**THE WAKEATRON: ACCELERATION OF ELECTRONS
ON THE WAKE FIELD OF A PROTON BUNCH**

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ABSTRACT

We explore in this note the idea of accelerating a low intensity electron or positron bunch, travelling through a linear rf structure, following at a short distance an intense proton bunch which leaves behind a wake field. This device acts like a transformer where two beams are involved: one made of protons at high current and low energy, the other made of either electrons or positrons, at low current and high energy. The two beams are coupled electromagnetically to each other by a specially designed rf structure made of a long sequence of cavities.

INTRODUCTION

We follow the approach already outlined in a previous paper¹. This note is divided into six sections. The first two sections deal with a theorem on the maximum energy gain, the transformation efficiency and related considerations. The other four sections describe the Wakeatron as a linear collider for electrons and positrons from 1 TeV to 10 TeV per beam and luminosity 10^{32} - 10^{33} $\text{cm}^{-2} \text{ s}^{-1}$.

The wake field accelerators are devices with two beams of charged particles that we can label beams #1 and #2. The first is a leading bunch moving in a structure made of a long sequence of rf cells and leaves behind a wake field to which loses a considerable amount of energy. The second beam (#2) travels behind, and it is accelerated by the wake field of the first. In order to have a positive net acceleration the second beam is of much weaker intensity than the first. A theorem has been established recently² which under rather stringent conditions establishes that a particle of beam #2 cannot gain more than twice the initial energy of a particle in beam #1. Therefore it seems that in order to accelerate electrons (beam #2) to say 1 TeV one needs a primary beam of particles, of so far unspecified nature, at 500 GeV. There are several ways to go around this theorem by not satisfying the preassumed conditions. It seems possible to have the two beams moving in two different parts of the same rf structure, for instance a hollow beam moving along the periphery of an axially symmetric structure which then generates a higher field gradient in the center of the structure where the second beam is located³. It is also possible to take the leading bunches with asymmetric particle distribution⁴, or as we shall propose here with a longitudinal extension large enough to cover several periods of the rf structure. With these techniques the original factor of two for the transformation ratio can be exceeded. In this note we estimate a possible ratio of 10. Thus to produce 1 TeV electrons now one

requires a leading intense bunch of 100 GeV particles. But so far there has been no need to say anything about the nature of the leading particles, they could have been electrons as well as protons or any others that can be easily produced with an energy of 100 GeV. We would like to point out that it is considerably easier, cheaper, and it requires less power to produce a 100 GeV proton beam than an electron beam of the same energy and intensity. Therefore in this note, as in the original one¹, we propose to use proton bunches to generate wake fields by which accelerate electrons. In other schemes where electrons are also used as leading bunches⁵, they start with an initial energy of 1 GeV, lose their total energy to the rf structure and are then accelerated again with conventional means to 1 GeV in parallel before they can be used again. This is equivalent to a sequence of 100 conventional linacs at 1 GeV each. If on the other hand protons are used, they can be generated initially at the required energy of 100 GeV in one single acceleration cycle in a conventional synchrotron.

Finally, we also point out that because of their larger mass, it is possible with protons to rearrange the longitudinal sequential order of the particles in the leading bunch since the amount of energy they lose or gain depends on their position within the bunch and this can be easily shifted compared to one another. It is because of this shuffling mechanism that it is actually possible to achieve a higher transformation ratio in a co-linear structure.

A MORE GENERAL THEOREM ON THE MAXIMUM ENERGY GAIN FOR THE WAKE FIELD CO-LINEAR ACCELERATORS (WAKEATRONS)

In a recent paper² a theorem was established about the maximum energy gain that a particle can receive under the accelerating effect of the wake field of an intense bunch of leading particles. According to this theorem the maximum energy gain cannot exceed twice the initial energy of a leading particle. This limitation is due to the fact that the primary bunch can travel only some distance L before it comes to a stop after having lost all its energy to the wake field. The derivation of the theorem mentioned above applies to the case where both bunches, the one that leaves the wake field and the one that is being accelerated, have zero longitudinal length. In this case the energy lost to the wake field is the largest and therefore the distance L travelled before coming to a stop is the shortest. Also in this case the acceleration gradient is the largest for a given number of leading particles.

