EFFECTS OF COMPRESSIVE FORCES IN PIPE TYPE CABLE

by

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CONTENTS

	Page
LIST OF FIGURES.	iv
LIST OF TABLES.	v
ACKNOWLEDGEMENT.	vi
ABSTRACT	vii
1. INTRODUCTION	1
2. THE CABLE.	4
3. TESTING PROCEDURE.	11
4. DISCUSSION	17
5. ANALYSIS	21
A. Equivalent Lateral Pressure.	21
B. Cable Insulation Model	25
6. EXPERIMENTAL RESULTS	35
A. Discussion of Results.	48
7. CONCLUSIONS.	52
8. RAW TEST DATA	55
9. REFERENCES	75
10. APPENDIX	77

LIST OF FIGURES

	Page
Figure 1 Detail of 345 kV Cable.	5
Figure 2 Pipe Type Cables.	8
Figure 3 Phelps Dodge Display of 230 kV Cable.	9
Figure 4 345 kV Cable Assembled in 10" Pipe.	10
Figure 5 Compression Test.	15
Figure 6 Computer Graphics System	15
Figure 7 Compression Specimen With Strain Gauges	16
Figure 8 Compression Specimen With Strain Gauges	16
Figure 9a Resultant Forces Acting on Helical Wire	23
Figure 9b Geometry of 6-Wire Cable.	23
Figure 10 Thick-Walled Tube Model	31
Figure 11 Compression Test for E_r	39
Figure 12 Tension Test for E_θ	41
Figure 13 Tension Test for E Metal.	43
Figure 14 Displacement Function	50
Figures E.1 - E.19 Compression Test Results	56
Figures A1 - A3 Early Flexure Test Results	78
Figure A4 Beam Diagram - Single Concentrated Load. . .	81
Figure A5 Beam Diagram - Two Concentrated Loads. . .	81
Figure A6 The Flexure Test System.	83
Figures A7 - A12 Recent Flexure Test Results.	89

LIST OF TABLES

	Page
Table 1 Design Characteristics of 500 kV Experimental Cable.	36

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ABSTRACT

Within the TMB phenomenon of underground power transmission systems, compressive cable forces give rise to increasing tape tensions in the cable insulation. Because of their substantial magnitude, these changes in tape tensions play an important role in the formation of soft spots which can eventually cause electrical failure.

This project report presents a theory of the mechanical behavior of power cables under compressive load, together with experimental verification of this theory. It may be noted that this theory is basic to the computation of gross moduli that are to be used in the in-situ analysis of cable deformation, and should also provide important additional design criteria for future cable manufacturing.

PART 1
INTRODUCTION

It is now generally accepted that the distress occurring in high voltage pipe-type cable systems has been associated with thermo-mechanical bending. This phenomenon (termed TMB failure) is a process through which electrical failure results from a mechanical action. Temperature change is accommodated through bending which in turn causes the formation of "soft spots" in the layers of cable insulation, through migration of the paper layers away from certain critical points.

The thermo-mechanical bending project at RPI was designed to investigate the potential problem of thermo-mechanical bending damage occurring within typical cable segments. The action of TMB is a direct result of thermal expansion of the cable. This thermal expansion occurs during (electric) load cycling, and must be accommodated within the line pipe if the joints are reinforced or restrained. Thermal expansion causes the cable to either develop compressive forces or to relieve the induced stresses through "snaking". Previous investigations and analysis of operating systems by Westinghouse Electric Corporation at Waltz Mill (1) and others all indicate that snaking does occur under usual operating conditions. The question under deliberation is what damage, if any, is caused by snaking or thermo-mechanical bending. The variables that must be considered are numerous. The list may include temperature differential, insulation thickness, conductor size, fill ratio, cable hardness, tape structure, oil viscosity, etc. The consideration of all these aspects requires a major research effort and the combination of all these effects leads to the

anticipation of countless complications.

It is the purpose of this paper to examine the response of the cable to axial load, which results from the thermal length changes in service. Once the data has been satisfactorily compiled and the theory has been developed, it is the anticipation of this author that the effects of compression, bending fatigue, and other mechanical testing procedures (e.g., torsion, thermal expansion) may be combined in physical testing as well as mathematical modeling.

Since the condition of axial loading is the first reaction of the cables to the thermal expansion, it is a most appropriate place to begin our analysis. My concern, therefore, is to construct a mathematical model of cable segment response to axial load. The development of these theoretical considerations will then be fitted to our own experimental verification, which has been quite extensive.

There have been many questions brought up concerning the characteristics of pipe type cable with respect to axial thrust. To begin with, many questioned whether the actual cabling process was necessary to preclude excessive thrust in service. This work examines these effects using general and specific elastic theory. Most other research efforts on this subject have tried to create a testing environment as close to the in-situ conditions as possible. This paper is based on a somewhat different approach.

Our tests are performed as simple and basic tests on relatively short cable segments. The hope is that it should be possible to model any condition encountered in general use if the behavior of cable segments can be determined. After a description of the testing procedures and cables used, an analysis is presented in which the cable is modeled as an orthotropic,

elastic, thick-walled tube. It is hoped that this type of analysis could lead to the prediction of in-situ cable behavior, given the dimensions and properties of the cable.

PART 2

THE CABLE

It is worthwhile to present a brief description of the cables we are studying, and perhaps, a more detailed description of the specific cable about which extensive compression data has been gathered.

Pipe-type cable systems are the most widely used in the United States (2) for high voltage underground transmission systems. The cables we have studied consist of copper conductors which are insulated by a thick layer of paper tapes wound helically around the conductor. This paper insulation is then bound by layers of metal tape which form the outer surface of the cable. The magnitude of the electrical load to be transmitted determines the conductor size, while the system voltage is what determines the insulation thickness.

The conductor of this type of cable is stranded for flexibility and, for the cases we have examined, consists of a four-cabled segmental construction (see Fig. 1). This technique is used to reduce the cable's a-c resistance (3). Each segment is made up of individual wires which have been drawn through dies to specified diameters. The segments are formed as a combination of two or three different diameter wires that are sized to form a compact and smooth-surfaced unit. Each segment is composed of many individual strands assembled in a series of concentric layers. These layers are then crushed into a "pie" shaped sector, and four of these sectors are put together to form the core of the cable. The construction of a four-segment conductor typically employs a higher degree of compaction for two or three of the segments, with these being insulated with two

Single Conductor Detail - Pipe Type Cable

Static Rating 3 PHASE 345 KV
Approximately 550 MVA. (920 AMPERES)

Figure 1

Detail of 345 kV Cable

or three layers of paper tape. Four segments are then mounted in a large cabling machine with the two insulated segments on diagonally opposite sides. They are then spiraled into a cylindrical section and bound with a layer of bronze or steel tape to form the completed current carrying conductor.

The taping operation is a very complex and highly sophisticated process. Extensive mathematical analysis is employed to determine the optimal dimensions and application specifications for each type of tape used. This achieves the required degree of cable compactness and mechanical flexibility. The taping is done on a taping machine capable of applying a wide range of tape tension and can adjust to any tape angle.

The remaining cable construction procedures consist of drying and impregnating, shielding, and final tests and sealing. The drying is accomplished by placing the cable in a vacuum tank. Direct current is passed through the conductor in order to heat the insulating paper uniformly, and the vacuum system removes the water vapor. After final checks for dryness have been made, oil is brought into the evacuated tank and absorbed by the cable as the tank is pressurized and cooled.

The shielding process involves the application of moisture barrier tapes (metalized synthetic), metal shielding tape and the skid wires. These are half round wires made of bronze, stainless steel, or polyethylene, which are wrapped around the cable to protect against damage from the pulling operation during placement.

All completed lengths of cable are put through a series of examinations for purposes of quality control. These include high voltage tests, capacitance, and power factor change with applied voltage. Segments are

examined for uniform taping and proper dimensions. Cables are usually fitted with pulling bolts at the factory to facilitate the field installation process. Once the cables are on the shipping reels, the reels are hermetically sealed (3) and many other precautions are taken with respect to moisture levels.

Three of the completed cables are pulled into a steel or PCV pipe which is then filled with insulating oil which is kept under pressure by a pumping station. For short distances the oil may be replaced by nitrogen gas which eliminates the need for a pumping station. Examples of the cable system cross-section may be seen in Figures 2 to 4.

