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Implementation of Two Geologic Constitutive Models in the HONDO Finite-Element Code

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Implementation of Two Geologic Constitutive Models in the HONDO Finite-Element Code

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Abstract

Two constitutive models for use with geologic materials have been incorporated into the HONDO finite-element program. Both models have the same behavior in tension, using a stress criterion to form cracks normal to the maximum principal stress. In compression, the two models give upper and lower bound solutions to the unconfined postfailure strength. The first model uses a Coulomb criterion to form explicit shear cracks, while the second model uses an elastic-plastic formulation developed by Krieg. Two sample applications, an indenter test and fracture of a borehole, are presented.

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Nomenclature

σ	Stress tensor
$\underline{s}$	Deviatoric stress tensor
ϵ	Strain tensor
$\underline{e}$	Deviatoric strain tensor
$\underline{C}$	Material stiffness tensor
$\underline{D}$	Material compliance tensor
J_2	Second invariant of stress deviator
E	Young's Modulus
ν	Poisson's Ratio
B	Bulk Modulus
G	Shear Modulus
S_0	Cohesion in Coulomb Model
μ	Internal friction in Coulomb Model
C_0	Unconfined compressive strength
r, z, t	Axisymmetric coordinates
p, n, t	Crack coordinates
α	Angle between minimum stress and Coulomb crack
γ	Orientation of principal stresses with global coordinates
θ	Crack orientation with global coordinates
p_c	Pressure on crack face
p_m	Mean pressure

Implementation of Two Geologic Constitutive Models in the HONDO Finite Element Code

1. Introduction

Many constitutive models have been proposed to represent geologic materials (Zienkiewicz and Pande¹ or Chen and Ting²). Because the behavior of geologic materials is extremely complicated, a complete constitutive description appears formidable. However, for many applications, a model sufficient for engineering use can be formulated if the following behavior is incorporated in the model:

- Brittle cracking in tension
- A pressure-dependent failure criterion in compression
- A postfailure material description.

This report describes the implementation into the HONDO³ finite element computer program of two geologic constitutive models that meet the above minimum criteria. Both constitutive models exhibit the same behavior in tension, using a stress criterion to form cracks normal to the maximum principal stress at the element integration points. Cracking occurs if the maximum tensile stress exceeds a specified fracture strength. When a crack forms, the strain normal to the crack is monitored in subsequent time steps to determine if the crack is open or closed. The normal stress on an open crack face is set to a specified pressure (or zero) and the shear is set equal to zero.

In compression, the two models give upper and lower bound solutions to the unconfined postfailure strength. By applying both models to the same problem, one can bound the postfailure response. The first model uses a Coulomb criterion to form explicit shear cracks at the element integration points. In subsequent time steps, closed cracks can carry a compressive normal stress, but the shear stress is limited to a value described by the Coulomb friction model. Open cracks act as described above.

The second model acts in compression as an elastic-plastic material. This model (without cracking) exists as Material 7 in HONDO and was developed by Krieg.⁴ The yield surface is described in principal stress space by a surface of revolution with a planar end cap. Plasticity is handled in two parts: a volumetric part and a deviatoric part. The plane cap, normal to the hydrostat in principal stress space, is movable because of volumetric work hardening. The deviatoric plastic strain rate vector is assumed normal to the hydrostat in principal stress space. In this model, when tensile cracks close, they are assumed to heal.

Figure 1 schematically illustrates the two yield surfaces in the principal stress space.

Figure 1. Schematic of Yield Surfaces in Principal Stress Space

The HONDO finite-element program into which these constitutive models have been added is designed to calculate the large deformation elastic and inelastic transient dynamic response of axisymmetric solids. The program uses an explicit time integration scheme to solve the dynamic problem. For each time step, the element stresses are used to obtain nodal accelerations, the accelerations are integrated to obtain displacements, the element strains are updated, new stresses are calculated, and the cycle is then repeated. Because this approach is very efficient for short-time dynamic loading of a continuum, models with a large number of elements can be used for analysis at a reasonable cost.

The user is cautioned that there are theoretical problems associated with the use of a stress criterion for crack growth (see Section 4). Nevertheless, the models have proven useful in several applications and have provided meaningful engineering results. They are presented with the hope that, when used with proper care, they may be helpful to other users.

2. Constitutive Relations

This section describes in detail the equations used in the constitutive routines. The reader may wish to examine the flow charts shown in Figures 2 and 3 to understand the sequence in which the equations are used. In HONDO, the constitutive routines are provided the following information: previous stresses, current strain increments, and material constants. The subroutines return new stresses. Both models follow the same basic sequence for each time step at each element integration point:

- Calculate an elastic trial stress based on the previous stresses and current strain increments.
- Using the strain increments, update the strains normal to any existing cracks.
- Modify the stresses for open cracks and check for new tensile failure.
- Modify stresses for closed cracks and check for shear crack formation (or do plasticity calculation).
- Correct strains normal to cracks for any sliding or plastic flow.

The approach taken for modifying the stresses at each crack is to store the crack orientation and strain normal to the crack. The strain normal to the crack is used to monitor whether the crack is open or closed. Stresses are then rotated to the crack orientation, modified for the crack status (whether open or closed), and then the modified stresses are rotated back to the global coordinates.

At each element integration point, a maximum of three cracks are allowed to form (two R-Z cracks and one hoop crack). The cracks rotate with the material.

