118

PPPL-2020

UC20-G

PPPL-2020

Dr. 1789-2

118
8-17-83 me①

I-11355

THE SPECTRUM OF RESISTIVE MHD MODES IN CYLINDRICAL PLASMAS

By

C.M. Ryu and R.C. Grimm

JULY 1983

DO NOT MICROFILM
COVER

PLASMA
PHYSICS
LABORATORY

PPL

MASTER

PRINCETON UNIVERSITY

PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America.

Available from:

National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy \$ * ; Microfische \$3.50

*PAGES

NTIS Selling Price

1-25	\$5.00
26-50	\$6.50
51-75	\$8.00
76-100	\$9.50
101-125	\$11.00
126-150	\$12.50
151-175	\$14.00
176-200	\$15.50
201-225	\$17.00
226-250	\$18.50
251-275	\$20.00
276-300	\$21.50
301-325	\$23.00
326-350	\$24.50
351-375	\$26.00
376-400	\$27.50
401-425	\$29.00
426-450	\$30.50
451-475	\$32.00
476-500	\$33.50
500-525	\$35.00
526-550	\$36.50
551-575	\$38.00
576-600	\$39.50

For documents over 600
pages, add \$1.50 for each
additional 25 page increment.

DO NOT MICROFILM
COVER

The Spectrum of Resistive ~~MHD~~ Modes
in Cylindrical Plasmas*

PPPL--2020

DE83 017985

C. M. Ryu[†] and R. C. Grimm

Plasma Physics Laboratory, Princeton University

P.O. Box 451, Princeton, N.J. 08544

July 1983

Abstract

A numerical study of the normal modes of a compressible resistive MHD fluid in cylindrical geometry is presented. Resistivity resolves the shear Alfvén and slow magnetosonic continua of ideal MHD into discrete spectra and gives rise to heavily damped modes whose frequencies lie on specific lines in the complex plane. Fast magnetosonic waves are less affected but are also damped. Overstable modes arise from the shear Alfvén spectrum. The stabilizing effect of favorable average curvature is shown. Eigenfunctions illustrating the nature of typical normal modes are displayed.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

[†]Present address Los Alamos National Laboratory, New Mexico.

*Work supported by United States Department of Energy Contract
No. DE-AC02-76-CHO3073.

§1. Introduction

The single fluid magnetohydrodynamic (MHD) equations are frequently used to describe the macroscopic behaviour of confined plasmas. In their simplest form (ideal MHD), these equations provide the most straightforward models which are capable of realistically including the effects of geometry and boundary conditions. However, since the basic equations are nonlinear and describe phenomena occurring over a wide range of characteristic spatial and temporal scales, additional simplification is necessary for detailed study.

Nonlinear phenomena are frequently studied from a reduced set of equations which do not describe all the characteristic motions of the magnetofluid system. On the other hand, information about different types of motion is best obtained by studying the linearized equations for the behaviour of small perturbations about an equilibrium of interest. While it seems clear that the detailed behaviour of most plasma configurations is nonlinear, an understanding of the properties of the linearized equations is essential, not only because it more clearly identifies the physics omitted from reduced models, but also because (for example, by identifying threshold conditions for instabilities) it identifies those situations where non-linear processes may be important.

Within the context of ideal MHD much has been written about the nature of the normal modes of the linearized equations, and several general remarks can be made. Most importantly, since the equations are self-adjoint (Bernstein, et al., 1958), the normal mode frequencies are either purely real (oscillatory wave motion) or purely imaginary (damped or growing instabilities). Thus most studies concentrate on either the stable or the unstable part of the MHD spectrum. Since the transition between these must involve the point of marginal stability ($\omega^2 = 0$), it plays a significant role in the theory.

For the stable part of the spectrum, consideration of an infinite

homogeneous plasma shows that there are three branches of wave motions (Alfvén and Falthammer, 1963). One of these branches decouples in the dispersion relation and involves incompressible transverse motion of the infinitely conducting fluid and the confining magnetic field. These are the Alfvén waves. The other two branches are commonly referred to as slow and fast magnetosonic waves. In the interesting limit where the fluid pressure is small compared with the magnetic field pressure, the slow waves propagate along the magnetic field lines with phase velocity essentially equal to the local sound speed, while the fast magnetosonic waves propagate isotropically with phase velocities larger than those of the Alfvén waves. For all branches the frequency spectrum of the normal modes is discrete and the corresponding wave functions are analytic and square integrable.

In an inhomogeneous medium this spectrum is considerably complicated since the governing eigenvalue problem becomes singular (Appert et al., 1974). These singularities occur for frequencies which correspond to the Alfvén and slow magnetosonic branches in an homogeneous medium and give rise to the possibility of continuous bands of eigenfrequencies called the shear Alfvén and slow magnetosonic continua. It is an essential feature of these continua that the corresponding eigenfunctions are not analytic in the vicinity of the singular point and, perhaps more important physically, are not square integrable. The third branch, fast magnetosonic waves, is not affected in this way.

Unstable modes, which arise in inhomogeneous current carrying configurations, are discrete and the corresponding eigenfunctions are Sturmian (Goedbloed and Sakanaka, 1974). The nature of the spectrum near the marginal point is strongly dependent on the existence of continua on the stable side. For many cases of practical interest the stable continuum may reach down to

the marginal point, $\omega^2 = 0$. While this may cause significant problems for the computational study of unstable modes (Grimm et al., 1983), it so profoundly affects the behaviour of solutions near $\omega^2 = 0$ (e.g., unstable modes may accumulate there) that it is responsible for the existence of criteria [e.g., Suydam (Suydam 1958)] which can be used to determine whether a system is stable or not.

Thus, although the specific details of the spectrum of ideal MHD must be determined [probably numerically (Chance et al., 1977)] for each configuration of interest, our basic mathematical understanding is fairly complete. There are, however, some fundamental difficulties associated with developing a good physical understanding. These arise from the interpretation of the stable continuum modes and from the emphasis placed on the point $\omega^2 = 0$ in stability studies. In both situations the corresponding eigenfunctions are not simply acceptable as physically meaningful quantities and one is led to ask whether the inclusion of additional physical effects would drastically change the interpretation. Amongst all the possible non-ideal effects, the addition of a small resistivity is often treated as being typical in the sense that it provides the simplest mathematical model which allows dissipative processes to resolve the singularities of the ideal MHD theory. It is with this phenomenological attitude that we approach this work. Our aim is to provide some qualitative understanding of how small resistivity modifies the ideal MHD spectrum by studying its effect explicitly for the simple cylindrical diffuse pinch configuration.

While it is well-known that resistivity permits the existence of additional unstable modes (Furth et al., 1963), its effect on the stable part of the spectrum has received considerably less attention. This is partly because of the priority associated with the design of stable plasma

confinement devices, but it is also because of the additional complexity of the resistive MHD equations which are not self-adjoint and in general possess normal mode solutions whose frequencies are complex. For very small resistivity a boundary layer approach, which asymptotically matches $\omega^2 = 0$ ideal MHD solutions away from the singularities to resistive boundary-layer solutions in the vicinity of the singular surfaces, has been most commonly adopted (Johnson et al., 1963). While it is reasonable to expect that this should be an interesting limit, such a study does not address the possibility that modes with finite real frequency could be destabilized by, for example, changes in the equilibrium parameters. Indeed the identification of overstable modes (Glasser et al., 1975), even in this limit, suggests that a more complete study of the resistive MHD spectrum may aid our understanding.