In this section we want to derive a more generalized theorem where the leading bunch has a finite but not vanishing longitudinal length. In this case the rate of energy lost to the wake field is lower and the leading bunch can travel a longer distance. Although the acceleration gradient is reduced, the maximum energy that the second beam can gain is no longer limited to only twice the initial energy of the primary particles.

The energy change of the leading bunch per unit length due to its own wake is given by

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$$\frac{d(N_1 E_1)}{dz} = - N_1^2 e^2 W(0) \exp(-\sigma^2/2g^2) \quad (1)$$

where all the symbols have the same meaning as in reference [2]. What has been added is the reduction of the amount of coherent radiation due to the length of the bunch. We are assuming gaussian distribution with rms length σ . Equation (1) is clearly model dependent and applies in particular to the case of wake fields left behind by a bunch travelling along the axis of an rf structure with periodicity of length g . Eq. (1) was derived a long time ago⁶ and it seems to be correct for the so-called "optical resonator" model made of a sequence of parallel planes of infinite extension. This case is shown in Fig. 1 and Eq. (1) applies more correctly when the conditions

$$a \ll g \ll b \quad (2)$$

are fulfilled, where, assuming cylindrical geometry, b is the outer radius of the structure and a is the radius of the opening on the axis. Also it is known⁶ that for the optical resonator model

$$W(0) = 1/2a^2 \quad (3)$$

Observe that Eq. (1) gives the total amount of the energy lost by the total charge in the bunch. Individual particles may lose a different amount of energy per unit length depending on their location within the bunch.

The energy change of the trailing bunch, which we assume without any longitudinal extension, is given by

$$\frac{d(N_2 E_2)}{dz} = -N_2^2 e^2 W(0) - N_1 N_2 W(y) \exp(-\sigma^2/4g^2) \quad (4)$$

The first term is of obvious origin, since the bunch has no longitudinal width. The second represents the positive energy gain provided $W(y) < 0$. Observe that the dependence on the rms bunch length σ of the first beam is weaker compared to the one shown in Eq. (1) due to the presence of a different coefficient in the exponential factor. A single explanation for it is that the energy loss is given by the integral of the square of the wake field on the rf structure volume⁶.

The energy conservation requires

$$[N_1^2 \exp(-\sigma^2/2g^2) + N_2^2] W(0) + N_1 N_2 W(y) \exp(-\sigma^2/4g^2) \geq 0 \quad (5)$$

Define

$$\bar{N}_1 = N_1 \exp(-\sigma^2/4g^2) \quad (6)$$

as the effective numbers of particles that can radiate coherently when the bunch length is included. Then Eq. (5) becomes

$$(\bar{N}_1^2 + N_2^2) W(0) + \bar{N}_1 N_2 W(y) \geq 0 \quad (7)$$

Since this relation is valid for any values of $\bar{N}_1$ and N_2 , we derive the following inequality

$$[-W(y)] \leq 2W(0) \quad (8)$$

which is the same result as obtained in reference [2]. Observe that in analogy to Eq. (3) we can also write

$$[-W(y)] = \epsilon/a^2 \quad (9)$$

where the form factor $\epsilon < 1$ as a consequence of (8). It can be thought to be made of the product of two quantities

$$\epsilon = \epsilon_{\text{geom}} \cdot \epsilon_{\text{loss}} \quad (10)$$

The first would represent the maximum accelerating gradient a trailing particle can receive even assuming a perfectly conductive material with no resistive or thermal losses. It depends on the distance y from the leading bunch and on the geometry (a, b, g) of the cavity structure. The second factor takes into account the resistivity of the material involved and the losses of field energy through this material that cannot possibly be recovered by the trailing particle.

We can now derive three important equations: the acceleration gradient for the trailing particles, the total length of the acceleration structure, given by the drift the leading particles can travel before they come to a stop or close to it, and the total energy gain per trailing particle which is given by the product of the former two quantities.

