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THE MOTION OF IONIZED ELECTRONS UNDER THE INTENSE
ELECTROMAGNETIC FIELD OF THE BEAM*

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The motion of electron ionized from residual gas by the beam becomes relativistic in such a case of the high - density beam as SLC, while the ions produced by the beam remain still nonrelativistic. The ionized relativistic electron will be dragged by the beam, and will go further away from the beam (electron beam) before the beam passes through, even for the very short bunch of SLC. The electron ejected from the beam will hit the surroundings around the beam (vacuum pipe, BPM, other detectors and so on) and would produce the noises, provided that the vacuum pressure is not so low. On the other hand, the ions produced by the beam almost do not move during this passage. The electric field of ions, which is not compensated by that of electrons, would have an effect on the beam. An estimation of the effect has shown that the tune shift caused by ions would not be significant under the vacuum condition expected in SLC (A. W. Chao; CN-121, and P. Norton; CN-89). Further, the ions directly produced by the beam as well as the ions ionized by the synchrotron radiation in the arc would not accumulate because of the low repetition rate of SLC. However it may be necessary to study the effect further, because the

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phenomena associated with ions are complicated as experienced in many electron storage rings (ion trapping) and have not been completely understood yet.

In this note, we will discuss only the motions of electrons ionized by the electron beam or by the positron beam. We assume that the density of the beam is uniform and the shape cylindrical (transversely round and longitudinally rectangular). Then the electric field produced by the beam is given in MKSA units as,

$$E(y) = \begin{cases} \frac{\rho_0 y}{2\epsilon_0} & (y \leq a) \\ \frac{\rho_0 a^2}{2\epsilon_0} \frac{1}{y} & (y \geq a) \end{cases} \quad (1)$$

where a is the radius of the beam, ρ_0 the density. In the arc of SLC, the external magnetic field of bending magnets will have an effect on the motion of the ionized electron. However, the external field is relatively small around the beam compared with the field induced by the beam. We will neglect the effect of the external field in this note. As the motion of the beam is extremely relativistic, the magnetic field induced by the beam is numerically almost equal in cgs units to the electric field. Therefore, if the ionized electron does not become so extremely relativistic as the beam, the magnetic strength can be put as equal to the electric strength. To observe a feature of the relativistic motion in such a field, we consider in Appendix a simple case that the electromagnetic field is uniform.

The motion of the electron in the electromagnetic field mentioned above is described by the equations,

$$\frac{dp_x}{dt} = \frac{e}{c} E(y) v_y, \quad v_x = \frac{c^2 p_x}{E}$$

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$$\frac{dp_x}{dt} = e E(y) \left(1 - \frac{v_x}{c}\right), \quad v_y = \frac{cp_y}{E}$$

$$\frac{dE}{dt} = e E(y) v_y,$$

where the beam is moving in the x-direction. From the above equations, we can get a constant of motion, $E - cp_x = \text{const.} (= \alpha)$, (see Appendix).

By putting $s = ct$, we have,

$$\frac{d(cp_x)}{ds} = e E \frac{(cp_y)}{E}, \quad \frac{dx}{ds} = \frac{cp_x}{E}$$

$$\frac{d(cp_y)}{ds} = e E \frac{a}{E}, \quad \frac{dy}{ds} = \frac{cp_y}{E}$$

$$E = \alpha + cp_x = \sqrt{m^2 c^4 + (cp_x)^2 + (cp_y)^2}.$$

The numerical solutions in the case of $a = 30 \mu\text{m}$, $l = 2 \text{ mm}$ and $N = 5 \times 10^{10}$ are shown in Fig. 1 to Fig. 5 for the electron beam and in Fig. 6 to Fig. 7 for the positron beam. Here l is the bunch length and N the number of particles. Fig. 1 shows the energy and momentum gains of the ionized electron versus the initial radial positions of y , in the case that the electron is ionized at the head of the beam and the initial velocity is zero. We should note that the beam have no effect if $y = 0$, but this position is the singular point of the motion. Fig. 2 shows the dragging of the electron in the longitudinal direction during the interaction with the beam, and also shows the transverse motion. The numbers attached to the lines in Fig. 2 denote the initial radial positions. In Fig. 3 we can see how quickly the ionized electron is swept out from

the beam. Fig. 4 shows the positions just after the interaction with the beam versus the initial longitudinal position measured from the head of the beam, where the electron has been ionized. Fig. 5 also shows the energy and momentum gains of the electron versus the initial longitudinal position. On the other hand, the electron ionized by the positron beam oscillates in the beam, as shown in Fig. 6. We can imagine from Fig. 6 that a large part of the ionized electrons will oscillate a few times in the positron beam. In this case the dragging is small compared with the case of the electron beam as shown in Fig. 7. The energy gain is also small but some electrons would become semi-relativistic, depending on when and where the electrons are ionized in the beam.

