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THEORIES WITHOUT AB EFFECT MISREPRESENT THE DYNAMICS OF THE
ELECTROMAGNETIC FIELD

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INTRODUCTION

The suggestion by Professor Bocchieri, Professor Loinger, and others¹ that the Aharonov-Bohm effect (AB) is not part of quantum mechanics or that it can be removed from quantum mechanics without ruining the theory, is an important one for many reasons. One reason which has perhaps been too little emphasized is that removing AB scattering from the theory would remove the justification for Dirac's charge quantization law

$$\frac{eg}{c} = \frac{n}{2} \hbar, \quad (1)$$

where e and g are the elementary units of electric and magnetic charge and n is an integer. In this discussion I shall summarize arguments that have been given in detail elsewhere^{2,3} to show why AB cannot in fact be removed from the theory except by spoiling the conservation and quantization of angular momentum. It is of course no coincidence that the most general proofs of Dirac's charge quantization law also rely mainly on the quantization of angular momentum.⁴ I will not deal with that in this talk, but I will return briefly in an appendix to the relation between AB and Dirac's original derivation of the charge quantization law from the now-familiar singular strings.

People who attempt to remove AB from the theory use a variety of mathematical devices such as multiple-valued wave functions or wave functions with discontinuities along a cut, or unusual boundary conditions. I shall avoid all questions which may be raised by those devices by speaking only in terms of the algebra of the observables. The wave functions will be mentioned only occasionally as an aside. I believe that all other approaches which obey the general principles of quantum mechanics are bound by the conclusions reached in this way.

MOTION IN A MULTIPLY-CONNECTED SPACE

First consider a spinless electron confined to move on a circle of radius r in the xy plane, but otherwise free. The dynamical variables are θ , or more properly $\exp\{i\theta\}$, and $L_z = Mr\dot{\theta}$. They obey

$$[L_z, \exp\{i\theta\}] = -i\hbar \exp\{i\theta\}. \quad (2)$$

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The Hamiltonian is given by

$$H = (1/2)M\dot{\theta}^2 = (1/2M)L_z^2. \quad (3)$$

The rotation operator R is generated by L_z and obeys

$$R(\phi_1)R(\phi_2) = R(\phi_1 + \phi_2)\exp\{i\delta(\phi_1, \phi_2)\}, \quad (4)$$

where ϕ_1 and ϕ_2 are rotation angles, and $\delta(\phi_1, \phi_2)$ is an arbitrary phase. From Eq. (4) and the requirement that a 2π rotation is physically equivalent to no rotation, it follows that the eigenvalues of L_z are given by

$$L_z = (\gamma + m)\hbar, \quad (5)$$

where m are the integers and γ is a fixed number in $0 \leq \gamma < 1$. Thus the spectrum of the Hamiltonian, given by

$$E_m = (\hbar^2/2M)(\gamma + m)^2, \quad (6)$$

depends non-trivially upon γ . In other words, the constant γ labels the inequivalent representations of the two-dimensional rotation group, and the physics is different for different values of γ .

Mathematically, this is old stuff. It has been applied in detail to AB.⁵ The same things can be said in another language by using multiple-valued wave functions that obey $\psi(2\pi) = \psi(0)\exp\{i2\pi\gamma\}$.

Now introduce a magnetic flux Φ , confined to a cylinder of radius less than r whose axis is the z axis, so that the electron moving on the circle of radius r never enters the magnetic field. The vector potential A outside the flux region is given, in the gauge where $\nabla \cdot A = 0$, by

$$A_\theta = \frac{\Phi}{2\pi r}; \quad A_z = A_r = 0 \quad (7)$$

(I shall stick to this gauge throughout. Other choices of the gauge do not change the physics.) Then the Hamiltonian and its eigenvalues become

$$H = (1/2)Mr^2\dot{\theta}^2 = (1/2M)[p - (e/c)A]^2 \quad (8)$$

$$E_m = (\hbar^2/2Mr^2)(m + \gamma - \alpha)^2, \quad (9)$$

where

$$\alpha = e\Phi/2\pi\hbar c. \quad (10)$$

On this level, there is no physical distinction between the effects of the magnetic field represented by α and those of the choice of representation labelled by γ .

