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GEPOIS: A TWO DIMENSIONAL NONUNIFORM MESH
POISSON SOLVER

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GEPOIS: A Two Dimensional Nonuniform Mesh Poisson Solver

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ABSTRACT

This report describes a computer code which solves Poisson's equation for the electric potential over a two dimensional cylindrical (r,z) nonuniform mesh which can contain internal electrodes. Poisson's equation is solved over a given region subject to a specified charge distribution with either Neumann or Dirichlet perimeter boundary conditions and with Dirichlet boundary conditions on internal surfaces. The static electric field is also computed over the region with special care given to normal electric field components at boundary surfaces.

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A. Introduction

Particle-in-cell (PIC) simulation codes have been used for several years ¹⁻⁵ to investigate the behavior of intense particle beams in diodes. These simulations have helped to explain the results of experiments which have been performed in the development and application of drivers for intense particle beam inertial confinement fusion research. A flow chart of a typical steady state diode code is shown in Figure 1. As can be seen from the flow chart, the steps involved in obtaining a self-consistent steady state solution for the current and charge distribution in a diode simulation are as follows: First the electric potential is found for a particular charge distribution and diode configuration. The electric field which results from this electric potential, together with the charge density is then used to determine how much charge should be emitted at that time step assuming space charge limited emission from all emitting surfaces. Simulation particles are introduced at these emission surfaces with charge corresponding to that specified by the emission law. These simulation particles are then pushed in the electric and magnetic fields one time step. New values of charge and current density are computed from the new simulation particle positions and velocities. The current density is integrated to produce the magnetic field component B_θ . Poisson's equation is then solved for the new charge distribution and the process is then repeated until a steady state is reached. The result is a description of the particle

Figure 1. Diode Simulation PIC Code Flow Chart.

beam behavior in its own self-consistent fields.

Solving Poisson's equation and obtaining the electric fields in the diode are integral parts of the above process. This report is a description of the latest version of the Poisson solver in the Sandia diode code. The earliest versions of the diode code¹ used a solver incorporating a fast Fourier transform (FFT) in the z direction and cyclic reduction in r . The FFT restricted the zoning of the finite difference mesh to be uniform in z and also required that the number of mesh lines in z be equal to an integer power of 2. A capacity matrix was needed to treat internal electrode points. This solver was extremely fast, but as diode configurations became more complex the need for nonuniform zoning in both the r and z directions became apparent. Although the new solver, which is based on an alternating-direction-implicit iteration method, is somewhat slower than the previous one, the added flexibility of internal surface description and spatial detail more than offset the increased run time.

Section B describes the method of solution of Poisson's equation and the computation of the electric field components. Section C describes the code in detail including pertinent inputs, dimensions, etc. There are three appendices to help the user. In Appendix I a summary of required inputs and dimensioning is presented in tabular form. Appendix II contains the input required for a sample problem which is discussed in the text. Finally, a complete listing of the code is given in Appendix III.

B. Method of Solution

In GEPOIS the first step in solving for the electric potential in a region is the definition of a finite difference mesh or grid. Figure 2 shows the p, m grid point with its surrounding mesh zones and grid points on a nonuniform mesh. In this figure, terms are defined which will be used in the following description of the method.

Poisson's equation in cylindrical coordinates for an axisymmetric system is

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = -\rho/\epsilon \quad (1)$$

To solve this equation by finite difference techniques, approximations to the first and second derivatives on a nonuniform mesh are needed. These are obtained from a Taylor series expansion of the function, viz:

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2} f''(x_0) + \dots \quad (2)$$

Now if $x = x_0 + \Delta_+$

$$f_+ = f_0 + \Delta_+ f'(x_0) + \frac{\Delta_+^2}{2} f''(x_0) + \dots \quad (3)$$

Figure 2. Grid Section Defining Terms.

or if $x = x_0 - \Delta_-$

$$f_- = f_0 - \Delta_- f'(x_0) + \frac{\Delta_-^2 f''(x_0)}{2} + \dots \quad (4)$$

where the notation

$$f_+ = f(x_0 + \Delta_+)$$

$$f_0 = f(x_0)$$

$$f_- = f(x_0 - \Delta_-)$$

has been used. Equations (3) and (4) can be solved simultaneously for the first and second derivatives of the function at x_0 in terms of the function values at x_0 , $x_0 - \Delta_-$, and $x_0 + \Delta_+$. The results are

$$f'(x_0) = \frac{\Delta_+^2 (f_0 - f_-) + \Delta_-^2 (f_+ - f_0)}{\Delta_+ \Delta_- (\Delta_+ + \Delta_-)} + O(\Delta_+ \Delta_-) \quad (5)$$

$$f''(x_0) = \frac{2\Delta_+ (f_- - f_0) + 2\Delta_- (f_+ - f_0)}{\Delta_+ \Delta_- (\Delta_+ + \Delta_-)} + O(\Delta_+ - \Delta_-) \quad (6)$$

Kalnay De Rivas⁶ has shown that if the grid intervals are varied using a stretched coordinate $x = x(\xi)$ in such a way as to insure that the $\Delta\xi$ are constant, $\frac{dx}{d\xi}$ is finite over the interval and $\frac{d^2x}{d\xi^2}$ is zero only where high resolution is desired, then equations (5) and (6) will give approximations to the first and second derivatives which are accurate to second order in the mesh spacing.

Using the notation of Figure 2 and the results of (5) and (6) a finite difference approximation for Poisson's

equation at the l, m grid point can be written as follows:

$$A_l \phi_{l+1,m} + B_l \phi_{l,m} + C_l \phi_{l-1,m} + D_m \phi_{l,m+1} + E_m \phi_{l,m} + F_m \phi_{l,m-1} = G_{l,m} \quad (7)$$

where

$$A_l = (2\Delta_- + \Delta_-^2/r_l)\alpha$$

$$B_l = -(A_l + C_l)$$

$$C_l = (2\Delta_+ - \Delta_+^2/r_l)\alpha$$

$$D_m = 2\delta_- \beta$$

$$E_m = -(D_m + F_m)$$

$$F_m = 2\delta_+ \beta$$

$$G_{l,m} = \rho_{l,m}/\epsilon$$

$$\alpha = \frac{1}{\Delta_+ \Delta_- (\Delta_+ + \Delta_-)}, \quad \beta = \frac{1}{\delta_+ \delta_- (\delta_+ + \delta_-)}$$

