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**SAMPLE PROBLEM CALCULATIONS RELATED TO
TWO-PHASE FLOW TRANSIENTS IN A PWR
RELIEF-PIPING NETWORK**

by

Yong W. Shin and Arne H. Wiedermann

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ABSTRACT

Two sample problems related with the fast transients of water/steam flow in the relief line of a PWR pressurizer were calculated with a network-flow analysis computer code STAC (System Transient-Flow Analysis Code). The sample problems were supplied by EPRI and are designed to test computer codes or computational methods to determine whether they have the basic capability to handle the important flow features present in a typical relief line of a PWR pressurizer. It was found necessary to implement into the STAC code a number of additional boundary conditions in order to calculate the sample problems. This includes the dynamics of the fluid interface that is treated as a moving boundary. This report describes the methodologies adopted for handling the newly implemented boundary conditions and the computational results of the two sample problems. In order to demonstrate the accuracies achieved in the STAC code results, analytical solutions are also obtained and used as a basis for comparison.

I. INTRODUCTION

ANL participated in an EPRI program in which a number of computer codes were used to calculate the sample problems that are designed to test the various codes in their ability to predict the two-phase flow transients in the relief piping network of a PWR. The computer code used for ANL participation is the STAC (System Transient-Flow Analysis Code) code that has been developed at ANL under a code development program for analysis of large-scale sodium-water reactions. The STAC code forms a module of the sodium-water reaction analysis code SWAAM-I (Sodium-Water Advanced Analysis Method - Version I) [1]. The STAC code is a homogeneous two-phase flow-transients analysis code for piping networks and is based on the finite-difference technique using the artificial viscosity concept for solution of interior flow field, but on the method of characteristics to treat the boundary conditions and junction conditions. During the STAC code development, it was recognized that the phase change, or more specifically the drastic change in the impedance (ρc) frequently existing at the boundary, required a special numerical treatment in order to successfully carry out the computation. Hence, a new technique is introduced in the STAC code to overcome the difficulty. The technique is a semi-analytical procedure based on the method of characteristics using a subsize time-step [2]. It has been proven that very accurate results can be obtained with the boundary technique.

In the beginning of this work of calculating the EPRI sample problems, a number of code modifications were necessary in order to carry out the work. A number of additional boundary condition options had to be implemented in the code, and that included the material interface boundary treated as a moving boundary. While the numerical technique implemented in the STAC code is accurate and stable, it requires an extensive amount of coding work. As a result of this inherent nature of the STAC code, the coding work involved in this present study was rather major. Nevertheless, the required coding work was completed and the sample problems calculated. In addition to the code calculations, analytical solutions were also obtained for reduced problems where certain second-order effects were neglected. The analytical solutions were used as a basis of comparison to demonstrate the accuracy of the numerical results for the sample problems.

II. NEW BOUNDARY CONDITIONS IMPLEMENTED IN STAC CODE

A. Specified Mass-Flow-Rate Boundary

It is often desirable to consider a boundary condition in which the mass-flow rate is specified, as in this present study. This boundary condition is especially important in analysis of test results where the mass flow rate is measured directly or indirectly and would like to maintain that condition at some point in the system.

Following the basic boundary-condition methodology [2], the entropy at the boundary, at the advanced time $(t + \Delta t)$, s_p , is first estimated by the upstream condition following the material characteristic equation (Eq. 10 of Ref. 2). If the upstream is a reservoir with a large inventory of mass, the entropy is set to the entropy for the reservoir. Once the entropy is determined, the remaining task is to compute the remaining two variables of the three-variable system on hand, namely, the pressure (p) and particle velocity (u). These are computed in the $p - u$ plane by a substep procedure as described schematically in Fig. 2. Figure 1 shows the physical configuration and the wave diagram for the subject boundary condition. Figure 2 shows the mass line and sonic line for a constant entropy $s = s_p$. The compatibility equations for the $u - c$ characteristics are also drawn in Fig. 2 for the neighboring states S . When the neighboring state S is at a high pressure, the solution state for the boundary is subsonic; i.e. P , in Fig. 2. But for a low enough pressure for state S , the solution state can be supersonic, namely, state P' . The supersonic state at the pipe inlet by a specified mass flow rate is only possible if the inlet section of the pipe is well contoured to permit such flow acceleration. Normally, this condition does not exist in the piping system, and hence it is assumed that the pipe area changes abruptly. The flow at the inlet then should be limited by choking. The solution state therefore is the state P_c for all sufficiently low pressures of the neighboring state S . Note that the compatibility equations are drawn schematically as straight lines in Fig. 2, but in the linear scale of p and u , they can be severely curved due to the large change in the impedance (ρc) between one time step (Δt). This is the reason the compatibility equations have to be numerically integrated by substeps ($\Delta t/N$) in order to locate the accurate solution state. This is not necessary when the impedance does not change much during the step calculation. The usual water-hammer transient is an example of a nearly constant impedance [3-5].

B. Material Interface as Moving Boundary

Often the piping system involves more than one fluid, and it is desirable to follow the dynamics of the material interface without allowing mixing of the two fluids. For the subject sample problems, for example, the piping had nitrogen gas in the line, and steam or water is to be introduced at one end of the piping to initiate the transient. Often when the interface tracking capability does not exist in a code, an attempt is made to approximate the two-fluid system by a single fluid at certain adjusted initial conditions. But this practice is not always very successful, as can be seen from the discussion later in this report. Also shown in this study is that, even for a single fluid system when the large change in fluid property is simulated by an interface, namely the water seal/steam situation in the PWR relief piping, very accurate results can be obtained.

