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Introduction to String Field Theory

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Abstract: We propose an action for an interacting closed bosonic string. Our formalism relies heavily on ideas discussed by Witten for the open bosonic string. We also obtain the gauge fixed quantum action for the fully interacting open bosonic string.

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1. Introduction

String theories may very well provide a unified description of all the known particles and their interactions- including gravity. The 17 or more phenomenological parameters of the standard model may, in principle, be determined in this fundamental theory with no free parameters. Such are the fervent hopes of particle theorists. Yet if we are to understand this new theory we must relinquish the paradigms of the point particle, local field theory and local gauge symmetries. These concepts are presumably relevant descriptions of nature only in a low energy approximation. For example, the principle of general coordinate invariance is probably not a useful concept as applied to the string. The Virasoro-Shapiro model contains a massless spin two "graviton". However this state has no local four point interactions as is the case for the graviton in general relativity.¹ Thus the "graviton" of the Virasoro-Shapiro model is not a simple piece of a tensor under general coordinate transformations.² A new symmetry principle based on the string is thus needed to supercede some old but valued concepts at distance scales where stringiness becomes relevant.

Recent work towards a covariant string field theory [1-27] is partly motivated by this desire to find the symmetry principle behind the theory. We feel that a significant step in this direction has recently been taken by Witten [3] in his creation of an interacting open bosonic string field theory using some ideas of Connes [29] on non-commutative differential geometry. In this paper we explore these ideas with respect

to the open bosonic string in some more detail. Using the intuition so obtained we propose an action for an interacting closed bosonic string field theory.

In section 2 we review Witten's theory of the open bosonic string. We use a notation, however, more akin to that of Siegel and Zwiebach [2]. We present the gauge fixed BRST [30-34] invariant action for Witten's theory. This result can be found in equation (1.33). In section 3 we describe our results on the closed bosonic string. We discuss some new axioms which can lead to a gauge invariant interacting closed bosonic string action. The relevant equations are (2.20) - (2.25). We then motivate the new set of axioms. In section 4 we summarize our results and discuss the problems which remain to be explored.

2. The Open Bosonic String

Following Siegel [1] we define a real string functional

$$\Phi[x_\mu(\sigma), c(\sigma), \tilde{c}(\sigma)] , \quad (1.1)$$

and a BRST generator Q , satisfying $Q^2 = 0$ [35,36], represented as a differential operator acting on Φ --

$$Q = \sqrt{\pi\alpha'} \int_{-\pi}^{\pi} d\sigma \, \hat{c}(\sigma) \left[\frac{1}{2} \hat{p}^2(\sigma) - \frac{1}{\alpha'} \hat{c}'(\sigma) \pi_{\hat{c}}(\sigma) \right] \quad (1.2)$$

where

$$\begin{aligned} \hat{p}_\mu(\sigma) &= \frac{1}{\sqrt{2}} \left[i \delta / \delta x^\mu(\sigma) - \frac{1}{\alpha'} x'^\mu(\sigma) \right] , \\ \hat{c}(\sigma) &= \alpha' / \sqrt{2} \left[\delta / \delta c(\sigma) + \frac{1}{\alpha'} \tilde{c}(\sigma) \right] , \\ \pi_{\hat{c}}(\sigma) &= \frac{1}{\sqrt{2}} \left[\delta / \delta \tilde{c}(\sigma) + \frac{1}{\alpha'} c(\sigma) \right] . \end{aligned} \quad (1.3)$$

The commuting (anti-commuting) coordinates $x_\mu(\sigma)$ ($c(\sigma), \tilde{c}(\sigma)$) with $\mu = 1, \dots, 26$ and $0 \leq \sigma \leq \pi$ satisfy the boundary conditions $x'_\mu(\sigma) \equiv dx_\mu(\sigma)/d\sigma = 0$, $c'(\sigma) = 0$, $\tilde{c}'(\sigma) = 0$ at $\sigma = 0, \pi$. (The coordinates for the region $-\pi \leq \sigma \leq 0$ are then obtained by analytic continuation. α' is the string tension.)

The string functional Φ can be expanded in terms of an infinite number of local component fields. This expansion is particularly useful when making contact with local field theory. We define the differential operators $a_{n\mu}, \beta_n, \tilde{\beta}_n$ for $-\infty \leq n \leq \infty$ as the normal modes of the operators given in (eq. 1.3)-

$$\begin{aligned} p_\mu(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum a_{n\mu} e^{-in\sigma}, \\ \dot{c}(\sigma) &= \sqrt{\alpha'}/2\pi \sum \beta_n e^{-in\sigma}, \\ \pi_{\tilde{c}}(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum \tilde{\beta}_n e^{-in\sigma}. \end{aligned} \quad (1.4)$$

They satisfy the canonical commutation and anti-commutation relations

$$\begin{aligned} [a_{n\mu}, a_{m\nu}] &= n \delta_{n+m,0} \eta_{\mu\nu}, \\ \{\beta_n, \tilde{\beta}_m\} &= \delta_{n+m,0}. \end{aligned} \quad (1.5)$$

We also define a ground state functional Φ_0 satisfying

$$a_{n\mu} \Phi_0 = \beta_n \Phi_0 = \tilde{\beta}_m \Phi_0 = 0 \text{ for } n \geq 0, n_1 > 0. \quad (1.6)$$

The component expression is then given by

$$\begin{aligned} \Phi &= (i \tilde{\beta}_1^\dagger B(x) + \dots + c (\varphi(x) + a_{1\mu}^\dagger A^\mu(x) + \beta_1^\dagger \bar{\eta}(x) + \tilde{\beta}_1^\dagger \eta(x) + \dots)) \Phi_0 \\ \text{where } c &= \sqrt{\alpha'}/\pi \tilde{\beta}_0 \text{ and } \partial_c = \sqrt{\pi/\alpha'} \beta_0. \end{aligned} \quad (1.7)$$

Φ is a commuting field. Thus the fields φ , A^μ , B are also commuting and correspond to the tachyon, abelian gauge field and Lautrup- Nakanishi auxiliary field. η and $\bar{\eta}$ are the anti-commuting Fadeev-Popov ghost and anti-ghost fields connected with A^μ and B .

