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McGlaun

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COMAG-III: A 2-D MHD CODE FOR HELICAL COIL GENERATORS*

J. M. McGlaun, S. L. Thompson, and J. R. Freeman

Sandia Laboratories, Albuquerque, New Mexico 87185

ABSTRACT: COMAG-III is a two-dimensional MHD code which was constructed to study the characteristics of compressed magnetic field generators which incorporate a helically wound output coil. The code is an improved and generalized version of COMAG-I, which was based on a self-consistent coupling of an Eulerian material response treatment with a magnetic field solver which included non-linear diffusion. COMAG-I also included Joule heating, Lorentz forces, a circuit equation to couple to an external load, a model for explosive burn, a detailed equation of state with a mixed phase capability, and a treatment of fracture and spall. In addition to these capabilities, COMAG-III can treat coils with turn-splitting (a technique used to reduce heating effects), and armatures having an arbitrary shape; it also can now include up to ten different materials and has a more general variable zoning capability. Sample results are described.

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1. INTRODUCTION

COMAG-III is a two-dimensional MHD code used to model compressed magnetic field (CMF) generators with helically wound coils. COMAG-III solves the coupled equations of electromagnetics and mechanics in a region of space containing the generator. A circuit equation couples the electromagnetic fields in the generator to the external load. The basic physical model and the finite difference methods used in COMAG-I were described in [1]. In this paper we discuss only the changes and improvements which have occurred since that time.

The compressed magnetic field (CMF) generator modeled by COMAG-III is constructed from a helically wound coil, a high explosive stick inside a metal sleeve (armature) and a crowbar circuit as shown in figure 1 (the crowbar circuit is not shown). The high explosive is selected to expand the armature rapidly. The armature is usually made of a copper cylinder or an aluminum cylinder inside a copper cylinder. The coil is encased in an epoxy housing and may be made from a single strand of wire or may be constructed in several sections, each having several wires in parallel (split-turn coil).

The sequence of events modeled by COMAG-III begins with a capacitor discharge generating a current in the coil thereby establishing an initial magnetic field between the

coil and armature, see figure 2. The explosive is detonated at one end causing the armature to expand, to sweep down the coil and to compress the initial magnetic field. As the armature begins to expand, the crowbar switch uncouples the generator from the circuit used to establish the initial current. The expanding armature transduces mechanical to electromagnetic energy resulting in an amplification of the initial current.

The hydro section of COMAG-III has been generalized to handle ten materials. The hydrocode CSC-II, which integrates the hydro portion of the problem and forms the basic framework for COMAG-III is fully documented in [2]. This new version of CSC allows, at least in principle, the study of layered armatures and layered high explosive sticks.

COMAG-III has been modified to handle a more general family of helically wound output coils. This more general family allows changes in wire diameter and splitting a coil wire into two parallel wires, each of which would carry half the current of the unsplit wire. These are used to decrease Ohmic heating energy losses in the coil. Splitting a wire into two wires is called turn-splitting and is often repeated several times. The final section may be constructed with 8-16 wires in parallel. The circuit equation and the field solver in COMAG-III have been generalized to treat generator coils with up to six sections of turn-splitting and six changes of wire diameter.

The field solver used in COMAG-III has been generalized to handle variable zoning in both the axial and radial directions, to use a more general set of acceleration parameters in the alternating direction implicit (ADI) algorithm and to model armatures which are not initially simple cylinders. These modifications reduce the number of zones required to model the generator, speed the convergence of the ADI algorithm and extend the family of generators which can be modeled.

The diffusion model in the coil wire has been changed from that used in COMAG-I. The new technique, described below, results in better resolution of the current densities on the boundaries of the wire, treats the diffusion process inside the wires more accurately and is more general than the algorithm used in COMAG-I.

Sample results for a generator with turn-splitting are also included. A companion paper [4] presents a comparison between a COMAG-III calculation and an experiment. This reference also discusses the relative importance of the competing loss mechanisms and presents results for the dependence of the current gain on injection current level.

A detailed 2-D MHD model like COMAG-III can include many non-linear effects and can provide quantitative results on loss mechanisms and sources. Codes of this sort tend to be too cumbersome and expensive to be used as a design tool for

parameter studies. It can provide guidance in the formulation of simpler lumped parameter [5] or analytic models [6] which can then provide important scaling results.

