24-81
24 Aug 81 NTIS

I-558 (1)

UCID--18947 Pt. 3 Rev. 1

DE82 006182

MASTER

PLASMAS IN PARTICLE ACCELERATORS:
A HYDRODYNAMIC MODEL OF
THREE-DIMENSIONAL
ELECTROSTATIC INSTABILITIES

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December 1, 1981

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under Contract W-7405-Eng-48.

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ABSTRACT

A hydrodynamic model is used to help isolate possible three dimensional space charge instabilities in beam plasmas of concern in designing heavy ion accelerators for inertial confinement fusion energy applications. The model provides an economic means for searching the large parameter space relevant to problems in which coupling of longitudinal and transverse motions is allowed. It is shown that the equilibrium axial hydrodynamic pressure of the beam plasma has a significant effect on the stability boundaries of a two-rotating-stream instability. When considering the resistive wall effect, this model shows a kink instability. The growth rate of some modes could be enhanced by increasing the equilibrium axial pressure.

I. INTRODUCTION

Induction linear particle accelerators are being designed for application as drivers for heavy ion inertial fusion energy power plants [1-4]. As part of the process, it is necessary to identify and avoid regimes of possible three-dimensional instabilities which involve coupling of longitudinal and transverse motions under anharmonic space charge forces which are now no longer negligible. This task has been considered prohibitive in the past partly because of the three spatial dimensions and partly because of the number of potential parameters involved in its solution. (e.g. even in the simplified discussion of Section III, there are nine parameters). A further hindrance is the fact that the realistic final discussion must involve the kinetic Vlasov description (cf. Ref. 5) in six-dimensional phase space and time with non-axisymmetric forces. Three dimensional particle codes with self-consistent space charge forces could in principle be used but are not realistic as a first attempt because of the computational expense needed to survey the extensive parameter space.

The above three dimensional problem is made more tractable to analytical efforts through approximations introduced by Mark and Smith [6; hereinafter referred to as Paper I]. Their scheme involved an ion beam subject to focusing forces which to the first approximation are axisymmetric. Thus we assume that the forces on an initially axisymmetric beam (Fig. 1) can be separated into a dominant averaged part $F_{av}(r, \theta, z, t)$ and a fluctuating part $F_{fx}(r, \theta, z, t)$ which is smaller in magnitude. Here (r, θ, z) are cylindrical polar coordinates and t is the time variable. The small parameter ϵ allows the averaged force F_{av} to vary in time, but

slower than the transverse betatron oscillation time-scale; it is also consistent with a finite beam bunch whose length is much greater than its width.

Some considerations for these (ϵ_z , ϵ_t) variations in F_{av} are given elsewhere [7]. Even omitting these variations, a "warm-beam" approximation is helpful. Only those particle orbits are retained whose amplitude of radial excursion (transverse to beam axis) is not greater than about one third of its average radius [6]. Within this model the Vlasov kinetic theory for wave disturbances is usually analytically tractable. This is the case provided one has some insight into the relative ordering of critical parameters (such as the equivalent radial mode number measuring how rapidly the mode varies with radius compared to the radial amplitudes of these orbits).

The hydrodynamic model of this paper is introduced in Section II partly in response to the need to identify relevant regimes of radial mode numbers, and partly to initiate the exploration of the nontrivial parameter space in the most efficient manner. If instabilities are found, they are further studied with the kinetic approach. Accordingly, since the electrostatic problem with conducting walls already involve nine parameters, we only make a preliminary examination of resistive wall instabilities while leaving until later discussions such topics as electromagnetic effects, etc.

Moreover this hydrodynamic model is a simplified version of the fluid equations derived directly for warm beams by the analyses of Mark [7] within relevant regimes. Not surprisingly then, the local dispersion relationship of our fluid model (Section II), in appropriate limits, has a close correspondence with the similar relation derived from the Vlasov kinetic equation. The global dispersion relation for electrostatic disturbances (Section III) and numerical results (Section IV-V) of our fluid model are some examples of what

could be done with this fluid model (cf. also Refs. [8] and [9]). Such discussions are now the "stepping stones" to a more extensive kinetic study of possible relevant instabilities (e.g. cf. Ref. [10]).

