The Modular Stellarator Fusion Reactor Concept

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by

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ABSTRACT

A preliminary conceptual study is made of the Modular Stellarator Reactor (MSR). A steady-state ignited, DT-fueled, magnetic fusion reactor is proposed for use as a central electric-power station. The MSR concept combines the physics of the classic stellarator confinement topology with an innovative, modular-coil design. Parametric tradeoff calculations are described, leading to the selection of an interim design point for a 4-GWt plant based on Alcator transport scaling and an average beta value of 0.04 in an $\ell = 2$ system with a plasma aspect ratio of 11. The physics basis of the design point is described together with supporting magnetics, coil-force, and stress computations. The approach and results presented herein will be modified in the course of ongoing work to form a firmer basis for a detailed conceptual design of the MSR. Neither an economic analysis nor a detailed conceptual engineering design is presented here, as the primary intent of this scoping study is the elucidation of key physics tradeoffs, constraints, and uncertainties with respect to the ultimate power reactor embodiment.

I. INTRODUCTION

A. Overview

The stellarator represents one of the earliest magnetic confinement concepts to receive attention¹⁻³ as a commercial fusion power reactor. Unlike the tokamak, the nonaxisymmetric stellarator achieves equilibrium in a toroidal geometry by externally inducing a rotational transform in the confining magnetic field. Ideally, no axial current need be supported by the toroidal plasma column, as is required in a tokamak, although until recently all stellarator experiments utilized such currents for Ohmic heating. The first stellarator reactor designs¹⁻³ proposed the use of separate toroidal and helical coil sets that were combined to form "figure-8" and racetrack configurations. Early calculations and cost estimates⁴⁻⁶ for toroidal stellarator reactors indicated the related potential problems of low power density and high magnet costs. These early survey studies were eventually

overshadowed by discouraging physics results for stellarators and concurrent progress in tokamak confinement. As a result reactor interest in the stellarator waned.

Consideration of the torsatron concept⁷ as a reactor allowed the elimination of the toroidal-field (TF) coil set and, when coupled with new understanding of stellarator/torsatron physics, has generated more recent interest in this truly steady-state device as a reactor.⁸⁻¹⁰ At the same time, the potential for moderate power output (≤ 5 GWt) was emphasized.⁸ Although the magnetic surfaces in a torsatron are topologically similar to those in the stellarator, the desired magnetic geometry can in principle be created by using only relatively "force-free" helical coils.⁸

Recognition that the helical coils can be eliminated in favor of toroidal coils that have been subjected to a periodic, lateral distortion (i.e., a twist) has given the stellarator the promise of greater and more realistic system modularity.¹¹ Specifically, implementation of the twisted TF-coil set produces the stellarator magnetic geometry while eliminating the helical coil set in favor of a modular device. In addition, more optimally oriented coil forces and lower coil stresses are anticipated for this modular stellarator approach. These new configurational advances have renewed interest in the reactor extrapolation of the stellarator/torsatron concept, a renaissance that coincides with recent experimental success in heating a low-Ohmic-current device¹² (the latter is a prerequisite for eventual steady-state reactor operation).

Qualitative advantages that in general can be invoked for the stellarator/torsatron reactor concept include:

- Steady-state magnetic fields and thermonuclear burn.
- Operation at ignition or with a high Q-value for low recirculating power.
- Plasma startup on existing magnetic surfaces with predictable particle and energy confinement at all times.
- Impurity and ash removal by means of a magnetic limiter and helical poloidal divertor that occur as natural consequences of the magnetic confinement topology.
- Evidence of operation without major plasma disruptions that could lead to an intense, local energy deposition on the first wall or in the blanket, shield, or coil regions.

- No auxiliary positioning or field-shaping coils and moderate plasma aspect ratio (≥ 10), both of which ease maintenance access.

These advantages remain to be quantified in the context of a comprehensive reactor study that self-consistently incorporates crucial physics issues (e.g., scaling of beta with aspect ratio and the required or optimal rotational transform, magnetic shear, and magnetic-well depth), engineering constraints (e.g., coil design, stresses, accessibility, and maintenance), and economics (e.g., power density, size, and capital and energy costs).

B. Scope of Study

As part of an overall program for assessing the reactor potential of alternative magnetic fusion concepts (AFCs), the Office of Fusion Energy of the US Department of Energy has funded systems studies according to a three-tier structure. These studies are categorized in order of decreasing level of effort and detail as Level I, Level II, and Level III. The highest level of study (Level I) includes, in a multi-man-year effort, considerable conceptual design and economic analysis in addition to sophisticated, state-of-the-art physics and operating-point analyses. The lowest level of study (Level III) would characterize less understood and developed confinement schemes by means of relatively simplified physics models and parametric analyses of potential reactor operating points. Generally, a Level III study would not project a reactor embodiment per se, and, because of obvious gaps in the physics understanding of these relatively unexplored concepts, only a range of potential reactor operating points may be parametrically identified. The stellarator/torsatron study described herein is a Level III study. This study has been conducted as a prelude to a more detailed conceptual design of the MSR at Level I.

Unlike other AFCs, the physics basis for the stellarator/torsatron is relatively well developed and has a long history. As noted previously, this physics basis led to earlier predictions¹⁻⁶ of a reactor that was large, expensive, and difficult to maintain. Recent understanding of stellarator/torsatron physics combines with a recently recognized potential for modular-coil configurations that may be more amenable to a commercial reactor in terms of size and maintainability, thereby warranting the present reassessment of the stellarator/torsatron. The wide range of configurational options available to the stellarator/torsatron combined with the general absence of a systems approach in past studies gives the rationale for

initiating this reassessment at Level III. Within this context, therefore, a systems model has been developed that is based on relatively simplified physics, but which nevertheless attempts to couple directly the complex interplay between overall reactor performance (i.e., total power, wall loading, power density), coil requirements (i.e., stresses, coil topology, interference, modularity, radiation shielding), and fundamental physics (i.e., density and temperature profiles, number of toroidal and poloidal field periods, rotational transform, shear, magnetic-well characteristics). The primary objective of this Level III study is to quantify by means of approximate but manageable models the key system tradeoffs. An interim design point is presented that is suggested as a starting point for a more detailed Level I study. The major objective of this Level III study, however, is to present quantified results from systems studies to the stellarator/torsatron physics community, in order that the Level I study be launched with a maximized input from all who best understand the intricacies of the stellarator/torsatron approach.

C. Study Approach

The approach adopted for this study involved several parallel efforts. First, magnetics and stress codes were applied to modular-coil parameters suggested by the Princeton Plasma Physics Laboratory (PPPL) group.¹³ Simultaneously, a simplified reactor survey code was developed in order to perform preliminary reactor tradeoff and optimization studies. The reactor survey results are used to identify a potentially attractive interim design for the MSR operating on the D/T/Li fuel cycle; this interim design is then subjected to detailed magnetics, stress, and other analyses. The interim design point will serve as the basis for an elaboration and extension of the Level III MSR study to the higher Level I, should continued study be justified by favorable results.

In applying a parametric systems approach to a concept that is as complex as the stellarator, no unique computational algorithm exists for the interrelated hyperspace that is formed from physics, engineering, and economic variables and constraints. Nevertheless, even for a reactor study that emphasizes physics issues, some indication of engineering (and economic) feasibility is desirable. Hence, although plasma modeling activities lie at the focus of this study, key engineering issues and constraints are addressed at the onset of and in conjunction with this Level III study. Figure I-1

Fig. I-1. Schematic diagram of the Level III MSR study approach, emphasizing key systems variables and interrelationships. Refer to Table I-I for definitions of notation used here and throughout this report.

illustrates schematically this preliminary but integrated approach; schematically this figure and the associated Table I-I also serve to identify and define key system variables that are used throughout this report. The major activities of this study involve interactions between the "plasma modeling/energy balance" and "engineering feasibility" sectors of the diagram in Fig. I-1. It is noted that two kinds of physics input are indicated:

TABLE I-I

DEFINITION OF KEY PARAMETERS USED IN FIG. I-1 AND THIS REACTOR SURVEY STUDY

Parameter	Symbol
Number of poloidal-field periods	ℓ
Number of toroidal-field periods	m
Average rotational transform ($1/2\pi$)	$\bar{q}$
Shear parameter ($d\bar{q}/dr$)	$\bar{q}'$
Magnetic-well (hill) magnitude [$\delta V(r)/V(0)$]	V''
Radial profile index [e.g. $p/p_0 = 1 - (r/r_p)^{\nu}$]	ν
Alpha-particle energy trapping fraction	f_α
Alpha-particle fractional pressure	p_α/p
Average plasma density (m^{-3})	$\langle n \rangle$
Average plasma temperature (keV)	$\langle T \rangle$
Average plasma beta	$\langle \beta \rangle$
Equilibrium beta limit	$\beta(EQ)$
Stability beta limit	$\beta(ST)$
Plasma major toroidal radius (m)	R_T
Average plasma minor radius (m)	r_p
Plasma aspect ratio, R_T/r_p	A
First-wall radius (m)	r_w
Radial filling factor, r_p/r_w	x
Combined blanket and shield thickness (m)	Δb
Blanket neutron energy multiplication	M_N
Maximum minor radius of separatrix (m)	r_s
Minor radius to coil current center (m)	r_c
Helical-field pitch parameter ($\pi r_c / \ell R_T$)	α
Maximum toroidal-coil lateral deformation	d
Coil thickness and width (m)	δ_c
Average coil current density (MA/m ²)	j
Coil current (MA)	I
Number of coils	N
Number of coils per toroidal-field period	N/m
Energy confinement time (s)	τ_E
Coil force (N)	f
Coil stress (Pa)	σ
On-axis magnetic field (T)	B_0
Peak magnetic field at coil (T)	B_c
Total thermal power output (MWt)	P_{TH}
Gross electrical power output (MWe)	P_{ET}
Net electrical power output (MWe)	P_E
Fusion-neutron first-wall loading (MW/m ²)	I_w

"analytic physics" and "numerical physics." This study has relied primarily on the former, with "numerical physics" being employed only to test specific results emerging from the analytical treatment. This study has emphasized the coupling between plasma performance, magnet-design constraints, and total system size. A complete system optimization based on minimizing the cost-of-electricity (COE) is a topic for future study that uses as an object function an extension of the reactor survey study presented herein.

After presenting a general background, interim results, and conclusions as an Executive Summary in Sec. II., the general principles of the stellarator and modular coils are briefly summarized in Sec. III. The detailed description of the systems model and results are given in Sec. IV. Section V. gives preliminary engineering estimates of the magnet design as well as more general reactor considerations. The summary and conclusions of the present study are given in Sec. VI. The basis of all analytic physics and engineering models used in this study can be found in Appendix B, and magnet force calculations and results for a specific coil geometry originally suggested by PPPL¹³ are given in Appendix A. The latter results were generated in the course of initiating the Los Alamos group into the stellarator concept, have little bearing on the MSR design per se, and are presented here only for historical completeness.

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II. EXECUTIVE SUMMARY

This Executive Summary presents the principle and rationale for the Modular Stellarator Reactor (MSR) approach to magnetic fusion energy. After a synoptic presentation of the stellarator and modular-coil principles, basic reactor considerations are summarized and interim results of a parametric reactor survey are given. This MSR study has evolved historically from electromagnetic force calculations of a modular-coil set originally proposed by the Princeton Plasma Physics Laboratory (PPPL) group¹; the results of these early calculations are given in Appendix A. A full development of the simplified physics and magnetics models used in the MSR survey and tradeoff calculations is given in Appendix B. The primary goal of this study is to examine past stellarator reactor approaches and, on the basis of simplified but self-consistent models, lay the groundwork for a more detailed engineering design study of the MSR approach. Directions suggested for future work can be found in the last subsection of this Executive Summary.

A. Stellarator Principles and Modular Coils

The status and history of the stellarator approach to magnetic confinement has been variously reviewed.²⁻⁶ The term "stellarator" is used generically in this study to describe those toroidal devices that produce closed magnetic surfaces by means of external conductors. This class of devices includes the classical stellarator, the torsatron, and the heliotron, all of which are depicted schematically in Fig. II-1. The torsatron and heliotron configurations have ℓ helical conductors with parallel current flow. As shown in Fig. II-1, the heliotron is essentially a torsatron with supplemental toroidal-field (TF) coils. Generally, an additional set of vertical-field windings (not shown) is required to obtain plasma equilibrium in the torsatron and heliotron. The classical stellarator, on the other hand, has ℓ pairs of helical conductors with alternately antiparallel current flow such that the net toroidal field is nulled on the magnetic axis, so a supplemental set of toroidal coils is required. No additional vertical-field windings are needed for the classical stellarator. A modular-coil configuration composed of laterally deformed TF coils can produce a magnetic topology that is essentially equivalent to that of the classical stellarator for minor radii considerably less than the coil radius, r_c . In the absence of a net toroidal current in the modular configuration, a supplemental set of vertical-field windings is not required. The simplest approach is a sinusoidal deformation as suggested by Rehker and Wobig.⁷⁻⁹ Figure II-2

STELLARATOR ($\ell=3$)

TORSATRON ($\ell=3$)

HELIOTRON ($\ell=3$)

Fig. II-1. Stellarator, torsatron, and heliotron coil configurations. For the torsatron and heliotron, additional vertical-field windings (not shown) are required to maintain plasma equilibrium.

Fig. II-2. Coil layout for a typical $l = 2$ MSR configuration. In this case, $m = 8$ with $N/m = 3$ coils per field period. The finite cross-section coils include internal support structure and thermal insulation.

illustrates the coil layout for a typical $\ell = 2$, $m = 8$ MSR configuration composed of $N = 24$ modular coils; $N/m = 3$ coils per field period results. A more detailed discussion of the application of the modular-coil configuration to stellarator confinement is presented in Sec. III. The general coil-configuration approach suggested by Fig. II-2 is adopted by this reactor study.

B. Reactor Considerations

The stellarator represents one of the earliest magnetic confinement concepts to receive attention as a commercial fusion power reactor. Early reactor calculations and cost estimates for stellarator reactors indicated the related potential problems of low power density and high magnet costs. These early survey studies were overshadowed for about a dozen years by discouraging physics results for stellarators and concurrent progress in tokamak confinement.

Consideration of the torsatron concept as a reactor allowed the elimination of the TF-coil set, and, when coupled with new understanding of stellarator and torsatron physics, has generated more recent interest in this truly steady-state device as a reactor. In a torsatron, the desired magnetic geometry can in principle be created by using only relatively "force-free" helical coils.¹⁰ Elimination of the helical coils in favor of toroidal coils that have been subjected to a periodic, lateral distortion (twist) allows the stellarator the promise of greater and more realistic system modularity. Specifically, implementation of the twisted TF-coil set produces the stellarator magnetic geometry while eliminating the helical coil set in favor of a modular device.⁹ In addition, more optimally oriented coil forces and lower stresses are anticipated for this modular stellarator approach (Appendix A).

Qualitative advantages that in general can be invoked for the stellarator/torsatron reactor concept include:

- Steady-state magnetic fields and thermonuclear burn.
- Operation at ignition or with a high Q -value for low recirculating power.
- Plasma startup on existing magnetic surfaces with predictable particle and energy confinement at all times.
- Impurity and ash removal by means of a magnetic limiter and helical poloidal divertor that occur as natural consequences of the magnetic confinement topology.

- Evidence of operation without major plasma disruptions that could lead to an intense, local energy deposition on the first wall or in the blanket, shield, or coil regions.
- No auxiliary positioning or field-shaping coils and moderate plasma aspect ratio (~ 10), both of which ease maintenance access.

These advantages remain to be quantified in the context of a comprehensive reactor study that self-consistently incorporates crucial physics issues (e.g., scaling of beta with aspect ratio and the required or optimal rotational transform, magnetic shear, and magnetic-well depth), engineering constraints (e.g., coil design and stresses), and economics (e.g., capital cost and power cost). The key limiting disadvantage for the stellarator/torsatron reactor concept identified to date is the marginally acceptable value of beta allowed by equilibrium and stability limits as presently understood and applied in this study (Sec. B.4. of Appendix B).

C. Summary Results

This preliminary scoping study of the MSR attempts to quantify the potential contribution of this innovative coil concept in the context of a classical and advantageous plasma confinement scheme with considerable merit for fusion reactor applications, where appropriate performance goals and constraints have been imposed. A broad survey of physics parameter tradeoffs has focussed onto the identification of a self-consistent interim design point for ignited reactor operation on the D/T/Li fuel cycle.

Figures II-3 and II-4 summarize typical results from the reactor survey calculations. Table I-I of Sec. I defines all notation used herein. Curves of the parameter $\langle \beta \rangle^{1/2} B_0$, where $\langle \beta \rangle$ is the average beta and B_0 is the on-axis magnetic-field strength, required for ignition are plotted as a function of the average plasma temperature, $\langle T \rangle$. In Fig. II-3 the first-wall fusion-neutron loading is fixed at $I_w = 1.5 \text{ MW/m}^2$, and the total thermal power, P_{TH} , is varied between 2-6 GWt. The total thermal power is fixed at $P_{TH} = 4 \text{ GWt}$ in Fig. II-4, and the first-wall fusion-neutron loading is varied between 1-4 MW/m^2 . In both Figs. II-3 and II-4 the MSR is taken to be an $\ell = 2$, $m = 8$ system composed of $N = 24$ modular coils with a plasma aspect ratio, $A = R_T/r_p = 11$. The rationale for this choice of stellarator parameters is based on a desire to maximize beta for a moderate aspect ratio, as indicated by simplified equilibrium and stability theories; the details of this analysis are given in Sec. IV.B.2. This study considered $\ell = 2$ and $\ell = 3$ stellarator configurations. An $\ell = 2$ configuration was found to yield

Fig. II-3. Dependence of the parameter, $\langle \beta \rangle^{1/2} B_0$, where $\langle \beta \rangle$ is the average beta and B_0 is the on-axis magnetic field strength, required for MSR ignition as a function of $\langle T \rangle$ for the fixed fusion-neutron wall loading, $I_w = 1.5 \text{ MW/m}^2$, for various fixed values of total output thermal power in the range $P_{TH} = 2-6 \text{ GWt}$. Alcator transport scaling is assumed.

generally higher values of beta than an $l = 3$ configuration on the basis of these theories for moderate aspect ratios. In addition, positioning of the maximum separatrix radius, r_s , near the coil radius, r_c , in order to accommodate pumped-limiter impurity control allows a higher plasma filling fraction within the first-wall radius, r_w . As indicated on Figs. II-3 and II-4, these parametric design curves are based on Alcator (empirical) transport scaling.¹⁰ If the parameter $\langle \beta \rangle^{1/2} B_0$ is essentially constant, as it is for average plasma temperatures in the vicinity of 8 keV, a higher allowed value for B_0 can compensate a lower value of $\langle \beta \rangle$ to give nearly constant overall reactor performance. A MSR with higher aspect ratio than allowed in an otherwise comparable tokamak reactor, therefore, can tolerate and remain competitive with higher values of B_0 and correspondingly lower values of $\langle \beta \rangle$,

Fig. II-4. Dependence of the parameter, $\langle \beta \rangle^{1/2} B_0$, required for MSR ignition as a function of $\langle T \rangle$ for the fixed total thermal power, $P_{TH} = 4$ GWt, for various fixed fusion-neutron first-wall loadings in the range 1-4 MW/m². Alcator transport scaling is assumed.

for a commonly imposed limit on the maximum magnetic field strength on the inboard side of the TF coils.

Selection of a plasma profile index, $\nu = 3$, gives a cubic variation of pressure with plasma minor radius. The separatrix radius, r_s , is located near the mean coil radius, r_c , such that the plasma is considered to be bounded by a limiter at the first-wall radius rather than by a separatrix surface. The rationale for this choice of separatrix surface is also given in Sec. IV.B.2. and is based on the desire to maximize the plasma filling fraction within the first-wall radius, r_w . In addition, the plasma edge is separated from any ergodicity in the magnetic field structure that is expected to be most severe near the separatrix. For a maximum allowable value of average beta, $\langle \beta \rangle \sim 0.04$, that is limited by equilibrium and stability constraints (Sec. IV.B.2.), the required on-axis confining magnetic field strength is

$B_0 \approx 6$ T. This value of B_0 is consistent with a maximum field at the coil, $B_c \approx 11$ T, for $A = 11$.

In summary, the selection of these key stellarator parameters (i.e., $\ell = 2$, $m = 8$, $N = 24$, $A = 11$, $r_s/r_c \approx 1$) is based on the desire to achieve a maximum beta value and an optimum plasma filling fraction within the first wall. It is emphasized that simplified stellarator expansions have been used in conjunction with the assumption that diffusion-driven currents are responsible for both equilibrium and stability limits (Sec. B.4. of Appendix B); these approximate models must be re-examined by more detailed numerical magnetics calculations during this and subsequent phases of this study. Preliminary comparisons between the analytic expansions and the numerical code results suggest that the interim coil configuration adopted for the CSR will require some modification (i.e., slightly greater first-harmonic distortion, additional higher-harmonic components, and a higher value of N/m leading to lower m for fixed $N = 24$) in order to achieve $\beta \sim 0.66$, as required to reach $\langle \beta \rangle = 0.04$ for $A = 11$. Alternatively, the simplified theory used to arrive at the relationships between $\langle \beta \rangle$, A , ℓ , and m may have to be abandoned (along with analytic "self-consistency") in order to allow the use of a lower- m , higher- N coil set to achieve a desired value of rotational transform while assuming a stable beta of $\gtrsim 0.04$. The development of more realistic stability and equilibrium limits on $\langle \beta \rangle$, however, remains a topic of future work.

Transport scaling also represents an uncertainty for this design study. It is noted that the particular scaling of I_w , P_{TH} , and r_p with the ignition condition, $\langle \beta \rangle$, and B_0 shown in Figs. II-3 and II-4 is a result of the assumed Alcator transport scaling. A range of other transport scaling options is examined in Sec. IV.A.2. and Sec. B.2. of Appendix B, including a stellarator-modified Alcator scaling, a Bohm-like scaling, and a neoclassical stellarator scaling. It is shown in Sec. B.2. that within relatively wide margins of uncertainty, all transport scalings considered give similar results, although the position of the design survey curves of the kind shown in Figs. II-3 and II-4 would vary. In order to narrow the parameter space in which this study was conducted, therefore, an Alcator transport scaling was adopted as a benchmark, although, as with the magnetics model, this assumption must also be examined by more comprehensive transport codes in subsequent phases of this study.

TABLE II-I
INTERIM MSR DESIGN POINT^(a)

<u>Parameter</u>	<u>Value</u>
Number of poloidal-field periods, ℓ	2
Number of toroidal-field periods, m	8
Average rotational transform, $\tau^{(b)}$	~ 0.66
First-wall neutron loading, I_w (MW/m ²)	1.5
Thermal power output, P_{TH} (GWt)	4.0
Plasma power density, p_p (MWt/m ³)	2.5
System power density, p_s (MWt/m ³) ^(c)	0.33
Major toroidal radius, R_T (m)	21.35
First-wall radius, r_w (m)	2.35
Plasma aspect ratio, $A = R_T/r_p$	11.0
Average plasma radius, r_p (m)	1.94
Plasma volume, V_p (m ³)	1586
Radial profile index, ν	3
Average plasma temperature, $\langle T \rangle$ (keV)	8.0
Average plasma density, $\langle n_i \rangle$ (10 ²⁰ /m ³)	1.72
Average beta, $\langle \beta \rangle$	0.04
Lawson parameter, $\langle n_e \rangle \tau_E$ (10 ²⁰ s/m ³)	4.12
On-axis magnetic field, B_0 (T)	6.37
Peak coil magnetic field, B_c (T)	~ 11.3
Mean coil current center radius, r_c (m)	4.6
Coil current density, j (MA/m ²)	12.5
Maximum coil lateral distortion, $d/r_c^{(b)}$	0.29
Coil current, I (MA)	28.5
Coil thickness, δ_c (m)	1.51
Coil volume, V_c (m ³ /coil)	70.
Stored magnetic energy, E_M (GJ)	$\sim 200.$

(a) Alcator transport scaling, $r_s \sim r_c$ implying pumped-limiter impurity control, $f_\alpha = 0.88$, $M_N = 1.1$, $\Delta b = 1.5m$, $N = 24$, $Z_{eff} = 1.1$, $x = r_p/r_w = 0.83$, and alpha-particle pressure contribution, $p_\alpha/p = 0.25$.

(b) As obtained from the conventional stellarator expansion [Eq. (B-52) of Appendix B]. Numerical results for the interim coil configuration predict lower values ($\tau \sim 0.15$), which will require improvement in subsequent design iterations if the required value of $\langle \beta \rangle = 0.04$ is to be achieved under the limits imposed in this study.

(c) Based on volume enclosed by and including the coils.

The reactor parameters developed for this interim design point are summarized in Table II-I. On the basis of generally conservative assumptions, and without the benefit of an economic analysis, this design point appears both viable and competitive with other approaches to magnetic fusion power. This prognosis is made on the basis of power density and technological requirements, however, and must ultimately be verified by a detailed design and associated cost estimate.

As stated previously, a major goal of this scoping study was to relate the results of simple plasma and magnetics calculations to the engineering requirements of the modular coils. Figure II-2 gives a schematic view of a sector of the modular-coil set that would provide the magnetic-field configuration of the design point given in Table II-I.

Results from three-dimensional magnetics computations (Sec. V.) for this design point, using the EFFI code¹¹, are summarized in Fig. II-5. In the coordinate system of Fig. II-2, the azimuthal angle $\theta = 0(\pi)$ represents the top (bottom) of the coil and $\theta = \pi/2$ ($3\pi/2$) corresponds to the outboard (inboard) sectors of the coil. The dominant mean force component is directed radially outward and can be supported externally. The lateral force component acts to increase the lateral deformation of the modular coil. There exists a net centering force which can easily be supported by structure in the hole of the torus. The corresponding mean stress is estimated analytically, pending full implementation of the SAP4 stress code.¹² The stress equals or is below a nominal stress limit of 200 MPa (~ 30 kpsi), corresponding to 0.1% strain in a stainless-steel internal coil support. Consequently, the modular-coil system proposed for this interim design point appears to satisfy basic mechanical and stress design criteria while meeting approximate constraints for modularity, accessibility, maintainability, and manufacturability for a coil set that can be assembled and operated at a conservative overall current density ($10\text{--}15 \text{ MA/m}^2$).

Preliminary calculations of magnetic surfaces and rotational transform profiles also have been made. Calculation of representative magnetic surfaces have been computed for the interim MSR design point. Preliminary results of this calculation are summarized in Fig. II-6. The magnetic surfaces have the quasi-elliptical cross section expected for the $\ell = 2$ stellarator and remain closed for radii approaching the coil radius, r_c , because the stagnation-point or maximum-separatrix radius, r_s , is close to r_c . The plasma boundary in this

Fig. II-5. Force and stress summary for a typical coil of the MSR interim design point of Table II-I.

Fig. II-6. Representative magnetic surfaces for the $\ell = 2$ interim MSR design point.

case would be defined by a limiter near the first-wall radius, r_w . The values of the rotational transform corresponding to these magnetic surfaces are computed in order to establish the radial profile of ι . As shown in Fig. II-7, the $\ell = 2$ modular-coil system produces a low-shear transform profile.

As noted previously, the interim MSR design point with $m = 8$ and $N/m = 3$ yields an insufficient value of rotational transform ($\iota \sim 0.15$), as calculated numerically or as obtained from the helix/antihelix model of Eqs. (B-46B), (B-49), and (B-51) of Appendix B, for $d/r_c \approx 0.3$ and a first-harmonic coil configuration when compared with the value $\iota \sim 0.66$ obtained from the conventional stellarator expansion [Eq. (B-52) of Appendix B]. This result has been confirmed by EFFI magnetics computations at Los Alamos and by separate calculations at PPPL¹³, also plotted in Fig. II-7. Improvement of the achievable transform, as dictated by the simplified equilibrium and stability imposed, can be expected from several optional modifications to the basic interim design point. The available options include:

- Small increases in the first-harmonic lateral coil distortion, ϵ_c , with a more carefully formulated coil-interference constraint than that developed in Sec. B.5. of Appendix B.
- Application of a higher harmonic lateral coil distortion with use of the same coil-interference constraint.
- Modest increases in the lateral coil distortion for the same values of m and N with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
- A higher value of N for the same value of m to increase N/m with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
- A lower value of m for the same value of N and lateral coil distortion to increase N/m at the expense of modifications in r_p/r_w and possibly a different optimal value of aspect ratio.
- Use of coils with noncircular bores (i.e., elliptical bore for $\ell = 2$ and trefoil bore for $\ell = 3$).
- Combinations of the above options.