The cable which has been used to provide the most compression data for this report is a 550 kV experimental cable obtained from the Pirelli Cable Corporation. This cable is manufactured with the same basic processes that have just been described. The upcoming section on Experimental Results contains a table depicting the general design characteristics of this cable. The similarity in nature between most pipe-type cables using paper lapped insulation should allow the analytical results of this report to be applied to most cables in existence today. However, for the sake of simplicity, the experimental verification will be limited to the above described cable.

Figure 2
PIPE TYPE CABLES

Figure 3 Phelps Dodge Display of 230 kV Cable

Figure 4

Assembled in 10" pipe

PART 3

TESTING PROCEDURE

The design, construction, and implementation of testing procedures was one of the most time consuming and often frustrating aspects of this research effort. My responsibility spread throughout all the mechanical testing equipment and methods with respect to all tests in progress. Unquestionably, the rate of progress achieved was directly controlled by the effectiveness and consistency of the testing procedures used.

For many of our tests, equipment and procedures went through many stages of re-design and re-implementation due to numerous unexpected problems or conflicts. These may have been mechanical, electrical, or simply a system that produces inconsistent or unacceptable data. These situations often resulted because of the somewhat unpredictable behavior of the cables under observation, while occasionally resulting from similar behavior experienced with our own systems. Fortunately, the methods and procedures used relating to our compression tests were reasonably successful.

The testing procedures used for compression tests are illustrated in Figures 5 - 8. A universal testing machine was used for this testing which had compressive load capabilities far beyond those necessary for our tests. The methods employed for these tests have progressed quite successfully over the time that has passed since the first compression test was attempted. The figures display the present version of our compression tests. When developing this procedure, the primary consideration was to insure that end effects would not destroy the accuracy or validity of the test data.

The performance of early tests were hampered by these end effects which were, at that time, unpredictable in nature. For these tests the cable stood free and the load was applied with flat surfaces at each end. The compression specimens were cut on a band saw which was not capable of cutting the cables in a precise manner. This caused local deformation at the ends until the cable seated properly and the load was being carried by the conductor. To alleviate this problem we designed a system for "capping" the ends of the cable. Although some seating would still take place at the outset of the tests, this system eliminated the local deformations and helped simulate the long length, fixed end conditions which the cable is normally subjected to.

A metal cutting saw which was much more effective than the band saw was used to cut the ends of the cable. The conductor was left extended at the ends to meet flush with the end of the grips. In this way, the load was initially distributed equally between the insulation and conductor. It should be noted that the use of this metal cutting saw greatly expedited the manual labor involved with every test, because of its relatively precise cutting angle and its greater speed of operation. This point may seem trivial but when working on a "hands-on" project such as ours, this type of mechanical advantage allows for less time spent on manual labor and more time spent on testing and analysis.

Once the tests seemed to be quite successful, the added dimension of tape strain measurement was implemented. This was accomplished by attaching four strain gauges in series to the outer metal tapes of the cable. The strain was measured on a Sanborn strip chart recorder which was equipped with a strain gauge amplifier. This procedure was difficult to perfect

at first, but soon became a routine process yielding excellent results.

The actual procedure of a typical test was as follows. Once the grips were tightened on the end of the specimen and the wiring was completed for the tape strain measurement, the test was ready for operation. The load was read off the dials of the compression machine and was controlled manually so that the rate and magnitude of the axial force was under our control at all times. The strain or displacement of the cable was measured by a standard dial gauge, usually a dial gauge with a one inch range and a scale of 0.0005 inches per division. Dial gauge readings were taken at previously chosen values of compressive load.

During the entire test the strip chart recorder is measuring the strain in the metal tapes. The strip chart was marked at each of the specified load points. When the test concludes, the result is three measured parameters: compressive load, cable strain, outer tape strain (stress). Since the tape strain measurements were graphic, these readings required a calibration of some kind. After the test was completed, this was accomplished by removing the section of tape where the strain gauges were mounted and placing it in the testing machine using special attachments. The metal tape was then loaded in tension and the corresponding strain was measured. In this way, a correlation could be made between tape strain measurements during the test and during the calibration. This process was repeated for every test to eliminate possible error caused by variations in metal tape properties.

The load range usually proceeded as an increase from zero to 5000 pounds, then unloading to zero. Then a re-load cycle took place to various maximum values (about 8000#) and then again unloading to zero. The

actual results of the tests will be discussed in the section on experimental results. However, it may be stated here that after working on virtually every test performed on this research project, it is this author's hope that all of our testing equipment and procedures will operate as smoothly and successfully as the compression test system.

As a final note, the data obtained from these tests was immediately converted into graphic output through the use of our own computer facilities. It was one of my accomplishments to write a computer program which would handle the graphic analysis requirements of all our testing procedures. This was done on a Tektronix 4051 graphics unit with an accompanying Tektronix 4610 hard copy unit. Upon entering the data as obtained from the test, the computer produces graphic output as is shown in the section containing raw test data. The use of this program and this system saved us countless hours which would have otherwise been spent plotting graphs by hand. The data from each test is stored on tape so that data from any test may be re-called and graphically displayed at anytime, using any desired scale or data range.

Figures 5 through 8 illustrate the procedure used for our compression tests and the computer system discussed above. These figures display the testing apparatus, compression specimens, and computer graphics system.

Figure 5 Compression Test

Figure 6 Computer Graphics System

Figure 7 Compression Specimen With Strain Gauges

Figure 8 Compression Specimen With Strain Gauges

PART 4

DISCUSSION

The fundamental cause of thermo-mechanical bending is the lengthening of a cable section as the temperature increases. As previously discussed, the cables are in segments and each segment is rigidly fixed at the manhole locations. Since these manholes are essentially a fixed distance apart, the thermal length change cycle must be accommodated by the cable within the conduit. It is the cyclic action of this thermal length change which results in the flexural fatigue of the cable and the formation of soft spots within the insulation tapes.

The degree of bending that may take place depends upon the relative sizes of the cable and its conduit. I have already discussed the usual configuration of the three power cables within their conduit (Figs. 3 and 4). The exclusion of constraints on the cable length in this type of arrangement was expected to provide a mechanism for thermal expansion, but actually provides added means for the propagation of thermo-mechanical bending and the formation of soft spots (4). In a conduit with a lot of room for the cables to move, the length change will be accommodated by an increase in bending. In a conduit of smaller diameter, the length change would induce more compressive stresses. In any case, it is safe to say that the existence of this thermal length change produces varying degrees of compressive stresses in the cable itself, depending on the dimensions of the components employed for the in-situ conditions.

The analysis that follows contains an attempt to model mathematically the effects of axial compressive forces on the cables under observation. The primary concern of this author is the influence of a radial pressure

exerted on the insulation tapes by the copper core, as a result of an axial load applied at its end points. Before attempting to analyze this phenomenon, it must be established that these axial compressive forces exist, and in fact, are of a large enough magnitude to develop the above mentioned stress state to a significant degree. This is important because the idea that stress levels induced by these compressive forces are not high enough to warrant consideration is a possible argument.

The existence of the axial compressive forces due to the thermal expansion of the cable is an established condition of the in-situ behavior and the primary cause of the TMB problem. Furthermore, the existence of resulting radial stresses through the insulation thickness may be proven significant by examining the physical construction of the cables. As stated earlier, the conductor of the cables are comprised of individual wires, with cross-sectional area of the conductor being approximately 600 times that of an individual wire. The resulting geometric configuration of the conductor wires is quite complex and to facilitate any mathematical analysis we must make a general assumption. Each individual wire of the conductor will be assumed helical in shape (4). This assumption is not far from physical reality and greatly facilitates the ensuing analysis. In addition, the existing work on twisted wire cables (6-12) may be employed because of its similarity to the problem at hand.

From test results it has been found that when a cable segment is loaded axially, most or nearly all of the force is carried by the conductor. The insulating tapes act as continuous bracing along the conductor length, but offer little resistance to the compressive load. However, it is this interaction between the conductor wires and the insulating

tapes which maintains the equilibrium state of the core. The helical configuration of the copper wires would never maintain its shape without the support of the surrounding paper wrapping. Knowing this, it must be concluded that a significant radial pressure is exerted on the insulation because of an applied axial load on the conductor, and because of its geometric properties. In fact, our tests have shown significant changes in stress on the insulating tapes produced by relatively low levels of axial load. Therefore, any analysis which proposes to include all the forces encountered during TMB must include the internal forces generated by this interaction.

