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FUSION POWER SYSTEMS TECHNICAL NOTE

STABILITY OF THE PLASMA IN A BUNDLE DIVERTOR

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ABSTRACT

Due to the pressure and magnetic field gradients and curvature of the magnetic field lines in a bundle divertor of a tokamak device, the plasma may be unstable to local interchange modes. Turbulent transport could be quite large and lead to a thick scrape-off layer which is as large as the radius of curvature of the diverted flux bundle. Such turbulence would be beneficial for lowering the energy and particle fluxes on the collector in a bundle divertor. The effect of a bundle divertor on the β limit resulting from the ballooning modes of instability in the central plasma is also estimated. The critical β is reduced by less than one percent.

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1.0 INTRODUCTION

Many theoretical analyses and experiments on divertors have shown that the plasma density and temperature drop sharply outside the separatrix in the scrape-off layer [1-6]. The common practice in the past has been to intercept the plasma in the divertor with a neutralizing or particle collection plate. The thermal power loading will concentrate in a very narrow region near the separatrix due to the high pressure gradient. The high power concentration presents a very difficult technology problem. A known method of spreading the thermal power is to expand the magnetic flux at the collection plate. Because a uniform expansion of the magnetic flux is not possible, this method often leads to a complicated and inefficient system. It would thus be useful to understand the plasma behavior when it is not intercepted by any plate in the divertor. In the case of a bundle divertor, we observe that the diverted flux bundle has a large field gradient and bad curvature. Thus, the plasma could be unstable to the interchange modes.

In the first part of this note, we analyze the interchange instability of the plasma in a bundle divertor when the particle collection plate is not in place. In the second part, the effect of local field ripple due to the bundle divertor on the ballooning mode instability of a tokamak is discussed.

2.0 LOCAL INTERCHANGE INSTABILITY IN BUNDLE DIVERTOR

DITE parameters [6-7] will be used as an example to demonstrate the instability mentioned in the introduction, and to estimate the diffusion coefficient. The flux pattern of the DITE bundle divertor has been reproduced and is shown in Figure 1a. A new flux configuration was also obtained by using a divertor coil angle of 45° and moving the coil outward by 5 cm. This configuration is shown in Figure 1b. Let us define the major radius which passes through the null point as the center line of the divertor. The diverted flux loop will intersect this center line at A. The field intensity $|B_\phi|$ at A is plotted as a function of the major radius for configurations (a) and (b) as shown in Figure 2. These curves show that the gradient of B_ϕ is negative in the bundle divertor. Now, the condition for instability for a closed field line is

$$\left(\frac{d}{d\Psi}\int_{(1)} \frac{dl}{B} + \frac{d}{d\Psi}\int_{(2)} \frac{dl}{B}\right)\frac{dp}{d\Psi} < 0; \tag{1a}$$

or

$$\left(\frac{d}{dr}\int_{(1)} \frac{dl}{B} + \frac{d}{dr}\int_{(2)} \frac{dl}{B}\right)\frac{dP}{dr} < 0 \tag{1b}$$

where (1) indicates the region of integration inside the tokamak and (2) the region inside the divertor. The separation points of these two regions can be chosen as the mirror points, MN or as the points, M'N', where the flux bundle is leaving the tokamak as described in reference (6). Let us indicate (2) as the path MAN and (2)' as M'AN'. The integrals $\int_{(2)} dl/B$ are computed by an integration over the lines of force. The dimensionless quantity $U = \int\left(\frac{dl}{B}\right)\frac{B_o}{R_o}$ for configurations (a) and (b) is plotted in Figures 3 and 4 respectively. From examination of these curves one will notice that the gradient of $U_{(2)}$ is positive except for $U_{(2)'}$ in Figure 3 which has a minimum close to the separatrix. Such a surface can be identified as a critical surface beyond which the plasma would be unstable. There is no stabilization from the

