Flow Shear Induced Compton Scattering of Electron Drift Instability

T. S. Hahm

Princeton Plasma Physics Laboratory, Princeton University
Princeton, N.J.

Abstract

Plasma flow shear effects on nonlinear saturation of electron drift waves are analyzed in the weak turbulence regime. Flow shear can enhance ion Compton scattering of long wavelength electron drift waves not only by modifying the beat wave-ion resonance condition, but also via the radial dependence of linear susceptibility. A nonlinear dispersion relation is obtained as a solution of the radially nonlocal nonlinear eigenmode equation. At nonlinear saturation, the spectral intensity of the fluctuations scales with flow shear as $(\partial V_\phi / \partial r)^{-2}$ in addition to the linear dependence on the linear growth rate.

RECEIVED

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

I. Introduction

The effect of flow shear on tokamak fluctuation and transport has recently become a topic of considerable theoretical interest. Several experiments¹⁻⁴ suggest that sheared flow at the edge may play a crucial role in quenching edge fluctuation^{5,6} and in the L to H transition observed in H-mode discharges.⁶⁻⁹ Furthermore, sheared toroidal rotation observed in the tokamak core region^{10,11} during neutral beam heating has motivated a number of investigations on possible anomalous toroidal momentum loss mechanisms,^{12,13} such as the ion temperature gradient driven mode and the trapped electron instability. Most theoretical studies of these electrostatic microinstabilities in the presence of sheared plasma flow have focused on the linear stability properties.¹⁴⁻¹⁷

Results from a recent study on a simple model of electron drift wave turbulence¹⁸ have indicated a significant modification of nonlinear coupling due to plasma flow shear. In this study, attention was focused on the strong turbulence regime. It was pointed out that plasma flow shear introduces a new feature in the renormalized propagator, affecting nonlinear wave-particle interaction.^{19,18} However, the weak turbulence regime has not been explored in detail.

In this paper, we investigate toroidal flow shear effects on the nonlinear saturation of electron drift waves in the weak turbulence regime. The strongly dissipative trapped electron mode (DTEM) with $\omega_{be} > v_{eff} > \omega$, $\omega_{*e} > \omega_{de}$ is considered for illustration.²⁰ Here, ω is the mode frequency, $\omega_{*e} = C_s \rho_s k_\theta / L_n$ is the electron diamagnetic drift frequency, $\omega_{te} = v_{Te} / qR$ and $\omega_{be} = \epsilon^{1/2} v_{Te} / qR$ are the electron transit and bounce frequencies, and ω_{de} is the orbit-averaged trapped electron precession drift frequency.

Since this mode is dispersive in the linearly most unstable k_y -range, a number of weak turbulence approaches have been attempted based upon the nonlinear wave-particle interactions.²¹⁻²⁴ The (dispersion-induced) ion Compton scattering,²⁵⁻²⁷ which transfers fluctuation energy to longer poloidal wavelengths, has been found to be the dominant nonlinear mechanism. However, at the longer poloidal wavelength side of the spectrum where fluctuation energy tends to accumulate, the dispersion-induced ion Compton scattering becomes weaker. Plasma flow shear, however, can enhance the ion Compton scattering of moderately long wavelength electron drift waves not only by modifying the beat wave-ion resonance condition,^{19,18} but also by introducing radial dependence into the linear susceptibility.²⁸ The focus of this paper is on this flow shear induced ion Compton scattering of moderately long wavelength drift waves.

The remainder of this paper is organized as follows. In Sec. II, a nonlinear eigenmode equation, which contains flow shear induced ion Compton scattering, is derived from the weak turbulence analysis of the nonlinear ion gyrokinetic equation. Here, the role of flow shear induced ion Compton scattering in introducing radial dependence in nonlinear coupling is discussed. The nonlinear dispersion relation is obtained as a solution of the nonlinear eigenmode equation in Sec. III. Finally, the implications of the principal results from the present studies are discussed in Sec. IV.

II. Flow Shear Induced Compton Scattering

In this section, the procedure for deriving the nonlinear evolution equation of fluctuations associated with drift instabilities in the presence of flow shear is developed in the weak turbulence regime. Rather than dealing with the wave kinetic equation which describes the nonlinear evolution of the radial-mode-width-averaged fluctuation spectral intensity, we derive the nonlinear eigenmode equation. This nonlocal approach allows us to examine the radial dependence of the Compton scattering cross section and the radial structure of the fluctuations. The emphasis is on a new nonlinear mechanism, the flow shear induced ion Compton scattering. Since the focus of this paper is on this new nonlinear mechanism, we consider the strongly dissipative trapped electron mode where nonlinearities in the electron dynamics can be ignored. As shown in the previous studies,²⁹⁻³¹ this simplification cannot be justified for the collisionless trapped electron mode

The theoretical model considered in this paper consists of the Boltzmann response for the untrapped electrons and the strongly dissipative trapped electron response which is characterized by the following formula.²⁰

$$\delta n_e/n_0 = (1 - i\delta) |e|\Phi/T_e, \quad (1)$$

where $\delta = \epsilon^{1/2} \omega_{*e} \{ 1 - (\omega_k/\omega_{*e}) + 3\eta_e/2 \} / v_{eff}$.