In a simple slab geometry the spectrum of stable resistive shear Alfvén waves was studied, both analytically and numerically, by Boris (1968). In contrast to the ideal MHD model, he found that the modes were strongly damped and that the frequencies of the normal modes lay on distinct lines in the complex plane. By an asymptotic analysis in the vicinity of the accumulation points of the ideal continuum he was able to explain qualitatively some of the features. His results are, however, not widely known and since they are confirmed here within the context of a more general confinement model, it is one of the motivations of this paper to correct this situation.

In a bounded configuration with finite compressibility the resistive normal mode equations constitute a sixth order boundary value problem, in contrast to the second order equation of ideal MHD. Aside from the study of special limits with asymptotic analysis, numerical methods must be used to determine the nature of the solutions. For the study of unstable resistive modes this has most usually been carried out as an initial value problem by

following the development of the most unstable mode in the system (Dibiase, and Killeen, 1977). This technique is not practical for determining other normal modes, especially if they are damped. Recent progress with a matrix shooting method developed by Freidberg and Hewett (1981) to study unstable modes, offers the capability of directly determining the complex eigenvalues of the normal mode equations. We use it in this study. Although there are limitations associated with numerical resolution difficulties at very small values of the resistivity, the method enables a qualitative assessment of the most important effects.

This paper is arranged as follows. In Sec. 2 we introduce the basic MHD model in cylindrical geometry, present the linearized equations and briefly describe the numerical method. The nature of the resistive shear Alfvén spectrum is presented in Sec. 3 and some results for the slow and fast magnetosonic waves are given in Sec. 4. A study of overstable modes is presented in Section 5. We summarize the results and indicate some issues for future study in the final section.

§2. Basic Equations and Numerical Method

2.1 Equations and boundary conditions

We begin with the single fluid resistive MHD equations. In the usual notation, these are the equations of continuity of mass and momentum

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{v} = 0 \quad , \quad (1)$$

and

$$\rho \frac{d\underline{v}}{dt} = \underline{J} \times \underline{B} - \nabla p \quad , \quad (2)$$

an adiabatic equation of state for the internal energy of the fluid,

$$\frac{d}{dt} (p\rho^{-\gamma}) = 0 \quad , \quad (3)$$

Ohm's law relating electromagnetic fields to the current flux

$$\underline{E} + \underline{v} \times \underline{B} = \eta \underline{J} \quad , \quad (4)$$

and Maxwells' equations for the fields,

$$\frac{\partial \underline{B}}{\partial t} = - \nabla \times \underline{E} \quad , \quad \nabla \times \underline{B} = \underline{J} \quad , \quad (5)$$

and $\nabla \cdot \underline{B} = 0$.

In cylindrical geometry we linearize these equations about the static equilibrium described by

$$\frac{d}{dr} \left(p + \frac{B^2}{r} \right) + \frac{B_\theta^2}{r} = 0 \quad (6)$$

where r is normalized to the wall radius $r = r_w$, and

$$\underline{B} = B_\theta(r) \underline{e}_\theta + B_z(r) \underline{e}_z. \quad (7)$$

To be more specific we consider two configurations representative of Reversed Field Pinch (RFP) and Tokamak geometries. These are; for the RFP model (Freidberg and Hewett, 1981)

$$B_{\theta} = 0.5 (r - 5\alpha_1 r^3/48), \quad B_z = (1 - \alpha_2 r^2/12)^{1/2} (1 - 5\alpha r^2/24)$$

$$p' = \alpha_3 (r - 5\alpha r^3/6)$$

$$\text{with } \alpha_1 = \alpha (8 - 5\alpha)/3, \quad \alpha_2 = (8 - 5\alpha)^2/9,$$

and

$$\alpha_3 = 25 (\alpha - 1) (\alpha - 0.4)/108; \quad (8)$$

and for the Tokamak (Furth et al., 1973) a peaked current profile

$$B_{\theta} = r/\alpha(1 + r^2), \quad B_z = (B_o^2 - 1 + 1/(1 + r^2)^2)^{1/2}/\alpha,$$

$$p' = (1 - \alpha^2) r/\alpha^2 (1 + r^2)^2. \quad (9)$$

(Here, and throughout this paper, a prime denotes differentiation with respect to r .) In both cases the parameter α can be adjusted to modify the pressure profile; when $\alpha = 1$ the equilibria are force-free. For the Tokamak profile, B_o measures the magnitude of the magnetic field on axis, when increased it has the effect of progressively flattening the B_z profile. Typical profiles are shown in Fig. 1.

In the Appendix we assume that perturbed quantities vary as $\exp i(\omega t + m\theta + kz)$, and reduce the linearized form of Eqs. (1)-(5) to three, coupled, second-order ordinary differential equations, similar to those derived by Freidberg and Hewett for the incompressible case. In terms of the radial

component of the plasma displacement, ξ_r , the radial component of perturbed current, j_r , and the residual part of the perturbed radial magnetic field associated with nonideal MHD behaviour,

$$v_r = b_r - F \xi_r, \quad (10)$$

with

$$F = k B_z + m B_\theta / r, \quad (11)$$

these equations are

$$\frac{d}{dr} \underline{A} \cdot \frac{d\underline{Z}}{dr} + \underline{B} \cdot \frac{d\underline{Z}}{dr} + \underline{C} \cdot \underline{Z} = 0, \quad (12)$$

where

$$\underline{Z} = \{r\xi_r, rv_r, rj_r\}^T, \quad (13)$$

and $\underline{A}(r)$, $\underline{B}(r)$, and $\underline{C}(r)$ are 3×3 matrices whose entries depend explicitly on ω , η , m , k and equilibrium quantities. These coefficients are given in detail in Eqs. (A1)-(A4) of the Appendix.

Solutions of the sixth-order system described by Eq. (12) must satisfy six boundary conditions. Three are given at $r = 0$ by demanding regularity of each of the components of $\underline{Z}$; $\underline{Z}(0) = 0$. The other three are determined by enclosing the plasma with an infinitely conducting wall at $r = r_w$. Then $b_r(r_w) = 0$. We use the requirement that plasma not penetrate the wall to set $\xi_r(r_w) = 0$. Finally we use Eq. (A8) in the limit $\eta \rightarrow 0$ to set $j_r(r_w) = 0$.

Thus $\underline{Z}(r = r_w) = 0$, also. The normal modes of the linearized resistive MHD equations are then the eigenfunctions of the set of three coupled two-point boundary-value equations described by Eq. (12), with $\underline{Z}$ vanishing at both boundaries.