The results of load cycling tests performed at the EPRI Waltz Mill Cable Test Facility (1) experimentally verify the above conjectures. Their tests simulated real conditions of cable loading by creating thermal expansion with conductor heating. Initially, they measured compressive forces in the range of 2500# to 4000#. Once the cable began "snaking" these levels dropped to 1500# to 2000#. In any case, compressive forces of these magnitudes, according to our test results, will cause significant changes in stress through the thickness of insulation.

The force levels encountered in compression may also be determined through an analysis of the Euler buckling load for the cable as performed by Spillers (5). In this analysis he makes use of a theory of bars with small initial curvatures. This type of approach is correct in theory, but would yield acceptable results if effects of the cable weight and the constraining effects of the pipe conduit were taken into account. This may be easier said than done, but successfully modeling this buckling behavior would certainly be worthwhile.

Now that the problem has been established we may proceed to the analysis. The behavior patterns to be analyzed may be summarized in the following way. When a cable conductor is loaded in tension, it remains in equilibrium through circumferential contact forces between the individual wires. However, when loaded in compression, the helical configuration of the conductor wires induce radial expansion rather than contraction. Therefore, for a cable subjected to axial load this circumferential force component must be provided by the insulating tapes. Since the tapes are relatively soft, they will expand when this load is applied. This indicates that the gross Young's modulus for the cable becomes highly dependent upon the properties of the insulating tape. It is this interaction between conductor and insulation that is to be examined.

PART 5

ANALYSIS

There are essentially two parts to the analysis presented in this project report. First of all it can be argued that the cable conductor is the primary load carrying component of a cable, but that the cable can only function in compression when restrained by the insulation tapes.

The first step in the analysis is to compute the lateral pressure which must be applied to the outside of the conductor by the insulation to maintain equilibrium of the helical wires of the conductor. This lateral pressure is, of course, also the pressure felt by the insulation at its inner surface. The second part of the analysis predicts the response of the insulation as a thick-walled tube to this internal pressure.

At a later stage in this report, forces measured in the external metal tapes of the cable as it is subjected to axial load are used to verify the analysis presented in this section. The first parameter to be examined here is the lateral pressure exerted by the conductor wires due to an applied load.

A. Equivalent Lateral Pressure

Consider the results of Reissner (6) for the equilibrium equations of a curved beam in space. These equations take the form

$$\underline{P}' + \underline{p} = 0$$

and

$$\underline{T}' + \underline{P} \times \underline{\xi} + \underline{q} = 0$$

where

$\underline{P}$ - force vector for the beam cross-section

$\underline{p}$ - applied force load intensity vector

$\underline{T}$ - moment vector for the beam cross-section

$\underline{q}$ - moment load intensity vector

$\underline{t}$ - tangent vector for the beam centerline

Now employing the work of Costello and Phillips (7,8,9) on the statics of twisted wire cables, the above equilibrium equations may be written in component form as: (see Fig. 9)

$$dN/ds - N' \tau_1 + k_1' + x = 0$$

$$dN'/ds - \tau k_1 + N \tau_1 + y = 0$$

$$dT/ds - N k_1' + N' k_1 + z = 0$$

$$dG/ds - G' \tau_1 + H k_1' - N' + k = 0$$

$$dG'/ds - H k_1 + G \tau_1 + N + k' = 0$$

$$dH/ds - G k_1' + G' k_1 + \theta = 0$$

where

N, N', T - components of the vector $\underline{P}$

G, G', H - components of the vector $\underline{T}$

$\underline{q}$ - assumed equal to zero

k_1, k_1', τ_1 - geometric properties of the wire profile

The critical assumption of the conductor wires taking the shape of a helix is now enforced. For a helix with radius r and helix angle α , the geometric properties are,

$$k_1 = 0$$

$$k_1' = \frac{1}{r} \cos^2 \alpha$$

Figure 9a Resultant Forces Acting on Helical Wire

Figure 9b Geometry of 6-Wire Cable

$$r_1 = \frac{1}{r} \sin \alpha \cos \alpha$$

For the case under investigation here, the conductor wire (helical beam) is subjected to an axial compressive force with no applied moments. Because of this, the moment equations are identically satisfied and the second and third component equations reduce to $Y = 0$ and $Z = 0$. It is the first equation which yields the circumferential force component x , which may be written as

$$T k_1' + x = 0$$

Since $k_1' = \frac{1}{r} \cos^2 \alpha$, we have

$$x = -T k_1' = -\frac{T}{r} \cos^2 \alpha$$

The following procedure is employed to arrive at the desired equivalent lateral pressure p_e , given an applied axial stress f_o . The circumferential force component x , for a wire of diameter D , relates to the lateral pressure as

$$p_e = x/D$$

This wire of diameter D or radius r may be considered equivalent to a hollow cylinder of thickness D and average radius r (4). Then the applied axial stress f_o is related to an axial wire force T in the following equation

$$T \sin \alpha = f_o D^2 / \sin \alpha$$

or

$$T = \frac{f_o D^2}{\sin^2 \alpha}$$

Combining all of the results above gives

$$p_e = \frac{x}{D} = \frac{T \cos^2 \alpha}{rD} = \frac{f_o D}{r} \operatorname{ctn}^2 \alpha \quad (P1)$$

We now have an equation yielding the lateral pressure exerted on the insulation by the conductor given an applied axial stress and the dimensions of the conductor. The next step, which is the heart of this effort, is to perform an elastic analysis of the cable's insulation as an orthotropic thick walled tube. The final result will enable a correlation between the applied axial load with the strain measurements in the external metal tapes of the cable.

B. Cable Insulation Model

An Orthotropic Elastic Thick-Walled Tube

The initiation of this approach was based upon the Pirelli model, in which the cable insulation is considered to be an orthotropic, elastic thick-walled tube (13,14,15).

An appropriate place to begin is with the general stress strain relations for elastic media.

$$\begin{aligned} \epsilon_{rr} &= a_{11} \sigma_{rr} + a_{12} \sigma_{\theta\theta} + a_{13} \sigma_{zz} \\ \epsilon_{\theta\theta} &= a_{12} \sigma_{rr} + a_{22} \sigma_{\theta\theta} + a_{23} \sigma_{zz} \\ \epsilon_{zz} &= a_{13} \sigma_{rr} + a_{23} \sigma_{\theta\theta} + a_{33} \sigma_{zz} \\ 2\epsilon_{\theta z} &= a_{44} \sigma_{\theta z} \\ 2\epsilon_{rz} &= a_{55} \sigma_{rz} \\ 2\epsilon_{r\theta} &= a_{66} \sigma_{r\theta} \end{aligned} \quad (1)$$

where

$\sigma_{rr}, \epsilon_{rr}$ - radial stress, strain

$\sigma_{\theta\theta}, \epsilon_{\theta\theta}$ - circumferential stress, strain

$\sigma_{zz}, \epsilon_{zz}$ - longitudinal stress, strain

$\left. \begin{array}{l} \sigma_{\theta z}, \epsilon_{\theta z} \\ \sigma_{rz}, \epsilon_{rz} \\ \sigma_{r\theta}, \epsilon_{r\theta} \end{array} \right\} - \text{shearing stresses, strains}$

a_{ij} - elastic coefficients

The coefficients a_{ij} are simply material constants, and it is advantageous to the problem at hand to write these stress strain relations in terms of the so-called engineering constants of elasticity (13)

$$\epsilon_{rr} = \frac{1}{E_r} \sigma_{rr} - \frac{\nu_{\theta r}}{E_\theta} \sigma_{\theta\theta} - \frac{\nu_{zr}}{E_z} \sigma_{zz}$$

$$\epsilon_{\theta\theta} = \frac{-\nu_{r\theta}}{E_r} \sigma_{rr} + \frac{1}{E_\theta} \sigma_{\theta\theta} - \frac{\nu_{z\theta}}{E_z} \sigma_{zz}$$

$$\epsilon_{zz} = \frac{-\nu_{rz}}{E_r} \sigma_{rr} - \frac{\nu_{\theta z}}{E_\theta} \sigma_{\theta\theta} + \frac{1}{E_z} \sigma_{zz}$$

