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EXPERIMENTAL STUDIES OF PLASMA
WAKE-FIELD ACCELERATION AND FOCUSING

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1. Introduction

More than four years after the initial proposal of the Plasma Wake-field Accelerator^[1] (PWFA), it continues to be the object of much investigation, due to the promise of the ultra-high accelerating gradients that can exist in relativistic plasma waves driven in the wake of charged particle beams. These large amplitude plasma wake-fields are of interest in the laboratory, both for the wealth of basic nonlinear plasma wave phenomena which can be studied, as well as for the applications of acceleration and focusing of electrons and positrons in future linear colliders. Plasma wake-field waves are also of importance in nature, due to their possible role in direct cosmic ray acceleration.^[2] The purpose of the present work is to review the recent experimental advances made in PWFA research at Argonne National Laboratory, in which many interesting beam and plasma phenomena have been observed. Emphasis is given to discussion of the nonlinear aspects of the PWFA beam-plasma interaction.

Experimental investigations of the PWFA at the Argonne Advanced Accelerator Test Facility^[3] (AATF) have produced a multitude of accomplishments in the short time since their inception in 1986. The results of these experiments have directly verified the existence of electron acceleration in plasma wake-fields by the injection of a test bunch of electrons into a beam-driven plasma wave, thus measuring the longitudinal dependence of the accelerating and deflecting wake-fields. These proof-of-principle experiments,^[4] which will be described below, validated major relevant predictions of linear wake-field theory,^[5-7] including the fundamental excitation of the electron plasma waves with their associated longitudinal and transverse electrostatic fields, the electromagnetic self-pinching forces of a high intensity driving beam, and the dependence of these phenomena on beam and plasma characteristics.

After the proof-of-principle experiments were completed, improvements in facility operation allowed a second round of experiments which explored an entirely different, nonlinear regime of the PWFA. High resolution measurements of the

wake-fields excited by more intense driving beams revealed large nonlinearities in the wake-fields, indicating that the driving beams became significantly self-pinched within the plasma by their transverse wake-fields.^[8] Both the nonlinearity and the self-pinching are exciting observations, as the nonlinear wake-fields can theoretically yield large transformer ratios,^[9] and the self-pinching wake-fields have been proposed for use in a powerful final focusing lens for a linear e^+e^- collider.^{[8][10]} The results and analysis of the nonlinear PWFA experiments will also be presented below.

A third set of experiments have also been recently completed, in which the focusing effect on the driving electron beam, which was inferred from the nonlinear PWFA measurements, was directly observed. A streak camera-based diagnostic allowed time-resolved measurement of the beam density transverse profiles at the end of the plasma column used in the PWFA experiments. These self-pinched profiles showed the development of Bennett-like^[11] equilibria, with radii smaller than a plasma skin-depth. The experimental results, as well as theoretical analysis of the self-focusing process, are discussed in detail. Particular attention is paid to analysis of the way Bennett-like equilibria can be approached through nonlinear phase space filamentation.

After the discussion of the experimental investigations which have been completed to date, we will turn to the subject of future efforts in this field. Prospects for future experiments, to be performed with a much more intense beam derived from a new laser photo-cathode source at the AATF, will be examined. Theoretical expectations for nonlinear wake-field amplitudes in these experiments will be estimated; they are predicted to be greater than 1 GeV/m.

2. The Argonne Advanced Accelerator Test Facility (AATF)

The AATF was constructed, for the purpose of studying the physics of wake-field acceleration, by the Argonne High Energy Physics Division Advanced Accelerator Research Group at the Argonne Chemistry Division 21 MeV, intense, short pulse electron linac. Beam pulses delivered from this L-band linac to the entrance of the AATF typically contain $N \simeq 4 \times 10^{10}$ electrons and are of length $\sim 15-25$ psec FWHM. The transverse emittance of these beam pulses is typically 7π mm-mrad. This beam is then used to create both a driving and a witness beam for wake-field studies.

A schematic of the AATF layout is shown in Fig. 1. The high intensity 21 MeV driving beam pulse, which is made up of the majority of the incident linac beam, traverses the upper beam line. A secondary, low intensity, 15 MeV witness beam, which is created by degrading a portion (typically 20-30 percent) the linac pulse on a carbon target just prior to the facility entrance, is transported through the lower beam line. The combination of scattering and energy spread in the energy-degraded electron population requires momentum and emittance collimation in the witness beam line; approximately 10^7 electrons per pulse are accepted into this transport line. The witness beam leg contains an adjustable "trombone" section which allows variation of the delay time between driver and witness from -0.2 to greater than 1.0 nanosecond. Both beam lines are designed to be isochronous in order to preserve the bunch time structures. The two beams are recombined at the end of the facility and pass through the wake-field experimental section. The minimum beam beta-function in the experimental section is $\beta^* \geq 30$ cm, giving a rms transverse beam size $\sigma_r \geq 1.4$ mm. Both beams are analyzed for energy spectra and transverse deflections by a high resolution $\Delta p/p \simeq 10^{-3}$, broad range spectrometer. The beam profiles at the focal plane of the spectrometer, as well as at the beam position monitors in the AATF, are monitored by CCTV imaging of phosphor plates. Beam currents are measured by use of Faraday cups. A 1-picosecond resolution streak camera allows deter-

mination of the beam pulse length at the entrance and exit of the facility, as well as measurement of the beam's longitudinal phase space distribution at an up-stream spectrometer magnet. All facility optics systems and data acquisition are under remote computer control.^[12]

The high intensity and short pulse length of the facility driving beam make it useful for driving plasma waves and their strong wake-fields. However, the crucial, unique experimental tool the AATF possesses is its witness pulse, which allows the probing of the plasma wake-fields at arbitrary positions behind the driver. Although this paper is concerned with description of the PWFA experiments, it should be mentioned that alternative wake-field schemes, employing disk-^[3] and dielectric-loaded^[13] cylindrical waveguides as a slow-wave structure, have also been investigated experimentally using the unique capabilities of the AATF.

3. Linear Plasma Wake-field Theory

The two dimensional linear theory of plasma wake-fields, as formulated by Chen,^[6] provides a simple model for calculating expected wake-fields in a PWFA experiment. This model has been used in optimizing the experimental parameters to match the beam characteristics at the AATF. The linear theory developed thus far applies to a cold plasma with density perturbation small compared to the unperturbed electron density, and assumes an unchanging beam density profile, which are good approximations for the experimental situation encountered in the proof-of-principle experiments.

We begin our discussion of the linear theory by reviewing the notion of a wake-field. Assuming the beam motion is mainly parallel to the z -axis, the transverse and longitudinal wake-fields are defined as

$$\mathbf{W}_{\perp} = q(\mathbf{E}_{\perp} + \frac{1}{c}\mathbf{v}_b \times \mathbf{B}) \quad \text{and} \quad (3.1)$$

$$W_z = qE_z, \quad (3.2)$$

respectively, and where the $\mathbf{E}$ and $\mathbf{B}$ fields are assumed to be beam generated. If we assume that the wake-fields are a function of the longitudinal coordinate and time only through the variable $\xi = z - v_b t$, then it is easy to show that $\nabla \times \mathbf{W} = 0$ and thus the wake-fields can be derived from a potential. This is merely the differential form of the Panofsky-Wenzel theorem.^[14] Since $v_b \simeq c$ for beams of interest we can write this wake potential as $\Omega = A_z - \phi$, where A_z is the longitudinal component of the electromagnetic vector potential and ϕ is the electrostatic scalar potential. The wake-fields are given in this approximation by $\mathbf{W} = \nabla \Omega$.