2.1 Basic Equations

This section provides the basic equations common to both constitutive models. Many of the equations are based on the work of Thomas.⁵

In these constitutive models, elastic trial stresses are calculated, assuming an elastic isotropic material. For the axisymmetric case, the stress increments are related to the strain increments by

$$\begin{Bmatrix} \Delta\sigma_r \\ \Delta\sigma_z \\ \Delta\sigma_t \\ \Delta\sigma_{rz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ C_{31} & C_{32} & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \Delta\epsilon_r \\ \Delta\epsilon_z \\ \Delta\epsilon_t \\ \Delta\epsilon_{rz} \end{Bmatrix} \quad (1)$$

For an isotropic material, the coefficients are given by

$$C_{11} = C_{22} = C_{33} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)},$$

$$C_{44} = \frac{E}{2(1+\nu)},$$

$$C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = \frac{E\nu}{(1+\nu)(1-2\nu)}.$$

Equation (1) may be written in matrix form

$$\Delta\sigma = \underline{C} \Delta\epsilon.$$

The elastic trial stresses for the time step are then given by

$$\sigma^{\text{trial}} = \sigma^{\text{old}} + \underline{C} \Delta\epsilon. \quad (2)$$

At times it is necessary to determine the elastic strains given the stresses. This can be accomplished by the inverse of Eq (1)

$$\begin{Bmatrix} \epsilon_r \\ \epsilon_z \\ \epsilon_t \\ \epsilon_{rz} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 \\ D_{21} & D_{22} & D_{23} & 0 \\ D_{31} & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & D_{44} \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_t \\ \sigma_{rz} \end{Bmatrix} \quad (3)$$

For an isotropic material, the coefficients are given by

$$D_{11} = D_{22} = D_{33} = \frac{1}{E},$$

Figure 2. Flow Chart for Tensile/Coulomb Model

Figure 3. Flow Chart for Tensile/Plasticity Model

$$D_{44} = \frac{2(1+\nu)}{E},$$

$$D_{12} = D_{21} = D_{13} = D_{31} = D_{23} = D_{32} = -\frac{\nu}{E}.$$

To evaluate failure in the R-Z plane, the principal stresses are calculated using

$$\sigma_{\max, \min} = \frac{\sigma_r + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_r - \sigma_z}{2}\right)^2 + \sigma_{rz}^2}. \quad (4)$$

The principal stresses are oriented at an angle γ to the global coordinate system

$$\gamma = \frac{1}{2} \tan^{-1} \left(\frac{2\sigma_{rz}}{\sigma_r - \sigma_z} \right). \quad (5)$$

Note that this calculated angle must be checked to see whether it corresponds to the maximum or minimum stress direction.

After cracks have formed, it is necessary to rotate global stresses to the crack orientation. The crack orientation is defined by the angle θ measured with respect to the global coordinates, as shown in Figure 4.

Figure 4. Definition of Crack Angle θ

The rotated stresses normal and parallel to the crack are given by

$$\begin{Bmatrix} \sigma_p \\ \sigma_n \\ \sigma_t \\ \sigma_{pn} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 0 & -2\sin\theta\cos\theta \\ 0 & 0 & 1 & 0 \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & 0 & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_t \\ \sigma_{rz} \end{Bmatrix}. \quad (6)$$

Strains can also be rotated to the crack coordinates using

$$\begin{Bmatrix} \epsilon_p \\ \epsilon_n \\ \epsilon_t \\ \epsilon_{pn} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 0 & -\cos\theta\sin\theta \\ 0 & 0 & 1 & 0 \\ 2\sin\theta\cos\theta & -2\sin\theta\cos\theta & 0 & \sin^2\theta - \cos^2\theta \end{bmatrix} \begin{Bmatrix} \epsilon_r \\ \epsilon_z \\ \epsilon_t \\ \epsilon_{rz} \end{Bmatrix}. \quad (7)$$

If the postfailure relations described in the next sections result in inelastic material behavior (either crack opening, crack sliding, or plastic flow), it is possible to calculate the inelastic strain increments given the trial stresses and the final modified stresses by assuming the total strain can be decomposed into elastic and inelastic parts

$$\Delta \epsilon^{\text{inelas}} = D (\sigma^{\text{trial}} - \sigma^{\text{final}}). \quad (8)$$

2.2 Tensile Failure and Open Crack Stress Calculation

Both constitutive models exhibit the same crack-opening behavior in tension. Tensile failure is assumed to occur if the maximum principal stress calculated using Eq (4) exceeds a specified tensile strength.

If failure occurs, the crack plane is normal to the maximum principal stress direction. The orientation can be determined using Eq (5). Note that failure can occur either in a plane that intersects the R-Z plane (Figure 5a) or in a plane normal to the circumferential direction (Figure 5b).

After a crack has formed and if it is open, in subsequent time steps the shear stress on the crack face is set equal to zero and the normal stress is set equal to a specified pressure (which may also be zero). Following Thomas,⁵ this can be accomplished using Eqs (6) and (7) to rotate the trial stresses and global-strain increments to the crack orientation. Writing Eq (1) in the crack coordinates, we have

$$\begin{Bmatrix} \Delta\sigma_p \\ \Delta\sigma_n \\ \Delta\sigma_t \\ \Delta\sigma_{pn} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ C_{31} & C_{32} & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \Delta\epsilon_p \\ \Delta\epsilon_n \\ \Delta\epsilon_t \\ \Delta\epsilon_{pn} \end{Bmatrix}. \quad (10)$$

We now require the normal stress to equal a specified pressure increment (Δp_c)

$$\Delta\sigma_n = \Delta p_c = C_{21} \Delta\epsilon_p + C_{22} \Delta\epsilon_n + C_{23} \Delta\epsilon_t. \quad (11)$$

Solving for the change in normal strain corresponding to the specified pressure, we obtain

$$\Delta\epsilon_n = \frac{\Delta p_c}{C_{22}} - \frac{C_{21}}{C_{22}} \Delta\epsilon_p - \frac{C_{23}}{C_{22}} \Delta\epsilon_t. \quad (12)$$

Substituting this relation into Eq (10) results in the stress increments in the other directions

$$\begin{aligned}\Delta\sigma_p &= \left(C_{11} - \frac{C_{12}C_{21}}{C_{22}}\right)\Delta\epsilon_p + \left(C_{13} - \frac{C_{12}C_{23}}{C_{22}}\right)\Delta\epsilon_t + \frac{C_{12}}{C_{22}}\Delta p_c, \\ \Delta\sigma_t &= \left(C_{31} - \frac{C_{32}C_{21}}{C_{22}}\right)\Delta\epsilon_p + \left(C_{33} - \frac{C_{32}C_{23}}{C_{22}}\right)\Delta\epsilon_t + \frac{C_{32}}{C_{22}}\Delta p_c. \quad (13)\end{aligned}$$