2.2 Numerical Method

We solve Eq. (12) numerically with a matrix shooting method similar to that employed by Freidberg and Hewett (1981). Introducing centered-differences, we obtain a second-order accurate representation of Eq. (12),

$$\underline{A}_i \cdot (\underline{Z}_{i+1} - 2\underline{Z}_i + \underline{Z}_{i-1}) + h(\underline{A}'_i + \underline{B}_i) \cdot (\underline{Z}_{i+1} - \underline{Z}_{i-1})/2 + h^2 \underline{C}_i \cdot \underline{Z}_i = 0, \\ i = 2, \dots, N, \quad (14)$$

where $N+1$ is the number of equally spaced grid points, $h = r_w/N$, $\underline{A}_i \equiv \underline{A}(r_i)$, $\underline{Z}_i \equiv \underline{Z}(r_i)$, etc. We have found that using the explicit analytic form for $\underline{A}'_i$ in the second term, rather than a difference approximation, can be very important for obtaining good numerical accuracy.

The shooting procedure is straightforward. We guess an ω , set $\underline{Z}(0) = j_r(r_w) = v_r(r_w) = 0$, and fix the derivative of ξ_r at the origin by setting $r\xi_r(r = h) = 1$. Then, solving the $3(N-1) \times 3(N-1)$ banded system of difference equations by a specially written Gaussian elimination routine which takes into account the sparse blocked nature of Eq. (14), we determine $r_w \xi_r(r_w)$. We iterate on ω until the boundary condition $\xi(r_{N+1}) = 0$ is satisfied to a prescribed tolerance, ϵ . All the calculations are carried out in complex arithmetic.

The eigenvalue iteration was carried out with an interactive, complex

root finding package which was originally developed by T. J. Martin of Culham Laboratory. Both Muellers' method and a technique based on Cauchy's integral formula have been used and give consistent results for isolated roots. Since a successful spectral study depends on locating all of the roots in the various regions of the complex plane, one might have expected the Cauchy technique to be particularly useful. However, it is a feature of this shooting method that there occur many poles in the vicinity of the eigenvalues so that it is not practically possible to choose appropriate contours of integration for determining multiple roots simultaneously. This weak point makes the search for the eigenmodes intrinsically difficult and tedious. It also makes it impossible to claim that we have determined the set of eigenvalues completely. Nevertheless, the rather special behaviour of the eigenvalues which we find indicates that this is not a serious problem for determining the overall spectral properties.

As one would expect, difficulties associated with numerical resolution occur when the resistivity becomes very small. For the unstable resistive modes we have found that a mesh of 1001 grid points is adequate for resistivities as small as $10^{-7} - 10^{-8}$. By this we mean that when the mesh size is halved the value of ω changes by $\lesssim 5\%$ with $\epsilon \lesssim 10^{-2}$. With these typical parameters it takes approximately one second of CRAY 1 computer time to locate a root.

The accurate determination of the shear Alfvén wave spectrum is, however, very sensitive to the value of the resistivity and, with the parameters described above, does not give really meaningful results when $\eta < 10^{-4}$. Undoubtedly this is because, as we shall see below, the eigenfunctions are not only highly localized, but are very oscillatory as a function of radius. When resolution difficulties begin as η is reduced, it may be possible to locate

the roots by choosing a larger value of ϵ , but eventually this becomes so inaccurate that there is no alternative to refining the mesh. We tried various zone packing schemes, but could not remedy the problem. Although these limitations make it difficult to make definitive statements about the interesting limit of ideal MHD ($\eta \rightarrow 0$), the rather simple structure of the spectra, coupled with the asymptotic analysis for small η , is sufficient to provide a good overall picture.

§3. Shear Alfvén Waves

Analytical and numerical studies of ideal MHD have revealed that both shear Alfvén and slow magnetosonic waves in inhomogeneous configurations have continuous spectra and that the corresponding eigenfunctions are singular and not square integrable. These singularities are associated with the vanishing of the coefficient D of the second order system for the normal modes (see Eq. A6), which occurs for the shear Alfvén waves when $\omega^2 = F^2(r)/\rho(r) \equiv \omega_A^2$, and for the slow waves when $\omega^2 = \gamma p(r)\omega_A^2/(\gamma p + B^2) \equiv \omega_s^2$. These continua arise since F^2/ρ is a continuous function of r for an inhomogeneous configuration. In the incompressible limit ($\gamma \rightarrow \infty$) only the shear Alfvén branch remains. The results in this section are obtained in that limit. In the next section we consider the compressible case.

There are two interesting configurations for $\omega_A^2(r)$. These are a) when ω_A^2 has a global maximum or minimum inside the plasma and b) when ω_A^2 varies monotonically for $0 < r < 1$. Both cases are useful for studying, but case (a) is probably most typical of RFP and Tokamak configurations. This is because, for reasonable values of k and m , the function $F(r)$ vanishes for an RFP (since B_z reverses near the edge of the plasma), while for a Tokamak configuration, interest usually concentrates on modes with $k = -n/2\pi R$ with R the major

radius of the equivalent toroidal configuration and n the toroidal mode number. Examples of typical $F(r)$ profiles are given in Fig. 2 for the RFP case (a) with parameters $\alpha = 1.05$, $m = 1$, $k = 0.6$, and $r_w = 3$ and for the Tokamak case (b) with $\alpha = 1.0$, $k = 0.2$, $m = 2$, $r_w = 2$, and $B_0 = 7$. In the following we concentrate on the results for case (a). We give an example of the spectrum for case (b) at the end of this section for completeness.

For the profile of Fig. 2a the continuum stretches from the marginal point to $\omega_A(r=0) = (1.05, 0)$ on the real axis of the complex ω -plane (see Fig 3a). In Figs., 3a and 3b we show the spectrum of modes for $\eta = 10^{-2}$ and 10^{-3} , respectively. In both cases we have plotted all the modes found in this region of the complex plane (since the roots appear in complex conjugate pairs we show only $R_e(\omega) \geq 0$ in most of these diagrams). For this configuration no roots are found along the real axis where the ideal continuum lies, and the only unstable mode is a purely growing tearing mode.

For these and other values of the resistivity which we have studied the spectra show several general properties. Most noticeably the eigenvalues lie on several specific lines. These lines meet the real axis at approximately $\omega = 1.1$ and $\omega = 0.3$ which correspond to the values of ω_A at $r = 0$ and $r = r_w$, respectively. The line approaching $\omega = 1.1$ very nearly makes an angle of 30° with the real axis. The one going to 0.3 is somewhat more complicated. The second readily observed feature is that, as the resistivity becomes smaller, more eigenvalues lie on the lines and the spacing between them becomes smaller. Modes move along the imaginary axis, downwards from the origin and upwards from $-\infty$, coalescing around the point A and then moving up towards the real axis. This line splits into two around the point B. One sequence moves up towards the large frequency accumulation point of the ideal continuum, C, and the other moves back towards the origin. This latter branch appears to

split again around the point D, with a sequence approaching the point, E, $\omega = \omega_A(r = r_W)$.

In Fig. 4 we show the effect of resistivity on the configuration of a typical mode in the sequence B-C. It is oscillatory and peaks around the value of r for which $R_e(\omega) = \omega_A(r)$. As the resistivity is reduced the mode becomes more highly localized around this surface. The eigenfunctions strongly suggest that an internal boundary layer forms as the resistivity is reduced.