Discussion of the implementation of these options is presented in Sec. B.8. of Appendix B. Implementation of these modifications must be made self-consistently with the imposed beta-limit scaling and the coil-interference and current-density constraints and are deferred to a subsequent phase of this study. Resolution of the discrepancy between desired and achieved values of β is the subject of ongoing magnetics computations. It is anticipated that the basic size and power output of the interim MSR design point will be preserved in the course of this work. The approach adopted by this scoping study is to retain the relatively optimal coil geometry indicated in Fig. II-2 for the MSR engineering design under the assumption that more favorable rotational-transform and magnetic-well properties can be found for coil configurations that can be fitted into the nominal geometry indicated in Fig. II-2. Simultaneously, the simplified equilibrium/stability theories used herein to specify the rotational transform will be re-examined with the goal of reducing the required rotational-transform values. The alternative approach would examine more complex plasma/coil configurations, an approach which would be adopted only if more extensive studies eliminate the simpler configurations examined thus far.

Fig. II-7. Rotational-transform profiles using Eq. (B-51) for the $\ell = 2$ MSR design point. The $d/r_c = 0.3$, $m = 8$ interim design point yields $\bar{\alpha} \sim 0.15$ (solid curve), as confirmed by EFFI (circles) and PPPL codes (squares and triangles). The required value, $\bar{\alpha} \sim 0.66$, is expected to be achievable by lowering m and by introducing a higher-harmonic coil distortion to give equivalent performance to that with $d/r_c \sim 0.5$, which corresponds to the analytic stellarator expansion of Eq. (B-46B) (dashed curve).

D. Conclusions

The following major conclusions have emerged from this Level III scoping study of the MSR.

- Marginally attractive values of average beta, as allowed by approximate and self-consistently applied equilibrium and stability limits, are a key limiting factor in MSR performance. Higher beta values would lead to lower confining fields, reduced coil forces and stresses, and smaller systems with higher power density, although even for $\langle \beta \rangle = 0.04$ the forces and fields at the coils do not appear unduly large for moderate aspect ratio systems.
- Application of conservative assumptions and constraints (e.g., $f_\alpha < 1$, $p_\alpha/p > 0$, $j = 12.5 \text{ MA/m}^2$) still allows the identification of potentially attractive MSR design points with moderate power output ($P_{TH} < 5 \text{ GWt}$) while self-consistently meeting key stellarator physics constraints in modular engineering configurations with maintenance and reliability advantages.

- Preliminary magnetics and coil stress computations indicate MSR systems with manageable structural requirements and accessibility. The conventional stellarator expansion should be used with caution for the modular-coil configuration, however, as it tends to overestimate the transform (Fig. II-7).
- A pumped-limiter impurity control scheme may improve MSR performance over that with the magnetic divertor that is traditionally associated with the stellarator/torsatron configuration.
- The MSR survey study is based on the applicability of Alcator (empirical) transport scaling, which was shown to give an energy confinement time that is a factor of ~ 60 greater than Bohm transport, a factor of ~ 2 greater than neoclassical plateau scaling, and a factor of ~ 10 less than classical transport. The level of energy loss predicted by Alcator scaling is viewed as an upper bound for MSR system viability.
- Adequate justification exists for continued study of the MSR configuration at a higher level of conceptual engineering design, in addition to continued parametric study using improved physics models in order to develop a better resolved and perhaps an even more attractive reactor design point. Detailed engineering design should proceed, however, only after the relationship between equilibrium/stability, beta, rotational transform, and coil geometry are better resolved.

On the basis of results generated by this preliminary scoping study, it is recommended that investigation of the Modular Stellarator Reactor configuration be continued at a higher level (Level I) pending a clear resolution of the $\langle \beta \rangle$, α , d/r_c , ℓ , m , and N coupling. Such a study can focus on those issues that were beyond the scope of this preliminary (Level III) analysis to refine and optimize further the MSR concept.

Specifically, the future direction of study of the MSR should emphasize a better resolution of the following issues.

- Refine and extend physics models:
  - beta scaling
  - transport scaling
  - confinement and beta of suprathermal alpha particles
  - startup and burn control.
- Exercise an extended survey model for refinement of the interim design point on the basis of cost.
- Establish a self-consistent internal coil design and external support structure as verified using appropriate magnetics (EFFI) and stress (SAP4) codes.

- Continue core component design emphasizing modularity and maintainability (i.e., first wall, blanket, shield, impurity control, heating, and fueling).
- Address in detail other plasma engineering issues (e.g., alpha-particle trapping, impurity-control physics).
- Systems integration and costing:
  - layout of nuclear island
  - design and support of realistic coils
  - better resolve coupled interaction of coil configuration, blanket and shield design, impurity-control requirements, and total system modularity and accessibility
  - quantify balance-of-plant (BOP) interfaces and layout
  - perform cost optimizations and identify key cost drivers.

For optimum use of limited resources, further reactor design work on the MSR should emphasize those features that follow directly from the concept itself rather than the development of design details that may be more generic in nature. Hence, the future Level I study should present a full, conceptual analysis and design of the reactor systems, with BOP issues being treated only at a level of detail required to give a reasonable estimate of plant capital cost and the cost-of-electricity (COE). The COE should, however, represent the primary "object function" used in arriving at an optimized MSR system.

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III. STELLARATOR AND MODULAR COILS

A. Stellarator Principles

The status and history of the stellarator approach to magnetic confinement has been variously reviewed.¹⁻⁴ The term "stellarator" is used generically in this study to describe those toroidal devices that produce closed magnetic surfaces by means of external conductors. This class of devices includes the classical stellarator, the torsatron, and the heliotron, all of which are depicted schematically in Fig. III-1.

A plasma cannot be stably confined in a pure toroidal magnetic field of finite aspect ratio because a $\vec{B} \times \vec{\nabla} B$ drift force acts on the plasma ions and electrons, leading to a spatial charge separation and a resultant electric field; the $\vec{E} \times \vec{B}$ drift causes loss of both ions and electrons from the toroidal plasma column. If, however, using additional helical field components, the magnetic field lines can be made to execute excursions around the minor circumference of the torus as a trajectory is traced around the major circumference of the torus, particles moving along the magnetic field lines will tend to experience alternating $\vec{B} \times \vec{\nabla} B$ drifts, which ideally average to zero such that charge separation and toroidal drifts do not occur. The locus of intersections of a given magnetic field line with a plane cutting the minor diameter of the torus will trace out in an average sense a closed magnetic surface. The average incremental poloidal angle, ι , between two successive intersections, normalized by the one-turn toroidal angle, 2π , is defined to be the rotational transform angle, $\iota = 1/2\pi$. The average rotational transform is a function of minor radius, r , and the geometry of the external current windings that produce the toroidal and the helical magnetic field components. The magnetic shear is given by the first derivative of the rotational transform, $d\iota/dr$. A given magnetic field line will close upon itself after traversing around the torus n times if $\iota = 1/n$. A shearless rotational transform may be induced by torsion of the magnetic axis into the nonplanar "figure-8" configuration used in early stellarator experiments.¹

The magnetic field strength near the magnetic axis is approximately given by

$$B \sim B_0 [1 - \epsilon_t \sin\theta - \epsilon_h \sin(\ell\theta - m\phi)] \quad , \quad (\text{III-1})$$

STELLARATOR ($\ell=3$)

TORSATRON ($\ell=3$)

HELIOTRON ($\ell=3$)

Fig. III-1. Stellarator, torsatron, and heliotron coil configurations. For the torsatron and heliotron, additional vertical-field windings (not shown) are required to maintain plasma equilibrium.

where B_0 is the on-axis magnetic field strength, ϕ is the toroidal angle, θ is the poloidal angle, and the perturbations in the field strength caused by the toroidal effect and the helical field component are given, respectively, by ε_t and ε_h . The torsatron configuration has ℓ helical conductors with unidirectional current flow. A separate set of vertical-field windings is required. The classical stellarator, on the other hand, has ℓ pairs of conductors with alternately antiparallel current flow such that the net toroidal field is nulled on the magnetic axis, and a separate set of toroidal coils is required. The pitch length, $1/\alpha = \ell R_T/m$, of the helical coils is defined by the pitch angle, α' , such that

$$\tan \alpha' = \frac{m r_c}{\ell R_T} \equiv \alpha r_c, \quad (\text{III-2})$$

where m is the number of field periods around the major circumference of the torus, r_c is the minor radius of the helical coil, and R_T is the major toroidal radius of the device. On the basis of simplified expansions for straight systems (i.e., infinite aspect ratio) and certain simplified winding laws (i.e., the relationship between θ and ϕ as a conductor is traced around the torus), analytic expressions can be derived that relate $\psi(r)$ to other quantities of interest. Those quantities in turn can be related to plasma size and shape and to the magnet coil parameters. These relationships are discussed in Appendix B and serve as the analytic magnetics basis of this reactor survey. Not shown for the schematic torsatron on Fig. III-1 is the set of auxiliary vertical-field coils that would normally be required to balance the net toroidal current and to establish plasma equilibrium. In a torsatron reactor⁵ such a coil set can add a significant cost component and may complicate the mechanical design. However, by properly modulating the helical pitch of the torsatron windings, the so-called "ultimate" torsatron configuration⁶ results in which the required magnetic topology can be produced without the addition of a separate vertical-field coil set. The "ultimate" torsatron is, therefore, a superior implementation of the helical coil configuration from the viewpoint that it eliminates the need for vertical-field windings.

B. Modular Coils

Under the assumption that the helical and toroidal coil windings of the classical stellarator (Fig. III-1) can be ideally represented by filamentary conductors located at a common coil radius, r_c , the coil set giving rise to the stellarator field configuration can be mapped into the θ - ϕ plane, as shown in Fig. III-2. If the number of toroidal field coils is chosen to assure that the current in each coil is equal to the current in the helical coils, an equivalent magnetic topology may be obtained by breaking and reconnecting the coil set, as shown in Fig. III-3. In this figure, a gap has been introduced between the modular coils. This conductor arrangement yields a modular coil configuration composed of circular-bore toroidal-field (TF) coils that have been subjected to a periodic lateral deformation; a modular-coil configuration results that is composed only of deformed TF coils without the encumbrances of poloidal or helical windings. Its rudimentary form consists only of a simple, sinusoidal deformation, corresponding to the modular coil as suggested by Rehker and Wobig.⁷⁻⁹ In that simple case the coil winding law is depicted in Fig. III-4 and is given by

$$y = d \sin \ell \theta , \quad (\text{III-3})$$

where y is the lateral distortion of a planar ($y = 0$) TF coil with a circular bore. Superposition of a smaller, higher-harmonic distortion has been proposed¹⁰ in order to improve the helical symmetry of the configuration and to obtain higher rotational transform with less distortion required in the fundamental mode. A typical corresponding winding law¹¹ is illustrated in Fig. III-5 and is given by

$$y = d' \sin 2\ell \theta , \quad (\text{III-4})$$

which is applied selectively in poloidal angle only where the winding crosses the coil midplane ($y = 0$). Other extensions and elaborations are, of course, possible.

Fig. III-2. Classical stellarator windings, assumed to lie, ideally, at a common radius.

Fig. III-3. Essentially equivalent modular-coil windings corresponding to the above stellarator configuration.

Fig. III-4. First-harmonic Rehker-Wobig winding with $r_c = 4.6$ m and $d/r_c = 0.29$, corresponding to the interim MSR design point.

Fig. III-5. Higher-harmonic winding (solid line) compared with the first-harmonic winding (dashed line) of Fig. III-4.

It is convenient to normalize the degree of maximum lateral deformation for a coil of radius r_c by the ratios d/r_c and d'/r_c . For a finite coil with square cross section of dimension, δ_c , and mean radius, r_c , Figs. III-6A and III-6B illustrate the geometry of $\ell = 2$ and $\ell = 3$ modular stellarator coils, respectively. The corresponding magnetic surfaces and average rotational transform profiles are illustrated² qualitatively in Fig. III-7 for both the classical stellarator and the analogous torsatron. Within the separatrix that bounds the last closed magnetic surface are shown the quasi-elliptical magnetic surfaces for $\ell = 2$ and the trefoil surfaces for $\ell = 3$. For vacuum-field configurations, the $\ell = 2$ stellarator has a finite rotational transform on axis, and the $\ell = 3$ (or higher- ℓ) system has zero rotational transform on axis (Sec. B.3. of Appendix B). The modular stellarator configuration can be expected to have a low-shear (positive) nonzero transform near the magnetic axis that drops off sharply for $r \sim r_s$.

For purposes of the analytic analysis of the magnetics of the modular-coil configuration, it is possible to represent¹¹ approximately the Rehker-Wobig coil configuration by the superposition of planar TF coils and two sets of stellarator helical windings with different pitch angles, α' and β' , and with antiparallel current flow giving opposite contributions to the net rotational transform. The pitch angles can be expressed¹¹ in the form of Eq. (III-2) as

$$\tan \alpha' = \frac{mr_c}{\ell R_T} \quad (\text{III-5A})$$

$$\tan \beta' = \frac{mr_c}{\ell R_T} \left[\frac{N}{m} - 1 \right], \quad (\text{III-5B})$$

where N is the number of modular coils. The α' -angle and β' -angle windings designate,¹¹ respectively, the helical lines and the antihelix, as illustrated in Fig. III-8. The helical lines connect the nodes of neighboring modular coils and represent the dominant components of the helical current. The pitch angle of the antihelix corresponds to a set of windings with toroidal-field period number $m' = m(N/m-1)$. Because of the higher toroidal pitch number (i.e., steeper pitch), the antihelix contributes relatively little to the opposing transform near the magnetic axis, but tends to flatten significantly or even

(A) $\ell=2$ REHKER-WOBIG COIL

(B) $\ell=3$ REHKER-WOBIG COIL

Fig. III-6. Views of the $\ell = 2$ and $\ell = 3$ Rehker-Wobig modular coils with circular bore.⁹

Fig. III-7. Schematic vacuum-magnetic surfaces and corresponding radial profiles of rotational transform, τ , in stellarators and torsatrons with $l = 2$ and $l = 3$.

Fig. III-8. Decomposition of the first-harmonic Rehker-Wobig coil configuration into the superposition of two sets of stellarator helical windings with different pitch angles α' and β' and antiparallel current flow.

reverse the transform profile at the plasma edge. The fewer the number of modular coils, N , the stronger is the opposing transform contributed by the antihelix. The lowest value of N/m that gives nonzero rotational transform is $N/m = 3$, because, for $N/m = 2$, $\beta' = \alpha'$ and there is no net transform for a sinusoidally deformed coil configuration.¹¹ The superposition of the helix and antihelix allows an approximate representation of the modular-coil rotational transform as the net contribution of two opposing stellarator expansions, as given by

$$\tau(r) = \tau(r, m) - \tau(r, m') \quad , \quad (\text{III-6})$$

where each term on the right-hand side is represented by Eqs. (B-46B) and (B-49) of Appendix B. This procedure, as adopted by this reactor survey study, is discussed in greater detail in Sec. B.3. of Appendix B.

Figure III-9 illustrates the coil layout for an $\ell = 2$, $m = 8$ MSR configuration made up of $N = 24$ modular coils, with $N/m = 3$ coils per field period. All of the geometric parameters developed in this subsection are indicated on Fig. III-9. As noted previously, $N/m = 3$ is the minimum number of coils per toroidal-field period that can be expected to give nonzero transform. The finite cross-sectional area of the coil includes allowance for internal structural support and thermal insulation consistent with an overall current density $j = 12.5 \text{ MA/m}^2$. In addition, the coil set must satisfy the coil interference condition

$$\frac{\delta_c + 2d}{2(R_T - r_c - \delta_c/2)\tan(\pi/N)} < 1, \quad (\text{III-7})$$

where δ_c is the thickness of the square cross-section coil. This condition is developed in Sec. B.5. of Appendix B. As will be seen, imposition of these constraints, applied in conjunction with the requirements of space for blanket/shield structure, gives a severe limitation on the maximum rotational transform achievable for a torus of limited aspect ratio (i.e., limited total fusion power).

The present study focuses on the circular-bore, modular-coil configuration. It is possible to envisage an elliptical-bore coil for the $\ell = 2$ case and a trefoil-bore coil for the $\ell = 3$ case to improve the utilization of magnetic volume and to obtain higher rotational transform for a given lateral coil distortion. Such coil modifications imply correspondingly noncircular cross sections for the first wall, blanket, and shield components. Although not foreclosing the option for improving MSR performance by using these modifications, the circular-bore coil is used in the present study as a first approach. Generally, this study has maintained the circular-bore option in order to avoid the use of more complex, shaped engineering configurations, although this constraint may have to be relaxed if future study shows that adequate plasma filling fractions or rotational-transform values with acceptably low coil deformations cannot be achieved.

Fig. III-9. Coil layout for a typical $l = 2$ MSR configuration. In this case, $m = 8$ with $N/m = 3$ coils per field period. The finite cross-section coils include internal support structure and thermal insulation. The scale used in this diagram ($R_T = 21.35$ m, $r_c = 4.60$) is appropriate for a combined blanket/shield thickness of $\Delta b = 1.5$ m, surrounding an $r_p = 1.94$ -m plasma and protecting a superconducting magnet supporting an average current density of $j = 12.5$ MA/m², consistent with the design point of Table II-I.

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IV. STELLARATOR/TORSATRON REACTOR SURVEY

The stellarator confinement scheme generally in the past has been synonymous with complex and, therefore, unattractive coil design. As noted in Sec. II.B., early stellarator reactor designs concluded that the coupled problems of high coil cost and low system power density (i.e., low beta) were severe for the classical stellarator. Although the more recent torsatron and modular-coil configurations show strong promise in alleviating the coil problem per se, efforts to obtain reliable beta-scaling relationships have begun only recently. For these reasons, a measure of coil performance has been retained as a major element in the present reactor survey and systems analysis. Figure IV-1 illustrates schematically the coupling between plasma, reactor, and coil performance. By means of simplified analytic models

Fig. IV-1. Schematic diagram illustrating the coupled performance of plasma, coil, and reactor. The systems model described in this section attempts to quantify this coupling, although economic considerations have not yet been included.

(Appendix B), this coupling has been implemented through a systems approach that is described and evaluated below. The dominance of coil electrical and mechanical considerations, in fact, has from the beginning characterized this Level III study. Incorporation of rudimentary plasma engineering constraints into this preliminary calculation of coil stresses led to the systems model upon which this study is based. Economic and costing constraints, however, have not yet been incorporated into this Modular Stellarator Reactor (MSR) survey study, but will be included in subsequent studies.

A. Past Studies and Design Bases

This subsection summarizes past conceptual designs¹⁻⁹ of stellarator/torsatron reactors. In order to define the context of the present study, it is instructive to consider the route by which fundamental assumptions are manifested as specific reactor design points, and to review the historical evolution of understanding regarding that which constitutes an acceptable and attractive design point. Table IV-I summarizes most of the proposed operating characteristics and device parameters available from the literature for six of the most thoroughly elaborated stellarator/torsatron conceptual design points.

The pioneering Princeton Model-D design¹ represents one of the most comprehensive extrapolations of a confinement concept to the reactor stage. Experimental and theoretical work during the 1950s, however, established that several of the physics and technology assumptions upon which the Model-D design was based were untenable (e.g., average beta $\langle\beta\rangle = 0.75$, "figure-8" topology, and copper coils).^{7,8} The Model-D design, however, set a precedent for stellarator-reactor studies that was prominent until the publication of the T-1 torsatron design⁶: the tendency for stellarators to produce unattractively large amounts of thermal power (i.e., >5 Gwt, in the contemporary view).

Attempts to impose equilibrium and stability limits on the plasma beta led to survey calculations made by the Culham group^{2,3} (Table IV-I). The assumed scaling of beta with plasma aspect ratio (equilibrium limit) in toroidal geometry, when coupled with the conservative transport scaling assumed, resulted in systems with high peak-magnetic-field strengths in the inboard region of the coils.

TABLE IV-I
SUMMARY OF PAST CONCEPTUAL STELLARATOR REACTOR DESIGN POINTS

Source	Spitzer, et al.	Gibson	Gibson, et al.	Iiyoshi and Uo	Georgievskii, et al.	Politzer, et al.
Date	1954	1969	1971	1974	1976	1978
Reference	1	2	3	4	5	6
Configuration	Figure-8	Stellarator	Stellarator	Heliotron	Torsatron	Torsatron
l		3	3	2	3	3
Transport scaling		$21 \tau_{E(BOHM)}^{(b)}$	$68 \tau_{E(BOHM)}^{(b)}$	$75 \tau_{E(BOHM)}^{(b)}$	Plateau	Alcator
First-wall loading, I_w (MW/m ²)	27.8	10	4.8	2.0	6.0	1.25
Thermal power output, P_{TH} (MWt)	17,300	3200	5830	8500	13,750	4340
Gross electric power, P_{GT} (MWe)	6000		2500	3700	5500	1520
Thermal conversion efficiency, η_{TH}	0.35		0.43	0.44	0.40	0.35
Plasma power density, P_D (MWt/m ³)	90.0	7.57	8.27	1.96	8.26	1.42
Major radius, R_T (m)	7.27(a)	7.0	10.9	28.0	37.5	29.2
First-wall radius, r_w (m)	0.66	2.19	2.26			~ 2.3
Blanket/shield thickness, Δb (m)	0.60	1.0	1.5	(c)	1.5	1.25
Blanket multi- plication, M_N	1.31			1.34	1.17	1.17
Plasma aspect ratio, $A = R_T/r_p$	NA	4.0	6.0	10.0	25.0	12.7
Plasma radius, r_p (m)	0.61	1.75	1.81	2.8	1.5	2.3
Plasma volume, V_p (m ³)	192	423	705	4333	1665	3049
Radial profile index, ν	flat	2	2	flat	flat	3
Average temperature, $\langle T \rangle$ (keV)	10	20	20	20	7.7	7.3
Average density, $\langle n \rangle$ (10 ²⁰ /m ³)	19.5	1.65	1.32	0.75	4.2	1.33
Average beta, $\langle \beta \rangle$	0.75	0.03	0.06	0.15	0.16	0.035
Lawson parameter, $\langle n \rangle \tau_E$ (10 ²⁰ s/m ³)	6.5	~ 1.0	~ 1.7	1.0	6.0	3.0
On-axis magnetic field, B_0 (T)	~ 5.0	10.0	6.33	2.9	4.0	5.0
Peak magnetic field, B_C (T)		~20	20	8.7	8.0	8.7
Coil radius, r_C (m)		4.75	4.51	4.0	3.9	4.0
Coil current, I (MA)			44	20	43.2	36.5
Number of turns, N			8	20	17	20

(a) Radius of end sections, $R_T = 7.27$ m (circumferential length, $L = 165$ m). All other systems are tori with major radius, R_T .

(b) $\tau_{E(BOHM)} \equiv 62.5 \langle T \rangle / B_0$.

(c) The blanket is not interposed between the plasma and the coils in this device.

The final three reactor design points listed on Table IV-I invoke the torsatron coil configuration. The Heliotron-C design⁴ is based upon a large beta value, which in a system of large major radius also leads to large power output. This situation also exists for the Kharkhov design point⁵, summarized on Table IV-I.

The T-1 torsatron design point⁶ reflects a recent attempt to reduce the total power output to ~ 4 Gwt under the constraint of a more conservative beta limit. The assumed Alcator (empirical) transport scaling allows moderate on-axis field strength, and the large aspect ratio reduces the corresponding peak field on the inboard region of the helical torsatron winding. Because of the monolithic, helical coils required of this $\ell = 3$ torsatron reactor, modularity could be achieved only by proposing demountable superconducting breaks within the coil structure and "modules" that weigh on the order of 1000 tonnes. The otherwise favorable performance of the T-1 reactor design was used as a benchmark for the present study, with an emphasis being placed on a more realistic definition of modular reactor construction.

B. Systems Model

1. Overview. The systems model that has been developed to quantify the relationship between plasma, coil, and reactor performance is depicted in Fig. IV-2. The analytic models used to evaluate a wide range of reactor parameters are described in Appendix B. With the exception of the coil analysis per se, more complex models needed to describe vacuum fields, rotational transform, hot-ion thermalization, and separatrix surfaces were deferred in this Level III study in favor of a simplified and more global approach. The algorithm described in Fig. IV-2 has been determined primarily by the analytic form of stellarator scaling and the desire to highlight the constraints imposed by the coil design. The use of more exact computer models, however, has been implemented to a limited extent in preparation for more detailed, follow-up studies. These numerical models have been used to monitor the validity of the results reported herein (Sec. V.).

The secondary variables identified in Fig. IV-2 are held fixed throughout the parameter search, whereas the primary variables have been subjected to extensive variation. Because of the highly nonlinear response of system parameters to plasma temperature, the scan through parameter space for a given set of primary and secondary variables used average plasma temperature, $\langle T \rangle$ (keV), as the "marching" variable. Generally, the fusion-neutron wall

PRIMARY VARIABLES:

- THERMAL POWER, P_{TH}
- WALL LOADING, I_w

SECONDARY VARIABLES:

- TRANSPORT SCALING
  - ALCATOR
  - BOHM-LIKE, $f_B \sim 100$
  - CLASSICAL
  - NEOCLASSICAL

- IMPURITY CONTROL
  - DIVERTOR, $r_s \sim r_w$
  - PUMPED LIMITER, $r_s \sim r_c$
- BLANKET/SHIELD THICKNESS, Δb (1.5 m)
- BLANKET ENERGY MULTIPLICATION, M_N (1.1)
- PROFILE INDEX, ν (3)
- COIL CURRENT DENSITY, j (10–15 MA/m²)
- STRESS LIMIT, $\hat{\sigma}$ (200 MPa)
- POLE NUMBER, ℓ (2 OR 3)
- COILS/FIELD PERIOD, N/m (3)

NOTATION:

COMPUTATION OF VARIABLE Y FROM
VARIABLES y_1, y_2 , AND y_3

Fig. IV-2. Logic flow diagram for the MSR systems model.

loading, $I_w(\text{MW/m}^2)$, and total thermal power, $P_{TH}(\text{MWt})$, are fixed for a given parameter variation, yielding known relationships between average plasma radius, $r_p(\text{m})$, on-axis magnetic field, $B_o(\text{T})$, and major toroidal radius, $R_T(\text{m})$. The ignition condition gives an explicit value for the product, $\langle\beta\rangle B_o^2 r_p$, for a given confinement scaling (e.g., Alcator, Bohm-like, neoclassical). Invoking a specific beta scaling, imposed by equilibrium and stability considerations, allows R_T/r_p to be computed for given values of ℓ , m , N , and plasma radial profile index, ν . Combination of these relationships yields explicit values of r_p , B_o , R_T , and $\langle\beta\rangle$ that are valid for the specific value of the marching variable, $\langle T \rangle$, and choice of the transport scaling. Details of the procedure are discussed in Sec. B.7. of Appendix B. The maximum lateral distortion of a twisted TF-coil configuration, d , can be computed (Sec. B.3. of Appendix B) for given values of poloidal-field number, ℓ , toroidal-field number, m , rotational transform, τ , plasma aspect ratio, $A = R_T/r_p$, ratio of coil radius to plasma radius, r_c/r_p , and ratio of separatrix to coil radius, r_s/r_c . The coil radius is determined from the computed value of r_p , the specified values of blanket and shield thickness, Δb , and the average plasma scrape-off thickness, $r_w - r_p \approx r_p(1/x - 1)$, where $x = r_p/r_w$ is a calculated variable used here as a measure of scrape-off thickness. On the basis of the coil dimensions that result when a homogenized overall coil current density, $j(\text{MA/m}^2)$, is specified, coil interference, forces, and local stresses can be quantified. As shown in Appendix B, a parameter space defined by $\langle T \rangle$ and B_o (or $\langle\beta\rangle^{1/2} B_o$) represents a convenient means to display the reactor performance and range of ignited operating points. As noted previously, quantitative costing algorithms, although available, have not been implemented at this level of study.

2. Transport Scaling. Radial transport of energy in a nonaxisymmetric stellarator/torsatron plasma is a complicated subject that presently is receiving theoretical attention. At this level of study it was judged imprudent to incorporate at the outset a detailed, state-of-the-art model of particle and energy transport. Instead, the reactor-survey calculations are performed using simplified, empirical or theoretical models in order that sensitive variables can be more directly identified. The resulting groundwork can be extended to more sophisticated models as the design progresses. For purposes of this study, therefore, transport is modeled using the Alcator (empirical) scaling relationship,

$$\tau_E(\text{Alcator}) \propto \langle n \rangle r_p^2 \propto \frac{\langle \beta \rangle B_o^2 r_p^2}{\langle T \rangle}, \quad (\text{IV-1})$$

or by the following Bohm-like scaling relationship:

$$\tau_B \equiv \tau_E(\text{Bohm}) \propto f_B \frac{B_o r_p^2}{\langle T \rangle}. \quad (\text{IV-2})$$

For the Bohm-like scaling relation, a factor, $f_B = 50\text{-}200$, is introduced as a measure of improvement over traditional Bohm scaling. In order to bring $\tau_E(\text{Bohm})$ into agreement in magnitude with $\tau_E(\text{Alcator})$, a factor $f_B \sim 60$ is typically required. Regardless of the details of the particular scaling relationship used, this preliminary survey suggests that the transport in a stellarator/torsatron reactor will have to be at least as good as that predicted by Alcator scaling if the reactor is to be competitive. For this reason, Alcator transport scaling is used in this parametric study. A more detailed elaboration of these transport scaling relationships is presented in Sec. B.2. of Appendix B, including a quantitative comparison between Alcator, Bohm-like, and neoclassical transport scaling.