The actual stresses passed to the open crack subroutine are the elastic trial stresses. Using Eqs (2), (11), and (13), we obtain the new stresses in terms of the trial stresses and the strain increments as

$$\begin{aligned}\sigma_p^{\text{new}} &= \sigma_p^{\text{trial}} - \frac{C_{12}C_{21}}{C_{22}}\Delta\epsilon_p - C_{12}\Delta\epsilon_n - \frac{C_{12}C_{23}}{C_{22}}\Delta\epsilon_t + \frac{C_{12}}{C_{22}}\Delta p_c, \\ \sigma_n^{\text{new}} &= p_c, \\ \sigma_t^{\text{new}} &= \sigma_t^{\text{trial}} - \frac{C_{32}C_{21}}{C_{22}}\Delta\epsilon_p - C_{32}\Delta\epsilon_n - \frac{C_{32}C_{23}}{C_{22}}\Delta\epsilon_t + \frac{C_{32}}{C_{22}}\Delta p_c, \\ \sigma_{pn}^{\text{new}} &= 0. \quad (14)\end{aligned}$$

The analysis for a hoop crack is similar and results in

$$\begin{aligned}\sigma_r^{\text{new}} &= \sigma_r^{\text{trial}} - \frac{C_{13}C_{31}}{C_{33}}\Delta\epsilon_r - \frac{C_{13}C_{32}}{C_{33}}\Delta\epsilon_z - C_{13}\Delta\epsilon_t + \frac{C_{13}}{C_{33}}\Delta p_c, \\ \sigma_z^{\text{new}} &= \sigma_z^{\text{trial}} - \frac{C_{23}C_{31}}{C_{33}}\Delta\epsilon_r - \frac{C_{23}C_{32}}{C_{33}}\Delta\epsilon_z - C_{23}\Delta\epsilon_t + \frac{C_{23}}{C_{33}}\Delta p_c, \\ \sigma_t^{\text{new}} &= p_c, \\ \sigma_{rz}^{\text{new}} &= \sigma_{rz}^{\text{trial}}. \quad (15)\end{aligned}$$

If both R-Z cracks are open, it is assumed that the material has no strength in the R-Z plane and that the hoop stress is decoupled from the R-Z strains. The stresses are given by

$$\begin{aligned}\sigma_p^{\text{new}} &= p_c, \\ \sigma_n^{\text{new}} &= p_c, \\ \sigma_t^{\text{new}} &= \sigma_t^{\text{old}} + C_{33}\Delta\epsilon_t, \\ \sigma_{pn}^{\text{new}} &= 0. \quad (16)\end{aligned}$$

This assumption preserves stress symmetry that would not be preserved if Eq (14) was simply used sequentially for two cracks.

In addition to modifying the stresses, it is necessary to calculate and accumulate the strain normal to a crack. This strain is used to indicate whether a crack is open or closed. On the initial formation of a crack, the total elastic strains are calculated and the crack opening strain is set equal to the elastic normal strain. In subsequent time steps, the normal strain increment is added to the crack opening strain. Then when the normal stress is released, the rebound in normal strain given by Eq (12) is used to correct the accumulated strain. The correction to the crack opening strain given by Eq (12) reflects the behavior that an increase in the gas pressure will tend to open the crack. Also, inspection of Eq (12) shows that a tensile strain parallel to the crack will tend to open the crack because of the Poisson effect.

a. Max Stress in R-Z Plane (R-Z crack)

b. Max Stress in Circumferential Direction (hoop crack)

Figure 5. Tensile Crack Orientations

2.3 Coulomb Failure and Closed Crack Stress Calculation

The two material models use different approaches for compressive failure. In the first model (described in this section), a Coulomb criterion is used to form explicit shear cracks at the element integration points. The failure criterion equations in this section are based on the discussion by Jaeger and Cook⁶ of the Coulomb criterion. Shear failure occurs if the shear stress on a plane exceeds a cohesion plus a linear function of the normal stress.

$$|\sigma_{\tau}| > S_0 - \mu \sigma_n. \quad (17)$$

The relation between cohesion (S_0), internal coefficient of friction (μ), and unconfined compressive strength (C_0) is given by

$$S_0 = \frac{C_0}{2[(\mu^2 + 1)^{1/2} + \mu]}. \quad (18)$$

If shear failure occurs, it is assumed that two cracks form simultaneously in the two equally probable shear failure directions inclined at angles $\pm \alpha$ on either side of the minimum (most compressive) stress direction.

$$\alpha = \pm \frac{1}{2} \tan^{-1} \left(\frac{1}{\mu} \right). \quad (19)$$

If Eq (17) is rewritten in terms of the principal stresses, shear failure occurs when

$$\sigma_{\max} [(\mu^2 + 1)^{1/2} + \mu] - \sigma_{\min} [(\mu^2 + 1)^{1/2} - \mu] \geq 2S_0. \quad (20)$$

Figure 6 illustrates the criterion, using a Mohr circle representation. As for the tensile crack case, it is possible to develop shear cracks that intersect a plane normal to the circumferential (T) axis or a plane parallel to the circumferential axis. These are illustrated in Figure 7. For hoop cracks, the crack plane is the orientation in the R-Z plane of either the minimum or maximum stress that interacted with the hoop stress to cause failure.

Figure 6. Mohr Circle Representation of Coulomb Criterion

At the time shear cracks form, the elastic strains normal to the crack faces are calculated using Eq (3) and the trial stresses in the crack orientation. In subsequent time steps, if a crack is closed and the Coulomb slip criterion is exceeded, the stresses are modified based on the following constraints:

- First, the shear stress is limited by the Coulomb criterion.

$$|\sigma_{\tau}| = S_0 - \mu \sigma_n. \quad (21)$$

As observed experimentally, the postfailure envelope may be different than the failure envelope.