Equation (7) is solved using an alternating-direction-implicit (ADI) method with a Wachspress iteration parameter sequence⁷, viz:

$$A_l \phi_{l+1,m}^* + (B_l - \eta^k) \phi_{l,m}^* + C_l \phi_{l-1,m}^* = G_{l,m} - D_m \phi_{l,m+1}^0 - (E_m + \eta^k) \phi_{l,m}^0 - F_m \phi_{l,m-1}^0 \quad (8)$$

where⁸

$$\eta^{k+1} = \frac{4 [\cos^2(\pi/2\text{MESHMX})]}{\Delta_{\min}^2 [\cos^2(\pi/2\text{MESHMX})] k/(k_0 - 1)} \quad (9)$$

$$k = 0, 1, \dots, k_0 - 1.$$

In Eq. (9), MESHMX is the maximum number of zones in either direction, $\Delta_{\min}$ is the minimum zone size, and k_0 is the least integer such that

$$k_0 - 1 + \frac{\ln\{\tan[(\pi/2\text{MESHMX})]\}}{\ln(\sqrt{2} - 1)}$$

On successive passes in the iterative solution of Eq. (8), the right-hand side of the equation is assumed to be known using previous values of potentials, and the starred values of potential are then found using a two-pass direct method. Next, new values of potential are found by solving

$$D_m \phi_{l,m+1}^n + (E_m - \eta^k) \phi_{l,m}^n + F_m \phi_{l,m-1}^n = H_{l,m}^*,$$

where

$$H_{l,m}^* = G_{l,m} - A_l \phi_{l+1,m}^* - (B_l + \eta^k) \phi_{l,m}^* - C_l \phi_{l-1,m}^*$$

with the same two-pass direct method. After each complete iteration, k is incremented by one until $k = k_0$, at which time k is set back to zero and the sequence begins again.

The two-pass direct method of solution is outlined below. It is desired to solve the equation

$$A_j \phi_{j+1} + (B_j - \eta^k) \phi_j + C_j \phi_{j-1} = H_j \quad (10)$$

where j is either k or m and A_j is either A_k or D_m , etc.

Suppose

$$\phi_j = O_j \phi_{j-1} + P_j \quad (11)$$

Then

$$\phi_{j+1} = O_{j+1} \phi_j + P_{j+1} \quad (12)$$

Substituting (12) into (10) yields

$$A_j [O_{j+1} \phi_j + P_{j+1}] + [B_j - n^k] \phi_j + C_j \phi_{j-1} = H_j$$

$$\text{or } O_j [A_j O_{j+1} + B_j - n^k] + C_j \phi_{j-1} = H_j - P_{j+1} A_j$$

$$\text{but } O_j - O_j O_{j-1} = P_j$$

so that

$$O_j = \frac{C_j}{A_j O_{j+1} + B_j - n^k} \quad (13)$$

$$P_j = \frac{H_j - A_j P_{j+1}}{A_j O_{j+1} + B_j - n^k} \quad (14)$$

Note that if O_{j+1} and P_{j+1} are known then O_j and P_j are determined by (13) and (14). Also, if all O_j and P_j are known then all ϕ_j are specified by (11). The first pass of the two pass direct method begins at the uppermost j boundary where two possible conditions exist; either ϕ_j max is specified or $\frac{\partial \phi}{\partial n} = 0$ there. If ϕ_j max is specified then from (11)

$$O_j \text{ max} = 0 \quad \text{and} \quad P_j \text{ max} = \phi_j \text{ max}$$

and (13) and (14) can be applied recursively until all O_j and P_j through $O_{j \min}$ and $P_{j \min}$ are determined. If $\frac{\partial \phi}{\partial n} = 0$ at the upper boundary then further analysis is required. Equation (5) can be solved for ϕ_{j+1} when $\frac{\partial \phi}{\partial n} = 0$ to yield

$$\phi_{j+1} = \left| \phi_{j-1} - \phi_j \right| \left[\frac{\Delta_+}{\Delta_-} \right]^2 + C_j \quad (15)$$

Substituting this relation into (10) and again solving for $O_j \max$ and $P_j \max$ gives

$$O_j \max = \frac{- \left[\Lambda_j \max \left(\frac{\Delta_+}{\Delta_-} \right)^2 + C_j \max \right]}{\left[B_j \max - \eta^k + \Lambda_j \max \left\{ 1 - \left(\frac{\Delta_+}{\Delta_-} \right)^2 \right\} \right]}$$

$$P_j \max = \frac{H_j \max}{\left[B_j \max - \eta^k + \Lambda_j \max \left\{ 1 - \left(\frac{\Delta_+}{\Delta_-} \right)^2 \right\} \right]}$$

Again, (13) and (14) can be applied recursively until $O_j \min$ and $P_j \min$ are determined. To obtain the potential ϕ_j for all j the second pass of the two-pass direct method begins at the lower boundary. If $\phi_j \min$ is specified, then all remaining ϕ_j are obtained from (12) by recursion. If, however, $\frac{\partial \phi}{\partial n} = 0$ on the lower boundary, then again further analysis is required. Equation (15) is now substituted into (11) and (12) with the result that

$$\phi_j \min = \frac{O_j \min P_{j \min+1} \left(\frac{\Delta_-}{\Delta_+} \right)^2 + P_j \min}{1 - O_j \min \left[\left(O_{j \min+1} - 1 \right) \left(\frac{\Delta_-}{\Delta_+} \right)^2 + 1 \right]} \quad (16)$$

Equation (16) is valid for a homogeneous Neumann boundary condition on a lower boundary except when the boundary is the z axis ($r = 0$). In this case the $\frac{1}{r} \frac{\partial \phi}{\partial r}$ term in Poisson's equation becomes indeterminate and must be removed using L'Hospital's rule. In actuality (13), (14) and (16) are still correct in this case but the expressions for A_z and C_z below (7) become

$$A_z \min = \frac{4\Delta_-}{(\Delta_+ + \Delta_-) \Delta_+ \Delta_-} = \frac{2}{\Delta_+^2}$$

$$C_z \min = \frac{4\Delta_+}{(\Delta_+ + \Delta_-) \Delta_+ \Delta_-} = \frac{2}{\Delta_+^2}$$

Here Δ_- is set equal to Δ_+ on all exterior boundaries. The two-pass method as described above is used on each successive row or column of the difference mesh until all values of $\phi_{i,m}$ or $\phi_{i,m}^*$ are determined.

