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HAIRPIN VORTICES, SINGULARITIES, AND TRANSITION TO TURBULENCE IN THREE-DIMENSIONAL SHEAR FLOWS†

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EXTENDED ABSTRACT

1. Introduction

A heuristic description of the vorticity distribution in transitional and fully turbulent flow that has emerged over the last decade is of a collection of stretched, folded, and pinched vortex tubes [1, 2]. As vortices stretch, the local velocity fields associated with them intensify, and conservation of energy dictates that they must fold so that cancellation can occur between adjacent patches of countersign vorticity. This informal picture has been supported by experimental observation of tightly curved (hairpin) vortices in turbulent boundary layers [3] and free shear layers [4]. Moin et al. [5] have simulated the emergence of a hairpin vortex from a perturbed vortex layer and have shown that a curved vortex filament can evolve into a vortex ring in the presence of viscosity and mean shear, thus linking experimental observations of hairpin vortices with observations of vortex rings in the outer regions of a turbulent boundary layer [3]. Hairpin vortices have been associated with turbulent bursts [6], and there has been speculation as to the connection between hairpin vortices and the formation of singularities [7, 8].

The existence and role of singularities in the Euler equations is an issue that has been subject to speculation and controversy [1, 9, 10, 11, 12]. There is no question that steep gradients exist on small length scales. Whether these lead to singular events in which the Euler equations break down (in a manner analogous to viscous regularization of Burgers' equation in the presence of a shock) remains a subject of vigorous debate. The importance of such singularities is obvious; they would dominate the vorticity dynamics and play a critical role in the transfer of energy from the inertial to the dissipative scales. Indeed, the existence of singularities would suggest that the macroscopic effect of dissipation is

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independent of the value of the viscosity, thus providing a heuristic link between the geometry of singularities and the dynamics of the inertial range. Therefore, the structure of a singularity, the possible scenarios leading to its formation, the time scale on which it occurs, and the vorticity dynamics in the post-singularity regime are all issues of significance and concern.

Chorin [1] observed that an inviscid vorticity distribution modeled as a collection of vortex tubes becomes singular in finite time, and that the Hausdorff dimension of the support of the vorticity collapses to a value of roughly 2.5. Siggia [13] performed further vortex filament calculations, with the conclusion that counter-sign vortices merge in finite time, becoming antiparallel at their point of closest approach. This has inspired several numerical studies of colliding vortex tubes and fueled speculation as to the role of vortex reconnection in the turbulent energy cascade [10,11,14,15]. Recently, Bell and Marcus [12] have provided strong numerical evidence that the evolving vorticity field of a transversely perturbed co-flowing jet becomes singular in finite time, accompanied by the emergence of a Kolmogorov range in the energy spectrum. This paper is a continuation and amplification of that work.

We study the evolution of a transversely perturbed co-flowing jet in a triply periodic box by means of a second-order projection method for the three-dimensional Euler equations [16,17]. Advection-diffusion equations are solved without enforcing the incompressibility constraint, and the resulting velocity field is projected onto a divergence-free subspace. The method is unique in its treatment of nonlinear advection; it incorporates a second-order, upstream-centered differencing procedure that provides a robust treatment of nonsmooth data without introducing spurious oscillations, even in the limit of vanishing viscosity. The flow is visualized by following the evolution of a tracer function initialized on the surface of the tube. We also carry out a volume rendering of the vorticity field with an opacity profile and color map which shows that hairpin vortices abound and that vorticity intensifies greatly at their tips. We present strong numerical evidence that the vorticity becomes unbounded, and show that accompanying the onset of the singularity is a decay in the mean kinetic energy. Following the onset, a Kolmogorov \((k^{-5/3})\) range emerges in the energy spectrum.

2. Governing equations and initial data

We solve the three-dimensional Euler equations for incompressible flow in a unit cube with triply periodic boundary conditions. The initial data \((u_0,v_0,w_0)\) represents a co-flowing jet, or cylindrical shear layer, subject to a small transverse perturbation in the form of a wide, flat Gaussian bump. Thus,

\[
u_0 = \tanh\left(\frac{r^2-\zeta^2}{\delta}\right),\]

1(a)
\[ v_0 = 0, \]
\[ w_0 = \varepsilon e^{-\beta(u^2 + y^2)}, \]
where \(-\frac{1}{2} \leq x \leq \frac{1}{2}, -\frac{1}{2} \leq y \leq \frac{1}{2}, -\frac{1}{2} \leq z \leq \frac{1}{2}\). The values of the numerical parameters \(\rho, \delta, \varepsilon, \) and \(\beta\) are, for the calculations presented herein, 0.15, 0.0333, 0.05, and 15.0. The tracer function, \(\phi\), satisfies an advection equation, \(\phi_t + u \cdot \nabla \phi = 0\), and is initialized with a Gaussian profile that peaks at the inflection point of the hyperbolic tangent in the \(u_0\) field, i.e. \(\phi_0 = e^{-\kappa(x^2+y^2)}\), where \(\kappa = 500\), and \(\rho\) is the same as in Equation 1(a).

3. Current Results

Figure 1 (a)-(l) shows volume renderings of the tracer field described in the previous section. The tracer was rendered with an opacity profile that was found, by experiment, to best retain the details of the evolving flow throughout the duration of the calculation. The calculations were carried out on a uniform 128\(^3\) grid.