One useful result may already be stated for the more general context of beam propagation and transport (including transport in gases). Usually in this subject, more involved problems are resolved using particle code simulations (cf. for example Refs. [1]-[4] and [11] for some Heavy-Ion Fusion applications). The possibility of using hydrodynamic simulations for certain problems is highly attractive because of the removal of velocity space from the simulation grids. Mark's derivation [7] of the fluid equations can be extended to such problems (including magnetic fields, relativistic effects, etc.). The recommendation from this paper for simulation of nonrelativistic beams is to use hydrodynamics in the transverse direction but use the drift-kinetic equation in the longitudinal direction (cf. Section III of Ref. [7]) whenever it is desirable to retain longitudinal Landau damping. For the example of relativistic beams, the particle motions are all at the velocity of light in the first approximation and longitudinal Landau damping is less relevant.

II. THE WARM BEAM FLUID MODEL

It is well known that the continuity and momentum moments of Vlasov's equation give exact hydrodynamic equations [cf. Ref. 5] even though their use poses the question of closure equations for the stresses. The model used here is based on retaining three different diagonal stresses p_r , p_θ , p_z , in the (r, θ, z) directions and no off-diagonal stresses. In particular for closure we assume a "barotropic fluid" type of adiabatic relation where each

stress component is a function of density. Thus the generalized "enthalpies" per unit mass are

$$h_j = \frac{dp_j}{\rho} = a_j^2 \frac{dn}{n}, \quad (j = r, \theta, z) \quad (11-1)$$

where ρ and n are the mass and number densities and a_j are the "equivalent acoustical speeds" corresponding to each stress component p_j .

We may note that in the warm beam kinetic equilibrium of paper I, applicable to anharmonic space charge forces, we assumed that particle orbits have radial excursions smaller than average orbit radius. Thus in order to retain orbits which have both signs of angular momentum, it is necessary to adopt two separate counter rotating streams. Zero total angular momentum was assumed. This approximation introduces a counter-streaming instability in the azimuthal direction, an unrealistic feature which could be turned off by decreasing the rotation frequency of each stream.

Each one of the above streams has in addition "temperatures" measured by the velocity dispersions c_i ($i = r, \theta, z$). In our above fluid model, it is natural to identify the acoustic speeds a_i with the dispersions c_i . In the fluid equilibrium, the radial acoustic speed a_{r0} is constrained by radial force balance.

Let us introduce the hydrodynamic velocity (v_r, v_θ, v_z) which characterizes each of the two counter-rotating streams with opposite signs of rotation frequency ω . The coasting beam equilibrium adopted here has infinite length in the axial or z direction. We need only consider radial force balance involving the equilibrium quantity $[\frac{n_0}{2}]$ (n_0 is the total equilibrium beam density of the two streams). Since

$$h_{r0} = \int (a_{r0})^2 \frac{dn_0}{n_0} \quad (II-2)$$

and

$$v_{\theta 0} = r\omega(r) \quad , \quad (II-3)$$

we obtain

$$\omega^2 r - \frac{dh_{r0}}{dr} = v_0^2 r - \frac{1}{r} \int_0^r \omega_p^2(r) r dr \quad , \quad (II-4)$$

where $\omega(r)$ is the rotation frequency of the streams each with its own sign (ω). Also $v_0^2 r$ is due to the averaged external focusing force and the ω_p^2 term represents the space charge of the combined beam system. The ion plasma frequency ω_p is

$$\omega_p^2 = \frac{4\pi n_0(r) q^2}{m} \quad , \quad (II-5)$$

where q and m are the ion charge and mass. If the equilibrium beam density n_0 is constant (simple harmonic space charge forces) then equation (II-4) is reduced to

$$\omega^2 - \frac{1}{r} \frac{dh_{r0}}{dr} = v_0^2 - \frac{1}{2} \omega_n^2 \quad . \quad (II-6)$$

Thus when $\omega_p^2 + 2v_0^2$, $[\omega^2 + \frac{1}{r} |dh_{r0}/dr|] + 0$, and the rotation necessarily stops.