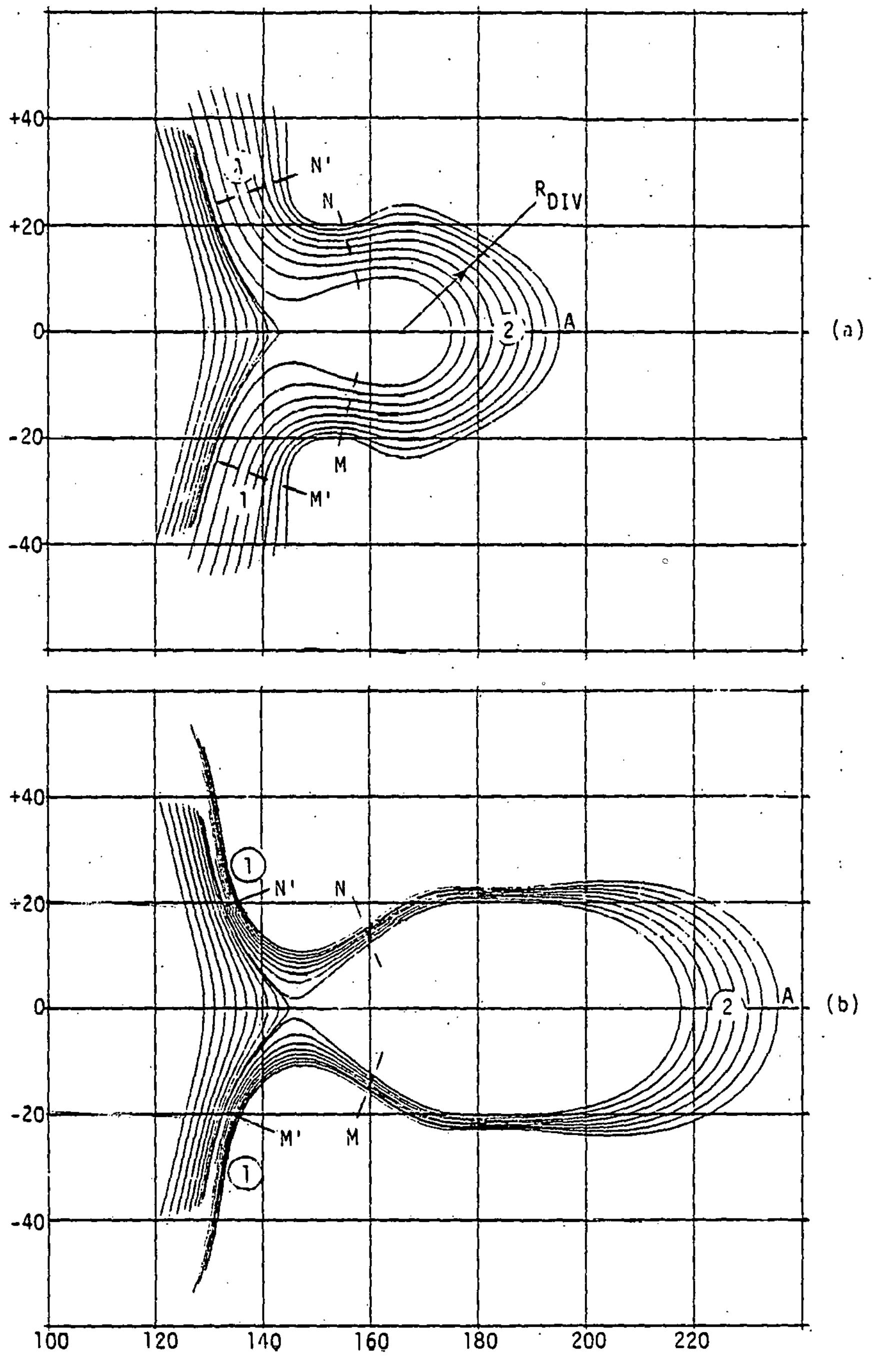

Figure 1. Flux Configurations of a Bundle Divertor
(a) Reproduced from Parameters Published for DITE
(b) New Flux Configuration for DITE for Divertor Coil Angle at 45°.