2. GOVERNING EQUATIONS AND MODELS IN COMAG-III

COMAG-III is an extensively modified version of the recently developed ten-material hydrocode CSO-II [2]. The CSO-II coding integrates the hydro portion of the problem, performs all equation of state calculations and performs all input and output. CSO-II employs the two-dimensional finite difference analogs of the Lagrangian equations of motion (a material fixed coordinate system) with a continuous rezoning to an Eulerian (space fixed) coordinate system. Ohmic heating and Lorentz force terms are added to the energy and momentum balance equations for COMAG-III calculations. Elastic-plastic and fracture behavior are treated. An energy flow calculation includes radiation and various forms of thermal conduction. Realistic material descriptions with phase transitions (solid-liquid, liquid-vapor, solid-solid, etc.) are available [7]. Up to ten materials may be included in a problem. Various high explosives and other energy sources are included.

The magnetic fields and current densities are defined in terms of the magnetic vector potential, A , thus assuring consistency between the magnetic fields and current densities. We assume the only nonzero component of the magnetic vector potential is in the azimuthal direction. Therefore the current density is also in the azimuthal direction and the magnetic fields are in the radial and axial directions. The hydro portion of the code supplies the

conductor positions, velocities and temperatures required for the solution of the electromagnetic field equations and the calculation of the temperature dependent electrical conductivity.

In COMAG-I we found that the temperature in the expanding armature increased too fast because of numerical diffusion in the mixed cells on the boundary between the high explosive and the armature. This resulted in artificially low electrical conductivity values. This problem was remedied by forcing the high explosive to burn at an artificially low temperature. This change affected only the temperature and left the other thermodynamic quantities, except the specific heat, unchanged.

The governing field equations used in the armature and vacuum, and the technique for determining the boundary values of the magnetic vector potential are the same as in COMAG-I [1]. The technique for calculating the current density in the coil wires has been changed. In COMAG-I the current density in the coil wires was allowed to diffuse. In COMAG-III the flux in the coil wires diffuses and the current density is calculated from the new flux values. We assume the flux in the coil wire diffuses only in the radial direction, a limitation forced by the small wire diameter. Using this assumption the diffusion equation for the flux in the stationary coil is, in practical cgs units,

$$\frac{\partial \psi}{\partial t} = \kappa_0 \left(\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \right)$$

$$\kappa_0 = \frac{10^9}{4\pi\sigma c} \quad (1)$$

Where ψ is the azimuthal component of the magnetic stream function (covariant component of the magnetic vector potential), $\psi = rA_\theta$ and σ_c is the electrical conductivity. Using Maxwell's equations, neglecting displacement currents and neglecting axial derivatives we find the current density, J , in practical cgs units, is given by

$$-0.4\pi r J = \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \quad (2)$$

This algorithm takes into account any radial variation in the electrical conductivity, whereas the algorithm used in COMAG-I did not.

The technique for making the current in the wire agree with the current calculated by the circuit equation has been changed. In COMAG-I the current density throughout the coil was scaled to give a current consistent with the current computed by the circuit equation. We found this algorithm exaggerated the rate of diffusion of the current density and

flux into the conductor. In COMAG-III only the current density on the boundary of the coil wire is redefined to give a current consistent with that calculated by the circuit equation. The new algorithm gives better resolution of the current density on the boundary of the coil wire and the flux and current density diffuse in the interior of the wire at the proper rate.

Several changes have been made to the terms appearing in the circuit equation used to couple the electromagnetic fields in the coil with the external load.

$$\frac{d}{dt} (\bar{\Phi} + I L_D) + \dot{\bar{\Psi}}_L + I(R_L + R_C) = 0 \quad (3)$$

Φ is the flux linkage of the generator circuit,

I is the total current,

R_C is the generator resistance,

L_L is the external load inductance,

R_L is the external load resistance,

$\dot{\bar{\Psi}}_L$ is a switching flux loss term.

The switching flux loss term compensates for flux linkage losses arising from time dependent changes in the generator circuit. The particular form of $\dot{\bar{\Psi}}_L$ used in COMAG-III is discussed in [4]. The equations for the generator inductance and resistance have been changed from those used in COMAG-I to handle split-turn coils. To

understand these changes, it is first helpful to consider the geometric model of a split-turn coil used in COMAG-III.

For a split-turn coil section with N wires in parallel we approximate each helical turn with N arcs of wire. The radial and axial coordinate of each arc is fixed and the azimuthal coordinate varies by $2\pi/N$, as shown in figure 3. These arcs are uniformly spaced and rotated along the curve originally occupied by the helical turn. After including all N parallel wires in the section we have a series of closed circular loops just as in the approximation to the single-wire coil, see figure 3. COMAG-III models split-turn coils with up to six sections. It assumes that there are 2^{N-1} parallel wires in the N -th section and each wire in that section carries the current $1/2^{N-1}$.