(2)

$$\epsilon_{\theta z} = \frac{1}{2G_{\theta z}} \sigma_{\theta z}$$

$$\epsilon_{rz} = \frac{1}{2G_{rz}} \sigma_{rz}$$

$$\epsilon_{r\theta} = \frac{1}{2G_{r\theta}} \sigma_{r\theta}$$

Because of the axial symmetric loading in this problem, the stress components $\sigma_{rr} = \sigma_r$, $\sigma_{\theta\theta} = \sigma_\theta$, $\sigma_{zz} = \sigma_z$ are the principal components,

while $\sigma_{r\theta} = \sigma_{\theta z} = \sigma_{zr} = 0$. Similarly for the strain components, $\epsilon_{rr} = \epsilon_r$, $\epsilon_{\theta\theta} = \epsilon_\theta$, $\epsilon_{zz} = \epsilon_z$ are the principal components, while $\epsilon_{r\theta} = \epsilon_{\theta z} = \epsilon_{zr} = 0$. Furthermore, since this is a case of plane stress $\sigma_z = 0$. In the plane stress case, the forms of Eqs. (1) reduce to

$$\begin{aligned}\epsilon_r &= a_{11} \sigma_r + a_{12} \sigma_\theta \\ \epsilon_\theta &= a_{12} \sigma_r + a_{22} \sigma_\theta \\ \epsilon_z &= a_{13} \sigma_r + a_{23} \sigma_\theta\end{aligned}\tag{3}$$

In the case of plain strain $\epsilon_z = 0$ which means that from the third equation of Eqs. (1)

$$\sigma_z = - (1/a_{33}) (a_{13} \sigma_r + a_{23} \sigma_\theta)\tag{4}$$

Through the use of Eq. (4), the first two of Eqs. (1) may be written as

$$\begin{aligned}\epsilon_r &= \alpha_{11} \sigma_r + \alpha_{12} \sigma_\theta \\ \epsilon_\theta &= \alpha_{12} \sigma_r + \alpha_{22} \sigma_\theta\end{aligned}\tag{5}$$

where

$$\begin{aligned}\alpha_{11} &= a_{11} - (a_{13}^2/a_{33}) \\ \alpha_{12} &= a_{12} - (a_{13} a_{23}/a_{33}) \\ \alpha_{22} &= a_{22} - (a_{23}^2/a_{33})\end{aligned}$$

For plain strain, Eqs. (3) may also be written as the stress components in terms of the three strain components, assuming the form

$$\begin{aligned}\sigma_r &= C_{11} \epsilon_r + C_{12} \epsilon_\theta \\ \sigma_\theta &= C_{12} \epsilon_r + C_{22} \epsilon_\theta \\ \sigma_z &= C_{13} \epsilon_r + C_{23} \epsilon_\theta\end{aligned}\tag{6}$$

where the C_{ij} terms are also elastic coefficients.

Now the application must be made to the problem at hand. The loading here is considered virtually symmetric and the insulation of the cable is taken as the thickness of the thick-walled tube. The tube is enclosed in an elastic shell which is represented by the outer metallic tapes of the cable. The radial stress is specified by the previously derived equation for p_e at the inner radius of the tube, or, at $r = a$. Since we are only considering a single radial displacement component u_r , the equations of equilibrium reduce to the single equation (13)

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (7)$$

The definition of the strain components, ϵ_r and ϵ_θ in terms of the radial displacement u_r are

$$\begin{aligned} \epsilon_r &= \frac{\partial u_r}{\partial r} \\ \epsilon_\theta &= \frac{u_r}{r} \end{aligned} \quad (8)$$

From this, the compatibility equations reduce to the single equation

$$\epsilon_r - r \frac{\partial \epsilon_\theta}{\partial r} - \epsilon_\theta = 0 \quad (9)$$

This problem could be solved by defining a stress function $f(r)$ in terms of the stress components such as

$$\sigma_r = \frac{1}{r} \frac{\partial f(r)}{\partial r}, \quad \sigma_\theta = \frac{\partial^2 f(r)}{\partial r^2}$$

However, this author chooses the alternative approach to the plain strain

problem by making use of Eqs. (7) and (9) and the definitions of Eqs. (8), and subsequently solving the differential equation that is derived.

By substituting Eqs. (8) into Eq. (9) a compatibility equation in terms of the displacements and the radius is yielded in the form

$$\frac{\partial u_r}{\partial r} - r \frac{\partial u_r}{\partial r} - \frac{u_r}{r} = 0 \quad (10)$$

Although a solution of this equation could be determined, this would not insure that equilibrium would be satisfied. It would be a more direct approach to solve the equilibrium equation itself [Eq. (7)]. This equation is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (7)$$

which requires the development of functions for σ_r and σ_θ which may be defined in terms of the displacements. This would mean solving Eqs. (2) for the stress components and plugging in displacement functions.

Before doing this, however, an important simplifying assumption must be enforced. From the experimental work of the Pirelli group (15), it is apparent that experiments have shown Poisson's ratios to be negligible for the paper insulation. This argument agrees with our own physical tests aimed at measuring Poisson's ratio, that is, the fact that it is small enough to be neglected in calculations. Remembering that $\sigma_z = 0$, the elastic equations may now be written as

$$\sigma_r = E_r \epsilon_r, \quad \sigma_\theta = E_\theta \epsilon_\theta$$

Employing Eqs. (8), these stress-strain relationships take the form

$$\sigma_r = E_r \epsilon_r = E_r \frac{\partial u_r}{\partial r} \quad (8.1)$$

$$\sigma_\theta = E_\theta \epsilon_\theta = E_\theta \frac{u_r}{r} \quad (8.2)$$

Knowing that

$$\frac{\partial \sigma_r}{\partial r} = E_r \frac{\partial^2 u_r}{\partial r^2}$$

substitution into Eq. (7) gives the desired differential equation as

$$E_r \frac{\partial^2 u_r}{\partial r^2} + \frac{E_r \frac{\partial u_r}{\partial r} - E_\theta \frac{u_r}{r}}{r} = 0$$

Multiplying through by r^2 gives

$$r^2 E_r \frac{\partial^2 u_r}{\partial r^2} + r E_r \frac{\partial u_r}{\partial r} - E_\theta u_r = 0 \quad (11)$$

Solving this equation will ultimately give the stress at any point in the insulation layers. The solution of this equation takes the form

$$u_r = A_1 r^k + A_2 r^{-k} \quad (11.1)$$

Evaluation of the subsequent derivatives gives

$$\frac{\partial u_r}{\partial r} = k A_1 r^{k-1} - k A_2 r^{-k-1} = k A_1 r^{k-1} - k A_2 r^{-(k+1)}$$

$$\frac{\partial^2 u_r}{\partial r^2} = k(k-1) A_1 r^{k-2} + k(k+1) A_2 r^{-(k+2)}$$

Considering the cable in the configuration shown below in Figure 10, there are two boundary conditions which may be applied to this problem.

Figure 10

Thick-Walled Tube Model

The first boundary condition is

$$\sigma_r = -p_e \text{ at } r = a$$

where p_e is the result of Eq. (P1) for the internal pressure exerted on the insulation by the expanding conductor. Therefore, at $r = a$,

$$\sigma_r = E_r \frac{\partial u_r}{\partial r} = -p_e$$

or

$$\frac{\partial u_r}{\partial r} = k A_1 a^{(k-1)} - k A_2 a^{-(k+1)} = -p_e/E_r$$

Applying the second boundary condition,

$$\sigma_r = \frac{-u_r E T}{(1-\nu^2)b^2} \text{ at } r = b$$

This means that at $r = b$,

$$E_r \frac{\partial u_r}{\partial r} = E_r (k A_1 b^{(k-1)} - k A_2 b^{-(k+1)}) = \frac{-u_r E t}{(1-\nu^2)b^2}$$

or

$$\frac{\partial u_r}{\partial r} = \frac{-u_r t E}{(1-\nu^2)b^2 E_r}$$

where E_r - modulus of elasticity for paper (radially)

E - modulus of elasticity for metal tapes

It is desirable to eliminate the u_r term from the above expression, and by substituting the right side of Eq. (11.1) this relationship takes the form,

$$\frac{\partial u_r}{\partial r} = \frac{-(A_1 b^k + A_2 b^{-k}) t E}{(1-\nu^2) b^2 E_r}$$