A global BRST transformation of Φ is given by

$$\delta\Phi = i\epsilon Q\Phi \quad (1.8)$$

which leaves the free action

$$S_0 = \int \Phi^T [Q, \rho] \Phi \quad (1.9)$$

invariant. ($\Phi^T \equiv \Phi [x_\mu(\pi-\sigma), c(\pi-\sigma), -\tilde{c}(\pi-\sigma)]$) ρ is the gauge fixing operator and the Feynman-Siegel gauge is obtained by the choice

$$\rho = [c, \partial_c]. \quad (1.10)$$

A particularly simple subspace to consider, for illustrative purposes, is that including the massless sector alone i.e., A^μ , B , η and $\bar{\eta}$. In this subspace the BRST generator Q assumes the form

$$Q_0 = [\beta_1 a_1^\dagger \cdot i\partial + \beta_1^\dagger a_1 \cdot i\partial - \partial_c \square - \beta_1^\dagger \beta_1 c] \quad (1.11)$$

and satisfies $Q_0^2 = 0$ (we have taken $\alpha'=2\pi$). It is then trivial to check that the BRST transformation (eq. 1.8) for the massless fields is given by

$$\begin{aligned} \delta A^\mu &= -\epsilon \partial^\mu \eta \\ \delta \eta &= 0 \\ \delta B &= \epsilon \square \eta \\ \delta \bar{\eta} &= \epsilon (\partial \cdot A + B) . \end{aligned} \quad (1.12)$$

Let us now consider the local gauge invariant theory of Witten. We must first define the physical subspace of Φ . Following Witten we

consider the ghost number operator given by

$$N_g = 1/2 [\partial_c, c] + \sum_{n \geq 0} (\beta_n^\dagger \beta_n - \beta_n^\dagger \beta_n). \quad (1.13)$$

Then $N_g \Phi_0 = 1/2 \Phi_0$

and the physical sector is defined to have ghost number $-1/2$. (We shall denote by capital Latin letters string functionals satisfying the constraint $N_g A = -1/2 A$.) The component expansion is given by

$$A = \{ i \beta_1^\dagger B(x) + \dots + c (\varphi(x) + a_{1\mu}^\dagger A^\mu(x) + \dots) \} \Phi_0. \quad (1.14)$$

The free field equations are then given by

$$Q A = 0 \quad (1.15)$$

which is invariant under the local gauge transformation

$$\delta A = Q \epsilon. \quad (1.16)$$

Note that $[N_g, Q] = Q$ and thus ϵ must have ghost number $-3/2$. In the massless subsector we find eqs. 1.15 and 1.16 imply

$$\square A_\mu + \partial_\mu B = 0, \quad \partial \cdot A + B = 0 \quad (1.17)$$

$$\delta A_\mu = \partial_\mu \lambda, \quad \delta B = -\square \lambda$$

where $\epsilon = \{ i c \beta_1^\dagger \lambda(x) + \dots \} \Phi_0$. Eqs. 1.17 are equivalent to the free Maxwell equations $\partial^\mu F_{\mu\nu} = 0$. The free action S_{W0} is given by

$$S_{W0} = \int A^\dagger Q A. \quad (1.18)$$

In order to generalize this theory to include interactions Witten relies on the axioms of the non-commutative differential geometry of Connes. In the rest of this section we shall

- 1) review these axioms, and
- 2) discuss an explicit representation of them

Abstract Axioms

We want the string functionals A to satisfy properties analogous to those of forms in differential geometry and for the BRST generator Q to act as a derivation. Q already satisfies the first axiom

$$i) \quad Q^2 = 0.$$

We define a product rule denoted by $*$ and an integral $\int$ which satisfy:

ii) **associativity** of $*$ product-

$$(A * B) * C = A * (B * C).$$

Although the product is non-commutative ($A*B \neq B*A$), we demand

$$iii) \quad \int A*B = (-1)^{AB} \int B*A$$

where $(-1)^{AB} = (-1)$, if A and B are Grassman odd; $+1$, otherwise).

The additional properties of a derivation are

$$iv) \quad \int Q \chi = 0 \text{ for any string functional } \chi \text{ and, } Q \text{ must satisfy}$$

v) **the Leibnitz rule-**

$$Q(A*B) = QA*B + (-1)^A A*QB$$

where $(-1)^A = (-1)$, if A is Grassman odd; $+1$, otherwise).

Given these properties we can then define the gauge field strength

$$F = QA + A*A \quad (1.19)$$

which under the local gauge transformation

$$\delta A = Q\epsilon + [A, \epsilon] \quad (1.20)$$

(where $[A, \epsilon] = A*\epsilon - \epsilon*A$) transforms by

$$\delta F = [F, \epsilon]. \quad (1.21)$$

[Note that the group structure implicit in this abstract algebra is encoded in the $*$ product. For example it is easy to see that the

transformation of A defined in eq. 1.20 satisfies the group property

$$[\delta_2, \delta_1] A = \delta_3 A \quad (1.22)$$

where the infinitesimal gauge functional parameters ϵ_i , $i=1,2,3$ are related by $\epsilon_3 = [\epsilon_2, \epsilon_1]$. Finally the action for the interacting theory is given by

$$S_W = \int (A^* Q A + \frac{2}{3} A^* A^* A) \quad (1.23)$$

which is invariant under (1.20). The equations of motion are given by

$$\delta S / \delta A = F = 0. \quad (1.24)$$

The ghost numbers of $\int$ and $*$ were determined by Witten to be $-3/2$ and $+3/2$, respectively. Moreover $\int \chi = 0$ unless χ has ghost number $+3/2$. Hence a non-trivial action has net ghost number zero.