3. Plasma Beta Scaling. That the primary difficulty of stellarator/torsatron as a reactor may be the relatively low attainable values of beta is widely recognized. Equilibrium and stability considerations impose upper limits on beta and thereby constrain the reactor to limited regimes of plasma aspect ratio, $A = R_T/r_p$. In addition, the beta limits are coupled to the magnetics performance through the rotational transform produced by the vacuum-magnetic-field topology. These considerations are discussed in greater detail in Sec. B.4. of Appendix B. For the purposes of this study, the simplified equilibrium/stability relationship between $\langle \beta \rangle$, m , A , ℓ , and ι is enforced in order to maintain a direct coupling between plasma performance (i.e., $\langle \beta \rangle$) and reactor feasibility (i.e., coil-set configuration needed to generate the ι required to achieve a given $\langle \beta \rangle$). It is recognized, however, that should difficulties be encountered in achieving "acceptable" rotational transforms for a given coil configuration (i.e., d/r_c , ℓ , m , N , etc.) as

predicted by the simplified equilibrium/stability theory employed, these imposed $\langle\beta\rangle$ versus m , ℓ , and A constraints must be re-examined.

With proper selection of ℓ , m , and A , it appears possible to attain $\langle\beta\rangle \approx 0.04$ using the simplified equilibrium/stability model. This value is anticipated to be marginally acceptable from the reactor viewpoint. As described in Sec. B.4. of Appendix B, however, these limits are based conservatively on the assumption that diffusion-driven currents establish both equilibrium (i.e., Pfirsch-Schlüter shift) and stability (i.e., Kruskal-Shafranov modes) constraints. Ongoing theoretical effort is aimed at providing more reliable beta scaling relationships. Specific combinations of convenient reactor parameters leading to this result are summarized in Table IV-II, the basis for which can also be found in Sec. B.4. of Appendix B. It is emphasized that unlike most magnetic fusion concepts, the maximum beta value for stellarator/torsatrons is intimately associated with coil configuration and magnet design (i.e., A , d/r_c , coil interference, current density, forces, etc.). For this reason, an approximate but analytically self-consistent model was used to relate $\langle\beta\rangle$ to such parameters as ℓ , m , d/r_c , and A , rather than to dictate a value of beta, in order to preserve this close coupling between plasma performance, coil design, and reactor design.

TABLE IV-II
PARAMETERS LIKELY TO MAXIMIZE $\langle\beta\rangle$ FOR THE MSR

<u>Parameter</u>	<u>Value</u>
Average beta value, $\langle\beta\rangle$	0.04
Radial profile index, ν	3
Plasma aspect ratio, $A = R_T/r_p$	11
Rotational transform, τ	0.6-0.7
Poloidal-field period number, ℓ	2
Toroidal-field period number, m	6-8
Pitch angle, α' (radians)	0.60
Number of coils per field period, N/m	3-4
Number of coils, N	18-24

Because the $\ell = 2$, $m = 8$, $A = 11$ case provides for $N = 24$ coils if $N/m = 3$ is fixed and leads to lower coil current density, lower peak fields at the coils, and less toroidal field ripple than the $m = 6$ case, it is used as the focus of the calculations reported in Sec. IV.C. This choice may have to be reexamined in subsequent phases of this study. In addition, the marginally acceptable value of $\langle\beta\rangle \approx 0.04$ is close to the value $\langle\beta\rangle \approx 0.035$ assumed in the T-1 torsatron conceptual design study.⁶ This commonality facilitates comparison of the mechanical design features of the modular-coil configuration with the continuous helical-coil configuration (i.e., torsatron) on the basis of an essentially common physics design point.

4. Configurational and Engineering Constraints. For a fixed power output, the size of the ignited DT plasma is determined by transport scaling and beta limits, as discussed in the preceeding two subsections. A self-consistent reactor design must impose subsequently the constraints required by magnetics, neutronics, and thermohydraulics considerations. These considerations include the standoff distance between the plasma edge and the reactor first wall, $r_w - r_p = r_p(1/x - 1)$, which affects the plasma volume utilization within the first wall and the magnitude of the first-wall thermal loading. The value of $x = r_p/r_w$ depends on the plasma cross section and hence on the stellarator parameters m , ℓ , r_c/r_p , and r_s/r_c . The blanket and shield located behind the first wall must be sufficiently thick ($\Delta b \approx 1\text{--}1.5$ m) to insure adequate tritium breeding and heat removal in a DT-fueled system, as well as providing thermal and radiation protection for the superconducting coil set. The coils themselves must carry sufficient current to provide the confining magnetic field required for ignition without exceeding force, stress, and current-density constraints. In the case of the modular-coil configuration, these constraints must be met while assuming that the lateral deformation is sufficient to provide the required rotational transform in the magnetic field without interfering with neighboring coils. The results from reactor survey calculations reported in Sec. IV.C. do not meet simultaneously all of these constraints in all cases. However, it is possible to identify viable, self-consistent, interim reactor design points that do satisfy these preliminary constraints (Sec. IV.D.) and that form the basis for more detailed engineering designs.

5. Computational Algorithm. The reactor survey results reported in Sec. IV.C. were obtained using a steady-state, zero-dimensional ("point") representation of an ignited DT plasma. Ignition implies that the self-heating of fusion-product alpha particles balances the radiation and transport losses; external auxiliary heating is not required except during startup of the device. The calculational algorithm is relatively simple (Sec. B.1. of Appendix B), but, nevertheless, incorporates the crucial physics models that are required to characterize magnetic fusion reactors in general and the stellarator/torsatron in particular. Details of the computational algorithm are presented in Sec. B.7. of Appendix B. The results of the algorithm (Sec. IV.C.) are expressed as values of $\langle \beta \rangle^{1/2} B_0$ or B_0 versus the average plasma temperature, $\langle T \rangle$, under conditions that are required to produce ignition in systems with specified values of total thermal power, $P_{TH}(MWT)$, and fusion-neutron first-wall loading, $I_w(MW/m^2)$. These results can then be checked against the imposed engineering constraints (Sec. V.A.4.) and integrated into the overall MSR systems code. Narrowing and/or optimization of the reactor survey results leads to the identification of interim MSR design points (Sec. IV.D.) that can then be subjected to more detailed elaboration and analysis using more sophisticated and expensive computational tools for magnetics and stress analyses (Sec. V.A.).

The algorithm displayed graphically in Fig. IV-2 is implemented as follows. All secondary variables are fixed to nominal values. Primary variables are swept over a range of desired values in order to assess MSR performance for various choices of total thermal power output and fusion-neutron first-wall loading. For a specified transport scaling relationship, the ignition condition is solved as a function of average plasma temperature in the range 5-30 keV. All derived reactor parameters (i.e., dimensions and characteristic time constants) are then calculated. The coil interference constraint (Sec. B.5. of Appendix B) is monitored to exclude design points that are not geometrically self-consistent. Attention focuses on those design points that achieve ignition for minimized values of $\langle \beta \rangle^{1/2} B_0$ or B_0 . A candidate design point that survives this scrutiny is then subjected to more detailed analysis using numerical magnetics models.

C. Results

1. Comparison with Previous Stellarator/Torsatron Reactor Studies. As a test of the reactor survey model developed for this study, a comparison was made with the results of the T-1⁶ and Heliotron-C⁴ reactor design points (Table IV-I). The results of this comparison are shown in Fig. IV-3. Where possible parameters are equated to the appropriate values reported for the respective reactor design points. For these parameters and as a function of average plasma temperature, $\langle T \rangle$, the ignition condition (Sec. B.1. of Appendix B) is solved to yield the on-axis magnetic field strength, B_0 , for the fixed thermal power output, P_{TH} , and average plasma radius, r_p , the latter being determined from the neutron wall-loading constraint, I_w . Transport for the T-1 reactor design is described by Alcator (empirical) scaling, and transport for the Heliotron-C requires that the energy confinement time, τ_E , equals 75 times the Bohm confinement time, where $\tau_{E(Bohm)} = 62.5 \langle T \rangle / B_0$. The Heliotron-C curve exhibits lower values of B_0 than does the corresponding

Fig. IV-3. Comparison of the results of the present MSR survey model with the specific T-1⁶ and Heliotron-C⁴ conceptual design points.

curve for the T-1 design point because of the higher value of $\langle\beta\rangle$ assumed for the Heliotron-C. Also, because the neutron first-wall loading is higher for the Heliotron-C and the plasma radius is larger, the total thermal output is nearly a factor of two greater than that for the T-1 design. The T-1 design point lies essentially at a plasma temperature that minimizes the required value of B_0 , which nevertheless occurs at a higher value than that for the Heliotron-C and is, therefore, more likely to be constrained by engineering limitations of the coil set (e.g., peak field and stresses).

Discrepancies between the curves computed from the present systems algorithm and the respective design point are considered minor. These differences are difficult to trace, because the assumptions and models used to obtain the T-1 and Heliotron-C design points are not always explicitly described and may, therefore, differ from those used by the present systems model. Also, additional minor differences in assumptions (i.e., the values of fusion reactivity, $\langle\sigma v\rangle$, radiation loss, etc.) can be identified. Nevertheless, the agreement between the systems model developed for this study and the results of previous independent reactor studies is good.

2. Results of MSR Survey Study. This subsection summarizes the results of MSR survey calculations using the systems model described in Appendix B and depicted in Fig. IV-2. These survey calculations form the basis for the selection of an interim MSR design point reported in Sec. IV.D.

2.1. Selection of Stellarator Physics Parameters. Anticipation of the equilibrium and stability constraints that impose maximum attainable limits on $\langle\beta\rangle$ allows a narrowing of the MSR parameter hyperspace by prior selection of several key parameters. In the first instance, only the $\ell = 2$ and $\ell = 3$ options are considered on the basis that higher ℓ values would lead to more complicated modular-coil windings; this assumption remains to be examined fully. The plasma radial profile index, ν , is taken to be 3 for all results reported herein. This choice is consistent with assumptions made for the T-1 conceptual design study.⁶ Using the conventional stellarator expansion to determine the rotational transform (Sec. B-3 of Appendix B) and implementation of equilibrium/stability theories that assume that diffusion-driven currents determine upper bounds on $\langle\beta\rangle$ (Sec. B.4 of Appendix B), it is possible to narrow attention to $\ell = 2$ systems with $m = 6$ or 8. Such systems tend to maximize $\langle\beta\rangle$ under the simultaneous application of the equilibrium limit as well as both the ideal and resistive stability limits at convenient

plasma aspect ratios. The next consideration is the positioning of the maximum separatrix radius, r_s , relative to the coil radius, r_c . If r_s is at the first-wall radius, r_w , the overall configuration is compatible with the magnetic divertor usually associated with the stellarator/torsatron. However, if $r_s \sim r_c$, the plasma radius, r_p , must still be constrained by r_w such that not all of the available closed magnetic surfaces are filled with plasma. This implies a limiter near the first wall to provide plasma-boundary control. Collateral benefits include a lower required rotational transform and higher volume utilization within the first-wall radius because of the fatter quasi-elliptical $\ell = 2$ plasma cross section. Additionally, the $r_s \sim r_c$ configuration yields a higher value of $\langle \beta \rangle$ (i.e., ~ 0.04) for a convenient plasma aspect ratio, $A = 11$, than can be obtained from the $r_s \sim r_w$ case, as is shown in Fig. IV-4.

The number of coils per field period, N/m , represents a choice that, at the level of this study, must be made on the basis of somewhat qualitative arguments.¹⁰ First, $N/m = 3$ is the lowest integer number giving nonzero rotational transform. Second, if N/m is too small, the amount of plasma azimuthal rotation from coil to coil would be too great, causing undesirably abrupt changes in field-line topology, formation of ergodic regions, and/or field-line disconnections. Third, if N/m is too large, the number of coils increases for a given value of m , and avoidance of coil interference would force the coil current density to increase. Expressed in terms of the pitch angle, α' , where $\tan \alpha' = (r_c/r_p)(1/A)(N/\ell)/(N/m)$, in order to preserve the same pitch for a given A , ℓ , and r_c/r_p , increasing the number of coils per field period, N/m , requires that N be correspondingly increased. Hence, on the basis of these qualitative arguments, $N/m = 3$ or 4 has been selected.

Preliminary numerical calculations of flux surfaces and rotational transform profiles given in Sec. V.A. indicate that $N/m = 3$ coils per field period may be adequate. Additional work in this area, however, is needed. Nevertheless, specification of three (four) coils per field period then gives $N = 24$ for $m = 8$ ($m = 6$). In either case, the maximum attainable value of $\langle \beta \rangle$ is ~ 0.04 , which is comparable to the value of 0.035 proposed for the T-1 torsatron conceptual reactor design,⁶ albeit the latter is at a higher aspect ratio. The $\langle \beta \rangle = 0.10$ value assumed for the Heliotron-C reactor⁴ exceeds the limits calculated and imposed in the course of this study. Accordingly, the Heliotron-C design point operates at ignition with a lower on-axis magnetic field.

Fig. IV-4. Dependence of combined equilibrium and stability limits on $\langle \beta \rangle$ as a function of plasma aspect ratio, $A = R_T/r_p$, for an $\ell = 2$ stellarator with radial profile index, $\nu = 3$. The configuration with maximum separatrix radius, r_s , near the coil radius, r_c , implying pumped-limiter impurity control yields $\langle \beta \rangle \sim 0.04$ for $A = 11$, superior performance to the case where r_s is near the first-wall radius, r_w , implying magnetic-divertor impurity control. The beta limits rely on the conventional stellarator expansion [Eq. (B-46)] representation of the rotational transform.

The maximum radial position of the separatrix, r_s , relative to the coil radius, r_c , is an important stellarator parameter choice. If $r_s \approx r_w$, then the device could operate with a magnetic divertor for impurity control, whereas if $r_s > r_w$ (i.e., $r_s \approx r_c$ in the extreme case), a pumped-limiter impurity control scheme would be required. The choice of $r_s \approx r_w$ or $r_s \approx r_c$ is made on the basis of beta-limit and rotational-transform arguments and the desire to maximize the plasma filling fraction within the first-wall radius as reflected by the parameter $x = r_p/r_w$. The parameter x depends on the ℓ value as well. Specifically, $x(\ell = 2) = 0.59-0.93$, whereas $x(\ell = 3) = 0.64$. Hence, in terms of maximizing $x(\ell = 2)$, it would be more advantageous for $r_s \approx r_c$

within the constraints imposed by the $\langle\beta\rangle$ versus A dependence depicted in Fig. IV-4. In addition, for the $\ell = 2$, $A = 11$ case, adequate values of $\tau(r_p)$ are achieved in the modular-coil configuration with lower values of lateral coil distortion, d/r_c , if $r_s \sim r_c$ (Fig. B-6 of Appendix B). Lower values of coil distortion are more likely to meet the coil-interference criterion (Sec. B.5. of Appendix B) for a given reactor aspect ratio and number of modular coils, N .

2.2. Selection of Transport Model. Various possibilities for transport scaling are considered in Sec. B.2. of Appendix B. Alcator (empirical) scaling leads to the convenient parameter grouping, $\langle\beta\rangle^{1/2}B_0$, which allows for a generalized characterization of MSR performance, as shown in Fig. IV-5 for the specified values $P_{TH} = 4.0$ Gwt and $I_w = 1.5$ MW/m². The values $A = 11$ and $x = 0.93$ are consistent with an $\ell = 2$ MSR system at a maximized value of β . Although curves of $\langle\beta\rangle^{1/2}B_0$ versus $\langle T \rangle$ are unique and general for Alcator (empirical) scaling, such curves are not unique for either Bohm-like or classical transport scaling laws. Nevertheless, ignoring small variations with $\langle\beta\rangle$, Fig. IV-5 also shows typical ignition thresholds as expressed in terms of $\langle\beta\rangle^{1/2}B_0$ for classical and Bohm-like transport scaling, where multipliers $f_B = \tau_E/\tau_B$ in the range 50-200 have been assumed. Alcator scaling is seen to be essentially equivalent to Bohm-like transport scaling, with $f_B \sim 60-80$ over the temperature range considered. Alcator scaling was assumed for the T-1 torsatron conceptual design study⁶ and, for purposes of benchmarking, is used in this study as well. A comparison of neoclassical scaling with Alcator and Bohm-like scaling is given in Sec. B.2. of Appendix B, but has not yet been incorporated into this study. Generally, neoclassical-plateau scaling is the most favorable model and can be approximately represented by Bohm-like scaling with $f_B \sim 100$, whereas helical-plateau scaling is the least favorable and can be represented by Bohm-like scaling with $f_B \sim 20$.

The quantity $\langle\sigma v\rangle/T^2$ for the DT reaction is constant to within 10% for temperatures in the range of 8-20 keV. Hence, $\langle\beta\rangle^{1/2}B_0$ is a weak function of $\langle T \rangle$ when P_{TH} and I_w are fixed for both Alcator and Bohm-like transport scalings. If the parameter $\langle\beta\rangle^{1/2}B_0$ is essentially constant, a higher allowed value for B_0 can compensate a lower value of $\langle\beta\rangle$ to give equivalent overall reactor performance. An MSR with higher aspect ratio than allowed in an otherwise comparable tokamak reactor, therefore, can tolerate and remain

Fig. IV-5. Dependence of the parameter $\langle \beta \rangle^{1/2} B_0$ required for MSR ignition as a function of $\langle T \rangle$ for the indicated specific parameters. Results for Alcator, Bohm-like, and classical transport scaling are presented.

competitive with higher values of B_0 and correspondingly lower values of $\langle \beta \rangle$, for a commonly imposed limit on maximum magnetic field strength, $B_c(T)$, on the inboard side of the TF coils. The value of $\langle \beta \rangle \approx 0.04$ allowed by the results of approximate calculations shown in Fig. IV-5 for the MSR may compare favorably with the value $\langle \beta \rangle = 0.067$ used in the STARFIRE tokamak conceptual

design study,¹¹⁻¹³ because for the MSR configuration, higher values of B_0 may be achieved for the same peak fields at the coils (i.e., higher A and more confinement by the poloidal field component).

2.3. Selection of Total Thermal Power and First-Wall Neutron Loading.

Figure IV-6 depicts curves of the parameter $\langle\beta\rangle^{1/2}B_0$ required for ignited MSR operation as a function of average plasma temperature, $\langle T \rangle$, for the indicated conditions and a range of neutron first-wall loadings. For purposes of Fig. IV-6 and all subsequent results from this MSR survey, Alcator transport scaling is assumed. The allowed neutron first-wall loading, I_w , is varied parametrically in the range 1-4 MW/m², with the total power output held fixed at $P_{TH} = 4$ GWt. As I_w increases, the required value of $\langle\beta\rangle^{1/2}B_0$ also increases; the plasma radius, r_p , decreases [i.e., Eq. (B-80) of Appendix B],

Fig. IV-6. Dependence of the parameter $\langle\beta\rangle^{1/2}B_0$ required for MSR ignition as a function of $\langle T \rangle$ for the fixed total thermal power, $P_{TH} = 4.0$ GWt, for various fusion-neutron first-wall loadings in the range $I_w = 1-4$ MW/m². Alcator transport scaling is assumed.

and B_0 must increase to restore the confinement time required for ignition. If B_0 is constrained below a maximum value determined by magnet technology, I_w may be limited to a relatively low value for any value of $\langle\beta\rangle$. Also, higher values of B_0 require larger coil cross sections when the coil current density, j , is fixed, and larger coils are more likely to violate the interference constraint for a fixed aspect ratio (Sec. B.5. of Appendix B).

In Fig. IV-7 the fusion-neutron first-wall loading is fixed at a modest value of $I_w = 1.5 \text{ MW/m}^2$, and the total thermal power, P_{TH} , is varied parametrically in the range $P_{TH} = 1\text{--}4 \text{ Gwt}$. Again, Alcator (empirical) transport scaling is assumed. Larger power systems require larger volumes of reacting plasma (i.e., larger values of r_p) and, because the Lawson parameter

Fig. IV-7. Dependence of the parameter $\langle\beta\rangle^{1/2} B_0$ required for MSR ignition as a function of $\langle T \rangle$ for the fixed fusion-neutron first-wall loading, $I_w = 1.5 \text{ MW/m}^2$, for various values of total output thermal power in the range $P_{TH} = 2\text{--}6 \text{ Gwt}$. Alcator transport scaling is assumed.

for ignition is proportional to $\langle \beta \rangle B_0^2 r_p$ for a fixed temperature, lower values of $\langle \beta \rangle^{1/2} B_0$ are required for ignition.

If the equilibrium and stability limit of $\langle \beta \rangle \sim 0.04$ is imposed explicitly, it is possible to display the ignition curves as B_0 versus $\langle T \rangle$. Figure IV-8, like Fig. IV-6, displays ignition thresholds for various fusion-neutron first-wall loadings. The on-axis magnetic field strength, B_0 , must lie in the range of 6-8 T for ignition at this specific value of $\langle \beta \rangle = 0.04$. Figure IV-9, like Fig. IV-7, displays the ignition thresholds for various total thermal powers.

Imposition of a fixed upper limit on B_0 , as dictated by magnet technology, and an upper limit on $\langle \beta \rangle$, from equilibrium and stability considerations, therefore, constrains the MSR to operate above a minimum

Fig. IV-8. Dependence of the on-axis magnetic field strength, B_0 , required for MSR ignition as a function of $\langle T \rangle$ for the fixed total thermal power, $P_{TH} = 4.0 \text{ GWt}$, for various fusion-neutron first-wall loadings in the range $I_w = 1\text{--}4 \text{ MW/m}^2$. Alcator transport scaling is assumed.

Fig. IV-9. Dependence of the on-axis magnetic field strength, B_0 , required for MSR ignition as a function of $\langle T \rangle$ for the fixed fusion-neutron first-wall loading, $I_w = 1.5 \text{ MW/m}^2$, for various values of total output thermal power in the range $P_{TH} = 2\text{--}6 \text{ GWt}$. Alcator transport scaling is assumed.

threshold in both total thermal output and corresponding physical size. A total thermal output power of $P_{TH} = 4.0 \text{ GWt}$ exceeds this minimum and is used as a target for this study in order to facilitate comparison with the STARFIRE tokamak¹³ and EBTR.^{14,15} This ex post facto justification explains the choice of P_{TH} used previously in Figs. IV-5 and IV-6. The remainder of this subsection examines the sensitivity of the ignition threshold curves to changes in secondary variables (e.g., Δb , Z_{eff} , r_p/r_c , etc.).

2.4. Variations of System Performance with Blanket Thickness. The effects of parametric variations in combined blanket and shield thickness, Δb , are displayed in Fig. IV-10. The nominal and conservative value $\Delta b = 1.5 \text{ m}$ is assumed throughout this study in order to assure adequate tritium breeding and magnet radiation and thermal protection. This choice is consistent with reactor design experience on many other magnetic fusion systems (Table B-II of

Fig. IV-10. Dependence of the on-axis magnetic field strength, B_0 , required for MSR ignition as a function of $\langle T \rangle$ for the nominal blanket and shield thickness, $\Delta b = 1.5$ m, and a thinner $\Delta b = 1.0$ m. The stagnation point radius is held at the coil radius. Alcator and Bohm-like transport scaling with $f_B = 100$ are compared. In both cases $r_s \sim r_c$ is consistent with pumped-limiter rather than magnetic-divertor impurity control.

Appendix B). Detailed thermohydraulics and nucleonics computations, which are beyond the scope of this study, may allow adequate magnet protection for thinner blanket and shield assemblies. Such improvement does not result in significantly lower values of required B_0 for the base-case parameters, as is shown in Fig. IV-10. The detailed design of the blanket and shield can, therefore, be decoupled from this reactor survey. For the $r_s \approx r_c$, $l = 2$ case considered here, a thinner blanket and shield allows a higher value of r_p/r_c ($\sim r_p/r_s$), thereby requiring less lateral distortion [Sec. B.3. of Appendix B] of the MSR coils and, for $l = 2$, a higher value of r_p/r_w [Sec. B.3. of Appendix B, Eq. (B-56)]. This change is equivalent to an increase in allowed I_w when viewed in terms of its influence on B_0 (Fig. IV-8).

2.5. Finite Alpha-Particle Pressure. The effect of the inclusion of the finite "partial pressure" of the fusion-product alpha particles is demonstrated in Fig. IV-11. The most optimistic assumption is that the equilibrium and stability limits imposed on $\langle\beta\rangle$ apply exclusively to the background plasma ions and electrons. However, for a suprathermal fraction of alpha particles with $T_\alpha/\langle T\rangle \sim 10$ (Sec. B.1. of Appendix B), such that without impurities the effective charge, Z_{eff} , of the plasma exceeds 1.0 [Eq. (B-5A) of Appendix B], additional confining magnetic field must be provided to contain the alpha-particle pressure contribution over and above the B_0 required to achieve ignition. A value of $Z_{\text{eff}} = 1.1$ corresponds to $n_\alpha/n_i = 0.055$ or a reduction in the productive beta value by a factor of $1/f_{Z2} = 0.75$, as seen from Fig. B-1 in Appendix B.

Fig. IV-11. Dependence of the on-axis magnetic field strength, B_0 , required for MSR ignition as a function of $\langle T \rangle$. Inclusion of the fusion-product alpha-particle partial pressure, p_α/p , must be compensated by additional confining magnetic field strength.

2.6. Position of Separatrix Radius. The position of the separatrix radius, r_s , relative to the coil radius, r_c , determines whether the last closed magnetic surface is defined by requiring a specific value of $r_s \sim r_w$ (i.e., magnetic-divertor operation) or by allowing r_s to approach r_c (i.e., pumped-limiter operation). In the latter case the closed magnetic surfaces overlap the blanket and shield region, and the plasma radius is determined by a limiter located at or near the first-wall radius. As discussed in Sec. B.3. of Appendix B, this choice affects the volume utilization of plasma within the first-wall radius, r_w , as measured by the parameter $x = r_p/r_w$, for $\ell = 2$ systems only. The value of $x = 0.64$ in $\ell = 3$ systems is independent of the ratio r_s/r_c . For the usual fixed parameters, Fig. IV-12 illustrates the consequences of the positioning of r_s . As the value of r_s is varied between r_w and r_c for $\Delta b = 1.5$ m, the value of x increases [Eqs. (B-53) and (B-56) of

Fig. IV-12. Sensitivity of ignited MSR performance to separatrix radius position for the indicated fixed parameters. Alcator transport scaling is assumed. Magnetic-divertor operation implies $r_s \sim r_w$, whereas pumped-limiter operation implies $r_s \sim r_c$.

Appendix B)] from 0.59 to 0.93; a corresponding reduction in the required value of B_0 for ignition results. If the total power is fixed and the plasma radius is thereby determined, a higher value for x allows for both a higher neutron first-wall loading and reduced coil radius, leading to higher power density and the potential for more economical systems. The modest improvement in B_0 obtained by moving r_s outward to r_c may be offset by the technical complexity of providing a viable limiter mechanism at the first-wall radius, r_w . It can only be concluded at this point that the pumped-limiter option is worthy of further investigation, in that preliminary prognoses for the pumped-limiter impurity control¹¹⁻¹⁵ are encouraging.

D. Interim Design-Point Selection

The results of the previous subsections have been used to examine the MSR parameter space quantitatively and to examine tradeoffs among the several key parameters. In this subsection attention is narrowed to the identification of an interim MSR design point that serves as the basis for review, evaluation, and ultimately a more detailed engineering design.

A competitive MSR candidate should have a physical size that is sufficient to satisfy the coil-interference constraint (Sec. B.5. of Appendix B) while not producing an excessively large power output. Parameters for an interim design point satisfying this general criterion are summarized in Table IV-III. This design point represents ignited operation under the assumption of Alcator transport scaling. The system is based on an $\ell = 2$, $m = 8$ stellarator configuration to allow a plasma aspect ratio of $A = 11$ and a corresponding average beta of $\langle\beta\rangle = 0.04$. The thermal power output is $P_{TH} = 4.0$ GWt, which is comparable to the recently-completed STARFIRE tokamak¹³ and the EBTR^{14,15} conceptual designs as well as contemporary fission-reactor experience. Table IV-IV presents a comparison of the interim MSR design point with the STARFIRE and EBTR designs. On the basis of this comparison, particularly with respect to the system power density, the MSR appears comparable to the other two steady-state, toroidal fusion concepts.