Thus, the two resulting equations in terms of A_1 and A_2 are

$$k A_1 a^{(k-1)} - k A_2 a^{-(k+1)} = \frac{-p_e}{E_r} \quad (12)$$

$$k A_1 b^{(k-1)} - k A_2 b^{-(k+1)} = \frac{-(A_1 b^k + A_2 b^{-k}) t E}{(1-\nu^2) b^2 E_r} \quad (13)$$

It now remains to solve for A_1 and A_2 . Solving for A_1 in Eq. (12) gives

$$k A_1 a^{(k-1)} = \frac{-p_e}{E_r} + k A_2 a^{-(k+1)}$$

$$A_1 = \frac{-p_e}{k E_r a^{(k-1)}} + A_2 a^{-2k} \quad (12.1)$$

Substituting this result into Eq. (13),

$$kb^{(k-1)} \left[\frac{-p_e}{kE_r a^{(k-1)}} + A_2 a^{-2k} \right] - kA_2 b^{-(k+1)} = \frac{-tE}{(1-v^2)b^2 E_r} \left(\left[\frac{-p_e}{kE_r a^{(k-1)}} + A_2 a^{-2k} \right] b^k + A_2 b^{-k} \right)$$

$$\frac{-p_e}{E_r} \left(\frac{b}{a} \right)^{k-1} + A_2 k a^{-2k} b^{(k-1)} - A_2 k b^{-(k+1)} = \frac{-tE}{(1-v^2)b^2 E_r} \left[\frac{-p_e b^k}{kE_r a^{(k-1)}} + A_2 (a^{-2k} b^{k+b-k}) \right]$$

$$\frac{-p_e}{E_r} \left(\frac{b}{a} \right)^{k-1} + A_2 k (a^{-2k} b^{(k-1)} - b^{-(k+1)}) = \frac{p_e t E b^k}{(1-v^2)b^2 E_r^2 k a^{(k-1)}} - \frac{tEA_2 (a^{-2k} b^{k+b-k})}{(1-v^2)b^2 E_r}$$

$$A_2 [k(a^{-2k} b^{(k-1)} - b^{-(k+1)}) + \frac{tE(a^{-2k} b^{k+b-k})}{(1-v^2)b^2 E_r}] = \frac{p_e t E b^{k-2}}{(1-v^2)E_r^2 k a^{(k-1)}} + \frac{p_e}{E_r} \left(\frac{b}{a} \right)^{k-1}$$

$$A_2 = \frac{\frac{p_e t E b^{k-2}}{(1-v^2)E_r^2 k a^{(k-1)}} + \frac{p_e}{E_r} \left(\frac{b}{a} \right)^{k-1}}{k(a^{-2k} b^{(k-1)} - b^{-(k+1)}) + \frac{tE(a^{-2k} b^{k+b-k})}{(1-v^2)b^2 E_r}}$$

It appears that simplification of this term is in order, but this will not take place until numerical values are substituted for the variables in the next section. Substituting A_2 into Eq. (12.1) gives

$$A_1 = \frac{-p_e}{kE_r a^{(k-1)}} + \frac{\frac{p_e t E a^{(1-3k)} b^{(k-2)}}{(1-v^2)E_r^2 k a^{(k-1)}} + \frac{p_e a^{-2k}}{E_r} \left(\frac{b}{a} \right)^{k-1}}{k(a^{-2k} b^{(k-1)} - b^{-(k+1)}) + \frac{tE(a^{-2k} b^{k+b-k})}{(1-v^2)b^2 E_r}}$$

The following list defines the terms used in the preceding equations and constants:

- p_e - equivalent lateral pressure (internal)
- E_r - modulus of elasticity for paper insulation (radial)
- E_θ - modulus of elasticity for paper insulation (circumferential)
- E - modulus of elasticity for outer metal tapes
- ν - Poisson's ratio for metal tapes
- t - thickness of elastic shell (metal tapes)
- u_r - radial displacement
- a - radius to conductor binder
- b - radius to metal tapes
- k - a constant

Eq. (11.1) may now be written in terms of the known quantities A_1 and A_2 . The u_r term may be related to the stresses (strains) through the use of Eqs. (8.1) and (8.2). Thus, we should be able to determine the stresses at any given radius with the cable insulation.

This accomplishment completes the analysis portion of this effort. It remains to apply the results of our physical testing to the above equations and ultimately, to draw conclusions from the outcome.

PART 6

EXPERIMENTAL RESULTS

The analysis of the preceding section gives us a model of the insulation behavior resulting from an applied axial load. This type of analysis, to my knowledge, has never been developed with respect to pipe-type cable. This may be because the stress variations encountered in the insulation due to compressive loads are considered (by conjecture) to be small enough in magnitude to be neglected.

This section will provide the only means of verification of the mathematical model at my disposal. The cable that has provided us with fairly extensive data relating to this phenomenon is a 550 kV ac cable, an experimental cable which we obtained from the Pirelli Cable Corporation. A paper prepared by the General Cable Corporation for the Electric Power Research Institute provided us with the specifics concerning this cable (16). An excerpt from this report describing the general design characteristics of the cable is shown in Table 1. Although there were many other types of cable under observation, I will limit the discussion of experimental results to this particular cable because of the generous amount of data obtained.

The tests performed on the cable gave us the following: For a given axial load, we know the strain in the cable in the vertical direction, and the stress or strain in the outer metallic tapes of the cable in the circumferential direction. Direct calculations give us a modulus of elasticity in the vertical or "Z" direction. The tape strain values will subsequently be used to provide the known stress value found in the second

Table 1
DESIGN OF FINAL 500 KV CABLE

Conductor:

Type - Tinned Copper Compact Segmental
4 Segments, 2 Insulated with Paper
Size - 2500 kcmil (1266 mm²)
Binder - Tinned Bronze Tape Intercalated with Cellulose Paper Tape
OD, Inches (cm) 1.82
(4.62)

Insulation Structure:

(Impregnant - Silicone Oil DC200-50 EG with 5% Dodecylbenzene
Additive)

Conductor Shield

4 - 5 mil (0.127 mm) Carbon Black + 1 - 5 mil (0.127 mm) Carbon
Black/Paper Duplex Tape

Insulating Tapes

2 - 3 mil (0.076 mm) x 1/2 Inch (1.27 cm) Cellulose Paper Tapes
13 - 3 mil (0.076 mm) x 1/2 Inch (1.27 cm) PPP Tapes
20 - 5 mil (0.127 mm) x 1/2 Inch (1.27 cm) PPP Tapes
20 - 5 mil (0.127 mm) x 3/4 Inch (1.90 cm) PPP Tapes
100 - 8 mil (0.203 mm) x 3/4 Inch (1.90 cm) PPP Tapes
18 - 8 mil (0.203 mm) x 1 Inch (2.54 cm) PPP Tapes
2 - 8 mil (0.203 mm) x 1 Inch (2.54 cm) Cellulose Paper Tapes

Insulation Thickness, Nominal, Inches (mm) 1.20
(30.5)
Insulation OD, Inches (cm) 4.28
(10.9)

Insulation Shield

1 - 5 mil (0.127 mm) Carbon Black/Paper Duplex Tape
2 - 5 mil (0.127 mm) Carbon Black Tape
2 - 5 mil (0.127 mm) Perforated Aluminum Foil Backed Carbon
Black Tape

Moisture Seal, Skid-Wire Assembly

2 - 2.5 mil (0.064 mm) Aluminum Foil Back Intercalated Polyester Tapes
2 - 5 mil (0.127 mm) Tinned Copper Tapes Intercalated with 2 mil
(0.051 mm) Polyester Tapes
2 - 0.150 Inch (0.38 cm) x 0.300 Inch (0.76 cm) Black High Density
Polyethylene Skid Wires
OD, Inches (cm) 4.67
(11.9)

boundary condition of the elastic analysis. The application of numbers derived from test results will be done by choosing somewhat arbitrary points during the tests and applying these numbers to the lateral pressure and internal stress equations which were previously derived.