The computational procedure developed for the material interface is based on the method of characteristics, as in all other boundary conditions. It involves flow dynamics in two material regions and their interaction. The interface condition is that the pressure and particle velocity must be continuous. The case of interface in an interior computing cell is discussed first, as shown in Fig. 3. The double dotted lines represent both sides of the locus of the interface during a unit time step Δt . The left side of interface (T-PT) is occupied by material "L," and the right side is occupied by material "R." A complete solution of interface for one cycle involves not only the determination of interface points, PT and $\bar{P}\bar{T}$, but the variables at the adjacent computing nodes PL and PR. The wave diagrams for the three calculational points are shown in Fig. 4. The interface first is translated by its local velocity u_T (or $u_{\bar{T}}$) to locate point PT (or $\bar{P}\bar{T}$) at the advanced time $t + \Delta t$. Next the entropies are calculated for both sides of the interface by advancing the material characteristics and averaging as follows:

$$s^R = \frac{1}{2} (s_{PT} + s_R) \quad (1)$$

and

$$s^L = \frac{1}{2} (s_{\bar{P}\bar{T}} + s_S), \quad (2)$$

where s^R and s^L are the average entropies for the R material and L material, respectively. Entropies at R and S are obtained by linear interpolations using adjacent node values at time t . The solution state P for the interface is the intersection of the two characteristics (R and S characteristics or

the compatibility equations) for the two fluid regions at the respective entropies, $s = s^R$ for the $u + c$ and $s = s^L$ for the $u - c$ as described in Fig. 5. Again the characteristics are represented schematically by straight lines, but can be severely curved. The same procedure applies to the adjacent nodes, node PL and PR, except that the two characteristics are for the same entropy, s^L or s^R , since both characteristics lie in the same fluid region. In fact, the representative entropies are averaged as follows:

$$s = \frac{1}{3} (s_R + s_{PL} + s_S) \quad \text{for node PL} \quad (3)$$

and

$$s = \frac{1}{3} (s_R + s_{PR} + s_S) \quad \text{for node PR,} \quad (4)$$

respectively. Note that the neighboring states R and S can lie on the locus of interface, such as the point R in Fig. 4c.

The case when the interface is located in the boundary cell is treated essentially the same way. Figure 6 shows the wave diagrams for the interface in the inlet and outlet cells. Here, too, the interface is first advanced and then the adjacent node. Since the interface is done before the boundary node, when the characteristic hits the boundary no interpolation can be made along the time line ($x = \text{const}$). In this case an approximation is made as shown in Fig. 6a, where the neighboring state R is assumed to collapse on point A. Besides this exception, the procedure is identical to the case of the interface in the interior cell. The adjacent nodes PR and PL are computed exactly the same way as discussed above for the interior cell. The solution methodologies for the boundary nodes, however, are slightly modified. The neighboring states, state R or S, are now obtained by interpolation along the interface path or the initial time line, but limited to the appropriate side of the interface.

C. New Set of Interpolation Formulas

The characteristics applied to a computing point of a finite-difference system requires evaluation of neighboring states that fall between computing nodes. As shown in Fig. 7, computation of node P requires determination of the neighboring states R, T, and S. A linear distribution of variables is assumed between the nodes. Moreover, the slope of characteristics is approximated by that of the initial plane, namely, at R, T, or S. Under these assumptions, the following can be written for point R, for example:

$$(c + u)_R = f \cdot (c + u)_C + (1 - f) \cdot (c + u)_A, \quad (5)$$

$$(c + u)_R \Delta t = f \cdot \Delta x, \quad (6)$$

and

$$f = \overline{RA}/\Delta x. \quad (7)$$

Solving for f by eliminating $(u + c)_R$ results in the desired interpolation formula for location of point R:

$$f = \frac{(c + u)_A}{\frac{\Delta x}{\Delta t} + (c + u)_A - (c + u)_C}. \quad (8)$$

Similarly the following formulas may be obtained for points T and S, respectively:

$$f = \frac{(u)_A}{\frac{\Delta x}{\Delta t} + (u)_A - (u)_C} \quad (9)$$

and

$$f = \frac{(c - u)_A}{\frac{\Delta x}{\Delta t} + (c - u)_A - (c - u)_B}. \quad (10)$$

These formulas apply to the neighboring states falling between the specified computing nodes at the initial plane and are used to evaluate the states at those points. For the interface procedures discussed above, however, the neighboring-state points do not always fall between computing nodes and special interpolation formulas must be developed.

1. Interpolation Formulas for Characteristics at Interface Location

The $u \pm c$ characteristics at the interface location are drawn in Fig. 8. The $u + c$ characteristic lies on the left of the particle path; the $u - c$ characteristic lies on the right. As discussed above, the particle path is advanced first, and hence DIXP of Fig. 8 is known, and the interpolation formulas are applied to locate the neighboring states R and S. There are two reasons why special formulas must be developed that differ from the standard formulas applicable to regular nodes (Eqs. 8-10). The first reason is that the interface lies between the nodes so that the distances DIXP and DIX enter in the formula. The other reason is that the interpolation to be made can be between the node point and the interface location, instead of between nodes only as in the standard case. Figure 8 shows the two possible cases for

each of the two characteristics $u \pm c$. For the case 1 configuration of Fig. 8, for example, the following two relations can be written:

$$(c + u)_R = f \cdot (c + u)_T + (1 - f) \cdot (c + u)_A \quad (11)$$

and

$$(c + u)_R \cdot \Delta t = -f \cdot DIX + DIXP, \quad (12)$$

where

$$f = \bar{AR}/DIX,$$

and from these relations the following interpolation can be obtained:

$$f = \frac{(c + u)_A - \frac{DIXP}{\Delta t}}{(c + u)_A - (c + u)_T - \frac{DIX}{\Delta t}}. \quad (13)$$

Similar formulas may be obtained, and the results are given in Table 1 for the four configurations shown in Fig. 8.