The finite difference mesh forces the wire cross-section in the code to be rectangular rather than circular. The difference in the wire areas is approximately compensated for by multiplying the calculated current density in the coil by the rectangular wire cross sectional area divided by the actual wire cross-sectional area.

The following formulas are used to compute the circuit parameters appearing in (3). Assume that there are N sections in the generator circuit, $n(j)$ parallel coil wires in the j -th section, $m(j)$ closed loops in the j -th section. Then, in practical cgs units,

$$\Phi = 2\pi \cdot 10^{-8} \sum_{j=1}^N \frac{1}{m(j)} \left(\sum_{i=1}^{m(j)} \psi(i,j) \right) . \quad (4)$$

where $\psi(i,j)$ is the stream function in the i -th loop of the j -th section of the generator. A coil inductance can be defined from $IL = \Phi$.

Assume the notation is as above and that there are $h(j)$ grid cells across the thickness of the wire in the j -th section of the coil. Then the coil resistance is defined by

$$I^2 R_c = 2\pi \sum_{j=1}^N \frac{S(j)}{n(j)} \left(\sum_{i=1}^{m(j)} \sum_{k=1}^{n(j)} J^2(i,j,k) r(k) A(i,j,k) / \sigma_c(i,j,k) \right) \quad (5)$$

$r(k)$ is the radius of the k -th coil wire cell.

$S(j)$ is the cross-sectional area of the rectangular wire used in the j -th section of COMAG-III's coil divided by the cross-sectional area of the wire in the j -th section of the CMF generator.

$J(i,j,k)$ is the current density in the k -th coil wire cell at the j -th coil loop of the i -th section.

$\sigma_c(i,j,k)$ is the electrical conductivity in the k -th coil wire cell at the j -th coil loop of the i -th section.

$A(i,j,k)$ is the cell area of the k -th coil wire cell at the j -th coil loop or the i -th section.

It is straightforward to show (4) reduces to the equation used in COMAG-I when there are no split-turn coil sections. Equation (5) also reduces to the equation used in COMAG-I, except for the $S(i)$ term, when there are no split-turn sections in the coil.

COMAG-I was limited to generators with armatures that were initially simple cylinders. More general armatures in which the radius varies as a function of the axial coordinate were also of interest. These armatures can be used to control the rate of rise of the current. They could not be adequately modeled by COMAG-I because the field solver needs to know the boundary of the armature when it is making a radial or axial pass in the ABL algorithm and the axial coordinate of the upper or lower boundary of the armature will be a multivalued function of the radial coordinate. COMAG-I would allow only one axial coordinate for each radial coordinate but COMAG-III has been modified to allow up to six axial coordinates for each radial coordinate.

3. NUMERICAL METHODS

Finite difference methods must be used to solve the discrete analogs to the differential equations in COMAG-III. The numerical methods used in the hydro portion of the code are described in [2]. The algorithms for solving the discrete approximation to the circuit equation and the flux diffusion equation in the armature are unchanged from COMAG-I.

The flux diffusion equation in the coil (1) is solved with an explicit finite difference approximation. The spatial differences are evaluated at the old time level and the time derivative is approximated with a forward difference. The time step is controlled by the hydro portion of the code and is small enough to assure stability.

The finite difference equations governing the value of the stream function in the vacuum and coil have been generalized from those in COMAG-I. The difference equations in COMAG-I allowed variable zoning in the radial direction but required uniform zoning in the axial direction. The difference equation in COMAG-III (6) allows variable zoning in both the radial and axial directions. This modification substantially reduces the number of zones required to model a problem.

$$\begin{aligned}
 - D(j) A(j+1, k) - C(k) A(j, k+1) + (B(j) + C(k) + D(j) + E(k)) A(j, k) \\
 - B(j) A(j-1, k) - E(k) A(j, k-1) = F(j, k)
 \end{aligned} \tag{6}$$

$$B(j) = (1 + \Delta_r^+ / 2r_j) / \Delta_r^- \Delta_r^{AV}$$

$$C(k) = 1 / \Delta_z^- \Delta_z^{AV}$$

$$D(j) = (1 - \Delta_r^- / 2r_j) / \Delta_r^+ \Delta_r^{AV}$$

$$E(k) = 1 / \Delta_z^+ \Delta_z^{AV}$$

$$\Delta_r^+ = r_{j+1} - r_j$$

$$\Delta_r^- = r_j - r_{j-1}$$

$$\Delta_r^{AV} = (\Delta_r^+ + \Delta_r^-) / 2$$

$$\Delta_z^+ = z_{k+1} - z_k$$

$$\Delta_z^- = z_k - z_{k-1}$$

$$\Delta_z^{AV} = (\Delta_z^+ + \Delta_z^-) / 2$$

$$F(j, k) = 0.4 \pi r_j J(j, k)$$

These difference equations are solved using the

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alternating direction implicit (ADI) algorithm [1,7]. The ADI algorithm is iterative and uses a sequence of acceleration parameters to speed convergence of the iterations. Several acceleration parameter sequences have been used. One sequence used is a modification of one described in [9] and is a generalization of the sequence used in COMAG-1 [1].