In Fig. 5 we show the eigenmode structure for the real and imaginary parts of the radial displacement ξ_r , the perturbed radial field b_r , the perturbed current j_r , and the residual field v_r for the mode 'a' in Fig. 3b. These quantities are seen to be smooth functions of r , peaking near the magnetic axis. As the resistivity is reduced the peaking increases and a boundary layer forms at $r = 0$. In the limit the asymptotic analysis of Boris can be used (see Appendix) and it is possible to write down an approximate analytic expression for some of the eigenvalues. From Eq. (A10) it can be seen that, close to the accumulation point, C, this describes the 30° characteristic of the segment B-C.

In Fig. 6 we show the eigenmode structure for the point 'l' of Fig. 3b. While mode 'a' was localized near the axis, this mode is mostly localized near to the wall. It has a very complex structure.

In Figs. 7 and 8 we demonstrate the change in structure of the modes as we move away from the ideal MHD accumulation points towards the intersection point B. We plot only the real part of ξ_r ; the other quantities undergo a similar change. In Fig. 7 we illustrate the branch C-B. It is seen that the number of radial nodes increases in an orderly manner and that the eigenfunctions are localized where their real frequencies correspond to the

ideal MHD singularities. As the real frequency becomes smaller the localized oscillatory region detaches from $r = 0$ and moves towards the wall. In Fig. 8 we show the branch D-B. Here the modes begin localized near the wall, but move inwards towards $r = 0$ as the real frequency increases. For both branches the peak amplitude of the modes increases as we move towards the point B.

In Fig. 9 we follow the modes along B-A, plotting the real parts of ξ_r for the points labelled 'f,g,h,i' in Fig. 3b. It is seen that as the frequency approaches the imaginary axis, the maximum amplitude of the mode decreases and moves towards $r = 0$. A boundary layer forms near $r = 0$ as we approach the point A. The structure of the modes 'j','k' and others on the purely damped imaginary ω axis are similar to that of mode 'i'.

As we have suggested previously, when the resistivity is reduced the eigenvalues generally move upwards along the characteristic lines towards the real axis. To see this more clearly, we trace several key modes in Fig. 3b as the resistivity is varied. In Fig. 10 we follow modes 'j' and 'k', varying η from 1.02×10^{-3} to 0.98×10^{-3} . The modes coalesce at A and head towards B. In Fig. 11 we follow mode 'd', which represents modes localized near the axis. Its locus clearly heads towards the accumulation point (1.1, 0) along the 30° characteristic B-C. What happens in the vicinity of point D is, however, not so clear. In Fig. 12 we show the locus of modes 'l', 'm', and 'n'. As η is decreased mode 'l' clearly moves towards $\omega_A(r_w) = (0.3, 0)$, while mode 'n' appears to approach the origin. Figure 12b, for mode 'm', indicates that the branches B-D and D-E are strongly connected. For the ranges of resistivity studied we have found no modes along the line D-O. While an asymptotic matching analysis around $\omega = 0$ suggests that there should be modes along a 30° line (OD) for sufficiently small resistivity (Grimm, et al., 1982), we can make no firm conclusion from this numerical method.

To complete this section we give the spectrum (Fig. 13) and several typical eigenfunctions (Fig. 14) for the Tokamak profiles of Fig. 2b. These are very similar to the RFP example given above, except that with ω_A^2 monotonic the behaviour near the walls is much simpler.

§4. Slow and Fast Magnetosonic Waves

We have seen in the last section that resistivity resolves the singularities of the ideal MHD shear Alfvén continuum and brings in interesting spectral properties. The results given there were obtained in the incompressible limit, where the slow wave continuum does not enter the problem. For the case of finite compressibility, it is reasonable to expect that resistivity might effect the slow magnetosonic continuum in a similar way to the shear Alfvén branch - indeed this is our basic finding.

To begin, it is necessary to determine if finite compressibility significantly modifies the results for the shear Alfvén waves of the last section. We consider the RFP model and repeat the calculations of Fig. 3b with $\gamma = 5/3$. We find essentially identical results for the eigenvalue branches. The most significant difference lies in the form of the eigenfunctions. As an example, we give a mode located just below the intersection point B (on line A-B) in Fig. 15. In contrast to Fig. 9 for the incompressible case, this mode shows a tendency to localize both around the radius resonating with the ideal shear Alfvén wave and near the point corresponding to the ideal slow magnetosonic singularity. This coupling between the ideal modes makes an accurate determination of the slow waves tedious when $\omega_A \approx \omega_S$. By increasing the parameter α of Eq. (8), we can raise the pressure and separate the modes.

In Fig. 16 we illustrate the spectrum for the parameters of Fig. 3b

with $\alpha = 1.15$. In addition to the shear Alfvén waves (segment B-C), we locate three waves which we identify as slow magnetosonic waves. Their eigenfunctions are illustrated in Fig. 17. Like the shear Alfvén waves they are localized and highly oscillatory. It is interesting that for these modes the j_r component displays a double resonance effect, while for the shear waves the other components ξ_r , b_r , and u_r do. This demonstrates that the eigenfunctions are nearly orthogonal to those of the shear Alfvén waves, as they would be in ideal MHD. Extrapolation of the eigenvalues of Fig. 16 to the real axis is in excellent agreement with the upper accumulation point for the ideal slow wave continuum, $\omega = \omega_s \approx 0.85$.

While the Alfvén and slow magnetosonic continua of ideal MHD have singular behaviour, the fast magnetosonic waves are discrete and their eigenfunctions are well-behaved. One would not expect the effect of resistivity to be as significant, and this is what we find. We plot the spectrum for a few fast waves in the RFP model in Fig. 18. Like the other modes we have found, these waves are damped and the damping rate increases with real frequency. A typical eigenfunction is shown in Fig. 19. The eigenmodes are not localized and the number of nodes increases with the frequency indicating that the Sturmian character of the ideal spectrum has not been distorted by finite resistivity.

§5. Unstable Modes

In the previous sections we have considered modes which are exponentially damped. Here we consider unstable modes. In addition to those modes which are unstable in the ideal MHD limit and whose growth rates and marginal points may be slightly modified by resistivity, new modes (tearing modes, resistive interchanges) arise because fluid motions are not frozen to the magnetic

field. These modes have been extensively studied. Here we report an interesting aspect of the unstable spectrum, the existence of overstable perturbations.

While unstable ideal MHD modes are discrete and purely growing, the MHD force operator is not Hermitian in the presence of resistivity and one has to consider the possibility of oscillatory, exponentially growing perturbations. In slab geometry it was shown that these could not exist for an incompressible, pressureless plasma with a single rational surface (Furth et al., 1963). In cylindrical geometry a similar result can be shown for a pressureless plasma with multiple rational surfaces (Manickam et al., 1982), but the situation is more complicated with finite plasma pressure (Coppi et al., 1966). Studies of resistive interchanges (Glasser et al., 1975) and resistive ballooning modes in toroidal geometry (Glasser et al., 1979) show that overstable modes are an essential feature of the spectrum, but an understanding of that case may be helped by a clarification of the cylindrical limit.