A more optimistic transport scaling than that provided by the Alcator case would allow operation at a lower on-axis magnetic field, which is minimized in both cases near $\langle T \rangle = 8$ keV. The lower on-axis field allows lower coil currents, which in turn reduces both the coil current density and cross-sectional area to better satisfy stress and access considerations. The required first-harmonic lateral coil distortion is estimated on the basis of

TABLE IV-III
INTERIM MSR DESIGN POINT^(a)

<u>Parameter</u>	<u>Value</u>
Number of poloidal-field periods, ℓ	2
Number of toroidal-field periods, m	8
Average rotational transform, τ ^(b)	~ 0.66
First-wall neutron loading, I_w (MW/m ²)	1.5
Thermal power output, P_{TH} (GWt)	4.0
Plasma power density, p_p (MWt/m ³)	2.5
System power density, p_s (MWt/m ³) ^(c)	0.33
Major toroidal radius, R_T (m)	21.35
First-wall radius, r_w (m)	2.35
Plasma aspect ratio, $A = R_T/r_p$	11.0
Average plasma radius, r_p (m)	1.94
Plasma volume, V_p (m ³)	1586
Radial profile index, ν	3
Average plasma temperature, $\langle T \rangle$ (keV)	8.0
Average plasma ion density, $\langle n_i \rangle$ (10 ²⁰ /m ³)	1.72
Average beta, $\langle \beta \rangle$	0.04
Lawson parameter, $\langle n_e \rangle \tau_E$ (10 ²⁰ s/m ³)	4.12
On-axis magnetic field, B_o (T)	6.37
Peak coil magnetic field, B_c (T)	~ 11.3
Coil current center radius, r_c (m)	4.6
Coil current density, j (MA/m ²)	12.5
Maximum coil lateral distortion, d/r_c	0.29
Coil current, I (MA)	28.5
Coil thickness, δ_c (m)	1.51
Coil volume, V_c (m ³ /coil)	70.
Stored magnetic energy, E_M (GJ)	~ 200 .

(a) Alcator transport scaling, $r_s \sim r_c$ implying pumped-limiter impurity control, $f_\alpha = 0.88$, $M_N = 1.1$, $\Delta b = 1.5$ m, $N = 24$, $Z_{eff} = 1.1$, $x = 0.83$, and alpha-particle pressure contribution, $p_\alpha/p = 0.25$.

(b) As obtained from the stellarator expansion [Eq. (B-52) of Appendix B]. Both numerical results and results from the antihelix-modified stellarator expansion for the interim coil configuration predict lower values ($\tau \sim 0.15$), which will require improvement in subsequent design iterations if the value of $\langle \beta \rangle = 0.04$ is to be achieved under the limits imposed in this study.

(c) Based on volume enclosed by and including the coils.

TABLE IV-IV

COMPARISON OF REACTOR DESIGN POINTS FOR THE STARFIRE,¹³ EBTR,^{14,15} and MSR

	STARFIRE	EBTR	MSR
Average beta, $\langle\beta\rangle$	0.067	0.17	0.04
First-wall radius, r_w (m)	~2.72	~1.10	2.35
Major toroidal radius, R_T (m)	7.0	35.0	21.35
Effective geometric aspect ratio, R_T/r_w	2.57	31.82	9.09
$\langle\beta\rangle B_o^2 r_p$ (T ² m)	5.36	4.25/0.86 ^(a)	3.0
Plasma volume, V_p (m ³)	781	691	1586
Plasma chamber volume, V_{pc} (m ³)	950	838	2327
Blanket volume, V_B (m ³)	543	1108	1095 ^(c)
Volume enclosed by coils, V_C (m ³)	13443	7978/16441 ^(b)	12110 ^(c)
Fusion power, P_F (MWt)	3510	2857	3703
Primary-coolant power, P_{pc} (MWt)	3800	3692	3750 ^(c)
Total thermal power, P_{TH} (MWt)	4033	4028	4000
Plasma power density, P_F/V_p (MWt/m ³)	4.50	4.13	2.33
Chamber power density, P_F/V_{pc} (MWt/m ³)	3.70	3.41	1.59
Blanket power density, P_{pc}/V_B (MWt/m ³)	7.00	3.33	3.42
Effective blanket power density, P_{TH}/V_B (MWt/m ³)	7.43	3.64	3.65
System power density, P_{TH}/V_C (MWt/m ³)	0.30	0.50/0.24 ^(b)	0.33
Neutron first-wall loading, I_w (MW/m ²)	3.6	1.4	1.5
Neutron energy multiplication, M_N	1.14	1.5	1.1 ^(c)
$(P_{TH}/V_B)/[I_w(M_N + 0.25)]$ (m ⁻¹)	1.48	1.49	1.80

^(a)Using coil-plane/midplane magnetic field.^(b)Without/with ARE-coil volume.^(c)Preliminary estimate pending detailed blanket/shield design.

approximate models (Sec. B.3. of Appendix B) to be $d/r_c \sim 0.3$, giving an edge-plasma rotational transform of $\iota(r_p) \sim 0.2$ ($r_p = 1.94$ m), which is below the value desired from the viewpoint of beta limits. Comparisons of the analytic and numerical calculations of the rotational transform are made in Sec. B.8. of Appendix B. Generally, the numerical computations do not yield as high values of ι as does the analytic stellarator expansion from which the beta limits are derived. As noted in Sec. B.3. of Appendix B, higher-harmonic coil distortions may be required in order to obtain self-consistently the required value, $\iota \sim 0.66$, that the presently imposed beta scaling requires for $\langle \beta \rangle = 0.04$ at $A = 11$ for $\ell = 2$. This interim design point specifies $r_s \sim r_c$, reflecting a plasma that is limited by the first wall (i.e., pumped-limiter impurity control) and does not fill all of the available closed (vacuum) magnetic surfaces. The neutron first-wall loading of $I_w = 1.5$ MW/m² is limited by constraints imposed on magnetic field and coil interference when the number of coils is fixed at a convenient $N = 24$ and the number of coils per field period is $N/m = 3$.

The interim MSR design point is tightly constrained by certain variables and insensitive to several others. The beta-limit versus aspect-ratio argument of Fig. IV-4 would allow only a somewhat higher value of $\langle \beta \rangle$ at a somewhat lower value of A (~ 10) if m is increased to 12. Conversely, if m is decreased to 6, only a slightly lower value of $\langle \beta \rangle$ results as A increases to ~ 15 . However, if N/m is fixed at 3, coil interference is encountered as m increases, and field ripple increases as m decreases. The $\langle \beta \rangle^{1/2} B_0$ or B_0 versus ignition curves exhibit a fairly broad minimum near $\langle T \rangle \sim 8$ keV and are, therefore, insensitive to modest changes in $\langle T \rangle$. Figure IV-10 suggests that a detailed blanket and shield optimization resulting in a blanket and shield thickness, $\Delta b \sim 1$ m, would not significantly improve the magnetic performance by allowing a lower value for B_0 . The use of finer step sizes in the P_{TH} and I_w searches might produce a slightly more optimized design point than that reported in Table IV-III. A higher allowed value for coil current density could lead to a smaller power output and physical size, but only at the expense of more severe coil stresses.

Compared to the collection of design points summarized in Tables IV-I and IV-IV, the interim MSR design point proposed here on the basis of generally conservative assumptions represents a potentially attractive system of moderate size and favorable performance. The key limiting parameter is the

marginally acceptable maximum allowable value of beta. Because of the relatively idealized models upon which this design is based, it appears certain that modifications will occur through future, more detailed study. A primary task of this Level III study, however, is to assess quantitatively the engineering potential for the coil structure required to make a MSR a reality. The magnet assessment is given in the following section and is based on the interim design that has emerged from this study (Table IV-III).

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V. ENGINEERING CONSIDERATIONS

Figure IV-1 illustrates the iterative interaction between plasma, reactor, and coil performance that has served as a guiding philosophy for this study. This Level III study has focused primarily on the interaction between plasma and coil performance for the Modular Stellarator Reactor (MSR), with considerations of reactor performance being more or less specified as desirable goals. This section addresses the engineering aspects of the MSR coil design as related to the physics design point selected in Sec. IV.D. On the basis of this magnetics design, a preliminary MSR engineering configuration is proposed. No quantitative costing has been performed by this study; this aspect of the study along with detailed conceptual engineering design must await future work. Although the focus of this section is placed on the magnetics and magnet design (Sec. V.A.), preliminary work on the overall MSR nuclear-island layout is reported (Sec. V.B.). Section V.C. also gives preliminary consideration of MSR operational modes.

A. Magnetics and Magnet Design

The performance of the MSR depends crucially upon the ability of the modular-coil configuration to produce the magnetic topology required for plasma confinement, under the constraints imposed by subsystems integration and structural mechanics. Although a detailed coil design is not within the scope of this reactor survey, attention has been given to the development of the computational tools required to perform the following calculations and to address related magnetic issues.

- magnetic forces on the coils
- $\text{mod-}|B|$ contours
- field-line tracing in the context of
  - field ripple
  - magnetic surfaces
  - rotational transform and shear
  - magnetic well (hill) magnitude.

The detailed and expensive magnetics computations are decoupled, where possible, from the reactor survey calculations reported in Sec. IV.C. Instead, approximations are used when possible. These methods and interim results for the MSR design point reported on Table IV-III are described in

this subsection. Preliminary results from the more detailed magnetics and magnet-design codes are also given.

The present study focuses on the circular-bore, modular-coil configuration. It is possible to envisage an elliptical-bore coil for the $\ell = 2$ case and a trefoil-bore coil for the $\ell = 3$ case in order to improve the utilization of magnetic volume and to obtain higher rotational transform for a given lateral coil distortion. Such coil modifications imply correspondingly noncircular cross sections for the blanket and shield components and the first wall. Although not foreclosing the option for performance improvement using these modifications, this study has focussed on the simpler, circular-bore configuration. The numerical tools developed for this study can readily be extended to the more complicated configuration should this refinement and the increased design complexity be deemed desirable or necessary.

1. Magnetics. A high-aspect-ratio MSR coil set will share many of the mechanical-design attributes of a linear system (e.g., the central cell of a Tandem Mirror Reactor¹). In the context of a stellarator/torsatron confinement approach, the MSR is anticipated to have more favorably oriented coil force components and improved access and maintainability compared to a continuous helical-winding configuration.²

1.1. Force and Field Calculations. The usual stellarator expansion gives the following expression for the magnetic field strength near the magnetic axis.

$$B \approx B_0 [1 - \epsilon_t \sin\theta - \epsilon_h \sin(\ell\theta - m\phi)] \quad , \quad (V-1)$$

where B_0 is the on-axis magnetic field strength, ϕ is the toroidal angle, θ is the poloidal angle, and the variations in the field strength caused by the toroidal effect and the helical field component are given by ϵ_t and ϵ_h , respectively. Figure V-1 gives the general coil configuration and geometry being examined here. A simplified representation for $B_0(T)$ is that for the torus of N turns and major radius, $R_T(m)$, with a uniform azimuthal surface current, $I(A)$, such that

Fig. V-1. Magnetic geometry used in MSR analysis and graphical definition of key variables.

$$B_o = \frac{\mu_o NI}{2\pi R_T} \quad . \quad (V-2)$$

The maximum field strength experienced by the coil on the inboard side of the torus (i.e., $\theta = 3\pi/2$) becomes an engineering design limit. The peak field, $B_c(T)$, at major radius $R = R_T - r_c$, therefore, is given by

$$B_c = \frac{\mu_o NI}{2\pi(R_T - r_c)} \quad . \quad (V-3)$$

Corrections have been applied^{3,4} to Eq. (V-3) in order to account for finite coil cross sections and discrete coil spacing. The corrected form of Eq. (V-3) becomes³

$$B_c(R) = \frac{\mu_o NI}{2\pi R} \left[1 + L''\left(\frac{Nc}{\pi R}\right) + \frac{R}{Nr_c} \ln\left(\frac{1.284R}{Nc}\right) \right] \quad , \quad (V-4)$$

where the radius, R , is evaluated at the inboard, inside edge of the coil, $R_T - r_w - \Delta b$, and $c = 0.573 \delta_c$, where $\delta_c(m)$ is the thickness of a square cross-section coil, $r_w = r_p/x$ is the first-wall radius, and Δb is the combined thickness of the blanket and shield. The auxiliary function, $L''(\gamma)$, is approximated³ to within 10% by

$$L''(\gamma) = \frac{1}{\pi\gamma} + \frac{\pi\gamma}{12} - \frac{1}{2} \quad (V-5)$$

for $\gamma < 0.7$, where the argument γ equals $Nc/\pi R$.

Using similar arguments, the mean force per unit length, $f_L(N/m)$, experienced by the coil at major radius, $R(m)$, can be approximated by

$$f_{\ell} = \frac{\mu_0 N I^2}{4 \pi R} \quad . \quad (V-6)$$

Adding corrections for finite coil cross section and discrete coil spacing, Eq. (V-6) becomes

$$f_{\ell} = \frac{\mu_0 N I^2}{4 \pi R} \left[1 + \frac{1}{N} \left(\cos\left(\theta + \frac{\pi}{2}\right) + \frac{R}{r_c} \ln \frac{1.284R}{Nc} \right) \right] , \quad (V-7)$$

where again, $c = 0.573 \delta_c$, but R is evaluated at $R_T + r_c \sin \theta$.

The above development is strictly valid only in the limit of zero lateral distortion (i.e., $d/r_c \rightarrow 0$), when applied to the MSR configuration. The reactor survey code (Sec. IV.B.) returns a value for B_0 required to achieve an ignited plasma burn. Equations (V-4) and (V-7) are used to monitor the performance of the design point relative to this engineering constraint. The basic computational tool available for this purpose is the three-dimensional computer code EFFI.⁵⁻⁷ For present purposes, each coil is represented by a homogeneous (including conductor, internal structure, and dewar), square-cross-section coil, as illustrated in Fig. V-2. More elaborate and expensive computational representations are possible, but the extensive use of these models was deemed unnecessary at this level of design. Each coil is subdivided azimuthally into a series of 45 general current elements⁶ in order to approximate closely the three-dimensional geometry. The EFFI code computes and reports $\vec{J} \times \vec{B}$ force components acting at the centroids of each of these elements.

Because the input data set required by EFFI to characterize the MSR geometry typically exceeds several thousand lines, a preprocessor code, EFFIN, has been developed and implemented in the course of this study to generate input data in a format that is compatible with EFFI. The EFFIN preprocessor code requires twelve input parameters to characterize the MSR coil set, thereby greatly simplifying the operation of EFFI. The force output from EFFI can be used in the three-dimensional stress calculations. Data transfer from EFFI to the SAP4 stress code⁸ is accomplished using adaptations of the codes MESHGEN⁹ and SHAPE,¹⁰ which have been integrated into the SAPIN code, the

Fig. V-2. MSR magnetics computations performed for this study assume a homogenized, square-cross-section coil with thickness δ_c . The coil is subdivided azimuthally into 45 finite elements. The internal coil schematic is not drawn to scale.

latter code being a postprocessor for EFFI and a preprocessor for SAP4 developed in the course of the present study.

Figure V-3 reports the results of an EFFI computation of $\text{mod-}|B|$ contours in the equatorial plane (Fig. V-1, $z = 0$) for a sector of the MSR torus such that $15.0 < x < 30.0$ m and $0.0 < y < 15.0$ m. The major radius of this MSR design point is $R_T = 21.35$ m. The first three coils of the $\ell = 2$ modular-coil set intersect the x-y plane at locations indicated by the squares in Fig. V-3. The coil radius is $r_c = 4.60$ m, and the coil thickness is $\delta_c = 1.51$ m, which corresponds to a coil current density of 12.5 MA/m^2 . The x-y grid resolution is $\Delta = 0.05$ m in both x and y directions, which is sufficiently fine to resolve the locations of peak magnetic field strength within the homogenized modular coils. Contours of constant $|B|$ in the range of 1-10 T are displayed. Figure V-4 presents the magnetic field strength, $|B|$, as a function of x for $y = z = 0.0$. This path cuts directly through the center of the first coil. The on-axis magnetic field is $B_0 \sim 5.9$ T at $x = R_T = 21.35$ m, and the peak

Fig. V-3. Mod- $|B|$ contours in the equatorial plane ($z = 0$) for a sector of the interim MSR design point and a typical vertical plane ($y = 0, \phi = 0$). The major radius is $R_T = 21.35$ m. Refer to Fig. V-1 for geometry and variable notations.

Fig. V-4. Local-magnetic field strength $|B|$ as a function of x at $y = z = 0$ at the MSR interim design point. Refer to Fig. V-1 for geometry information.

magnetic field strength is $B_c \approx 10.6$ T for $x = R = 17.50$ m at the inner surface of the inboard side of the coil. The on-axis value is $\sim 7\%$ below the value required for ignition, as listed in Table IV-III, and the peak value is also $\sim 7\%$ under the estimate made using Eq. (V-4) and reported in Table IV-III. These discrepancies can be eliminated by slight increases in either the coil current density or cross-sectional area. The on-axis field ripple, as calculated by EFFI, is $\pm 2\%$ from the average on-axis value of ~ 5.9 T, with the highest field strength values occurring under each coil and the lowest values occurring at the midplanes between coils.

The mean force per unit length, f_ℓ , computed by EFFI will similarly tend to underestimate results from the reactor survey code. Figure V-5 displays f_ℓ as a function of azimuthal angle, θ , determined from the EFFI computation for the interim $\ell = 2$ design point. In the coordinate system of Fig. V-1, the angle $\theta = 0(\pi)$ corresponds to the top (bottom) of the coil and $\theta = \pi/2(3\pi/2)$ represents the outboard (inboard) sectors of the coil. Shown is the lateral force component, which tends to increase the lateral deformation of the coil and is smaller than the radial force component. Also shown is the vector sum of these two force components and the result predicted by Eq. (V-7), which is seen to be of modest value. The peak value of the radial component of f_ℓ occurs along the inboard section of the modular coil, but is displaced by $\pm\pi/4$ from $\theta = 3\pi/2$. Figure V-6 displays the corresponding mean force, f , as a

Fig. V-5. EFFI-generated mean coil force per unit length, f_ℓ , as a function of azimuthal angle for a typical coil of the interim MSR design point. The dotted curve is the Eq. (V-7) result.

Fig. V-6. EFFT-generated mean coil force, f , as a function of azimuthal angle, θ , for a typical coil of the interim MSR design point.

function of azimuthal angle, θ , which is also evaluated at the centroids of the EFFT finite-element representation of the modular coil.

Table V-I summarizes the results of the magnetics/force calculations made for the interim MSR design point. As a point of reference, the value of maximum force per unit length reported¹¹ for the $\ell = 3$ T-1 torsatron conceptual design point with helical coils is ~ 30 MN/m for $R_T = 29.2$ m, $B_0 = 5$ T, $r_c = 4.0$ m, $I = 36.5$ MA, $j = 30$ MA/m², and $N = 20$ turns.

1.2. Flux Surfaces and Rotational Transform. The EFFT code has been used to follow the trajectories of a magnetic field line originating at a point $P(x,y,z) = P(x,0,0)$ (Fig. V-1). The point of origin is positioned at various values of minor radius, r , from near the magnetic axis to well into the blanket and shield annulus, but within the maximum separatrix radius, where $r_s \sim r_c$. The total length followed along the field line is $L = 2000$ m, compared to the nominal toroidal circumference of ~ 135 m. If the intersection of the field line with a given plane (e.g., $y = 0$) is tracked, a

TABLE V-I
INTERIM MSR DESIGN-POINT MAGNETICS RESULTS (CIRCULAR BORE)

<u>Parameter</u>	<u>Value</u>
Major toroidal radius, R_T (m)	21.35
Coil current center radius, r_c (m)	4.60
Coil thickness and width, δ_c (m)	1.51
Coil current density, j (MA/m ²)	12.5
Coil current, I (MA/coil)	28.5
First-order coil distortion, d/r_c	0.3
Higher-order coil distortion, d'/r_c	0.0
Number of poloidal-field periods, l	2
Number of toroidal-field periods, m	8
Total number of coils, N	24
Number of coils per field period, N/m	3
On-axis magnetic field, B_0 (T)	6.37/5.9±2%(EFFI)
Peak field at coil, B_c (T) [Eq. (V-4)]	11.3/10.6(EFFI)
Coil volume, V_c (m ³ /coil)	70.
Summary of mean of coil forces per unit length:	
$\theta = 0(\pi)$ (top):	
Radial, f_r (MN/m)	68.
Lateral, f_y (MN/m)	1.1
B (T), $r = r_c$	--
$\theta = \pi/2$ (outboard):	
Radial, f_r (MN/m)	56.
Lateral, f_y (MN/m)	-9.0
B (T), $r = r_c$	~2.0
$\theta = 3\pi/2$ (inboard):	
Radial, f_r (MN/m)	86.
Lateral, f_y (MN/m)	-6.5
B (T), $r = r_c$	~3.5
Net force on coil (Fig. V-1):	
F_x (MN) (centering force)	-210.
F_y (MN)	-0.5
F_z (MN)	-3.6

magnetic surface is traced out. As a demonstration of this procedure, the University of Wisconsin XPL0T code¹² was used as an EFEL postprocessor to generate sample magnetic surfaces. Figure V-7 displays representative nested magnetic surfaces generated for the interim MSR design point. The quasi-elliptical shape is characteristic of the $\ell = 2$ stellarator configuration. These nested magnetic surfaces are used to obtain radial profiles of ψ and magnetic-well properties. The outer two magnetic surfaces appear not to close upon themselves only because the field lines used to obtain them were not tracked a sufficient number of times around the major axis of the torus to close the surfaces. Figure V-8 displays the results of rotational transform calculations for the interim design point. The dashed curve with $\psi(0) \sim 0.62$ is the result of the conventional stellarator expansion [Eq. (B-52) of Appendix B] that is the basis of the equilibrium/stability limit development leading to the value $\langle \beta \rangle \approx 0.04$ for $\ell = 2$, $m = 8$, and

Fig. V-7. Typical magnetic surfaces for the $\ell = 2$ interim MSR design point with the separatrix radius $r_s \sim r_c$.

$A = 11$. The solid curve with $\alpha(0) \approx 0.15$ is the result of the application of the superposed helix/antihelix representation of the modular-coil configuration for $d/r_c = 0.3$ and $N = 24$, as described by Eqs. (B-46), (B-49), and (B-51) of Appendix B. For the parameters of the interim design point, the modular-coil configuration does not introduce sufficient helical current and corresponding rotational transform to reproduce the results of the conventional stellarator expansion with the same ℓ and m values. Also plotted in Fig. V-8 are the results of the EFFI magnetics computation (circles), the results of a single-filament PPPL numerical model (triangles), and a PPPL

Fig. V-8. Rotational-transform profiles using Eq. (B-51) for the $\ell = 2$ MSR design point. The $d/r_c = 0.3$, $m = 8$ interim design point yields $\alpha \sim 0.15$ (solid curve) as confirmed by EFFI(circles) and PPPL codes (squares and triangles). The required value, $\alpha \sim 0.66$, is expected to be achievable by lowering m and by introducing a higher-harmonic coil distortion to give equivalent performance to that with $d/r_c \sim 0.5$, which corresponds to the analytic stellarator expansion of Eq. (B-46B) (dashed curve).

series expansion model (squares) that confirm the lower transform results of the superposed helix/antihelix representation.

Using the EFFI results, the properties of the MSR magnetic well (hill) can be examined by usual techniques.¹³ The quantity

$$V'(r) = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L \frac{dl}{B}, \quad (V-8)$$

which denotes the specific volume enclosed by a flux surface, is evaluated for each flux line at an average minor radius, r , and integrated along a distance, L (typically ~ 2000 m). In the limit $L \rightarrow \infty$, the magnetic-well (hill) magnitude is given by $V'' = \delta V'(r)/V'(0)$, where $\delta V'(r) = V'(0) - V'(r)$, and $V'(0)$ in Eq. (IV-8) is evaluated near the magnetic axis ($r = 0$). Using the

Fig. V-9. Magnetic-hill magnitude for the interim MSR design point of Table IV-III as generated from EFFI code results.

four innermost magnetic surfaces of Fig. V-7, Fig. V-9 indicates the presence of a weak magnetic hill for the interim MSR design point.

Generally, most of the preliminary magnetics computations made for the interim MSR design point of Table IV-III confirm the indications of simple analytic models. The only serious discrepancy identified to date is the computation of the rotational transform using the conventional stellarator expansion versus models developed specifically for the modular-coil configuration. The discussion in Sec. B.3. of Appendix B, as well as other treatments of the Rehker-Wobig coil configuration,^{14,15} are based on the usual stellarator expansions¹⁶ and are strictly applicable to the modular-coil configuration depicted in Fig. III-3. A more careful representation¹⁷ of the Rehker-Wobig coil using the superposition of two sets of helical windings of different pitch and antiparallel current flow yields rotational transform values that are considerably lower (i.e., $\iota \approx 0.15$) for a single-harmonic distortion and are confirmed by the EFFI code results (Fig. V-8). The transform profile has a positive but low shear near the magnetic axis, which drops off as the minor radius approaches r_c . The incorporation of finite cross sections for the coil also tends to lower the allowable lateral coil distortion and thus reduces the resultant transform. It is expected that, for a given first-harmonic lateral coil distortion of $d/r_c \approx 0.3$, the rotational transform can be raised by the addition of higher-harmonic components without significantly perturbing the basic interim MSR engineering layout. A preliminary, numerical treatment of the higher-harmonic winding law is considered in Sec. B.8. of Appendix B. In addition, it may be necessary to raise the value of N/m by either decreasing m or increasing N . Last, the simplified (and conservative) theory used to relate $\langle \beta \rangle$ to ℓ , m , and A can be re-examined in order to determine whether more exact stability and equilibrium constraints will permit $\langle \beta \rangle \gtrsim 0.04$ at lower values of m . Many of these latter predictions are dependent upon pressure and current profiles, which must also be examined in more detail; as shown in Sec. B.4.2. of Appendix B, the simplified stability limits applied here assume a flat current profile associated with pressure-driven currents. A considerably more sophisticated analysis is required to resolve better these important issues.

In summary, the approach adopted by this scoping study is to retain the relatively optimal coil geometry indicated in Fig. V-1 for the MSR engineering design under the assumption that more favorable rotational-transform and

magnetic-well properties can be found for coil configurations that can be fitted into the nominal geometry indicated in Fig. V-1. Simultaneously, the simplified equilibrium/stability theories used herein to specify the rotational transform will be re-examined with the goal of reducing the required rotational-transform values somewhat. The alternative approach would examine more complex plasma/coil configurations, an approach that would be adopted only if more extensive studies eliminate the simpler geometries examined thus far.

1.3. Stress Analysis of Design-Point Coil Configuration.

1.3.1. Preliminary Stress Analysis. Before performing a three-dimensional stress analysis using the SAP4 code,⁸ which requires specification of coil support locations derived from an integrated systems design, stress analyses of the MSR coil configuration were limited to estimates of the effective stress at the centroids of the finite elements used in the force calculations (Sec. V.A.1.1.). Material properties¹⁸ and design goals¹⁹ have been developed and implemented in the context of the overall Magnetic Fusion Energy program and can be adapted for use in the present study. Magnet technology required for the MSR does not appear to be particularly exotic.

For present purposes the mean stress, $\langle \sigma \rangle$ (Pa), experienced by the MSR coil is estimated from the approximation

$$\langle \sigma \rangle = \frac{f_{\ell} r_c}{\delta_c^2}, \quad (V-9)$$

where r_c (m) is the mean coil radius, δ_c (m) is the thickness of the square-cross-section coil, and f_{ℓ} (N/m) is the mean force per unit length as calculated from analytic approximations or from the EFFI magnetics code (Sec. V.A.1.2.). A nominal coil stress of $\hat{\sigma} = 200$ MPa (~ 30 kpsi) corresponds to 0.1% strain in the stainless-steel internal support, and is selected as a conservative limit for use in monitoring the structural performance of the MSR coil configuration for any particular interim design point. Figure V-10 displays the dependence of the mean stress as a function of the azimuthal angle, θ , for the interim MSR design point of Table IV-III. The stress curve is calculated using the mean force per unit length, f_{ℓ} (Fig. V-5), generated by the EFFI code. The dashed curve in Fig. V-10 represents the mean stresses as calculated from the force approximation represented by Eq. (V-7). The mean

Fig. V-10. Mean coil stress, $\langle \sigma \rangle$, results from EFFI calculations and Eq. (V-9) as a function of azimuthal angle, θ , for a typical coil of the interim MSR design point. The dotted curve is the estimate using Eqs. (V-7) and (V-9).

stresses fall at or below the imposed limit, but no additional design margin is available to accommodate variations in stress across the coil thickness or for the usual engineering safety factors. The angle $\theta = 0(\pi)$ corresponds to the top (bottom) of the coil and $\theta = \pi/2$ ($3\pi/2$) represents the outboard (inboard) sector of the coil.