$$\xi^{n+1} = 4 \cos^2 \left(\frac{\pi}{2m} \right) / \left(\xi^2 \left(\cos \left(\frac{\pi}{2m} \right) \right)^{\frac{n}{N}} \right), \quad n = 0, \dots, N \quad (7)$$

m is the maximum number of meshes in either the radial or axial direction,

δ is the minimum average mesh length in either the radial or axial direction,

$$N = \begin{cases} 6 & \text{if } m < 129 \\ 7 & \text{if } 129 \leq m < 312 \end{cases}$$

Another sequence used is the sequence for 2^N , $N=0,1,2,\dots$ acceleration parameters described in [9].

The first set of acceleration parameters works well in the problems we have considered. The algorithm typically converges in 50 to 100 iterations on an approximately 100×100 mesh. The second set of parameters seem to work either very

well, or very poorly. In some of the problems considered, the algorithm did not converge, in others, it converged in less than 10 iterations. In some problems the algorithm did not converge when we used eight acceleration parameters but converged when we used sixteen. The theory behind this phenomenon is presented in [9].

1. SAMPLE CALCULATION

COMAG-I and COMAG-III calculations have been compared to a number of experiments. In this section we compare the results of a COMAG-III calculation to an experiment with a split-turn coil generator. A comparison between COMAG-III and a single-wire coil generator is presented in [4].

A GMP generator with a split-turn coil was used to generate a current history for comparison with a COMAG-III calculation. The circuit parameters of the external load were 0.024Ω and $136 \mu\text{H}$. The armature was made of an aluminum cylinder inside a copper cylinder and was initially tapered. The high explosive was PBX-9404 with an outer diameter of 15.5 mm and the armature and high explosive stick were 103. mm long.

The coil had four sections. The first was constructed from twenty-nine turns of a single strand of wire. The second section had two parallel wires and was modeled as fourteen loops. The third section had four parallel wires and was modeled as fourteen loops. The fourth section had eight wires in parallel and was modeled as thirty-three loops. The wire diameter changed from 0.574 mm to 0.7239 mm at the fourteenth loop of the fourth section. The inner radius of the coil was 30.48 mm.

For an initial current of 1480. A, the peak experimental current was 121,600 A. The peak calculated current was

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11.4 μs giving a 34% overprediction, see figure 4. Although there was a substantial difference between the peak currents, the experimental and calculated current histories coincide up to peak experimental current.

Comparing the COMAG-III calculations and the experimental current history we find the experimental current peaks with 60% of the fourth section still in the circuit. The experimental current peak occurred at approximately 14.7 μs after initiation of dot action. At this time COMAG-III indicates there are twenty-one turns still left in the circuit. The calculated current at this time is 133,300 A, which is 9.6 % higher than the experimental current. It should be noted that the external load used in this experiment had a larger impedance than typical loads for this generator and this may account for the early peak current.

It is interesting to consider the variation of the coil inductance as a function of the position of the armature. The initial inductance of the generator was 85. μH . Just before the first coil loop was switched out at 10.4 μs , the coil inductance had decreased to 25.7 μH . The decrease was due to the change in the radial position of the armature. At 11.65 μs the first section of the coil was switched out and the coil inductance was down to 3.7 μH . Most of the inductance of this generator was switched out of the circuit early in the operation and this resulted in a lower rate of rise of the current than for a single wire coil generator,

see [4].

This experiment demonstrates that COWAG-III can be used to model GDF generators with split-turn coils. The agreement up to peak experimental current is excellent. The overprediction of peak current and early peak current in the experiment suggests there is an important loss mechanism not correctly modeled or not included. Possible loss mechanisms are turn-to-turn proximity effects, turn-to-turn breakdown or turn-to-armature breakdown along the inner surface of the coil.

These calculations were done using only the simplest model for the switching flux loss term. Only the flux trapped in the conductors was switched out of the circuit. A discussion of the effects and possible mechanisms for a larger flux loss are discussed in the following paper [4].

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FIGURE 1

FIGURE 2

HELICAL TURNS

APPROXIMATION OF
HELICAL TURNS

FIGURE 3

FIGURE 4