Resistive instabilities are usually studied analytically, in the limit of small resistivity, by using an asymptotic boundary layer approach to construct approximations to the normal mode solutions which vary rapidly in the vicinity of rational surfaces where the solutions of the ideal, marginally stable, MHD equations are singular (Johnson, et al., 1963). The method involves asymptotically matching these marginally stable solutions away from the rational surfaces, to solutions of the resistive MHD equations inside a boundary layer located at the singular surface. The result of this analysis is a dispersion relation for the frequency and growth rate of the normal modes. These analyses have provided most of our qualitative understanding of resistive instabilities, much of which has been obtained from studying

special, simple, equilibrium configurations. In general, a complete solution for quantitative results requires detailed numerical calculations which recently have been reported elsewhere (Grimm et al., 1982). In the process of this work we have carried out a comparison of the numerical boundary layer approach and the numerical techniques described in this paper, and have obtained excellent agreement in the limit of small resistivity, both for actual growth rates and for their scaling with resistivity (Ryu, 1982). Here we use the matrix shooting code, on a case where the asymptotic analysis indicates overstability, to illustrate the way in which overstability arises and to emphasize that, in more general configurations than are usually studied, the determination of the stability of a configuration requires some understanding of the nature of the whole spectrum, in contrast to the case of ideal MHD, where eigenmodes can only become unstable by passing through the marginal point $\omega = 0$.

The asymptotic boundary layer analysis of resistive modes shows that two conditions must be satisfied for the existence of overstable modes (Coppi et al., 1966, Glasser et al., 1975). These are that the equilibrium parameter $D_R = -2q^2 p/B_z^2 q^2 r_s$ must be negative, and that Δ' , the ratio of small to big odd- ξ singular solutions at the rational surface (r_s), must be larger than some positive value, Δ_c . For the model equilibria introduced in Sec. 2, both of these conditions can be satisfied if the parameter $\alpha < 1$, since then $p'(r) > 0$. Although such a pressure profile is unrealistic, this choice provides a useful model to simulate the effect of favorable average-curvature in the small, aspect-ratio limit of Tokamak geometry. Hollow pressure profiles in RFP geometry may not be as unrealistic. In either case, our intention is not to make quantitative statements but to understand the nature of the spectrum and eigenfunctions.

In Fig. 20 we illustrate the behaviour of the unstable eigenvalues of the RFP equilibrium at fixed η as α is decreased from 1, making the parameter D_R increasingly stabilizing. For $\alpha \gtrsim 0.97$ there are two purely growing modes; the root 'a' which is a pure tearing mode at $\alpha=1$ and a second resistive mode, 'b'. As α is decreased the tearing mode growth rate is reduced while that of the second mode is increased rapidly. They coalesce and become overstable for $\alpha \lesssim 0.97$. As α is varied further the growth rate decreases until finally the modes become damped when $\alpha \lesssim 0.96$. The eigenfunctions corresponding to purely growing and overstable modes are shown in Figs. 21 and 22. The tearing mode and second mode are both odd around the mode rational surface. As they coalesce their eigenfunctions mix. After the coalescence the real parts of the eigenfunctions recover the shape of the tearing mode, but the imaginary part is more complicated. In Fig. 23 we plot the value of $p'(r_s)$ corresponding to the coalescence point as a function of resistivity. As η becomes smaller, overstable modes can occur for smaller values of $|D_R|$.

To investigate the effect of resistivity further, in Fig. 24 we fix α and vary η from 10^{-3} to 10^{-7} . The behaviour is similar to that of Fig. 20, but we follow the locus further into the damped regime. The roots are seen to approach the marginal point as η approaches zero. Numerical resolution difficulties prevented a definitive demonstration of the asymptotic $\eta^{1/3}$ behaviour, but as can be seen from Fig. 24 a linear extrapolation of the value at $\eta = 10^{-7}$ is close to 30° , in good agreement. In Fig. 25 we show the extreme localization of the eigenfunctions around the rational surface as η is made very small.

It is useful to relate the modes of this section to the stable part of the spectrum described in the previous two sections. Since the results presented here, Figs. 20-25, were all obtained with the incompressible

equations, it is reasonable to associate the modes with the physics of the shear Alfvén branch of the MHD equations. This is consistent with studies of the relation between stable shear Alfvén waves and unstable Suydam modes in ideal MHD, where plasma compressibility does not play a major role. We have repeated these calculations with finite compressibility ($\gamma = 5/3$) and found only minor changes. The basic features of coalescence, overstability, and stabilization seem to be essentially unaffected.

Since the damped slow magnetosonic modes have spectral properties similar to the damped shear Alfvén waves, with finite compressibility one might anticipate additional growing modes and overstable modes which vanish in the incompressible limit. We searched for such modes but failed to find any.

§6. Discussion

In this paper we have presented a numerical study of the properties of the spectrum of the linearized, resistive, MHD equations in cylindrical geometry. While it is difficult to make any strong claims for completeness, such a study serves to give an overall synthetic perspective of the model by illustrating various general properties and identifying aspects for which separate analysis might be rewarding.

We have found that the presence of a small resistivity resolves the singularities associated with the shear Alfvén continuum of ideal MHD and produces a spectrum of discrete, heavily damped modes which lie on specific lines originating from accumulation points in the ideal continuum. As the resistivity is reduced, the eigenvalues move along these lines towards the accumulation points and the spectral density increases. In the vicinity of the accumulation points the lines make characteristic 30° angles with the real (oscillatory) axis, regardless of the geometry employed. The corresponding

eigenfunctions are, in general, oscillatory and are localized around the plasma surface which would be singular in the ideal MHD limit. The number of radial nodes increases as we consider eigenmodes located further away from the ideal continuum, and the modes become more localized as the resistivity is reduced. While plasma compressibility modifies the eigenfunctions, it does not affect the basic spectral properties.

These results confirm and extend the results of Boris (1968) for an incompressible slab model. They have stimulated new interest in obtaining an analytic understanding of the spectral properties of the resistive MHD model; for example, do the spectral splitting points B and D of Fig. 3 have any special relevance in addition to the accumulation points E and C, and, is the picture, suggested here, in the limit $\eta \rightarrow 0$ consistent with the usual interpretation of continuum modes as the long time asymptotic limit of time-dependent solutions? Since the damping rates are large these results might provide an explanation for the experimental observation of the large attenuation of Alfvén waves and provide some additional understanding of Alfvén wave heating experiments. They might also introduce some new mechanisms for nonlinear MHD processes.

Resistive slow magnetosonic waves which, like the shear Alfvén waves, have a continuous spectrum in ideal MHD, show similar spectral properties. The modes are discrete, damped, and have a 30° characteristic behaviour, and the eigenfunctions are localized and oscillatory. The fast magnetosonic waves are discrete and damped. Their eigenfunctions have a smooth oscillatory behaviour and are not localized.

Unstable modes in the model appear to be associated only with the shear Alfvén branch. Overstable modes occur only under circumstances predicted by the asymptotic boundary layer analysis (Glasser et al., 1975), which is

confirmed here for quite large values of the resistivity. In this cylindrical model compressibility does not seem to play a significant role in the stabilizing effect of average-favorable curvature.

Thus, as expected, we have seen that the properties of the resistive MHD normal modes are quite different from those of ideal modes, especially for the shear Alfvén and slow magnetosonic branches. While the frequencies of specific modes depend on the value of the resistivity, the spectra show certain global properties which are independent of its magnitude. What is important is the presence of a dissipative mechanism which resolves the singularities of the ideal MHD model. Thus, while other non-ideal dissipative mechanisms might be more important in various circumstances (Connor et al., 1982), it is reasonable to conjecture that, although the results may differ in quantitative details, these overall properties would remain a central feature.

