} \left(1 + \frac{2}{2.31}\right) \\ = 466.5 \text{ Gauss}$$

and from equations (11) and (16) we obtain

$$K = 0.093 \times 0.4665 \times 1.072 \\ = 0.047$$

for an undulator wavelength $\lambda_u = 1.072$ cm and an operating wavelength of 2 cm.

We may calculate R/Q from equation (24) as

$$\frac{R}{Q} = \frac{4 \times 377 \times 1}{\pi \sqrt{4 + (2.31/2)^2}} = 207.84 \Omega$$

The Q value for a 100 cm long undulator ($n = 200/2.31 = 87$) is given by

$$Q = \frac{\pi 377 (4 + (\frac{2.31}{2})^2)^{3/2}}{4 \times 2.61 \times 10^{-7} \sqrt{15 \times 10^9} \{(\frac{2.31}{2})^3 + \frac{2^3}{87} + \frac{2 \times 2.31}{4} (4 + (\frac{2.31}{2})^2)\}}$$

$$= 14747$$

The shunt resistance $R = 3.065 \times 10^6 \Omega$ and the shunt resistance per unit length $= R/b = 306.5 \text{ M}\Omega/\text{m}$.

For a peak electric field $E_0 = 15 \text{ MV/m}$ this gives a power requirement for from equation (22) as

$$P = \frac{(15 \times 10^6)^2}{2 \times 306.5 \times 10^6} = 367 \text{ kw/m}$$

This is above the average power limit for a $2 \times 1 \text{ cm}$ waveguide operating in air and about one half of the peak power limit; though such an undulator could operate in a pulsed mode with maximum 50% duty factor. A properly water-cooled undulator operating in vacuum could be run in a continuous mode provided the radiofrequency power was available. Note that in short ($< 1 \text{ }\mu\text{sec}$) pulsed operation in vacuum peak electric fields of 100 to 150 MV/m have been attained at a frequency of 15 GHz, so much higher fields and hence K factors are attainable at the cost of higher pulsed microwave power.

Conclusion

The microwave undulator represents a viable option for undulator wavelength down to about 1 cm where peak voltage and available microwave power considerations limit its effectiveness. For undulator wavelengths in the 5 to 10 cm region a pulsed microwave undulator is an attractive proposition for either storage ring or linac, single pass, operation.

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APPENDIX I

Calculation of Q and Shunt Resistance for a Simple Half Wavelength Resonator ($l = \lambda_g/2$)

For a simple half-wavelength resonator using standard rectangular waveguide we have a length d given by:

$$d = \frac{\lambda_g}{2} = \frac{\lambda_0}{2\sqrt{1 - (\lambda/2a)^2}} \quad (1)$$

$$H_z = (B e^{-j\beta z} + B' e^{j\beta z}) \cos \frac{\pi x}{a} \quad (2)$$

$$H_x = (B e^{-j\beta z} - B' e^{j\beta z}) j \frac{2a}{\lambda_0} \sqrt{1 - \left(\frac{\lambda_0}{2a}\right)^2} \sin \frac{\pi x}{a} \quad (3)$$

$$E_y = -(B e^{-j\beta z} + B' e^{j\beta z}) j Z_0 \left(\frac{2a}{\lambda_0}\right) \sin \frac{\pi x}{a} \quad (4)$$

From the boundary conditioners, for $E_y = 0$ at $z = 0$ the sign and magnitude of E_y for the incident and reflected waves must be equal and opposite so that $B' = -B$.

If we define $E_0 = -2B Z_0 \frac{2a}{\lambda_0}$ we obtain

$$H_z = j \frac{E_0}{Z_0} \cdot \frac{\lambda_0}{2a} \sin \frac{2\pi z}{\lambda_g} \cos \frac{\pi x}{a} \quad (5)$$

$$H_x = j \frac{E_0}{Z_0} \cdot \frac{\lambda_0}{\lambda_g} \cos \frac{2\pi z}{\lambda_g} \sin \frac{\pi x}{a} \quad (6)$$

$$E_y = E_0 \sin \frac{2\pi z}{\lambda_g} \sin \frac{\pi x}{a} \quad (7)$$

The total current flow on all sides of the waveguide is the integral of J_y , the current distribution

For the narrow sides $J_y(N) = -j \frac{E_o}{Z_o} \frac{\lambda_o}{2a} \sin \frac{2\pi z}{\lambda_g}$ (8)

and for the end walls $J_y(E) = -j \frac{E_o}{Z_o} \frac{\lambda_o}{\lambda_g} \sin \frac{\pi x}{a}$ (9)

Current flows along the narrow walls and across the ends so that total current on narrow side and end equals

$$I_y = -\frac{2jE_o\lambda_o}{Z_o} \int_0^a \frac{1}{2a} \sin \frac{2\pi z}{\lambda_g} dz + \frac{2jE_o}{Z_o\lambda_g} \int_0^a \sin \frac{\pi x}{a} dx$$

i.e., $I_y = -2j \frac{E_o\lambda_o}{Z_o} \left(\frac{\lambda_g}{\pi} \cdot \frac{1}{2a} + \frac{2a}{\pi} \cdot \frac{1}{\lambda_g} \right)$

$$= -\frac{2jE_o\lambda_o}{\pi Z_o} \left(\frac{\lambda_g}{2a} + \frac{2a}{\lambda_g} \right) \quad (10)$$