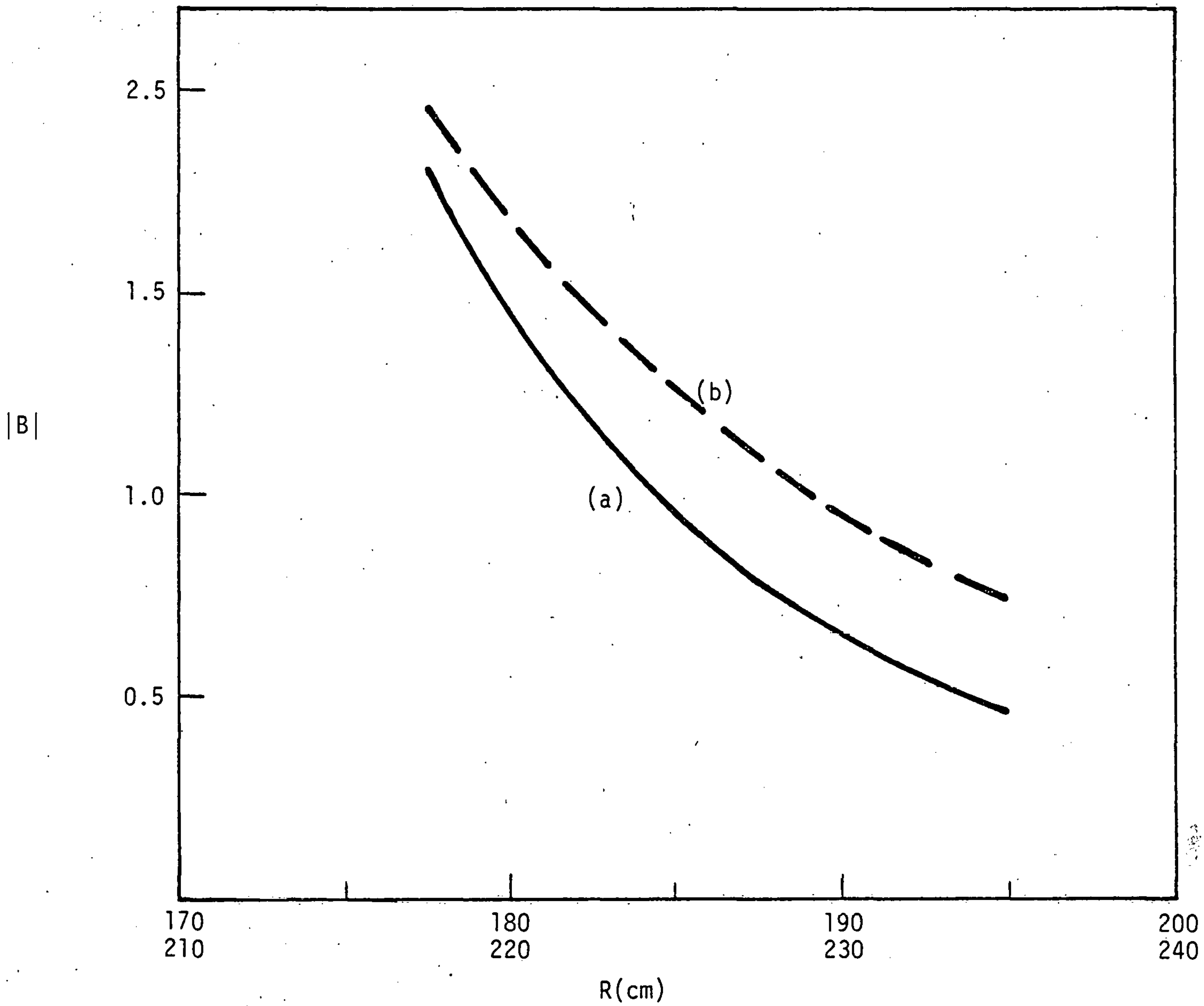

Figure 2. Plot of |B| Versus Major Radius on the Line Passing Through the Null Point. Solid Curves and Upper Scale on Abscissa are for Figure 1a. Dashed Curves and Lower Scale are for Figure 1b.