From the cold plasma fluid equations with the ions assumed stationary, a linearized equation has been derived^[6] which describes small amplitude plasma electron oscillations driven by an extremely relativistic electron beam. The equation can be written as

$$\frac{\partial^2 n_1}{\partial \xi^2} + k_p^2 n_1 = -k_p^2 n_b, \quad (3.3)$$

where $n_1 = n - n_0$ is the deviation of the plasma electron density from equilibrium density n_0 , $k_p \equiv 2\pi/\lambda_p = \omega_p/c = \sqrt{4\pi e n_0}$ is the plasma wave number, and n_b is the beam density. The linearization of the equations requires that $n_1 \ll n_0$.

If we assume that the beam density distribution can be expressed in separable form as

$$n_b = \rho_b f(r) g(\xi), \quad (3.4)$$

where ρ_b is the maximum beam density, r is the radial coordinate, and ξ is the longitudinal coordinate measured from the center of the bunch, the solution of Eq. (3.3) is given by^[6]

$$n_1 = -k_p \rho_b f(r) \int_{\xi}^{\infty} g(\xi') \sin(k_p(\xi' - \xi)) d\xi' \equiv -\rho_b f(r) G(k_p \xi). \quad (3.5)$$

Note that the plasma charge density $-en_1$ has the same transverse dependence as the beam density $f(r)$. The longitudinal response, however, is described by

$G(k_p \xi)$, a convolution integral over the beam density ahead of the point in question with the causal Green's function solution to Eq. (3.3).

Also notice that for an infinitesimally short driving beam of surface charge density Σ_b the plasma wave left in the wake of the beam has the form $n_1 = -k_p \Sigma_b f(r) \sin(k_p \xi)$. This is the most efficient (impulse) excitation of the plasma wave; if the beam is not short compared to a plasma wavelength then the amplitude of the excited wave is smaller, as the plasma electrons have time to react to and partially neutralize the beam space charge density. We quantify this effect by defining the wake plasma wave longitudinal excitation efficiency to be

$$\eta_l = \frac{\|n_1\|}{k_p \rho_b f(r) \int g(\xi) d\xi}, \quad (3.6)$$

which is the density perturbation amplitude normalized to that obtained for an infinitesimally short driver beam of the same transverse profile. This factor can be calculated by performing the convolution integral in Eq. (3.5). For a beam with a longitudinal Gaussian profile of rms length σ_z , we have

$$\eta_l = \exp [(-k_p \sigma_z)^2 / 2] \quad (3.7)$$

Once one knows the plasma response, the wake-fields are easily found. Chen derives the wake potential corresponding to the plasma response by solving the following equation:

$$(\nabla_{\perp}^2 - k_p^2) \Omega = -4\pi e n_1. \quad (3.8)$$

This equation yields approximate solutions to the wake-fields which ignore the terms of order γ^{-2} , i.e. the net force due to the beam vacuum fields.^[18] The causal solution to this equation is given by

$$\Omega = \frac{4\pi e \rho_b}{k_p^2} F(k_p r) G(k_p \xi) \quad (3.9)$$

where $F(k_p r)$ is the radial part part of the solution. The radial part is obtained alternatively from another Green's function integral expression.^[6] Note from the

above discussion that the wake-fields have both electrostatic and electromagnetic components. The homogeneous plasma waves in the wake region are purely electrostatic oscillations. Inside the beam, the electromagnetic portion of the wake-field is due to the self-magnetic field of the beam.

In the limit that the beam is wide compared to the plasma wave length $k_p \sigma_r \gg 1$, $F(k_p r) \simeq f(r)$, and the plasma motion and wake-field is mainly longitudinal. In the opposite limit, $k_p \sigma_r \ll 1$, the plasma motion and wake-field is largely radial. We thus define a radial field efficiency as follows $\eta_r = F(0)$ which is near unity for $k_p \sigma_r \gg 1$ and becomes logarithmically small for $k_p \sigma_r \ll 1$. The maximum accelerating wake-field driven on axis can now be written in terms of these two geometrical efficiencies,

$$eE_{z,max} = 4\pi r_e m_e c^2 \Sigma_b \eta_l \eta_r.$$

If the beam is disk-like, with $\sigma_z \ll \sigma_r$, then a working point in plasma density can be found in which η_l and η_r are near unity, and the largest amplitude plasma waves can be driven. On the other hand, if the beam is sausage-like, and $k_p \sigma_r \ll 1 \ll k_p \sigma_z$, the plasma responds to nearly completely neutralize the space charge density of the beam ($G(k_p \xi) \simeq g(\xi)$), while the plasma return current flows inside a cylinder of radius $\sim k_p^{-1} \gg \sigma_r$. Thus the beam's repulsive self-electric force is neutralized while the focusing self-magnetic force is left unbalanced.^[16] This focusing, which can be termed the plasma lens effect, is potentially quite powerful and can lead to self-pinching of the beam in the plasma. A self-pinched beam profile can in turn drive larger amplitude plasma waves, as will be emphasized when we discuss the nonlinear PWFA experiments.

4. Measurement of Linear Plasma Wake-fields

It is clear from the discussion above that the maximum longitudinal wake-fields are generated if $k_p \sigma_z \ll 1 \ll k_p \sigma_r$, which means a beam much shorter than it is wide. In the proof-of-principle linear PWFA experiments the beam was approximately round ($\sigma_z \simeq \sigma_r \simeq 2.4$ mm) and is thus not quite optimum. The total geometrical excitation efficiency $\eta_l \eta_r$ for this case has a maximum value of about 0.45, at a point where k_p would indicate use of a plasma density of $n_e = 8 \times 10^{12} \text{ cm}^{-3}$ for these beam dimensions. We may consider this an optimum point in plasma density to observe large accelerating gradients in the linear PWFA. Experimentally, there are two complications in this case over a disk-shaped beam: the transverse fields are larger relative to the longitudinal fields, and with a witness beam length not small compared to a plasma wavelength, the resolution of the measurement is degraded. In the linear PWFA measurements, the total beam charge per bunch is approximately 2.1 nC ($N = 1.3 \times 10^{10} e^-$).

The DC plasma source used for the PWFA experiment is a hollow cathode arc constructed at the University of Wisconsin. A schematic, not-to-scale drawing of the arc source is shown in Fig. 2. The arc electrical characteristics are typically $V_a \simeq 50$ V and $I_a \simeq 30 - 60$ A. The interelectrode plasma column length L_p is 20 - 35 cm, and density n_e is variable about our optimum working point, from approximately $0.5 - 8.0 \times 10^{13} \text{ cm}^{-3}$, with an electron temperature of 2 - 8 eV. An axial magnetic field 50 - 700 G is applied to confine the plasma column. The feed gas is argon, and the background pressure is limited to approximately 1-2 μm by use of a 750 l/sec turbo-molecular pump. Plasma diagnostics include microwave interferometry (line integral of plasma density across the column) and electrostatic triple probes. The probe measurements give local measurements of n_e (n_0) and T_e ; the density is also determined to excellent accuracy ($\sim 1\%$) by the PWFA measurement of the plasma frequency. A comparison of probe and PWFA plasma density measurements is shown in Fig. 3. The probe measurements agree with the more precise and unambiguous PWFA data to within 20%.