The ADI procedure is repeated until convergence is reached. The convergence criterion may be adjusted by the user to the required accuracy. The criterion used in GEPOIS is that

$$\sum_{\text{mesh}} (O - N)^2 / \sum_{\text{mesh}} (N)^2 \leq \text{CONV} \quad (17)$$

where O and N refer to old and new values of $\phi_{i,m}$ respectively. Here CONV is typically $\sim 10^{-13} - 10^{-17}$.

With the potential computed, the electric field is calculated by using $E = -\nabla \phi$. This implies that

$$E_r = - \frac{\partial \phi}{\partial r} ,$$

$$E_z = - \frac{\partial \phi}{\partial z} .$$

The derivatives above are computed using the approximation given in Eq. (5). On boundaries, the normal electric field component is computed using a quadratic fit to the potential and differentiating this quadratic function. This avoids the one sided difference and its inherent inaccuracy. An additional advantage of the ADI method is its generality when compared with direct solvers. Many other elliptic equations can be treated by this technique.⁷

C. Code Description

Subroutine GEPOIS, together with its companion routine GENSLV, computes the electric potential over a specified 2-d, nonuniform, cylindrical (r,z) mesh, subject to a known charge distribution and rather general boundary conditions. The r and z components of the electric field can also be obtained by setting the appropriate flag when calling GEPOIS. The parameter list for calling GEPOIS is

```
CALL GEPOIS (ISET, IEF, IMX, CONV, NR, NZ, R, Z,
             INELPT, PHI, ER, RHOEZ, DUMABC, DUMAOP,
             DUMERC, DUMEZC, DUMDR, DUMDZ)
```

Each parameter will be explained below.

1) INPUT

a) ISET. The first call to GEPOIS sets constants.

ISET must be equal to 1 during this first

call. ISET must not equal 1 during subsequent calls to GEPOIS. The first call to GEPOIS is a dummy call and does not compute potentials.

- b) IEF. This is a flag which when 1 results in the computation of both potential and electric field. If IEF is not equal to 1 only the potential is computed.
- c) IMX. The maximum number of iterations to try in the ADI process before stopping and printing a convergence failure message.
- d) CONV. The convergence criteria as defined in (17).
- e) NR. The array size in the r direction. NR is equal to the number of r grid lines plus 2.*
- f) NZ. The array size in the z direction. NZ is equal to the number of z grid lines plus 2.*
- g) R. The array containing the r_g values of the grid lines.*
- h) Z. Array containing the z_m values of the grid lines.*
- i) INELPT. Array specifying which grid points are to be held at fixed potentials. If INELPT(L,M) equals 1 then the L, M grid point is held fixed at PHI(L,M) volts. If INELPT(L,M) equals 0 then the L,M grid point potential will be computed.*

*(See grid description below)

2) INPUT/OUTPUT

- a) PHI. The array which contains the value of potential at each grid point. $PHI(L,M)$ must be set on all Dirichlet boundaries and at all points where $INELPT(L,M) = 1$ prior to the call to GEPOIS.
- b) RHOEZ. The array which contains the value of $-p/\epsilon$ at each grid point before the GEPOIS call and which contains the value of E_z (when $IEF = 1$) at each grid point after the GEPOIS call.

3) OUTPUT

- a) ER. The array which contains the value of E_r (when $IEF = 1$) after the call to GEPOIS.

4) DUMMY VARIABLES

- a) All parameters with names starting with DUM are only dimensioned in the main program. This provides storage locations for constants defined in GEPOIS and makes subroutines useable without themselves having to be redimensioned with each new problem.

In addition to the above calling parameters, the user must define a common block in the calling program as

```
COMMON/SURF/IDS(4,4,5), NOS(4), NOSS, IBC(4)
```

Each of these variables relate to surface description and boundary conditions. They are explained in the mesh and surface description below.

The first consideration in using these routines is the definition of a differencing mesh. Figure 3 details a sample problem which will be used to describe the code features in all that follows. Figure 4 shows a nonuniform mesh which has been chosen to solve this problem. The r direction index is k and the z direction index is m . The mesh is defined from 1 to NR in r and 1 to NZ in z . The actual problem boundaries are 2 to NR-1 and 2 to NZ-1 allowing for one extra zone around the entire mesh, a feature which is not used in this code but has applications in particle-in-cell codes. The (r,z) location of the grid lines must be specified by defining R(L) and Z(M) arrays in the calling program. Care should be taken in choosing these grid lines to insure accuracy in the final potential solution. (Refer to Kalnay De Rivas⁶ for guidance in choosing acceptable variations in grid spacings.) In the sample problem Δr increases from bottom to top by the power 1.15 and Δz increases from right to left by the power 1.20. The reader can verify that this choice of zoning does satisfy the restrictions to insure the desired accuracy in the Poisson solver.⁶

In the sample problem of Figure 3 there are internal surfaces shown. (Note: Internal surfaces are any surfaces which fall inside the region defined by the mesh. The four boundaries of the region, top, bottom, right and left are referred to here as perimeter surfaces or boundaries. An

Figure 3. Sample Problem.

Figure 4. Location of Surfaces on the Nonuniform Mesh for Sample Problem.

upper surface is a surface whose normal vector pointing into the region of solution is in the $-r$ direction. A lower surface has its normal vector pointing in the $+r$ direction. Similarly, the normal vector for a right surface is in the $-z$ direction and for a left surface in the $+z$ direction.) All surfaces must be described to GEPOIS via the array $IDS(I,J,K)$ which is defined in the calling program. $IDS(I,J,K)$ is a three dimensional array. The I index runs from 1 to 4 and identifies the surface as either 1) an upper surface 2) a lower surface 3) a right surface or 4) a left surface. The J index identifies which surface of type I the particular surface is (for example $IDS(2,3,K)$ is the third lower surface). Each surface has five pieces of information stored in IDS which describe the surface. The K index varies from 1 to 5. If K equals: 1) $IDS(I,J,1)$ contains the grid line on which the J^{th} surface of the I^{th} type falls 2) $IDS(I,J,2)$ contains the lower extent of the surface in terms of grid lines 3) $IDS(I,J,3)$ contains the upper extent of the surface in terms of grid lines. If a boundary surface is to be a symmetry boundary then $IDS(I,J,4) = -1$ otherwise $IDS(I,J,4) = 0$. $IDS(I,J,5)$ contains a surface identifying number. Each surface has a unique number associated with it. The surfaces are labeled 1, 2, ..., NOSS with all upper surfaces numbered first, then lower surfaces, then right surfaces and finally left surfaces. Figure 5 shows the sample problem with surfaces appropriately numbered. A final requirement in

Figure 5. Surface Identification for Sample Problem.

describing the surfaces is providing the array NOS(N). The array NOS(N) contains the number of each type of surface NOS(1) = 2 means that there are 2 upper surfaces. N = 2, 3 or 4 apply to lower, right and left surfaces respectively.