2. Other Special Interpolation Formulas

Calculation of the interface by the method of characteristics, namely, points PT and $\bar{P}\bar{T}$ of Fig. 3, requires similar calculations for the two adjacent nodes PL and PR. Again for these calculations, the characteristics may fall between points that are not always the grid points that are fixed. For computation of PL, for example, the $u - c$ characteristic may fall between points A and T, and a special formula is needed. Likewise $u + c$ characteristic for PR may fall between $\bar{T}$ and $\bar{A}$. In addition, the characteristics can fall on the interface line, namely, the line connecting T and PT or the line connecting $\bar{T}$ and $\bar{P}\bar{T}$.

Nine different configurations are identified for PL-node calculations for the three characteristics, $u + c$, $u - c$, and u , as shown in Fig. 9. The procedure for determining the interpolation formulas is similar to that used for the interface and hence is not repeated here, but the formulas are listed in Table 2. There are two possible configurations, however, as shown in Figs. 9e and 9i where the $u \pm c$ characteristics intersect the interface and, hence, require interpolations not only along distance (x), but also along time (t). The interpolation formula for configuration case 5 is derived below. The $(u + c)$ characteristic crosses the interface and, hence, extends to the other side of the material region. In determining the state $\bar{R}$, a continuity is assumed of the right-hand-side-material states, even into the left of the

interface, so that an extrapolation is made for state $\tilde{R}$. Then the following two relations can be written:

$$(c + u)_{\tilde{R}} = (c + u)_{\bar{A}} + f \cdot [(c + u)_{\bar{T}} - (c + u)_{\bar{A}}] \quad (14)$$

and

$$(c + u)_{\tilde{R}} \cdot \Delta t = f \cdot DIX - \Delta x \quad (15)$$

From these equations the following formula can be written:

$$f = \frac{(c + u)_{\bar{A}} + \frac{DIX}{\Delta t}}{(c + u)_{\bar{A}} - (c + u)_{\bar{T}} + \frac{DIX}{\Delta t}} \quad (16)$$

Since the neighboring state in this configuration is the state R located on the interface, linear interpolations are applied along the interface and the $c + u$ characteristic. Since both lines are now defined, the intersection of the two lines are solved, the interpolation formula defining g (see Fig. 9e) can be obtained:

$$g = \frac{(f - 1) \cdot DIX}{(f - 1) \cdot DIX + DIXP - \Delta x} \quad (17)$$

Similar formulas are obtained for Case i of Fig. 9 and are included in Table 2.

Another nine configurations are shown in Fig. 10 for node PR, and the corresponding formulas are derived and shown in Table 3. In essence the procedure used in deriving the formulas for node PR is the same as that used for node PL.

III. DESCRIPTION OF SAMPLE PROBLEMS

The sample problems considered here for STAC code calculation are the blowdown transient from a large pressure vessel and through a connected piping. The other end of the piping is the low-pressure end exposed to the ambient at 14.7 psia pressure. The piping is filled initially with nitrogen gas at room temperature and ambient pressure. Instead of specifying the valve area, the mass flow rate $\dot{m}$ is given. The desired results of the calculation are the pressure-time histories at points B and D of Fig. 11, and the forces acting on the pipe segment AC (including the elbow).

A. Sample Problem No. 1

Sample problem no. 1 is described in Fig. 11. It consists of the blow-down of dry saturated steam at 2400 psia through the safety valve and the downstream piping, 9.564-in. ID and 25 ft long. The valve opening is replaced by a specified mass flow rate $\dot{m} = 560,000$ lbm/hr. The piping is filled with low-pressure nitrogen gas, and hence, the initial transient involves the shock waves generated at the valve (initial material interface location) and their propagation and interaction in the pipe. The initial and boundary conditions are defined, and the problem is ready for numerical solution by STAC code with the code extensions as discussed above. The code models the pipe in between the two boundaries, namely, the specified mass-flow-rate boundary and the out-flow (or break-end) boundary.

B. Sample Problem No. 2

Sample problem no. 2 is identical to the sample problem no. 1 in geometric configuration, but considers the flow of cold water instead of dry saturated steam as in the sample problem no. 1. The water condition is at 100°F (0.9 psia saturation pressure); hence, the water is expected to remain as water (i.e., no phase change). The water flow rate is 200 gpm. Aside from this difference, the two sample problems are identical.

IV. ANALYTICAL SOLUTIONS

Seeking analytical solutions for the sample problems was an important part of the task. The analytical solutions not only served as a basis for comparison with the code results, but also helped in learning the physical nature of the problems. The STAC code is based on a semianalytical approach and is capable of producing very accurate results. The accuracies of the STAC code results are demonstrated by direct comparisons of the results with analytical solutions as is discussed later in this report. The wave system involved in the two sample problems were simple, and it was possible to obtain analytical solutions when certain secondary effects were neglected. The effects of gravity and friction are neglected in the analytical solutions.

A. Sample Problem No. 1

Sudden opening of the valve develops a shock wave traveling down the pipe followed by the material interface moving at the particle velocity. Depending on whether the pipe inlet geometry is well-contoured or sharp-edged, the inlet state of flow can be locally either supersonic or choked. First, the case of supersonic inlet condition is considered.