Acceleration Gradient

The rate of energy gain per particle is obviously given by Eq. (4) when both sides are divided by N_2 . We will make use of the more explicit definitions (3) and (9) for $W(0)$ and $W(y)$. Observe that Eq. (4) is made of two terms; the first is the rate of energy lost to its own wake field and the second the rate of gain from the wake field of the leading bunch. Let us define a loss-to-gain energy ratio for the trailing particle,

$$\eta = \frac{\text{Energy Lost to Own Wake Field}}{\text{Energy Gain from Leading Bunch Wake Field}}$$

that is

$$\eta = N_2 \exp(\sigma^2/4g^2)/2\epsilon N_1 \quad (11)$$

then we can write for the acceleration gradient

$$G \equiv dE_2/dz = \epsilon N_1 \frac{r_0 E_0}{a^2} (1 - \eta) \exp(-\sigma^2/4g^2) \quad (12)$$

where $r_0 E_0 = e^2$ and, for instance,

$$\begin{aligned} r_0 &= 1.535 \times 10^{-18} \text{ m, classical radius of a proton} \\ E_0 &= 0.93826 \text{ GeV, rest energy of a proton} \end{aligned}$$

Total Acceleration Length

This can be derived from Eq. (1) which gives the total energy loss rate from all the particles of the leading bunch. As we have already observed the actual loss rate of an individual particle depends on its location within the bunch and on the dependence of the form factor ϵ on the distance between particles. It is obvious that the particles at the head of the leading bunch do not suffer appreciable energy loss. In reference [2] it was assumed that ϵ is practically constant (or the equivalent of it) so that the energy loss rate for a particle increases steadily with the distance from the head of the bunch. In reality for the structure we are considering an oscillatory behaviour of ϵ with a period closely related to the gap width g is expected, and if the bunch has an extension larger than g then the energy loss will also have an oscillatory behaviour along the length of the bunch. In particular there will be other locations within the bunch where there are no energy losses, and some sections of the bunch will actually gain energy instead of losing it. It is clear that the estimate and understanding of the factor ϵ , which we do not claim here, is crucial to estimate the spread of energy loss rates. For the moment we will assume that all the particles have an average energy loss given by Eq. (1) after having divided both sides by N_1 . Then the total acceleration length is

$$L = 2\Delta E_1 a^2 \exp(\sigma^2/2g^2)/N_1 E_0 r_0 \quad (13)$$

where ΔE_1 is the average energy lost per particle in the leading bunch. It is convenient to dump the primary beam before the velocity of the particles is slightly reduced below the velocity of light. Indeed an energy spread ΔE in the beam due to a gradient of energy loss rate would correspond to a velocity spread Δv which in turn would give the following increase in length

$$\Delta s = \frac{e}{\beta \gamma^3} \frac{\Delta E}{E_0}$$

after travelling a distance l short enough so that γ is about constant. This might cause a change from the originally assumed gaussian distribution, but also a mixing mechanism by which the longitudinal position of the particle is continuously changed so that we can indeed speak of an average energy loss per particle, a quantity which is then common to all the particles. A convenient parameter to describe this effect is the mixing rate Δ which we can define as the change per unit length of the distance between a particle which suffers the maximum rate of energy loss and a particle which does not suffer any at all. The mixing rate is a

function of the distance z travelled, it is zero at the beginning and it will constantly increase since then. It is convenient to estimate Δ half-the-way of the rf structure, that is for $z = L/2$, in which case one has simply

$$\Delta = E_0^2/E_1^2 \quad (14)$$

This quantity depends only on the initial energy value E_1 and on the particle next energy E_0 . It is clearly seen that because of their mass, protons would allow considerably more mixing than electrons.

Total Energy Gain

This is given by the product of Eq. (12) with Eq. (13). One obtains

$$\Delta E_2 \equiv G \cdot L = 2e\Delta E_1(1-n)\exp(\sigma^2/4g^2) \quad (15)$$

As one can see, by making the length of the primary bunch longer, the total amount of energy gain can increase well beyond the factor of two proven in the theorem of reference [2]. Of course the distance to travel increases also and at a faster rate as one can observe by inspecting Eq. (13). Similarly the acceleration gradient reduces. Observe that if one desires $\Delta E_2 \rightarrow \infty$ than also $L \rightarrow \infty$ and $G \rightarrow 0$. For the special case the primary bunch acts like a point charge, that is $\sigma = 0$, we recover the results of the theorem of reference [2].

Finally it should be noted that only two of the three equations (12, 13, 15) are essential since any one of them can be derived from the other.