Representation of the Axioms

Witten has shown that an associative $*$ product requires treating the point $\sigma=\pi/2$ special. Thus $A*B$ is defined by a process of "sewing" the right half of string A with the left half of string B as depicted in fig.1. This statement will be made more precise shortly. If $A*B$ is given by fig.1, then integral $\int$ might be expected to "sew" the remaining halves of the string to each other, as in fig.2. In this way we obtain the "sewing condition"

$$\int A^T B \text{ (eq. 1.18)} = \int A * B \quad (1.25)$$

where $A^T = A[x_\mu(\pi-\sigma), c(\pi-\sigma), -\tilde{c}(\pi-\sigma)]$.

The Integral - $\int$

We shall now obtain an explicit representation of $\int$. The two main

properties it must satisfy are Axiom (iv) and ghost number $-3/2$. The "naive" integral is represented by

$$\int = \int D \delta \quad (1.26)$$

where $D \equiv \prod_{\sigma=0}^{\pi} dx_{\mu}(\sigma) dc(\sigma) d\tilde{c}(\sigma)$, $\delta \equiv \prod_{\sigma=0}^{\pi/2} \delta(x^0) \delta(c^0) \delta(\tilde{c}^0)$ and

$$\begin{aligned} x_{\mu}^0(\sigma) &\equiv x_{\mu}(\sigma) - x_{\mu}(\pi - \sigma) \\ c^0(\sigma) &\equiv c(\sigma) - c(\pi - \sigma) \\ \tilde{c}^0(\sigma) &\equiv \tilde{c}(\sigma) - \tilde{c}(\pi - \sigma). \end{aligned} \quad (1.27)$$

The ghost number of the factors are $D [1/2]$, $\delta [0]$. Thus the "naive" integral has the wrong ghost number. Moreover we find that $\int D \delta Q \chi \neq 0$ in general, due to complications associated with the special point $\sigma = \pi/2$. Thus an insertion is needed. The unique integral with ghost number $-3/2$ and satisfying Axiom (iv) is given by

$$\int = \int D \delta I \quad (1.28)$$

where $I \equiv c(\pi/2) \delta / \delta \tilde{c}(\pi/2)$. We then find $\int Q \chi = 0$ only in 26 dimensions.

The * product

The most difficult constraint the * product must satisfy is the Leibnitz rule. Recall that the BRST charge Q (eq.1.2) contains products of up to three derivatives. A "naive" definition of the * product is given by the expression (see fig.1)

$$\begin{aligned} A * B &= \int \prod_{\sigma=0}^{\pi} dx^1_{\mu}(\sigma) dc^1(\sigma) d\tilde{c}^1(\sigma) \prod_{\sigma=0}^{\pi/2} dx^2_{\mu}(\sigma) dc^2(\sigma) d\tilde{c}^2(\sigma) \\ &\quad \times \prod_{\sigma=0}^{\pi/2} \delta(x^2(\sigma) - x^1(\pi - \sigma)) \delta(c^2(\sigma) - c^1(\pi - \sigma)) \delta(\tilde{c}^2(\sigma) + \tilde{c}^1(\pi - \sigma)) A[1] B[2] \\ &\equiv \int \mu^* AB \end{aligned} \quad (1.29)$$

(where we use the shorthand notation $A[1] = A[x^1_\mu(\sigma), c^1(\sigma), \tilde{c}^1(\sigma)]$).

Since Q is defined by an integral over a local function of σ , it is possible to define a new charge Q' with the point $\sigma=\pi/2$ removed. It is trivial to show that Q' satisfies a Leibnitz rule on the "naive" $\ast$ product.

Consider $Q' (A \ast B) = \int \mu^\ast [(Q'_- A)B + (-1)^A A(Q'_+ B)]$ where $Q'_\pm$ are defined by integrals over the $\pm$ sectors of strings A or B (see fig. 1). However, using the following identity, which is a consequence of integration by parts, $\int \mu^\ast [(Q'_+ A)B + (-1)^A A(Q'_- B)] = 0$ and

$Q'A = (Q'_+ + Q'_-)A$ we find $Q'(A \ast B) = Q'A \ast B + (-1)^A A \ast Q'B$. The difficulty in satisfying the Leibnitz rule is thus all contained in the treatment at the special point $\sigma=\pi/2$.

We have not yet solved this difficult problem. Nevertheless we may state some general properties the $\ast$ product must satisfy in a notation which will be useful later in our discussion of the closed string. In general we may define a $\ast$ product by the expression

$$A \ast B[3] = \int D_1 D_2 K[312] A[1] B[2] \quad (1.30)$$

where $D_i = \int_{\sigma=0}^{\pi} dx^1_\mu(\sigma) dc^1(\sigma) d\tilde{c}^1(\sigma)$ and the Kernel K must satisfy

$$(a) \text{ "sewing condition" (eq.1.25) -- } \int D_3 \delta_3 I_3 K[312] = \delta[12] ; \quad (1.31a)$$

$$(b) \text{ Leibnitz rule - } \sum_{i=1}^3 Q_i K[312] = 0 \text{ , and} \quad (1.31b)$$

$$(c) \text{ associativity - } \int D_3 K[534] K[312] = - \int D_3 K[513] K[324]. \quad (1.31c)$$

Property (a) fixes the ghost number of K to be $+1/2$ for the open string and hence the ghost number of the $\ast$ product is $+3/2$. It can also be used to show that K is Grassman even, using D_3 [odd] , δ_3 [even] , I_3 [even] and $\delta[12]$ [odd].