This fluid model does not specify the acoustic speeds a_θ or a_z . From the discussion of heavy-ion accelerators, we do not expect a_z and a_r to be related. But our kinetic equilibrium results [6] inform us that the corresponding kinetic equilibrium implies

$$\frac{a_{\theta 0}^2}{a_{r 0}^2} = \left(\frac{v}{2\Omega}\right)^2, \quad (II-7)$$

where

$$v^2 = \frac{1}{r^3} \frac{d}{dr} (\Omega^2 r^4), \quad (II-8)$$

is the generalized betatron frequency. This frequency characterizes the smaller radial (and accompanying θ) oscillations of a typical particle orbit of the kinetic equilibrium. This oscillation we called [b] betatron vortex oscillation, the latter word being consistent with the terminology of Davidson [11]. Note that in the presence of anharmonic space charge forces, we must distinguish between v and (2Ω) . With simple harmonic forces, the betatron frequency $v = 2\Omega$ and $a_r = a_\theta$.

We now consider wave perturbations with $n_1, v_{r1}, v_{\theta 1}, v_{z1}$ for each stream defined according to

$$n(r, \theta, z, t) = \frac{1}{2} n_0(r) + \text{Re}[n_1(r) e^{i(\lambda\theta + kz - \omega t)}], \quad (II-9)$$

$$v_r(r, \theta, z, t) = \text{Re}[v_{r1}(r) e^{i(\lambda\theta + kz - \omega t)}], \quad (II-10)$$

$$v_\theta(r, \theta, z, t) = v_{\theta 0}(r) + \text{Re}[v_{\theta 1}(r) e^{i(\lambda\theta + kz - \omega t)}], \quad (II-11)$$

$$v_z(r, \theta, z, t) = v_{z0} + \text{Re}[v_{z1}(r) e^{i(\lambda\theta + kz - \omega t)}], \quad (II-12)$$

where λ and k are the azimuthal and axial wave number. The continuity equation for these perturbations implies

$$2(i\omega) \frac{n_1}{n_0} = \frac{1}{r n_0} \frac{d}{dr} [r n_0(r) v_{r1}] + i \left(\frac{\lambda}{r} v_{\theta 1} + k v_{z1} \right), \quad (II-13)$$

where

$$\bar{\omega} = \omega - kx_0 - kv_{z0} \quad (11-14)$$

The three momentum equations give

$$i\bar{\omega}v_{r1} + 2\omega v_{r1} = \frac{q}{m} \frac{d\phi_1}{dr} + 2 \frac{d}{dr} \left(\frac{a_r^2 n_1}{n_0} \right) \quad (11-15)$$

$$i\bar{\omega}v_{\theta 1} - \frac{v^2}{2\omega} v_{r1} = \frac{ikq}{mr} \phi_1 + 2 \frac{ika_{\theta 0}^2}{n_0 r} n_1 \quad (11-16)$$

$$i\bar{\omega}v_{z1} = ik \left(\frac{q}{m} \phi_1 + 2 \frac{a_{z0}^2}{n_0} n_1 \right) \quad (11-17)$$