TRANSFORMATION EFFICIENCY AND RELATED CONSIDERATIONS

The device we describe here can be thought of as a transformer where high energy particles at low intensity are obtained from a larger number of particles at lower energy. A transformation efficiency can therefore be defined as follows:

$$\alpha = \frac{\text{Total Energy Gain for Trailing Bunch}}{\text{Total Energy Lost from Leading Bunch}}$$

that is

$$\alpha = N_2 \Delta E_2 / N_1 \Delta E_1 \quad (16)$$

By making use of Eq. (15), Eq. (16) becomes

$$\alpha = 4e^2 \eta (1 - n) \quad (17)$$

where we have made use of the definition (11) for the loss-to-gain ratio η .

In order for α to be positive, one has to require $\eta < 1$ which condition sets a limit on the maximum number of trailing particles

that can be accelerated, that is

$$N_2 < 2\epsilon N_1 \exp(-\sigma^2/4g^2) \quad (18)$$

At this limit, $\alpha = 0$, there is no acceleration for the trailing bunch. In this case the amount of energy gained from the wake field of the leading bunch is offset by the energy lost to its own wake field. If the number of trailing particles is increased beyond the limit (18) so that $\eta > 1$, then a net energy loss (deceleration) will result also for the second bunch.

The optimum transformation conditions are obtained, as one can see by inspecting (17) for $\eta = 1/2$, in which case $\alpha = \epsilon^2$, the best of the total energy transformation possible, as it is possible to see in the special case $\epsilon = 1$ in which case also $\alpha = 1$.

The optimum condition $\eta = 1/2$ corresponds to the following distribution of the numbers of particles between the two bunches

$$N_2 = \epsilon N_1 \exp(-\sigma^2/4g^2) \quad (19)$$

Consider the special case $\sigma = 0$ and $\epsilon = 1$, with maximum energy transformation, that is the total energy lost by the first beam is gained in all of the second beam. Of course since the two beams have the same intensity, the second one will end up with the same initial energy of the first.

One more useful parameter which describes the performance of the proposed device is the transformer ratio that is the ratio r of the rate of the energy gain per particle of the trailing bunch to the rate of energy loss per particle of the leading bunch. From Eq. (1), after having divided both sides by N_1 , and from (12) we obtain

$$r = 2\epsilon(1 - \eta) \exp(\sigma^2/4g^2) \quad (20)$$

Once more, for the case of point distribution ($\sigma = 0$) and no resistive losses ($\epsilon = 1$), we recover the result of the theorem of Ref. [2] which states that the ratio of energy gain cannot exceed twice the rate of energy loss per particle. Nevertheless it is pointed out here that by lengthening the primary bunch to several widths g of the rf gaps it is possible to enhance the transformer ratio.

OUTLINE OF THE DEVICE

The Wakeatron which we describe next is intended as a linear collider for electron and positron beams each with energy of 1-10 TeV and luminosity of 10^{32} - 10^{33} cm⁻²s⁻¹. In this device electrons and positrons are accelerated on the wake field of intense and relatively short proton bunches which these days we know how to produce in efficient and reliable ways. All the bunches involved travel along the axis of an RF structure as shown in Fig. 1. Therefore the analysis in the previous two sections above apply to this case where the leading bunch is made of protons and the trailing bunch of either electrons or positrons.

A possible lay-out of the Wakeatron is given in Fig. 2. It is made of two parts which are identical to each other but arranged symmetrically to each other around the crossing point where the two beams collide. One part is to accelerate electrons and the other positrons. Each part is made of a proton source which generates tight bunches in a conventional way. There is an electron beam source at one side and a positron beam source at the other. The acceleration of electrons and positrons takes place in the two sections of the Wakeatron itself which are identical to each other and of the same rf structure. The mode of operation we conceive is that one proton bunch is extracted from each side and injected into their respective sections of rf structure immediately followed by either an electron or a positron bunch. This will occur at some repetition rate at which all sources are to be adjusted to.

As a comparison, we show in Fig. 2 a circle which represents an SSC ring, a hadron collider for energies of 20 TeV per beam and luminosity of $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$. With today's superconducting magnet technology, this ring is expected to have a circumference of about 100 Km, that is a diameter of 30 Km. One can presume, and we shall here, that given the performance of the two possible colliders, circular and linear, they can occupy roughly the same side of real estate, as shown in Fig. 2. Thus we can take 30 Km for the total length of the linear collider, that is an acceleration length $L = 15 \text{ Km}$ for each RF structure that makes the Wakeatron.