Before pointing out what's wrong with all this, let me first generalize it to three dimensions by freeing the electron from its circle in the xy plane and allowing it to move freely outside a cylinder whose axis is the z axis. All the physics is unchanged from the usual case except that now the centrifugal term in the radial part of the Hamiltonian becomes

$$V_c = (1/2)Mr^2\dot{\theta}^2 = (1/2Mr^2)(L_z - \alpha\hbar)^2. \quad (11)$$

Once again we have the inequivalent representations characterized by γ . The allowed values of the centrifugal barrier height

$$V_c = (\hbar^2/2Mr^2)(m + \gamma - \alpha)^2, \quad (12)$$

depend upon the flux parameter α , and so therefore do cross sections, diffraction patterns, and energy eigenvalues. Just as in the case of the electron confined to a circle, the effects of the flux can be removed by using a different representation of the rotations γ for each flux value α , that is by choosing $\gamma=\alpha$ in every case. In the language of multiple-valued wave functions, one must choose different Hilbert spaces for electrons in the presence of different magnetic fields. As far as I understand them, all the papers which remove AB from conventional quantum mechanics by introducing multiple-valued wave functions or their equivalent without other radical changes do exactly that.

However, there is no physics in that. The quantum theories so devised misrepresent the dynamics associated with the magnetic field. This can be seen by considering the angular momentum in the electromagnetic field.

ANGULAR MOMENTUM AND THE RETURN FLUX

Detailed proofs of the assertions in this section can be found in Refs. (2 and 3). Here I will only try to emphasize the generality of the results. My point is that, regardless of what you assume about choices of gauge or about multiple-valued wave functions, AB cannot be removed from the theory except by paying the following price:

Either the total angular momentum in the theory, mechanical plus electromagnetic, is not quantized in integer units for spinless electrons and angular momentum is not conserved,

or angular momentum does not correspond to its classical analogue in the usual way.

To see this, return to the centrifugal barrier.

$$V_c = (1/2Mr^2)(L_z - \alpha\hbar)^2 \quad (13)$$

$$L_z/\hbar = 0, \pm 1, \pm 2, \dots \quad (14)$$

It was shown above that the dependence of V_c upon the flux through α is equivalent to AB. That flux dependence of V_c requires only that the eigenvalues of L_z are independent of α . The following general argument gives the physical basis for the flux independence of L_z .

Real magnetic fields have return fields somewhere; in the case of the field generated by a long solenoid along the z axis, the return field may be arbitrarily remote from the solenoid, but the return flux is equal and opposite to the flux through the cylinder from which the electron is excluded. In principle, to demonstrate AB, the electron must also be excluded from the remote return field region. However, that creates no problem. Examples have been given by Profesor Nagel at this meeting⁶ and by others⁷ to show that the physical effects remain when the excluded return magnetic fields are correctly taken into account in solving the Schroedinger equation. When it is assumed that the vector potential goes to zero fast enough at infinity, which physically requires that the return field not be neglected, the canonical angular momentum L in the gauge where $\nabla \cdot A = 0$ obeys

$$L = r \times mv + \int r' \times P(r') d^3 r' \quad (15)$$

where $v = dr/dt$ and

$$P(r') = (1/4\pi c) E(r') \times B(r') \quad (16)$$

is the Poynting vector formed by the external magnetic field $B(r')$ and the electric field at r'

$$E(r') = e \frac{r' - r}{|r' - r|^3} \quad (17)$$

whose source is the electron at r . Then the canonical angular momentum equals the total angular momentum, mechanical plus electromagnetic. Any theory which quantizes the total angular momentum in integer units does the same for the canonical angular momentum in the chosen gauge and therefore in any gauge. It is noteworthy that the presence of the return flux is essential for this proof. In the case at hand, where the magnetic field is symmetric about the z axis, all of the angular momentum in the crossed fields resides in the return flux region, and none in the excluded cylinder centered on the z axis.