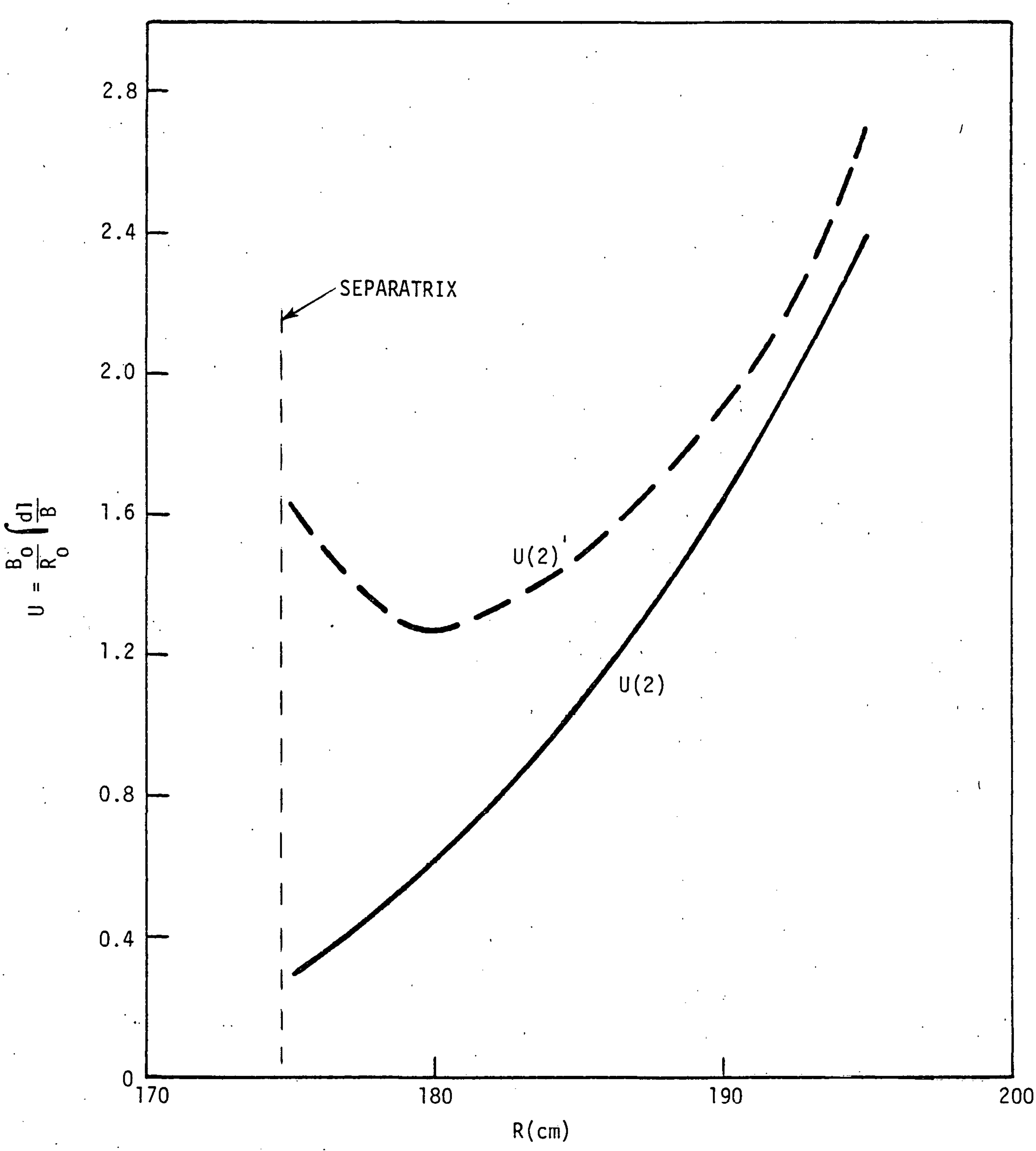

Figure 3. Plot of $U = \frac{B_o}{R_o}\int\frac{dl}{B}$ as a Function of Major Radius for Region (2) (Solid Curves) and region (2)' (Dashed Curve) for configuration (a).