Appendix

Here we summarize some of the algebra associated with the resistive normal modes study described in this paper. We first briefly describe the derivation of Eq. (12) for the compressible model and list the coefficient matrices for completeness. We then reduce these equations to the ideal MHD limit. Finally we reduce the resistive equations to a single fourth-order boundary layer equation in the small resistivity, incompressible limit and briefly describe its asymptotic behavior.

The derivation of Eqs. (12) is straightforward and follows Freidberg and Hewett (1981). We linearize Eqs. (1), (2), (3), and (5), using Eq. (4) with constant η to eliminate $\underline{E}$, introduce the plasma displacement, $\partial_t \underline{\xi} \equiv \underline{v}$, and assume that all perturbed quantities vary as $\exp i(\omega t + m\theta + kz)$. This gives a set of ten coupled equations for the ten perturbed variables, $\underline{\xi}$, $\underline{b}$, $\underline{j}$, and δp . Taking components of these equations in the directions of the unit vectors $\underline{e}_r$, $\underline{e}_\tau \equiv \nabla\tau/|\nabla\tau|$, and $\underline{e}_s \equiv \underline{e}_r \times \underline{e}_\tau$, with $\tau = m\theta + kz$, simplifies the elimination of the τ and s components of the perturbed displacement, magnetic field and current vectors, and provides expressions for these and for δp in terms of ξ_r , b_r , and j_r . After some algebra there results a set of three coupled second-order equations for the solution, $\underline{Y} \equiv \{r\xi_r, rb_r, rj_r\}$, similar to Eq. (12).

Introducing the residual perturbed magnetic field v_r , as in Eq. (10), we transform $\underline{Y}$ to $\underline{Z}$ of Eq. (13) using $\underline{Z} = \underline{T}^{-1} \cdot \underline{Y}$ with

$$\underline{T} \equiv \begin{pmatrix} 1 & 0 & 0 \\ F & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

Equation (12) follows straightforwardly by forming the Euler equations associated with the quadratic form obtained by forming the inner product of the normal mode equation for $\underline{y}$ with $\underline{y}^+$. The coefficients in Eq. (12) are found to be;

$$A \equiv \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix},$$

$$A_{11} = (F^2 - \rho\omega^2/H)/rk_0^2, \quad A_{12} = F/rk_0^2, \quad A_{21} = \eta A_{12}, \quad A_{22} = A_{21}/F, \quad A_{33} = A_{22}, \quad (A1)$$

$$B \equiv \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & 0 & 0 \\ B_{31} & 0 & 0 \end{pmatrix}$$

$$B_{11} = -(\rho\omega^2/H)(p'/\gamma p)/rk_0^2 + F(1 - 1/H)M/rk_0^2,$$

$$B_{12} = [(1-1/H)M - F']/rk_0^2,$$

$$B_{13} = -(1-1/H)G/rk_0^2, \quad B_{21} = \eta F'/rk_0^2,$$

$$B_{31} = -\eta\rho(i\omega)^3 G/\gamma p H rk_0^4, \quad (A2)$$

$$C \equiv \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix},$$

$$C_{11} = (\rho\omega^2 - F^2)/r + B_{11}' + F[(F'-M)/rk_0^2]' ,$$

$$C_{12} = -F/r - (M/Hrk_0^2)' ,$$

$$C_{13} = 2mkF/k_0^4 r^3 + (G/Hrk_0^2)' + L/rk_0^2 ,$$

$$C_{21} = \eta[(F'/rk_0^2)' - F/r] , C_{22} = -(\eta + i\omega/k_0^2)/r ,$$

$$C_{23} = 2\eta mk/k_0^4 r^3 ,$$

$$C_{31} = i\omega[-L(1-F^2/\rho\omega^2) + 2kB_\theta/r + Gp'/\gamma H\rho + FGM/\gamma H\rho k_0^2]/rk_0^2 ,$$

$$C_{32} = -i\omega[FL/\rho\omega^2 + 2mk/r^2 k_0^2 - GM/\gamma H\rho k_0^2]/rk_0^2 ,$$

and

$$C_{33} = -\eta/r - i\omega[1 - F^2/\rho\omega^2 + G^2/\gamma H\rho k_0^2]/rk_0^2 , \quad (A3)$$

with

$$k_0^2 \equiv k^2 + m^2/r^2 , F \equiv kB_z + mB_\theta/r , G \equiv mB_z/r - kB_\theta ,$$

$$L \equiv k(rB_\theta)' / r - mB_z' / r , M \equiv m(rB_\theta)' / r^2 + kB_z ,$$

and

$$H \equiv 1 - \rho\omega^2/\gamma p k_o^2 \quad . \quad (A4)$$

When the resistivity vanishes, the second of Eq. (12) yields the ideal MHD constraint $v_r = 0$, the third allows j_r to be eliminated in terms of ξ_r , and the first gives

$$\frac{d}{dr} (A_{11} \frac{d}{dr} (r\xi_r)) + Q \frac{d}{dr} (r\xi_r) + Rr\xi_r = 0 \quad , \quad (A5)$$

$$\text{where } Q \equiv B_{11} - B_{13}C_{31}/C_{33}$$

and

$$R \equiv C_{11} - C_{13}C_{31}/C_{33} - B_{13}(C_{31}/C_{33})' \quad .$$

The singularities of Eq. (A5) occur where the quantity,

$$D/\gamma C_2 \equiv A_{11} \exp \int dr(Q/A_{11}) \quad , \quad (A6)$$

vanishes. Here we have introduced the usual notation [Chance et al., 1977, Eq. (A1)]

$$D \equiv (\rho\omega^2 - F^2)[\rho\omega^2(B^2 + \gamma p) - \gamma p F^2]$$

and

$$C_2 = i\omega p(\rho\omega^2 - \gamma p k_o^2) r k_o^2 C_{33} \quad .$$

Additional simplification occurs in the incompressible limit, $\gamma \rightarrow \infty$; then $H \rightarrow 1$, $B_{11} \rightarrow 0$, $B_{13} \rightarrow 0$ and Eq. (A5) reduces to

$$\begin{aligned} \frac{d}{dr} \left(\frac{\rho \omega^2 - F^2}{r k_o^2} \right) \frac{d}{dr} (r \xi_r) - [(\rho \omega^2 - F^2) - 2mr \left(\frac{B_\theta F}{k_o^2 r^3} \right)' - 2B_\theta (B_\theta/r)'] \\ - 4k^2 B_\theta^2 / k_o^2 r^2 - F^2 / (\rho \omega^2 - F^2)] \xi_r = 0 . \end{aligned}$$

In the incompressible, but resistive, limit some progress can be made by expanding about the solutions of the ideal MHD case in powers of η . Since η appears linearly in the coefficients $\underline{A}$, $\underline{B}$, and $\underline{C}$ we write each in the form,

$$C_{22} \equiv \bar{C}_{22} + \eta \tilde{C}_{22}, \quad z_i = z_i^{(0)} + \eta z_i^{(1)} + \dots, \quad i = 1, 2, 3 .$$