- The volume is preserved; therefore, the hydrostatic stress is maintained constant.

$$\sigma_p + \sigma_n = \text{constant}. \quad (22)$$

- Finally, to maintain symmetry of stress, it is necessary that the principal stress orientation of the cracks be maintained.

$$\frac{2\sigma_{pn}}{(\sigma_p - \sigma_n)} = \text{constant}. \quad (23)$$

a. Minimum and Maximum Stress in R-Z Plane (R-Z crack)

b. Minimum Stress in R-Z Plane and Maximum in Hoop Plane (hoop crack)

c. Minimum Stress in Hoop Plane and Maximum in R-Z Plane (hoop crack)

Figure 7. Orientation of Shear Cracks

Equations (21), (22), and (23) may be written for both the trial and new stresses. The equations may then be solved to give the new stresses in terms of the old stresses. If the shear is positive, one obtains

$$\sigma_n^{\text{new}} = \frac{S_0 (\sigma_p^{\text{trial}} - \sigma_n^{\text{trial}}) - (\sigma_p^{\text{trial}} + \sigma_n^{\text{trial}}) \sigma_{pn}^{\text{trial}}}{\mu(\sigma_p^{\text{trial}} - \sigma_n^{\text{trial}}) - 2\sigma_{pn}^{\text{trial}}},$$

$$\sigma_p^{\text{new}} = \sigma_p^{\text{trial}} + \sigma_n^{\text{trial}} - \sigma_n^{\text{new}},$$

$$\sigma_{pn}^{\text{new}} = S_0 - \mu \sigma_n^{\text{new}}, \quad (24)$$

and if the shear is negative, the equations become

$$\sigma_n^{\text{new}} = \frac{S_0 (\sigma_p^{\text{trial}} - \sigma_n^{\text{trial}}) + (\sigma_p^{\text{trial}} + \sigma_n^{\text{trial}}) \sigma_{pn}^{\text{trial}}}{\mu(\sigma_p^{\text{trial}} - \sigma_n^{\text{trial}}) + 2\sigma_{pn}^{\text{trial}}},$$

$$\sigma_p^{\text{new}} = \sigma_p^{\text{trial}} + \sigma_n^{\text{trial}} - \sigma_n^{\text{new}},$$

$$\sigma_{pn}^{\text{new}} = -S_0 + \mu \sigma_n^{\text{new}}. \quad (25)$$

The mechanics of modifying stresses for a hoop crack are the same as for an R-Z crack. More steps are required, however, since the R-Z stresses must first be rotated to the crack plane orientation. Then the rotated R-Z stress and the hoop stress are rotated to the crack and modified. Finally, the reverse rotations are performed.

After the stresses are modified for a closed crack, it is necessary to update the elastic strain normal to the crack. As an example, if (because of sliding) the elastic trial stress normal to the crack is reduced, this reduces the elastic strain normal to the crack. Correcting the strains normal to the cracks is accomplished by using Eq (8) and the stresses before and after the closed crack stress calculations to obtain the inelastic (sliding) strain increments. The sliding strain increments are then used to update the strains normal to the cracks.

2.4 Plasticity Stress Calculation

The soils plasticity model used here is a standard material subroutine in the HONDO II code. The model was derived and coded by Krieg. Detailed descriptions of the model are given both in the HONDO manual³ and in the development report by Krieg.⁴

In order to minimize changes to the existing plasticity model coding, it was decided to decouple the tensile cracking part of the routine from the plasticity part. This is accomplished by adding to the plasticity model the calculations described in Section 2.2 that evaluate the changes in the trial stresses resulting

from the existence of open cracks. The modified stresses are not used directly, but are used with the trial stresses to calculate crack opening strains (Eq (8)). These strains are then subtracted from the total strain increments, and the new modified strain increments are passed to the plasticity calculation. The plasticity routine then performs the stress calculation as originally programmed; however, since the crack opening strains have been subtracted from the total strain increments, the calculated stresses reflect the presence of cracks. The final step is to calculate the plastic strains and use these to update the crack normal strains for any plastic flow. In this model, if a crack closes, the crack is assumed to heal and the material retains no memory of previous cracks.

The advantage of decoupling is that it easily allows flexibility in incorporating cracking into existing constitutive routines. The disadvantage of decoupling is that it can introduce residual stresses normal to an open crack if plastic flow occurs. In most cases this is an acceptable approximation, since this error is corrected in the next time step when the normal stress is updated to reflect the crack. Also, in most cases, the residual stresses are of small magnitude because of the small time steps used to explicitly integrate the accelerations.

Only a brief description of the plasticity model will be given.* A yield surface is assumed for the material that is a surface of revolution about the hydrostat in principal stress space, as shown in Figure 8. The radius of the surface about the hydrostat is taken to be a quadratic function of the mean pressure (p_m) defined as

$$p_m = -\frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3). \quad (26)$$

The plasticity model includes both deviatoric and volumetric plasticity. The volumetric plasticity defines the position of the end cap on the normally open (most compressive) end of the surface of revolution.

Figure 8. Plasticity Yield Surface in Principal Stress Space

The yield surface is most easily described as two functions; one describing the paraboloid and one describing a plane that is normal to the hydrostat

$$\theta_s = J_2 - (a_0 + a_1 p_m + a_2 p_m^2), \quad (27)$$

$$\theta_p = p_m - f, \quad (28)$$

where p_m is again the mean pressure (positive); a_0 , a_1 , and a_2 are material constants; and J_2 is the second invariant of the deviatoric stresses ($\underline{s}$). The quantity (f) is a function of the mean volumetric strain and defines the volumetric stress-strain curve for the material as shown in Figure 9.

Figure 9. The Pressure vs Finite Volume Strain Behavior

The strain increment, which includes the corrections to account for crack opening strains, is decomposed into a mean volumetric strain increment ($\Delta \epsilon_v$) and a deviatoric strain increment ($\Delta \underline{\epsilon}$).

The plasticity theories for the volumetric and deviatoric parts are now taken to be completely independent. The volumetric part is treated first.

*I am indebted to Lee Taylor who allowed me to use this description of the plasticity model. He originally wrote it for one of his reports.

pendent. The volumetric part is treated first.