Fully implemented,²⁰ the SAP4 code calculates the stress components that constitute the effective (Von Mises) stress,

$$\sigma_{\text{eff}} =$$

$$[\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{yy}\sigma_{xx} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)]^{1/2}, \quad (\text{V-10})$$

where σ_{xx} , σ_{yy} , and σ_{zz} are the normal stresses and τ_{xy} , τ_{xz} , and τ_{yz} are the

shear stresses. These results are then compared to the yield stress of the coil structural material. Before the SAP4 code can be applied to this problem, a means for supporting the MSR coil structure must be postulated. Such a support structure is postulated in the following subsection in order to lay groundwork for SAP4 calculations.

1.3.2. Postulated Coil Support Structure. The MSR coil configuration produces radial forces, which are directed exclusively outward, and lateral forces, which tend to increase the nominal lateral distortion of the modular coils. External support structures that are compatible with the modular nature of the device must be provided to react these forces. Preliminary consideration of this issue suggests that each modular coil be equipped with a number (e.g., ≤ 6) of gimbal mounts spaced equally about the outer circumference. These mounts may be used to couple the coil to external load-bearing structures that take up the net centering force on each coil as well as providing support from below to react the gravitational loads. In addition, these gimbal mounts can be used to connect demountable intercoil structural members between adjacent coils to react the lateral force components. The structural components of the MSR must be configured to minimize interference with heating, vacuum, and coolant access requirements and must be conveniently and quickly demountable if the modularity and replaceability advantages of the device are not to be unduly compromised. The specific implementation of this concept is considered in the following subsection.

B. Preliminary Indications of Reactor Configuration

The MSR interim design point embodies the steady-state, ignited operation of a DT thermonuclear plasma in a toroidal device of moderate aspect ratio ($A = R_T/r_p = 11$). Except for startup power requirements, therefore, an ignited burn implies operation with low recirculating power beyond that required for major power users, such as the cryogenic refrigeration of the superconducting coils. Steady-state operation without plasma disruptions can be expected to minimize thermal cyclic fatigue of reactor components. Modularity of the coil set allows exoreactor testing of components to improve reliability and to assure more rapid change-out in the event of coil failure. No obstacles to plasma fueling by means of pellet injection and impurity and ash removal by means of either a magnetic divertor or a pumped limiter have been identified in this study. A marginal value for the average plasma beta

can be offset by a higher on-axis confining magnetic field in a device of moderate aspect ratio without exceeding peak magnetic field limitations at the inboard side of the coils. Coil forces appear manageable in both magnitude and orientation. Operation of the MSR coil at a peak field of ~ 11 T will require Nb_3Sn superconducting magnet technology or the supercooling of a NbTi conductor.

Modularity for the MSR may imply the ability to remove and to replace efficiently a single coil with minimal disturbance to the neighboring coils. Based on a nominal average coil density of 2.5 tonnes/m^3 (Ref. 1), the mass of a coil with $V_c = 70 \text{ m}^3$ is estimated at ~ 175 tonnes. An additional desirable feature in promoting high plant availability would be the ability to replace blanket and shield modules without moving the coils.² This issue for the MSR has not received detailed attention to date, but may be difficult in the MSR configuration even for $d/r_c \lesssim 0.3$. In the worst case the unit module would consist of a single modular coil with the blanket and shield modules situated within; the total mass of the integrated module would be approximately 300 tonnes.

The major circumference of the interim design point is ~ 135 m, such that, for $N = 24$ coils, each module would measure approximately 5.6 by 5.4 by 5.4 m in length, width, and height, respectively. Increasing the number of coils to $N = 32$, which is the next highest multiple of $m = 8$ allowed when the number of coils per field period, N/m , is fixed at 3, would reduce both the size and mass of the individual module and would also reduce the magnetic field ripple. However, in order to fit 32 coils with $d/r_c = 0.3$ into the same reactor volume for a fixed $P_{\text{TH}} = 4.0$ Gwt, the average coil current density must be increased from 12.5 MA/m^2 to 32.5 MA/m^2 . Retention of the lower current density as a constraint forces the $N = 32$ system to a larger major radius and a power output that would be in excess of 6 Gwt. In addition, the overall reliability of the coil set is degraded somewhat as the number of independent components is increased.

Figure V-11 illustrates a highly schematic layout of the MSR module. The coils are supported against the net centering forces by leaning against a solid central core. Gimballed supports at the top and bottom of the coil are indicated. Removal of modules would entail decoupling of the support structure at the gimbal mounts followed by a radially outward translation. Although not yet investigated in detail, access for vacuum, fueling,

Fig. V-11. Preliminary reactor layout based on the interim MSR design point for use in examining coil support structure and intercoil forces. Elevation and equatorial-plane views of a sector of the reactor are shown.

electrical leads, and coolant pipes in this moderate-aspect-ratio device appears straightforward and flexible. One option would be to concentrate all access requirements into wedge-shaped submodules (Fig. V-11) that would serve as interfaces between right-circular-cylindric coil, blanket, and shield modules. The wedge-shaped region could itself be considered a moveable module or could be fixed to an adjacent coil/blanket and shield module. The wedge-shaped region would contain the pumped-limiter impurity-control mechanism²¹⁻²² and all heating/fueling/vacuum/coolant penetrations and external connections. This preliminary module layout is compatible with the structural support considerations discussed in Sec. V.A.1.3.2. It is emphasized that the primary intent of the reactor layout given in Fig. V-11 is to provide a relatively "self-consistent" model for the modular-coil supports with which the SAP4 stress calculations can be made; considerably more conceptual design of all MSR components is required before a totally self-consistent design can be proposed.

An electric generating plant with a total thermal power output, $P_{TH} = 4.0$ Gwt, will produce a gross electric power output, $P_{ET} = 1.4$ GWe, for a nominal thermal conversion efficiency, $\eta_{TH} = 0.35$. A fraction, f_{AUX} , of the gross electric power must be recirculated within the plant to drive auxiliary systems such as coil refrigeration, vacuum systems, and coolant pumps. An allowance²³ of $f_{AUX} = 0.08$ for these purposes in an ignited MSR systems leaves a net power output of $P_E \approx 1.3$ GWe. No unique requirements for the balance of plant (BOP) are anticipated, although, again, more detailed conceptual design of key MSR systems is required.

C. Preliminary Consideration of Modes of Operation

The question of operational mode for any magnetically confined fusion reactor is centered around the following issues: pulsed versus steady-state plasma, driven high-Q versus ignited operation, refueling mechanism, impurity-control scheme, and plasma startup and shutdown procedure. Each of these issues will strongly impact the reactor design and cost of electricity (COE), and each is determined by physical phenomena that to date are computed or extrapolated from experimental observation. The uniqueness of the stellarator/torsatron approach in this respect rests with the generation of the full magnetic-field topology solely by external electrical conductors. Even the steady-state ELMO Bumpy Torus approach²⁴ relies on significant, local perturbations of the field topology by plasma processes (i.e., the energetic

electron rings). Furthermore, because the stellarator/torsatron reactor may be an inherently low-beta device (i.e., $\langle \beta \rangle < 0.05$), the vacuum field topology is expected to be left relatively unperturbed by the presence of plasma. These factors impinge on the reactor mode of operation by suggesting a steady-state plasma that would more than likely be achieved through a low-density startup on existing, relatively unperturbed field lines.

The nature and level of the external power requirement needed to achieve a desirable steady state is determined primarily by the particle and energy transport scaling, which also determines the thermal stability of that steady state. Both Alcator and Bohm-like scalings, when used in conjunction with the pressure-balance requirement, predict for a given beta that $\tau_E \propto B_0^\eta r_p^2 / T$, where $\eta = 2$ for Alcator scaling and $\eta = 1$ for Bohm-like scaling; both transport scalings predict a $1/T$ dependence of τ_E , a dependence that presents a sufficiently strong negative feedback to allow a thermally stable ignition as well as promising lower startup power requirements. On the other hand, neoclassical transport theories in certain collisionality regimes can lead to a positive temperature coefficient, thermal instability, and perhaps the need to operate the plasma in a subignited but high-Q (where Q is the ratio of fusion power output to driver input power) driven mode. The nature of the transport scaling during the low-density initiation and subsequent density buildup to an ignited or high-Q driven mode will also impact the operating mode, in that the delivery systems for the startup power can affect the reactor design while not necessarily impacting the steady-state energy balance per se.

When combined with a goal to generate electricity at power levels of approximately 1 GWe with a fusion-neutron first-wall loading of $\geq 1\text{--}2 \text{ MW/m}^2$, the stability and equilibrium scaling of beta and plasma aspect ratio used for stellarator/torsatron reactors generally leads to plasma densities of $\geq 1(10^{20}) \text{ m}^{-3}$, moderate temperatures (8-15 keV), and large minor radii ($\geq 2 \text{ m}$). For these conditions, edge refueling is not possible and high-velocity ($> 10^4 \text{ m/s}$) pellet injectors may be needed.

The issue of transport in plasmas with these parameters also impacts on the method by which impurity and helium-ash levels will be controlled. Impurity and ash control at the plasma edge in stellarator/torsatron reactors can be achieved by either pumped limiters or magnetic divertors. Although the latter approach can be a natural consequence of the stellarator/torsatron

magnetic-field topology, the engineering convenience and feasibility of extracting open field lines to an adequately engineered divertor plate and vacuum region depends crucially on the coil configuration. Furthermore, location of the separatrix at or near the coil, rather than within the vacuum first wall, may offer some advantage in maximizing the plasma volume utilization within the first wall and minimizing the complexity of the blanket and shield design. Hence, the use of pumped limiters versus (natural) magnetic divertors appears strongly dependent upon the specific stellarator/torsatron configuration and remains to be fully quantified.

In summary, those unique characteristics of the stellarator/torsatron approach that are related to low-beta plasma confined within an externally produced field topology almost certainly will lead to a steady-state operating mode for the reactor. The issue of driven high-Q versus ignited operation depends crucially upon the energy transport scaling, the related thermal stability of the burn, and the ability to refuel and control impurity levels by external means. As for most approaches to magnetic fusion, the latter issue remains to be understood and quantified fully in the reactor context, although the choice between (natural) magnetic divertors (i.e., separatrix near the first wall) versus pumped limiters (i.e., separatrix near the coils) represents an option for the stellarator/torsatron. Improvements in reactor performance for $r_s \sim r_c$ (Fig. IV-12) have prompted consideration of pumped-limiter impurity control for purposes of this study.

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VI. SUMMARY AND CONCLUSIONS

This preliminary (Level III) survey study of the Modular Stellarator Reactor (MSR) is the first phase of an assessment that quantifies parametrically the reactor potential for this innovative coil concept where appropriate performance goals and constraints have been imposed. A broad survey of physics parameter tradeoffs has been made and has been focused onto the identification of a self-consistent interim design point. Table VI-I summarizes the key reactor parameters that have emerged from this study. On the basis of generally conservative assumptions, but without supporting economic analyses, the design point appears to be both viable and competitive with other approaches to magnetic fusion. A number of numerical computational tools have been integrated into a systems code for detailed analysis and optimization of the MSR concept.

On the basis of the results obtained in this preliminary study, it is recommended that investigation of the modular stellarator reactor configuration be continued at a higher level (e.g., Level I). Such a study can focus on those issues that were beyond the scope of this Level III analysis to further refine and optimize the concept. The effort expended in

TABLE VI-I
INTERIM DESIGN POINT FOR AN $\ell = 2$, $m = 8$, $N = 24$
MODULAR STELLARATOR REACTOR

<u>Parameter</u>	<u>Value</u>
Major radius, R_T (m)	21.35
First-wall radius, r_w (m)	2.35
Average plasma temperature, $\langle T \rangle$ (keV)	8.0
Average plasma ion density, $\langle n \rangle$ ($10^{20}/\text{m}^3$)	1.72
Average plasma beta, $\langle \beta \rangle$	0.04
On-axis magnetic field, B_0 (T)	~ 6.37
Coil radius, r_c (m)	4.6
Coil distortion, d/r_c	0.29
Fusion-neutron first-wall loading, I_w (MW/m^2)	1.5
Thermal power, P_{TH} (Gwt)	4.0
System power density, p_s (MWt/m^3)	0.33

this study on numerical methods development can be used to advantage in any follow-on effort.

The timing of this study is significant, occurring as it does in a period of enthusiasm and optimism generated by recent encouraging experimental results from the Wendelstein VII-A and the Heliotron-E experiments. This study is aimed at drawing the attention of the experimental and theoretical physics communities to those critical issues that will ultimately determine reactor attractiveness and which, therefore, should be addressed as soon as is practicable. The following major conclusions are drawn from this Level III scoping study of the MSR.

- Marginally attractive values of average beta, as allowed by approximate and self-consistently applied equilibrium and stability limits, are a key limiting factor in MSR performance. Higher beta values would lead to lower confining fields, reduced coil forces and stresses, and generally smaller systems with higher power density. The stability and equilibrium beta limits used in this study are based on a simplified theory of diffusion-driven (toroidal) currents and may not be generally applicable. Nevertheless, these beta limits are considered to represent conservatively low bounds on beta. In spite of the application of these limits, the MSR design point reported herein appears to present no insurmountable coil problems of a technological nature.
- Application of other conservative assumptions and constraints (e.g., $f_\alpha < 1$, $p_\alpha/p > 0$, $j = 12.5 \text{ MA/m}^2$) still allows the identification of potentially attractive MSR design points with moderate power output ($P_{TH} \leq 5 \text{ GWt}$), while self-consistently meeting key stellarator physics constraints in modular engineering configurations with maintenance and reliability advantages.
- Preliminary magnetics and coil-stress computations indicate MSR systems can be constructed with manageable structural requirements and accessibility. The conventional stellarator expansion should be used with caution for the modular-coil configuration, as it tends to overestimate the transform available from modestly-deformed coils. This coil design used as an engineering model for this study, however, falls short by a factor of ~ 4 in producing the transform predicted to be necessary on the basis of simplified theories of equilibrium/stability beta limits. Recommendation for future work in rectifying these differences include:
  - Small increases in the first-harmonic lateral coil distortion, d/r_c , with a more carefully formulated coil interference constraint than that developed in Sec. B.5. of Appendix B.
  - Application of a higher-harmonic lateral coil distortion with use of the same coil-interference constraint.

- Modest increases in the lateral coil distortion for the same values of m and N with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
  - A higher value of N for the same value of m to increase N/m with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
  - A lower value of m for the same value of N and lateral coil distortion to increase N/m at the expense of modifications in r_p/r_w , and possibly a different optimal value of aspect ratio.
  - Use of coils with noncircular bores (i.e., elliptical bore for $\ell = 2$ and trefoil bore for $\ell = 3$).
  - Combinations of the above options.
- The use of higher harmonic distortions shows little improvement in increasing the rotational transform from ~ 0.15 to the design goal of $0.6-0.7$. Greater coil distortions ($d/r_c \approx 0.5$) would increase the transform to ~ 0.66 , but coil interference constraints then drive the coil current density above the nominal design limit of 15 MA/m^2 .
  - The use of smaller values of m and higher values of N , in conjunction with higher harmonic distortions and larger values of d/r_c , would lower both the needed rotational transform and allowable beta, while the aspect ratio would be increased somewhat (i.e., increased total power). These questions, as well as the adequacy of the simplified equilibrium/stability theory used to generate them, remain unresolved.
  - Properly adjusted analytical and numerical magnetics models used to estimate vacuum rotational transform profiles showed good agreement.
  - A pumped-limiter impurity-control scheme may improve MSR performance over that with a magnetic divertor that is traditionally associated with the stellarator/torsatron configuration. A detailed tradeoff study of the feasibility and problems of leading open field lines to a divertor plate versus the advantage of higher plasma filling fraction and uncertainties associated with the pumped-limiter approach remains to be performed, however.
  - The MSR survey study is based on the applicability of Alcator (empirical) transport scaling, which was shown to give an energy confinement time that is a factor of ~ 60 greater than Bohm transport, a factor of ~ 2 greater than neoclassical-plateau scaling, and a factor of ~ 10 less than classical transport. The level of energy loss predicted by Alcator scaling is viewed as an upper bound for MSR system viability.
  - Adequate justification exists for continued study of the MSR configuration at a higher level of conceptual engineering design, in addition to continued parametric study using improved physics models in order to develop a better resolved and perhaps an even more attractive reactor design point. Detailed engineering design should proceed, however, only after the relationship between equilibrium/stability, beta, rotational transform, and coil geometry are better resolved.

In order to achieve a "self-consistent" analytic determination of the MSR design point, both equilibrium and stability beta limits were imposed using simplified theories based on diffusion-driven plasma currents. Although subject to criticism, only this theory is presently available in analytic form and, therefore, useable in a parametric trade-off study. This theory predicts a maximum beta of ~ 0.04 for $m = 6-8$ when $l = 2$ and $A = 10-11$. The $l = 2$ case with $r_s = r_c$ was selected to maximize plasma filling fraction, and A in the 10-11 range is desirable from the viewpoint of total power ≤ 5 GWt at a neutron first-wall loading of 1-2 MW/m². For these conditions, the average rotational transform must be $\sim 0.6-0.7$, and the lateral coil distortion is limited to 0.3-0.4 by both superconductor current-density and coil-interference constraints. With these constraints imposed, however, the required $q \approx 0.66$ could not be achieved (i.e., $q \approx 0.15-0.30$) even with strong second-harmonic coil distortions. The magnetics solution to this problem would increase N and decrease m , an approach that would lead to lowered $\langle \beta \rangle$ limits if the simplified (diffusion-driven currents) equilibrium and stability model is retained. It is at this point where more exact stability and equilibrium computations, including plasma profile effects, must be used; future physics studies, performed in conjunction with the MSR study, will focus on this important area.

The approach adopted by this scoping study is to retain the relatively optimal coil geometry indicated in Fig. V-1 for the MSR engineering design under the assumption that more favorable rotational-transform and magnetic-well properties can be found for coil configurations that can be fitted into the nominal geometry indicated in Fig. V-1. Simultaneously, the simplified equilibrium/stability theories used herein to specify the rotational transform will be re-examined with the goal of reducing the required rotational-transform values somewhat. The alternative approach would examine more complex plasma/coil configurations, an approach that would be adopted only if more extensive studies eliminate the simpler geometries examined thus far.

Other key areas also remain to be examined prior to detailed engineering design of the MSR system. Specifically, future studies should emphasize the following areas.

- Refine and extend physics models:
  - beta scaling
  - transport scaling
  - confinement and beta of suprathreshold alpha particles
  - startup and burn control.
- Exercise an extended survey model for refinement of the interim design point on the basis of cost.
- Continue self-consistent coil design and further verification using appropriate magnetics and stress codes.
- Continue core component design emphasizing modularity and maintainability (i.e., first wall, blanket and shield, impurity control, heating, fueling).
- Address in detail other plasma engineering issues (e.g., alpha-particle trapping, impurity-control physics).
- Systems integration and costing:
  - layout of nuclear island
  - design and support of realistic coils
  - better resolve coupled interaction of coil configuration, blanket and shield design, divertor requirements, and total system modularity and accessibility
  - quantify balance-of-plant interfaces and layout
  - perform cost optimizations and identify key cost drivers.

For optimum use of limited resources, further reactor design work on the MSR should emphasize those features that follow directly from the concept itself, rather than the development of design details that may be more generic in nature. Specifically, if the generic reactor and BOP systems developed in conjunction with the more elaborate engineering studies of the STARFIRE tokamak^{1,2} and EBT^{3,4} are used for the MSR, a more meaningful comparison on the basis of cost and physics assumptions could be made.

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APPENDIX A:

FORCE CALCULATION FOR COIL PARAMETERS SUGGESTED BY PPPL

Researchers at Princeton Plasma Physics Laboratory (PPPL) have made a preliminary examination of modular stellarator systems using a filamentary approximation to characterize the coil set.¹ An early subtask of the present study was to assess the favorable results predicted by the PPPL calculations. In fact, the development of the simple physics models that have served as a basis for this Level III reactor survey (Appendix B) were generated in order to make more realistic these early force calculations of the PPPL configurations. This assessment of the PPPL modular stellarator coils used the EFFI-based code system,² which allows for finite-cross-section coils. Results of calculation for coil-set parameters provided³ by PPPL are reported in this Appendix primarily for historical reasons. These coil-set parameters were used as test cases for the development of the magnetic and stress code system used in the present MSR study while the parallel reactor survey was being developed.

For a circular-bore coil with a mean current radius, $r_c(m)$, and a square cross section of thickness, $\delta_c(m)$, the magnetic field strength, $B(T)$, as a function of major radius, $R(m)$, is approximately given by

$$B = \frac{\mu_0 N (j \delta_c^2)}{2 \pi R} . \quad (A-1)$$

The homogenized overall coil-current density is $j(A/m^2)$, such that the coil current is $I = j \delta_c^2$. The mean radial force per unit length, $f_l(N/m)$, at radius R in the limit $d/r_c \rightarrow 0$ is then approximately given by

$$f_l = \frac{\mu_0 N (j \delta_c^2)^2}{4 \pi R} , \quad (A-2)$$

and the corresponding mean stress, $\langle \sigma \rangle$ (Pa), can be estimated from

$$\langle \sigma \rangle = \frac{f_l r_c}{\delta_c^2} . \quad (A-3)$$

The three cases examined are summarized in Table A-I. These cases were used to test the simple relationship for the mean force per unit length [Eq. (A-2)] against the EFFI code results. The f_l results from the EFFI code are used to compute the stress estimates directly from Eq. (A-3) rather than from a subsequent SAP4 computation,⁴ which can be computationally expensive and, furthermore, requires the detailed specification of an external coil-support system. All three test cases preserve the common parameters recommended by

TABLE A-I
MSR^(a) TEST-CASE PARAMETERS SUGGESTED BY PPPL

<u>Case</u>	<u>1</u>	<u>2</u>	<u>3</u>
Bore	circular	circular	elliptical
Poloidal-field period number, ℓ	--	2	2
Toroidal-field period number, m	--	6	6
Number of coils, N	36	36	36
Major toroidal radius, R_T (m)	10.0	10.0	10.0
Coil current center radius, r_c (m)	2.25	2.25	2.25
Mean coil radius, $\langle r_c \rangle$ (m)	2.25	2.25	1.59
Ellipticity, e	1.0	1.0	0.87
Pitch angle, α' (radians)	--	0.44	0.44
Coil aspect ratio, $R_T/\langle r_c \rangle$	4.44	4.44	6.28
First-order coil deformation, d/r_c	0	0.10	0.20
Second-order coil deformation, d'/r_c	0	0.05	0.022
Coil current density, j (MA/m ²)	10.0	15.0	10.0
Coil thickness, δ_c (m)	0.82	0.58	0.82
Coil current, I (MA/coil)	6.6	5.0	6.6
On-axis magnetic field, B_0 (T)	4.8	3.6	4.8

^(a) PPPL researchers have used the parlance "Modular Twisted Torus (MTT)."

the PPPL group³: $N = 36$, $R_T = 10$ m, and $r_c = 2.25$. The lateral distortion of the coil is given by

$$y = d \sin \ell \theta + d' \sin 2\ell \theta, \quad (A-4)$$

where $\ell = 2$ for these cases, and θ is the azimuthal angle such that $\theta = 0(\pi)$ corresponds to $R = R_T$ and the top (bottom) of the coil and $\theta = \pi/2$ ($3\pi/2$) represents the outboard (inboard) sectors of the coil. Case 1 is the degenerate case of $d/r_c = d'/r_c = 0$ that yields a pure toroidal field, for which Eqs. (A-1) and (A-2) are most applicable. Case 2 describes a circular-bore coil with modest lateral coil distortion ($d/r_c = 0.10$, $d'/r_c = 0.05$). Case 3 represents a generalization to an elliptical bore coil, which for $\ell = 2$ is topologically compatible with the quasi-elliptical cross section of the plasma. A detailed investigation of this latter option was beyond the scope of this study. Also, the three test cases have small coil radii ($r_c = 2.25$ m), which together with a nominal blanket and shield thickness ($\Delta b = 1$ -1.5 m) are not sufficient to be of interest for a commercial reactor. For purposes of the EFFI computation, the coil is subdivided into 45 azimuthal segments of approximately equal length. Mean forces and stresses are reported at the centroids of the segments.

Figure A-1 summarizes the mean force per unit length, f_ℓ , and corresponding mean stress, $\langle \sigma \rangle$, as a function of azimuthal angle, θ . A positive radial force component is directed outward. Again, in the coordinate system of Fig. II-2, the angle $\theta = 0(\pi)$ corresponds to the top (bottom) of the coil and $\theta = \pi/2$ ($3\pi/2$) represents the outboard (inboard) sectors. Because lateral coil distortions for this case are zero, the lateral force per unit length is zero. The mean stress is well below a conservative stress limit of $\hat{\sigma} = 200$ MPa.

For the second test case given in Table A-I, the lateral coil distortion is $d/r_c = 0.10$ and $d'/r_c = 0.05$. For a higher average coil current density ($j = 15$ MA/m²) and smaller coil thickness ($\delta_c = 0.58$ m), the on-axis field strength is lower ($B_0 = 3.6$ T). The radial mean force per unit length and mean stress are correspondingly lower. As shown in Fig. A-2, the lateral force per unit length acts to increase the $\ell = 2$ lateral coil deformation.

Fig. A-1. Mean coil force per unit length, f_ρ , and corresponding mean coil stress, $\langle \sigma \rangle$, as functions of azimuthal angle for Test Case 1 of Table A-I.

Fig. A-2. Mean coil force per unit length, f_ℓ , as a function of azimuthal angle for Test Case 2 of Table A-I.

Fig. A-3. Mean coil force per unit length, f_ℓ , as a function of azimuthal angle for the elliptical bore Test Case 3 of Table A-I.

The lateral forces are smaller in magnitude than the radial forces. Equation (A-2) is no longer a good approximation to the radial f_r results.

For Test Case 3 of Table A-I (elliptical-bore coil), mean force per unit length results are summarized in Fig. A-3. Equations (A-2) and (A-3) do not apply to this case. The superposition of the $\ell = 2$ periodicity of the coil deformation onto the elliptical coil gives more variation to the mean force per unit length, f_ℓ , than was found for the circular-bore Test Case 2 of Table A-I. In addition, the maximum value (at $\theta = 3\pi/2$) of the radial f_r is slightly higher than the peak value for the comparable circular-bore Test Case 1 of Table A-I. Table A-II summarizes additional mean force and mean stress results from the EFFI computations made for the three Test Cases of Table A-I.

TABLE A-II
MSR TEST-CASE RESULTS FOR PARAMETERS SUGGESTED BY PPPL¹

Case	1	2	3
Bore	circular	circular	elliptical
On-axis magnetic field, B_0 (T)	4.8	3.6	4.8
Coil arc length (m)	14.0	14.1	20.1
Force/stresses for $\theta = 0(\pi)$ top (bottom):			
• Radial, f_r (N/m)	$1.54(10)^7$	$1.26(10)^7$	$1.96(10)^7$
• Lateral, f_y (N/m)	$6.20(10)^2$	$1.02(10)^5$	$-3.19(10)^6$
• Mean, σ_r (Pa)	$5.2(10)^7$	$8.4(10)^7$	---
• R (m)	10.0	10.0	---
• B (T)	5.7	4.5	---
Force/stresses for $\theta = \pi/2$ (outboard):			
• Radial, f_r (N/m)	$1.34(10)^7$	$1.20(10)^7$	$1.57(10)^7$
• Lateral, f_y (N/m)	$7.50(10)^2$	$-6.78(10)^5$	$6.08(10)^5$
• Mean, σ_r (Pa)	$4.5(10)^7$	$8.0(10)^7$	---
• R (m)	12.25	12.25	---
• B (T)	2.6	2.1	---
Force/stresses for $\theta = 3\pi/2$ (inboard):			
• Radial, f_r (N/m)	$1.91(10)^7$	$1.38(10)^7$	$2.10(10)^7$
• Lateral, f_y (N/m)	$-2.85(10)^3$	$2.31(10)^5$	$-2.58(10)^5$
• Mean, σ_r (Pa)	$6.4(10)^7$	$9.2(10)^7$	---
• R (m)	7.75	7.75	---
• B (T)	3.3	2.5	---
Net force on coil:			
• F_x (N) (centering force)	$-2.00(10)^7$	$-5.87(10)^6$	$-2.33(10)^7$
• F_y (N)	$-3.36(10)^3$	$-4.32(10)^4$	$-8.70(10)^5$
• F_z (N)	$-6.24(10)^3$	$-4.72(10)^3$	$-3.10(10)^4$

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APPENDIX B:
DESCRIPTION OF COMPUTATIONAL BASES AND SYSTEMS MODELS
USED TO EVALUATE THE MODULAR STELLARATOR REACTOR

This appendix describes the computational bases used in the preliminary assessment of the reactor potential of the Modular Stellarator Reactor (MSR). The approach adopted in this study uses approximate analytic models. The merit of this approach rests with an ability to quantify parameter tradeoffs and to identify trends; ultimately, this simplified approach must be reconciled with the predictions of more complex, numerical models that are generally necessary to describe the stellarator configuration. Section B.7. of this appendix describes the reactor model per se, with preceeding sections giving a brief description of specific physics models and assumptions. Section B.8. of this appendix gives the status of numerical model development for the more detailed, ongoing description of stellarator magnetics.

