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PLASMA HEATING BY EXTERNALLY
LAUNCHED ION BERNSTEIN WAVES

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This work was supported by the U. S. Department of Energy
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Plasma Heating by Externally Launched

Ion Bernstein Waves

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Externally launched ion Bernstein waves are shown to provide highly effective means for heating reactor plasmas. In a low-density plasma ($n_e \sim 10^{14} \text{ cm}^{-3}$), an ion Bernstein wave with a parallel index of refraction $n_{\parallel} \sim 1$ is transformed into a high-density ion Bernstein wave with electron Landau damping several orders of magnitude smaller than that of the original ion Bernstein wave. Because of this reduced damping, an externally launched ion Bernstein wave with $n_{\parallel} \sim 1$ can penetrate to and deposit most of its wave power in the core of even a reactor-grade plasma ($n_e \sim 10^{14} \text{ cm}^{-3}$, $T_{e0} = T_{i0} \sim 7 \text{ keV}$, $\rho_0 \sim 60 \text{ kg/m}^3$).

NOTES
1. This paper is part of a series of papers on the theory of ion Bernstein waves in a plasma. The first paper in the series is "Ion Bernstein Waves in a Plasma," J. Geophys. Res., 76, 10, 24, 1971, pp. 5000-5010.
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Radiofrequency heating near the ion cyclotron frequency is one of the most promising approaches for heating fusion plasmas to ignition temperatures. In this frequency range, RF power is readily available and relatively inexpensive. Conventional ion cyclotron plasma heating schemes employ various electromagnetic waves as principal energy carriers¹⁻³, while externally generated electrostatic waves in the lower frequency range are usually considered infeasible, as heavy damping prevents accessibility to the high-density center of reactor plasmas. This letter shows, however, that an externally launched ion Bernstein wave has properties which lead to exceptionally favorable penetration to the power deposition in the core of a reactor-grade plasma.

The mode of particular interest here is a high-density counterpart of the ion Bernstein wave which we call here, a high-density ion Bernstein wave. This mode becomes dominant when $\frac{\omega^2}{\omega_{pe}^2}$ becomes large compared to $\frac{\omega^2}{\omega_{pi}^2} = \frac{\omega_{pe}^2}{\omega_{pi}^2} \frac{n_e}{n_i}$. This condition would usually be satisfied in a fusion plasma with $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = T_i = 1 \text{ keV}$. With n_e so high, the wave dispersion relation can be written⁴ to the lowest order in $\frac{\omega_{pe}^2}{\omega_{pi}^2} \frac{n_e}{n_i}$ as:

$$\frac{n_{\perp}^2}{n_{\parallel}^2} = \frac{K_{xy}^2 - (K_{xx} - n_{\parallel}^2)^2}{n_{\parallel}^2} = \frac{K_{xy}^2}{n_{\parallel}^2 - K_{xx}} \quad (1)$$

where

$$K_{xx} = 1 + \frac{\omega_{pe}^2}{\omega^2} + \sum_i \frac{\omega_{pi}^2 \exp(-\frac{1}{2})}{\omega^2} \sum_{n=1}^{\infty} I_n(\frac{\omega}{\omega_{ci}}) \frac{2n^2}{(n^2 - \frac{\omega^2}{\omega_{ci}^2})}$$

and

$$i K_{xy} = + \frac{\omega_{pe}^2}{\omega} - \frac{\omega_{pi}^2}{\omega} \exp(-\gamma_i) \left(\frac{\gamma_i}{\omega} \right) \sum_{n=1}^{\infty} \frac{2n^2}{(n^2 - \frac{1}{2})^2} [I_n(\gamma_i) - I_{n+1}(\gamma_i)].$$

Here, $\gamma_i = k_{\perp}^2 T_i / (m_i v_i^2)$ and I_n is the modified Bessel's function. The propagation of such a wave is possible because of the finite ion Larmor radius effect which maintains $(n_{\perp}^2 - K_{xx})$ a positive quantity. For this wave, typically $K_{xy} \sim K_{xx} \sim n_{\perp}^2$, and $n_{\perp}$ is relatively large compared to that of the magnetosonic wave which usually satisfies $K_{xy} \sim K_{xx} \sim n_{\parallel}^2$. These orders break down, of course, near the cut-off frequencies ($\omega = 2\gamma_i$ for example) where the dispersion curves coalesce³. It should be emphasized that the propagation of the high density ion Bernstein wave requires K_{xx} to be a small quantity ($\sim n_{\perp}^2$) which for $\omega_{pi}^2 / \omega^2 \gg 1$ requires the finite ion Larmor radius terms to nearly cancel each other (i.e., $\omega \approx \omega_{ci}$ where $K_{xx} = \omega_{pi}^2 / \omega^2$). This high density mode can be classified as an electrostatic wave since the wave vector, $\vec{k} = k_{\perp} \hat{x}$, is nearly parallel to the wave electric field, yet it is electromagnetic in a sense $B^2/8\pi$ is usually larger than $E^2/4\pi$.