Both the witness and driver beam profiles are monitored on CCTV at the focal plane of the spectrometer. The video images are digitized through use of a frame grabber and stored on disk for off-line analysis. The witness beam data in PWFA experiments is complicated by the large and rapidly fluctuating background of light generated by the driver beam halo which precludes a simple subtraction of a background frame from the data. Instead, an algorithm is employed which extracts the majority of the witness beam spot from the rest of the image, and calculates the resulting centroids of the image distribution in the dispersive and deflection planes. Background baseline shifts due to changing witness beam transmission optics are then subtracted. Thus, at each point in witness beam delay we determine the centroids of the witness beam distribution in the energy analyzing and deflecting planes. The witness beam wake-field scans in these experiments consist of many such points incremented in delay by fine time steps over a range of many wake-field oscillation lengths.

Witness beam delay scans, displaying beam energy centroid as a function of the time delay between driver and witness beam, are shown in Figs. 4 (Scan 1) and 5 (Scan 2). The dashed lines indicate the predictions of the linear PWFA theoretical model, plotted for comparison to the data. The theoretical predictions include the experimental effects of the resolution of the witness beam, which we discuss below. These effects make the predictions approximate due to the shot-to-shot fluctuation of the beam length and charge measurements.

All theoretical predictions consistent with these uncertainties would form a band about the dashed line shown in each figure. Only the central prediction is shown for clarity. Ahead of the driver, we observe small fluctuations in the energy centroid due to background and linac fluctuations. The centroid then lowers while passing through the region near zero delay, as it feels the decelerating wake on the driver. Behind the driver we observe oscillations characteristic of the plasma wake-fields, harmonic with the plasma frequency. These oscillations persist for many wavelengths, and maintain a nearly constant frequency. In both scans the driver beam charge, measured by using the spectrometer as a Faraday cup, was

determined to be 2.1 nC, with shot-shot fluctuation of approximately 5%. The plasma characteristics were: scan 1 - $L_p = 28$ cm and $n_e = 8.6 \times 10^{12} \text{cm}^{-3}$, and scan 2 - $L_p = 33$ cm and $n_e = 2.3 \times 10^{13} \text{cm}^{-3}$. The densities quoted above were deduced from FFT calculation of the oscillation frequency.

The maximum accelerating wake-field amplitude W_m can be calculated from the maximum energy centroid excursions δE_m . We must first average over the longitudinal wake-field felt by different parts of the witness beam spatial distribution, and divide by the resulting resolution function. This function can be divided into two factors, a radial resolution function $R_r \simeq \sigma_{r,d}^2 / (\sigma_{r,w}^2 + \sigma_{r,d}^2)$, where $\sigma_{r,d(w)}$ is the radial beam size of the driver (witness), and a longitudinal resolution function $R_l = \exp -[(k_p \sigma_z)^2 / 2] = \eta_l$. For both scans the radial resolution R_r is about 0.5, but with the higher density in scan 2 one obtains less longitudinal resolution due to the shortening of the plasma wavelength. For the beam and plasma characteristics of Scan 1, $R_l = 0.43$, and for Scan 2, $R_l = 0.11$. The maximum accelerating wake-field is thus given by the expression

$$W_m = \frac{\delta E_m}{R_r R_l L_p}, \quad (4.1)$$

which gives $W_m \simeq 0.94_{-0.19}^{+0.27}$ MeV/m for Scan 1 and $1.60_{-0.30}^{+0.63}$ MeV/m for Scan 2. The uncertainty in the determination is due mainly to the 20% uncertainty in the beam length (as determined by streak camera measurements) from shot-to-shot, which strongly affects the calculated wake-field amplitudes through the quickly varying longitudinal resolution function. The agreement with the simple theoretical model is quite good, as indicated in Figs. 4 and 5.

Scanning further back in delay shows a decay in amplitude of the measured energy centroid shifts. Since the plasma is not collisional ($\omega_p \gg \nu_{e-i}$, the collision frequency), we attribute this decay to the longitudinal density gradient along the plasma column. The phase delay between the two beams is related to the time delay by $\Delta\phi = \omega_p(z)\Delta t$, so that a variation in the plasma density changes the phase delay more dramatically at larger time delays. These larger changes in

the phase delay cause the total wake-fields integrated by the witness beam to be diminished. Assuming the plasma density gradient along the column is small and constant, the total density variation $\delta n_e/n_e$ in the plasma can be related to the observed damping of the measured wake-field amplitude $\delta W_m/W_m$ by

$$\frac{\delta W_m}{W_m} \simeq \left| \text{sinc}\left(\frac{\omega_p \Delta t}{4} \frac{\delta n_e}{n_e}\right) \right|. \quad (4.2)$$

For a scan shown in a following section (Scan 4), in which we observed a degradation in amplitude $\delta W_m/W_m \simeq 0.6$ after eighteen plasma wavelengths of delay we deduce a density variation of about 6%. This also is consistent with electrostatic probe measurements.

The existence of strong transverse plasma wake-fields is of great interest due to their possible application as a final focusing scheme for linear colliders. (This subject is explored in great detail in the literature.^{[6][10][16][17,18]}) Evidence for such forces were observed in transverse deflections in the witness beam non-bend plane position centroid when the witness beam and driver were not collinear. One such scan (Scan 3) is shown in Fig. 6, which plots the position centroid versus delay time. For this scan the driver beam charge is 2.2 nC, the plasma length is 33 cm and the plasma density is $n_e = 3.4 \times 10^{-13} \text{ cm}^{-3}$. The deflection is quite strong at zero delay, where the driver magnetic fields provide a strong pinching force. The plasma excitation efficiency is very low for this case due to the high plasma density, as the plasma lens regime has been entered, so the focusing wake-fields are expected to be non-negligible only inside the driver, as is observed. The 5 mm maximum position change on the focal plane corresponds to approximately 2.5 mrad of deflection, or 37.5 keV of transverse kick, which is of the same order of magnitude as the acceleration amplitudes observed in the linear on-axis scans.

A typical driver beam energy loss spectrum is shown in Fig. 7, with a comparison to the case of no plasma present. Qualitatively, we observe the most total driver energy loss at lowest plasma density, which is expected, as the total energy

loss should scale as η, η_l^2 , and $\eta_l \sim \exp(-n_0)$. However, we are unable to quantify the total energy loss well with the present spectrometer geometry, because the self-pinching of the decelerating driver beam electrons deflects a portion of the beam distribution off the focal plane detectors. This is further evidence, of course, for the strong focusing wake-fields inside the driver beam.

5. Review of Nonlinear Plasma Wave Theory

Much of the theoretical effort to date has concentrated on the linear regime of the PWFA, in which assumption of small amplitude electron density perturbations $\|n_1/n_0\| \ll 1$ allows linearization of the fluid equations. The predominance of linear theory summarized above is due to its usefulness – it allows calculation of the general three-dimensional variation of the transverse and longitudinal wake-fields if the driving electron beam current distribution is a known quantity, unchanging in time. This linear treatment is of course inadequate to deal with self-pinching of the driver beam,^[18] the effects of which have been previously simulated for early proposed experimental scenarios.^[19] In addition, previous theoretical analyses of the nonlinear regime of the PWFA have treated only the one-dimensional case,^{[9] [20–22]} due to the mathematical intractability of the three-dimensional problem.