APPENDIX

In this appendix, we first consider the motion in the uniform electromagnetic field such that $E = H$ and $E \perp H$. The equations of motion are,

$$\frac{dp_x}{dt} = \frac{e}{c} E v_y$$

$$\frac{dp_y}{dt} = e E \left(1 - \frac{v_x}{c} \right) \quad (A-1)$$

$$\frac{dp_z}{dt} = 0$$

The solution is given as follows (Landau & Lifshitz; Classical Field),

$$p_z = \text{const.}, \quad E_{\text{kin}} = c p_x = a, \quad c^2 = m^2 c^4 + c^2 p_z^2, \\ 2eEt = \left(1 + \frac{e^2}{a^2} \right) p_y + \frac{c^2 p_y^3}{3a^2},$$

$$\begin{aligned}
 \epsilon_{kin} &= \frac{\alpha}{2} + \frac{c^2 p_y^2 + \epsilon^2}{2\alpha} , \\
 p_x &= -\frac{\alpha}{2} + \frac{c^2 p_x^2 + \epsilon^2}{2\alpha c} , \\
 x &= \frac{c}{2eE} \left(-1 + \frac{\epsilon^2}{\alpha^2} \right) p_y + \frac{c^3}{6\alpha^2 eE} p_y^3 , \\
 y &= \frac{c^2 p_y^2}{2eE} , \quad z = \frac{c^2 p_z^2}{eE\alpha} p_y ,
 \end{aligned} \tag{A-2}$$

If we consider the initial condition such that

$$p_z = 0 , \quad p_x = 0 .$$

Then the solution is simplified as,

$$\begin{aligned}
 p_z &= 0 , \quad \alpha = mc^2 , \quad \epsilon^2 = m^2 c^4 , \\
 2eEt &= 2p_y + \frac{p_y^3}{3m^2 c^2} , \quad \epsilon_{kin} = mc^2 + \frac{p_y^2}{2m} , \\
 p_x &= \frac{p_y^2}{2mc} , \\
 x &= \frac{p_y^3}{6m^2 ceE} , \quad y = \frac{p_y^2}{2meE} , \quad z = 0
 \end{aligned} \tag{A-3}$$

If the motion is nonrelativistic ($v/c \ll 1$), we have

$$\begin{aligned}
 p_y &= eEt , \quad p_x = \frac{(eEt)^2}{2mc} , \quad \epsilon_{kin} = mc^2 + \frac{(eEt)^2}{2m} , \\
 x &= \frac{e^2 E^2}{6m^2 c} t^3 , \quad y = \frac{eE}{2m} t^2 , \quad z = 0
 \end{aligned} \tag{A-4}$$

In the extremely relativistic case ($v/c \approx 1$), we have

$$p_y^3 = 6eEm^2 c^2 \quad ,$$

$$p_x = \frac{(6eEm^2 c^2)^{2/3}}{2mc} \quad , \quad p_y = (6eEm^2 c^2)^{1/3} \quad , \quad (A-5)$$

$$x = ct \quad , \quad y = \frac{6^{2/3}}{2} \frac{c}{(eE)^{1/3}} (mc)^{1/3} t^{2/3} \quad .$$

In this case, the motion in the longitudinal direction would become much larger than the motion in the transverse direction after a sufficient time.

We will next suppose that the electromagnetic field above is produced by the imaginary beam with the velocity of light and the finite length of $c\Delta t$ as shown in Fig. a. Since the particle accelerated relativistically is dragged in the longitudinal direction, the time duration of interaction is different from Δt (see Fig. b).

From Eq. A-3 and Fig. b, we have,

$$\Delta t = \frac{p_y}{eE} \quad .$$

Fig. a

And then we have the result,

$$p_y = eE\Delta t, \quad p_x = \frac{(eE\Delta t)^2}{2mc},$$

$$x = \frac{e^2 E^2}{6m^2 c} (\Delta t)^3,$$

$$y = \frac{eE}{2m} (\Delta t)^2$$

$$\epsilon_{kin} = mc^2 + \frac{(eE\Delta t)^2}{2m}.$$

(A-6)

Fig. b

It should be noted that the expressions of (A-6) are very similar to those in the nonrelativistic case. The magnetic field H induced by the real beam is not equal to E , but $H = \beta E$. However, if the Lorentz factor of the particle is much less than that of the beam ($\gamma_p \ll \gamma_b$), the H can be assumed as equal to E .