Table I gives the summary for the choice of the proton beam energy and intensity for three energy cases: 1, 3 and 10 TeV per beams. These values correspond to $L = 15 \text{ Km}$ as previously stated, and have been derived from Eqs. (13 and 15). For reasons also explained above we have assumed a low efficiency, that is $\eta \ll 1$. Finally we can see from both Eqs. (12) and (13) that a very small opening radius a is desirable, on the other hand the opening must be large enough to accommodate both beams dimensions and we have chosen $a = 1 \text{ mm}$. The ratio σ/g is taken as a variable parameter in Table I as well as the form factor ϵ , for which we have considered only the two values $\epsilon = 1$ and $\epsilon = 0.5$.

It can be seen from Table I that the proton beam total energy ($N_1 \Delta E_1$) does not depend on σ/g for a fixed acceleration length L but increases with the square of both ϵ and the electron/positron final energy ΔE_2 . Therefore for a given ϵ and ΔE_2 , the number of protons per bunch increases with the inverse of the initial proton beam energy ΔE_1 . An interesting case is the one circled in by a dashed line in Table I, which corresponds to $\exp(\sigma^2/4g^2) = 5$. In the following we will continue our analysis for this special case.

The performance of the collider is described by the parameters listed in Table II. These parameters are consistent with those agreed upon by the community at the 1984 Frascati Meeting.⁷ They are based upon the repetition rate of 4000 bunch-bunch encounters. For a luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ this corresponds to 3×10^9 electrons or positrons per bunch and to a normalized rms emittance of 10^{-6} mrad for the case of 1 TeV per beam. These parameters have been chosen so that the disruption parameters D is about unit and the energy spread due to Beamstrahlung is not too large, in this

case 3%. These conditions are satisfied for the same beam dimensions and intensity at all three values of the collider energy we have chosen. The luminosity increases from $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ at 1 TeV to $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ at 10 TeV.

It is our understanding that the performance of the linear collider described here is equal or even superior to that of the circular collider for the hadron beams. The SSC performance lies somewhere between the 1 TeV and 3 TeV case shown in Table II. The high energy 10 TeV case, is certainly a project for the very far future although it does not quite make the level of luminosity that would be desired⁸ (around $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$) at least in the context of this paper.

As we have already said, the Wakeatron is made of a long sequence of RF cells which closely resemble the structure of a conventional linac. The energy of the proton bunches is used to power the device and generate the accelerating field, rather than klystrons or other power sources. There is a basic difference between a conventional linac and the device we describe here. The linac operates at one single frequency mode, whereas the Wakeatron we propose is capable of generating many modes in the wake field of the proton bunches, and the sum of all the modes together can generate a higher accelerating gradient. It is then necessary that the particles which are to be accelerated follow the primary bunch at close distance, likely to be no more than a few times the period length g . In fact most of the higher order mode will damp rather fast and at a longer range, only the fundamental mode will predominate.

We observe that the bunch rms length σ and the gap width g enter only as a ratio σ/g in all our considerations so far. The only other quantity that might depend on σ and g independently is of course the form factor c but this still remains a subject of further study. Therefore we are free to choose the gap width g which is large enough to allow the proton bunches to be long enough to support a considerably large number of particles. The dimension of the rf cavities we have chosen are shown in Table III. It is worth noting that proton bunches, each with 1.3×10^{11} particles, have been obtained in the CERN SPS at 270 GeV and with a rms bunch length of 10 cm. We have therefore chosen a bunch length and a gap width g to roughly match this configuration for a proton beam energy of 100 GeV (1 TeV for electrons and positrons). Since the larger energies also have higher intensity, we have chosen to increase the bunch length σ and the gap width g accordingly. We believe that condition (2) needs to be satisfied and we have chosen the outer radius of the cavities $b = 5g$. The thickness of the walls can be set to 1 or 2 mm, large enough compared to skin depth in the material at any frequency of interest.