For extremely large amplitude plasma waves driven by electron beams the theoretical models developed thus far predict many interesting physical phenomena, which result from relativistic, convective, and modulational effects ignored in linear theory. The most intriguing prediction is the aforementioned possibility of obtaining high transformer ratios by driving waves with a long electron beam ($l_b \gg \lambda_p$) of density $n_b = n_0/2$; the peak value of n_1/n_0 in the wake of such a beam can be much greater than unity. In addition, the problems of background electron trapping and thermal effects,^{[20] [21]} ion motion, and continuous beam loading^[23] have all been explored using the one-dimensional theory.

For moderately nonlinear one-dimensional waves (the plasma electrons become only slightly relativistic^[20]), steepened wave profiles develop which can be decomposed into Fourier harmonics of the fundamental (plasma) frequency. The harmonic decomposition of the perturbed plasma electron density can be written as^[24]

$$\frac{n_e - n_0}{n_0} = \sum_{m=1}^{\infty} \left(\frac{n_1}{n_0}\right)^m \frac{m^m}{2^{m-1}m!}, \quad (5.1)$$

where n_1/n_0 is henceforth taken to be the amplitude of the *fundamental* component of the electron plasma wave; the coefficients of the harmonics form a power series expansion in this quantity, and are nontrivial when n_1/n_0 is not small compared to unity. Nonlinear beat-wave excited plasma waves have been indirectly observed experimentally^[24] by laser scattering off these harmonics in the electron density perturbation and predicted in computer simulations of plasma wake-fields.^[19]

A one-dimensional treatment of the nonlinear PWFA is expected to be valid only if the driving beam is transversely uniform in density and wide compared to a plasma wavelength. We have already seen from the linear experiments that this is not the case for beams at the AATF. As there exist no two- or three-dimensional nonlinear treatments of the PWFA, and since we predict quite dramatic changes in the transverse profile of the driving beam, we must therefore employ a hybrid of linear and nonlinear plasma wave theories, as well as plasma focusing theory to understand the experimental results we present below. Using theoretical arguments originally derived for the study of beam dynamics^[10] in the plasma lens, we can estimate the expected reduction in beam size for the beam and plasma conditions present, in order to address the issue of driver beam pinching analytically.

This predicted pinched beam profile can then be used in the approximation of linear self-focusing forces to calculate the amplitude of the fundamental frequency excitation of the plasma wake-fields from Eq. (3.5). Using the harmonic decomposition of the one-dimensional density wave in Eq. (5.1) as a basis for a perturbation treatment, we are able to estimate the expected amplitude of the higher harmonics in the narrow ($k_p \sigma_r < 1$) 3-D wake-fields. Use of the non-relativistic results is justified even for fairly large n_1/n_0 in our case, as plasma electron velocities for radially dominated waves are not nearly as large as for wide beams of equivalent n_1/n_0 . Despite the lack of rigor in the perturbation treatment, this analysis will be shown to give good qualitative agreement with the experimental results.

6. Experimental Measurement of Nonlinear Plasma Wake-fields

In the second round of PWFA experiments performed at the AATF, several changes in the experimental scenario allowed observation of plasma wake-fields with the clear signature of nonlinearity: wave steepening, with its accompanying harmonic generation. In particular, the raising of the driving beam current by a factor of two, coupled with the tighter initial focusing of the driver and higher resolution measurements (scans finer in time step) of both longitudinal and transverse plasma wake-fields yield dramatic demonstrations of the nonlinear effects.

The 21 MeV driving electron beam pulse used in the nonlinear experiments has the following characteristics: number of electrons per pulse $N = 2.5 \times 10^{10}$ ($Q = 4$ nC), rms pulse length $\sigma_z = 2.1$ mm, and initial rms radius $\sigma_r = 1.4$ mm. The plasma source length was set at $L_p = 33$ cm, and variable plasma densities were explored, as before.

To illustrate most of the observed relevant nonlinear PWFA phenomena we show two high resolution wake-field scans. Scan 4, shown in Figs. 8 - 10, is taken with a relatively low plasma density $n_0 = 7.3 \times 10^{12} \text{ cm}^{-3}$. The witness and driver beams are horizontally misaligned in this scan to allow observation of the longitudinal dependence of the transverse wake-fields. Several qualitative remarks can be made upon inspection of Figs. 8(a) and 9. The first is that both the longitudinal and transverse wake-fields are stable, oscillatory functions of the distance behind the driving beam $\xi = ct - z$. This is surprising, as narrow nonlinear plasma wave may be subject to differences in frequency as a function of transverse position. Secondly, the longitudinal wake-fields W_z have taken on a more saw-tooth appearance, as would be expected from the one-dimensional nonlinear theory. The transverse wake-fields show a form consistent with the differential form of the Panofsky-Wenzel theorem, which implies that $W_z = \partial_z \int_0^r dr W_r$ for a cylindrically symmetric driver. It is apparent from inspection that the measured longitudinal wake-fields are to a good approximation proportional to the longitudinal derivative of the measured transverse wake-fields. In addition to these

qualitative remarks on the wave-forms, we have found it of use to employ the the Fourier spectrum of the longitudinal wake-fields in attempting to quantify the physical basis for the nonlinearity of these waves using our perturbation treatment. The FFT of the longitudinal wake-field shown in Fig. 8(a) is displayed in Fig. 8(b); note that the ratio of first harmonic to the fundamental amplitude in the wake-fields is about 0.3.

Using Eq. (5.1) for the amplitude of the m -th wave harmonic, the longitudinal electric field on axis associated with this charge density wave is, ignoring phase factors,

$$E_z = \sum_{m=1}^{\infty} E_m e^{imk_r \xi} = 4\pi e n_0 \sum_{m=1}^{\infty} \frac{n_m}{n_0} \cdot \frac{\eta_{rm}(mk_p \sigma_r)}{mk_p} e^{imk_r \xi}, \quad (6.1)$$

where $\eta_{rm}(mk_p \sigma_r)$ is the radial efficiency factor defined above, generalized for all the wave harmonics. This factor is a monotonically increasing function of its argument, and thus is an increasing function of harmonic number m . The ratio of first harmonic E_2 to fundamental E_1 amplitude is simply $1/2(n_1/n_0)(\eta_{r2}/\eta_{r1})$. This implies, for example, that the density perturbation at the fundamental frequency in Scan 4 is $n_1/n_0 \simeq 0.33$.

This wave amplitude is not consistent with the predictions of linear theory if one ignores possible pinching of the driver beam. This is easily seen, as in the linear theory the wake wave amplitude for a bi-Gaussian driver is given by^{[4][7]} $n_1/n_0 = 2Nr_e \exp[-(k_p \sigma_z)^2/2]/k_p \sigma_r^2$. For the present case, with σ_r taken as its initial value, this yields $n_1/n_0 = 0.08$, which is much smaller than the estimate from harmonic content. Thus we are led to suggest that significant self-pinching of the driver must have occurred.