The perimeter boundary surfaces which surround the mesh region are described as either Dirichlet or homogeneous Neumann boundaries by defining IBC(N) to be 1 or 2. N again specifies the boundary surface as upper, lower, right or left for N = 1, 2, 3, or 4 respectively. If IBC(N) equals 1 then the surface is a Dirichlet boundary (and PHI(L,M) must be specified on this boundary). If IBC(N) equals 2 then the surface is a homogeneous Neumann boundary and $\frac{\partial \phi}{\partial n} = 0$ will be enforced there.

Subroutine GEPOIS must be called once as a dummy call to set all constants which are used subsequently. This dummy call is accomplished by calling GEPOIS with ISET = 1. No potential computation occurs during this call to GEPOIS. Subsequent calls to GEPOIS should be made with ISET $\neq$ 1. If any of the following inputs to GEPOIS change

IMX, CONV, R, Z, IDS, NOS, NOSS, IBC

another dummy call with ISET = 1 must be made. Of course changes in PHI or RHOEZ are allowed between subsequent calls to GEPOIS without an intervening dummy call. In addition IEF may change from 1 to 0 or back, but only

if IEF was set to 1 during the last dummy call to GEPOIS, otherwise IEF must remain 0 until changed by another dummy call.

Appendix I summarizes each of the above parameters with their required dimensioning if any. Appendix II gives the appropriate input used to solve the sample problem of Figure 3. The equipotential plot which results from the solution of the sample problem is shown in Figure 6.

As a final example of the code flexibility Figure 7 shows an equipotential plot for a complicated system. This problem was solved with $IEF = 0$ because of the surfaces which do not lie on grid lines. Computing the proper normal E field component on slanted surfaces is beyond the scope of this code because of the difficulty in making such a routine general enough to handle all possible cases. Normal electric fields have been computed on slanted surfaces for specific problems in the past using this routine with minor modifications⁵. When $IEF = 0$ the information in the IDS and NOS arrays is not used. Poisson's equation is solved for the exterior boundary conditions given by the IBC array. Any grid points which have INELPT equal to 1 will be held at the value of potential specified in the PHI array before the call to GEPOIS. In Figure 7, a piecewise linear approximation to a quarter circle (which results in a hemisphere when revolved about the r axis) is held at V volts. A slanted surface structure is held

Figure 6. Equipotential Plot for Sample Problem.

Figure 7. Equipotential Plot for More Complicated Geometry.

at 0 volts. Equipotential lines which result from the solution of this problem are shown. This configuration could represent a hemispherical target mounted on the anode in a tapered cathode diode.

Appendix I. Summary of Input Parameters and Dimensioning.

A. Dimensioning

In the calling program the user must dimension the following variables

R(NR), DUMERC(3,NR), DUMDR(NR)
Z(NZ), DUMEZC(3,NZ), DUMDZ(NZ),
DUMABC(6,NMAX), DUMAOP(3,NMAX)
INELPT(NR,NZ), PHI(NR,NZ), RHOEZ(NR,NZ),
ER(NR,NZ)

where NMAX is the larger of NR or NZ and NR and NZ are specified below. (Note: ER and RHOEZ must be dimensioned even if E field computations are not desired; IEF = 0)

B. Calling Parameters

ISET	= 1	This is a dummy call to GEPOIS used to set constants
	= 0	Not a dummy call - Compute ϕ
IEF	= 1	Compute E fields too
	= 0	Don't compute E fields
IMX		Number of iterations before printing convergence failure message
CONV		Convergence criterion as defined in equation (17)
NR		Number of r grid lines plus 2
NZ		Number of z grid lines plus 2

R(L)		Array of r grid line locations
Z(M)		Array of z grid line locations
INELPT(L,M)	= 1	L,M is a point at which potential should be held fixed
	= 0	L,M is not a fixed potential point
PHI(L,M)		Array containing values of $\phi_{l,m}$
RHOEZ(L,M)		Array containing values of $-\rho_{l,m}/\epsilon$ before the call to GEPOIS and containing values of $E_{z,l,m}$ after the call to GEPOIS (If IEF = 1)
ER(L,M)		Array containing values of $E_{r,l,m}$ (IF IEF = 1)
DUM---		Arrays which are dimensioned in the main program but not defined

C. Common Block

The calling program must contain the common block SURF

COMMON/SURF/IDS(4,4,5), NOS(4), NOSS, IBC(4)

where IDS(I,J,K) describes the surfaces in the problem:

I	=	1	upper surface
	=	2	lower surface
	=	3	right surface
	=	4	left surface

N = 1 first surface of type 1
 = 2 second surface of type 1
 etc.
 k_i = 1 IDS contains the surface location:
 ℓ value of surface for I = 1 or 2
 m value of surface for I = 3 or 4
 = 2 IDS contains the lower limit of the
 surface extent:
 surface leftmost m for I = 1 or 2
 surface bottom ℓ for I = 3 or 4
 = 3 IDS contains the upper limit of the
 surface extent:
 surface rightmost m for I = 1 or 2
 surface upper ℓ for I = 3 or 4
 = 4 If IDS(I,J,4) = -1 the surface is a
 symmetry boundary. If IDS(I,J,4) = 0
 the surface is not a symmetry boundary.
 = 5 IDS contains the surface ID
 (Note: Number all upper surfaces
 sequentially first, then lower
 surfaces, the right surfaces and
 finally left surfaces)

NOS(N) is an array which contains the number of each type of
 surface in the problem:

NOS(1) is the number of upper surfaces
 NOS(2) is the number of lower surfaces
 NOS(3) is the number of right surfaces
 NOS(4) is the number of left surfaces

NOSS is the total number of surfaces

$$NOSS = \sum_{N=1}^4 NOS(N)$$

IBC(N) is an array which describes the perimeter boundary surfaces as either Dirichlet or homogeneous Neumann

$$\begin{aligned} IBC(N) &= 1 && \text{Dirichlet} \\ &= 2 && \text{Homogeneous Neumann} \end{aligned}$$

where N = 1, 2, 3 or 4 stands for upper, lower, right and left boundaries respectively.