The mean volumetric strain is first updated as

$$\epsilon_v^{t+\Delta t} = \epsilon_v^t + \Delta \epsilon_v. \quad (29)$$

A check is then made to see if

$$\epsilon_v^{t+\Delta t} > \epsilon_u, \quad (30)$$

where ϵ_u is the most negative volumetric strain previously experienced by the material (initialized to zero). If Eq (30) is satisfied, the step is elastic and,

$$p_m^{t+\Delta t} = p_m^t - B \Delta \epsilon_v, \quad (31)$$

where B is the elastic bulk modulus for the material. If Eq (30) is not satisfied,

$$p_m^{t+\Delta t} = f(\epsilon_v^{t+\Delta t}), \quad (32)$$

and we set

$$\epsilon_u = \epsilon_v^{t+\Delta t}. \quad (33)$$

The deviatoric part uses almost conventional plasticity theory. The deviatoric stress rate is written as a function of the deviatoric strain rate in a manner that is consistent with the constitutive assumptions in the plasticity model. These include the assumption that the total strain rate is decomposed into an elastic and plastic part, that linear elasticity is used for the elastic part, and that the plastic part is given by a normality condition. The deviatoric stress rate is then:

$$\underline{\dot{s}} = 2 G \underline{\dot{e}} - \frac{2 G \underline{s} \cdot \underline{e} \dot{e} - (a_1 + 2a_2 p_m) \dot{p}_m}{\underline{s} \cdot \underline{s}} \underline{s}. \quad (34)$$

Assuming that the strain rate is constant throughout the time step, Eq (34) can be integrated exactly so that the final stress state lies on the yield surface. The details of this exact integration are found in Krieg.⁴

3. Implementation in HONDO

The two new constitutive models have been added as two new materials in HONDO, MT1, and MT2. Input parameters for these two models are given in Tables 1 and 2.

For MT1 (the tensile/Coulomb model), the post-fracture surface may be described as different than the failure surface. A simple rate-dependent strength

ratio can also be input. As shown in Figure 10, the maximum principal strain rate is calculated. A strength ratio is interpolated in a linear log-log manner and used to multiply the input strength parameters. If the strain rate is below the lowest specified rate, the input values for strength are used in the calculation. If the highest specified strain rate is exceeded, a message is printed and the solution is terminated.

Figure 10. Interpolation of Strength Ratio (MT1)

The input for MT2 (the tensile/plasticity model) is the same as described in the HONDO manual for MT7, except for the addition of the tensile strength parameter. As for MT7, the parameters a_0 , a_1 , and a_2 specify the deviatoric yield stress as a function of pressure. That is

$$J_2 = a_0 + a_1 p_m + a_2 p_m^2,$$

where

$$J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2],$$

$$p_m = -\frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3). \quad (35)$$

The yield stress is related to J_2 by

$$\sigma_y^2 = 3J_2. \quad (36)$$

Thus, given triaxial test data, it is possible to calculate the appropriate input parameters describing the yield surface.

Based on calculations to date, it is recommended that 4-Gauss point integration be used for both the volumetric and deviatoric stiffness components. This gives the most accurate elastic solution for a given mesh and results in the best crack formation prediction.

Table 1. Input for Material Type 1 – Tensile Fracture/Coulomb Model

Card VIa. Material Identification Card (2I5, E10.0)

Col's	Fmt	Var	Item
1-5	I	N	Material identification number
6-10	I	MTYPE(N)	Material type number
11-20	E	RO(N)	Mass density of material

Card VIb. Material Heading Card (80H)

Col's	Fmt	Var	Item
1-80	H	—	Any title suitable for describing the material or the table which follows

Card VIc. Material Stress Strain Information (5E10.0) (6 cards)

Card No.	Col's	Fmt	Var	Item
1	1-10	E	EE(1,1,N)	Young's modulus (E)
2	1-10	E	EE(1,2,N)	Poisson's ratio (ν)
3	1-10	E	EE(1,3,N)	Tensile fracture strength (T_0)
4	1-10	E	EE(1,4,N)	Unconfined strength of virgin material (C_0)
4	11-20	E	EE(2,4,N)	Postfailure unconfined strength (C_0^*) (Note 1)
5	1-10	E	EE(1,5,N)	Internal friction coefficient for virgin material (μ)
5	11-20	E	EE(2,5,N)	Postfailure internal coefficient of friction (μ^*) (Note 1)
6	1-10	E	EE(1,6,N)	First strain rate ($\dot{\epsilon}$)
6	11-20	E	EE(2,6,N)	Strength ratio at ϵ_1 (R_1)
6	21-30	E	EE(3,6,N)	Second strain rate ($\dot{\epsilon}_2$)
6	31-40	E	EE(4,6,N)	Strength ratio at ϵ_2 (R_2)

(Note 2)

Note 1: If not specified, the virgin material value is used.

Note 2: If not specified, no strain rate effect included.

Table 2. Input for Material Type 2 - Tensile Fracture/Plasticity Model

Card VIa. Material Identification Card (2I5, E10.0)

Col's	Fmt	Var	Item
1-5	I	N	Material identification number
6-10	I	MTYPE(N)	Material type number
11-20	E	RO(N)	Mass density of material

Card VIb. Material Heading Card (80H)

Col's	Fmt	Var	Item
1-80	H	—	Any title suitable for describing the material or the table which follows

Card VIc. Material Stress Strain Information (5E10.0) (6 cards)

Card No.	Col's	Fmt	Var	Item
1	1-10	E	EE(1,1,N)	Shear modulus (G)
1	11-20	E	EE(2,1,N)	Bulk unloading modulus (B)
1	21-30	E	EE(3,1,N)	Yield function constant (a_0)
1	31-40	E	EE(4,1,N)	Yield function constant (a_1)
1	41-50	E	EE(5,1,N)	Yield function constant (a_2)
2	1-10	E	EE(1,2,N)	Volumetric strain ($\ln(\rho_0/\rho)$)
2	11-20	E	EE(2,2,N)	Pressure (p)
2	21-30	E	EE(3,2,N)	Volumetric strain ($\ln(\rho_0/\rho)$)
2	31-40	E	EE(4,2,N)	Pressure (p)
2	41-50	E	EE(5,2,N)	Tensile fracture strength (T_0)
3	1-10	E	EE(1,3,N)	Volumetric strain ($\ln(\rho_0/\rho)$)
3	11-20	E	EE(2,3,N)	Pressure (p)
3	21-30	E	EE(3,3,N)	Volumetric strain ($\ln(\rho_0/\rho)$)
3	31-40	E	EE(4,3,N)	Pressure (p)
	.			
	.			
	.			

4. Theoretical Limitations and Applicability of the Models

There are at least three limitations associated with use of a stress criterion for tensile cracking: crack propagation is mesh-size dependent, cracks tend to follow mesh lines, and spurious cracks may be introduced if the input tensile failure strength is relatively low.