Once the $\ast$ product (or the Kernel K) is determined we may then obtain the interacting field equations in component form. Work in this direction is still in progress. Throughout the remainder of this section we shall assume that there exists a K satisfying (a-c).

The $\ast$ product is also significant on a deeper level. It defines a sort of matrix multiplication where the string functional A (a connection on some fibre bundle) can be thought of as an infinite dimensional matrix. Gauge transformations δA (1.20) , as usual, describe the motion of A along the fibre. However the base manifold of this fibre bundle is trivial; it's just a point.

We end this section with a discussion of the gauge fixed quantum action for the fully interacting theory. Consider first the free field case. Witten's gauge invariant action $S_{W0} = \int A \ast Q A$ is invariant under $\delta A = Q\epsilon$. A has ghost number $-1/2$. The quantum string functional (where by "quantum" we mean it includes the BRST ghost and anti-ghost fields which are necessary for a quantum description of the theory) $\Phi = A + \Xi$ where the field Ξ includes all ghost numbers, excluding $-1/2$. Siegel's gauge fixed action may then be rewritten using the notation of $\int$ and defined previously. We have

$$S_0 = \int \Phi \ast [Q, \rho] \Phi \quad (1.32)$$

$$= \int A \ast Q A + \int A \ast ([Q, \rho] - Q) A + \int \Xi \ast [Q, \rho] \Xi$$

where the three terms may be identified as the gauge invariant action, the gauge fixing term and the Fadeev-Popov ghost action, respectively.

S_0 is invariant under global BRST transformations $\delta \Phi = i\epsilon Q\Phi$. Note the physical sector transforms according to $\delta A = Q(-i\epsilon \Xi|_{-1/2})$.

BRST Invariant Action

Let us now generalize Siegel's free gauge fixed quantum action to the fully interacting theory. We contend that

$$S_{qu} = \int (\Phi^* [Q, \rho] \Phi + 2\rho\Phi^*\Phi\Phi) \quad (1.33)$$

is invariant under the non-linear global BRST transformation

$$\delta\Phi = i\epsilon (Q\Phi + \Phi^*\Phi) \quad (1.34)$$

satisfying $\delta^2 = 0$. The physical sector obeys

$$\delta A = Q\xi + [A, \xi] + i\epsilon \Xi_{n_1}^* \Xi_{n_2} |_{n_1+n_2=-2} \quad (1.35)$$

where $\xi = -i\epsilon \Xi |_{-3/2}$. Except for the last term, this is just a generalization of global BRST transformations for non-abelian gauge theories.

3. The Closed Bosonic String

Following Siegel (once more) we define a real closed string functional

$$\Phi[x_\mu(\sigma), c(\sigma), \tilde{c}(\sigma)] \quad (2.1)$$

and a BRST generator Q , satisfying $Q^2 = 0$. Q is given by

$$Q = \sqrt{2\pi\alpha'} \int_{-\pi}^{\pi} d\sigma \left(\hat{c}(\sigma) \left[\frac{1}{2} \hat{p}^2(\sigma) - \frac{1}{\alpha'} \hat{c}'(\sigma) \pi_{\hat{c}}(\sigma) \right] \right. \\ \left. + \hat{\bar{c}}(\sigma) \left[\frac{1}{2} \bar{p}^2(\sigma) + \frac{1}{\alpha'} \hat{\bar{c}}'(\sigma) \bar{\pi}_{\hat{c}}(\sigma) \right] \right) \quad (2.2)$$

where

$$\hat{p}_\mu(\sigma) = \frac{1}{\sqrt{2}} \left[\delta / \delta x^\mu(\sigma) - \frac{1}{\alpha'} x'_\mu(\sigma) \right], \\ \hat{c}(\sigma) = \alpha' / \sqrt{2} \left[\delta / \delta c(\sigma) + \frac{1}{\alpha'} \tilde{c}(\sigma) \right], \\ \pi_{\hat{c}}(\sigma) = \frac{1}{\sqrt{2}} \left[\delta / \delta \tilde{c}(\sigma) + \frac{1}{\alpha'} c(\sigma) \right] \quad (2.3)$$

$$\bar{p}_\mu(\sigma) = 1/\sqrt{2} [i \delta / \delta x^\mu(\sigma) + 1/\alpha' x'_\mu(\sigma)] ,$$

$$\tilde{c}(\sigma) = \alpha' / \sqrt{2} [\delta / \delta c(\sigma) - 1/\alpha' \tilde{c}(\sigma)] ,$$

$$\bar{\pi}_c(\sigma) = 1/\sqrt{2} [- \delta / \delta \tilde{c}(\sigma) + 1/\alpha' c(\sigma)] .$$

The commuting (anti-commuting) coordinates $x_\mu(\sigma)$ ($c(\sigma), \tilde{c}(\sigma)$) with $\mu = 1, \dots, 26$ and $-\pi \leq \sigma \leq \pi$ satisfy the periodic boundary conditions $x_\mu(\sigma) = x_\mu(\sigma + 2\pi)$, $c(\sigma) = c(\sigma + 2\pi)$, $\tilde{c}(\sigma) = \tilde{c}(\sigma + 2\pi)$. Note that $\tilde{c}(\sigma)$ now has a zero mode, unlike the case of the open string.