Appendix II. Input for sample problem.

The input required for the sample problem of Figure 3 is given here

	K				
	1	2	3	4	5
IDS (1,1,K)	42	2	32	0	1
IDS (1,2,K)	12	2	8	0	2
IDS (2,1,K)	22	2	8	0	3
IDS (2,2,K)	6	26	32	0	4
IDS (2,3,K)	2	2	32	-1	5
IDS (3,1,K)	32	2	42	0	6
IDS (3,2,K)	26	2	6	0	7
IDS (4,1,K)	8	12	22	0	8
IDS (4,2,K)	2	2	42	-1	9

	N			
	1	2	3	4
NOS(N)	2	3	2	2
IBC(N)	1	2	1	2

$$\text{NOSS} = 9$$

$$\text{IMX} = 3000$$

$$\text{CONV} = 10^{-15}$$

$$\text{NR} = 43$$

$$\text{NZ} = 33$$

$$R(L) = \left(\frac{L-2}{40} \right)^{1.15}$$

$$Z(M) = 1. - \left(\frac{32-M}{30} \right)^{1.20}$$

INELPT(L,M) = 0 for all L and M except

$$\text{INELPT(L,M)} = 1 \left\{ \begin{array}{l} 12 \leq L \leq 22 \text{ and} \\ 2 \leq M \leq 8 \\ \text{or} \\ 2 \leq L \leq 6 \text{ and} \\ 26 \leq M \leq 32 \end{array} \right.$$

PHI(L,M) = 0. for all L and M except

$$\text{PHI(L,M)} = 1. \left\{ \begin{array}{l} M = 32 \\ \text{or} \\ L = 42 \\ \text{or} \\ 2 \leq L \leq 6 \text{ and } 26 \leq M \leq 32 \end{array} \right.$$

RHOEZ(L,M) = 0. for all L and M.