Before any calculations may be performed, the terms appearing as variables in the analysis equations must be determined as real numerical quantities. The first of these parameters to be defined are the values of "a" and "b" which were first encountered with the introduction of the boundary conditions. As previously defined:

a - radius to conductor binder

b - radius to metal tapes

Table 1 contains these radius values, or at least contains the dimensions necessary to compute them. The diameter of this conductor assembly is given as 1.82 inches. Therefore, the radius value is,

$$a = 0.91 \text{ inches}$$

The outer diameter of the entire cable assembly including the skid wires is given as 4.67 inches. Subtracting twice the thickness of the skid wires (0.150 inches) from this value, the value of b is determined as,

$$b = 2.185 \text{ inches}$$

The variable t which is introduced with the second boundary condition is simply the thickness of the elastic shell, or in this case, the outer metal tapes. Table 1 indicates that there are two layers of metal tapes each with a thickness of 0.005 inches. The very thin polyester tapes that are intercalated with the metal will not be included since they are removed before the compression test is attempted. This is done in order to allow application of the SR-4 strain gauges to the metal tapes. In any case,

the value of t to be used is,

$$t = 0.01 \text{ inches}$$

There are three different Young's modulus values to be determined in order to employ the analysis equations. The first to be considered here is E_r , the modulus of elasticity for the paper insulation. There are a variety of paper types used between the conductor binder and the outer metal tapes of the Pirrelli cable. However, the insulation is predominantly composed of PPP tapes, and it was this tape which was used to determine the effective E_r value for the cable insulation.

A simple compression test was performed on 100 layers of PPP tape. A plot of this test is shown in Fig. 11. Since each layer of this tape is 8 mils (0.008") thick, the effective depth of the tapes was 0.8 inches. The load was applied to a surface area of one square inch. The load range was extended far beyond the necessary extent and the tapes began to attain a high degree of compaction. For the purposes of this test, the middle range of compressive loading was used to compute a value for E_r .

The slope of the middle compressive load range was determined and a value for E_r was calculated. A factor that must also be considered is the effect of butt spaces on the actual modulus of the insulation as fabricated in the cable. This effect would tend to decrease the effective modulus, and so the value obtained was proportionally decreased with respect to the size of the gaps. The final number arrived at was

$$E_r = 1.5 \times 10^4 \text{ psi}$$

It should be noted that this value falls within the range established by the Pirrelli group (15) for the E_r value for paper lapped insulation.

Figure 11. Compression Test for E_r

Although much more accurate procedures are necessary to determine this value, this approximation will suffice here. It may be stated here that the error encountered in determining these values should certainly cause inconsistency in the numerical results of the equations. Therefore, the purpose here shall be more an illustration of the use of the preceding analysis. More accurate measurement of material constants should, of course, render more accurate results when using the stress equations.

The next value to be considered is E_θ , the modulus of elasticity in the circumferential direction. Again employing an assumption first developed in the Pirelli paper (15), the paper lapped insulation shall be assumed as layers of contiguous rings. This means that E_θ can be taken as Young's modulus for the paper in tension. If the actual helical configuration of the paper lapping was taken into account, the modulus value would be slightly different than the value for linear tension. To my knowledge this type of analysis including the helix effect has never been undertaken for calculations of E_θ , and it is the author's opinion that this factor would have a very small effect on the final outcome. In any case, a test run on the PPP tapes in tension (Fig. 12) delivered a reasonable value of

$$E_\theta = 1.85 \times 10^6 \text{ psi}$$

This value is also within the range of generally expected magnitudes for this modulus value. One should note the difference in magnitude between E_r and E_θ , which is to the order of 10^2 .

The third Young's modulus value necessary to perform this analysis was simply denoted as E (unsubscripted), which is the modulus of elasticity for the outer metal tapes. This value was introduced with boundary

Figure 12 Tension Test for E_0

condition number two, and its determination was performed identically to that for E_θ . Figure 13 illustrates the results of the actual test on the tapes, and the resulting value was

$$E = 17 \times 10^6 \text{ psi (metal tape)}$$

This value is essentially the same as that of copper, which is the primary material in the composition of the metal tapes.

The final material constant to be determined is Poisson's ratio for the outer shell (assumed zero for the paper). Since the value of E was so close to that of copper, it appears adequate to use copper values here as well.

$$\nu = 0.33$$

Finally, a numerical value for k must be established. From the analysis of Bieniek, Spillers, and Freudenthal (14) it is established that Eq. (11) takes the same form when solved in terms of unknown constants rather than material properties. Upon substitution of the general solution into this equation, the value of k^2 may be established as a combination of constants. By inspection, the case studied here has k^2 corresponding to the value of E_θ/E_r . Therefore,

$$k^2 = \frac{1.85 \times 10^6}{1.5 \times 10^4} = 1.23 \times 10^2$$

or

$$k = 11$$

The values of A_1 and A_2 may now be calculated. These values were solved for earlier in terms of the variables discussed here. It is now worthwhile to present a somewhat less formal, but simpler and equally

Figure 13 Tension Test for E Metal

accurate approach to finding A_1 and A_2 . Although direct substitution into the derived expressions is certainly feasible, the following method eventually employs less mathematical calculations.

Equation (13) may be written in the form

$$E_r k (A_1 b^{k-1} - A_2 b^{-(k+1)}) = \frac{-E t}{(1-v^2)b^2} (A_1 b^k + A_2 b^{-k})$$

This equation may be manipulated to yield the ratio of A_1 and A_2 .

$$A_1 (E_r k b^{k-1} + \frac{E t b^k}{(1-v^2)b^2}) = A_2 (E_r k b^{-(k+1)} - \frac{E t b^{-k}}{(1-v^2)b^2})$$

$$A_1 b^{k-1} (E_r k + \frac{E t}{(1-v^2)b}) = A_2 b^{-(k+1)} (E_r k - \frac{E t}{(1-v^2)b})$$

$$A_1 = A_2 \frac{b^{-(k+1)} (E_r k - \frac{E t}{(1-v^2)b})}{b^{k-1} (E_r k + \frac{E t}{(1-v^2)b})}$$

$$A_1 = A_2 \frac{b^{-2k} (E_r k - \frac{E t}{(1-v^2)b})}{E_r k + \frac{E t}{(1-v^2)b}}$$

Substituting the numerical values,

$$A_1 = A_2 \frac{(2.185)^{-22} [(1.5 \times 10^4)(11) - \frac{(17 \times 10^6)(0.01)}{(1-0.33^2)(2.185)}]}{(1.5 \times 10^4)(11) + \frac{(17 \times 10^6)(0.01)}{(1-0.33^2)(2.185)}}$$

$$A_1 = 1.090 \times 10^{-8} A_2$$

Introducing Eq. (12.1),

$$A_1 = \frac{-p_e}{k E_r a^{(k-1)}} + A_2 a^{-2k}$$

$$1.09 \times 10^{-8} A_2 = \frac{-p_e}{(11)(1.5 \times 10^4)(.91)^{10}} + A_2 (.91)^{-22}$$

$$A_2 = 1.954 \times 10^{-6} p_e$$

and

$$A_1 = (1.954 \times 10^{-6} p_e) (1.09 \times 10^{-8})$$

$$A_1 = 2.13 \times 10^{-14} p_e$$

Equation (11.1) may now be written in its final form as

$$u_r = 2.13 \times 10^{-14} p_e r^{11} + 1.954 \times 10^{-6} p_e r^{-11} \quad (14)$$

The value of u_r may be used to find the stresses in the radial and circumferential direction for any given radius. Before application to real test data is made, some further development of Eq. (P1) is necessary. This was the equation for the internal pressure produced by the conductor and took the form

$$p_e = \frac{f_o D}{r} \csc^2 \alpha$$

The variables of this equation shall now be discussed, the first of which shall be the angle α . The configuration of the conductor is, to a close approximation, a simple helix in shape. The value of α is the helix

angle of this conductor configuration. Through physical measurement and simple calculations it was found that for the 550 kV experimental cable,

$$\alpha = 75^\circ$$

The variable f_o is simply the applied axial stress on the conductor. The area of the conductor used in this cable is 1.96 in^2 (calculated from Table 1). Therefore, the value of f_o may be found as

$$f_o = \frac{P}{1.96}$$

where P is the applied axial load. This value is different, of course, for each data point, and it will not be calculated until actual test results are used.

The final consideration is the values of D and r . This is the critical point for Eq. (P1) since the configuration of the conductor is being approximated as a helical wire model, but does not actually retain this exact shape. Since there are many layers of wires in the conductor, the D/r term should be written with this term as a summation. Therefore, Eq. (P1) takes the form

$$p_e = f_o \csc^2 \alpha \sum_{i=1}^n \frac{D_i}{r_i} \quad (\text{P2})$$

where n = the number of individual wires.