1. Contoured-Inlet-Geometry Case

The solution is obtained by a simultaneous solution of the constant mass line ($\dot{m} = \text{const}$) and the nitrogen Hugoniot in the $p - u$ diagram at a constant entropy, as shown in Fig. 12. The intersection of the two lines is the state of the shocked region. The solution obtained is used in the wave diagram shown in Fig. 13. There are three regions of constant state: the undisturbed nitrogen region at state S_∞ (14.7 psia and 70°F), the shocked nitrogen region at state S_1 , and the expanded steam region S_2 . The interface separates the two materials across which the interface conditions $u_1 = u_2$ and $P_1 = p_2$ are maintained. The solution is summarized below:

$$\left. \begin{aligned} p_1 &= p_2 = 64.64 \text{ psia,} \\ u_1 &= u_2 = 1408.4 \text{ ft/s} \\ U &= 2770.5 \text{ ft/s} \\ c_1 &= 1483.3 \text{ ft/s} \\ T_1 &= 424.8^\circ\text{F} \\ c_2 &= 1215 \text{ ft/s} \\ T_2 &= 297.6^\circ\text{F} \end{aligned} \right\} \quad (18)$$

A narrow fan rarefaction system develops at the end of the pipe, since the particle velocity in region S_1 is slightly less than the sonic speed when the shock wave reflects from the outlet end. In the steam region, S_2 , however, the state is supersonic, and hence, the rarefaction system blows out of the pipe. The rarefaction system is shown in Fig. 13, and the flow remains choked at the outflow end during the rarefaction.

a. Similarity Solution for Rarefaction Fan

Consider first an expansion fan at $x = 0$ propagating into a quiescent state of a gas region. The applicable characteristics are the $c - u$ characteristics and the compatibility condition for the $c + u$ characteristic as shown in Fig. 14. For an assumed ideal gas, the applicable equations can be written as

$$\frac{x}{t} = c - u \quad (19)$$

and

$$c + \frac{k-1}{2} u = c_o, \quad (20)$$

or in a dimensionless form

$$\zeta = \chi - \phi \quad (21)$$

and

$$\chi + \frac{k-1}{2} \phi = 1, \quad (22)$$

where

$$\zeta = \frac{x}{c_o t}, \quad (23)$$

$$\chi = \frac{c}{c_o}, \quad (24)$$

$$\phi = \frac{u}{c_o} \quad (25)$$

Eliminating ϕ and χ from Eqs. 21 and 22 results in the desired solutions for χ (or ϕ after substitution) in terms of the similarity variable ζ as follows:

$$\chi = \frac{k-1}{k+1} \left(\frac{2}{k-1} + \zeta \right) \quad (26)$$

and

$$\phi = \frac{2}{k+1} (1 - \zeta). \quad (27)$$

The pressure then can be obtained from the relationship

$$\frac{p}{p_0} = \left(\frac{c}{c_0} \right)^{\frac{2k}{k-1}}. \quad (28)$$

Equations 26-28 represent the complete solution for the entire field.

These solutions apply when the free field is in a quiescent state. The initial field is a shocked state and a slight modification has to be made. The characteristic and compatibility equations are

$$\chi + \frac{k-1}{2} \phi = 1 + \frac{k-1}{2} \phi_s \quad (29)$$

and

$$\zeta = \chi - \phi \quad (30)$$

And the solutions are:

$$\chi = \frac{k-1}{k+1} \left(\frac{2}{k-1} + \phi_s + \zeta \right) \quad (31)$$

and

$$\phi = \frac{2}{k+1} \left(1 + \frac{k-1}{2} \phi_s - \zeta \right) \quad (32)$$

The characteristics and the compatibility condition for the shocked initial state are shown schematically in Fig. 15. This solution is used to obtain the details of the rarefaction system shown in Fig. 13.

b. Comparison of Analytical Solutions with Code Calculation

Selected plots of time histories, spatial distributions and loci are generated from the analytical solutions as given above in Eqs. 18, 28, 31, and 32 to compare against code results obtained for the same boundary and initial conditions. The comparisons are given in Figs. 16-24. Figure 16 shows the loci of the shock wave and the interface, and the agreement shown between the analytical solution and the code calculation is nearly perfect. Figures 17-21 show profiles or spatial distributions at $t = 3.85, 9.66,$ and 25 ms to show the discontinuities at the shock wave and interface. Agreement is again excellent, except near the discontinuities, where slight overshoots are shown in the code results. These overshoots are typical of finite-difference

techniques using artificial viscosity and results from the interior field procedure that is based on the Lax-Wendroff scheme (see Ref. 2). The interface discontinuities are well represented in the code results, and this indicates the general capability of the semianalytical boundary technique implemented in STAC code. The procedure used in the interface calculation is based on the same general boundary procedure, and the accuracy shown in these figures near the interface is to be expected. Pressure-history plots are given in Figs. 22 and 23. The history plot for point D (outlet end) shown in Fig. 22b shows the effect of the rarefaction system. Again the slight numerical overshoots are evident. Finally, the fluid forces acting on the segment A-C including the elbow are shown in Fig. 24. The longitudinal force F_x is identical to the lateral force F_y in the absence of gravity and friction as in the case here.

It was of interest to see the effects of gravity and friction, although they are considered second-order effects, at least for the early time transients, when most important wave effects take place. An additional code calculation was therefore made for the same left-end boundary condition allowing the supersonic inflow. The results are presented in Figs. 25-31. These results are similar to the results discussed above where the second-order effects of gravity and friction are neglected. Therefore no additional discussion is necessary, except to note the effects of the gravity and friction. Comparison of the plots with the corresponding ones for the simplified problem (no gravity-no friction) indicates that the relaxation is longer with gravity and friction than without. This is shown by the wavy nature of the plots, as can be seen in Figs. 25 and 26. The secondary effects add long-wavelength oscillation to the solution.