B.1. Ignition Condition

The MSR concept is characterized by a point plasma model that determines the self-consistent parameters of an ignited, steady-state, DT thermonuclear burn. Ignition is defined when the self-heating power density, $p_{\alpha}(W/m^3)$, of alpha-particle fusion product energy that is trapped in the plasma with an efficiency, f_{α} , just offsets the combined Bremsstrahlung loss, $p_{BR}(W/m^3)$, and radial transport loss, $p_l(W/m^3)$. That is,

$$f_{\alpha} p_{\alpha} = p_{BR} + p_l \quad . \quad (B-1)$$

Bremsstrahlung is the dominant radiation loss in the temperature range of interest to DT fusion reactor applications. The terms in Eq. (B-1) can be expressed as follows, using SI units, with the exception of temperature expressed in keV units.

$$p_{\alpha}(W/m^3) = 1.41(10)^{-13} n_1^2 \langle \sigma v \rangle \quad , \quad (B-2)$$

$$p_{BR}(W/m^3) = 5.35(10)^{-37} n_e^2 Z_{eff} T^{1/2} \quad , \quad (B-3)$$

$$P_R(W/m^3) = \frac{4.806(10)^{-16} n_e^2 T}{n_e \tau_E} \quad (B-4)$$

The expression for the Bremsstrahlung term uses a Gaunt factor of 1.1, $n_i(m^{-3})$ is the plasma ion density, $n_e(m^{-3})$ is the plasma electron density ($n_e = n_i + 2n_\alpha$), $T(keV)$ is the plasma temperature ($T_i = T_e = T$), and the energy confinement time is $\tau_E(s)$. Ignoring impurities other than the alpha-particle fusion products, the effective charge of the plasma, Z_{eff} , is given by

$$Z_{eff} = \frac{n_i + 4n_\alpha}{n_i + 2n_\alpha} \quad \text{and} \quad (B-5A)$$

$$\frac{n_\alpha}{n_i} = \frac{Z_{eff} - 1}{2(2 - Z_{eff})} \quad (B-5B)$$

The DT fusion reactivity, $\langle\sigma v\rangle(m^3/s)$, can be approximated as a function of T to within 10% of nominal values¹ by

$$\langle\sigma v\rangle = 1.1(10)^{-24} T^2 \quad , \quad (B-6)$$

in the range $8 < T < 20$ keV. A number of scalings for the energy confinement time is considered in Sec. B.2. The alpha-particle heating efficiency, f_α , can be varied parametrically in the range $0 < f_\alpha < 1$, pending the development of suitable scaling relationships (Sec. B.7.) obtained from numerical simulation studies of transport and orbit tracking.

A peaked radial pressure profile reduces the confinement required for ignition. In order to assess parametrically the influence of radial density and temperature profiles, the following profile functions are assumed, using a plasma of average radius, $r_p(m)$, such that

$$n(r) = n_o [1-(r/r_p)^v]^{1/2} \quad \text{and} \quad (B-7A)$$

$$T(r) = T_o [1-(r/r_p)^\nu]^{1/2} . \quad (B-7B)$$

The corresponding radial pressure profile is given by

$$p(r) = p_o [1-(r/r_p)^\nu] \quad (B-8)$$

$$= 2n_o k_B T_o [1-(r/r_p)^\nu] ,$$

where $k_B = 1.602(10)^{-16}$ J/keV. The subscript "o" refers to on-axis ($r = 0$) quantities, and ν ($\nu = 1, 2, 3, \dots, \infty$) is a parametric radial-profile index. Low values of ν represent sharply peaked profiles, and larger values of ν give increasingly flatter profiles.

The radially averaged value of the density (or analogous temperature) profile is defined by

$$\langle n \rangle = \frac{\int_0^{r_p} n(r) 2\pi r dr}{\int_0^{r_p} 2\pi r dr} = \frac{2}{r_p^2} \int_0^{r_p} n(r) r dr . \quad (B-9)$$

Substitution of Eqs. (B-6) and (B-7) into Eqs. (B-2) and (B-3) gives expressions for profile-averaged power densities related to alpha-particle heating and Bremsstrahlung losses. An accurate estimate of the alpha-particle heating power density, therefore, is restricted to the same temperature range ($8 < T < 20$ keV) as was used in Eq. (B-5). If this convenient approximation proves inaccurate, the profile averages can be calculated numerically for the general case. The volume-averaged density, temperature, and pressure are related to the on-axis values by

$$\langle n \rangle = n_o \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu]^{1/2} r dr = n_o I_{1/2} , \quad (B-10A)$$

$$\langle T \rangle = T_0 \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu]^{1/2} r dr = T_0 I_{1/2} \quad , \text{ and} \quad (\text{B-10B})$$

$$\langle p \rangle = p_0 \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu] r dr = p_0 I_1 \quad , \quad (\text{B-10C})$$

where the following simplifying notation for radial profile integrals is introduced.

$$I_x \equiv \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu]^x r dr \quad . \quad (\text{B-11})$$

The profile-averaged power densities related to alpha-particle heating and Bremsstrahlung can be written as

$$\langle p_\alpha \rangle = p_\alpha I_{1/2}^{-4} \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu]^2 r dr = p_\alpha I_{1/2}^{-4} I_2 \quad \text{and} \quad (\text{B-12A})$$

$$\langle p_{BR} \rangle = p_{BR} I_{1/2}^{5/2} \frac{2}{r_p^2} \int_0^{r_p} [1-(r/r_p)^\nu]^{5/4} r dr = p_{BR} I_{1/2}^{5/2} I_{5/4} \quad . \quad (\text{B-12B})$$

In Eqs. (B-12A) and (B-12B), p_α and p_{BR} are obtained by substituting the profile-averaged values of Eqs. (B-11) into Eqs. (B-2) and (B-3). Values of the profile integrals $I_{1/2}$, I_1 , $I_{5/4}$, and I_2 , as functions of the profile index, ν , are summarized in Table B-I.

For an on-axis vacuum magnetic field, $B_0(T)$, the on-axis beta is given by

$$\beta_0 = \frac{\sum_{j=1,e,\alpha} n_{0j} k_B T_{0j}}{B_0^2 / 2\mu_0} = \frac{p_0}{B_0^2 / 2\mu_0}, \quad (B-13)$$

where μ_0 is the permeability of free space $(4\pi(10)^{-7} \text{ h/m})$. A pressure-balance relationship can be written in consistent units as

$$\sum_{j=1,e,\alpha} n_{0j} T_{0j} = 2.48(10)^{21} \beta_0 B_0^2, \quad (B-14A)$$

TABLE B-I
RADIAL PROFILE INTEGRALS [EQ. (B-11)]

ν	$I_{1/2}$	I_1	$I_{5/4}$	I_2
1	0.5332	0.3333	0.2735	0.1667
2	0.6667	0.5000	0.4444	0.3333
3	0.7390	0.6000	0.5515	0.4500
4	0.7851	0.6667	0.6243	0.5333
5	0.8173	0.7143	0.6769	0.5952
6	0.8410	0.7500	0.7166	0.6429
7	0.8592	0.7773	0.7476	0.6806
8	0.8737	0.8000	0.7725	0.7111
9	0.8854	0.8182	0.7930	0.7364
10	0.8951	0.8333	0.8101	0.7576
.				
.				
.				
∞	1.0	1.0	1.0	1.0

which upon substitution of the profile-averaged parameters becomes

$$\sum_{j=i,e,\alpha} \langle n_j \rangle \langle T_j \rangle = 2.48(10)^{21} (I_{1/2}^2 / I_1) \langle \beta \rangle B_0^2 . \quad (B-14B)$$

It is noted that $\langle \beta \rangle = \beta_0 I_1 = \beta_0 v / (v+2)$. The remainder of this discussion is presented on the basis of homogeneous profile-averaged parameters.

The accumulation of impurities and alpha-particle ash is neglected for the purposes of this study. If alpha particles are allowed to build up to an equilibrium level characterized by Z_{eff} , Eqs. (B-5), (B-3), (B-4), and (B-14), can be recast as follows.

$$\langle p_\alpha \rangle = 1.41(10)^{-13} \langle n_i \rangle^2 \langle \sigma v \rangle (I_2 / I_{1/2}^4) , \quad (B-15A)$$

$$\langle p_{BR} \rangle = 5.35(10)^{-37} \langle n_i \rangle^2 \left[1 + \frac{Z_{\text{eff}} - 1}{2 - Z_{\text{eff}}} \right]^2 Z_{\text{eff}} \langle T \rangle^{1/2} (I_{5/4} / I_{1/2}^{5/2}) , \quad (B-15B)$$

$$\langle p_\ell \rangle = \frac{4.806(10)^{-16} \langle n_i \rangle^2 \left[1 + \frac{Z_{\text{eff}} - 1}{2 - Z_{\text{eff}}} \right]^2 \langle T \rangle}{\langle n_e \rangle \tau_E} , \text{ and} \quad (B-15C)$$

$$\langle n_i \rangle \langle T \rangle \left[1 + \frac{(Z_{\text{eff}} - 1)}{2(2 - Z_{\text{eff}})} \left(1 + \frac{1}{2} T_\alpha / \langle T \rangle \right) \right] = 1.24(10)^{21} (I_{1/2}^2 / I_1) \langle \beta \rangle B_0^2 . \quad (B-15D)$$

These relationships eliminate the explicit use of the electron and alpha-particle densities in favor of the ion densities. The ratio $T_\alpha / \langle T \rangle$ is taken to be ~ 10 based on Fokker-Planck computations of burning DT plasma in other systems. The following variables are defined in order to simplify the notation.

$$f_{Z1} = \left[1 + \frac{Z_{\text{eff}} - 1}{2 - Z_{\text{eff}}} \right]^2 \text{ and} \quad (B-16A)$$

$$f_{Z2} = 1 + \frac{(Z_{\text{eff}} - 1)}{2(2 - Z_{\text{eff}})} \left(1 + \frac{1}{2} T_{\alpha}/\langle T \rangle\right) \quad (\text{B-16B})$$

such that $\langle n_e \rangle = \langle n_i \rangle f_{Z1}^{1/2}$. The presence of suprathermal alpha particles may be interpreted as a reduction in the productive $\langle \beta \rangle$ of the fusion plasma when the confining magnetic field strength is fixed. Figure B-1 displays $1/f_{Z2}$ as a function of Z_{eff} for several ratios of $T_{\alpha}/\langle T \rangle$. The quantity $1/f_{Z2}$ may be seen from Eq. (B-15D) to be the ratio of $\langle \beta \rangle$ for $Z_{\text{eff}} > 1.0$ to the value of $\langle \beta \rangle$ when the pressure contribution, p_{α}/p , of alpha particles is neglected

Fig. B-1. Dependence of f_{Z2} and n_{α}/n_i on Z_{eff} in the Z_{eff} range of 1-2.

(i.e., $Z_{\text{eff}} = 1.0$). Equations (B-15) reduce to the following set of working relationships.

$$\langle p_\alpha \rangle = 1.41(10)^{-13} \langle n_i \rangle^2 \langle \sigma v \rangle (I_2 / I_{1/2})^4, \quad (\text{B-17A})$$

$$\langle p_{\text{BR}} \rangle = 5.35(10)^{-37} \langle n_i \rangle^2 f_{Z1} Z_{\text{eff}} \langle T \rangle^{1/2} (I_{5/4} / I_{1/2})^{5/2}, \quad (\text{B-17B})$$

$$\langle p_\alpha \rangle = \frac{4.806(10)^{-16} \langle n_i \rangle^2 f_{Z1} \langle T \rangle}{\langle n_e \rangle \tau_E}, \text{ and} \quad (\text{B-17C})$$

$$\langle n_i \rangle \langle T \rangle f_{Z2} = 1.24(10)^{21} (I_{1/2}^2 / I_1) \langle \beta \rangle B_o^2. \quad (\text{B-17D})$$

Once a specific radial transport scaling for $\langle n_e \rangle \tau_E$ or τ_E is adopted and a pressure balance is enforced, Eqs. (B-17) can be used in the ignition condition [Eqs. (B-1)] to give $\langle n_e \rangle \tau_E$ as a function of $\langle T \rangle$. Appropriate application of beta limits (Sec. B.4.), coil-configuration constraints (Secs. B.3. and B.5.), total-power, and fusion-neutron wall-loading constraints leads to an interrelated computational algorithm (Sec. B.7.) that is used for the parametric tradeoff studies reported in Sec. IV.

B.2. Transport Scaling

Radial energy transport is conveniently, if approximately modeled in this study using one of two alternatives: Alcator (empirical) scaling and Bohm-like scaling. Each of these scaling models can be expressed in terms of the $\langle n_e \rangle \tau_E$ Lawson parameter used in Eq. (B-17C). Pending the experimental verification or elimination of one or more of these alternatives for reactor-grade plasma, several options are carried along for purposes of the reactor survey calculations. These transport scaling laws are also related to the developing theories of neoclassical transport in stellarator geometries,^{2,3} although it is recognized that ultimately this issue must be more thoroughly examined in a reactor context. Furthermore, use of the term "Bohm-like", when coupled with the necessary pressure balance, implies a thermally stable scaling, $\tau_E \propto r_p^2 B / T$, which generally is taken to be a factor of ~ 100 greater in

magnitude than the absolute value of this stochastic transport scaling originally hypothesized by Bohm.⁴

Alcator (empirical) transport scaling is given by⁵

$$\begin{aligned} \langle n_e \rangle \tau_{E(\text{Alcator})} &= 3.0(10)^{-21} \langle n_e \rangle^2 r_p^2 \\ &= 4.613(10)^{21} \left[\frac{(I_{1/2}^2/I_1) f_{Z1}^{1/2}}{f_{Z2}} \right]^2 \left[\frac{\langle \beta \rangle B_o^2 r_p^2}{\langle T \rangle} \right]^2, \end{aligned} \quad (\text{B-18})$$

where $r_p(m)$ is the average plasma radius. Using pressure balance based on centerline beta and magnetic-field values, but expressing the result in terms of profile-averaged quantities, an explicit expression for the Alcator confinement time is given by

$$\tau_{E(\text{Alcator})} = 3.72(I_{1/2}^2/I_1)(f_{Z1}^{1/2}/f_{Z2}) \frac{\langle \beta \rangle B_o^2 r_p^2}{\langle T \rangle}. \quad (\text{B-19})$$

For a Bohm-diffusion coefficient given by $D_{\text{Bohm}} \approx 62.5 \langle T \rangle / B_o$, and defining an energy-loss time, $\tau_{E(\text{Bohm})}$, as some factor, f_B , multiplied by r_p^2 / D_{Bohm} , the following Bohm-like scaling relation results.

$$\langle n_e \rangle \tau_{E(\text{Bohm})} = 2.0(10)^{19} f_B (I_{1/2}^2/I_1)(f_{Z1}^{1/2}/f_{Z2}) \frac{\langle \beta \rangle B_o^3 r_p^2}{\langle T \rangle^2}, \quad (\text{B-20})$$

where, in both Eqs. (B-19) and (B-20), a centerline pressure balance ($\sum n_{oj} k_B T_{oj} = \beta_o B_o^2 / 2\mu_o$) is used in conjunction with profile-average quantities (i.e., $\langle n \rangle / n_o = I_{1/2}$, $\langle T \rangle / T_o = I_{1/2}$, $\langle \beta \rangle / \beta_o = I_1$). Expressed explicitly, the Bohm-like confinement time appears as

$$\tau_{E(\text{Bohm})} = 0.016 f_B \frac{B_o r_p^2}{\langle T \rangle}. \quad (\text{B-21})$$

No inference is made here that energy transport in stellarators would in

magnitude equal that predicted by Bohm scaling. Rather, only the functional form of this scaling is proposed, with the required scaling factor, f_B , being inferred from experiment. For instance, the ratio $\tau_E(\text{Alcator})/\tau_E(\text{Bohm})$ would equal $233(I_{1/2}^2/I_1)(f_{Z1}^{1/2}/f_{Z2})\langle\beta\rangle B_0/f_B$, which, for a cubic ($\nu = 3$) profile (Table B-I) and $Z_{\text{eff}} = 1.0$ equals $212\langle\beta\rangle B_0/f_B$; hence, for Alcator and Bohm-like scalings to be equal in magnitude for typical reactor beta and field values ($B_0 \approx 6$ T, $\langle\beta\rangle \approx 0.04$), the Bohm factor, f_B , would equal ~ 50 . When the Bohm-like scaling is used parametrically in this study, the factor f_B is generally taken in the range 50-200 and gives results that are similar to or more favorable than Alcator scaling.

An empirical scaling has recently been proposed by Chu⁶ for both Ohmically heated stellarators and tokamaks. This scaling is based on the assumption that cross-field thermal transport is driven in proportion to the classical parallel-field momentum transport associated with the Ohmic currents. For $T_e = T_i = \langle T \rangle$, this empirical scaling takes the form

$$\tau_E(\text{Chu}) = 5.47(10)^{-21} q_a (R_T/r_p) \frac{\langle T \rangle \langle n \rangle r_p^2}{B_0} \quad , \quad (\text{B-22})$$

where $q_a = (r_p/R_T)(B_\phi/B_\theta)$ is the effective safety factor at the plasma edge, and the toroidal and poloidal magnetic-field components are, respectively, B_ϕ and B_θ . If the toroidal current that is assumed to drive this cross-field transport is related to pressure-gradient-driven currents, the effective vertical field is given by⁷

$$B_z = \beta_0 B_0 f(\nu, \ell) / \tau_0 \quad , \quad (\text{B-23})$$

where a pressure profile of the form given by Eq. (B-19C) has been assumed, the vacuum rotational transform is approximated by $\tau(r) = \tau_0 (r/r_p)^{2\ell-4} \Delta(r)$, $\tau_0 \Delta(0)$ is the on-axis rotational transform, and the function $f(\nu, \ell)$ is given by⁷

$$f(\nu, \ell) = \frac{(\nu - 2\ell + 6)\nu}{2[(\nu - 2\ell + 5)^2 - 1]} \quad (B-24)$$

Relating q_a in Eq. (B-22) to the pressure-driven currents results in $q_a(R_T/r_p) = (I_{1/2}/f(\nu, \ell))(\tau_o/\langle\beta\rangle)$, which when substituted into Eq. (B-22) along with the pressure balance results in the following expression for $\tau_{E(\text{Chu})}$.

$$\tau_{E(\text{Chu})} = 6.79[I_{1/2}^2/f(\nu, \ell)] \tau_o B_o r_p^2 \quad (B-25)$$

For both ν and ℓ equal to 3 and $f(3,3) = 3/2$, τ_E equals $2.47\tau_o B_o r_p^2$. The magnitude of $\tau_{E(\text{Chu})}$ relative to Bohm-like scaling is $424\tau_o I_{1/2}^2 \langle T \rangle / f(\nu, \ell) f_B = 155 \langle T \rangle \tau_o / f_B$; for $\langle T \rangle \approx 10$ keV and $\tau_o \approx 1.0$, $\tau_{E(\text{Chu})}/\tau_{E(\text{Bohm})} \approx 1500/f_B$, leading to a Bohm factor, f_B , that is somewhat larger than estimated previously for Alcator (empirical) scaling in order to make both comparable.

The classical cross-field energy confinement time is defined in terms of the ion-ion collision time, $\tau_{ii} = 1/\nu_{ii}$, such that $\tau_{E(\text{Classical})} = \tau_{ii}(r_p/\rho_i)^2$, where again r_p is the average plasma radius, and ρ_i is the ion gyroradius evaluated at the internal magnetic field, $B_o(1-\langle\beta\rangle)^{1/2}$, such that, for an average DT ion (i.e., average mass equals 2.5 amu),

$$\tau_{E(\text{Classical})} = 3.858(10)^4 (1-\langle\beta\rangle) \frac{B_o^2 r_p^2 \tau_{ii}}{\langle T \rangle} \quad (B-26)$$

Substitution of the pressure-balance relationship [Eq. (B-17D)] and the explicit definition of τ_{ii} allows the classical confinement time to be written in the form

$$\langle n_e \rangle \tau_{E(\text{Classical})} = \frac{2.761(10)^{22} f_{Z1}^{1/2}}{Z_{eff}^2 \ln \Lambda} \langle T \rangle^{1/2} (1-\langle\beta\rangle) B_o^2 r_p^2, \quad (B-27)$$

where $\ln \Lambda \approx 20$ is the usual Coulomb-logarithm term. The ratio $\tau_{E(\text{Classical})}/\tau_{E(\text{Bohm})}$ is $\sim 30,000/f_B$ for typical reactor parameters.

The ratio $\tau_E(\text{Chu})/\tau_E(\text{Alcator})$ equals $1.80(I_1/f(\nu, \ell) + \epsilon_0 \langle T \rangle) / \langle \beta \rangle B_0$, which for both ν and ℓ equal to 3, becomes $0.72 + \epsilon_0 \langle T \rangle / \langle \beta \rangle B_0$; for $\langle T \rangle \approx 10$ keV and $B_0 \approx 6$ T, $\tau_E(\text{Chu})$ is about 25 times greater than $\tau_E(\text{Alcator})$. In order to limit the parameter space, this study focuses on Alcator scaling. In the presence of other conservatively chosen parameters and constraints, Alcator transport scaling is minimally required for competitive MSR performance.

Before leaving this issue of stellarator transport scaling, it is useful to relate the forgoing results to the confinement-time scaling predicted by neoclassical theory. The usual presentation of neoclassical transport scaling in toroidal magnetic confinement devices^{3,7} is summarized schematically in Fig. B-2. Different transport rates are encountered as a function of collision frequency, giving rise to distinct transport scaling relationships.

Fig. B-2. Dependence of the diffusion coefficient, D , on electron-ion collision frequency, ν_{ei} , following Ref. 3. $D_{ps} \equiv$ Pfirsch-Schlüter, $D_p \equiv$ plateau, $D_H \equiv$ helical-ripple, $D_{HP} \equiv$ helical plateau, $D_V \equiv$ trapped superbanana, $D_E \equiv$ nonzero electric field, $D_{GS} \equiv$ Galeev-Sagdeev, and $D_B \equiv$ Bohm diffusion.

Like most steady-state, magnetic fusion devices, the MSR reactor is anticipated to operate in the collisionless regime. As ν decreases the rightmost region of Fig. B-2 indicates that $D_{PS} \propto \nu$ (Pfirsch-Schlüter) until $\nu < \nu_p$ and the plateau regime (D_p independent of ν) is encountered. Continued application of tokamak axisymmetric theory would suggest that for $\nu < \nu_b$ the diffusion would resume the $D_{GS} \propto \nu$ (Galeev-Sagdeev) scaling. However, in nonaxisymmetric stellarators ($\nu_3 < \nu_b$) and Torsatrons ($\nu_3 > \nu_b$), helical ripple diffusion dominates for $\nu < \nu_4$ to give $D_H \propto \nu^{-1}$. If ν is less than the helical-banana precession frequency, $\nu_2 = \epsilon_h^2 \omega_o$, where $\omega_o = -T/eB_o r_p^2$, vertical drifts are compensated and a helical plateau regime (D_{HP} independent of ν) is encountered. Finally, when ν is less than the superbanana bounce frequency, ν_1 , trapped superbananas allow $D_V \propto \nu$. In the presence of radial electric fields, helical bananas may become untrapped for $\nu < \nu_e < \nu_3 \equiv \epsilon_h \omega_e$ ($\omega_e = E_r/B_o r_p$), such that $D_E \propto \nu^{1/2}$. In the limit of infinite plasma aspect ratio ($\epsilon_t \rightarrow 0$), axisymmetric theory would continue to apply for $\nu < \nu_b$ such that $D_{GS} \propto \nu$. Bohm diffusion D_B is formally independent of ν and is much more rapid than the various neoclassical diffusion regimes discussed above.

The various quantities indicated in Fig. B-2 can be defined in terms of reactor parameters. The collision frequency may be expressed as either the electron-ion or ion-ion collision frequency, assuming the average DT ion mass is 2.5 amu, as follows.

$$\nu_{ei} = 5.397(10)^{-17} Z_{eff}^2 \frac{\langle n_i \rangle}{\langle T \rangle^{3/2}} \quad \text{and} \quad (B-28)$$

$$\nu_{ii} = 1.399(10)^{-18} Z_{eff}^2 \frac{\langle n_i \rangle}{\langle T \rangle^{3/2}} \approx \nu_{ei}/68 \quad . \quad (B-29)$$

Applying the pressure-balance relationship [Eq. (B-17D)] while neglecting the alpha-particle pressure contribution to $\langle \beta \rangle$ (i.e., $Z_{eff} = f_{Z2} = 1$), Eqs. (B-28) and (B-29) can be recast as follows.

$$\nu_{ei} = 6.69(10)^4 \ln \Lambda (I_{1/2}^2 / I_1) \langle \beta \rangle \frac{B_0^2}{\langle T \rangle^{5/2}} \quad \text{and} \quad (B-30)$$

$$\nu_{ii} = 1.1(10)^3 \ln \Lambda (I_{1/2}^2 / I_1) \langle \beta \rangle \frac{B_0^2}{\langle T \rangle^{5/2}} . \quad (B-31)$$

Assuming a typical value for $\langle \beta \rangle = 0.04$ (Sec. B.4.), lines of constant ν_{ei} and $\langle n_i \rangle = \langle n_e \rangle$ are displayed in Fig. B-3 as a function of B_0 and $\langle T \rangle$. Typical reactor parameters can be expected to lie in the ranges of $2 < B_0 < 8$ T and $5 < \langle T \rangle < 40$ keV such that $10^2 < \nu_{ei} < 10^5$ s⁻¹. It is now necessary to evaluate explicitly the several transition collision frequencies displayed in Fig. B-2 in order to establish the appropriate transport scaling regime.

Fig. B-3. Lines of constant $\langle n_i \rangle$ and electron-ion collision frequency, ν_{ei} , in B_0 versus $\langle T \rangle$ parameter space for $Z_{eff} = 1.0$, radial profile index $\nu = 3$, and typical $\langle \beta \rangle = 0.04$.

Following Refs. 3 and 7 as well as references cited therein, these transition collision frequencies can be expressed as follows.

$$\nu_b = \nu_{th}/qR_T = 1.19(10)^7 + \langle T \rangle^{1/2}/R_T \quad (B-32)$$

$$\nu_p = \epsilon_t^{3/2} \nu_p = 1.19(10)^7 + \langle T \rangle^{1/2} \epsilon_t^{3/2}/R_T \quad (B-33)$$

$$\nu_4 = (\epsilon_h/\epsilon_t)^{3/2} \nu_b = 1.19(10)^7 + \langle T \rangle^{1/2} (\epsilon_h/\epsilon_t)^{3/2}/R_T \quad (B-34)$$

$$\nu_3 = \epsilon_n \omega_e = \epsilon_h E_r / (r_p B_o) \quad (B-35)$$

$$\nu_2 = 3\epsilon_t \epsilon_h^{1/2} \omega_o = 3.0(10)^3 \epsilon_t \epsilon_h^{1/2} \langle T \rangle / (r_p^2 B_o) \quad (B-36)$$

$$\nu_1 = \epsilon_t \epsilon_h \omega_o = 1.0(10)^3 \epsilon_t \epsilon_h \langle T \rangle / (r_p^2 B_o) \quad (B-37)$$

The average rotational transform, $\bar{\iota}$, is evaluated at the radius where $T(r) = \langle T \rangle$ for self-consistency. Retaining the explicit ν_{ei} dependence where possible, the diffusion coefficients, $D(m^2/s)$, of interest become

$$D_p^e = 0.067 \frac{\langle T \rangle^{3/2}}{(\bar{\iota} R_T B_o^2)} \quad (\text{electron plateau}) \quad (B-38)$$

$$D_p^i = 7.2 \frac{\langle T \rangle^{3/2}}{(\bar{\iota} R_T B_o^2)} \quad (\text{ion plateau}) \quad (B-39)$$

$$D_H = 10^6 \epsilon_t^2 \epsilon_h^{3/2} \frac{\langle T \rangle^2}{\nu_{ei} r_p^2 B_o^2} \quad (\text{helical ripple}) \quad (B-40)$$

$$D_{HP} = \frac{16}{3} \epsilon_t \epsilon_h D_B \quad (\text{helical plateau}) \quad (B-41)$$

$$D_V = \frac{1}{3} r_p^2 \nu_{ei} \quad (\text{trapped superbanana}) \quad (B-42)$$

$$D_E = \epsilon_t^2 \left(\frac{\nu_{ei}}{\omega_e} \right)^{1/2} \frac{\omega_o}{\omega_e} \frac{r_p}{B} \quad (\text{nonzero electric field}) \quad (B-43)$$

$$D_B = 62.5 \frac{\langle T \rangle}{B_0} \quad (\text{Bohm}) \quad . \quad (\text{B-44})$$

A Monte Carlo numerical simulation⁸ of torsatron transport at $\langle T \rangle = 8$ keV, $B_0 = 5.5$ T, $\tau = 0.25$, $r_p = 2.1$ m, and $R_T = 48$ or 24 m indicates a value of $D \sim 0.1$ to 1.0 m²/s that is essentially independent of $\langle n \rangle$ in the range of 10^{18} to 10^{21} m⁻³, corresponding to $\langle \beta \rangle$ in the range of $2(10)^{-4}$ to 0.2 for $\epsilon_t = 1/24$ and $1/12$. This numerical result casts doubt on the applicability of Eq. (B-40), that predicts the "1/ ν catastrophe". The corresponding values of the other diffusion coefficients that are independent of ν for $R_T = 24$ m are $D_p^e = 0.008 = D_B/1100$, $D_p^i = 0.9 = D_B/100$, $D_{HP} = 485\epsilon_t\epsilon_h$, and $D_B = 90.9$ m²/s. The value of D_p^i corresponds well to the numerical result. For $\epsilon_t = 0.07$ and $\epsilon_h = 0.12$, $D_{HP} = 4.7$ m²/s $\sim D_B/20$. Only D_{HP} has the same functional dependence as D_B . A factor $f_B \sim 100$ yields transport results comparable in magnitude to plateau scaling in the reactor regime.