Then the second and third components of Eq. (12) can be written as

$$\bar{C}_{22} z_2 = -\eta [(\tilde{A}_{2\alpha} z'_\alpha)' + \tilde{B}_{21} z'_1 + \bar{C}_{2i} z_i]$$

and

$$\bar{C}_{3i} z_i = -\eta [(\tilde{A}_{33} z'_3)' + \tilde{C}_{33} z_3] , \quad (A8)$$

where $\alpha = 1, 2$, $i = 1, 2, 3$ and summation convention is implied. Equations (A8) allow $z_2^{(1)}$ and $z_3^{(1)}$ to be expressed in terms of the ideal MHD solution $z_1^{(0)}$ to first order in η . Substituting these expressions into the first component of Eq. (12) and collecting all resistive corrections to the right hand side yields

$$\begin{aligned} \frac{d}{dr}(\bar{A}_{11} \frac{d}{dr} z_1^{(0)}) + (\bar{C}_{11} - \bar{C}_{13} \bar{C}_{31}/\bar{C}_{33}) z_1^{(0)} = - \eta \left[\frac{d}{dr} (\bar{A}_{12} \frac{d}{dr} z_2^{(1)}) \right. \\ \left. + \bar{B}_{12} \frac{d}{dr} z_2^{(1)} + \bar{C}_{12} z_2^{(1)} + \bar{C}_{13} z_3^{(1)} \right] + O(\eta^2) . \end{aligned}$$

Thus, under the assumption that only the highest order derivatives contribute to the right-hand side, we find the "boundary layer" equation

$$\frac{d}{dr} A_{11} \frac{d}{dr} z_1 + (C_{11} - C_{31} C_{13}/C_{33}) z_1 = \eta \frac{i F^2}{\omega k_o^2} \frac{d^4 z_1}{dr^4} . \quad (A9)$$

In slab geometry Boris (1968) analyzed asymptotic solutions of Eq. (A9) near the boundaries of the slab. He found discrete localized solutions with eigenvalues

$$\omega \approx \omega_A(0) \{ 1 - \epsilon/2 (a_1/\epsilon)^{2/3} e^{i\pi/6} [3\pi/2(n - 3/4)]^{2/3} + \dots \} \quad (A10)$$

where $\epsilon = \eta^{1/3}$, $a_1 = 2F(0)$ and $n = 1, 2, \dots$.

ACKNOWLEDGMENTS

The authors are grateful for illuminating discussion with Dr. J. M. Greene who initiated our interest in this problem and to J. Delucia for his help in obtaining Eq. (12). We thank Dr. J. L. Johnson who provided constant encouragement and constructive criticism during the course of this work, and also Drs. J. P. Friedberg, D. W. Hewitt, and R. Nebel who were helpful in sharing their experiences with the matrix shooting method. We gratefully acknowledge many useful discussions with PPL Theory Division Staff, especially Drs. L. Chen and M. S. Chance, who have had a long interest in this problem.

This work was supported by U. S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

Figures

Fig. 1. Typical equilibrium profiles for reversed field pinch ($\alpha = 1.05$, $r_w = 3.0$) and Tokamak ($B_0 = 7$, $\alpha = 1.0$, $r_w = 2$) configurations. The scales have been adjusted to make $B_z(0) = 1$.

Fig. 2. Typical $F(r)$ profiles. $\omega_A^2 \equiv F^2/\rho$. a) $\alpha = 1.05$, $k = 0.6$, $m = 1$, $r_w = 3$; b) $\alpha = 1.0$, $k = 0.2$, $m = 2$, $r_w = 2$, $B_0 = 7$.

Fig. 3. Spectrum of resistive shear Alfvén waves for Fig. 2a with $\eta = 10^{-2}$ (Fig. 3a) and $\eta = 10^{-3}$ (Fig. 3b).

Fig. 4. Effect of resistivity for a typical mode on B-C in Fig. 3. a) $\eta = 4 \times 10^{-3}$, b) 2×10^{-3} , c) 1×10^{-3} , and (d) 5×10^{-4} .

Fig. 5. Eigenmode structures for the mode 'a' of Fig. 3b.

Fig. 6. Eigenmode structures for the mode 'l' of Fig. 3b.

Fig. 7. Variation in $\text{Re}(\xi_r)$ along the segment C-B of Fig. 3b.

Fig. 8. Variation in $\text{Re}(\xi_r)$ along the segment D-B of Fig. 3b.

Fig. 9. Variation in $R_e(\xi_r)$ along the segment B-A of Fig. 3b.

Fig. 10. Locus of modes 'j' and 'k' of Fig. 3b as η is reduced from 1.02×10^{-3} to 0.98×10^{-3} .

Fig. 11. Locus of mode 'd' of Fig. 3b as η is reduced.

Fig. 12. Locus of modes 'l' (Fig. 12a), 'm' (Fig. 12b), and 'n' (Fig. 12c) as η is reduced.

Fig. 13. Spectrum of resistive shear Alfvén waves for the Tokamak equilibrium of Fig. 2b with $\eta = 10^{-3}$.

Fig. 14. Variation of $\text{Re}(\xi_r)$ for several eigenmodes in Fig. 13. Note that $\xi_r(0) = 0$ for the $m = 2$ case considered here.

Fig. 15. Eigenfunction just below the intersection point, R, of Fig. 3b for finite compressibility, $\gamma = 5/3$.

Fig. 16. Spectrum of damped resistive modes for the compressible model ($\gamma = 5/3$) of Fig. 2a with $\alpha = 1.15$.

Fig. 17. Real parts of the resistive slow magnetosonic mode 'a' of Fig. 16.

Fig. 18. Spectrum of resistive fast magnetosonic waves for the configuration of Fig. 2a with $\alpha = 1.05$.

Fig. 19. Real parts of the eigenmode of the resistive fast magnetosonic wave 'e' of Fig. 18.

Fig. 20. Loci of overstable modes for the equilibrium of Fig. 2a as D_R is made increasingly stabilizing by increasing the pressure gradient (reducing α). $\eta = 10^{-4}$.

Fig. 21. Eigenfunctions of the tearing mode and second unstable mode of Fig. 20 with $\alpha = 0.98$.

Fig. 22. Real (solid) and imaginary (dotted) parts of the eigenfunction of an overstable mode of Fig. 20 for $\alpha = 0.965$.

Fig. 23. The value of p' corresponding to coalescence as a function of resistivity for the equilibrium of Fig. 2a.

Fig. 24. The loci of overstable modes as resistivity is varied from 10^{-3} to 10^{-7} for the equilibrium of Fig. 2a. $\alpha = 0.96$.

Fig. 25. Real and imaginary parts of a damped eigenmode of Fig. 24 for $\eta = 10^{-5}$.

References

Alfven, H. and Falthammer, C. G. 1963 "Cosmical Electrodynamics," Oxford Univ. Press.

Appert, K., Gruber, R., and Vaclavik, J. 1974 Phys. Fluids 17, 1471.

Bernstein, I. B., Freiman, E. A., Kruskal, M. D., and Kulsrud, R. M. 1958 Proc. Roy. Soc. Ser. A244, 17.

Boris, J. P., Ph.D dissertation 1968 Princeton University.

Chance, M. S., Greene, J. M., Grimm, R. C., and Johnson, J. L., 1977 Nucl. Fusion 17, 65.