2. Abrupt-Inlet-Geometry Case

Abrupt geometry is more common in piping systems than the contoured geometry. The abrupt-geometry assumption was therefore adopted for STAC code development. In the sample problems under investigation, the inlet of the pipe (downstream section of the valve) may be closely modeled by an abrupt-inlet geometry and the basic assumption is well justified. The difference between this inlet boundary condition and the contoured-inlet case is that the flow chokes at the inlet with the expansion taking place in the pipe for the case of the abrupt inlet while the inlet flow is supersonic in the contoured-inlet case. This difference is clearly seen by comparison of Figs. 13 and 33.

First the shocked state is to be determined. As shown in the centered rarefaction wave system developed at the pipe inlet as shown in Fig. 33, the sonic condition at the inlet expands further into a supersonic state. The choked inlet state S_1 is the intersection of the constant mass-flow line and sonic line as shown in Fig. 32. Further expansion from the inlet choked state follows C^+ characteristic in the solution plane (the compatibility condition for $u + c$ characteristic). The shocked state or the solution state is then the simultaneous solution of the C^+ characteristic for steam and the nitrogen Hugoniot. This means that the solution state is the intersection of the C^+ characteristic and nitrogen Hugoniot in the $p - u$ diagram Fig. 32. The pressure and velocity are continuous across the material interface. The solution is as follows:

$$\left. \begin{aligned} p_1 &= p_2 = 63.97 \text{ psia} \\ u_1 &= u_2 = 1396.5 \text{ ft/s} \\ U &= 2259 \text{ ft/s} \\ c_1 &= 1217.7 \text{ ft/s} \\ T_1 &= 296.9^\circ\text{F} \\ c_2 &= 1479.8 \text{ ft/s} \\ T_2 &= 420.6^\circ\text{F} \end{aligned} \right\} \quad (32)$$

The solution is plotted in terms of the similarity variable x/t as shown in Figs. 34-36. The inlet expansion fan region is solved by computer. The expansion waves expressed by

$$\frac{x}{t} = u - c \quad (33)$$

correspond to points on the C^+ characteristic of Fig. 32. Therefore a complete solution is obtained for the rarefaction wave system at the inlet. The small rarefaction system at the outflow end is also solved. The procedure used here is similar to the one discussed in Sec. 1 above. The complete analytical solution is given later in the report where the code result is discussed and compared with the analytical result.

B. Sample Problem No. 2

Because of the low flow rate, the disturbance level is very low. For this low-disturbance system, the water-hammer waves in the water region may be

neglected, and the interface may simply be treated as a constant-velocity boundary like a piston. For this simple system, the wave system is simply a reverberation of waves between the piston at one end and the constant pressure boundary at the other. The solution obtained is shown in the wave diagram of Fig. 37 and the $p - u$ diagram of Fig. 38. It is essentially an acoustic system with pressure disturbance $\Delta p = 0.163$ psi and corresponding velocity disturbance $\Delta u = 9.14$ fps.

V. CODE CALCULATIONS AND COMPARISON WITH ANALYTICAL SOLUTIONS

As indicated above, the effects of friction and gravity are small. This is especially true in sample problem no. 2; where the velocity involved is small and the water moves into the tube only a short distance, and hence, never occupies the vertical pipe section. The gravity effect therefore is also negligible. Because of these two effects, which are shown to be small, it was decided to neglect these effects in the code calculations in order to make critical comparisons of the code results with the analytical solutions.

A. Sample Problem No. 1

The code-calculation results and their comparison with the analytical results are presented in Figs. 39-44. The inlet condition is assumed to be an abrupt-inlet geometry and the flow chokes at the inlet. The loci of shock wave and the material interface are shown in Fig. 39. Spatial profiles at selected times are given in Figs. 40 and 41; time histories at points B ($x = 2.5$ ft) and D ($x = 25$ ft) are given in Figs. 42 and 43. Finally, the force-calculation result is shown in Fig. 42. The agreement shown in the comparison of code results and analytical solutions in all the figures is nearly perfect, except near the shock wave front where some wave smearing and overshoots are evident in the code results. This phenomenon is purely numerical and typical of results obtained by finite-difference techniques employing artificial viscosity. The STAC code uses a finite-difference method for field-equations solution and the overshoots stem from this part of the STAC code. Note that the discontinuities at the interface in such properties as temperature and sonic speed are represented nearly exactly in the code results (see Fig. 41). This again demonstrates the ability of the STAC-code boundary methodology to produce accurate results. The interface is a moving boundary, and its calculational procedure is based on the basic semianalytical method implemented in the STAC code for all other boundary conditions.