The dependence of crack propagation on mesh size has been discussed by Bazant and Cedolin⁷ and by Saouma.⁸ Figure 11 illustrates the cause of the mesh dependence when a stress criterion is used for crack propagation. For a crack in a plate, the analytic elastic solution predicts an infinite stress at the tip of a sharp crack. In the finite-element solution, the crack is smeared over an element. The stress in the element ahead of the crack tip governs the condition of cracking, yet these stresses can be made to have any value, depending on mesh size. As the mesh size is reduced, the finite element calculation captures the stress singularity better (i.e., the element stresses at the crack tip become larger as the mesh size is reduced). As a result, the crack is predicted to grow under lower loads as the mesh is reduced. In the limit, the solution does not converge to the theoretical solution, but instead predicts the crack will propagate under near zero load.

One way to remove the mesh-size dependence is to use a fracture mechanics criterion for crack growth. Saouma, Ingraffea, and Catalano⁹ have successfully applied linear elastic fracture mechanics to concrete, while Schmidt and Huddle¹⁰ and Schmidt and Lutz¹¹ have used the same approach for Indiana Limestone and Westerly Granite. However, implementation of a fracture approach is cumbersome for arbitrary cracks whose direction is not known. The method of moving singularity elements through the mesh as a crack propagates requires continuous rezoning of the mesh. Since cracks can propagate arbitrarily across mesh lines and multiple cracks can form, the rezoning can become difficult. As discussed by Basant and Cedolin, the approaches they propose (energy method, equivalent strength, and fitting an asymptotic series to nodal displacements) all have advantages and disadvantages. As such, they offer the potential of simpler implementation, but require further development for implementation to predict arbitrary crack propagation.

a. Coarse Mesh → High Propagation Load

b. Fine Mesh → Low Propagation Load

Figure 11. Mesh Dependence of Crack Propagation

It can be argued that in many applications a stress criterion is adequate for materials that have universally distributed flaws (i.e., many geologic materials). The tensile strengths of these materials are low (because of the inherent flaws) and, therefore, from a global perspective, tensile stresses are low and do not contribute significantly to the total internal energy of the structure. As a result, the overall response of the structure can be predicted using a stress criterion, although the detailed location and lengths of the

fractures can not be accurately predicted. It should also be noted that both the fracture mechanics and stress criterion approaches use stresses to predict crack initiation (for the fracture approach, the initiation stress is calculated by assuming a flaw size), the difference in the approaches arises during crack propagation.

A second theoretical limitation is that cracks tend to propagate parallel to mesh lines. This is a result of the element discretization, where displacements are assumed to be continuous. As a consequence, as illustrated in Figure 12 for 4-Gauss point integration, a greater volume of material must fail for a crack to pass diagonally through a mesh than for a crack to run parallel to the mesh lines. Thus, extending a diagonal crack is more difficult than extending a parallel crack. The opposite occurs when one-point integration is used. Because of the zero energy bending mode of the one point integration element, it becomes easiest to propagate diagonal cracks.

The tendency of cracks to propagate with mesh lines can be reduced by reducing element size. This makes it easier for cracks to propagate in the desired direction, but also reduces the loads to propagate the cracks. One alternate method, which may be worthy of development, would be to make the tensile strength a function of orientation relative to the mesh lines.

A third limitation of the model is the formation of spurious cracks. Typically, the tensile failure strength is specified as a relatively low value in relation to the other calculated stresses in the problem. As a result spurious cracks can form due either to the stress spikes caused by the sudden reduction of stresses when a new crack forms, or by numerical approximations in the calculation. An approach that reduces this problem in HONDO is to add some artificial damping to help smooth stress spikes. A second approach is to release only a fraction of the stresses when a crack forms (i.e., cut the stress normal to the crack by one-half rather than setting it to zero). The most successful approach seems to be the addition of some artificial damping, although this area requires further investigation.

The postfailure behavior of the two models is different when unconfined. As illustrated in Figure 13, the tensile/Coulomb model has a minimum unconfined postfailure strength, while the tensile/plastic model has the maximum postfailure strength. Thus, the models give upper and lower bound solutions to postfailure response.

a. Parallel Propagation

b. Diagonal Propagation

Figure 12. Gauss-Point Fractures Required for Crack Propagation

The appropriateness of these models to any particular problem must be judged by the user. In general, they seem to work best when applied to problems with a confining stress (such as an in situ stress). One should not expect the models to predict exact fracture

locations; rather, one should look at the overall response. They should definitely not be used for materials capable of sustaining a significant tensile stress; fracture mechanics must be used for those problems.

Figure 13. Postfailure Behavior of Models

5. Sample Applications

Two sample applications will be used to illustrate analyses with these material models. The first uses both models to predict the results of an indenter test on a block of sandstone. The second is a borehole fracture analysis using the tensile/plasticity model.

5.1 Indentor Test

As part of model verification, indenter tests were performed on block specimens of Berea Sandstone. The block dimensions were 3.75 x 3.75 x 6 in. with a 1-in.-wide flat indenter loading the specimen that rested on a flat surface. Two tests were performed: one in which no lateral prestress was applied to the specimen and one where a lateral prestress of -2100 psi was applied by clamping the sides of the specimen.

Figure 14 shows the fracture patterns visible to the naked eye. The effect of the prestress can clearly be seen. Without a prestress, cracks extended to the bottom of the specimen. With a prestress, rock was crushed under the indenter and essentially no cracking occurred. The load at failure also increased with increasing prestress.

Figure 14. Indenter Specimen Fracture Patterns

A plane strain analysis was used to model these tests. The indenter was displaced downward at a slow constant velocity to give an essentially static solution. Slide lines with a friction coefficient of 0.6 represented contact between the specimen and steel. For the case with prestress, the specimen was given an initial stress and the restraining plates were fixed on their outer surfaces.