Connor, J. W., Tang, W. M., and Taylor, J. B. 1982 Princeton University, Plasma Physics Laboratory Report PPPL-1869, Phys. Fluids (1983).

Coppi, B., Greene, J. M., and Johnson, J. L., 1966, Nucl. Fusion 6, 101.

Dibiase, I. A., and Killeen, J. 1977 J. of Compt. Phys. 24, 158.

Freidberg, J. P., and Hewett, D. W. 1981 J. Plasma Phys. 26, 177.

Furth, H. P., Rutherford, P. H., and Selberg, H. 1973 Phys. Fluids 16, 1054.

Furth, H. P., Killeen, J., and Rosenbluth, M. N. 1963 Phys. Fluids 6, 459.

Glasser, A. H., Greene, J. M., and Johnson, J. L. 1975 Phys. Fluids 18, 875.

Glasser, A. H., Chance, M. S., and Dewar, R. L. 1979 19th European Conference on Controlled Fusion and Plasma Physics, Oxford, Vol. 1, p8.

Goedbloed, J. P., and Sakanaka, P. H. 1974 Phys. Fluids 17, 908.

Grimm, R. C., Dewar, R. L., and Manickam, J. 1983 J. Comput. Phys. 49, 94.

Grimm, R. C., Dewar, R. L., Manickam, J., Jardin, S. C., Glasser, A. H., and Chance, M. S., 1982 in "Plasma Physics and Controlled Nuclear Fusion Research, IAEA-CN-41/P-2-1.

Johnson, J. L., Greene, J. M., and Coppi, B. 1963 Phys. Fluids 6, 1169.

Manickam, J., Grimm, R. C., and Dewar, R. L., 1982 Proceedings of the 10th
IMACS World Congress on System Simulation and Scientific Computation,
Montreal, Vol. 4, 175.

Ryu, C.-M., Ph.D dissertation, 1982, Princeton University.

Suydam, B. R., 1958 Proceedings of 2nd United Nations International Conference
(United Nations; Geneva) Vol. 31, 157.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

#83T0180

Fig. 9₄₀¹

#83T0009

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

83T0179

Fig. 16

#83T0035

Fig. 17

83T0178

Fig. 18

83T0182

Fig. 19

83T 0024

Fig. 20

Fig. 21

83T0028

Fig. 22

83 T 0023

Fig. 23

83T 0027

Fig. 24

Fig. 25

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

Plasma Res Lab, Austra Nat'l Univ, AUSTRALIA
Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
Prof. I.R. Jones, Flinders Univ., AUSTRALIA
Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
Prof. F. Cap, Inst Theo Phys, AUSTRIA
Prof. Frank Verheest, Inst theoretische, BELGIUM
Dr. D. Palumbo, Dg XII Fusion Prog, BELGIUM
Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
Dr. C.R. James, Univ of Alberta, CANADA
Prof. J. Telchmann, Univ of Montreal, CANADA
Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
Prof. S.R. Greenivasan, University of Calgary, CANADA
Prof. Tudor W. Johnston, INRS-Energie, CANADA
Dr. Hannes Barnard, Univ British Columbia, CANADA
Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
Zhengwu Li, SW Inst Physics, CHINA
Library, Tsing Hua University, CHINA
Librarian, Institute of Physics, CHINA
Inst Plasma Phys, SW Inst Physics, CHINA
Dr. Peter Lukac, Komenskeho Univ, CZECHOSLOVAKIA
The Librarian, Culham Laboratory, ENGLAND
Prof. Schatzman, Observatoire de Nice, FRANCE
J. Radet, CEN-BP6, FRANCE
AM Dupas Library, AM Dupas Library, FRANCE
Dr. Tom Mui, Academy Bibliographic, HONG KONG
Preprint Library, Cent Res Inst Phys, HUNGARY
Dr. A.K. Sundaram, Physical Research Lab, INDIA
Dr. S.K. Trehan, Panjab University, INDIA
Dr. Indra, Mohan Lal Das, Banaras Hindu Univ, INDIA
Dr. L.K. Chavda, South Gujarat Univ, INDIA
Dr. R.K. Chhajlani, Var Ruchi Marg, INDIA
P. Kaw, Physical Research Lab, INDIA
Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
Prof. S. Cuperman, Tel Aviv University, ISRAEL
Prof. G. Rostagni, Univ DI Padova, ITALY
Librarian, Int'l Ctr Theo Phys, ITALY
Miss Ciella De Palo, Assoc EURATOM-CNEN, ITALY
Biblioteca, del CNR EURATOM, ITALY
Dr. H. Yamato, Toshiba Res & Dev, JAPAN
Prof. M. Yoshikawa, JAERI, Tokai Res Est, JAPAN
Prof. T. Uchida, University of Tokyo, JAPAN
Research Info Center, Nagoya University, JAPAN
Prof. Kyuji Nishikawa, Univ of Hiroshima, JAPAN
Prof. Sigeru Mori, JAERI, JAPAN
Library, Kyoto University, JAPAN
Prof. Ichiro Kawakami, Nihon Univ, JAPAN
Prof. Satoshi Itoh, Kyushu University, JAPAN
Tech Info Division, Korea Atomic Energy, KOREA
Dr. R. England, Ciudad Universitaria, MEXICO
Bibliothek, Fom-Inst Voor Plasma, NETHERLANDS
Prof. B.S. Lilley, University of Waikato, NEW ZEALAND
Dr. Suresh C. Sharma, Univ of Calabar, NIGERIA
Prof. J.A.C. Cabral, Inst Superior Tech, PORTUGAL
Dr. Octavian Petrus, ALI CUZA University, ROMANIA
Dr. R. Jones, Nat'l Univ Singapore, SINGAPORE
Prof. M.A. Hellberg, University of Natal, SO AFRICA
Dr. Johan de Villiers, Atomic Energy Bd, SO AFRICA
Dr. J.A. Tagle, JEN, SPAIN
Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
Dr. Lennart Stenflo, University of UMEA, SWEDEN
Library, Royal Inst Tech, SWEDEN
Dr. Erik T. Karlson, Uppsala Universitet, SWEDEN
Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
Dr. W.L. Weise, Nat'l Bur Stand, USA
Dr. W.M. Stacey, Georg Inst Tech, USA
Dr. S.T. Wu, Univ Alabama, USA
Prof. Norman L. Oleson, Univ S Florida, USA
Dr. Benjamin Ma, Iowa State Univ, USA
Prof. Magne Kristiansen, Texas Tech Univ, USA
Dr. Raymond Askew, Auburn Univ, USA
Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
Dr. D.D. Ryutov, Siberian Acad Sci, USSR
Dr. M.S. Rabinovich, Lebedev Physical Inst, USSR
Dr. G.A. Eliseev, Kurchatov Institute, USSR
Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
Prof. T.J. Boyd, Univ College N Wales, WALES
Dr. K. Schindler, Ruhr Universitat, W. GERMANY
Nuclear Res Estab, Julich Ltd, W. GERMANY
Librarian, Max-Planck Institut, W. GERMANY
Dr. H.J. Kaeppler, University Stuttgart, W. GERMANY
Bibliothek, Inst Plasmaforschung, W. GERMANY