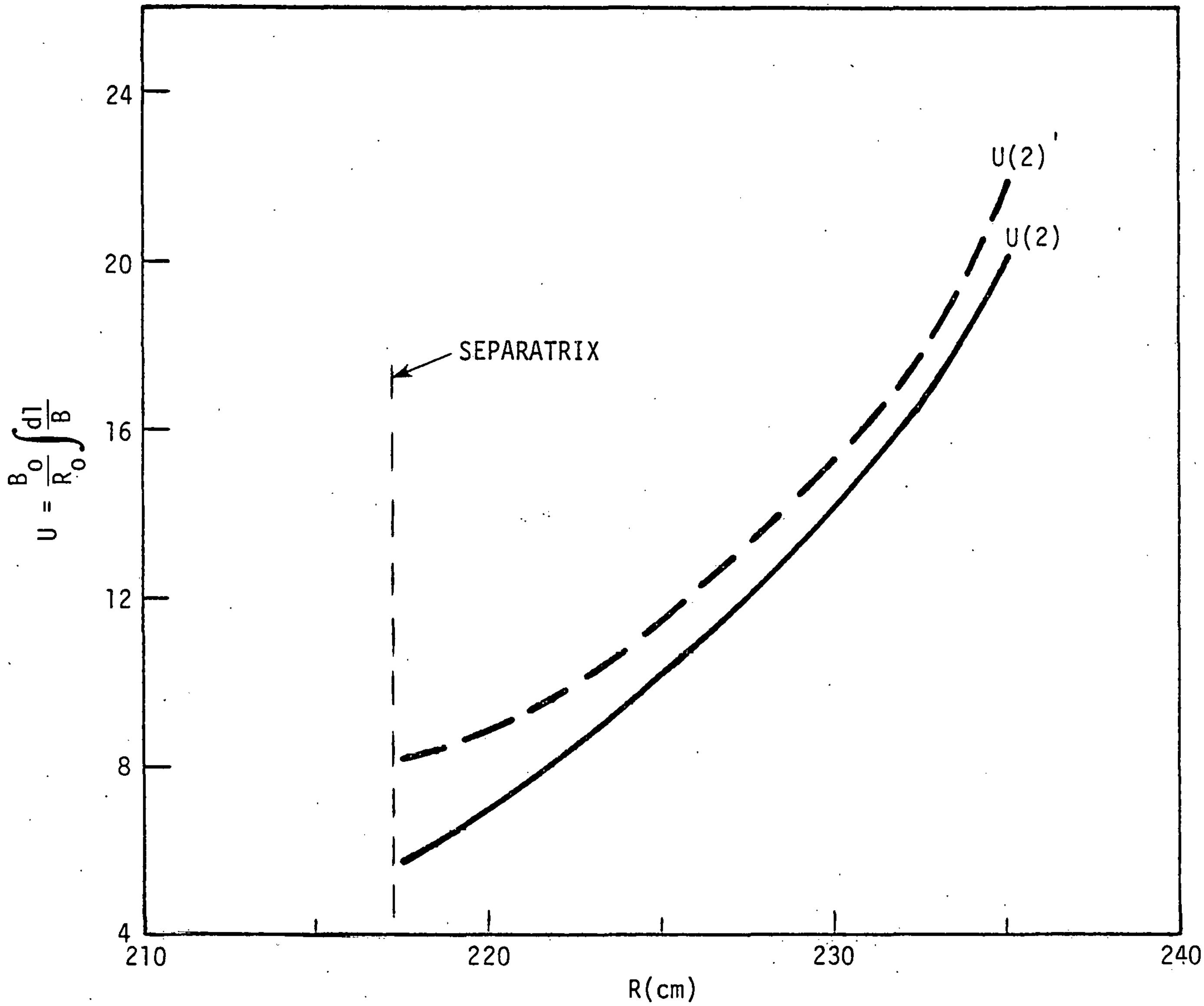

Figure 4. Plot of $U = \frac{B_o}{R_o}\int \frac{dl}{B}$ as a Function of Major Radius for Region (2) (Solid Curve) and Region (2)' (Dashed Curve) for Configuration (b).

from the plasma column region if the second term in equation (1) dominates. The dimensionless quantity $U' = \frac{d}{dx} U$, where $x = \frac{r}{R_o}$, for each region is: $U'_{(2)} \cong 10$ for the configuration (a), and $U'_{(2)} \cong 100$ for the configuration (b). The gradient of $U_{(1)}$ inside the tokamak is discussed in the Appendix. It is given by equation (A-12):

$$U'_{(1)} = -2\pi\varepsilon^2 \frac{r}{R} (4\Delta'_a \frac{R}{a} + 1) < 0$$

So that the plasma is stable inside the tokamak because $\frac{dP}{dr} < 0$. From equation (A-11) we have $\Delta'_{(1)} \sim \varepsilon$ for low Beta. Using DITE parameters (b) we can estimate

$$U'_{(1)} \sim -0.15$$

Therefore $|U'_{(1)}| << |U'_{(2)}|$ in equation (1); i.e., the destabilization associated with the divertor dominates and the plasma is interchange unstable. One also notices that $U'_{(2)}$ is an order of magnitude larger for the configuration (b) than for the configuration (a). Therefore the plasma is more unstable in configuration (b).

