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EXACT INVARIANTS IN RESONANCE FORM FOR TIME-DEPENDENT POTENTIALS

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We have developed a framework for the momentum-resonance formulation of Lewis and Leach [1] that casts new light into the nature of exact, explicitly time-dependent invariants for one-dimensional time-dependent potentials and produces additional examples of such invariants. The momentum-resonance formulation postulates that the invariant be a rational function of momentum with simple poles, which are called momentum resonances. We have shown that an invariant of resonance type can be written as a functional of the potential in terms of the solution of a system of linear algebraic equations; and we have obtained a single necessary and sufficient condition for a potential to admit an invariant of resonance type. These results were obtained by reformulating the problem in terms of a set of discrete moments that satisfy two separate recursion formulae. Invariants for new time-dependent potentials were obtained and previously known invariants were recovered. A detailed description of this work will be published later [2,3].

We consider the Hamiltonian for a particle moving in a potential that depends on a coordinate q and time t ,

$$H = \frac{1}{2} p^2 + V(q, t) \quad (1)$$

The momentum-resonance ansatz of Lewis and Leach [1] postulates that the invariant be a rational function of momentum expressed as a sum of terms whose singularities in momentum are only distinct simple poles ("momentum resonances").

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$$I(q, p, t) = c(q, t) + \sum_{n=1}^N \frac{v_n(q, t)}{p - u_n(q, t)} \quad (2)$$

The functions of position and time that appear in the expression for the invariant in resonance form satisfy conditions that are decoupled to a remarkable degree. Necessary and sufficient conditions on the functions c , v_n and u_n such that $I(q, p, t)$ be an invariant are

$$\frac{\partial c}{\partial q} = 0 \quad (3a)$$

$$\frac{\partial c}{\partial t} + \sum_{n=1}^N \frac{\partial v_n}{\partial q} = 0 \quad (3b)$$

$$\frac{\partial v_n}{\partial t} + \frac{\partial}{\partial q} (u_n v_n) = 0 \quad (3c)$$

$$\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial q} = - \frac{\partial v}{\partial q} \quad (3d)$$

The momentum-resonance ansatz can also be formulated in terms of certain discrete momentum moments. The k^{th} moment $g_k(q, t)$ is defined by

$$g_k(q, t) = \sum_{n=1}^N u_n^k v_n \quad (4)$$

By direct manipulation of (3c)-(3d), it can be shown that these moments satisfy the differential recursion relation

$$\frac{\partial g_k}{\partial q} = - \frac{\partial g_{k-1}}{\partial t} - (k-1) g_{k-2} \frac{\partial v}{\partial q}, \quad k \geq 1 \quad (5a)$$

with the initial condition

$$g_0(q, t) = -\dot{c}(t)q + \alpha_0(t) \quad (5b)$$

In addition to the differential recursion relation (5a), the moments also satisfy an algebraic recursion relation of the form

$$g_l = - \sum_{n=1}^N a_n g_{l-n} \quad , \quad l \geq N \quad (6)$$

We have proved the following theorem:

Linearization Theorem If $V(q, t)$ admits an invariant with N resonances, then this invariant can be expressed by

$$I(q, p, t) = c(t) + \frac{\sum_{n=1}^N p^{N-n} \sum_{k=1}^n a_{k-1} g_{n-k}}{p^N + \sum_{n=1}^N a_n p^{N-n}} \quad , \quad a_0 = 1 \quad (7)$$

in terms of moments g_k and coefficients a_n . The g_k can be calculated from (5a)-(5b) in terms of the potential and $2N+2$ unspecified functions of t ("integration constants"). The a_n are the solution of a system of N linear algebraic equations whose coefficients are also moments g_k . For a specified potential, the entire q dependence of the moments can be determined a priori from (5a)-(5b). Only the functions of t that arise as the "integration constants" for (5a) are undetermined from (5a)-(5b). A necessary and sufficient condition for the existence of an N -resonance invariant for the Hamiltonian (1) is that moments satisfying (5a)-(5b) exist such that a certain Hankel matrix constructed from the moments is singular.

New examples of invariants have been found by using the discrete-moment formulation [2,3].

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