All other parameters can then be easily calculated and are also shown in Table 3. In particular we give the acceleration rate which is ideally 200 MeV/m at 3 TeV. The energy lost per unit length for the protons is 10 times smaller. The dependence of these parameters with the beam energy is just a consequence of our rule of the exercise by which the total acceleration length, L has been fixed

for all cases to 15 Km. We observe that indeed the efficiency factor η is very small for all cases, and that the transformation coefficient α is about 12%. This is to produce for example a 2 MW electron beam as required for the collider at $1 \times 1 \text{ TeV}^2$, one needs 16 MW in the proton beam.

THE SOURCES FOR ELECTRONS AND POSITRONS

The electron and positron sources are each made of a 1 GeV linac and a damping ring of the same energy. To generate positrons a target is inserted after the early stages of the linac. Parameters are given in Table IV and are intended as just as example. A more careful analysis is required to optimize the design of the two sources. Anyway, in principle, the linac will generate bunches at the rate $f = 4000$ per second and could be made operable in the S-band mode like SLAC which would give an overall length of about 100 m at a gradient of 10 MV/m. We have found that the energy of 1 GeV is an optimum which provides fast radiation damping and small natural emittance at the same time.

The electron/positron bunches are transferred in the damping ring where they are kept circulating for a while. The primary function of the damping ring is to hold the beam until it is "cooled" effectively by synchrotron radiation. We propose that there be 100 bunches at any time circulating in the ring; since bunches are to be extracted at a rate of $f = 4000$ per second, the time each bunch will spend in the ring is $100/f = 25 \text{ msec}$. For the radiation effects to take over effectively we require that the typical radiation damping time correspond to a fraction of the circulating time, for example $1/3$ which is 8 msec. The parameters given in Table IV would roughly yield an equilibrium rms emittance $\epsilon = \sigma^2/\beta_L = (10^{-6} \text{ m})/\gamma$ similar to the proton beam. For the electron/positron bunch length, we have taken an rms value $\sigma_e = 1 \text{ mm}$ smaller than the length of the proton bunches. The design of these sources should be rather straight-forward and conventional.

THE PROTON SOURCE

Each proton source is made of a Linac, a Booster, Accelerator and Shuttle Ring. The energy and size for the components are given in Table V. Table VI summarizes the beam parameters for the Shuttle Ring. The Accelerator Ring has a cycle of one ramp per second and will deliver 4000 proton bunches to the Shuttle Ring where they will be stored for a period of one second during which they will be extracted one by one until the Shuttle Ring is empty and ready to receive a new load of 4000 bunches from the Accelerator Ring. This mode of operation will provide a constant rate of $f = 4000$ encounters/second. To fill up the Accelerator Ring with 10 consecutive pulses over a short front porch of one second cycles, we require the Booster to operate at 30 cycles per second and to have dimensions about one tenth of the Accelerator and Shuttle Rings. For this last mode of operation we impose a maximum limit bending field of 5 KG. However, the separation of bunches in the Shuttle

Ring will be 5 nsec at 1 TeV and 50 nsec at 10 TeV, obviously of some concern in designing the extraction kicker. The proton sources have to deliver, for obvious reasons, very intense, short bunches. The short bunch length can be achieved by raising the transition energy and choosing large accelerating rf frequency and voltage. The design of the entire proton source should clearly await a more detailed study which will require some of these parameters to change in order to meet the beam specifications. In particular we believe that an individual bunch area of 0.5 eV-sec for 1 TeV and correspondingly higher at larger energies is adequate for the beam to stabilize against microwave coherent excitation, assuming a coupling impedance $|Z/\eta| \sim 1$ ohm. Also the rms emittance, the same in both planes, $\epsilon = \sigma^2/\beta_L = (10^{-6})/\gamma$ is consistent with the assumptions being made for other current projects (SSC, LHC).

Finally, observe that because of the assumed maximum field of 5 KG the dimension of the proton rings increases linearly with the electron beam energy i.e., a radius of 1 km for 1 TeV and 10 km for 10 TeV. We are confident that with more careful analysis we will find means to optimize these parameters in future.