The growth rate of the interchange instability modes is approximately

$$\gamma_{inst.} \cong V_S/\sqrt{R_c R_p}\,, \tag{2}$$

where $V_S = \sqrt{T_e/M_i}$ is the sound speed, $\frac{1}{R_p} = \frac{1}{p}\frac{dp}{dr}$, $\frac{1}{R_c} \cong \frac{d}{dr} \ln \int_{(2)} \frac{dl}{B}$. The inverse of the transit time of the plasma flowing through the divertor is

$$\gamma_{transit} \sim V_S/2\pi R_{div}, \tag{3}$$

where R_{div} is the mean radius of the divertor flux loop. Thus, we see that it is possible that

$$\gamma_{inst.} \geq \gamma_{transit}$$

for $R_c \geq R_{div} \geq R_p$.

The cross-field diffusion coefficient of the turbulent transport can be estimated by

$$D_\perp \sim \int d\tau \left. \frac{\tilde{\vec{E}} \times \vec{B}}{B^2} \right|_t \cdot \left. \frac{\tilde{\vec{E}} \times \vec{B}}{B^2} \right|_{t+\tau}$$

$$\sim \frac{1}{\gamma_{inst}} \left(\frac{k_y \tilde{\Phi}}{B} \right)^2$$

$$\sim \left(\frac{\sqrt{R_c R_p}}{V_S} \right) \left(k_y \rho_s \right)^2 \left(\frac{e\tilde{\Phi}}{T} \right)^2 V_S^2 \ . \qquad (4)$$

The values $k_y \rho_s \lesssim 1$ and $e\tilde{\Phi}/T \lesssim 1$. Thus,

$$D_\perp \lesssim V_S \sqrt{R_c R_p} \ . \qquad (5)$$

For this maximum perpendicular diffusion rate, the scrape-off layer thickness is approximately

$$\Delta x \sim \sqrt{D_\perp \tau_{||}}$$

$$\sim \left[2\pi R_{div} (R_c R_p)^{1/2} \right]^{1/2}$$

$$\gtrsim R_{div} \qquad (6)$$

Thus, the scrape-off layer thickness could be as large as the mean radius of the diverted flux loop. Therefore, the plasma may be turbulently dispersed over the entire bundle divertor chamber, thereby leading to a relatively low power loading and particle flux on the divertor chamber wall. Since the interchange modes would be unstable over the entire length of a field line, the plasma could also be turbulently dispersed to the entire toroidal plasma chamber wall, if it is close enough to the separatrix.

The relationship given in Equation (6) indicates that it is better to obtain a flux pattern with a larger radius of curvature. Such a configuration is shown in Figure 1b with an appropriate choice of divertor coil angle and location. The mean radius of the loop, R_{div}, is about doubled compared to the standard DITE case (Figure 1a).

3.0 BALLOONING MODE WITH LOCAL FIELD RIPPLE

It was shown in the last section that the bad curvature in the divertor region will make all the magnetic field line passing through the divertor coils interchange instable. It remains to investigate the effect of the enhanced curvature due to the divertor on the flux surfaces that remain in the column. This can be done by considering ballooning modes.

The effect can be estimated using the potential energy associated with small perturbations from equilibrium [8].

$$2\delta W = \int d\tau \left[Q_{\perp}^2 + \left(\vec{Q}_{||} - \vec{B} \frac{\vec{\xi} \cdot \overline{\nabla} p}{B^2} \right)^2 + \gamma p (\overline{\nabla} \cdot \vec{\xi})^2 \right.$$

$$\left. + \frac{\vec{J} \cdot \vec{B}}{B^2} \vec{B} \times \vec{\xi} \cdot \vec{Q} - 2(\vec{\xi} \cdot \overline{\nabla} p)(\vec{\xi} \cdot \vec{\kappa}) \right] \quad (7)$$

Based on the balancing of the shear term of $Q_{\perp}^2$ with the curvature term of $2(\vec{\xi} \cdot \overline{\nabla} p)(\vec{\xi} \cdot \vec{\kappa})$, a limit on β was given in Reference [9] as $\beta^* \leq a/q^2R$ for axisymmetry. Better estimates on the β limit have been given elsewhere [9-13]. We will use the method given in Reference [9] to estimate the relative change in β due to the local field ripple created by a bundle divertor. The approximate connection length and radius of curvature are qR and R in the axisymmetric case and become $q \cdot (R + \Delta R)$ and $(R - \Delta R)$ respectively with a bundle divertor. Here, ΔR is the local deviation in radius as shown in Figure 1. The shear term in the ripple region is

$$Q_{\perp}^2 \simeq \left[\frac{B - \Delta B}{q(R + \Delta R)} \right]^2 \xi^2$$

$$\simeq \frac{B^2}{q^2 R^2} \xi^2 \left[1 - 4 \frac{\Delta B}{B} \right]. \quad (8)$$

