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THE Z LIMIT OF ELEMENTS IN THE UNIVERSE

By

S. Yoshikawa

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PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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THE Z LIMIT OF ELEMENTS IN THE UNIVERSE

Shoichi Yoshikawa

Princeton University

Plasma Physics Laboratory and Department of Astrophysical Sciences
Princeton, New Jersey 08544Abstract

From a general consideration of atomic models, the Z of elements cannot exceed $1/\alpha$, where α is the fine structure constant. Combined with a knowledge of nuclear physics, we may conclude that the limit on strong interaction is that no long-lived nucleus exists beyond $Z = 1/\alpha$.

Eddington et al., according to Flint,¹ postulated that the limit of a naturally occurring element, i.e., $Z = 92$, is related to the fact that the ground state of a hydrogen-like atom theoretically cannot exceed $Z \sim 100$. They pointed out that there exists a conflict between quantum mechanics and the theory of relativity. The argument was based on the prevailing, but somewhat incomplete, knowledge of nuclear and atomic structures. However, the concept is worth reexamining in light of our present knowledge of physics, astrophysics, etc.

First, we note that the orbit of a particle in gravitational potential or in a Coulomb potential field, $\phi(r)$, becomes fundamentally different if

$$m_0 c^2 \sim |\phi(a_c)|, \quad (1)$$

where m_0 is the rest mass of a particle and the relativistic mass is assumed to be of the same order of magnitude as the rest mass. For $m \gg m_0$, the potential field, $\phi(a_c)$, can obviously be treated as a perturbation. The radius, a_c , is the critical radius and in the case of a gravitational field,

is half the Schwarzschild radius. In a Coulomb field, a_0 equals Za_0 . Here a_0 is the (classical) relativistic radius of electrons and Z is the charge of the hydrogen-like nucleus. In this case, Sommerfeld² showed that the precessional motion of the electron around the nucleus is so large that the orbit does not stay quasi-elliptical anymore and the particle (without quantum effect) spirals into the nucleus. Alternatively, this can be considered as the effect of the virtual potential barrier due to the centrifugal force disappearing because of the relativistic effect. (In Sommerfeld's notation, $p < p_0$).

The relativistic radius of electrons, a_0 , is smaller than the Bohr radius of hydrogen atom, a_H , by

$$\frac{a_0}{a_H} = \alpha^2 = \left(\frac{e^2}{\hbar c}\right)^2 = \left(\frac{1}{137}\right)^2, \quad (2)$$

where α is the fine structure constant. Since for the nucleus of charge Z , the Bohr radius, a_B , would be a_H/Z , it follows that

$$\frac{a_0}{a_B} = \alpha^2 Z^2. \quad (3)$$

Thus one would expect a major, fundamental change at $Z_0 = 137$. In fact, there is a peculiar singularity at Z_0 in the energy levels of a hydrogen-like atom in the Sommerfeld-Dirac^{2,3} model. (We closely follow Dirac's notation.) Dirac showed that

$$\frac{H'}{m_0 c^2} = \left(1 + \frac{\alpha^2 Z^2}{[n + (j^2 - \alpha^2 Z^2)^{1/2}]^2}\right)^{1/2}. \quad (4)$$

Here H' is the energy level including the rest mass of electrons, and j and n are quantum numbers. The lowest three energy levels are

$$\frac{H'}{m_0 c^2} \approx (1 - \alpha^2 Z^2)^{1/2}, \quad |j| = 1, n = 0, 1S_{1/2} \quad (5)$$

$$\frac{H'}{m_0 c^2} = \frac{1}{2^{1/2}} [1 + (1 - \alpha^2 Z^2)^{1/2}]^{1/2} \quad |j| = 1, n = 1, 2S_{1/2}, 2P_{1/2} \quad (6)$$

$$\frac{H'}{m_0 c^2} = [1 - (\alpha^2 Z^2/4)]^{1/2} \quad |j| = 2, n = 0, 2P_{3/2} \quad (7)$$

Graphs are shown in Fig. 1 along with the observed characteristic X-ray data of some heavy elements.^{4,5}

As far as we can determine, Formula (4) is accurate. At $\alpha Z = 1$, the Dirac solution predicts the higher probability of the distribution function near the origin. Thus the electron-nucleus interaction (such as K-electron capture) increases.

Within the quantum electrodynamics correction and the electron-nucleus interaction, the energy levels of Eq. (4) agree with the experimentally obtained values of high-Z, hydrogen-like ions.^{6,7}

Thus if Formula (4) is correct experimentally, we must accept the breakdown of $|j| = 1$ states at $\alpha Z = 1$, such as Eqs. (5) and (6). Superficially, for $\alpha Z > 1$, H' becomes imaginary or complex, so no stationary solution exists. This conclusion agrees with the one taken from the old quantum theory, because at $\alpha Z = 1$, electrons start to spiral into the nucleus.

Also, if a fully stripped $Z = 138$ nucleus were to exist, a free electron with the angular momentum $\hbar$ would hit the nucleus.

If one insists that H' must be real for $\alpha Z > 1$, this difficulty can be removed by including the finite extent of the nucleus. However, there still exists a limit to Z , since for the energy $H'/m_0 c^2 < -1$, there is no confined wave function.⁸⁻¹⁰ The limit in Z is estimated to be ~ 170 .

Alternatively, if one seeks solutions to the Dirac equations for $\alpha Z > 1$, H' solutions with H' , either imaginary or complex, can be possible. These solutions are logarithmically singular at the origin. Thus the finite size of the nucleus removes the singularities at the origin. If we choose this solution, we are interpreting that the 1S level electron orbit is not time-independent and the electron falls into the nucleus in the time

$$\tau \equiv (\text{Im } H'/h)^{-1} . \quad (8)$$

This interpretation agrees with the conclusion drawn from the old quantum theory.²

Therefore, one is tempted to take the view that a long-lived nucleus with $Z = 138$ is incompatible with the fundamental postulates of quantum mechanics. (Or if one prefers, incompatible with the theory of relativity.) Thus a $Z > 137$ nucleus should not exist except briefly when two nuclei ($Z_1 + Z_2 > 137$) collide with the appropriate velocity.

Thus Eddington's postulate may be formulated as: The atomic model predicts that the $Z > 137$ nucleus does not exist. Hence strong interaction is limited so that a $Z > Z_0$ does not exist in the universe.

From observed experimental facts, the author is tempted to say that the coupling constant of the strong interaction equals the limit mentioned above.

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Figure Caption

Fig. 1. Three energy levels of hydrogen-like ions using the Sommerfeld-Dirac formula. Also the energies of characteristic X-rays of lead, uranium, and curium are shown.

Fig. 1

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