The first frame is at \(t=0.8\); up until this time, the growth of the perturbation has been slow and unspectacular. Patches of strain-induced secondary vorticity have begun to form on the top and bottom of the jet, offset, respectively, to the right and left of the origin (governed by the direction of the mean flow). These structures intensify as the jet further deforms, acquiring definition, interweaving, and wrapping themselves around the primary jet in a helical manner. The secondary vortices continue to intensify and begin to flatten into ribbon-like structures, themselves exhibiting a tendency towards helical deformation. By this time, the primary jet has taken the shape of an inverted "U" at each end, with thin ribbon-like parallel walls thickening to tubular structures at the base and tips, rising in the middle to form a topologically complicated knot of tightly curved vorticity (Figure 1(f)). The main features of the flow at this time are a predominance of hairpin structures and a tendency towards flattening and helical deformation of vortex tubes. This tendency towards "sheetification" of vorticity has been observed in other numerical simulations [11]. Figure 1(h) shows the flow field at the approximate time of onset of the first singularity (\(t=1.5\)). There is a predominance of highly attenuated structures, and regions of intense curvature. In the final frames, the main tubular structure reconnects into a series of rough, staggered rings.

Figures 2, 3, and 4 show the vorticity field at times 1.3, 1.4, and 1.5. In this color map, the yellow indicates a high concentration of vorticity. In these renderings, the predominance of hairpin vortices and the progressive intensification of vorticity at their tips is clearly seen. It can also be seen that as some structures intensify, others diminish in intensity.
Figures 5 and 6 show the time evolution of the $L^\infty$ norm of the vorticity and the enstrophy, respectively, for $32^3$, $64^3$, and $128^3$ grids. The maximum vorticity first peaks at approximately $t=1.5$, its value roughly doubling as the grid refinement is doubled, suggesting persuasively that in the limit of infinite resolution it would be unbounded. Similarly Figure 5 indicates that this singularity is not $L^2$-integrable. However, this first singularity appears to be highly localized. The continued growth in enstrophy after the occurrence of the first $L^\infty$ peak indicates a subsequent broadening of the singularity. At the highest resolution we observe a second $L^\infty$ peak at roughly $t=2.5$. This suggests a possible scenario for transition to turbulence as a cascade of singular events, and lends credence to speculation regarding the connection between singularities, turbulent bursts, and hairpin vortices.

The mean kinetic energy is shown in Figure 7 for the different levels of grid resolution. It remains constant in the early stages of the growth of the perturbation, then begins to decay as the vortex tube begins to strongly self-interact. The onset of the decay is delayed as the grid is refined; the steeper slope at late time in the high resolution ($128^3$) case can be attributed to the influence of the second $L^\infty$ peak, which is under-resolved on the coarser grids. The onset of decay in the kinetic energy appears to be coupled, then, to the onset of the singularity. This suggests an intimate connection between singularities and energy transfer to the small scales. Dissipation must occur; as adjacent patches of countersign vorticity merge, the energy is fluxed into increasingly high wave numbers, which cannot be represented on a finite lattice. The energy in these modes is then dissipated numerically, analogous to the physical mechanism of dissipation at the viscous length scales. To analyze the distribution of energy as a function of wave number we define an integrated energy spectrum $\Phi(K) = \int_{|K|=K} E(\vec{k}) \, dk$ where $E(\vec{k})$ is the energy at wave number vector $\vec{k}$, which is obtained by an FFT of the velocity field. Thus,

$$\frac{d\Phi(K)}{dK} = \int_{|\vec{k}|=K} E(\vec{k}) \, dk$$

(2)

is the traditional energy spectrum defined as the integral of the energy over spherical shells in $k$-space. The choice of $\Phi$ to represent the spectral data avoids numerical artifacts arising from using finite thickness shells to evaluate (2) on a discrete lattice. Figure 8 shows $\Phi(K)$ at times 0.62, 1.65, and 2.67, corresponding to before, near, and after the onset of the first singularity. We observe the growth of a region in which $\Phi(K) = O(K^{-2/3})$, and a steepening drop-off at high wave numbers, indicating the emergence of a Kolmogorov ($K^{-5/3}$) range in the energy spectrum.
4. Anticipated results

The final paper will include comparison with a similar calculation of a transversely perturbed plane shear layer, comparison with calculations at finite Reynolds number, and determination of the fractal properties of the support of the vorticity as the flow evolves.

References


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Figure 4. Vorticity field at \( t=1.5 \).

Figure 5. Maximum vorticity vs. time: \( 32^3, 64^3, \) and \( 128^3 \) grids.

Figure 6. Enstrophy vs. time: \( 32^3, 64^3, \) and \( 128^3 \) grids.

Figure 7. Kinetic energy vs. time: \( 32^3, 64^3, \) and \( 128^3 \) grids.

Figure 8. Integrated energy spectrum (normalized). a) \( t=0.62 \), b)\( t=1.65 \), c) \( t=2.67 \)
Figure 5

MAXIMUM VORTICITY VS. TIME

- заказ

TIME

MAXIMUM VORTICITY

- заказ

- заказ
Figure 6
Figure 7
Figure 8