The curvature term is

$$2\vec{\xi} \cdot \vec{\nabla} p \; \vec{\xi} \cdot \vec{k} \cong 2 \frac{P}{a + \Delta a} \; \frac{\xi^2}{R + \Delta R}$$

$$\leq \frac{2P}{aR} \xi^2$$

The ripple occupies a region of angular width $\Delta\phi$. The volume ratio of the ripple region to the entire plasma column is approximately $\Delta\phi/2\pi$. The integral of the shear and curvature terms is approximately

$$\int dsR \int_{-\pi}^{\pi} \left(\frac{B^2}{q^2 R^2} - \frac{2P}{aR} \right) \xi^2 \, d\phi$$

$$\cong -\int_{-\Delta\phi/2}^{+\Delta\phi/2} \left[\frac{B^2}{q^2 R^2} \left(\frac{4\Delta B}{B} \right) - \frac{2P}{aR} \left(\frac{\Delta R}{R} - \frac{\Delta a}{a} \right) \right] \xi^2 \, d\phi \tag{10}$$

Then, the critical β becomes

$$\beta_c \simeq \frac{a}{q^2 R} \left[1 - \frac{\Delta\phi}{2\pi} \left(\frac{4\Delta B}{B} \right) - \frac{\Delta\phi}{2\pi} \left(\frac{\Delta R}{R} - \frac{\Delta a}{a} \right) \right] \tag{11}$$

or

$$\frac{\Delta\beta_c}{\beta_c} \lesssim \frac{\Delta\phi}{2\pi} \; \frac{4\Delta B}{B} \tag{12}$$

The ratio $\frac{\Delta\phi}{2\pi}$ is approximately the inverse of the number of TF coils which is typically less than 1/16. The ratios $\frac{\Delta R}{R}$ and $\frac{\Delta B}{B}$ are a few percent. Therefore, the β limit is reduced only by a few tenths of a percent due to the local ripple created by a bundle divertor.

APPENDIX

DERIVATION OF $V' = \int \frac{dl}{B}$, $\frac{d}{d\Psi}V'$ or $\frac{d}{dr}V'$

Consider an axisymmetric (r, o, ϕ) coordinate system as shown by figure A-1 and a magnetic field of the form

$$B = R_0 B_0 [f(r) \nabla\phi x \nabla r + g(r)\nabla\phi] \tag{A-1}$$

The toroidal flux function given by Greene, Johnson and Weimer [14] is

$$\Psi(r) = \pi B_0 \varepsilon^2 r^2 \left[1 + \varepsilon^2\left(\frac{r^2}{4R^2} + \frac{\Delta(r)}{R} - \frac{2P}{r} + \frac{2}{r^2}\right)\int_0^r rg(2)\,dr\right] \tag{A-2}$$

$\varepsilon = a/R$ is the inverse aspect ratio and $\Delta(r)$ denotes the shift fo the center of the surface from the magnetic axis

$$\Delta(r) = \frac{1}{R}\int_0^r \frac{dr}{rf^{(1)^2}(r)} \int_0^r \left(f^{(1)^2}(r) - \frac{2rP^{(2)'}(r)}{B_0^2}\right) rdr, \tag{A-3}$$

and

$$g^{(2)}(r) = -\frac{P^{(2)}(r)}{B_0^2} + \int_0^a \frac{f^{(1)}}{r}\left[rf^{(1)}(r)\right]'dr, \tag{A-4}$$