Appendix III. Code Listing.

```

SUBROUTINE GEPOIS (ISET,IEF,IM),CONV,NP,NZ,R,Z,INELPT,PHI,ER,RHOEZ
1,ABCODEF,DUM,ERC,EZC,OR,CZ)
*****
**
**      CODE DESCRIPTION
**
**      SUBROUTINE GEPOIS, TOGETHER WITH ITS COMPANION ROUTINE
**      GENSLV, COMPUTES THE ELECTRIC POTENTIAL AND THE R AND
**      Z COMPONENTS OF THE ELECTRIC FIELD OVER A SPECIFIED
**      NONUNIFORM CYLINDRICAL (R,Z) MESH, SUBJECT TO A KNOWN
**      CHARGE DISTRIBUTION AND RATHER GENERAL BOUNDARY
**      CONDITIONS. THE USER SHOULD CONSULT SANDIA REPORT
**      NUMBER 79-0939 FOR INFORMATION ON THE USE OF THESE
**      ROUTINES. THIS CODE WAS WRITTEN AND IS MAINTAINED BY
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**
*****
DIMENSION ABCDEF(6,1), ERC(3,1), P(1), DP(1), EZC(3,1), Z(1), DZ(1
1), DUM(3,1)
DIMENSION INELPT(NP,1), IRC(1)
COMMON /POIS/ XI(12),NRM1,NZM1,NRM2,NZM2,NOP1,NI,ISM1,ISL,ISM1,IE
1,1,NZM3,NPM3,LS1,LSX,MSX,MSY
COMMON /SURF/ IDS(4,4,5),NOS(4),NOSS,IRC(4)
DIMENSION PHI(NR,1), FR(NP,1), RHOEZ(NR,1)
DIMENSION LV(6), ML(6), MU(6), FV(6), MV(6), LL(6), LU(6), I1(6,2)
1, AA(6,3), BB(6,3), GG(6,3), EE(6,3), CC(6,3,2)
*****
THE FIRST PASS THROUGH GEPOIS COMPUTES ALL REQUIRED
CONSTANTS ONCE AND STORES THEM FOR SUBSEQUENT USE.
FOR THIS REASON *** A DUMMY CALL TO GEPOIS IS REQUIRED ***
BEFORE THE FIRST POISSON SOLVE.
*****
IF (ISET.NE.1) GO TO 380
*****
COMPUTE REQUIRED INDICES
*****
CO 30 N=1,4
IF (NOS(N).GT.4.AND.IEF.EQ.1) 10,20
10 PRINT 5A1
STOP
20 CONTINUE

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30 CONTINUE
   LG=4-IBC(2)
   LSX=NR-3+IBC(1)
   MSX=NZ-3+IBC(3)
   NR=M2-NZ-2
   NZM2=NZ-2
   NR=M3-NZ-3
   NZM3=NZ-3
   NZM1=NZ-1
   LST=LSX-1
   MST=MSX-1
   IE=M1=NZ+IBC(4)-5
   IEL1=NZ+IBC(7)-5
   ISM1=4-IBC(4)
   ISL1=4-IBC(2)
   NR=M1-NZ-1
   *****
   COMPUTE WACHSPRESS ACCELERATION PARAMETERS FOR THIS MESH
   *****
   DO 40 L=3, NR*M2
     DP(L)=P(L+1)-R(L)
40  CONTINUE
   DO 50 M=2, NZ*M2
     DZ(M)=Z(M+1)-Z(M)
50  CONTINUE
     DR(1)=DR(2)
     DR(NRM1)=DR(NRM2)
     DZ(1)=DZ(2)
     DZ(NZM1)=DZ(NZM2)
     R(1)=R(2)-DR(2)
     R(NR)=R(NRM1)+DR(NRM1)
     Z(1)=Z(2)-DZ(2)
     Z(NZ)=Z(NZM1)+DZ(NZM1)
     RP=R(NRM1)
     CZM=Z(NZ-1)
     LM1=2
     DO 60 L=3, NRM1
       DRM=AMIN1(DRM,DP(LM1))
60  LM1=L
       MM1=2
       DO 70 M=3, NZM1
         DZM=AMIN1(DZM,DZ(MM1))
70  MM1=M
       IF (DRM.LE.DZM) 80,90
80  SQDEL=DRM*DRM
       GO TO 100
90  SQDEL=DZM*DZM

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100 CONTINUE
  MESHMZ=(7*(NZ-1)-Z(2))/DZM+1
  MESHMP=(R(NRM1)-R(2))/DPM+1
  MESHMX=MAX0(MESHMP,MESHMZ)
  N0=12
  IF (MESHMX.LT.10560) N0=11
  IF (MESHMX.LT.4377) N0=10
  IF (MESHMX.LT.1813) N0=9
  IF (MESHMX.LT.751) N0=8
  IF (MESHMX.LT.312) N0=7
  IF (MESHMX.LT.129) N0=6
  IF (MESHMX.LT.54) N0=5
  IF (MESHMX.LT.23) N0=4
  IF (MESHMX.LT.10) N0=3
  IF (MESHMX.LT.5) N0=2
  PI=3.141592654
  PTC2=PI/2.
  ARG=PTC2/MESHMX
  N0P1=N0+1
  N0M1=N0-1
  TCON=COS(ARG)
  TCON1=4.*(TCON*TCON)
  TCON1=TAN(ARG)
  TCON1=1./(TCON1*TCON1)
  DO 110 M=1,N0
    N=M-1
    FN=N
    FNM1=N0M1
    EX=FN/FNM1
110  XI(M)=TCON/(TCON1**EX*SOBEL)
*****
COMPUTATION OF COEFFICIENTS IN ADI SOLUTION OF FINITE
DIFFERENCE APPROXIMATION TO POISSONS EQUATION
*****
  LG=3
  CNX=DR(1)*DP(2)*(DP(1)+DR(2))
  ABCDEF(1,2)=4.*(DR(1)/CNX
  ABCDEF(2,2)=-4.*(DP(1)+DR(2))/DNX
  ABCDEF(3,2)=4.*DP(2)/CNX
120 CONTINUE
  DO 130 L=LG,LSX
    LM1=L-1
    FN=(DP(L)+DR(LM1))*DP(L)*DP(LM1)
    ABCDEF(1,L)=(2.+DP(LM1)/R(L))*DR(LM1)/DN
    ABCDEF(2,L)=(DR(L)*(DP(L)/R(L)-2.)-DR(LM1)*(DR(LM1)/R(L)+2.))/DN
130  ABCDEF(3,L)=(2.-DP(L)/R(L))*DR(L)/CN
    DO 140 M=2,NZM1
      NM1=M-1
      FN=(DZ(M)+DZ(NM1))*DZ(M)*DZ(NM1)
      ABCDEF(4,M)=2.*DZ(NM1)/CN
      ABCDEF(5,M)=-2.*(DZ(M)+DZ(NM1))/DN

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140 ABCDEF(6,M)=2.*DZ(M)/DH
IF (IEF.EQ.0) GO TO 370
*****
COMPUTATION OF COEFFICIENTS FOR DERIVATIVES ON A NONUNIFORM
MESH, USED IN THE COMPUTATION OF THE ELECTRIC FIELD FROM
THE POTENTIAL
*****
LS=2
IF (R(2).EQ.0.) 150,160
150 LS=3
160 CONTINUE
CO 170 L=LS,NPM1
LM1=L-1
SUM=CR(L)+DP(LM1)
ERC(3,L)=-DR(LM1)/(SUM*DP(L))
ERC(2,L)=(CR(LM1)-DP(L))/(CR(L)+DP(LM1))
170 ERC(1,L)=DR(L)/(SUM+DP(LM1))