Eliminating v_{r1} , $v_{\theta 1}$, and v_{z1} from equations (II-13) and (II-15)-(II-17) gives an equation in the perturbed density n_1 and the electric potential ϕ_1 :

$$\begin{aligned} \frac{n_1}{n_0} \left\{ 1 - \frac{(a_{\theta 0} x/r)^2}{\bar{\omega}^2 - v^2} - \left(\frac{a_{z0} k}{\bar{\omega}} \right)^2 \right\} - \frac{x}{\bar{\omega} r n_0} \frac{d}{dr} \left[\frac{2\omega a_{\theta 0}^2}{\bar{\omega}^2 - v^2} n_1 \right] + \frac{xv^2}{2\omega r \bar{\omega} (\bar{\omega}^2 - v^2)} \frac{d}{dr} \left(\frac{a_r^2 n_1}{n_0} \right) \\ = - \frac{1}{\bar{\omega} n_0 r} \frac{d}{dr} \frac{\bar{\omega} r n_0}{\bar{\omega}^2 - v^2} \left[\frac{q}{2m} \frac{d\phi_1}{dr} + \frac{d}{dr} \left(\frac{a_{r0}^2 n_1}{n_0} \right) \right] \\ + \frac{kx_0}{r \bar{\omega} (\bar{\omega}^2 - v^2)} \left[1 - \frac{v^2}{(2\omega)^2} \right] \frac{q}{m} \frac{d\phi_1}{dr} \\ + \frac{q}{m} \phi_1 \left\{ \frac{x}{\bar{\omega} r n_0} \frac{d}{dr} \left[\frac{\Omega n_0}{\bar{\omega}^2 - v^2} \right] + \frac{(x/r)^2}{2(\bar{\omega}^2 - v^2)} + \frac{1}{2} \left(\frac{k}{\bar{\omega}} \right)^2 \right\} \quad (11-18) \end{aligned}$$

which must be solved in conjunction with Poisson's equation for electrostatic waves

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\phi_1}{dr} \right) - \left(k^2 + \frac{\omega_p^2}{r^2} \right) \phi_1 = - 4\pi q \sum_{\text{sgn}(\omega)} n_1. \quad (11-19)$$

The sum over $\text{sgn}(\omega)$ occurs because Poisson's equation involves the total perturbed charge density of the two counter rotating streams.

III.A GLOBAL DISPERSION RELATION

Equations (11-18)-(11-19) could in general be solved numerically for beams in a cylindrical conductor (Fig. 1) to determine the eigenvalue (ω/ν_0) . However, the solution depends on a number of parameters, they are (ν/ν_c) , (ω/ν_0) , (ω_p/ν_0) , $(a_r/\nu_0 R_p)$, $(a_z/\nu_0 R_p)$, $(\nu_{z0}/\nu_0 R_p)$, (R_p/R_w) , and the mode numbers (κR_p) , κ and a radial mode number obtained by solving the eigenvalue problem in the radial direction. Thus it is judicious to first consider the case where the equilibrium has a constant plasma density n_0 inside the plasma radius (cf. Fig. 2). This results in harmonic radial forces where $\nu = 2\omega$. The main advantage of harmonic forces is that equations (11-18) and (11-19) have solutions in terms of Bessel functions $J_\kappa(\Gamma r)$ and all that remains of the eigenvalue problem is a pair of simultaneous global dispersion relations; one of these determines the radial mode number Γ in terms of the frequency ω while the other relation determines ω itself.

The constancy of the particle density n_0 implies $\nu = 2\Omega$ and ω_p are all constants. If we also assume that a_r and a_z are constants for $r < R_p$, it is easy to see that equations (11-18)-(11-19) admit solutions where n_1 and

Φ_1 are both proportional to $J_k(Tr)$ where T is a (possibly complex-value radial mode number. Making such a substitution gives us

$$T^2 \left[1 - \sum_{\text{sgn} \omega} \frac{\omega_p^2 / 2 (\bar{\omega}^2 - v^2)}{1 - \frac{a_r^2 T^2}{\bar{\omega}^2 - v^2} - \frac{a_z^2 k^2}{\bar{\omega}^2}} \right] \\ = -k^2 + \sum_{\text{sgn} \omega} \frac{\omega_p^2 k^2 / 2 \bar{\omega}^2}{1 - \frac{a_r^2 T^2}{\bar{\omega}^2 - v^2} - \frac{a_z^2 k^2}{\bar{\omega}^2}}$$

which can be viewed as an equation giving the radial mode number T in the frequency ω . Especially when $(a_r T/v)$ is not large, iterative numerical solution for $T(\omega)$ is rather straightforward.