In addition to the favorable result of Ref. 8, several other numerical studies⁹⁻¹¹ of stellarator/torsatron transport have recently become available. Incorporation of these new results and resolution of any discrepancies between them is beyond this phase of the MSR study. Retaining the pessimistic Bohm-like functional dependence of D_{HP} can be interpreted as a conservative approach to the design of the stellarator reactor for $f_B < 100$. In terms of Alcator (empirical) transport scaling, which has served as a basis for this MSR reactor survey, the neoclassical results in the helical-plateau regime are about a factor of two more pessimistic and the results in the ion-plateau regime are about a factor of two more optimistic in magnitude. The issue of transport will, of course, be central to a reliable selection of a credible MSR design point.

B.3. Analytic Approximations to Stellarator Field and Rotational Transform

Perhaps the most difficult task associated with a "self-consistent" reactor survey of a stellarator is the identifying relationship between the vacuum-field configuration [i.e., through the parameters $\tau(r)$, $d\tau/dr$, magnetic well (hill) magnitude], the transport/equilibrium/stability, and the physical and mechanical layout of magnets, blanket and shield, and impurity-control system (i.e., divertor or limiter). This study utilizes primarily analytic stellarator expansions and, therefore, is so limited; detailed numerical analyses are beyond the scope of this parametric study, but the results

presented here should at least give both trends as well as future guidance for more detailed numerical analyses. This section summarizes the analytic stellarator expansions used by this reactor survey calculation.

The stellarator creates magnetic surfaces and a rotational transform by means of an array of external coils. An exact determination of the topology of these magnetic surfaces in toroidal geometry and the complex interrelation between the field structure, transport, equilibrium, stability, and physical layout of key reactor systems (i.e., magnets, blanket and shield, impurity control), have only been touched upon by this study. Nevertheless, the quantification of these interrelationships, even by approximate analytic models, was deemed necessary even for this preliminary study. Towards this goal analytic expressions were used to describe a "linear" stellarator field.

In this approximation, the magnetic field vector, $\vec{B}$, is expressed in terms of a scalar potential, ϕ_B , which, upon expansion into a harmonic series satisfying Laplace's equation ($\nabla^2 \phi_B = 0$), is given by^{7,12}

$$\phi_B = B_0 z + \frac{1}{\alpha} \sum_{\ell=1}^{\infty} b_{\ell} I_{\ell}(\ell \alpha r) \sin \ell(\theta - \alpha z) \quad , \quad (B-45)$$

where b_{ℓ} is the magnitude of the ℓ th helical-field harmonic, α is the helical pitch parameter, and $I_{\ell}(x)$ is the modified Bessel function of the first kind of ℓ th order and argument x . The bumpiness of the magnetic field in the z direction (or ϕ direction in the toroidal case) caused by discrete coils is ignored for present purposes. The pitch length, $1/\alpha$, equals $(\ell/m)R_T$, where R_T is the major toroidal radius, and consequently $\ell \alpha r_p = m \epsilon$, with $\epsilon = r_p/R_T$ being the inverse plasma aspect ratio. Using the "average method," the average rotational transform as a function of minor radius, r , becomes^{7,12}

$$\tau(r) = \frac{1}{2} \left[\frac{b_{\ell}}{B_0} \right]^2 \frac{R \ell^3}{r} \frac{d}{dx} \left[\frac{I_{\ell}(x) I'_{\ell}(x)}{x} \right]_{x=\ell \alpha r} \quad (B-46A)$$

$$= \frac{m}{\ell} \left[\frac{b_{\ell}}{B_0} \right]^2 \left[\frac{2}{\ell!} \left(\frac{\ell}{2} \right)^{\ell+1} \right]^2 \left[\left(1 - \frac{1}{\ell} \right) (\alpha r)^{2\ell-4} + \left(\frac{\ell}{2} \right) (\alpha r)^{2\ell-2} + \dots \right] \quad (B-46B)$$

The dominate harmonic term in Eqs. (B-45) and (B-46) for a stellarator with poloidal-field-period number ℓ is b_ℓ , hereafter written as b with the subscript dropped for notational convenience.⁷ For the conventional stellarator, each harmonic of the helical-field strength relative to the toroidal-field strength is related to the maximum separatrix radius, r_s , by⁷

$$\frac{\ell b}{B_0} = \frac{1}{[1 + (\alpha r_s)^{-2}] I_\ell(\ell \alpha r_s)} . \quad (B-47)$$

Equation (B-47) is substituted into Eq. (B-46) to give the rotational transform in terms of the separatrix radius. The corresponding helical current component, I_h , is related to the coil radius, r_c , by the relationship⁷

$$\frac{2\mu_0 I_h}{\pi r_c B_0} = \frac{\ell b}{B_0} \frac{[1 + (\alpha r_c)^2]^{1/2}}{(\ell \alpha r_c)^2 K'_\ell(\ell \alpha r_c)} , \quad (B-48)$$

where $K'_\ell(x)$ is the first derivative of the ℓ th-order modified Bessel function of the second kind, $K_\ell(x)$, with respect to the argument x .

The corresponding modular-coil configuration in the toroidal case has been considered.¹³ For the scalar potential given by Eq. (B-45), the dominant component of the helical-field strength relative to the toroidal-field strength is given by

$$\frac{\ell b}{B_0} = 2(\ell \alpha r_c) J_1[(\ell \alpha r_c)(d/r_c)] K'_\ell(\ell \alpha r_c) , \quad (B-49)$$

where $J_1(x)$ is the first-order Bessel function of the first kind with argument x , and d/r_c is the maximum lateral distortion, d , of a modular (Rehker-Wobig) coil relative to the mean coil radius, r_c . The corresponding effective helical-current component becomes

$$I_h = \frac{[1 + (\alpha r_c)^2]^{1/2}}{2} \left(\frac{N}{m}\right) I J_1[(\ell \alpha r_c)(d/r_c)], \quad (B-50)$$

where I is the current carried by each of N modular coils. Equation (B-47) is retained to determine the separatrix radius, but Eq. (B-49) is substituted into Eq. (B-46) to obtain the rotational transform as a function of the coil distortion parameter, d/r_c .

It is not always possible to reproduce a rotational transform value achievable with a conventional stellarator by means of a modular-coil analogue because the modular coil necessarily tends to give a smaller value of I_h . A more recent description of the modular-coil configuration¹⁴ suggests the use of two superposed solutions of Eqs. (B-46) and (B-49) representing a helical current component and an antihelical current component of different pitch. The treatment of Ref. 13 using only one harmonic tends to model the configuration illustrated in Fig. III-3, whereas the superposed solution of two antiparallel harmonics more closely approaches the Rehker-Wobig coil configuration illustrated in Fig. III-6. The superposed solution, using the expansion in Eq. (B-46), can be written in the form¹⁴

$$\tau(r) = [\tau(r, m) - \tau(r, m')] , \quad (B-51)$$

where $m' = N/m - 1$. The second term in Eq. (B-51) tends to "unwind" the transform of the first term to given low-shear transform profiles characteristic of modular-coil configurations, as confirmed by numerical computations (Sec. B.8.).

Equation (B-46) may be rewritten in the form

$$\tau(r) \approx \tau_0 (r/r_p)^{2(\ell-2)} \Delta(r) , \quad \text{where} \quad (B-52A)$$

$$\tau_0 \epsilon = \left[\frac{\ell(\ell-1)^{1/2}}{2^{\ell} \ell! I_{\ell}(\ell \alpha r_s)} \frac{(\alpha r_s)^2}{1 + (\alpha r_s)^2} \right]^2 (\ell \alpha r_p)^{2\ell-3} , \quad (B-52B)$$

$$\Delta(r) = 1 + \frac{(\ell \alpha r_p)^2}{2(\ell-1)} (r/r_p)^2 + \dots, \text{ and } m = \ell \alpha R_T, \quad (\text{B-52C})$$

in order to separate the functional dependence of $\psi(r)$ on the minor radius, r . This form is convenient because the beta limits discussed in the next section depend on $\psi_0 \Delta(r)$ for the conventional stellarator or on $\psi_0 (\Delta(r)=1)$ for the low-shear, modular-coil configuration, whereas in Eq. (B-51) ψ_0 is taken to be the superposed solution of Eq. (B-52B). It is emphasized that all expressions given in this section are valid only at high coil aspect ratio, R_T/r_c , and generally apply only near the magnetic axis (i.e., $\ell \alpha \epsilon \ll 1$).

On the basis of this simplified representation for the stellarator-field topology, the following plasma engineering issues can be identified.

- What is the position of the separatrix radius, r_s , relative to an effective first-wall radius, r_w , and the coil radius, $r_c \approx r_w + \Delta b$, where Δb is an effective blanket and shield thickness?
- What is the effective plasma radius, r_p , in terms of r_s and r_w ?
- How can these relationships be used to relate the rotational transform to the number, N , and lateral distortion, d/r_c , of a modular-coil (Rehker-Wobig) winding?
- What is the interaction between the above parameters and the stability and equilibrium beta limits?

The latter question is addressed in Sec. B.4. With respect to the first issue, this study implicitly assumes that magnetic-divertor action would require $r_s \approx r_w$. Specifically, a "scrape-off" parameter, $x = r_s/r_w$, is defined that ultimately must be related to the stellarator geometry in an averaged sense. The relationship between r_p and r_s will depend on the value of ℓ . For $\ell = 2$ the ratio of separatrix radius to minor plasma radius of an $\ell = 2$ elliptical plasma cross section is given⁷ by $(3^{1/2}/\alpha r_s)^{1/2}$. Hence, taking a geometric mean of the major and minor radii of the $\ell = 2$ ellipse gives the following relationship between r_p and $r_s (\sim r_w)$, recalling that $\alpha = m/\ell R_T$.

$$x = r_p/r_w = [m\epsilon/3^{1/2}\ell]^{1/3} = 0.66(m\epsilon)^{1/3}. \quad (\text{B-53})$$

For the $\ell = 3$ trefoil plasma cross section, the area of the circle πr_p^2 is equated to the area of an equivalent-area triangle with sides equal to $3^{1/2} r_s$. The following relationship between the average plasma and separatrix radii results.

$$x = r_p/r_w = (3^{3/2}/4\pi)^{1/2} = 0.64 \quad . \quad (B-54)$$

Figure B-4 illustrates the relationships implied by Eqs. (B-53) and (B-54) for $\ell = 2$ and $\ell = 3$ configurations, using a magnetic divertor at the first-wall radius to define the plasma boundary at the last closed magnetic surface. It is emphasized that all expressions given in this section are limited to high aspect ratio and apply only near the magnetic axis (specifically, for

Fig. B-4. Schematic relation of separatrix radius, r_s , to first-wall radius, r_w , and equivalent plasma radius, r_p , for $\ell = 2$ and $\ell = 3$ configurations, assuming a magnetic divertor acting at r_w .

$\ell a \epsilon < 1$). A divertor ($r_s \leq r_w$) rather than a limiter ($r_s > r_w$) has been assumed historically to be a more natural impurity-control approach for the steady-state stellarator. Nevertheless, the limiting case where the separatrix radius approaches the coil radius ($r_s \sim r_c$) is of interest in quantifying the incremental amount of coil distortion of the modular coil required to push the separatrix radius from r_c (limiter) to r_w (divertor), where again $r_c \sim r_w + \Delta b$. For an $\ell = 2$ system with the semimajor radius of the elliptical plasma cross section determined by a "limiter" surface placed near radius r_w , the average plasma radius is given approximately by $(r_w r_o)^{1/2} = [(r_s - \Delta b) r_o]^{1/2}$. For this case, $r_s \sim r_c$ and the semiminor radius of the elliptical plasma, r_o , is approximately related⁷ to that for the separatrix surface by $r_o/r_s = (\alpha r_o/3^{1/2})^{1/2}$. Because $\ell \alpha r_p = m \epsilon$, the following expressions result for r_p and $x = r_p/r_w$ for the $\ell = 2$ ellipse when $r_s \sim r_c$.

$$r_p = r_c \left[\frac{(m/\ell)\epsilon(1 - \Delta b/r_c)}{3^{1/2}} \right]^{1/3} \quad (B-55)$$

$$x = r_p/r_w = \left[\frac{(m/\ell)\epsilon}{3^{1/2}(1 - \Delta b/r_c)^2} \right]^{1/3} . \quad (B-56)$$

As expected, the utilization of volume within the first-wall radius for an $\ell = 2$ system with $r_s \sim r_c$ is greater by a factor, $1/(1 - \Delta b/r_c)^{2/3}$, than for the case where $r_s \sim r_w$. For the $\ell = 3$ case with $r_s \sim r_c$, it can similarly be shown that $x = 0.64$ [Eq. (B-54)], except τ_o from Eq. (B-52B) must be evaluated at r_p with $r_s \approx r_c = r_w + \Delta b$. Figure B-5 illustrates the configuration implied by a limiter located near r_w to define the plasma boundary.

The foregoing qualitative discussion can now be examined quantitatively. Figure B-6 displays the dependence of the helix/antihelix superposition representation of Eqs. (B-46B), (B-49), and (B-51) for modular $\ell = 2$ and $\ell = 3$ systems. Nominal parameters for plasma radius, $r_p = 2.0$ m, $\nu = 3$, and combined blanket and shield thickness, $\Delta b = 1.5$ m, are selected along with the choice of $N = 24$ coils. First, for fixed maximum lateral coil distortion, $d/r_c = 0.3$, the dependence of edge-plasma rotational transform, $\tau(r_p)$, on plasma aspect ratio, $A = R_T/r_p$, is depicted. Considered are $\ell = 2$ and $\ell = 3$

Fig. B-5. Schematic relation of separatrix radius, r_s , to first-wall radius, r_w , and equivalent plasma radius, r_p , for $\ell = 2$ and $\ell = 3$ configurations, assuming a limiter acting at r_w .

configurations for $m = 4, 6$, and 8 . Generally, the lower the m -number, the higher is the rotational transform for a fixed value of d/r_c . Also, for a given m -number, $\ell = 2$ yields higher transforms until an m -dependent plasma aspect ratio is reached at which the $\ell = 3$ edge transform surpasses the $\ell = 2$ value. Also shown in Fig. B-6 is a comparison between the conventional stellarator expansion [Eq. (B-52)] for $r_s \sim r_w$ and $r_s \sim r_c$ options compared with the helix/antihelix superposition model [Eqs. (B-46B), (B-49), and (B-51)] of the first-harmonic modular-coil configuration for $\ell = 2$ and $m = 8$ for several values of d/r_c . Generally, the higher values of d/r_c give higher values of $\iota(r_p)$. However, values of d/r_c sufficient to return the transform values of the conventional stellarator expansion may be inconsistent with the modular-coil interference constraint developed in Sec. B.5. of this appendix. The $r_s \sim r_c$ option results in a lower value of edge transform for any given aspect ratio and thus a value of d/r_c required to give a value of $\iota(r_p)$ is

Fig. B-6. Dependence of edge-plasma rotational transform, $t(r_p)$, on plasma aspect ratio, $A = R_T/r_p$, for fixed lateral coil distortion, $d/r_c = 0.3$, for $\ell = 2$ and 3 and $m = 4, 6$, and 8 . The second graph compares the results of the conventional stellarator expansion [Eq. (B-52)], for $r_s \sim r_w$ and $r_s \sim r_c$, with the modular-coil helix/antihelix superposition model results (solid curves) for various values of d/r_c .

consistently lower for the $r_s \sim r_c$ (pumped-limiter impurity control) case than for the $r_s \sim r_w$ (magnetic-divertor impurity control) case. Thus, in a given device with A and the number of coils, N, fixed, the $r_s \sim r_c$ case is more likely to avoid coil interference (Sec. B.5.). A limiter ($r_s \sim r_c$) rather than a divertor ($r_s \sim r_w$) is suggested as the impurity-control technique from the viewpoint of minimizing the lateral coil distortion for a maximum $\iota(r_p)$, but limits imposed by equilibrium and stability considerations on beta must first be examined.

The $m = 8$, $N = 24$ case considered in Fig. B-6 corresponds to the interim design point of Table IV-III. The rotational transform value, $\iota \sim 0.15$, for $d/r_c = 0.3$ at $A = 11$ falls short of the conventional stellarator expansion result, $\iota \sim 0.66$. Options required to bridge this gap in a self-consistent design are considered in Sec. B.8. of this appendix.

B.4. Equilibrium and Stability Beta Limits

The thrust of the developing theoretical and numerical studies of stellarator/torsatron equilibrium and stability beta limits over the last twenty years suggests that $\langle \beta \rangle \propto \iota^2 \epsilon < 0.1$. The relatively low value of $\langle \beta \rangle$ is potentially the key stellarator/torsatron reactor disadvantage. Although efforts to determine the potential for "beta-enhancement" will continue, for purposes of this study, the attainment of a marginally acceptable value of $\langle \beta \rangle$ as suggested by contemporary understanding is a fundamental assumption. For this reason the equilibrium and stability beta limits imposed by this study, although conservative, are based on the effects of diffusion-driven currents.

B.4.1. Equilibrium Beta Limits. As noted by Miyamoto,⁷ exact and simple equilibrium solutions for a toroidal stellarator do not exist. Within the context of this study, therefore, it is assumed that Pfirsch-Schlüter currents associated with a finite pressure give rise to a vertical field equal to⁷

$$B_v/B_o = \frac{(\nu - 2\ell + 6)\nu}{2[(\nu - 2\ell + 5)^2 - 1]} \frac{\beta_o}{\iota_o \Delta(r_p)} \quad , \quad (B-57)$$

where the pressure profile and rotational transform used to arrive at Eq. (B-57) are given by Eqs. (B-8) and (B-52). The shift of the magnetic axis caused by this vertical field is equal to $R(B_v/B_o)/\iota_o \Delta(r_p)$, where ι_o and $\Delta(r_p)$ are given by Eqs. (B-52) for the low-shear modular-coil configuration with

$\Delta(r) \approx 1$. Equating this Shafranov shift to the plasma radius and utilizing Eq. (B-57) leads to the following equilibrium constraint on the centerline beta value.

$$\beta_0(\text{EQ}) \leq (r_s/r_p) [\tau_0 \Delta(r_p)]^2 \leq \frac{2[(\nu - 2\ell + 5)^2 - 1]}{(\nu - 2\ell + 6)\nu} . \quad (\text{B-58})$$

As has been shown previously (Sec. B.1.), the profile-averaged beta, $\langle \beta \rangle$, equals the on-axis value, β_0 , reduced by the factor $\nu/(\nu + 2)$, where ν is the radial-profile index. It is recalled from Eq. (B-52C) that the quantity $\Delta(r_p) = [1 + (m\epsilon)^2/2(\ell-1)]$ is a second-order correction, which under certain circumstances can be important, particularly with respect to shear in an $\ell = 2$ system. Equation (B-58) is used as the primary equilibrium constraint for purposes of this study.

B.4.2. Stability Beta Limits. A wide range of theoretical stability beta limits have been applied to the stellarator:^{7,15-27} resistive and nonresistive Kruskal-Shafranov limits on bootstrap currents, ballooning modes, Suydam modes, etc. This study applies a simplified theory of diffusion-driven currents to establish a stability-related beta limit imposed by gross kink modes. Specifically, the toroidal current density driven by a radial pressure gradient is given by

$$j_\phi \approx - \frac{1}{B_\theta} \left[\frac{r}{R_T} \right]^{1/2} \frac{dp}{dr} . \quad (\text{B-59})$$

This expression reduces to the following form if the radial pressure profile given by Eq. (B-8) is used.

$$j_\phi = \frac{\epsilon^{1/2}}{B_\theta} \frac{\nu p_0}{r_p} (r/r_p)^{\nu-1/2} . \quad (\text{B-60})$$

If I_ϕ is the total net toroidal current encompassed by the plasma at a minor radius, r , the effective poloidal field at radius, r , is given by

$$B_{\theta} = \frac{\mu_0 I_{\phi}}{2\pi r} + \alpha(r) B_{\phi}(r/R_T) \quad . \quad (B-61)$$

Because $j_{\phi} = (dI_{\phi}/dr)/2\pi r$, defining $\rho = r/r_p$, $y = I_{\phi}/I_{\phi 0}$, and $I_{\phi 0} \equiv 2\pi \epsilon_0 \epsilon B_{\phi} r_p / \mu_0$, reduces Eq. (B-60) to

$$\frac{dy}{d\rho} = \frac{v\beta_0}{2\epsilon^{3/2}\epsilon_0^2} \frac{\rho^{v+3/2}}{y + \rho^{2(l-1)}\Delta(\rho)} \quad . \quad (B-62)$$

In arriving at Eq. (B-62), Eq. (B-52) was used for $\alpha(\rho) = \epsilon_0 \rho^{2(l-2)}\Delta(\rho)$. Although Eq. (B-62) can be solved numerically to yield the current profile that results for the assumed pressure profile, for the purposes of this study j_{ϕ} is specified to be constant across the plasma radius. In this case the following expression results for the centerline beta when $r = r_p$ (i.e., $\rho=1$).

$$\beta_0 = \frac{4\epsilon^{3/2}\epsilon_0^2}{v} y(r_p) [\Delta(r_p) + y(r_p)] \quad . \quad (B-63)$$

If finite-resistivity ($\eta \neq 0$) modes occur, the stability condition, as applied at $r = r_p$, requires that the total rotational transform at the plasma edge, $\alpha_{TOT} = \epsilon_0 [\Delta(1) + y(r_p)]$, satisfy the following constraint.²⁷

$$\epsilon_0 [\Delta(1) + y(r_p)] \leq n/q_a, \quad (n-1 \leq \epsilon_0 \Delta(r_p) \leq n) \quad , \quad (B-64)$$

where q_a is a safety factor, n is an integer, and $\epsilon_0 \Delta(r_p)$ is the vacuum rotational transform at the plasma edge. Applying this restriction for the resistive kink mode to yield $y(r_p) \leq n/q_a \epsilon_0 - \Delta(r_p)$ for $n-1 \leq \epsilon_0 \Delta(r_p) \leq n$, the following resistive stability limit on β_0 results.

$$\beta_0(\text{ST}) \leq \frac{4\epsilon^{3/2}}{\nu q_a^2} [n - q_a \tau_0 \Delta(r_p)] n, \quad (n \neq 0) \quad . \quad (\text{B-65})$$

If resistive kink modes are not important,²⁷ the system is stable if the vacuum transform at the edge, $\tau_0 \Delta(r_p)$, exceeds the transform associated with pressure-driven currents, $y(r_p) \tau_0$. Hence, applying the condition $y(r_p) \leq \Delta(r_p)$ to Eq. (B-63), the following stability-related beta limit results for the case where resistance is not important.

$$\beta_0(\text{ST}) \leq 8\epsilon^{3/2} [\tau_0 \Delta(r_p)]^{2/\nu}, \quad (n = 0) \quad . \quad (\text{B-66})$$

Fig. B-7. Equilibrium and stability limits on $\langle \beta \rangle$ for the $\ell = 2$, $r_s \sim r_c$ stellarator case with radial profile index, $\nu = 3$, as a function of plasma aspect ratio, $A = 1/\epsilon = R_T/r_p$, for various n -numbers. Resistive ($\eta \neq 0$) and ideal ($\eta = 0$) stability limits are shown.

Fig. B-8. Equilibrium and stability limits on $\langle \beta \rangle$ for the $\ell = 3$, $r_s \sim r_c$ stellarator case with radial profile index, $\nu = 3$, as a function of plasma aspect ratio, $A = 1/\epsilon = R_T/r_p$, for various m -numbers. Resistive ($\eta \neq 0$) and ideal ($\eta = 0$) stability limits are shown.

Equations (B-58), (B-65), and (B-66) have been evaluated parametrically and results are shown in Fig. B-7 for $\ell = 2$ and Fig. B-8 for $\ell = 3$. Average rather than central beta values are displayed for consistency with the model development in Secs. B.1. and B.2. of this appendix. Figure B-7 shows the results of the $\ell = 2$ case for profile index, $\nu = 3$, and $r_s \sim r_c$, implying pumped-limited impurity control. It is recalled from Sec. B.3. that $r_s \sim r_c$ configurations are expected to require less lateral coil distortion (i.e., are more likely to meet the coil interference constraint of Sec. B.7. of this appendix) and result in better plasma volume utilization within the first-wall radius, r_w . Limiting average beta values are plotted as a function of plasma aspect ratio, $A = R_T/r_p$, in the range, $A = 3-40$, for $m = 4, 6$, and 8 . Generally, the higher the m -number, the higher are the equilibrium and ideal ($\eta = 0$) stability limits. Application of the resistive ($\eta \neq 0$) stability limit sharply restricts the available parameter space, however. For $n = 1$, the $\beta(\text{ST}, \eta \neq 0)$ curves drop sharply with increasing A , allowing $\langle \beta \rangle \sim 0.02$ in

combination with the equilibrium limit. For $n = 2$, a sawtooth region is encountered for $m = 6$ and 8 whose peak is bounded by the corresponding $\beta(\text{ST}, \eta=0)$ curves. Simultaneous application of all three limits yields peak beta values, $\langle\beta\rangle \sim 0.04$, at $A = 11$ for both $m = 6$ and $m = 8$ systems, under the assumption $r_s \sim r_c$ for $\ell = 2$.

Figure B-8 shows the results of the $\ell = 3$ case with profile index, $\nu = 3$ and $r_s \sim r_c$. For this case, equilibrium beta limits are lower than the comparable $\ell = 2$ results, the ideal ($\eta=0$) stability limits fall below 0.01 and are therefore not of reactor interest, and the resistive ($\eta \neq 0$) stability limits do not enter the $n = 2$ sawtooth regime, but do allow $\langle\beta\rangle$ values near 0.02 in combination with the equilibrium limits. Comparison of Figs. B-7 and B-8 clearly dictates a focus of consideration onto $\ell = 2$ rather than $\ell = 3$ systems from the viewpoint of maximizing $\langle\beta\rangle$ at convenient values of A . The relationship $\beta'(EQ) = A/64$ has been proposed by Gibson^{27,28} for the equilibrium limit expected for an $\ell = 3$ stellarator of an unspecified number of field periods, m , and profile index, ν . It has been observed that $\beta'(EQ) = A/64$ describes well the locus of limiting values of $3\beta(EQ)$ for both $\ell = 2$ and $\ell = 3$ at arbitrary values of m and is thus generally consistent with the present results. The regions where stability and equilibrium beta limits are simultaneously satisfied are shown in Fig. B-9 for $\ell = 2$ and $m = 6$ and 8 for $r_s \sim r_c$ (implying pumped-limiter impurity control) and $r_s \sim r_w$ (implying magnetic-divertor impurity control). The $r_s \sim r_c$ configuration yields a higher value of $\langle\beta\rangle$ (i.e., ~ 0.04) for a convenient plasma aspect ratio, $A = 11$, than can be obtained from the $r_s \sim r_w$ case for $\ell = 2$ and $m = 6$ or 8 , when all three limits on $\langle\beta\rangle$ are imposed simultaneously. It is emphasized that the application of these simplified equilibrium and stability theories is a key driver in the selection of an interim design point. Refinement of the theories that allows a different scaling or magnitude on any of the three distinct limits will change the intersection of the respective curves and will alter the optimum $\langle\beta\rangle$ - A pair. The issue of beta limits is an important one for further investigation.