POWER REQUIREMENTS, OVERALL EFFICIENCY AND COST ESTIMATES

We can make an estimate of the power requirements scaling from well known proton accelerators like those existing in CERN or Fermilab. A summary is given in Table VII. There are two major contributions; one is the power required to operate the magnets, and this increases with the size of the ring, and the other is the power which goes into the rf accelerating system. This is about twice the power of each proton beam since the same amount is dissipated in the rf cavities themselves. The operation of the electron linacs and damping rings as well as of the focussing systems along the length of the Wakeatron do not take a large amount of power. In the end we estimate about 100 MW to operate the entire facility for a 1×1 TeV² collider; this requirement increases almost quadratically with the collider energy. We can then estimate the overall efficiency, that is the ratio between the power in the electron beam and the total power required to operate the facility. We expect an overall efficiency of 4% at 1 TeV which then decreases linearly with the beam energy. We believe that an overall efficiency of a few percent is a significant figure and deserves careful consideration.

It is relatively easy to make an approximate cost estimate of the device from existing proton facilities. There are few items like the electron source (linac and damping ring) and the accelerating structure itself (Wakeatron) that have a cost independent of the choice of the collider energy. On the other hand because of the different sizes, the proton source has a cost that should increase linearly with the collider energy. One should also add the cost for beam transfer and focussing along the accelerating structure; we believe this cost also increases with the energy. The summary of a very rough cost estimate is given in Table VIII. Although a more detailed study is required before one can reach a more definite conclusion, it nevertheless seems that from a cost and

performance point of view the device we have described here has some advantages over a circular collider like the SSC.

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Table I Proton Beam Parameters vs. Electron Beam Energy

r	σ/g	$\Delta E_2 = 1 \text{ TeV}$		$\Delta E_2 = 3 \text{ TeV}$		$\Delta E_2 = 10 \text{ TeV}$	
		ΔE_1	N_1	ΔE_1	N_1	ΔE_1	N_1
2.	0	0.5 TeV	$0.45 \cdot 10^{11}$	1.5 TeV	$1.4 \cdot 10^{11}$	5 TeV	$4.6 \cdot 10^{11}$
		1.0	0.93	3.0	2.8	10	9.3
4.	1.665	0.25	0.93	0.75	2.8	2.5	9.3
		0.5	1.9	1.5	5.6	5	19.
10.	2.537	0.1	2.3	0.3	6.9	1.	23.
		0.2	4.6	0.6	14.	2.	46.
20.	3.035	0.05	4.6	0.15	14.	0.5	46.
		0.1	9.3	0.3	28.	1.	93.
first line:		$\epsilon = 1$		$\alpha = 1 \text{ mm}$		$L = 15 \text{ Km}$	
second line		$\epsilon = 0.5$		$\eta \ll 1$			

Table II Collider Parameters

Energy/Beam	1 TeV	3 TeV	10 TeV
Repetition Rate	4,000 encounters/sec		
No. of $e^\pm$ /Bunch	3×10^9		
β^* (H and V)	5mm		
β -emittance σ^2/β_L (H and V)	$10^{-6} \text{ m}/\gamma$		
Rms Bunch Length	1mm		
Rms Beam Spot, $\sigma_{H,V}^*$	500 A°	300 A°	150 A°
Disruption Parameters, D	1		
Energy Spread due to Beamstrahlung	3%		
Luminosity, $\text{cm}^{-2}\text{s}^{-1}$	1.0×10^{32}	3.0×10^{32}	1.0×10^{33}

Table III The Wakeatron RF Structure

Energy/Beam	1 TeV	3 TeV	10 TeV
No. of p's/Bunch	2.3×10^{11}	6.9×10^{11}	2.3×10^{12}
p-Beam Energy	100 GeV	300 GeV	1000 GeV
σ/g , ($r = 10$)		2.537	
Rms p-Bunch Length	12.7 cm	38.1 cm	127 cm
Gap Width, g	5 cm	15 cm	50 cm
Iris Radius, a	1 mm	1 mm	1 mm
Outer Radius, b	0.5 m	1.5 m	5 m
Wall Thickness	1-2 mm	5 mm	1-2 cm
Total Length		2 x 15 Km	
Accelerat. Rate	67 MeV/m	200 MeV/m	667 MeV/m
Energy Loss (p)	6.7 MeV/m	20 MeV/m	67 MeV/m
Energy Deposition	0.25 J/m	2.25 J/m	25 J/m
Form Factor, c		1.	
Loss-to-Gain Ratio, η	0.03	0.01	0.003
Transformation Coefficient, α	0.12	0.04	0.012
Power/Beam: electrons	2 MW	6 MW	20 MW
protons	16 MW	144 MW	1600 MW