Since D is the diameter of a single wire, this value will remain constant and assume the value of

$$D = 0.11"$$

for this particular problem. The r_i values are the average radii of successive shells of thickness D . Therefore, approximating the conductor

as eight consecutive shells ($n = 8$) the summation term becomes

$$\sum_{i=1}^8 \frac{D_i}{r_i} = .11 \left(\frac{1}{.050} + \frac{1}{.165} + \frac{1}{.275} + \frac{1}{.385} + \frac{1}{.495} + \frac{1}{.605} + \frac{1}{.715} + \frac{1}{.825} \right)$$

$$= 4.04$$

Equation (P2) may now be written in terms of the internal pressure p_e and the applied load P .

$$p_e = \frac{P}{1.96} (\cot^2 75^\circ) (4.04)$$

$$p_e = 0.5523 P \quad (P3)$$

Using Eq. (P3), the internal pressure may be calculated knowing the applied axial load. It is now possible to use experimental data from actual compression tests in these expressions.

From the raw test data presented at the end of this section, Fig. (E.3) illustrates the results of a test performed on February 20, 1979. From the graph it can be seen that at a compressive load of 2000 #, the force in the outer metal tape was approximately 2 #. This 2 # is of course the increase in force beyond any force that already exists in the tape due to pre-tensioning.

Employing Eq. (P3), for a load of 2000 #,

$$p_e = (0.5523) (2000)$$

$$p_e = 1104.6 \text{ psi}$$

Returning to Eq. (14) with $p_e = 1104.6 \text{ psi}$,

$$u_r = (2.13 \times 10^{-14})(1104.6)(2.185)^{11} + (1.954 \times 10^{-6})(1104.6)(2.185)^{-11}$$

$$u_r = 5.26 \times 10^{-7}$$

This can immediately be recognized as a much smaller value than that of the experimental results. Remembering relations (8.2),

$$\sigma_\theta = E_\theta \epsilon_\theta = E_\theta \frac{u_r}{r}$$

Therefore,

$$\sigma_\theta = (1.85 \times 10^6) \frac{(5.26 \times 10^{-7})}{2.185}$$

$$\sigma_\theta = 4.45 \times 10^{-1} \text{ psi}$$

or with the tape cross-sectional area of 0.01 in^2 ,

$$\begin{aligned} \text{Tape Force} &= (4.45 \times 10^{-1} \text{ psi}) (0.01 \text{ in}^2) \\ &= 4.45 \times 10^{-3} \# \end{aligned}$$

A. Discussion of Results

The fact that the measured tape tension varies by three orders of magnitude from the computed tape tension is not surprising whatsoever to this author. There are many factors still in need of consideration which can explain this difference.

First of all, the displacement function is very dependent upon the previously discussed material constants. The most important of these constants are the Young's modulus values, particularly the E_r value for the paper insulation. This value was measured experimentally and several

assumptions were made towards the determination of a final numerical value. The variation of E_r is most critical because it appears in the original differential equation of displacement [Eq. (11)]. The resulting solutions are highly dependent upon the value of k , which is a ratio of E_θ and E_r . Since k is used as an exponent as well as a coefficient, the accuracy of the computed displacement is dominated by the precision with which the material constants were obtained.

Figure (14) contains a plot of the displacement equation [Eq. (14)] for an internal pressure of 1000 psi. The shape of this function is highly sensitive to the exponent k in the displacement function. Our tests tend to overestimate k and thus our computations tend to underestimate stresses in the exterior metal tapes under applied axial load. It will simply be necessary to revise our test procedures in order to produce a more representative set of material constants.

The large value of k also gives rise to some computational difficulties. Since this value is used throughout the analysis, the coefficients in the solution of the differential equation are also governed by the size of k . The resulting values for A_1 and A_2 in Eq. (14) are comparatively very small in magnitude. This occurrence invites loss of accuracy in computations, although this problem should be considered of minor consequence. However, because of the high value of k and the small coefficients, Eq. (14) tends to be dominated by only one of its two terms. Only for a very small range are both terms contributing significantly to the result. This effect is quite obvious when examining the shape of the displacement function in Fig. (14). Had the ratio between E_θ and E_r been greater, this effect would be even more pronounced.

Figure 14 Displacement Function

In any case, upon examination of the test data, the existence of these stress variations within the tapes due to axial load becomes quite evident. The graphs presented as raw test data are a sample of our results. All tests that were performed with tape strain measurements indicated significant force changes within the outer metal tapes. On occasion, these forces would reach as high as 12 to 14 pounds [Fig. (E.14)]. The reason for the variation in results is most probably due to the degree of initial tensioning on the tapes at the start of each test. When using such relatively short segments, it is difficult to maintain the original amount of tension in the outer tapes while cutting the cable. Therefore, many of the test segments may not have maintained the degree of initial tape tensioning that would otherwise exist in the in-situ conditions. This is why some of the tests may have taken longer to build up significant outer tape stresses. But whatever the magnitude, the test results positively confirm the existence of the phenomenon under analysis in this project report. The remaining section will draw conclusions concerning these results and the proposed analysis techniques.

PART 7

CONCLUSIONS

The results of compression tests performed over the last year indicate that the insulation tapes of a 550 kV experimental cable undergo increases in stress as a result of an applied axial load. This axial load causes the radial expansion of the copper conductor which then causes radial displacement of the insulating tapes and increasing stresses. Since the TMB phenomenon is initiated by axial compressive forces, this stress condition must always occur during TMB.

In theory, the analysis presented here should provide a suitable model for this behavior. However, the results indicate that more extensive work must be done in obtaining the material properties of the insulation tapes. The accuracy of these values has a critical effect upon the resulting equations of the elastic analysis presented earlier. Therefore, strong consideration must be given to the development of improved methods for determining these material constants. The high k value provided by our previous tests caused the extreme slopes in the displacement function as illustrated in Fig. (14). It is expected that this effect would be reduced by more accurate measurement of these material constants.

The analysis of the cable insulation as an orthotropic material must be considered the most appropriate type of analysis approach. Furthermore, the representation of the cable configuration as a thick-walled tube within an elastic shell seems to be a natural extension. The effort here was to modify the existing work on this subject to the specific problem at hand. This analysis appears quite successful, and if all assumptions

were correct, the displacement functions should eventually provide accurate predictions of tape stress variations. The only assumption made that may warrant further examination is the assumption that Poisson's ratio is of negligible magnitude.

This idea was taken from the work of the Pirelli group (15) who stated simply that experiments have proven that Poisson's ratios for the paper are negligible. Accepting this statement allowed for great simplifications of the stress-strain relationships at the outset of the analysis. However, if Poisson's ratio is at all significant, the relationships that have been derived would be considerably altered. The confirmation of this assumption is another topic which warrants further investigation, or more specifically, more accurate testing procedures.

The mathematical model of the conductor expansion under axial load appears quite acceptable as a first approximation. However, since the cable conductor configuration is not exactly that of a helical wire rope, further modification of these relations is also warranted. Attempts were made to corroborate the computed expansion pressures with data from physical tests. This was to be accomplished by removing the conductor from a segment of cable and creating an internal pressure artificially, using a method that could control and measure the amount of pressure applied. As yet, this test procedure has not been successful.

In summary, the behavior of pipe-type cable under axial compressive loads appears suitably modeled through the use of anisotropic thick-walled tube theory. True accuracy in predicting actual stress values hinges upon the correct determination of the material constants. Hopefully this accomplishment will reduce the extreme slopes of the displacement function, thus

enabling stress-strain computations to be performed with some precision. Finally, more consideration should be given to the initial tape tensions, which have a direct effect upon experimental test results.

PAET 8

RAW TEST DATA

The ensuing pages contain a representative sample of the graphic output from various compression tests performed over the last year.

They include:

1. Compressive Load vs. Strain in Cable (axial)
2. Tape Force (metal) vs. Strain in Cable
3. Compressive Load vs. Tape Force

These plots were all produced by our own graphics program using the Tektronix system previously discussed.