MS=2
IF (Z(2).EQ.0.) MS=3
CO 180 M=MS,NZM1
MM1=M-1
SUM=DZ(M)+DZ(MM1)
EZC(3,M)=-DZ(MM1)/(SUM*DZ(M))
EZC(2,M)=(DZ(MM1)-DZ(M))/(DZ(M)+DZ(MM1))
180 EZC(1,M)=DZ(M)/(SUM+DZ(MM1))
*****
SURFACE INFORMATION COMPILED
*****
INIX=1
NEPB=0
CO 210 J=1,2
NSS=NCOS(J)
GO 215 NS=1,NSS
IF (IDS(J,NS,4).LT.0) GO TO 210
NEPB=NEPB+1
L=IDS(J,NS,1)
LV(NEPB)=L
ML(NEPB)=IDS(J,NS,2)
MU(NEPB)=IDS(J,NS,3)
PV(NEPB)=R(L)
IF (R(L).EQ.C.) 190,200
190 NEPB=NEPB-1
GO TO 215
200 CONTINUE
I1(NEPB,1)=L
IF (J.EQ.1) I1(NEPB,1)=L-2
210 CONTINUE
NMX=NEPB
GO TO 240

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220 INOX=2
    NEZ9=0
    CO 230 J=3,4
    NSS=NOSS(J)
    DO 230 NS=1,NSS
    IF (IOS(J,NS,4).LT.9) GO TO 230
    NEZ9=NEZ9+1
    I=IOS(J,NS,1)
    MV (NEZ3)=M
    LL (NEZ7)=IOS(J,NS,2)
    LU (NEZ8)=IOS(J,NS,3)
    FV (NEZ6)=Z(M)
    I1 (NEZ3,2)=M
    IF (J.EQ.1) I1 (NEZ3,2)=M-2
230 CONTINUE
    NMX=NEZ9
    .....
    COMPUTATION OF COEFFICIENTS FOR QUADRATIC FIT OF POTENTIAL
    AT CONDUCTING SURFACES, USED IN COMPUTING THE NORMAL
    COMPONENT OF THE ELECTRIC FIELD AT CONDUCTING BOUNDARIES
    .....
240 IF (NMX.GT.6) 250,250
250 PRINT 59)
    STOP
260 IF (NMX.EQ.0) GO TO 360
    DO 350 I=1,NMX
    II=I1(I,INOX)
    IF (INOX.EQ.1) 270,280
270 XXX=R(II)
    YYY=R(II+1)
    ZZZ=R(II+2)
    GO TO 290
280 XXX=Z(II)
    YYY=Z(II+1)
    ZZZ=Z(II+2)
290 AA(I,2)=1./(YYY*YYY-YYY*ZZZ-XXX*YYY+XXX*ZZZ)
    AA(I,3)=1./7ZZZ*ZZZ-YYY*ZZZ-XXX*ZZZ+XXX*YYY)
    AA(I,1)=-AA(I,2)-AA(I,3)
    BB(I,1)=-(XXX+YYY)*AA(I,1)-1./(YYY-XXX)
    BB(I,2)=-(XXX+YYY)*AA(I,2)+1./(YYY-XXX)
    BB(I,3)=-(XXX+YYY)*AA(I,3)
    IF (INOX.EQ.1) 300,310
300 XXX2=XXX*XXX
    GG(I,1)=1.-AA(I,1)*XXX2-BB(I,1)*XXX
    GG(I,2)=-AA(I,2)*XXX2-BB(I,2)*XXX
    GG(I,3)=-AA(I,3)*XXX2-BB(I,3)*XXX
    THOP=2.*FV(I)
310 TWOFV=2.*FV(I)
    DO 320 J=1,3
320 CC(I,J,INOX)=-(TWOFV*AA(I,J)+BB(I,J))
    IF (INOX.EQ.1) 330,350

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330 DO 340 J=1,3
340 EE(I,J)=THREP*AA(I,J)+2.*B3(I,J)+GG(I,J)/FV(I)
350 CONTINUE
360 CONTINUE
IF (INDX.EQ.1) 220,370
370 CONTINUE
RETURN
*****
RETURN FROM GUMMY CALL TO GEPOIS
*****
380 CONTINUE
*****
SUBSEQUENT CALLS TO GEPOIS ENTER HERE
*****
*****
POISSON EQUATION SOLVED BY ADI WITH WACHSPRESS ACCELERATION
IN GENSLV
*****
CALL GENSLV (IMX,CONV,PHOEZ,PHI,NP,INELPT,ER,ABCOEF,DUM,DP,DZ)
IF (IEF.EQ.0) RETURN
*****
E FIELDS COMPUTED OVER THE MESH
*****
DO 390 L=LS,NPM1
LP1=L+1
LM1=L-1
EP(L,1)=ERC(3,L)*PHI(LP1,1)+ERC(3,L)*PHI(L,1)+ERC(1,L)*PHI(LM1,1)
EP(L,NZ)=ERC(3,L)*PHI(LP1,NZ)+ERC(2,L)*PHI(L,NZ)+ERC(1,L)*PHI(LM1,
1,NZ)
DO 390 M=MS,NZM1
ER(L,M)=ERC(3,L)*PHI(LP1,M)+ERC(2,L)*PHI(L,M)+ERC(1,L)*PHI(LM1,M)
390 PHCEZ(L,M)=EYC(3,M)*PHI(L,M+1)+EYC(2,M)*PHI(L,M)+EYC(1,M)*PHI(L,M-
1)
*****
NORMAL E FIELD COMPONENTS COMPUTED ON CONDUCTING SURFACES
*****
DO 410 I=1,NERR
LE=LV(I)
LLP=I1(I,1)
MMIN=M_(I)
MMAX=M0(I)
DO 400 M=MMIN,MMAX
400 ER(LE,M)=CC(I,1,1)*PHI(LLP,M)+CC(I,2,1)*PHI(LLP+1,M)+CC(I,3,1)*PHI
1(LLP+2,M)
410 CONTINUE
DO 430 I=1,NFZR
ME=MV(I)
MMP=I1(I,2)
LMIN=LL(I)
LMAX=LU(I)
DO 420 L=LMIN,LMAX

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420 RHOCZ(L,ME)=CC(I,1,2)*PHI(L,MMP)+CC(I,2,2)*PHI(L,MMP+1)+CC(I,3,2)*
1 PHI(L,MMP+2)
430 CONTINUE
*****
SETS NORMAL COMPONENT OF E FIELD TO ZERO ON SYMMETRY BOUNDARY
*****
DO 450 J=1,4
NS=NCS(J)
DO 455 NS=1,NSS
IF (IDS(J,NS,4).GE.3) GO TO 450
IXV=IDS(J,NS,1)
IXL=IDS(J,NS,2)
IXH=IDS(J,NS,3)
DO 440 IXX=IXL,IXH
IF (J.LE.2) ER(IXV,IXX)=0.
IF (J.GE.3) RHOCZ(IXX,IXV)=0.
440 CONTINUE
450 CONTINUE
IF (LS.EQ.3) 460,430
460 DO 470 M=MS,NZM2
470 RHOCZ(2,M)=EZO(3,M)*PHI(2,M+1)+EZO(2,M)*PHI(2,M)+EZO(1,M)*PHI(2,M-
11)
480 IF (MS.EQ.3) 490,510
490 DO 500 L=LS,NPM2
500 ER(L,2)=ERC(3,L)*PHI(L+1,2)+ERC(2,L)*PHI(L,2)+ERC(1,L)*PHI(L-1,2)
510 CONTINUE
IF (IBC(4).EQ.2) 520,540
520 DO 530 L=LS,NZM1
530 RHOCZ(L,2)=0.
540 IF (IBC(2).EQ.2) 560,570
550 DO 560 M=MS,NZM1
560 ER(2,M)=0.
570 CONTINUE
RETURN
*****

580 FORMAT (33H MORE THAN 4 OF ONE TYPE SURFACE ,/,37H REDIMENSION IDS
1 IN ALL COMMON BLOCKS,/,214 AND REMOVE THIS TEST)
590 FORMAT (33H DIMENSIONING IN GEPOIS TOO SMALL)
END

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SUBROUTINE GENSLV (IMX, CONV, G, PHI, NT, INELPT, PPT, ABCDEF, AOP, DR, DZ)