As in the case of the open string, we may expand ϕ in terms of an infinite number of local component fields. We define the differential operators $a_{n\mu}, \beta_n, \tilde{\beta}_n, \bar{a}_{n\mu}, \bar{\beta}_n, \bar{\tilde{\beta}}_n$ for $-\infty \leq n \leq \infty$ by

$$\begin{aligned} p_\mu(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum a_{n\mu} e^{-in\sigma} , \\ \hat{c}(\sigma) &= \sqrt{\alpha'}/2\pi \sum \beta_n e^{-in\sigma} , \\ \pi_c(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum \tilde{\beta}_n e^{-in\sigma} , \\ \bar{p}_\mu(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum \bar{a}_{n\mu} e^{in\sigma} , \\ \bar{\tilde{c}}(\sigma) &= \sqrt{\alpha'}/2\pi \sum \bar{\beta}_n e^{in\sigma} , \\ \bar{\pi}_{\tilde{c}}(\sigma) &= 1/\sqrt{2\pi\alpha'} \sum \bar{\tilde{\beta}}_n e^{in\sigma} \end{aligned} \quad (2.4)$$

satisfying the canonical commutation and anti-commutation relations

$$\begin{aligned} [a_{n\mu}, a_{m\nu}] &= n \delta_{n+m,0} \eta_{\mu\nu} , \\ (\beta_n, \tilde{\beta}_m) &= \delta_{n+m,0} , \\ [\bar{a}_{n\mu}, \bar{a}_{m\nu}] &= n \delta_{n+m,0} \eta_{\mu\nu} , \\ (\bar{\beta}_n, \bar{\tilde{\beta}}_m) &= \delta_{n+m,0} \end{aligned} \quad (2.5)$$

with all others zero. We also define a ground state functional Φ_0 satisfying

$$a_{n\mu} \Phi_0 = \beta_m \Phi_0 = \bar{\beta}_m \Phi_0 = \bar{a}_{n\mu} \Phi_0 = \bar{\beta}_m \Phi_0 = \bar{\beta}_m \Phi_0 = 0$$

for $n \geq 0, m > 0$. (2.6)

We also have

$$\partial_c \Phi_0 = \partial_{\tilde{c}} \Phi_0 = 0 \quad (2.7)$$

where $\partial_c = \sqrt{\pi/\alpha'} (\beta_0 + \bar{\beta}_0)$ and $\partial_{\tilde{c}} = \sqrt{\pi/\alpha'} (\bar{\beta}_0 - \beta_0)$.

The component expansion of Φ includes a tachyon, massless graviton, Kalb-Ramond anti-symmetric tensor, dilaton and an infinite number of massive fields. It also includes the necessary Fadeev-Popov ghosts, auxiliary fields and many more unphysical fields.

Global BRST transformations are again given by

$$\delta \Phi = i\epsilon Q \Phi. \quad (2.8)$$

We now want to discuss a gauge invariant description for the closed bosonic string. The physical sector of Φ may be defined a la Witten by its ghost number content. We define the ghost number operator

$$N_g = 1/2 [\partial_c, c] - 1/2 [\partial_{\tilde{c}}, \tilde{c}] + \sum_{n \geq 0} (\beta_n^\dagger \bar{\beta}_n - \bar{\beta}_n^\dagger \beta_n + \bar{\beta}_n^\dagger \bar{\beta}_n - \bar{\beta}_n^\dagger \bar{\beta}_n)$$

with $N_g \Phi_0 = 0$. (2.9)

Then the physical sector is defined to have ghost number -1 . (We shall use capital Latin letters for physical string functionals which satisfy $N_g A = -A$.) We shall impose two additional constraints on Φ in order to define the physical sector. The first is the standard requirement that the string functional is invariant under constant reparametrizations of σ

i.e. the "origin" of the string has no physical significance, or

$$A[x_\mu(\sigma+f), c(\sigma+f), \tilde{c}(\sigma+f)] = A[x_\mu(\sigma), c(\sigma), \tilde{c}(\sigma)] \quad (2.10)$$

for constant shift f . Finally we constrain the dependence of A on the ghost zero mode $\tilde{c}$. We demand that A be expressible in the form

$$A = \partial_{\tilde{c}} \chi \quad \text{which implies} \quad \partial_{\tilde{c}} A = 0. \quad \text{To summarize, the physical}$$

sector of Φ is defined to satisfy

$$N_g A = -A, \quad \Delta N A = 0, \quad A = \partial_{\tilde{c}} \chi \quad (2.11)$$

where $\Delta N = N - \bar{N}$ (the number of right minus left movers of the string) is the generator of constant σ reparametrizations.

The free field equations of motion are given by

$$Q A = 0 \quad (2.12)$$

with the corresponding gauge invariance

$$\delta A = Q\epsilon. \quad (2.13)$$

The component expansion of A is given by

$$A = \left(i \beta_1^\dagger \bar{a}_{1\mu}^\dagger \bar{B}^\mu(x) + i \bar{\beta}_1^\dagger a_{1\mu}^\dagger B^\mu(x) + \dots + c(\phi(x) + \bar{a}_{1\mu}^\dagger a_{1\nu}^\dagger \eta^{\mu\nu}(x) + \beta_1^\dagger \bar{\beta}_1^\dagger \chi(x) + \bar{\beta}_1^\dagger \beta_1^\dagger \chi(x) + \dots) \right) \phi_0. \quad (2.14)$$