As the density is reduced, $n_{\perp}^2$ becomes large compared to ω_{pe}^2/ω^2 , and one can show that the high density mode is transformed continuously into the well-known ion Bernstein wave⁴,

$$n_{\perp}^2 K_{xx} + n_{\parallel}^2 K_{zz} = 0 \quad (2)$$

where

$$K_{zz} = 1 + 2 \frac{\omega_{pe}^2}{\omega^2} y^2 (1 + yZ(y)) - \frac{\omega_{pe}^2}{\omega^2}$$

Here $y = \omega/k_{\perp} (m_e/2 \cdot T_e)^{1/2}$ and Z is the plasma dispersion function⁶. We should note that the finite ion Larmor radius effect is responsible here for also the propagation of the ion Bernstein wave above ω_i for $\omega/k_{\perp} > V_{Te} (2 \cdot T_e/m_e)^{1/2}$. Propagation is apparent from Eq. (2) since $K_{zz} = \omega_{pe}^2/\omega^2$, and K_{xx} must be a positive quantity to satisfy the dispersion relation. This possibility was recognized and investigated by Swanson⁷ in conjunction with the experimental observations of Kristiansen and Dougal⁸.

Although the dispersion relations for the two modes (Eqs. (1) and (2)) are quite different, the dispersion relations for $n_{\perp}$ yield similar numerical values. As an illustration, we show in Fig. 1(a), numerically calculated electrostatic wave dispersion curves for several densities (as labeled). In the calculation, a non-drifting Maxwellian plasma is assumed, and the full hot-plasma dielectric tensor is utilized. The following plasma parameters are used: $B_0 = 60$ kG, $T_e = T_i = 7$ keV, $n_{\perp} = 3$, $\omega/\omega_D = 1-2$, and $N(T)=N(D)$. As shown in the figure $k_{\perp}$ remains real for very low to high density values. Qualitatively speaking, in the low density regime ($n_0 < 10^{11} \text{ cm}^{-3}$), the wave is the electron plasma wave⁹; for intermediate densities ($n_0 = 10^{11}-10^{13} \text{ cm}^{-3}$) it becomes the ion Bernstein wave, and finally in the high density regime ($n_0 \geq 10^{14} \text{ cm}^{-3}$),

it fits the high-density mode description. One can indeed see from the figure, the dispersion curve for the high density mode almost overlaps that of the ion Bernstein wave. In spite of this similarity, the high density mode has important structural differences in the wave damping, electric fields, and propagation characteristics.

We now consider electron Landau damping of the high density mode. One can estimate the wave damping by retaining the terms with $O(\epsilon)$ in Eq. (1), and obtain the following simple approximate expression for $\text{Im } k_{\perp}$:

$$\text{Im } k_{\perp} \approx |k_{\perp}| \frac{\text{Im } K_{zz}}{|K_{zz}|} \approx A, \quad (3)$$

where $A = n_e^2 / (k_{\perp} \partial K_{xx} / \partial k_{\perp})$. This should be compared with the damping of the ion Bernstein wave which can be computed from Eq. (2) to be,

$$\text{Im } k_{\perp} \approx |k_{\perp}| \frac{\text{Im } K_{zz}}{|K_{zz}|} D, \quad (4)$$

where $D = (2 + k_{\perp}^2 \ln K_{xx} / \partial k_{\perp})^{-1}$. In a high density plasma, ϵ and A are small numbers both of which are inversely proportional to the plasma density. Typically, in a plasma with $n_0 = 10^{14} \text{ cm}^{-3}$, $\epsilon A \approx 10^{-3} - 10^{-4}$. On the other hand, the quantity D in Eq. (4) is usually $O(1)$. Therefore the electron Landau damping of the high density mode can be several orders of magnitude smaller than that of the ion Bernstein wave. The wave electric field for a given wave energy density generally decreases as the plasma density increases. In particular, E_z/E_x for the high

density mode is smaller by r from that of the ion Bernstein wave. This large reduction in E_z with respect to the wave energy density is responsible for the weak electron Landau damping of the high density ion Bernstein wave. As an illustration, we show in Fig. 1(b) the imaginary part of the wave dispersion relation shown in Fig. 1(a). In the calculation, electron Landau damping and the ion cyclotron damping are included. We show by dotted curves where the high density mode becomes relevant. One can see a generally decreasing damping rate as a plasma density increases. For a fixed density, the damping increases as the wave approaches the resonance frequency. This behavior can be easily understood from Eq. (3) where $\text{Im}k_{\perp}$ is roughly proportional to $k_{\perp}^2/n_0^2$. The sudden increase in $\text{Im}k_{\perp}$ near the cutoffs ($\omega/\Omega_D = 2.0$ and 1.33) is due to the ion cyclotron damping.