In a following section we will document actual measurements of the beam self-pinching effects. As the measurements were performed nine months after the observation of nonlinear wake-fields, we present the list of evidence accumulated at the time of analysis of the nonlinear wake-field data to support the hypothesis

of beam self-focusing. The first observation, of course, is the previously discussed measurement of large witness beam deflections at zero delay, which indicate the existence of large focusing fields. Also recall our discussion of the expansion of the driver beam image in the spectrometer nonbend plane with plasma present, which implies strong overfocusing of the beam in front of the focal plane. In addition, we note that the driving beam is aperture limited by the plasma anode with the plasma source off, and 20% of the beam current is not transmitted to the spectrometer. When the plasma is turned on, the full current is transmitted through the previously limiting aperture. Finally, we can examine a different range of Scan 4, shown in Fig. 10, one in which the beam charge is diminished a factor of 0.72 from 4.0 to 2.9 nC, and we observe a disproportionate reduction in the wake-field amplitude, to a factor of approximately 0.35. This indicates that the pinch effect has been taken below a threshold for significant reduction in beam size, with its resultant enhancement of the wake-field amplitude. Note also that the longitudinal wake-fields are much more sinusoidal at the lower amplitudes, and that the measured and predicted amplitudes (assuming no driving beam pinching) of n_1/n_0 in this case are 0.09 and 0.06, respectively, in much closer agreement than with the full beam charge present.

To study beam self-pinching effects in greater detail, we move on to the analysis of a Scan 5, shown in Fig. 11, in which the plasma is nearly four times as dense as in Scan 4. The nonlinearity of these waves is even more pronounced than in the previous scan. The plasma wavelength in this case is twice as short as before (0.65 mm), and is now shorter than the full beam length $4\sigma_z = 0.84$ mm. This case is geometrically, as in the linear scan shown in Fig. 6, in the regime of the plasma lens, where the self-focusing forces on the beam can be well approximated by assuming total space charge neutralization of the beam and evaluating the magnetic self-forces of the electron beam distribution. Although this produces non-ideal focusing forces, an approximate equation for the radius of the beam envelope can be derived that gives some insight to the beam dynamics.^[10] Assuming laminar beam electron flow (ignoring the finite emittance of the beam)

the equation of motion for the beam radius can be written approximately as

$$r'' + \frac{C(r_0)}{r} = 0, \quad (6.2)$$

where $C(r_0) \simeq (\nu/\gamma)(r_0/\sigma_r)^2$ near the center of a Gaussian transverse profile, and ν is the Budker parameter, the number of beam electrons per unit length, measured in units of r_e . The distance inside the plasma required to achieve a pinch (one-half of a betatron wavelength) s can be calculated by integrating this equation:

$$s = \frac{1}{\sqrt{2C(r_0)}} \int_0^{r_0} \frac{dr}{\sqrt{\log(r_0/r)}} = r_0 \sqrt{\frac{\pi}{2C(r_0)}}. \quad (6.3)$$

For a Gaussian initial profile we have $s \simeq \sqrt{\pi \epsilon_n \beta_0 / 2\nu}$, where $\epsilon_n = \gamma \epsilon$ is the normalized emittance. If we assume a Gaussian beam profile approximately in equilibrium with its magnetic self-pinch forces left unshielded by the plasma (a more exact treatment of the beam self-pinch equilibrium is presented in a following section), the equilibrium rms beam radius is $\sigma_{eq} \simeq \epsilon_n / \sqrt{\gamma \nu}$. The maximum current as a function of ξ in our beam corresponds to $\nu(0) = \nu_0 = 1.3 \times 10^{-2}$. Thus, with $\beta_0 = 0.3$ m and $\epsilon_n = 3 \times 10^{-4}$ m-rad we have $\sigma_{eq} = \epsilon_n / \sqrt{\gamma \nu_0} = 0.44$ mm and $s \simeq 8$ cm. Thus we expect the beam to pinch to approximately one-third its original radius well within the plasma column. This pinching is not uniform along the length of the beam, as in our model it depends on the enclosed current at a given point in ξ , a fact we can reflect by writing the invariant as a function of ξ , that is $\nu(\xi) = \nu_0 \exp(-\xi^2 / 2\sigma_z^2)$. We then have $\sigma_{eq}(\xi)^{-2} = (\nu_0 \epsilon) \exp(-\xi^2 / 2\sigma_z^2)$, and the equilibrium density profile on axis is proportional to $\exp(-\xi^2 / \sigma_z^2)$, i.e. the bunch is effectively shortened on axis by a factor of $\sqrt{2}$.

The FFT of the scan in Fig. 11 gives a relative first harmonic amplitude value of $E_2/E_1 \simeq 0.48$. The predicted value, using $\sigma_r = \sigma_{eq} = 0.44$ mm and $\sigma_z = 2.1/\sqrt{2}$ mm in the linear response formula, is $E_2/E_1 = 0.38$, in excellent agreement when compared to the prediction assuming no self-pinching of

$E_2/E_1 = 0.04$. In the first scan, the equilibrium pinched beam radius necessary for the theoretical estimate to agree with the measured harmonic content $\sigma_{eq} = 0.77$ mm is not as small as our calculated value for the second scan. This is due to the fact that the beam length is shorter than the plasma wavelength, and the beam does not encounter maximum strength transverse wake-fields, as the plasma does not react quickly enough to completely neutralize the beam charge. The driving beam thus takes longer to pinch and does not achieve such small spot sizes.

If we apply the same deconvolution analysis as before, using Eq. (4.1) to give the maximum accelerating wake-field amplitude, we obtain for Scan 4 $W_m = 1.48$ MeV/m, and for Scan 5 $W_m = 5.3$ MeV/m. In the presence of large nonlinearities, transverse forces and the transverse motion induced by these forces, we expect this analysis to be much less reliable, however. Simulations performed at UCLA^[25] indicate that the maximum average accelerating gradient for the conditions in Scan 5 is 7 MeV/m, in fairly good agreement with the experimental result. The prediction for Scan 4 is more problematic, however, as the simulation predicts a maximum average accelerating gradient approximately one order of magnitude greater than we have calculated from the data. This discrepancy is probably due to a combination of effects which are difficult to quantify. This difficulty is in part due to a lack of complete diagnosis of the witness beam distribution in this scan, as good measurement of maximized intensity driver beam is not compatible with the sensitive witness beam diagnostics. This is because the background level at the lower energy region of the spectrometer focal plane increases with the added driver beam energy loss. Thus the degree of misalignment, and the transverse witness beam profile are uncertain in this scan. The effects of the transverse motion in these wake-fields provide further uncertainty. Another, related source of diminished wake-field amplitude measurement is the probable loss of the tails of the witness beam distribution either in the background level of the focal plane, or completely off the focal plane. All these uncertainties mainly impact on the normalization of the amplitude of the wake-field measurement;

they do not greatly affect our conclusions concerning the form (harmonic content) or stability of the longitudinal and transverse nonlinear wake-fields in this scan. The observed form of the wake-fields is reproduced well in the simulations. The stability of the plasma oscillations in the simulations appears to be limited by numerical, rather than physical effects.

We have dealt successfully with the data here in a rather phenomenological manner. A more rigorous theoretical understanding of the physics issues would be desirable. The main questions raised by these experimental results have to do with the nonlinear plasma response. In our nonlinear PWFA measurements, the beam is much narrower than a plasma wavelength, and thus the plasma electron response and associated wake-fields cannot be well explained by a one-dimensional theory. Conversely, one-dimensional theory predicts improved transformer ratios in very nonlinear waves driven by long, wide beams of density $n_0/2$; it is not possible to confirm this prediction by extrapolating our results. On the other hand, the status of the theory and simulation of beam focusing in plasmas is more well understood than the three-dimensional nonlinear plasma motion. These experimental results are consistent with the theoretical picture of the self-pinch dynamics, and provided motivation for further experimental investigation of plasma focusing, some results of which are presented below.