In the massless subspace the BRST generator takes the simple form

$$Q_0 = \left(\beta_1 a_{1\cdot}^\dagger \partial + \beta_1^\dagger a_{1\cdot} \partial + \bar{\beta}_1 \bar{a}_{1\cdot}^\dagger \partial + \bar{\beta}_1^\dagger \bar{a}_{1\cdot} \partial - \partial_c \square - (\beta_1^\dagger \beta_1 + \bar{\beta}_1^\dagger \bar{\beta}_1) c \right) \quad (2.15)$$

satisfying $Q_0^2 = 0$ (with $\alpha' = 4\pi$). It is then trivial to check that in the

massless subsector eqs. 2.12 and 2.13 give the equations of motion

$$\begin{aligned}
\Box h^{\mu\nu} + \partial^\nu \bar{B}^\mu + \partial^\mu B^\nu &= 0, \\
\Box \chi - \partial \cdot B &= 0, \quad \Box \bar{\chi} + \partial \cdot \bar{B} = 0 \\
\partial_\nu h^{\mu\nu} + \bar{B}^\mu + \partial^\mu \chi &= 0, \\
\partial_\mu h^{\mu\nu} + B^\nu - \partial^\nu \bar{\chi} &= 0;
\end{aligned} \tag{2.16}$$

and local gauge transformations

$$\begin{aligned}
\delta h^{\mu\nu} &= \partial^\nu \xi^\mu + \partial^\mu \xi^\nu \\
\delta \bar{B}^\mu &= -\Box \xi^\mu + \partial^\mu \eta, \quad \delta B^\nu = -\Box \xi^\nu - \partial^\nu \eta \\
\delta \bar{\chi} &= \partial \cdot \xi - \eta, \quad \delta \chi = -\partial \cdot \xi - \eta
\end{aligned} \tag{2.17}$$

where $\epsilon = (\bar{\beta}_1^\dagger \beta_1^\dagger \eta(x) + i c(\beta_1^\dagger \bar{a}_{1\mu}^\dagger \xi^\mu(x) + \bar{\beta}_1^\dagger a_{1\mu}^\dagger \xi^\mu(x)) + \dots) \phi_0$.

These are just the equations of motion for linearized gravity, the Kalb-Ramond gauge field and a massless dilaton.

An action may be constructed for the free theory. We find³

$$S_0 = \int A^T[\tilde{C}, Q] A \tag{2.18}$$

where $A^T = A[x_\mu(-\sigma), c(-\sigma), -\tilde{C}(-\sigma)]$. Note the insertion of the ghost zero mode coordinate $\tilde{C}$. This is necessary; otherwise the integral would vanish upon $\tilde{C}$ integration. The equations of motion derived from this action are

$$\partial_{\tilde{C}}[\tilde{C}, Q] A = 0 \tag{2.19}$$

which (using the constraints eq. (2.11)) is equivalent to eq. (2.12). (Note that Q may generally be written in the form $Q = Q' + \sqrt{\pi/\alpha'} \Delta N \tilde{C} + \sqrt{\alpha'/\pi} \Delta T \partial_{\tilde{C}}$ where we have made the $\tilde{C}$ dependence explicit. It is then trivial to prove the useful identity $QA = 1/2 \partial_{\tilde{C}}[\tilde{C}, Q]A$.)

Let us now discuss our proposal for the fully interacting closed bosonic string theory. We want to write an action which is similar to that for the open bosonic string (eq. 1.23). In order to accomplish this we must change some of the axioms of section 2. We shall assume that we can define an integral $\int$ and $\ast$ product which satisfy the following axioms. Although we have not yet completely defined a representation for the new axioms, we shall try to motivate each one by comparison with our knowledge of the open string. It is not inconceivable to us that the axioms we discuss may, in fact, already be represented by the path integral expressions defined by Witten.

Abstract Axioms

$$Q^2 = 0$$

We define a product rule which is Grassman odd and satisfies $\Delta N(A \ast B) = 0$ if $\Delta N A = \Delta N B = 0$, but $\partial_{\tilde{C}} A \ast B \neq 0$, even if $\partial_{\tilde{C}} A = \partial_{\tilde{C}} B = 0$. The "associativity" axioms of the $\ast$ product are

ii) "associativity" -

$$(A \ast B) \ast C = -(-1)^A A \ast (B \ast C)$$

$$\partial_{\tilde{C}} (\partial_{\tilde{C}} (A \ast B) \ast C) = \partial_{\tilde{C}} (A \ast \partial_{\tilde{C}} (B \ast C)).$$

The integral is defined to satisfy

$$\text{iii) } \int A \ast B = -(-1)^{AB} \int B \ast A.$$

Q satisfies the additional axioms

iv) $\int Q\chi = 0$ for any string functional χ , and

v) the "Leibnitz rule" -

$$-Q(A \ast B) = QA \ast B + (-1)^A A \ast QB.$$

Note that the axioms (i) and (iv) are identical to those of the open string. (ii),(iii) and (v) have been altered. We shall also assume that the integral $\int$ and $*$ product can be defined to have ghost number -3 and $+3$, respectively.

Before we attempt to justify our apparently bizarre set of axioms, let us first discuss some results which follow from their use. We are able to define a field strength

$$F = \partial\tilde{C} (1/2 [\tilde{C}, Q] A + A^* A) \equiv QA + \partial\tilde{C} (A^* A) \quad (2.20)$$

which under the local gauge transformation

$$\delta A = Q\epsilon + \partial\tilde{C} [A, \epsilon] \quad (2.21)$$

(where $[A, \epsilon] \equiv A^* \epsilon - \epsilon^* A$) transforms by

$$\delta F = \partial\tilde{C} [F, \epsilon] . \quad (2.22)$$

The fully interacting theory is then defined by the action

$$S = \int (A^* 1/2 [\tilde{C}, Q] A + 2/3 A^* A^* A) . \quad (2.23)$$

[Note that as a result of the axioms $\int (A^* A)^* A = \int A^* (A^* A)$.] It is invariant under the transformation (2.21). A proof of this statement requires using the following identities which follow from integration by parts and the "sewing condition" :

$$\text{"sewing condition"} - \int A^* B = \int A^T B \quad (2.24)$$

(where A^T is defined after eq.(2.18))

identities -

$$\int (\partial\tilde{C} A)^* B = (-1)^A \int A^* \partial\tilde{C} B$$

$$\int A^* [\tilde{C}, Q] B = \int ([\tilde{C}, Q] A)^* B .$$

The equations of motion are given by

$$\delta S / \delta \chi = F = 0 \quad (2.25)$$

where $A \equiv \partial \tilde{c} \chi$. For the free theory these equations reduce to those of eq.(2.12) which has been shown to include the equations of linearized gravity.