A second equation which determines ω is obtained when we match the solution inside the plasma ($r \leq R_p$) to the outside solution. Assuming a conducting wall at $r = R_w > R_p$, the boundary condition for solution outside of beam is

$$\Phi(r = R_w) = - \frac{\omega \mathcal{Z}}{ck^2} \left(\frac{\partial \Phi}{\partial r} \right)_{r=R_w}$$

where $\mathcal{Z}$ is the wall impedance in units of $\mathcal{Z}_0 = 377$ ohm. The wall impedance in general, can be a complex number, but for a perfect conducting wall is zero. Matching the solutions inside and outside of the beam at the beam surface, we obtain

$$\begin{aligned}
& \left\{ 1 - \sum_{\text{sgn} \omega} \frac{\omega_p^2 / 2 (\tilde{\omega}^2 - v^2)}{(1 - k^2 a_{z0}^2 / \tilde{\omega}^2)} \right\} \frac{TR_p J'_x(TR_p)}{J'_x(TR_p)} + \sum_{\text{sgn} \omega} \frac{k a_{z0}^2 / (\tilde{\omega}^2 - v^2)}{\tilde{\omega} (1 - k^2 a_{z0}^2 / \tilde{\omega}^2)} \\
& = k R_p \left\{ \frac{I'_x(kR_p) [K_x(kR_w) + \frac{i\omega}{ck} K'_x(kR_w)] - K'_x(kR_p) [I_x(kR_w) + \frac{i\omega}{ck} I'_x(kR_w)]}{I_x(kR_p) [K_x(kR_w) + \frac{i\omega}{ck} K'_x(kR_w)] - K_x(kR_p) [I_x(kR_w) + \frac{i\omega}{ck} I'_x(kR_w)]} \right\} \quad (III-3)
\end{aligned}$$

In deriving equation (III-3), we have first assumed (cf. Fig. 2) that there is a very thin boundary layer $(R_p - \epsilon) < r < (R_p + \epsilon)$ where the density $n_0(r)$ falls smoothly to zero from its inner constant value for $r < R_p - \epsilon$. In this layer we also assumed that $a_{r0} \rightarrow 0$ more rapidly than n_0 as would be required for a proper warm beam kinetic equilibrium (cf. Ref. [6]).

IV. DISCUSSION OF NUMERICAL RESULTS

A. Beam inside of a Perfect Conducting Pipe

For a beam inside of a perfect conducting pipe, equation (III-1) and (III-3) are solved with $\mathcal{J} = 0$.

In Fig. 3 we have plotted the square of the Doppler shifted wave frequency $\omega - kv_{z0}$ (relative to beam frame) versus plasma frequency squared, ω_p^2 of the ions for a typical set of parameters: the axial wavenumber $kR_p = 1$, wall to plasma radii $R_w/R_p = 2$, azimuthal wavenumber $x = 3$ and several values of the axial dispersion speed parameter $\alpha_z = a_{z0}^2/v_0^2 R_p^2$. These frequencies in the figure are normalized relative to magnitude of v_0 , the external force betatron frequency undepressed by space charge effects. Figure 3 shows that there is a weak instability when $(\omega_p/v_0)^2$ is sufficiently large; the $x = 3$ mode being the most unstable. Figure 4 shows the situation with larger axial wavelengths ($kR_p = 0.2$) which are more comparable to the bunch lengths.

The instability found here has been shown to be due to the two counter-rotating streams in this model; when the particle density in one of the streams is made to be zero, the instability vanishes. That the first unstable mode occurs at $k = 3$ in a two-rotating-stream beam was demonstrated previously [12]. The shifting of the threshold density when α_z increases as shown in Figs. 3 and 4 has not been discussed before.