In one-dimensional nonlinear theory, one would expect plasma electron relativistic effects to cause an increase in the plasma period in waves as steepened as we have presented here. We expect less severe relativistic effects in our cases, however, as the waves are predominately radial, which causes less energy to be transferred to the plasma electron motion and a concomitant degradation of the longitudinal wake-field amplitude. In fact, we did not observe significant dependence of the plasma wave frequency in this and subsequent experiments. A related aspect of these experiments is that the longitudinal and transverse wake-fields are quite well behaved under these conditions of moderate nonlinearity – the wake-fields in Scan 4 were observed out to 18 wavelengths behind the driver with little degradation in form or amplitude. A wave frequency dependence on

transverse position, which would be due to relativistic plasma effects, would tend to cause incoherence which could destroy the wake plasma waves. The stability of the observed wake-fields also argues for the absence of relativistic effects in these narrow, nonlinear plasma waves.

7. Theory of Beam Self-pinching in Plasma

Before moving on to the discussion of the measurement of self-pinched beam profiles in plasma, we must undertake a deeper discussion of the plasma response and the beam dynamics involved in this interaction. In the case where the double inequality $k_p \sigma_r \ll 1 \ll k_p \sigma_z$ holds, the assumption of space charge neutralization holds to a good approximation, and the beam electrons act under the influence of only their collective magnetic field. These beams were termed magnetic streams by Bennett in his original investigation (in 1934!) of the equilibrium of such a system.^[11] The Bennett profile can be derived by looking for a cylindrically symmetric equilibrium solution to the Vlasov equation,

$$-\frac{\partial f}{\partial t} = \frac{p_r}{\gamma m} \frac{\partial f}{\partial r} + F_r \frac{\partial f}{\partial p_r} = 0, \quad (7.1)$$

Assuming the distribution is separable in coordinate and momentum, $f(r, p_r) = R(r)P(p_r)$, and substituting in the dependence of the magnetic self-force F_r , we have the momentum equation

$$\frac{\partial P}{\partial p_r} = -\frac{\alpha p_r}{\gamma m} P, \quad (7.2)$$

where $-\alpha$ is the separation constant, and the radial equation

$$\frac{\partial R}{\partial r} = -R \frac{2\alpha e^2 \beta^2}{r} \int_0^r R(r') r' dr'. \quad (7.3)$$

The solution to the Eq. (7.2) is a Gaussian (thermal) distribution,

$$P = \sqrt{\frac{\alpha}{2\pi\gamma m}} \exp[-\alpha p_r^2 / 2\gamma m]. \quad (7.4)$$

The solution to the radial equation corresponding to this thermal equilibrium in

momentum space is a Bennett profile which has the form

$$R = \frac{n_b}{[1 + (r/a)^2]^2} \quad , \quad (7.5)$$

where a is the Bennett radius. The usual expression for the Bennett radius relates it to the beam Debye length λ_D , with $\beta \simeq 1$,

$$a^2 = 8\lambda_D^2 = \frac{2kT}{\pi e^2 n_{b0}} = \frac{2kT}{\nu m c^2} a^2 \quad , \quad (7.6)$$

which we see only specifies the relationship between the beam temperature and current and not the radius, which cancels out of the equation. This uncertainty must be removed by invoking another constraint, as illustrated below.

It has been known for some time that the Bennett profile, being a state in thermal equilibrium, can evolve from a different initial state due to the thermalizing influence of multiple scattering of beam particles off the background plasma ions.^[26] It has been observed in particle-in-cell simulations^[27] that a near equilibrium self-focused state develops from an unfocused initial profile after a few betatron oscillations past the initial focusing point. This distribution displays a Bennett-like spatial profile and a Gaussian-like momentum profile. The near-equilibrium state evolves from the initial pinch by filamentation of the beam phase space as it spirals under the influence of its own nonlinear self-fields. The tails of the distribution come into equilibrium less quickly than the core, as the betatron wavelength is a dramatically increasing function of amplitude. This non-thermal approach to equilibrium, whereby the coherent radial beam motion dissipates, was in fact initially sketched out by Bennett in 1955, who termed the effect 'mixing'.^[11] This effect is similar to the phenomenon of decoherence observed in the experimental study of nonlinear transverse dynamics in synchrotrons.^[28]

From the form of the self-fields of a cylindrically symmetric beam and the resultant Vlasov equation, we can see that the phase space density must be

constant at $(r, p_r) = (0, 0)$, as $\partial f / \partial t = 0$ there. If we take an original (four-dimensional) phase space density corresponding to a cylindrically symmetric bi-Gaussian profile, then this constraint gives

$$a^2 = \frac{4\epsilon_n}{\gamma\nu}. \quad (7.7)$$

Note that a betatron wavelength for small amplitude motion $r \ll a$ is $\lambda_\beta = \sqrt{2}\pi\epsilon_n/\nu \simeq 9\text{--}11$ cm for our experimental conditions, while from Eq. (6.2) we have that the initial pinch occurs in 7–9 cm. Thus, we expect a near-equilibrium state to develop after approximately 20 cm of plasma for our experimental parameters, which is shorter than the length of our column.

8. Direct Observation of Self-focused Beam Profiles in Plasma

While all of the experimental evidence presented above makes an excellent circumstantial case for the self-pinching of the driving beam in plasma, it is of great interest to examine the physics of this process through direct observation. To this end, we have implemented a streak camera-based diagnostic that allows measurement of the transverse profile of the beam as a function of longitudinal position in the beam. Physically, this scheme entails placing an opaque thin walled (to minimize multiple scattering effects) Cerenkov cell inside the anode, in direct contact with the downstream end of the plasma column. The design of this cell is heavily constrained by the difficulty of survival of this relatively fragile device in such a hostile environment as inside of an arc electrode. The wall in contact with the plasma is 450 μm (.005 radiation lengths) of aluminum at its narrowest region, a horizontal slit on which the beam is vertically centered. The beam profile at this horizontal slit is converted to a photon profile by Cerenkov radiation due to the beam electrons' interaction with 2 mm of xenon gas at atmospheric pressure. The beam electrons and Cerenkov photons exit the cell through a synthetic quartz window. The photon profile is then transported through a telescopic optics system and focused onto the horizontal slit of the

streak camera, giving a swept image that reflects the transverse profile of the beam in the horizontal direction versus time in vertical (streak) direction. The resolution of the profile obtained is $12\ \mu\text{m}$ in the transverse dimension and 1 psec in time.

As a witness beam is not necessary for the plasma focusing experiments, the witness beam target at the input to the AATF was removed and the full intensity, 21 MeV linac beam allowed to pass through the AATF and plasma source. Typically, the number of electrons per pulse was $N = 3.2 \times 10^{10}$, the beam length $\sigma_z = 2.1\ \text{mm}$ and width $\sigma_r = 1.4\ \text{mm}$ in these experiments. For these beam characteristics, the Budker parameter $\nu_0 = 1.6 \times 10^{-2}$.

The time-resolved pictures of the self-pinch transverse beam profile reveal a great deal about the physical processes involved – the plasma and beam dynamics. We show in Fig. 12 contour plots of the streak camera picture of the spatially resolved beam intensity, where the vertical axis is time and the horizontal axis is the transverse (radial) dimension. The plot in Fig. 12(b) is the profile with no plasma, and the plot in Fig. 12(a) shows the self-pinch profile in the case where the plasma density $n_e = 6.2 \times 10^{13}\ \text{cm}^{-3}$. In terms of the initial beam profile, this corresponds to a beam length in plasma radians $k_p \sigma_z \simeq 3$, and is thus in the regime where magnetic self-focusing occurs. The dependence of the focusing strength on enclosed current density is apparent from the contour shapes. With no plasma present, the iso-intensity contours are approximately elliptical, as expected for a beam Gaussian in both longitudinal and transverse dimensions. The self-pinch profile, however, shows a transverse distribution which narrows noticeably towards the longitudinal midplane. The self-pinch longitudinal profile measured near the propagation axis is also steepened and effectively shortened in time, as mentioned above in connection with the driving of nonlinear wake-fields. This phenomenon is also apparent from inspection of the contours near the transverse beam center, as they also narrow noticeably near the current maximum.