where

$$f^{(1)}(r) = \frac{r}{Rq} \tag{A-5}$$

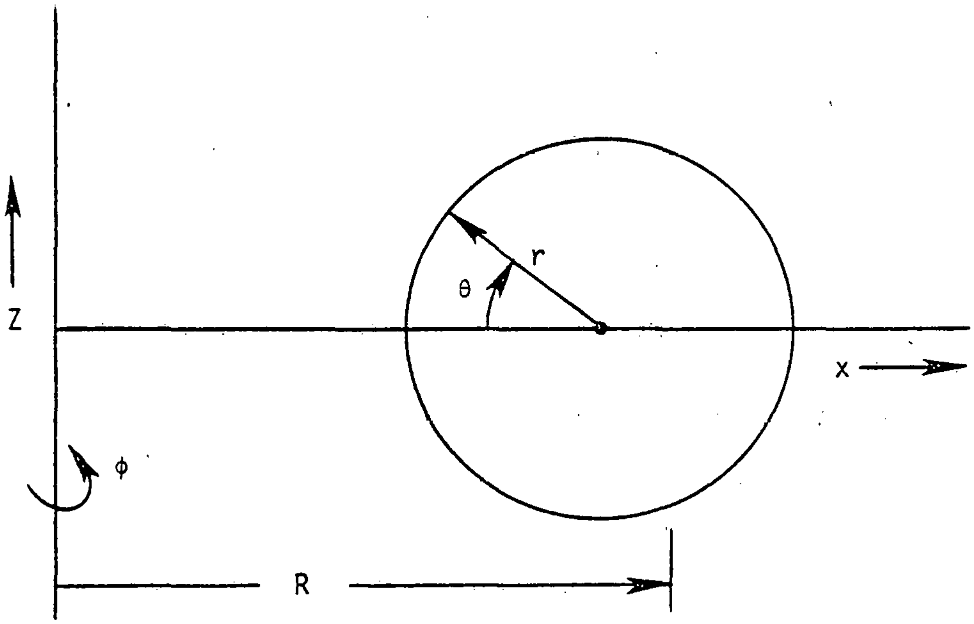

Figure A-1. Coordinate System

The volume inside the magnetic surface is [14]

$$V(r) = 2\pi^2 R\varepsilon^2 r^2 \left[1 - \varepsilon^2\left(\frac{\Delta}{R} + \frac{2P}{r}\right) + \ldots\ldots\right] \quad \text{(A-6)}$$

From equations (A-2) and (A-6) we obtain

$$\frac{dV}{d\Psi} = \frac{dr}{d\Psi}\,\frac{dV}{dr}$$

$$\cong \frac{2\pi R}{B_o}\left[1 - \varepsilon^2\left(2\,\frac{\Delta' + r\Delta^{1/2}}{R} + \frac{r^2}{2R^2} + 2rg^{(2)}\right) + \ldots\right] \quad \text{(A-7)}$$

and

$$\frac{d}{dr}\,\frac{dV}{d\Psi} = \frac{d\Psi}{dr}\,\frac{d^2V}{dr^2}$$

$$= \frac{2\pi R}{B_o}\left[-\,\varepsilon^2\left(\frac{2\Delta' + r\Delta''}{R} + \frac{r}{R^2} + 2g^{(2)} + 2r\,\frac{dg^{(2)}}{dr}\right) + \ldots\right] \quad \text{(A-8)}$$

Our interest is in the scrape-off layer outside the plasma surface $r = a$ (vacuum region) where $P^{(2)} = g^{(2)}(r) = 0$ and

$$f^{(1)}(r) = f_a^{(1)}\,\frac{a}{r}\,; \quad \text{(A-9)}$$

Equation (A-3) gives

$$\Delta(r) = \Delta_a + \frac{(r^2 - a^2)}{4}\left(\frac{2\Delta_a'}{a} - \frac{1}{R}\right) + \frac{r^2}{2R}\ln\frac{r}{a}\,, \quad \text{(A-10)}$$

where

$$\Delta_a' = \frac{a}{R}\left(\beta_\Theta + \frac{1}{a^2 f_a(1)^2}\int_o^a rf(1)^2 dr\right) \tag{A-11}$$

Therefore we have:

$$\frac{d}{dr}\frac{dV}{d\Psi} = -\frac{2\pi R\varepsilon^2}{B_o}\left[\frac{2r}{R}\left(\frac{2\Delta_a'}{a} + \frac{1}{2R}\right) + \frac{4r}{R^2}\ln\frac{r}{a}\right] \tag{A-12}$$

or

$$\frac{d^2V}{d\Psi^2} = -\frac{1}{RB_o^2}\left[\frac{4\Delta_a' R}{a} + 1 + 4\ln\frac{r}{a}\right] \tag{A-13}$$

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