Next consider the consequences of varying the external flux as a function of time. The torque on the electron is related to the rate of change of the flux through the induced electric field E_i .

$$\frac{d}{dt}(r \times mv) = r \times e E_i = -\frac{e}{c} \left(\frac{d\phi}{dt} \right), \quad (18)$$

The rate of change of the field angular momentum is given by

$$\frac{d}{dt} \int r' \times p(r') d^3 r' = +\frac{e}{c} \left(\frac{d\phi}{dt} \right). \quad (19)$$

Then, from Eq. (15),

$$\frac{dL_z}{dt} = 0. \quad (20)$$

Therefore the eigenvalues of L_z are unchanged by turning on and off the magnetic flux if the theory conserves the total angular momentum, mechanical plus electromagnetic. This restriction cannot be evaded by moving the electron out past the remote return flux while the magnetic field is being changed because the integrated torque on the electron during its passage out through the return flux is just equal to $(e/c)(d\phi/dt)$, and the torque which it in turn exerts on the electromagnetic field is $-(e/c)(d\phi/dt)$.

CONCLUSION

I think I've shown in a very general way that to remove AB from quantum mechanics, you must either neglect the return flux, surrender the quantization and the conservation of the total angular momentum, or somehow break the relation between the electromagnetic field angular momentum and the operator which generates the rotations in quantum mechanics. This proof is very simple and appears to depend only upon the most general principles. I think it's a fair challenge to anyone who presents a quantum mechanical theory without AB that he or she should answer the question as to whether or not angular momentum is conserved and quantized in that theory or, if the theory has somehow gotten around that problem, to point out how and at what price. Specifically, has the dynamics of the electromagnetic field itself been changed, and if so, how? I do not believe that challenge has been accepted for the theory of Professor Bocchieri and Professor Loinger.

The arguments given above neither require nor exclude the possibility that some hole in the space has a fixed γ unequal to zero and independent of any magnetic flux. That idea has recently been suggested⁷ and it leads to many interesting speculations, all of which obey the general principles discussed here if one recognizes that introducing a magnetic flux through the hole with no return flux is physically represented by γ in this discussion, not by α . In other words, there is no physical basis for ascribing the

phenomena involving such a hole to any magnetic field, and no need to do so. But regardless of the language used, there is no conflict with AB.

APPENDIX: CHARGE QUANTIZATION

That AB lies at the root of Dirac's original derivation⁸ of the charge quantization law (1) is implicit in that derivation, and I believe that is well known to most people who have interested themselves in AB, but it is seldom remarked upon and perhaps worth repeating in this session which is focussed upon the possibility of removing AB from quantum mechanics. The Dirac vector potential due to a magnetic monopole charge g fixed at the origin can be chosen to be

$$A_{\phi} = \frac{g(1 - \cos\theta)}{r \sin\theta} \quad ; \quad A_{\rho} = A_{\theta} = 0 . \quad (21)$$

For general g , this does not represent the pure monopole field $B = (g/r^2)\hat{r}$, but rather that monopole field plus a delta function field along the negative z axis carrying flux equal to $4\pi g$ from minus infinity to the origin. Dirac showed that when $eg/c = n\hbar/2$, the incoming flux thread can be moved by a gauge transformation from the negative z axis to any other line from infinity to the origin. Then the choice of the incoming flux thread has no physical effect, and Dirac's vector potential represents a monopole field. For other values of eg/c , AB tells us that electrons are scattered by the flux thread itself, at an arbitrary distance from the monopole at the origin. Then Dirac's potential does not represent a simple monopole field. If we could remove AB from the theory, that difficulty for general values of eg/c would not arise.

Dirac's discussion differs from the usual discussions of AB in that Dirac does not confine the electron away from the negative z axis. However, for a flux thread of zero diameter, that does not matter. If the electron is excluded from the interior of a cylinder of radius ϵ about the negative z axis, then the limit as ϵ approaches zero gives all the same physical results as those that obtain with no excluded volume.²

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REFERENCES

1. Representative statements of that point of view are given by Professor Bocchieri in the previous paper and by W. Henneberger in Phys. Rev. Lett. 52, 573 (1984).
2. M. Peshkin, I. Talmi, and L. J. Tassie, Ann. Phys. 16, 177 (1961).
3. M. Peshkin, Phys. Rep. 80, 376 (1981).
4. H. J. Lipkin, W. I. Weisberger, and M. Peshkin, Ann. Phys. 53, 203 (1969), and references therein.
5. L. J. Tassie and M. Peshkin, Ann. Phys. 16, 177 (1961).
6. B. Nagel, this volume.
7. F. Wilczek, Phys. Rev. Lett. 48, 1144 (1982); and 49, 957 (1982).
8. P. A. M. Dirac, Proc. Roy. Soc. (London) A133, 60 (1931).