Representation of the Axioms

The Integral - $\int$

Our definition of the integral for the closed string is a simple generalization of our results for the open string. The "naive" integral is defined by folding the closed loop in half (see fig. 3) and "sewing" the two halves together. There are now two singular points at $\sigma = 0$ and π , which requires an insertion in order to satisfy axiom (iv). The correct measure is given by

$$\int \equiv \int D \delta I \quad (2.26)$$

where $D \equiv \prod_{\sigma=0}^{\pi} dx_{\mu}(\sigma) dc(\sigma) d\tilde{c}(\sigma)$, $\delta \equiv \prod_{\sigma=0}^{\pi} \delta(x^0) \delta(c^0) \delta(\tilde{c}^0)$,

$$x^0_{\mu}(\sigma) \equiv x_{\mu}(\sigma) - x_{\mu}(-\sigma),$$

$$c^0(\sigma) \equiv c(\sigma) - c(-\sigma),$$

$$\tilde{c}^0(\sigma) \equiv \tilde{c}(\sigma) + \tilde{c}(-\sigma),$$

and $I \equiv c(0) \delta / \delta \tilde{c}(0) \times c(\pi) \delta / \delta \tilde{c}(\pi)$. The ghost number of the factors are $D [0]$, $\delta [1]$ and $I [-4]$. Thus the integral has ghost number -3. It satisfies $\int Q\chi = 0$ for all χ (when the dimension of space-time is 26).

The * product

We shall define a * product by the expression

$$A * B[3] = \int D_1 D_2 K[312] A[1] B[2] \quad (2.27)$$

where $D_1 = \prod_{\sigma=-\pi}^{\pi} dx_{\mu}^1(\sigma) dc^1(\sigma) d\tilde{c}^1(\sigma)$. The "naive" kernel K is a product of delta functions which "sew" together half of string A to half of string B as shown in fig. 4.

We assume that the integral and $*$ product satisfy the "sewing condition" (eq. 2.24) which implies that K satisfies

$$\int D_3 \delta_3 I_3 K[312] = \delta[12] . \quad (2.28)$$

The "sewing condition" determines two important properties of K and thus of the $*$ product. It fixes the ghost number of K to be $+3$, since $\delta[12]$ has zero ghost number. Thus the $*$ product has ghost number $+3$, as desired. It also fixes the kernel K to be Grassman [odd], using D_3 [even], δ_3 [odd], I_3 [even] and $\delta[12]$ [even]. Most of the changes in the axioms can be traced back to the fact that for the closed string the kernel is

Grassman [odd], where as for the open string K is Grassman [even]. To illustrate why this change occurs consider one of the factors, for example D . For the open string $D = \prod_{\sigma=0}^{\pi} dx_{\mu}(\sigma) dc(\sigma) d\tilde{c}(\sigma)$ which in component form is given by $D = \prod_{n=0}^{\infty} dx_{n\mu} \prod_{n=0}^{\infty} dc_n \prod_{n=1}^{\infty} d\tilde{c}_n$. The product $\prod_{n=1}^{\infty} dc_n d\tilde{c}_n$ is Grassman even; thus the extra zero mode for $c(\sigma)$ makes D

Grassman odd. For the closed string $D = \prod_{\sigma=-\pi}^{\pi} dx_{\mu}(\sigma) dc(\sigma) d\tilde{c}(\sigma)$ which in component form is given by $D = \prod_{n=-\infty}^{+\infty} dx_{n\mu} \prod_{n=-\infty}^{+\infty} dc_n \prod_{n=-\infty}^{+\infty} d\tilde{c}_n$. Since now there are an equal number of modes for c and $\tilde{c}$, D is Grassman even. Similar considerations have been used to obtain the Grassmanality of all the other factors. Since K is Grassman odd, one immediately concludes that $A * A$ is also Grassman odd.

Let us now reconsider the axioms.

ii) "associativity" -

The expression $(A*B)*C = -(-1)^A A*(B*C)$ is equivalent to the following familiar relation for K : $\int D_3 K[534] K[312] = - \int D_3 K[513] K[324]$ (see eq.1.31c for the open string). Since K is Grassman odd it is not possible for the $*$ (eq.2.27) to be associative for both even and odd Grassman valued fields.