On the basis of the foregoing discussion attention can be narrowed to the following two representative cases for $\nu = 3$, $n = 2$, and $\ell = 2$: $m = 8$ yielding $\langle\beta\rangle = 0.04$ at $A = 11$ and $m = 6$ yielding $\langle\beta\rangle = 0.04$ at $A = 11$. A common value of $\langle\beta\rangle$, which is incidentally close to the value $\langle\beta\rangle = 0.035$ assumed for the T-1 torsatron conceptual design,⁵ is attainable for both cases. If the number of

Fig. B-9. Dependence of combined equilibrium and stability limits on $\langle\beta\rangle$ as a function of plasma aspect ratio, $A = 1/\epsilon = R_T/r_p$, for various values of m for a fixed radial profile index, $\nu = 3$. The $r_s \sim r_c$ case (implying pumped-limiter impurity control) yields higher $\langle\beta\rangle$ values at a convenient plasma aspect ratio, $A = 11$, than the $r_s \sim r_w$ case (implying magnetic-divertor impurity control).

modular coils, N , is fixed or the number of coils per field period is fixed for either case (e.g., $N/m = 3$), then the $m = 6$ case is benefitted by taking a lower value for m (i.e., $m = 6$ as opposed to $m = 8$). The maximum allowable values for $\langle\beta\rangle$ are not changed by this difference. In any event the two cases cited above ($m = 8$, $A = 11$ and $m = 6$, $A = 11$) for maximized $\langle\beta\rangle \sim 0.04$ are selected for further parametric reactor analysis (Sec. B.7.), with the former being adopted as the interim MSR design configuration. According to the discussion given in Sec. B.3., the $r_s \sim r_c$ (limiter) configuration is used for both cases.

Three-dimensional computer modeling²⁹ of MHD equilibrium and stability suggests that a combination of $\ell = 2$ and $\ell = 3$ windings may lead to more favorable beta limits (central $\beta_0 = 0.076$ or $\langle\beta\rangle \sim 0.05$ for $A \sim 10$) than either poloidal-field-period number taken individually. Such a combined configuration might be implemented in the MSR by simply alternating $\ell = 2$ and $\ell = 3$ coils around the major circumference; some modulation of the minor toroidal axis on which the coil centers are positioned may be required. This possibility as applied to a reactor, however, must await future study.

B.5. Modular-Coil Interference Constraint

The lateral deformations of the MSR coils relative to the coil plane, as measured by the ratio d/r_c , are used in Sec. B.3. to determine the number, N , of modular coils. The approach in Sec. B.3. assumed filamentary windings. This issue is briefly addressed in this section in order to characterize more carefully the modular coil interference constraint.

Imposing a nominal blanket and shield thickness, $\Delta b(m)$ (typically ~ 1.5 m), and a square-cross-section magnetic coil thickness, $\delta_c(m)$, allows the mean radius of the coil, $r_c(m)$, to be determined as

$$r_c = r_w + \Delta b + \delta_c/2 \quad (B-67A)$$

$$= (r_p/x) + \Delta b + \delta_c/2 \quad (B-67B)$$

The local magnetic field strength, $B(T)$, expressed as a function of major radius, $R(m)$, for a system of N coils carrying a fixed, homogenized current density, $j(A/m^2)$, is approximately given by

$$B = \frac{\mu_0 N (j \delta_c^2)}{2\pi R} . \quad (B-68)$$

Equation (B-68) can be used to determine δ_c once the ignition condition has been solved for a given transport scaling to give the required value of $B_0 = B(R_T)$ once beta is determined. This MSR coil envelope will interfere with itself on the inner side of the torus unless the following condition is satisfied.

$$\frac{\delta_c + 2d}{2(R_T - r_c - \delta_c/2) \tan(\pi/N)} < 1 . \quad (B-69)$$

This expression treats the coil envelope as a regular polygon of N sides, each with length $\delta_c + 2d$, circumscribed about a circle of radius $R_T - r_c - \delta_c/2$. Figure B-10 illustrates this constraint. This condition is more restrictive than the condition that $r_c + \delta_c/2 < R_T$, the violation of which means that the axial hole in the torus has been closed off. For large N , $\tan(\pi/N)$ is approximately π/N .

B.6. Coil Stress Constraint

Coil stress considerations imposed an additional engineering constraint on the reactor design. A simple model is described here pending an analysis using more sophisticated stress codes. The mean force per unit length, f_ℓ (N/m), at a major radius, R (m), experienced by a modular coil with small values of d/r_c is approximated by

$$f_\ell = \frac{\mu_0 N (j \delta_c^2)}{4\pi R} . \quad (B-70)$$

Fig. B-10. The MSR coil-interference constraint monitors potential overlap of the adjacent coil envelopes on the inboard side of the torus.

The corresponding mean stress, $\langle \sigma \rangle$ (Pa), can be estimated by

$$\langle \sigma \rangle = \frac{f_k r_c}{\delta_c^2} . \quad (\text{B-71})$$

Letting $R = R_T$ and $B = B_0$, Eqs. (B-68), (B-70), and (B-71) can be combined to give the following upper limit on the parameter grouping, $r_c B_0$, for a maximum allowed value of mean stress, $\hat{\sigma}$ (Pa).

$$(r_c B_0)^* < \frac{2\hat{\sigma}}{j} . \quad (\text{B-72})$$

For a given value of $\hat{\sigma}$, the limit $(r_c B_0)^*$ may be raised by lowering the

current density, j . A lowering of j will tend to increase δ_c to keep the current, $I = j\delta_c^2$, fixed in order to produce a given on-axis field strength for a fixed number of modular coils. Ultimately the modular-coil interference constraint as represented by Eq. (B-69) is violated. A self-consistent design, as a minimum, must satisfy

$$r_c B_0 < (r_c B_0)^* . \quad (B-73)$$

This constraint is less restrictive than monitoring the stress level on the inboard side of the torus, an estimate of which can be approximated from the EFFI results or more rigorously using the SAP4 code (Sec. V.1.3.).

B.7. Computational Algorithm

On the basis of the simplified and approximate models described in the preceding subsections, a comprehensive, "self-consistent" computational algorithm has been generated that evaluates a range of ignited MSRs. Figure B-11 gives a logic flow diagram for this algorithm, which is described quantitatively below.

Combining Eqs. (B-1) and (B-17), once the $\langle n_i \rangle^2$ terms have been cancelled, the ignition condition can be written in the form of the Lawson criterion as

$$\langle n_e \rangle \tau_E = \frac{4.806(10)^{-16} f_{Z1} \langle T \rangle}{[f_\alpha 1.41(10)^{-13} \langle \sigma v \rangle (I_2/I_{1/2}^4) - 5.35(10)^{-37} f_{Z1} Z_{eff} \langle T \rangle^{1/2} (I_{5/4}/I_{1/2}^{5/2})]} . \quad (B-74)$$

The alpha-particle energy trapping efficiency, f_α , can be selected parametrically, or, as will be seen below, can be modeled directly. If Z_{eff} is known, the quantity f_{Z1} on the right-hand side of Eq. (B-74) is determined by Eq. (B-16A). Also, if the profile index, ν , is known, the various I_x profile integrals are determined (Table B-I). The right-hand side of Eq. (B-74) is, therefore, a pure function of $\langle T \rangle$ once these auxiliary

PRIMARY VARIABLES:

- THERMAL POWER, P_{TH}
- WALL LOADING, I_w

SECONDARY VARIABLES:

- TRANSPORT SCALING
  - ALCATOR
  - BOHM-LIKE, $f_B \sim 100$
  - CLASSICAL
  - NEOCLASSICAL
- IMPURITY CONTROL
  - DIVERTOR, $r_s \sim r_w$
  - PUMPED LIMITER, $r_s \sim r_c$
- BLANKET/SHIELD THICKNESS, Δb (1.5 m)
- BLANKET ENERGY MULTIPLICATION, M_N (1.1)
- PROFILE INDEX, ν (3)
- COIL CURRENT DENSITY, j (10–15 MA/m²)
- STRESS LIMIT, $\hat{\sigma}$ (200 MPa)
- POLE NUMBER, ℓ (2 OR 3)
- COILS/FIELD PERIOD, N/m (3)

NOTATION:

COMPUTATION OF VARIABLE Y FROM
VARIABLES y_1, y_2 AND y_3

Fig. B-11. Computational algorithm for the parametric analyses of ignited MSRs.

parameters are known. Equation (B-74) is plotted in Fig. B-12 as the dashed curve. The dashed curve reflects the approximations contained in Eqs. (B-6) and (B-12A) that are built into the reactor survey model. The solid curve in Fig. B-12 is the result of a numerical integration, which does not depend on these approximations, of the first term (representing alpha-particle heating) in the denominator of Eq. (B-74). The agreement between the two results is considered good and justifies the use of the approximate model. The MSR design point at $\langle T \rangle = 8$ keV is also shown in Fig. B-12. The $\langle n_e \rangle \tau_E$ curves in Fig. B-12 exhibit a shallow minimum in the vicinity of $\langle T \rangle = 20$ keV. The design point is not selected to minimize the required Lawson parameter, rather, if the Alcator transport scaling expression [Eq. (B-18)] is substituted for the left-hand side of Eq. (B-74), the parameter grouping, $\langle \beta \rangle B_{0p}^2 r_p$, can be plotted as a function of $\langle T \rangle$ as in Fig. B-13. The interim MSR design point at $\langle T \rangle = 8$ keV is near the minimum of this parameter, reflecting low-field and compact reactor performance for a given $\langle \beta \rangle$.

The alpha-particle energy trapping efficiency, f_α , has been estimated³⁰ to be a function of plasma aspect ratio, $A = R_T/r_p$. This dependence is approximated here by the expression

$$f_\alpha \sim 1 - e^{-A/5.22} , \quad (B-75)$$

which is shown graphically in Fig. B-14. The interim design point ($A = 11$) gives the value of $f_\alpha = 0.88$ used in Figs. B-12 and B-13. If f_α is too small, the denominator on the right-hand side of Eq. (B-74) will become negative, and physical solutions to the ignition condition will not exist. This condition corresponds to the ideal ignition condition (i.e., trapped alpha-particle power just balances Bremsstrahlung losses) and defines an ideal ignition threshold, $\langle T \rangle^*$, as a function of f_α ; this condition is also shown in Fig. B-14.

The total thermal power output of the ignited reactor, $P_{TH}(W)$, is given by

$$P_{TH} = \langle P_I \rangle + \langle P_{BR} \rangle + \langle P_N \rangle M_N + (1 - f_\alpha) \langle P_\alpha \rangle = \langle P_N \rangle M_N + \langle P_\alpha \rangle . \quad (B-76)$$

Fig. B-12. Lawson parameter, $\langle n_e \rangle \tau_E$, required for ignition as a function of average plasma temperature, $\langle T \rangle$, for $\nu = 3$, $Z_{eff} = 1.1$, and $f_\alpha = 0.88$. The interim MSR design point at $\langle T \rangle = 8$ keV is shown. The dashed curve is a plot of Eq. (B-74) that reflects the approximations contained in Eqs. (B-6) and (B-12A). The solid curve is the result of a numerical integration of the alpha-particle power density without these approximations.

Fig. B-13. Dependence of the parameter $\langle \beta \rangle B_0^2 r_p$ required for ignition, assuming Alcator transport scaling [Eq. (B-18)], as a function of average plasma temperature, $\langle T \rangle$, for $\nu = 3$, $Z_{eff} = 1.1$, and $f_\alpha = 0.88$. The interim MSR design point at $\langle T \rangle = 8$ keV is shown.

Fig. B-14. Approximate dependence³⁰ of the alpha-particle heating efficiency, f_α , on the plasma aspect ratio, $A = R_T/r_p$, and the ideal ignition temperature, T^* , on f_α .

Each component of P_{TH} is related to its corresponding power density, $\langle p \rangle$, by $P_j = \langle p_j \rangle V_p = \langle p_j \rangle (2\pi^2 R_T r_p^2)$, where the subscript "j" $\equiv$ " α ", "BR", or "g". The average neutron power density, $\langle p_N \rangle (W/m^3)$, is related to the alpha-particle power expression [Eq. (B-17A)] by

$$\langle p_N \rangle = \frac{14.06}{3.52} \langle p_\alpha \rangle \quad (B-77A)$$

$$= 5.63(10)^{-13} \langle n_i \rangle^2 \langle \sigma v \rangle (I_2/I_{1/2}^4) \quad (B-77B)$$

Allowing for neutron energy multiplication in the blanket by the factor, M_N (typically $M_N \sim 1.1$), the total thermal power, $P_{TH}(W)$, can be written as

$$P_{TH} = 5.63(10)^{-13} \langle n_I \rangle^2 \langle \sigma v \rangle (M_N + 0.25) (I_2/I_{1/2}^4) V_p . \quad (B-78)$$

Elimination of the $\langle n_I \rangle^2$ term by using the pressure-balance relation [Eq. (B-17D)], and rearrangement of Eq. (B-78) gives

$$R_T \langle \beta \rangle^2 B_o^4 r_p^2 = 5.85(10)^{-32} \frac{f_{Z2}^2}{(I_2/I_1^2)} \frac{\langle T \rangle^2}{\langle \sigma v \rangle} \frac{P_{TH}}{(M_N + 0.25)} . \quad (B-79)$$

Incorporation of results from Sec. B.4. suggests fixing the plasma aspect ratio in order to maximize its corresponding value of $\langle \beta \rangle$. The total thermal output can also be fixed to a convenient value. Equation (B-79) is then rewritten in the form

$$r_p^3 \langle \beta \rangle^2 B_o^4 = 5.85(10)^{-32} \frac{f_{Z2}^2}{(I_2/I_1^2)} \frac{P_{TH}}{A(\langle \sigma v \rangle / \langle T \rangle^2) (M_N + 0.25)} , \quad (B-80)$$

where the right-hand side is a weak function of $\langle T \rangle$, because $\langle \sigma v \rangle / \langle T \rangle^2$ is approximately constant for plasma temperatures of interest [Eq. (B-6)]. For $M_N = 1.1$, $v = 3$, and $Z_{eff} = 1.0$, it follows that the total power constraint gives $r_p^3 A \langle \beta \rangle^2 B_o^4 / P_{TH} = 55.8$ for P_{TH} expressed in Gwt units.

An additional constraint of engineering and economic interest is the average neutron first-wall loading, $I_w (W/m^2)$, which takes the following form.

$$I_w = \frac{P_{TH}}{(2\pi)^2 R_T r_w (M_N + 0.25)} \quad (B-81A)$$

$$= \frac{P_{TH} x}{(2\pi)^2 A r_p^2 (M_N + 0.25)} . \quad (B-81B)$$

If the value of $x = r_p/r_w$ is sufficiently large, the poloidal deviations of the local neutron wall loading can be expected to be small, particularly for

Fig. B-15. Variation of local neutron wall loading, I_w , as a function of azimuthal angle, θ , for various aspect ratios, $A = R_T/r_p$, and for a fixed plasma filling fraction, $x = r_p/r_w$ (Ref. 31).

large-aspect-ratio systems. These deviations, expressed in percent changes from the average first-wall neutron loading, are plotted³¹ in Fig. B-15 as a function of azimuthal angle, θ , for various plasma aspect ratios and for a typical fixed value of the filling fraction, $x = r_p/r_w = 0.8$. The angle $\theta = 0(\pi)$ corresponds to the top (bottom) of the torus (i.e., $R = R_T$).

For fixed P_{TH} and I_w Eq. (B-81B) can be solved for r_p , and this value is used in Eq. (B-80) to solve for B_0 (or $\langle\beta\rangle^{1/2}B_0$) as a function of $\langle T \rangle$ and the other fixed quantities. Specification of the energy transport scaling {i.e., Alcator [Eq. (B-18)] or Bohm-like [Eq. (B-20)]} as the right-hand side of Eq. (B-74) allows complete characterization of the plasma operating point as a function of $\langle T \rangle$ for the fixed parameters indicated above. As indicated in Fig. B-11, implementation of this basic computational procedure is accomplished in conjunction with: a) the imposed beta limits for both equilibrium and stability, b) assumed transport scaling, c) coil-interference

constraints, and d) coil stress limits. The reactor survey calculations of Sec. IV. are performed using $\langle T \rangle$ as the "marching" variable and with P_{TH} and I_w used as primary parametric variables. On the basis of the studies summarized in Table B-II, Δb and M_N are taken as 1.5 m and 1.10, respectively, for purposes of the MSR survey calculations. Pending detailed nucleonics evaluation of an elaborated blanket and shield design, the blanket and shield performance of the MSR is characterized in this study by just two parameters: the blanket and shield thickness, $\Delta b(m)$, and the blanket fusion-neutron energy multiplication, M_N .

Another parameter of considerable economic interest is the system power density, p_s , defined here as $P_{TH}(MW)$ divided by the total volume enclosed by and including the coils, $V_C = 2\pi^2 R_T(r_w + \Delta b + \delta_C)^2$. The system power density is given by

$$p_s = \frac{P_{TH}}{V_C} = 2(M_N + 0.25)I_w \frac{r_w}{(r_w + \Delta b + \delta_C)^2}, \quad (B-82)$$

which has been expressed in terms of the neutron first-wall loading, $I_w(MW/m^2)$. Recognizing that M_N is relatively insensitive to blanket thickness for $\Delta b \gtrsim 0.3-0.4$ m, Eq. (B-82) shows that P_{TH}/V_C shows an optimum for $r_w = \Delta b + \delta_C$ equal to $(M_N + 0.25)I_w / 2(\Delta b + \delta_C)$, which equals $0.34 MW/m^3$ for the parameters suggested in Table IV-III. This optimum value is close to the $0.33 MW/m^3$ value reported in Table IV-III, but falls short by a factor of $\gtrsim 30$ from the power density within the pressure vessel of a light-water fission reactor.³² Although P_{TH}/V_C is not monitored explicitly by the systems code, it is worth noting that LWR-like power densities can be achieved by: a) increased M_N (i.e., fissioning blankets), b) increased I_w (and first-wall heat load), and/or c) decreased Δb (i.e., normal-conducting, water-cooled copper coils, reduced shielding, but probably increased δ_C). If P_{TH} is to be maintained at or below 4 Gwt, however, both r_w and R_T must be decreased, which for acceptable values of I_w implies both increased $\langle \beta \rangle B_0^2$ and better confinement, neither of which can be justified at this point. As seen from Table IV-IV, however, the P_{TH}/V_C values projected for the MSR are low compared to LWRs, but are nearly identical to values computed for the STARFIRE³³⁻³⁵ and the EBTR^{36,37} designs.

TABLE B-II

SUMMARY OF TYPICAL BLANKET/SHIELD CONFIGURATIONS

	<u>STARFIRE</u> ³³⁻³⁵	<u>NUWMAK</u> ³⁸	<u>TMR</u> ³⁹	<u>WITAMIR-I</u> ⁴⁰	<u>EBTR</u> ^{36,37}	<u>RFPR</u> ⁴¹
First-wall neutron loading, I_w (MW/m ²)	3.6	4.34	2-4	2.4	1.4	2.7
Blanket thickness (m)	0.37	0.2-0.5	0.75	0.73	0.37-0.72	0.55
Shield thickness (m)	0.83-1.1	0.85-1.05	1.0	0.88	0.67-0.57	1.85
Overall blanket/shield thickness, Δb (m)	1.2-1.5	1.05-1.55	1.75	1.61	1.04-1.28	2.4
Breeding material	α -LiAlO ₂	LiPb	Li ₂ O	LiPb	α -LiAlO ₂	Li ₂ O
Coolant material	H ₂ O	H ₂ O	He	LiPb	H ₂ O	H ₂ O
Structural material	SS	Ti-6Al-4V	Inconel-718	HT-9	SS	SS
Shield material	Fe, B ₄ C, Pb, Ti	TiB ₄ C, Pb	lead-cement	Fe, B ₄ C	SS304, H ₂ O, TiB ₂ , TiH ₂	H ₂ O, B
Neutron multiplier	ZrPb ₃ , Be	NA	NA	NA	Be	NA
Tritium breeding ratio	~ 1.1	1.54	1.07-1.27	1.07	1.06	~1.1
Energy multiplication, M_N	1.14	1.22	1.02-1.09	1.37	1.5	1.16

B.8. Numerical Computation of Rotational Transform

A "self-consistent" MSR design point requires that the coil-set geometry yields the rotational-transform value that was assumed by the equilibrium and stability scaling described in Sec. B.4. Verification of the self-consistency of the design involves the cross-checking of the analytic representations of the rotational transform against the numerical results of the EFFI magnetics code.⁴²

The interim MSR point design developed in this report reflects the search for desirable reactor parameters on the basis of ignited operation at moderate power output and neutron first-wall loading. The beta-limit scaling, in the simplified form adopted by this study, drives the choice of an $m = 6$ or 8 , $\ell = 2$ system in order to achieve a $\langle\beta\rangle = 0.04$ plasma at $A = 11$ with acceptable filling fraction, x . The coil-set configuration is imposed subsequently, subject to the constraints of fitting a convenient number of coils, N , with finite cross section and adequate lateral distortion into the given toroidal configuration with moderate power output ($P_{TH} \lesssim 5$ GWt).

Rotational transform results for the interim design point of Table IV-III are summarized in Fig. B-16. The equilibrium and stability scaling that leads to $\langle\beta\rangle \approx 0.04$ requires that $\iota(r) \approx 0.62$ for $m = 8$. The first-harmonic (Rehker-Wobig) coil set with $m = 8$, $N = 24$, $d/r_c = 0.3$, and $A = 11$ yields $\iota(r) \approx 0.15$, which is substantially below the required performance. This result is confirmed by the EFFI numerical computation and by subsequent application of Princeton Plasma Physics Laboratory (PPPL)⁴³ analytic expansions and numerical codes. Because lowering the beta value to a level (< 0.01) consistent with the rotational transform predicted to be necessary by the simplified equilibrium/stability theories, for the specific coil set of the interim design point, would lead to unacceptable degradation of the attractiveness of the reactor, it becomes necessary to investigate modifications in the coil-set parameters that would allow a higher value of rotational transform to be obtained without substantially altering the basic MSR configuration. The available options include:

1. Small increases in the first-harmonic lateral coil distortion with use of a more carefully formulated coil-interference constraint than that developed in Sec. B.5.

Fig. B-16. Rotational transform profiles for the interim design point as required by the equilibrium and stability scaling (dashed curve) and as predicted by Eq. (B-51) (solid curve). The corresponding EFFI results (circles) are plotted. In addition, a PPPL⁴³ analytic expansion result (squares) and numerical result (triangles) are plotted for comparison.

2. Application of a higher-harmonic lateral coil distortion, with the same coil-interference constraint developed in Sec. B.5.
3. Modest increases in the lateral coil distortion for the same values of m and N with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
4. A higher value of N for the same value of m to increase N/m with higher current-density, smaller cross-section coils at the expense of higher coil forces and stresses.
5. A lower value of m for the same values of N and d/r_c to increase N/m at the expense of modifications in r_p/r_w and possibly a different optimal value of $A = R_T/r_p$.

6. Use of coils with noncircular bores and/or a modulated minor toroidal axis.

7. Combinations of the above options.

Should applications of the above potential modifications prove insufficient or unattractive, the final resort of relaxation of some or all of the conservative physics assumptions and engineering constraints remains. Also, the nominal power output could be allowed to increase to ~ 6 GWT in a larger system. Option 1 is anticipated to yield only slight gains in the rotational transform and could be used only to "fine-tune" an otherwise acceptable coil configuration. In the absence of analytic models to describe the higher-harmonic configuration, it is difficult to assess parametrically the improvement in the transform that might result from the implementation of Option 2. Numerical results⁴⁴ for specialized single-filamentary-coil configurations (i.e., $N = 36$, $m = 4$, $d/r_c = 0.2$, $\epsilon = 0.5-0.7$) are encouraging. Options 3 and 4 preserve the physics scaling of the $m = 8$ system, but require a relaxation of the coil technology constraints and can be expected to yield only modest improvement in the rotational transform. Option 5, a lowering of m from 8 to 6, allows a higher value of $N/m = 4$, if the number of coils is fixed at $N = 24$, while still giving a value of $\langle\beta\rangle = 0.04$ at $A = 11$. Particularly when combined with Option 2, the implementation of a higher-harmonic lateral coil distortion, Option 5 is presently viewed as the most likely route to a self-consistent design that meets the set of technology constraints and design goals imposed for this study.

Results of the investigation of an $m = 6$, $N = 24$, $d/r_c = 0.3$, $A = 11$ configuration are summarized in Fig. B-17. The rotational transform for this configuration as obtained from the analytic expression [Eq. (B-52)] is $\epsilon(r) \approx 0.22$, which, although representing an improvement over the $m = 8$ case previously considered, is still insufficient when compared with the value $\epsilon(r) \approx 0.66$ required to justify the beta value of 0.04 on the basis of simplified equilibrium/stability theories used in this study. The EFFI code results for the first-harmonic case yield $\epsilon(0.56 \text{ m}) \approx 0.22$, $\epsilon(1.56 \text{ m}) \approx 0.22$, and $\epsilon(2.79 \text{ m}) \approx 0.22$, in agreement with the analytic result. The analytic expression yields $\epsilon(r) \approx 0.66$ if it is assumed that $d/r_c = 0.5$; however, such a large lateral coil distortion is incompatible with a finite-cross-section coil set. Thus, to bring the rotational-transform value up to $\epsilon(r) \approx 0.66$, it is necessary to apply a higher harmonic lateral coil distortion to a

finite-cross-section coil set that effectively acts as a filamentary, first-harmonic coil set with $d/r_c \approx 0.5$. The result of this implementation for a $d/r_c = 0.3$ higher-harmonic coil set is displayed in Fig. B-17 as $\epsilon(0.48) \approx 0.27$, $\epsilon(1.58 \text{ m}) \approx 0.28$, and $\epsilon(2.50 \text{ m}) \approx 0.27$, representing only a small improvement over the first-harmonic case results. Results of a PPPL computation⁴⁵ using a single-filament model of the higher-harmonic coil configuration are also displayed in Fig. B-17 and yield $\epsilon \approx 0.35$, somewhat more optimistic than the distributed-coil EFFI results. The tendency of single-filament representations to give higher transform estimates than those

Fig. B-17. Rotational transform profiles for the $m = 6$ variant of the interim MSR design point as required by the equilibrium and stability scaling (dashed curve) and as predicted by Eq. (B-52) (solid curves) for $d/r_c = 0.3$ and $d/r_c = 0.5$. The corresponding EFFI results for the first-harmonic coil configuration (squares) and higher harmonic case (circles) are plotted showing small performance improvement. Results from a PPPL computation⁴⁵ (triangles) using a single-filament coil model of the higher harmonic case are also shown.

of either multi-filamentary or distributed-coil models has been noted.^{46,47} Table B-III summarizes the EFFI-code rotational transform results. An auxiliary EFFI computation of the column B case of Table B-III, for which the coil dimension, δ_c , was decreased by a factor of ten and the current density was increased accordingly by a factor of one hundred, was made in order to test the coil representation model. No detectable difference in rotational

TABLE B-III
SUMMARY OF ROTATIONAL-TRANSFORM ENHANCEMENT (EFFI) RESULTS

<u>Case</u>	<u>A</u>	<u>B</u>	<u>C</u>
Number of poloidal-field periods, ℓ	2	2	2
Number of toroidal-field periods, m	8	6	6
Number of modular coils, N	24	24	24
Lateral coil distortion, d/r_c	0.3	0.3	0.3
Higher harmonic coil distortion	no	no	yes
Major toroidal radius, R_T (m)	21.4	20.2	20.2
Plasma aspect ratio, $A = R_T/r_p$	11.0	11.0	11.0
Flux surface #1			
semiminor axis (m)	0.62	0.46	0.38
semimajor axis (m)	0.77	0.69	0.60
effective radius (m)	0.69	0.56	0.48
rotational transform, τ	0.144	0.22	0.27
ellipticity, e	1.24	1.50	1.56
Flux surface #2			
semiminor axis (m)	1.15	1.31	1.08
semimajor axis (m)	1.46	1.85	2.31
effective radius (m)	1.30	1.56	1.58
rotational transform, τ	0.16	0.22	0.28
ellipticity, e	1.27	1.41	2.14
Flux surface #3			
semiminor axis (m)	1.69	2.31	1.85
semimajor axis (m)	2.15	3.38	3.38
effective radius (m)	1.91	2.79	2.50
rotational transform, τ	0.17	0.22	0.27
ellipticity, e	1.27	1.46	1.83

transform results from the fatter-coil result reported in Table B-III was observed. Contrary to expectation, the higher harmonic coil configuration, for which no analytic theory exists presently, does not result in the required improvement of the rotational transform over the result of otherwise consistent first-harmonic configurations from either the analytic expression [Eq. (B-52)] or EFFI numerical computations. Work to resolve this situation continues.

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