  DIMENSION INELPT(NT,1), PPT(NT,1), DR(1), DZ(1)
  DIMENSION ABCDEF(6,1)
  DIMENSION G(NT,1), PHI(NT,1), AOP(3,1)
  COMMON /POIS/ XI(12), NTM1, NSM1, NTM2, NSM2, NOP1, NI, ISM1, ISL1, IEM1, IE
1 L1, NSM3, NTM3, LST, L3X, MSX, MST
  COMMON /SUPP/ IDSOUM(4,4,5), NOSSOUM(4), NOSSOUM, IBC(4)
  N=0
  DO 290 NI=1, IMX
    SUMN=0.
    SUMD=0.
    N=N+1
    IF (MOD(N, NOP1).EQ.3) N=1
    DO 12 L=2, NTM1
10  AOP(1,L)=PHI(L, IBC(4)+1)
      DO 140 M=ISM1, MSX
        MP1=M+1
        IF (IDC(1), N=.2, OP, INELPT(NTM1, M).EQ.1) GO TO 50
        DR2=OP(NTM1)*OP(NTM1)/(CR(NTM2)*CR(NTM2))
        CN=ABCDEF(2, NTM1)-XI(N)+ABCDEF(1, NTM1)*(1.-DR2)
        AOP(2, NTM1)=- (ABCDEF(1, NTM1)*OP2+ABCDEF(3, NTM1))/DND
        IF (M.EQ.NSM1) GO TO 30
20  HST=G(NTM1, M)-ABCDEF(4, M)*PHI(NTM1, NSM2) - (ABCDEF(5, M)+XI(N))*PHI(N
1 TM1, M)-ABCDEF(6, M)*AOP(1, NTM1)
        GO TO 40
30  CONTINUE
        HST=-ABCDEF(4, M)*PHI(NTM1, MP1) - (ABCDEF(5, M)+XI(N))*PHI(NTM1, M)-ABC
1 DEF(6, M)*AOP(1, NTM1)+G(NTM1, M)
40  CONTINUE
        AOP(3, NTM1)=HST/DND
        GO TO 60
50  CONTINUE
        AOP(2, NTM1)=0.
        AOP(3, NTM1)=PHI(NTM1, M)
60  CONTINUE
        IXP1=NTM1
        DO 120 L=1, ISL1
          IX=NTM1-L
          IF (INELPT(IX, M).EQ.1) GO TO 80
70  AOP(2, IX)=0.
          AOP(3, IX)=PHI(IX, M)
          GO TO 120
80  CONTINUE
          CN=ABCDEF(1, IX)*AOP(2, IXP1)+ABCDEF(2, IX)-XI(N)
          IF (M.EQ.NSM1) GO TO 100
90  HST=G(IX, M)-ABCDEF(4, M)*PHI(IX, NSM2) - (ABCDEF(5, M)+XI(N))*PHI(IX, M)
1 -ABCDEF(6, M)*AOP(1, IX)
          GO TO 110
100 CONTINUE
          HST=G(IX, M)-ABCDEF(4, M)*PHI(IX, MP1) - (ABCDEF(5, M)+XI(N))*PHI(IX, M)-
1 ABCDEF(6, M)*AOP(1, IX)

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110 CONTINUE
  AOP(2,IX)=-ABCODEF(3,IX)/DN
  AOP(3,IX)=(HST-ABCODEF(1,IX)*AOP(3,IXP1))/DN
120 IXP1=IX
  AOP(1,2)=PHI(2,M)
  IF (IRG(2).EQ.2.AND. INELPT(2,M).EQ.0) PHI(2,1)=(AOP(2,2)*AOP(3,3)+
1 AOP(3,2))/(1.-AOP(2,2)*AOP(2,3))
  DO 130 L=2,LST
    LP1=L+1
    AOP(1,LP1)=PHI(LP1,M)
    IF (INELPT(LP1,M).EQ.0) PHI(LP1,M)=AOP(2,LP1)*PHI(L,M)+AOP(3,LP1)
130 CONTINUE
140 CONTINUE
  DO 150 M=2,NSM1
    AOP(1,M)=PHI(IRG(2)+1,M)
    DO 200 L=ISL1,LSX
      LP1=L+1
      IF (IRG(3).NE.2.OF.INELPT(L,NSM1).EQ.1) GO TO 190
      DZ2=DZ(NSM1)*DZ(NSM1)/(DZ(NSM2)*DZ(NSM2))
      DNC=AR(DEF(5,NSM1)-XI(N)+ABCODEF(4,NSM1))*(1.-DZ2)
      AOP(2,NSM1)=-(ABCODEF(4,NSM1)*DZ2+ABCODEF(6,NSM1))/DND
      IF (L.EQ.NTM1) 160,170
160 HST=G(L,NSM1)-ABCODEF(1,L)*PHI(NTM2,NSM1)-(ABCODEF(2,L)+XI(N))*PHI(L
1,NSM1)-ABCODEF(3,L)*AOP(1,NSM1)
      GO TO 180
170 CONTINUE
      HST=G(L,NSM1)-ABCODEF(1,L)*PHI(LP1,NSM1)-(ABCODEF(2,L)+XI(N))*PHI(L
1,NSM1)-ABCODEF(3,L)*AOP(1,NSM1)
180 CONTINUE
      AOP(3,NSM1)=HST/DND
      GO TO 200
190 CONTINUE
      AOP(2,NSM1)=0.
      AOP(3,NSM1)=PHI(L,NSM1)
200 CONTINUE
      IXP1=NSM1
      DO 260 M=1,NTM1
        IX=NSM1-M
        IF (INELPT(L,IX).EQ.1) 210,220
210 AOP(2,IX)=0.
        AOP(3,IX)=PHI(L,IX)
        GO TO 260
220 CONTINUE
        DN=ABCODEF(4,IX)*AOP(2,IXP1)+ABCODEF(5,IX)-XI(N)
        IF (L.EQ.NTM1) 230,240
230 HST=G(L,IX)-ABCODEF(1,L)*PHI(NTM2,IX)-(ABCODEF(2,L)+XI(N))*PHI(L,IX)
        1-ABCODEF(3,L)*AOP(1,IX)
        GO TO 250

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240 CONTINUE
    MST=G(L,IX)-ABCODEF(1,L)*PHI(LP1,IX)-(ABCODEF(2,L)+XI(N))*PHI(L,IX)-
    1ABCODEF(3,L)*AOP(1,IX)
250 CONTINUE
    AOP(2,IX)=-ABCODEF(5,IX)/DN
    AOP(3,IX)=(MST-ABCODEF(4,IX)*AOP(3,IXP1))/DN
260 IXF1=IX
    AOP(1,2)=PHI(L,2)
    IF (IRC(4).EQ.2.AND.INELPT(L,2).EQ.0) PHI(L,2)=(AOP(2,2)*AOP(3,3)+
    1AOP(3,2))/(1.-AOP(2,2)*AOP(2,3))
    DO 270 M=2,MST
        MP1=M+1
        AOP(1,MP1)=PHI(L,MP1)
        IF (INELPT(1,MP1).EQ.1) GO TO 270
        PHI(L,MP1)=AOP(2,MP1)*PHI(L,M)+AOP(3,MP1)
        SUMD=SUMD+PHI(L,MP1)*PHI(L,MP1)
        SUMN=(PRT(L,MP1)-PHI(L,MP1))**2+SUMN
        PPT(L,MP1)=PHI(L,MP1)
270 CONTINUE
280 CONTINUE
    IF (SUMD.EQ.0.) RETURN
    IF (SUMN/SUMD.LT.CONV) RETURN
290 CONTINUE
    PRINT 300
    STOP
    C
    C
    C
300 FORMAT (1H ,19HCONVERGENCE FAILURE)
    END

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