The two counter-rotating streams are the artifact of the warm beam model considered here and may not appear in a realistic plasma. However, in reality a spread of rotational angular momentum among beam particles is expected. From that point of view, it is worth while to mention a crude comparison between the present model and the popular kinetic theory model of Kapchinskij and Vladimirovskij [13] (abbreviated K-V). A beam with a K-V distribution reveals two-hump character in both the radial and azimuthal momenta, which is known to be more unstable than a plasma with distribution function decreasing monotonically in momenta. In fact, instabilities in the higher radial modes in a beam having a K-V distribution have been found [14]. *The warm beam model discussed here has two counter-rotating streams but no similar two-hump or two-stream behaviors in the radial direction. Hence we expect that some instabilities found in a beam with K-V distribution will not occur in a warm beam. Both of these models may be different from reality, but they do provide conservative beam current limitations perhaps for somewhat different reasons.*

B. Beam in a Pipe with Resistive Surface

For a pipe with resistive surface, the wall impedance is taken to be a purely real number. We have observed that for a given nonzero value of β_1 , this is the case in which the largest growth rate occurs. For

accelerators in Heavy Ion Inertial Fusion, the values of the factor (ω_{ck}) could range from 0 to 0.5.

When the resistive wall impedance is included, the $k = 0$ and $k = 1$ modes of perturbations, which are stable for $\beta = 0$, become unstable. The first two roots of equations (III-1) and (III-3) for these two modes are found to have the largest imaginary parts. For $\beta = 0.2$, Fig. 5 shows the imaginary parts of the two branches of $k = 0$, $k = 1$ and modes. Within the parameter range shown in Fig. 5, all branches have comparable radial wave number T and the upper branch of the $k = 1$ mode, the kink mode, has the highest growth rate, about a factor of 5 higher than the largest growth rate in $k = 0$ mode, the axis-symmetric mode. Note that when the axial acoustic speed a_{z0} increases, the growth rate in the upper branch of the kink mode and axis-symmetric mode decrease slightly but the growth rate in the low branch of the kink mode increases almost linearly with the pressure. It might be interesting to recall that the growth rate of the fire hose instability in a nonneutral plasma column has also a linear dependence on the longitudinal pressure.

Not shown in the Fig. 5 are the first two damped branches for $k = 0$ and $k = 1$ modes among the solutions of equation (III-1) and (III-3). These two branches have negative imaginary parts and are less affected by the variation of a_{z0}^2 . Also, a limit that should be mentioned here is that when ω_z diminishes, the upper branch approaches the result in reference [12] with a resistive boundary.

In concluding the discussion, we notice that within the context of the warm beam model, a hydrodynamic approximation is likely to be more conservative than the Vlasov-Kinetic description in certain regimes because

the Landau damping is omitted. However, discussions of a hydrodynamic model gives insight as to the regimes where possible instabilities might lie amidst the nontrivial extent of parameter space in this three dimensional problem. Besides, the discussions of hydrodynamic approximation also help in exploring the limitations of hydrodynamic simulation for beams propagating in gases (Cf Section I, last paragraph). The more involved kinetic discussion of the instabilities found in this paper is being pursued, especially on the competitive effects between Landau damping due to the longitudinal energy spread and the resistive kink destabilizing tendency caused by the axial pressure. The complexity of the kinetic results is due to these cancelling effects, making it worthwhile to first exhibit the destabilizing tendency by the fluid model without Landau damping.

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Figure 1. The beam considered here has radius R_p situated within a cylindrical cavity of radius R_w ; its average axial speed is v_{z0} and particle density is n_0 . Although not essential, the beam length is taken to be infinite.

Figure 2. The particle density profile adopted for numerical results in this paper; the dashed curve is for convenience of matching boundary conditions across the plasma radius R_p .

Figure 3. Doppler shifted frequency squared ($\tilde{\omega} = \omega - k_z v_{z0}$) versus beam particle density. Quantity normalized by zero density betatron tune ν_o .

Figure 4. Doppler shifted frequency squared versus beam particle density.

Figure 5. Resistive wall instability growth rates obtained from fluid model, where $\beta = .02$.