The plots in Fig. 13 shows the transverse cross-section (a projection of the most intense 0.5 mm long transverse strip) of both the cases from Fig. 12. In the case of no plasma present shown in Fig. 13(b), we also show a best fit of the data, to a Gaussian of width $\sigma_r = 1.4$ mm, with a dashed line. With dense plasma present we obtain the self-pinched profile shown in Fig. 13(a), with a best fit of the data, to a Bennett profile of radius $a = 0.91$ mm, indicated by the dashed line. The Bennett radius that we predict from our distribution function analysis is

$$a = \frac{2\epsilon_n}{\sqrt{\gamma\nu_0}} = 0.72. \quad (8.1)$$

That the experimentally measured value is larger can be attributed to several effects. The first is that the beam is not perfectly symmetric, and deviations from cylindrical symmetry undoubtedly allow more filamentation and effective dilution of the phase space density of the beam. The other major effect is due to the plasma response; even in the largest density case the product $k_p\sigma_z$ which results on axis after pinching (due to the effective pulse shortening described above) is about 2.2, which is just on the border of the adiabatic regime where one can assume approximate charge neutralization of the beam. The focusing in the core of the beam can be lowered by the response lag of the plasma electrons to the larger gradients in the beam charge density.

Parametric studies of the effects on the self-focusing of varying the plasma density and the beam intensity have been performed and are being analyzed as of this writing. These results will be presented in a later publication.

9. Prospects for Future Experiments

For the future, upgrades of the AATF are planned which should greatly enhance the intensity and brightness of the driving beams available for future wake-field studies. These improvements include a RF laser photocathode source which should provide 100 nC of electrons per pulse in 6 psec FWHM. If this source is used at the Chemistry linac, then the final driving beam energy can be raised to 30 MeV. Alternatively, a new RF acceleration system may be built as part of a general upgrade, which would allow a driving beam energy of 150 MeV. It is of interest to extrapolate the results obtained in the nonlinear PWFA experiments to predict what sort of accelerating gradient can be expected in an experiment at the upgraded facility.

From plasma wake-field theory, we know that for a beam to efficiently self-pinch in the plasma its length must be at least on the order of a plasma wavelength. Experimentally, we deduced very powerful self-pinching forces and observed large accelerating gradients in the case where $k_p \sigma_z \simeq 2$, which is on the border of this regime. The longitudinal efficiency for plasma wave excitation a Gaussian longitudinal profile with $k_p \sigma_z \simeq 2$ is $\eta_l \simeq 0.125$. For the purpose of simplifying the remainder of the discussion we will take $k_p \sigma_z = 2$ and $\eta_l = 1/8$.

The equilibrium Bennett radius size for a self-pinched beam is given by $a = 2\epsilon_n/\sqrt{\gamma\nu_0}$. If this equilibrium state has been approached in the plasma, then the maximum accelerating field behind the beam is approximately given by

$$W_m = \frac{N r_e m_e c^2 \eta_r(k_p a)}{2a^2} = \frac{N^2 r_e^2 \gamma m_e c^2 \eta_r(k_p a)}{8\pi^{1/2} \sigma_z \epsilon_n^2}, \quad (9.1)$$

where $\eta_r(k_p \sigma_{r,eq})$ is the radial field efficiency of the in the case of the self pinched beam. Note that the wake-field amplitude appears to have an N^2 dependence, which is quite dramatic. For cases of interest, however, we often have $a \ll \sigma_z$ and thus $k_p a \ll 1$, a regime where $\eta_r(k_p a) \ll 1$. In this limit Eq. (9.1) becomes

$$W_m = \frac{r_e m_e c^2}{2\sigma_z^2} \left[N \ln N - N \ln \left(\frac{\sqrt{8\pi} \epsilon_n^2}{\gamma r_e \sigma_z} \right) \right]. \quad (9.2)$$

Now the leading functional dependence of the wake-field amplitude on particle beam population at large N is $N \ln N$, which looks suspiciously like an entropy! As an example, we take the recent experiments, with $\gamma = 42$, $\epsilon_n = 3 \times 10^{-4}$ m-rad, $N = 2.5 \times 10^{10}$, and $\sigma_z = 2.1$ mm, and calculate $W_m = 6$ MeV/m. The experimentally determined value is $W_m = 5.3$ MeV/m, in good agreement.

For the planned RF photo-cathode source upgrade, the proposed beam characteristics change to $\epsilon_n = 3 \times 10^{-5}$ m-rad, $N = 6 \times 10^{11}$, and $\sigma_z = 0.75$ mm, which gives an estimate of $W_m = 4$ GeV/m. Unfortunately, the formula no longer applies, as the assumption to this point has been that the plasma response is linear both for the self-focusing and the wake-field amplitude calculations. We have already seen moderate nonlinearities, and while our assumed plasma density has been raised a factor of four, the proposed self-pinch beam is 5000 times denser than before.

A criterion for determining when one is in the regime where the linear focusing theory will be invalid is, with $k_p \sigma_z = 2$, approximately $N r_e / \epsilon_n > 1$, and for the upgraded beam parameters $N r_e / \epsilon_n \simeq 56$. In this new regime the focusing is linear, with strength dependent only on the plasma electron density n_e , and the equilibrium beam radius is $\sigma_r^2 = \epsilon_n / \sqrt{2\pi r_e n_e \gamma} = \sqrt{(2/\gamma) \epsilon_n / k_p}$. This is an underdense plasma lens regime,^[29] and the focusing of the beam should be weaker but much more uniform. If we extrapolate linear wake-field theory to estimate the amplitude of the nonlinear wake-fields we obtain

$$W_m \simeq \frac{N r_e m_e c^2}{2\sigma_z^2} [-\ln(\sqrt{(8/\gamma) k_p \epsilon_n})]. \quad (9.3)$$

For the upgraded source parameters we estimate $W_m = 1.45$ GeV/m, with the plasma density $n_e \simeq 2.7 \times 10^{14} \text{ cm}^{-3}$, which is within reach of a hollow cathode arc plasma source similar to the one presently used. Note that even though these waves are dominated by radial effects, the maximum wake-field is almost equal to the so-called wave breaking limit, given by $eE_{wb} = m_e c \omega_p \simeq \sqrt{n_e (\text{cm}^{-3})} \text{ eV/m} \simeq 1.65 \text{ GeV/m}$. We thus expect that the wake-fields will display a wealth of

interesting nonlinear behavior as well as extremely large accelerating gradients in these experiments.