Using the above theory, we indicate how one might heat a reactor plasma with externally launched ion Bernstein waves. We consider a tokamak plasma with equal concentrations of deuterium and tritium ions, a major radius of 6 m, a minor radius of 2 m, and at the plasma center, $T_{e0} = T_{i0} = 7$ keV, $n_0 = 3 \times 10^{14} \text{ cm}^{-3}$, and $B_0 = 60$ kG. A Gaussian temperature profile with 70 eV edge temperature and a $\cos(-r/2a)$ density profile with $3 \times 10^{12} \text{ cm}^{-3}$ edge density are assumed. The wave with frequency of 65 MHz is launched at the outer edge of the plasma ($r = 2$ m) where $\omega/\Omega_D \approx 1.9$ and propagates toward the resonance, $\omega/\Omega_T = 2.0$, which is located near the center, $r = -0.38$ m (Fig. 2(a)). We have numerically calculated the wave

trajectory⁴ and damping using the full dielectric tensor in a slab geometry. In Fig. 2(a), we show the trajectory of waves with $n_{\parallel}$ ranging from 1 to 9 corresponding to $\omega/k_{\parallel}V_{Te} \approx 6-0.7$. As can be seen from the figure, the wave reaches the plasma center relatively rapidly with the axial travel distance $z \approx 1.5 n_{\parallel} r$. For these parameters, waves mainly propagate as the high density modes, and only near the plasma edge (where the density is low) and near cyclotron resonance (where $n_{\perp}$ is large), waves propagate as the ion Bernstein waves. Consequently, the wave trajectories show rapid penetration except near the edge and near the resonance where the propagation is much shallower with respect to the magnetic field line. This behavior can be seen qualitatively in Fig. 2(a) and $n_{\parallel} = 1$ and 3.

In Fig. 2(b), we show the fractional wave power transmitted as a function of radial position for various $n_{\parallel}$. For $n_{\parallel} = 1$, the wave undergoes ion cyclotron damping near the resonance layer. One can see for a wave with $n_{\parallel}$ up to 6, most of the power deposition occurs near the plasma center. However, the damping occurs relatively near the surface for $n_{\parallel} = 9$. These results indicate that the present heating scheme would have a relatively large window in $n_{\parallel}$ available for heating reactor plasmas.

Finally, we shall briefly discuss the method of wave excitation. As mentioned earlier, in a very low density regime near

the exciter, the ion Bernstein wave becomes an electron plasma wave.⁹ Therefore, the wave excitation mechanism for the ion Bernstein wave is very similar to that of the lower hybrid wave.¹⁰⁻¹⁴ The similarity of the excitation for the two modes has been partially confirmed by an experiment which verified the excitation of the cold electrostatic ion cyclotron wave¹⁵ by an external electrostatic slow wave structure.^{10,11}

In a reactor size device, use of a waveguide system becomes an attractive possibility. Since the available window in $n_{\parallel}$ is relatively large ($n_{\parallel} \leq 6$), the waveguide phasing requirement can be very modest. We envision here a simple waveguide system with the aperture openings of ~50 cm width and ~250 cm height operating in TE_{10} mode with a fixed frequency. The rf coupling efficiency of such a system is expected to be similar to that of lower hybrid wave where the high efficiency of ~90% has been reported.¹²⁻¹⁴ However, a detailed calculation similar to the one carried out in Ref. 14 is necessary to obtain the condition for the optimum coupling.

In conclusion, our theoretical analysis shows that externally launched ion Bernstein waves may be effective in heating plasmas to ignition temperatures. In a high density plasma, the ion Bernstein wave mode-transforms into a weakly damped high density ion Bernstein wave whose electron Landau damping can be several orders of magnitude smaller than that of the ion Bernstein wave. For a reactor plasma, the wave trajectory calculation shows favorable penetration and power deposition characteristics for the waves with $n_{\parallel} \leq 6$. In a reactor size

device, it may be even possible to use a waveguide system to couple rf into the plasma.

The author would like to thank Drs. W. M. Hooke and T. H. Stix for discussions and helpful suggestions. This work is supported by U.S. Department of Energy Contract EY-76-C-02-3073.

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FIGURE CAPTION II.

Fig. 1(a). Perpendicular wave dispersion relation for ion Bernstein waves with $n_{\perp} = 2$ for various plasma densities (as labeled). $B_0 = 60$ kG, $T_0 = T_i = 7$ keV, $N(\text{Deuterium}) : N(\text{Tritium}) = 5:5$. (b). Imaginary part of the wave dispersion relation shown in Fig. 1(a).

Fig. 2(a). Wave trajectories of ion Bernstein waves for various $n_{\perp}$ (as labeled). At the center ($r = 0$), $B_0 = 60$ kG, $T_0 = T_i = 7$ keV and $n_0 = 3 \cdot 10^{14}$. At the edge ($r = 25$ cm) $B_0 = 45$ kG, $T_0 = T_i = 70$ eV, and $n_0 = 3 \cdot 10^{12}$. $\omega = 65$ MHz and $N(\text{Deuterium}) : N(\text{Tritium}) = 5:5$. (b). Fractional wave power transmitted as a function of radial position with the same plasma parameters as in Fig. 2(a).

Fig. 1. 796015

Fig. 2. 796046

ACKNOWLEDGMENTS

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