The expression $\partial\tilde{c}(\partial\tilde{c}(A*B)*C) = \partial\tilde{c}(A*\partial\tilde{c}(B*C))$ is equivalent to the relation for K : $\int D'_3 K_1[534] K_1[312] = - \int D'_3 K_1[513] K_1[324]$ where $K_1[312] = -\partial\tilde{c}_3\partial\tilde{c}_1\partial\tilde{c}_2 K[312]$ and $D = D' \partial\tilde{c}$. Note since K_1 is Grassman even, the above associativity rule does not distinguish between even and odd Grassman valued fields.

iii) $\int A*B = -(-1)^{AB} \int B*A$ This axiom is a direct consequence of the "sewing condition" and the identity for the closed string $\delta[12] = -\delta[21]$ where $\delta[12] = \prod_{\sigma_2=-\pi}^{\pi} \delta(x^1_\mu(\sigma) - x^2_\mu(-\sigma)) \delta(c^1(\sigma) - c^2(-\sigma)) \delta(\tilde{c}^1(\sigma) + \tilde{c}^2(-\sigma)) = \prod_{\sigma_2=0}^{\pi} \delta(x^1_{e\mu} - x^2_{e\mu}) \delta(x^1_{o\mu} + x^2_{o\mu}) \delta(c^1_e - c^2_e) \delta(c^1_o + c^2_o) \delta(\tilde{c}^1_e + \tilde{c}^2_e) \delta(\tilde{c}^1_o - \tilde{c}^2_o)$ and $x_{e(o)}(\sigma) = (x(\sigma) + (-) x(-\sigma))$. Since there is one additional mode in the expansion of c_e , i.e. the zero mode, than there is for $\tilde{c}_o$ we obtain the above identity.

v) the "Leibnitz rule" -

$$-Q(A*B) = QA*B + (-1)^A A*QB$$

Although this rule appears quite unusual it is nevertheless equivalent to the simple condition on K : $\sum_{i=1}^3 Q_i K[312] = 0$. Once again the strange sign in this axiom is a direct consequence of the odd Grassmanality of K .

4. Summary and Conclusions

We have explored the gauge invariant string field theory introduced by Witten. We have tried to find a representation of the axioms keeping the anti-commuting coordinates. In particular we were able to find an explicit representation of the integral $\int$ for both the open and closed strings. Unfortunately our representation of the $*$ product was incomplete due to the special problems associated with the point $\sigma = \pi/2$ for the open string and $\sigma = 0$ and π for the closed string. We feel however that this technical problem can certainly be overcome [8]. We chose to work with the anti-commuting coordinates in order to be able to use the intuition gained from the component expansions, as described in the text. We believe however that our representation of $\int$ and $*$ is nevertheless equivalent to the path integral representation discussed by Witten. A proof of this assertion would require an explicit representation of the kernel K . One would then be able to explicitly check whether the axioms are satisfied.

In the case of the open string we have assumed that Q , $\int$ and $*$ satisfy the requisite axioms. We have then used these axioms to present the BRST invariant action for the interacting string.

In the case of the closed string we have found the free action and

demonstrated that the equations of motion for the massless sector includes the equations of linearized gravity , as well as the equations for the Kalb-Ramond gauge field and a dilaton. We have then obtained an abstract set of axioms which can be used to construct the gauge invariant action for the interacting closed bosonic string. These axioms, though new, are motivated by properties satisfied by the path integral representation of $\int$ and $*$ discussed by Witten. We have assumed that $\int$ has ghost number -3 and $*$ has ghost number $+3$. The "sewing condition", which is satisfied by the path integral representation , was used to motivate many of the new axioms. In particular, we used the "sewing condition" to prove the important result that the kernel K for the closed string was Grassman odd. Combining this result with Witten's result $\sum_{i=1}^3 Q_i K[312]=0$ we obtain the new "Leibnitz rule". There is one property of the $*$ product which was crucial but not sufficiently motivated. We have assumed that $\partial\tilde{C}(A*B) \neq 0$ when $\partial\tilde{C} A = \partial\tilde{C} B = 0$. This relation is probably satisfied by the path integral representation. The two dimensional manifold on which the $*$ product is defined violates ghost number by $+3$. By the Reimann-Roch index theorem there are 3 nontrivial ghost zero modes on this two dimensional surface. Thus $\hat{c}$ and $\bar{\hat{c}}$ are emitted by the manifold. $\tilde{c}$ is contained in $\hat{c}$ and $\bar{\hat{c}}$ and is probably also emitted. As a result we find $\partial\tilde{C}(A*B) \neq 0$.

If our axioms for the closed string are indeed satisfied by the path integral representation of Witten, we should be able to obtain the scattering amplitudes of the Virasoro-Shapiro model using our action. This calculation should be done.

We are not satisfied with the constraints $\Delta N A = 0$ and $A = \partial_{\tilde{C}} \chi$ which we found necessary to obtain the interacting closed string action. Perhaps these should be thought of as gauge conditions in a theory with an enlarged symmetry group. These constraints are even more unappealing in the context of the closed superstring as discussed by Banks et. al. [13].

Finally, work is now in progress to understand the symmetry principle embodied in the transformation laws of equations (1.20) for the open string and (2.21) for the closed string.

Footnotes

1. General relativity is only valid in the effective low energy theory defined in the zero slope limit.
2. Perhaps a complicated field redefinition of the "graviton" in the Virasoro-Shapiro model does actually transform as $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ in general relativity- private communication- B. Zwiebach.
3. A similar result for the free closed bosonic string has been obtained by Banks et. al. [13].

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Figure Captions

- 1) This figure illustrates the $*$ operation for open strings. The '+' half of string A is sewn with the '-' half of string B . The remaining halves become the string $A*B$.
- 2) The integral $\int$ sews two halves of the same string. This is illustrated for the string $A*B$. The final result, $\int A*B$, could have been obtained by simply sewing strings A and B , with opposite orientations, along their entire lengths.
- 3) The integral $\int$ for closed strings also sews two halves of the same string. This results in two singular points, labelled $\sigma=0$ and $\sigma=\pi$ in the figure.
- 4) This figure illustrates the $*$ operation for closed strings. Half of string A is sewn with half of string B . The remaining halves become the closed string $A*B$.

(1)

(2)

(3)

(4)