The stored energy in the electric fields is given by

$$\begin{aligned} U_E &= \int_0^a \int_0^b \int_0^{\lambda_g/2} \frac{\epsilon_o E_o^2}{2} dx dy dz \\ &= \int_0^a \int_0^b \int_0^{\lambda_g/2} \frac{\epsilon_o E_o^2}{2} \sin^2 \frac{\pi x}{a} \sin^2 \frac{2\pi z}{\lambda_g} dx dy dz \\ &= \frac{\epsilon_o E_o^2}{2} \frac{a}{2} \cdot b \cdot \frac{\lambda_g}{4} \\ &= \frac{\epsilon_o ab\lambda_g}{16} E_o^2 \end{aligned} \quad (11)$$

Power loss on the walls is given by $\frac{|J|^2}{2} R_s$ where R_s is the skin effect resistivity.

$$\begin{aligned} \text{Power loss on narrow sides} = P_N &= 2 \int_0^b \int_0^{g/2} \frac{R_s}{2} \frac{E_o^2}{Z_o^2} \frac{\lambda^2}{4a^2} \sin^2 \frac{2\pi x}{\lambda_o} dy dz \\ &= \frac{R_s b \lambda_o^2}{16 Z_o^2 a^2} E_o^2 \end{aligned} \quad (12)$$

$$\begin{aligned} \text{Power loss on end walls} = P_E &= 2 \int_0^a \int_0^b \frac{R_s}{2} \frac{E_o^2}{Z_o^2} \frac{\lambda_o^2}{\lambda_g^2} \sin^2 \frac{\pi x}{a} dx dy \\ &= \frac{R_s \lambda_o^2 ab}{4 Z_o^2 \lambda_g^2} E_o^2 \end{aligned} \quad (13)$$

$$\text{Power loss on the wide wall } P_W = 2 \int_0^a \int_0^{g/2} \frac{R_s}{2} ((J_x)^2 + |J_z|^2) dx dz$$

$$\begin{aligned} P_W &= \frac{R_s E_o^2 \lambda_o^2}{4 Z_o^2} \int_0^a \int_0^{g/2} \left(\frac{4}{\lambda_g^2} \sin^2 \frac{\pi x}{a} \cos^2 \frac{2\pi z}{\lambda_o} + \frac{1}{a^2} \cos^2 \frac{\pi x}{a} \sin^2 \frac{2\pi z}{\lambda_g} \right) dx dz \\ &= \frac{R_s E_o^2 \lambda_o^2}{16 Z_o^2} \left(\frac{2a}{\lambda_g} + \frac{\lambda_g}{2a} \right) \end{aligned} \quad (14)$$

The total power loss $P_L = P_N + P_E + P_W$

$$= \frac{R_s \lambda_o^2}{4 Z_o^2} E_o^2 \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right) \quad (15)$$

The quality factor of a resonant cavity (Q) is defined as

$$Q = \frac{\omega (\text{energy stored})}{\text{average power loss}}$$

$$= \frac{4\omega\epsilon_0 ab\lambda_g E_0^2 Z_0^2}{16R_s \lambda_0^2 E_0^2 \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right)}$$

Remembering that $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = f_0 \lambda_0$, $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ and $\lambda_0 = \frac{\lambda_g}{\sqrt{1 + (\lambda_g/2a)^2}}$

we obtain

$$Q = \frac{\pi Z_0}{4R_s} \frac{(a^2 + (\frac{\lambda_g}{2})^2)^{3/2}}{\{(\frac{\lambda_g}{2})^3 + a^3 + \frac{a\lambda_g}{4b} (a^2 + (\frac{\lambda_g}{2})^2)\}} \quad (16)$$

For a cavity of length $l = n \lambda_g/2$ where n is a positive integer, we have

$$Q_n = \frac{\pi Z_0}{4R_s} \frac{(a^2 + (\frac{\lambda_g}{2})^2)^{3/2}}{\{(\frac{\lambda_g}{2})^3 + \frac{a^3}{n} + \frac{a\lambda_g}{4b} (a^2 + (\frac{\lambda_g}{2})^2)\}} \quad (17)$$

since the side wall losses and stored energy increase linearly with the number of half wavelengths and the end losses remain the same as for a single half wavelength cavity.

We may express the power losses in terms of the voltage $V = E_0 b$ at the center of the resonator and the shunt resistance R_0 as

$$P_L = \frac{V^2}{2R_0} = \frac{(E_0 b)^2}{2R_0}$$

$$\begin{aligned}
 R_o &= \frac{E_o b^2}{\frac{2R_s \lambda_o^2 E_o^2}{4Z_o^2} \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right)} \\
 &= \frac{2Z_o^2 b^2}{R_s \lambda_o^2 \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right)} \quad (18)
 \end{aligned}$$

From the expressions (16) and (18) for R_o and Q we may obtain a geometric factor R_o/Q for the $\lambda_g/2$ resonant cavity as

$$\begin{aligned}
 \frac{R_o}{Q} &= \frac{\frac{2Z_o^2 b^2}{R_s \lambda_o^2 \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right)}}{\frac{\omega E_o ab \lambda_g Z_o^2}{4R_s \lambda_o^2 \left(\frac{b\lambda_g}{4a^2} + \frac{2ab}{\lambda_g^2} + \frac{a}{2\lambda_g} + \frac{\lambda_g}{8a} \right)}}
 \end{aligned}$$

or

$$\frac{R_o}{Q} = \frac{4Z_o b}{\pi \sqrt{a^2 + (\lambda_g/2)^2}} \quad (19)$$

Figure 1. Rectangular Waveguide Resonator.

Figure 2. TE_{10n} Mode in Rectangular Waveguide.

Figure 3. Definition of Observation Angles Θ and ϕ .
(Electrons undulate in the $\phi = 0$ plane.)

Figure 4. Ridged Waveguide Cross Section.