B. Sample Problem No. 2

The results for the sample problem no. 2 are presented in Figs. 45-49. This problem has features to test the code capability for two special aspects, namely, the extremely small disturbance and the large gradient in fluid impedance at the water/nitrogen interface. The disturbance fronts at two selected times, $t = 10.4$ and 30.4 ms, are presented in Figs. 45 and 46, where spatial distributions of pressure and particle velocity are plotted and compared

between the code calculation and the analytical solution. The agreement is generally good in that the amplitude and time of the disturbance are well represented in the code result. The overshoots and undershoots behind the disturbance fronts are evident, and as discussed earlier, these are numerical phenomena associated with the finite-difference numerical technique employed in STAC for field solution. Another observation to be made is the smearing of the disturbance front, which is more pronounced than the steam-inflow case of the sample problem. The increased smearing shown here is due to the interpolation error associated with the finite-difference technique employing fixed spatial nodes and time steps. Since the Courant condition dictates the time step a time step much smaller than the steam/nitrogen system (Sample Problem 1) was used for the water/nitrogen system (Sample Problem 2). This introduces numerical errors known as interpolation errors. These errors are the basic limitation of the large-scale computer codes in which the fixed noding and constant time step are employed. However, the code results as shown in the figures should be good for practical purposes. Time histories are shown in Figs. 47 and 48, and the force-calculation result is shown in Fig. 49. Again, the agreement is generally good, other than the two numerical aspects noted above. In the force-calculation result, F_x is the longitudinal fluid force acting on the segment A-C including the elbow for which F_y is the transverse force. The slight difference shown in Fig. 49 between F_x and F_y is due to the friction force acting along the pipe. Other than the friction force, the forces in the two directions are identical. All the calculations made for the two sample problems used 41 nodes.

VI. COMPUTATIONAL DIFFICULTY AT MATERIAL INTERFACE WITH LARGE IMPEDANCE GRADIENT

The numerical schemes employing fixed computing points are common in large computer codes and the STAC code is not an exception in this respect. It uses the constant spatial noding system ($\Delta x = \text{constant}$), and the method of characteristics is applied for the boundary or junction cells including the interface as a moving boundary. As described earlier in the discussion of the interface-boundary-treatment methodology, the neighboring states of characteristics are evaluated by linear interpolations in the initial plane (e.g., state S by interpolation between $\bar{T}$ and $\bar{A}$ of Fig. 50). This interpolation would be safe as long as the state distribution in the initial plane is linear. However, the distribution can deviate from the linear assumption and introduces interpolation errors. Normally, this error is not significant and remains bounded, as long as the fluid impedance does not change too rapidly. In the interface boundary case such as sample problem no. 2, however, the impedance of the low-temperature water is many orders of magnitude greater than that of nitrogen gas and there exists a large discontinuity in impedance at the interface. The small interpolation errors in this case were found to grow without bounds, and it was necessary to introduce a special technique to remedy the difficulty. In the following, the details of the numerical difficulty are discussed and the technique to overcome the difficulty is explained.

Consider an interface situation where the left-hand material is very high in impedance in comparison to the right-hand material as shown in Fig. 50, i.e.,

$$\rho_L c_L \gg \rho_R c_R. \quad (34)$$

When the state between $\bar{T}$ and $\bar{A}$ is uniform, as in the sample problem for $0 \leq t \leq 43$ ms (see Fig. 37), the interpolation for calculation of state S is exact, and hence, there is no problem. But when the reflected wave comes back to the interface, $t \approx 43$ ms, namely, the disturbance front lies between $\bar{T}$ and $\bar{A}$ (see Fig. 50), the linear interpolation introduces some error in both the pressure and velocity calculations for the interface. This small error in velocity, which originated from the soft-material region (material "R"), induced large errors in pressure in the stiff-material region (material "L"), specifically, in the PL node calculation. The error amplification is as follows:

$$\delta p_L \sim \frac{\rho_L c_L}{\rho_R c_R} \delta p_R \quad (35)$$

This large error in pressure for the PL node calculation is fed back to the interface calculation and in return creates large errors in velocity for PR node. The error therefore can grow without bound.

The growth in the error described above is purely numerical and stems from the inherent numerical procedures. The remedy used in correcting the problem is one that suppresses the nonphysical response of pressure in the stiff material. The interaction of the two materials at the interface is maintained through the velocity while limiting the pressure interaction. The constancy requirement for pressure across the inlet face is extended to include nodes PR and PL. Specifically, the technique employed in the STAC code is equating pressure at PL with that of the PR node. The result of this corrective technique for sample problem no. 2 is shown in the STAC code result presented in Figs. 45-49. It is clear that the corrective technique removed the problem while maintaining all physical phenomena in the results.

VII. SUMMARY AND CONCLUSIONS

A number of code modifications have been implemented to result in an extension of STAC code capabilities. The modifications include the boundary conditions peculiar to the two sample problems solved and the material interface boundary as a moving boundary. With the extended STAC code, the two sample problems were successfully solved and the accuracies in the code results are shown excellent. To demonstrate the accuracies in the code results, analytical solutions were also obtained with certain second-order effects neglected. The comparison shown between the code and analytical results indicated that the STAC results are nearly exact, except for the numerical smearing and over- or under-shoots near sharp discontinuities. The close comparison presented in this report once again demonstrates that the hybrid technique which forms the basis of the STAC code is the most accurate numerical method known today for the fast transient, two-phase-flow phenomena in a piping system.

In addition to the extension of code capabilities described above, a special technique is developed to overcome the numerical problems arising from the interface boundary scheme where a large gradient exists in fluid impedance at the interface. The newly introduced, corrective scheme is shown to suppress the error which otherwise can grow without bounds, and, more importantly, the scheme maintains all important physical features in the results. Large gradients in fluid impedance exist in the water seal of the PWR relief line, and the new technique introduced in the STAC code during this study should form the basis for handling the blowdown transients of the PWR relief piping with a cold-water seal.