Figure E.1 Compression Test Results

Figure E.2 Compression Test Results

Figure E.3 Compression Test Results

Figure E.4 Compression Test Results

Figure E.5 Compression Test Results

Figure E.6 Compression Test Results

Figure E.7 Compression Test Results

Figure E.8 Compression Test Results

Figure E.9 Compression Test Results

Figure E.10 Compression Test Results

Figure E.11 Compression Test Results

Figure E.12 Compression Test Results

Figure E.13 Compression Test Results

Figure E.14 Compression Test Results

Figure E.15 Compression Test Results

Figure E.16 Compression Test Results

Figure E.17 Compression Test Results

Figure E.18 Compression Test Results

Figure E.19 Compression Test Results

PART 9

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PART 10

APPENDIX

The scope of my work on this research effort encompassed much more than the primary subject of this project report. The author's responsibilities ranged throughout every aspect of this research project, and it is the purpose of this appendix to briefly review another area of cable research work.

A. The Flexure Test

Our flexure test originated as a segment of cable, simply supported, and loaded at the center with a single point load. The supports used were flat knife-edged supports. Although there were expectations of local deformation at these points, there was no surface damage observed after tests were completed and so this was assumed to produce very little error.

The results of the tests were plots of load versus deflection at the load point. Results from these tests proved to be fairly consistent, and plots of these early tests are shown in Figures (A1) to (A3). Despite the consistency in test results, this data offered little insight to the moment-curvature properties of the cable. Then, the following idea was developed:

Consider the moment diagram of a simply supported beam acted upon by a concentrated load at midspan shown in Fig. (A4). Because there is no region of constant moment there is also no region of constant curvature. Realizing this type of loading does not facilitate the establishment of a moment curvature relationship, the loading scheme of Fig. (A5) was considered.

FLEXURE TEST #2

29 1/2" Span

Figure A1 Early Flexure Test Results

FLEXURE TEST #3

26" Span

Figure A2 Early Flexure Test Results

Figure A3 Early Flexure Test Results

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

Figure (A4)

Beam Diagram - Single Concentrated Load

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Figure (A5)

Beam Diagram - Two Concentrated Loads

Employing this type of loading would enable us to directly measure curvature values for any given moment because of the region of constant moment between the load points.

We chose quarter-point loading for our tests so that there would be a large enough region of constant curvature to facilitate its measurement. The tests were performed by supporting the cable above the platform of our compression machine. The load was applied using the compression machine

and a dual flat knife-edged loading system. The entire flexure test system is illustrated in Fig. (A6).

The apparatus hanging below the cable in the figure is a device for measuring the deflection. When the computations for curvature are performed, the only deflection of interest is the relative displacement between the load points and the midspan point of the cable. This displacement is what is necessary to employ the derived equations of curvature which follow. Therefore, the system hanging below the cable was designed so that this deflection, and only this deflection, would be measured during the test.

This mechanism is very simple in principle. The entire apparatus is supported by the cable at the load points so that as the cable deflects, so does the measuring device. The system is sufficiently light in weight so that it has no effect on the deflection of the cable. A dial gauge is mounted on the side of the cable at midspan, the side mounting being used so that the deflection measured is as close as possible to the centerline of the cable. The other end of the dial gauge is in contact with the frame which is suspended from the cable at the quarter-points. Therefore, the dial gauge measures the relative displacement between the midspan point and the quarter-points or load points.

B. Analysis

Given the load and the relative displacement between the midspan and the quarter-points, values for the moment and curvature may be calculated. The analysis employed is fundamental in nature. Since the interest lies within the region between the two point loads, only the moment in this region (the maximum moment) need be found. This may be accomplished using

Two-point (quarter point) loading

Figure A6 The Flexure Test System

the formula

$$M = P a$$

where P and a are as shown in Figure (A5). The data given to the computer program is the list of P values, which is then converted to moment values.

An equation yielding curvature values is derived using the classical approach. Consider the relative deflection of the critical region to be equal to the variable D , and having length L . Let the curvature be denoted as

$$y'' = a$$

Therefore

$$y' = ax + b$$

$$y = \frac{ax^2}{2} + bx + c \quad (A.1)$$

Using Eq. (A.1) as the equation for the curve of the cable under load, boundary conditions may be applied to solve for the three unknown constants. The origin is taken as the initial position of the midspan point. The analysis then proceeds as follows:

B.C. # 1 at $x = 0$, $y = -D$

which implies $c = -D$

B.C. # 2 at $x = 0$, $y' = 0$

which implies $b = 0$

B.C. # 3 at $x = L$, $y = 0$

which implies $D = \frac{a(L/2)^2}{2}$

or

$$a = \frac{2D}{(L/2)^2} = y''$$

Therefore, the equation for the curvature is

$$y'' = \frac{8D}{L^2} \quad (A.2)$$

where D - relative deflection

L - distance between load points

Equation (A.2) is written into the code of the graphics program so that given the values of D and L , the curvature may be computed at any data point. The result is a plot of moment versus curvature. The cable was loaded to a certain point and then unloaded, this pattern being repeated with the maximum load (moment) being sequentially increased.

The results show the relationship between moment and curvature for the cable to be quite linear. The establishment of this fact should provide some simplifications when serious mathematical bending analysis is undertaken. The test results of this modified flexure test are given in Figures (A7) through (A12).

A simple comparison may be made here between the behavior of the cable and that of twisted wire rope. Costello and Sinha (1) provide the following expression for the flexural rigidity of a helical spring in the case of pure bending.

$$B = \frac{\sin^2 \alpha}{\frac{1 + \sin^2 \alpha}{\frac{E \pi R^4}{2}} + \frac{\cos^2 \alpha}{\frac{E \pi R^4}{2(1+\nu)}}} \quad (A.3)$$

where

α - helix angle

E - Young's modulus

ν - Poisson's ratio

R - radius of a single wire

Furthermore, the same source provides us with the bending stiffness for the cross-section displayed earlier in Fig. (9) as

$$A = \frac{m \sin \alpha}{\frac{1 + \sin^2 \alpha}{\frac{E \pi R^4}{2}} + \frac{\cos^2 \alpha}{\frac{E \pi R^4}{2(1 + \nu)}}} \quad (\text{A.4})$$

where

m - the number of individual wires

For the 550 kV experimental cable previously discussed, we may again take the wire diameter to be 0.11", or

$$R = 0.055"$$

The helix angle is again,

$$\alpha = 75^\circ$$

and the material constants remain,

$$E = 17 \times 10^6 \text{ psi}$$

$$\nu = 0.33$$

The cross-sectional area of the conductor is 1.92 in^2 , so that the number of individual wires is computed as

$$m = \frac{1.92 \text{ in}^2}{\pi(0.055")^2} = 202$$

Therefore, Eq. (A.4) becomes

$$A = \frac{202 \sin 75^\circ}{\frac{1 + \sin^2 75^\circ}{\frac{(17 \times 10^6)(\pi)(0.055)^4}{2}} + \frac{\cos^2 75^\circ}{\frac{(17 \times 10^6)(\pi)(0.055)^4}{2(1 + 0.33)}}}$$

$$A = 23573 \# - \text{in}^2$$

This is the calculated stiffness of the wire conductor for flexure. The stiffness value may also be obtained from the slope of our moment-curvature graphs derived from the test results.

There are essentially two slopes to be considered on these graphs. The initial loading slope and the re-loading slope. However, the slopes of the initial load paths contain the effects of the paper insulation and most likely represent yielding of the copper strands. The above calculation may be compared to the re-loading slope with some validity.

From Figure (A7) we may obtain the re-loading slope as

$$A^* = \frac{(120 \text{ ft-lb})(12 \frac{\text{in.}}{\text{ft.}})}{(0.12 \frac{1}{\text{in.}})}$$

$$A^* = 12000 \# - \text{in}^2$$

This shows a stiffness essentially one-half that calculated for a helical wire rope of 202 wires. This result is not at all surprising since the conductor is not actually in a helical rope configuration, but is composed of four separate quadrants, which are individually and jointly formed into

a helical shape.

It should be noted that these calculations do not take into account the effect of the paper insulation on the copper. The combined stiffnesses and flexural properties of the cable system shall be determined in future analysis.

In addition, a value of 17×10^6 psi was used in Eq. (A.4) for Young's modulus of the conductor. This is a generally accepted value for copper and has been used previously. However, we have found the properties of the copper conductors to vary with different types of cable, which may decrease the accuracy of the above calculations. In any case, the comparison is still noteworthy, requiring more extensive research and analysis.

Figure A7 Recent Flexure Test Results

Figure A8 Recent Flexure Test Results

Figure A9 Recent Flexure Test Results

Figure A10 Recent Flexure Test Results

Figure A11 Recent Flexure Test Results

Figure A12 Recent Flexure Test Results

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