To confirm this assumption, we next try another approach, which will result in the same equations as in (A-6). We use the co-moving frame of the beam, which is expressed by the Lorentz transformation as,

$$x = \gamma(x' + Vt') \quad , \quad t = \gamma(t' + \frac{V}{c^2} x')$$

(A-7)

$$p_x = \gamma(p'_x + \frac{V}{c^2} \epsilon') \quad , \quad \epsilon = \gamma(\epsilon' + Vp'_x)$$

where the superscript dash means the co-moving frame, and $V(= \beta c)$ is the velocity of the beam. The electric field E' on the co-moving frame is also Lorentz-transformed as,

$$E = \gamma E' .$$

If the initial velocity of the ionized electron is zero on the laboratory frame, the momentum on the co-moving frame at $t' = 0$ is given by,

$$p'_x = -mc\beta\gamma = -\alpha , \quad p'_y = 0 , \quad p'_z = 0 \quad (A-8)$$

The equations of motion on the co-moving frame are

$$\frac{dp'_x}{dt} = 0 , \quad \frac{dp'_y}{dt} = eE' , \quad \frac{dp'_z}{dt} = 0 \quad (A-9)$$

We can easily find the solution,

$$p'_x = -\alpha , \quad p'_y = eE't , \quad p'_z = 0$$

$$\epsilon'_{kin} = \sqrt{\epsilon_0^2 + e^2 E'^2 t'^2 c^2} \quad (A-10)$$

$$\epsilon_0^2 = m^2 c^4 + \alpha^2 c^2 .$$

And next from the following equations,

$$\frac{dx'}{dt'} = \frac{c^2 p'_x}{\epsilon'_{kin}} = \frac{-\alpha c^2}{\sqrt{\epsilon_0^2 + (eE'ct')^2}} \quad (A-11)$$

$$\frac{dy'}{dt'} = \frac{c^2 p'_y}{\epsilon'_{kin}} = \frac{c^2 eE't'}{\sqrt{\epsilon_0^2 + (eE'ct')^2}}$$

we have,

$$\begin{aligned}x' &= \frac{-\alpha c}{eE'} \sinh^{-1} \left(\frac{ceE't'}{\epsilon_0} \right) \\y' &= \frac{1}{eE'} \left(\sqrt{\epsilon_0^2 + (eE'ct')^2} - \epsilon_0 \right).\end{aligned}\tag{A-12}$$

As the beam is extremely relativistic on the laboratory frame, the ionized electron is so extremely relativistic on the co-moving frame. Therefore the motion of the electron on the co-moving frame is changed only a little bit through the interaction with the beam, and the electron passes through with almost constant velocity of light, if the electric field of the beam on the co-moving frame (not on the lab-frame) is not extremely strong. Then we can use an approximation that the time duration of interaction $\Delta t'$ on the co-moving frame is equal to the proper-length of the beam divided by the velocity of light,

$$\Delta t' \approx \gamma \Delta t \quad .$$

Using the inverse Lorentz transformation, we can get the position and momentum of the electron on the lab-frame just after the interaction,

$$\begin{aligned}p_y &= eE\Delta t \\p_x &= \gamma \left(-mc\beta\gamma + \frac{\beta}{c} \sqrt{(mc^2\gamma)^2 + (eE\Delta t)^2} \right) \\x &= \gamma \left(-\frac{mc^2\beta\gamma^2}{eE} \sinh^{-1} \left(\frac{ceE\Delta t}{mc^2\gamma} \right) + \beta c\gamma\Delta t \right) \\y &= \frac{\gamma}{eE} \left(\sqrt{(mc^2\gamma)^2 + (eE\Delta t)^2} - mc^2\gamma \right) \\\epsilon_{kin} &= \gamma \left(\sqrt{(mc^2\gamma)^2 + (eE\Delta t)^2} - mc^2\beta^2\gamma \right) .\end{aligned}\tag{A-13}$$

If the electric field of the beam satisfies the relation,

$$eE\Delta t/mc\gamma \ll 1 \text{ (as in SLC),}$$

(A-13) can be reduced to,

$$p_y = eE\Delta t \quad , \quad p_x = \frac{(eE\Delta t)^2}{2mc} \quad (A-14)$$

$$y = \frac{eE}{2m} (\Delta t)^2 \quad , \quad x = \frac{B(eE)^2}{6m^2 c} (\Delta t)^3 \approx \frac{(eE)^2}{6m^2 c} (\Delta t)^3 \quad ,$$

where we have retained only the first term in the expansion with respect to the parameter $eE\Delta t/mc\gamma$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

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