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REFERENCES

1. P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, *Phys. Rev. Lett.*, **54**, 693 (1985).
2. V. N. Tsytovich, *Plasma Astrophysics*, Pergamon Press, Oxford (1962).
3. H. Figueroa, W. Gai, R. Konecny, J. Norem, P. Schoessow, and J. Simpson, submitted to *Phys. Rev. Lett.* (1988).
4. J. B. Rosenzweig, D. B. Cline, B. Cole, H. Figueroa, W. Gai, R. Konecny, J. Norem, P. Schoessow, and J. Simpson, *Phys. Rev. Lett.* **61**, 98 (1988).
5. P. Chen, *Particle Accelerators* **20** (1985) 171.
6. T. Katsouleas, *Phys. Rev. A* **33** (1986) 2056.
7. R. D. Ruth, A. Chao, P. L. Morton, and P. B. Wilson, *Particle Accelerators* **17** (1985) 171.
8. J. B. Rosenzweig, P. Schoessow, B. Cole, W. Gai, R. Konecny, J. Norem and J. Simpson, *Phys. Rev. A - Rapid Comm.*, **39**, 1586 (1989).
9. J. B. Rosenzweig, *Phys. Rev. Letters*, **58** (1987) 555.
10. J. B. Rosenzweig and P. Chen, SLAC-PUB-4571 (1988), to be published in *Phys. Rev. D*, April 1989.
11. W. H. Bennett, *Phys. Rev.* **45**, 890 (1934), and *Phys. Rev.* **98**, 1584 (1955).
12. P. Schoessow, submitted to the *Proceedings of the 1989 Particle Accelerator Conference*, IEEE, Chicago (March 1989).
13. W. Gai, P. Schoessow, B. Cole, R. Konecny, J. Norem, J. B. Rosenzweig and J. Simpson, *Phys. Rev. Lett.* **61**, 2756 (1989).
14. W. K. H. Panofsky and W. A. Wenzel, *Rev. Sci. Instr.*, **27**, 967 (1956).
15. R. Keinigs and M. Jones, *Physics of Fluids* **30**, 252 (1987).

16. J. B. Rosenzweig, B. Cole, D. B. Cline, and D. J. Larson, *Particle Accelerators*, **24**, 11 (1988).
17. D. B. Cline, B. Cole, J. Rosenzweig, and J. Norem, *Proceedings of the 1987 Washington Particle Accelerator Conference*, IEEE (1987) 154.
18. P. Chen, J.J. Su, T. Katsouleas, J. M. Dawson, R. Keinigs, and M. Jones, *IEEE Trans. Plasma Sci.* **PS-15** (1987).
19. R. Keinigs, M. Jones and J.J. Su *IEEE Trans. Plasma Sci.*, **PS-15**, 199 (1987).
20. J. B. Rosenzweig, *Phys. Rev. A*, **38**, 3634 (1988).
21. T. Katsouleas and W. B. Mori, *Phys. Rev. Lett.*, **61**, 90 (1988).
22. A.Ts. Amatuni, S.S. Sekhpossian and E.V. Elbakian, Yerevan Preprint YerPhi-935(86)-86 Yerevan, USSR (1986).
23. J. B. Rosenzweig, Fermilab Preprint FERMILAB-Pub-88/212 (1989), submitted to *Phys. Rev. A*.
24. D. Umstadter, R. Williams, C. Clayton, and C. Joshi, *Phys. Rev. Lett.*, **59** 292 (1987).
25. J.J. Su, presented at the Lake Arrowhead Workshop on Advanced Accelerator Concepts.
26. E.P. Lee, *Phys. Fluids* **19**, 60 (1976).
27. J. Simpson, private communication.
28. A. Chao, D. Johnson, S. Peggs, J. Peterson, C. Saltmarsh, L. Sachinger, R. Meller, R. Siemann, R. Talman, D. Edwards, D. Finley, R. Gerig, N. Gelfand, M. Harrison, R. Johnson, N. Merminga, and M. Syphers, *Phys. Rev. Lett.* **61**, 2752 (1988).
29. P. Chen, S. Rajagapalan and J. B. Rosenzweig, SLAC-PUB-4746 (October 1988), to be published in *Phys. Rev. D*, August 1989.

FIGURE CAPTIONS

Figure 1. Schematic of Argonne Advanced Accelerator Test Facility layout.

Figure 2. Schematic drawing (not to scale) of hollow cathode arc plasma source used in PWFA experiments.

Figure 3. The plasma electron density n_e from probe measurements for various running conditions (arc current I_a and applied axial magnetic field B) of the arc source. The darkened symbols show the density as determined from ω_p in PWFA experiments, and open symbols are the corresponding probe measurements.

Figure 4. Scan 1 - Witness beam energy centroid change δE versus time delay behind driver. Total driver beam charge $Q = 2.1$ nC, plasma parameters $L_p = 28$ cm and $n_e = 8.6 \times 10^{12} \text{ cm}^{-3}$. Theoretical predictions are given by the dashed line.

Figure 5. Scan 2 - Witness beam energy centroid change δE versus time delay Total driver beam charge $Q = 2.1$ nC, plasma parameters $L_p = 33$ cm and $n_e = 2.3 \times 10^{13} \text{ cm}^{-3}$. Theoretical predictions are given by the dashed line.

Figure 6. Scan 3 - Witness beam centroid position Δy in the non-bend plane of the spectrometer versus delay, for off-axis injection of witness beam. The large deflection near zero delay is expected from the linear theory, due to the self-focusing magnetic wake-fields at the driver. For this scan $Q = 2.2$ nC, $L_p = 33$ cm and $n_e = 3.4 \times 10^{13} \text{ cm}^{-3}$.

Figure 7. Driver beam energy spectrum for $Q = 2.5$ nC, $L_p = 28$ cm, and $n_e = 2.9 \times 10^{13} \text{ cm}^{-3}$ plotted with solid line, along with case of no plasma present in dashed line.

Figure 8. (a) Scan 4 - Longitudinal wake-field scan, witness beam energy centroid ΔE motion versus time delay, with plasma density of $n_0 = 7.3 \times 10^{12} \text{ cm}^{-3}$.
(b) FFT amplitude function $w_z(f)$ for longitudinal wake-fields.

- Figure 9. Transverse wake-field scan - witness beam deflection plane Δy centroid versus time delay for Scan 4.
- Figure 10. Longitudinal wake-field for Scan 4, with a different delay range. Below 250 psec in delay, the driving beam charge is 2.9 nC. Between 250 and 280 the charge is raised to 4 nC, where it remains for the rest of the scan. The disproportionate rise in the wake-field amplitude is attributable to stronger self-pinching of the driver beam.
- Figure 11. Scan 5 - Longitudinal wake-field scan, with plasma density of $n_0 = 2.8 \times 10^{13} \text{ cm}^{-3}$.
- Figure 12. Contour plots of the spatially resolved beam intensity, with x being the transverse (radial) coordinate and y the longitudinal (time) coordinate. (a) Beam density contour with plasma of density $n_e = 2.9 \times 10^{13} \text{ cm}^{-3}$ present. (b) No plasma present.
- Figure 13. Plots of the transverse intensity profiles (a projection of the most intense 0.5 mm long transverse strip) of the contours in from Fig. 12. (a) With dense plasma present - the self-pinched profile (solid line) with a best fit of the data to a Bennett profile of radius $a = 0.91 \text{ mm}$ (dashed line). (b) With no plasma - the unpinched profile (solid line) with a best fit of the data to a Gaussian profile of width $\sigma_r = 1.4 \text{ mm}$ (dashed line).

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8(a)

Figure 8(b)

Figure 9

Figure 10

Figure 11

Figure 12(a)

Figure 12(b)

PROJECTED INTENSITY

x (mm)
Figure 13(a)

PROJECTED INTENSITY

Figure 13(b)