Early in this study, before the code modification for material interface was completed, it was of interest to examine what the error would be if the low-pressure nitrogen gas in Sample Problem No. 1 were simulated by a superheated steam. The properties of superheated steam at three different temperatures (still at the ambient pressure, 14.7 psia) were compared with the nitrogen gas. As shown in Table 4, in all the parameters that influence the wave characteristics of the fluid, the superheated steam at 220°F simulates the nitrogen gas the best and is chosen for further examination. The wave system for the simulated system for sample problem no. 1 was solved analytically and the results are compared to the analytical solution obtained earlier for the true system. The comparisons given in Figs. 51-54 indicate that the

response of the simulated system is quite different quantitatively from that of true system, although qualitatively the responses are similar. It was therefore decided that the interface treatment be implemented in the STAC code.

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TABLE 1. INTERPOLATION FORMULAS FOR THE CHARACTERISTICS CONFIGURATIONS OF
OF MATERIAL INTERFACE
(Refer to Fig. 8 for specific configurations)

<u>Configuration</u>	<u>Interpolation Formula</u>
Case 1	$f = \frac{(c + u)_A - \frac{DIXP}{\Delta t}}{(c + u)_A - (c + u)_T - \frac{DIX}{\Delta t}}$
Case 2	$f = \frac{(c + u)_A - \frac{DIXP}{\Delta t}}{\frac{\Delta x}{\Delta t} + (c + u)_A - (c + u)_B}$
Case 3	$f = \frac{(c - u)_A - \frac{DIXP}{\Delta t}}{(c - u)_A - (c - u)_T - \frac{DIX}{\Delta t}}$
Case 4	$f = \frac{(c - u)_A - \frac{DIXP}{\Delta t}}{\frac{\Delta x}{\Delta t} + (c - u)_A - (c - u)_B}$

TABLE 2. INTERPOLATION FORMULAS FOR THE CHARACTERISTICS CONFIGURATIONS OF
NODE PL
(Refer to Fig. 9 for specific configurations)

<u>Configuration</u>	<u>Interpolation Formula</u>
Case 1	$f = \frac{(u)_A}{-\frac{DIX}{\Delta t} + (u)_A - (u)_T}$
Case 2	$f = \frac{-\frac{\Delta x}{\Delta t} + (u)_{\bar{A}}}{-\frac{DIX}{\Delta t} + (u)_{\bar{A}} - (u)_{\bar{T}}}$
Case 3	$f = \frac{(c + u)_A}{\frac{\Delta x}{\Delta t} + (c + u)_A - (c + u)_B}$
Case 4	$f = \frac{(c + u)_A}{-\frac{DIX}{\Delta t} + (c + u)_A - (c + u)_T}$
Case 5	$f = \frac{\frac{\Delta x}{\Delta t} + (c + u)_{\bar{A}}}{\frac{DIX}{\Delta t} + (c + u)_{\bar{A}} - (c + u)_{\bar{T}}};$
	$g = \frac{(f - 1) \cdot DIX}{(f - 1) \cdot DIX + DIXP - \Delta x}$
Case 6	$f = \frac{(c - u)_A}{\frac{DIX}{\Delta t} + (c - u)_A - (c - u)_T}$
Case 7	$f = \frac{-\frac{\Delta x}{\Delta t} + (c - u)_{\bar{A}}}{-\frac{DIX}{\Delta t} + (c - u)_{\bar{A}} - (c - u)_{\bar{T}}}$

TABLE 2. INTERPOLATION FORMULAS FOR THE CHARACTERISTICS CONFIGURATIONS OF
NODE PL
(Refer to Fig. 9 for specific configurations)(Cont'd)

<u>Configuration</u>	<u>Interpolation Formula</u>
Case 8	$f = \frac{(c - u)_A}{-\frac{\Delta x}{\Delta t} + (c - u)_A - (c - u)_B}$
Case 9	$f = \frac{(c - u)_A}{\frac{DIX}{\Delta t} + (c - u)_A - (c - u)_T} ;$
	$g = \frac{(f - 1) \cdot DIX}{(f - 1) \cdot DIX + DIXP}$

TABLE 3. INTERPOLATION FORMULAS FOR THE CHARACTERISTICS CONFIGURATIONS OF
NODE PR
(Refer to Fig. 10 for specific configurations)

<u>Configuration</u>	<u>Interpolation Formula</u>
Case 1	$f = \frac{(u)_A}{\frac{DIX}{\Delta t} + (u)_A - (u)_T}$
Case 2	$f = \frac{\frac{\Delta x}{\Delta t} - (u)_A}{\frac{DIX}{\Delta t} - (u)_A + (u)_T}$
Case 3	$f = \frac{(c + u)_A}{\frac{DIX}{\Delta t} + (c + u)_A - (c + u)_T}$

TABLE 3. INTERPOLATION FORMULAS FOR THE CHARACTERISTICS CONFIGURATIONS OF NODE PR

(Refer to Fig. 10 for specific configurations)(Cont'd)

<u>Configuration</u>	<u>Interpolation Formula</u>
Case 4	$f = \frac{\frac{\Delta x}{\Delta t} - (c + u)_A}{\frac{DIX}{\Delta t} - (c + u)_A + (c + u)_T}$
Case 5	$f = \frac{(c + u)_A^-}{-\frac{\Delta x}{\Delta t} + (c + u)_A^- - (c + u)_B^-}$
Case 6	$f = \frac{(c + u)_A^-}{\frac{DIX}{\Delta t} + (c + u)_A^- - (c + u)_T^-};$ $g = \frac{(f - 1) \cdot DIX}{(f - 1) \cdot DIX + DIXP}$
Case 7	$f = \frac{(c - u)_A^-}{\frac{\Delta x}{\Delta t} + (c - u)_A^- - (c - u)_B^-}$
Case 8	$f = \frac{(c - u)_A^-}{-\frac{DIX}{\Delta t} + (c - u)_A^- - (c - u)_T^-}$
Case 9	$f = \frac{\frac{\Delta x}{\Delta t} + (c - u)_A}{\frac{DIX}{\Delta t} + (c - u)_A - (c - u)_T}$ $g = \frac{(f - 1) \cdot DIX}{(f - 1) \cdot DIX - DIXP}$