Figure 15 shows the calculated fracture patterns for zero prestress for both material models. The load/displacement curve for the indenter is shown in Figure 16. The fracture patterns predicted by both models approximate the experimental results. As expected, the two models bound the postfailure response. The tensile/Coulomb model shows a failure load that is below that measured experimentally, while the tensile/plasticity model continues to carry load after

fracture. Similar results are shown in Figures 17 and 18 for the case with prestress. Again, the predicted fractures approximate the observed fractures, while the two models bound the observed load/displacement behavior.

a. Tensile/Coulomb Model

b. Tensile/Plasticity Model

Figure 15. Predicted Indentor Fracture Patterns (zero prestress)

Figure 16. Comparison of Predicted and Measured Indentor Loads (zero prestress)

More details of this calculation may be found in Reference 12.

5.2 Borehole Fracture

The second sample problem is the analysis of gas fracture experiments performed at Sandia.¹³⁻¹⁵ In these experiments, a pressure pulse is applied to a borehole to form radial cracks extending from the borehole into the surrounding rock. The objective is to stimulate recovery from natural gas wells. The experimentally observed behavior is that the fracture patterns are a function of the pressure-loading rate on the borehole. Under slow-loading rates, hydraulic fractures (two radial fractures normal to the minimum in situ stress) are observed, multiple radial fractures result from intermediate loading rates, and the borehole is crushed under rapid loading rates.

The plane strain finite element model is shown in Figure 19. In situ stresses were included in the calculation by initializing all the elements to a uniform stress state. A zero pressure was applied to the borehole and the initial stresses were allowed to come to equilibrium before the pressure pulse was applied. It is recognized that gas pressurization of the cracks plays an important role in the fracture process both by extending existing cracks and by putting adjacent material in compression and inhibiting the formation of new cracks. This was included in the analysis by setting the pressure in the open cracks to a fraction of the borehole pressure. Appendix A illustrates the method used to calculate material parameters and to initialize in situ stresses.

a. Tensile/Coulomb Model

b. Tensile/Plasticity Model

Figure 17. Predicted Indentor Fracture Patterns
(-2100 psi prestress)

Figure 18. Comparison of Predicted and Measured
Indentor Loads (-2100 psi prestress)

Figure 19. Finite Element Mesh

Three calculations were performed using the tensile/plasticity model with generic low, intermediate, and fast loading rates shown in Figure 20. The calculated fracture patterns and minimum stress contours are shown in Figures 21 through 23. For the slow and intermediate loading cases, fractures form in the expected patterns, with hydraulic fracture for the slow case and multiple fracture for the intermediate case. For the rapid-loading case, no fractures form; instead, the material flows plastically, leaving a residual stress cage. This corresponds to observed behavior.

Further details of this calculation are given in References 15 and 16. These results illustrate that the material models can provide a meaningful prediction for at least some engineering calculations. This method is now being used to predict the loading rates required to obtain multiple fractures in different conditions than those for which test data is available.

6. Summary

The implementation of two geologic constitutive models in the HONDO computer program has been described in detail. Both models exhibit the same tensile cracking behavior. In compression, one model uses a Coulomb criterion to form shear cracks, while the second model performs an elastic-plastic calculation. Theoretical limitations of the models have been discussed, including the mesh size dependence, tendency to propagate parallel to mesh lines, and the formation of spurious cracks. Despite these limitations, two applications of the models are shown where the models provided meaningful engineering results.

CASE	RISE TIME t_p (sec)	MAXIMUM PRESSURE P_{MAX} MPa (psi)	LOADING RATE GPa/sec ($\times 10^6$ psi/sec)
SLOW	0.00500	52 (7500)	10.4 (1.5)
INTER.	0.00050	103 (15000)	206 (30)
FAST	0.00005	207 (30000)	4140 (600)

Figure 20. Borehole Loading Conditions

a. Initial Conditions

b. At Peak Pressure

c. After Transient

Figure 21. Calculated Fractures and Minimum Stress Contours for Slow Loading Rate

a. Initial Conditions

b. At Peak Pressure

c. After Transient

Figure 22. Calculated Fractures and Minimum Stress Contours for Intermediate Loading Rate

Figure 23. Calculated Fractures and Minimum Stress Contours for Fast Loading Rate

7. References

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APPENDIX A

Input of Elastic/Plastic Material Model Parameters

Specification of the input parameters for the standard soils plasticity model in HONDO typically causes confusion. Since the same input is required for the tensile/plasticity model, this appendix illustrates the calculation of input parameters using typical available data. Assume we are given the following data (typical for Ash Fall Tuff):

Young's Modulus: $E = 7.25 \times 10^5 \text{ lb/in}^2$
 Poisson's Ratio: $\nu = 0.26$
 Unconfined Compressive Strength: $C_0 = 4350 \text{ lb/in}^2$
 Internal Coeff. of Friction $\mu = 0.4$

The material will be described, using elastic volumetric behaviour and a pressure-dependent yield.

The shear modulus is calculated by

$$G = \frac{E}{2(1+\nu)} = 2.877 \times 10^5 \text{ lb/in}^2. \quad (\text{A1})$$

The bulk unloading modulus is given by

$$B = \frac{E}{3(1-2\nu)} = 5.035 \times 10^5 \text{ lb/in}^2. \quad (\text{A2})$$

The parameters a_0 , a_1 , and a_2 describe the deviatoric yield surface as a function of J_2 . Where

$$J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2],$$

$$J_2 = \frac{1}{3} \sigma_y^2 = a_0 + a_1 p_m + a_2 p_m^2,$$

$$p_m = -\frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3). \quad (\text{A3})$$

Observe that in HONDO compressive stresses are negative, while a compressive pressure is positive. We now need to calculate the parameters a_0 , a_1 , and a_2 . The most general way to do this is to evaluate J_2 at three pressures and fit the quadratic equation to the three points. For a triaxial test, σ_1 is the stress in the loading direction, while the principal stresses in the

other two directions are equal to the confining pressure, p_{conf} . We know the stresses for the unconfined case.