Table IV Electron/Positron Source

Linac:

Output Energy	1 GeV
RF	3 GHZ
Repetition Rate	4 KHZ

Damping Ring:

Energy	1 GeV
Average Radius	10m
Packing Factor	50%
Dipole Field	8 KG
Betatron Tune, (H,V)	15
No. of Bunches	100
No. of $e^{\pm}$ /Bunch	3×10^9
Radiat. Damping Time	8 msec
Time between two consecutive bunch extractions	25msec
σ^2/β_2 (H and V)	$10^{-6} \text{ m}/\gamma$
Rms Bunch Length	1mm
Energy Loss	20 KeV/turn

RF:

Frequency	500 KHZ
Voltage	100 KV

Table V Parameters for the Proton Source

Energy/Beam	1 GeV	3 GeV	10 GeV
Components:			
Linac (200 MHz)	1 GeV	3 GeV	10 GeV
Booster	10 GeV/100m	30 GeV/300m	100 GeV/1km
Accel. Ring	100 GeV/1Km	300 GeV/3Km	1 TeV/10Km
Shuttle Ring	100 GeV/1Km	300 GeV/3Km	1 TeV/10Km
Cycle Rate:			
Booster		30Hz	
Acceler. Ring		1Hz	
RF:			
Booster	175-200 MHz/2MV	67MHz/1MV	20 MHz/0.2MV
Accel. Ring (h = 4000)	200 MHz/7MV	67MHz/2MV	20 MHz/0.7MV
Shuttle Ring (h = 4000)	200 MHz/10MV	67MHz/3MV	20 MHz/1.0MV
Bending Field:			
Booster		0.5-5KG	
Accel. Ring		0.5-5KG	
Shuttle Ring		5KG	

Table VI Beam Parameters in the Shuttle Ring

Energy/Beam	1 TeV	3 TeV	10 TeV
Average Radius	1Km	3Km	10Km
Bending Field		5KG	
RF: Frequency	200 MHz	67 MHz	20 MHz
Harmonic No.		4000	
Voltage	10 MV	3MV	1MV
No. of Bunches		4000	
No. of p's/bunch	2.3×10^{11}	6.9×10^{11}	2.3×10^{12}
Rms Bunch Length	12.7cm	38.1cm	127cm
Bunch Separation	5nsec	15nsec	50nsec
Total No. of Protons	9.2×10^{14}	2.8×10^{15}	9.2×10^{15}
Longit. Phase Space Area(95%)	0.5 eV.s	1.5 eV.s	5 eV.s
β -Emittance σ^2/β_L (H and V)		10^{-6} m/γ	

Table VII Power Requirements

Energy/Beam	1 TeV	3 TeV	10 TeV
Power/Beam: electrons	2MW	6MW	20 MW
protons	16MW	144MW	1600MW
α	12%	4%	1.2%
Each p-Source:			
Magnet Power	10MW	30MW	100MW
RF Power (x2)	32MW	288MW	3200MW
Each e^+ -Source	6 MW	6MW	6MW
Transv. Focus/Section	2MW	6MW	20MW
Total Power for			
Entire Facility	100MW	660MW	6.65GW
Overall Efficiency	4%	1.8%	0.6%

Table VIII Cost Estimate (approximate)

Energy/Beam	1 TeV	3 TeV	10 TeV
Each Proton Source	200 M\$	600 M\$	2000 M\$
Each Electron Source	25	25	25
Each Section of RF Structure	100	100	100
Transport, Transfer and Focussing for each side	25	75	250
Total each side	350	800	2375
	x 2	x 2	x 2
Total Project(*)	0.7 B\$	1.6 B\$	4.75 B\$

(*)1985 US dollars. No contingency and no escalation added.

Fig. 1 CONCEPTUAL DESIGN OF AN RF STRUCTURE TO ACCELERATE ELECTRONS OR POSITRONS ON THE WAKE FIELD OF AN INTENSE PROTON BUNCH (PROTON KLYSTRON, WAKETRON).

Fig. 2 LAY-OUT OF AN ELECTRON-POSITRON LINEAR COLLIDER WHICH MAKES USE OF THE IDEA OF ACCELERATING ELECTRONS AND POSITRONS ON THE WAKE FIELD OF INTENSE PROTON BUNCHES.