TABLE 4. COMPARISON OF PROPERTIES OF SIMULANT
SUPERHEATED STEAM AND NITROGEN GAS

<u>Fluids</u>	<u>Properties</u>			
	<u>ρ, lbm/ft³</u>	<u>s, ft/s</u>	<u>ρc, lbm/ft²-s</u>	<u>k</u>
N ₂	0.0725	1148	83.23	1.4
Superheated Steam (T = 220°F)	0.0368	1551	57.08	1.3
Superheated Steam (T = 250°F)	0.0350	1587	55.55	1.3
Superheated Steam (T = 400°F)	0.029	1753	50.85	1.3

Fig. 1. Specified-mass-flow Boundary

Fig. 2. Solution Methodology for Specified-mass-flow Boundary

Fig. 3. Material Interface in an Interior Computing Cell

Fig. 4. Wave Diagrams for the Three Computing Points for Interface in an Interior Cell

Fig. 5. Solution Methodology for the Interface

Fig. 7. Wave Diagram for a Typical Computing Node

(a) LEFT BOUNDARY

(b) RIGHT BOUNDARY

Fig. 6. Wave Diagrams for the Interface in the Boundary Cell

(a) CASE 1

(b) CASE 2

(c) CASE 3

(d) CASE 4

Fig. 8. Characteristics Configurations for the Material Interface

(a) CASE 1

(b) CASE 2

(c) CASE 3

(d) CASE 4

(e) CASE 5

(f) CASE 6

(g) CASE 7

(h) CASE 8

(i) CASE 9

Fig. 9. Characteristics Configurations for Adjacent Node PL

(a) CASE 1

(b) CASE 2

(c) CASE 3

(d) CASE 4

(e) CASE 5

(f) CASE 6

(g) CASE 7

(h) CASE 8

(i) CASE 9

Fig. 10. Characteristics Configurations for Adjacent Node PR

Fig. 11. Sample Problem No. 1

Fig. 12. Solution for the Shocked State (Well-contoured Inlet)

Fig. 13. Wave Diagram (Well-contoured Inlet Case)

Fig. 14. Applicable Characteristics and Compatibility Condition for Expansion Fan (QUIESCENT INITIAL STATE)

Fig. 15. Applicable Characteristics and Compatibility Condition for Expansion fan (SHOCKED INITIAL STATE)

Fig. 16. Shock Wave and Interface Locations

Fig. 17. Pressure and Velocity Profiles at $T = 3.85$ ms

Fig. 18. Temperature and Sonic Speed Profiles at $T = 3.85$ ms

Fig. 19. Pressure and Velocity Profiles at $T = 9.66$ ms

Fig. 20. Temperature and Sonic Speed Profiles at $T = 9.66$ ms

Fig. 21. Pressure and Velocity Profiles
at $T = 25$ ms

Fig. 22. Pressure Histories

Fig. 23. Pressure Histories

Fig. 24. Forces on Segment A-C ($F_x = F_y$)

Fig. 25. Pressure History at Location B (x = 2.5 ft)

Fig. 26. Velocity History at Location B (x = 2.5 ft)

Fig. 27. Pressure History at Location D ($x = 25$ ft)

Fig. 28. Velocity History at Location D ($x = 25$ ft)

Fig. 29. Pressure and Velocity Profiles at
T = 9.7 ms

Fig. 30. Temperature and Sonic Speed Profiles at
T = 9.7 ms

Fig. 31. Force on Segment on A-C (F_x)

Fig. 32. Solution for the Shocked State (Abrupt Inlet Case)

Fig. 33. Wave Diagram (Abrupt Inlet Case)

Fig. 34. Similarity Solution - Pressure Result

Fig. 35. Similarity Solution - Particle Velocity Result

Fig. 36. Similarity Solution - Temperature Result

Fig. 37. Wave Diagram for Sample Problem No. 2

Fig. 38. p - u Diagram for Sample Problem No. 2
($\Delta p = 1.63 \text{ PSI}$; $\Delta u = 9.14 \text{ FPS}$)

Fig. 39. Shock Wave and Interface Locations

Fig. 40. Pressure and Velocity Profiles at $T = 0.69$ ms

Fig. 41. Temperature and Sonic Speed Profiles at $T = 9.69$ ms

Fig. 42. Pressure History at $x = 2.5$ ft Fig. 43. Pressure History at $x = 25$ ft

Fig. 44. Force on Segment A-C

Fig. 45. Pressure and Particle Velocity Profiles at $T = 10.4$ ms

Fig. 46. Pressure and Particle Velocity Profiles at $T = 30.4$ ms

Fig. 47. Pressure History at $x = 25$ ft

Fig. 48. Particle Velocity History at
 $x = 25$ ft

Fig. 49. Force on Segment A-C

Fig. 50. Characteristics Configuration with Disturbance Front at the Interface

Fig. 51. p-u Diagram for Simulant Steam Compared to Nitrogen Gas

Fig. 52. Wave Diagram for the Simulant Superheated Steam

Fig. 53. Pressure Histories Comparison:
Steam-N₂ vs Simulated System

Fig. 54. Comparison of Forces on Segment A-C:
Steam N₂ vs Simulated System