$$\sigma_1 = -4350.0 \text{ lb/in}^2, \sigma_2 = \sigma_3 = 0.0.$$

Using Eq (A3), we obtain

$$J_2 = 6.30750 \times 10^6 \text{ lb}^2/\text{in}^4, \sigma_y = 4350.0 \text{ lb/in}^2,$$

$$p_m = 1450.0 \text{ lb/in}^2.$$

We now need to calculate J_2 and the pressure at two other points. We can do this, using Eqs (18) and (20) of Section 2.3. Using Eq (18), the cohesion is given by

$$S_0 = \frac{C_0}{2[(\mu^2 + 1)^{1/2} + \mu]} = 1472.55 \text{ lb/in}^2.$$

Then using Eq (20) at a confining pressure of 5000 lb/in² ($\sigma_2 = \sigma_3 = \sigma_{\text{max}} = -5000 \text{ lb/in}^2$)

$$\begin{aligned} \sigma_1 = \sigma_{\text{min}} &= \frac{\sigma_{\text{max}} [(\mu^2 + 1)^{1/2} + \mu] - 2S_0}{[(\mu^2 + 1)^{1/2} - \mu]} \\ &= -15,258.1 \text{ lb/in}^2. \end{aligned}$$

As before, using Eq (A3), we obtain

$$J_2 = 3.50762 \times 10^7 \text{ lb}^2/\text{in}^4, \sigma_y = 10,258.1 \text{ lb/in}^2,$$

$$p_m = 8419.37 \text{ lb/in}^2.$$

Similarly, at a confining pressure of 10,000 lb/in², we obtain

$$\sigma_1 = -26,166.3 \text{ lb/in}^2,$$

$$J_2 = 8.71164 \times 10^7 \text{ lb}^2/\text{in}^4, \sigma_y = 16,166.3 \text{ lb/in}^2,$$

$$p_m = 15,388.8 \text{ lb/in}^2.$$

We then write Eq (A3) in matrix form for the three points

$$\begin{Bmatrix} 6.30750 \times 10^6 \\ 3.50762 \times 10^7 \\ 8.71164 \times 10^7 \end{Bmatrix} = \begin{bmatrix} 1 & 1450.0 & (1450.0)^2 \\ 1 & 8419.37 & (8419.37)^2 \\ 1 & 15,388.8 & (15,388.8)^2 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix}.$$

And solve

$$a_0 = 3.2465 \times 10^6,$$

$$a_1 = 1763.7 \approx 1764.0,$$

$$a_2 = 0.23955.$$

These values can now be used for analysis. Two notes of caution are appropriate. First, the program requires that $(a_1)^2 - 4(a_0)(a_2) \geq 0$, and prints a warning message if this is not met. Second, to be consistent with tensile fractures, the absolute value of the bulk fracture pressure (that is, the intersection of the yield surface with the pressure axis) should be greater than the specified tensile fracture strength. The bulk-fracture stress is printed out at execution and in this example is PFAC = -3619 lb/in².

If a conical yield surface (linear with pressure) yield surface is satisfactory, one can use Figure A1 to derive direct expressions for a_0 , a_1 , and a_2 .

Figure A1. Linearly Dependent Yield Stress

Then, the yield stress is a linear function of pressure

$$|\sigma_y| = C_1 + C_2 p_m, \quad (A4)$$

where

$$C_1 = \frac{-p_0 \sigma_{y1}}{(p_1 - p_0)},$$

$$C_2 = \frac{\sigma_{y1}}{(p_1 - p_0)}.$$

Squaring Eq (A4) and dividing by 3

$$\frac{1}{3} \sigma_y^2 = \frac{1}{3} (C_1^2 + 2C_1 C_2 p_m + C_2^2 p_m^2).$$

Equating terms to Eq (A3) and substituting for C_1 and C_2

$$a_0 = \frac{C_1^2}{3} = \frac{p_0^2 \sigma_{y1}^2}{3(p_1 - p_0)^2},$$

$$a_1 = \frac{2C_1 C_2}{3} = \frac{-2p_0 \sigma_{y1}^2}{3(p_1 - p_0)^2},$$

$$a_2 = \frac{C_2^2}{3} = \frac{\sigma_{y1}^2}{3(p_1 - p_0)^2}. \quad (A5)$$

Equation (A5) is often the simplest way to evaluate the desired coefficients.

Finally, it is necessary to specify the bulk behavior. This data is input as pairs of pressure and corresponding logarithmic bulk strain. For an elastic material

$$\frac{\rho_0}{\rho} = 1 - \frac{p_m}{B}.$$

Typical input points then are

p_m	$\ln(1 - p_m/B)$
0	0
5000	-0.009980
10000	-0.020061
15000	-0.030244
20000	-0.040532
100000	-0.221407

For some problems, it is desired to initialize stresses. When using Material 2 (and Material 7), it is necessary to also initialize the material history storage parameters. The following lines of code illustrate the changes necessary to do this for four-point integration with:

Bulk Modulus = 0.503 E6

Bulk Fracture Strength (intersection of P axis) = -1974.2

```

*INSERT,HONDO.1425
C      INITIALIZE STRESSES
      SIGINIX = -1500.
      SIGINIY = -1250.
      SIGINIT = -780.
      DO 35 I=1,480
        SIG4( 1,I) = SIGINIX
        SIG4( 5,I) = SIGINIX
        SIG4( 9,I) = SIGINIX
        SIG4(13,I) = SIGINIX
        SIG4( 2,I) = SIGINIY
        SIG4( 6,I) = SIGINIY
        SIG4(10,I) = SIGINIY
        SIG4(14,I) = SIGINIY
        SIG4( 3,I) = SIGINIT
        SIG4( 7,I) = SIGINIT
        SIG4(11,I) = SIGINIT
        SIG4(15,I) = SIGINIT
35 CONTINUE
      PINIT = -(SIGINIX+SIGINIY+SIGINIT)/3.
      BINIT = (ALOG(1.-PINIT/.503472E6))/3.
      ST5INIT = BINIT+(PINIT+1974.2)/.503472E6
      DO 36 I=1,NUMEL
        EPX4(6,I) = BINIT
        EPX4(8,I) = BINIT
        EPX4(7,I) = ST5INIT
        EPX4(16,I) = BINIT
        EPX4(19,I) = BINIT
        EPX4(17,I) = ST5INIT
        EPX4(26,I) = BINIT
        EPX4(28,I) = BINIT
        EPX4(27,I) = ST5INIT
        EPX4(36,I) = BINIT
        EPX4(38,I) = BINIT
        EPX4(37,I) = ST5INIT
36 CONTINUE

```

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