Ions are treated by the collisionless gyrokinetic equation. The relevant frequencies in the linear analysis are ordered as $\omega_{te} > \omega_{be} > v_{eff} > \omega$, $\omega_{*e} > \omega_{de}$, and $\omega, \omega_{*e} > \omega_{di}, \omega_{ti}$. Here, ω_{ti}

$= v_{Ti}/qR$ is the ion transit frequency, the ion magnetic drift frequency is ω_{di} , and the orbit-averaged trapped electron precession drift frequency is ω_{de} .

The collisionless nonlinear gyrokinetic equation^{28,32} for ions in sheared slab geometry is

$$(\partial_t + (\mathbf{V}_0 + c_{\parallel} \mathbf{b}) \cdot \nabla - B^{-1} \nabla \Phi \times \mathbf{b} \cdot \nabla) F - (e/M)(\mathbf{b} - B^{-1} \mathbf{b} \times \nabla V_{0\parallel}) \cdot \nabla \Phi (\partial F / \partial c_{\parallel}) = 0, \quad (2)$$

where $c_{\parallel} = v_{\parallel} - V_{0\parallel}$. The parallel shear flow drive term^{33,12} is contained in the last term of Eq. (2). After defining $g = f_{i1} + F_m e \phi / T_i$ and adopting the ordering of Frieman and Chen,³² we have

$$\begin{aligned} & -i(\omega - \mathbf{k} \cdot \mathbf{V}_0 - k_{\parallel} c_{\parallel} / L_s) g + i\tau[\omega - \mathbf{k} \cdot \mathbf{V}_0 - \omega_{*i} \{1 + (u^2 - 3)\eta_i / 2\} + k_{\parallel} V_{0\parallel} c_{\parallel} / \Omega_i L_v] \Phi J_0 F_m \\ & = \nabla \Phi J_0 \times \mathbf{b} \cdot \nabla g, \end{aligned} \quad (3)$$

where g is the nonadiabatic part of the perturbed distribution function; $F_m = n_0 (M_i / 2\pi T_i)^{3/2} \exp(-u^2/2)$, $u^2 = M_i v^2 / T_i$, $\mathbf{b} = \mathbf{B} / |\mathbf{B}|$, $\Phi = e \phi / T_e$, and ϕ is the electrostatic potential, and $J_0(k_{\perp} \rho_i)$ is a Bessel function, $\tau = T_e / T_i$. In this paper, concentrating on core fluctuation, we consider only toroidal rotation for simplicity. Of course, $V_0 \phi$ has a finite component in the direction perpendicular to $\mathbf{B}$.

Starting from Eq. (3), we carry out the usual weak turbulence expansion²⁵ up to third order, to get the following nonlinear ion response. In the expansion, we have kept the only dominant "bare" contribution,

$$g_{\mathbf{k}}^{(3)} = -\sum_{\mathbf{k}'} (C_s \rho_s \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 L_{i\mathbf{k}} L_{i\mathbf{k}'} \{ \chi_{\mathbf{k}}^{(1)} - \chi_{\mathbf{k}'}^{(1)} \} J_0^2 J_0'^2 F_m |\Phi_{\mathbf{k}'}|^2 \Phi_{\mathbf{k}}, \quad (4)$$

where the part of the linear response which leads to the linear susceptibility after velocity space integration is given by

$$\chi^{(1)}_{\mathbf{k}} = \tau[\omega - \mathbf{k} \cdot \mathbf{V}_0 - \omega_{*i}] / (\omega - \mathbf{k} \cdot \mathbf{V}_0 - k_{\parallel} c_{\parallel} / L_s). \quad (5)$$

In this paper, we call this quantity the linear susceptibility. In Eq. (5), we have ignored ion temperature gradient and parallel shear flow modifications^{12,33} for simplicity, and we are keeping only the resonances from the (driven) beat mode particle interactions, $L_{i\mathbf{k}} = i(\omega - \mathbf{k} \cdot \mathbf{V}_0(x) - k_{\parallel} c_{\parallel} / L_s)^{-1}$. Other possible wave-particle resonant interactions at $\mathbf{k}$ or $\mathbf{k}'$ are neglected in the nonlinear response since only the high energy ions participate. Therefore, we use the following approximate formula in Eq. (4),

$$\chi^{(1)}_{\mathbf{k}} - \chi^{(1)}_{\mathbf{k}'} \equiv \{(\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'})\} + \{(\omega_{*e} \mathbf{k} \cdot \mathbf{V}_0(x)/\omega_{\mathbf{k}}^2) - (\omega_{*e}' \mathbf{k}' \cdot \mathbf{V}_0(x')/\omega_{\mathbf{k}'}^2)\}, \quad (6)$$

where $\omega_{\mathbf{k}} = \omega - \mathbf{k} \cdot \mathbf{V}_0$ and $\omega_{\mathbf{k}'} = \omega - \mathbf{k}' \cdot \mathbf{V}_0$ are the Doppler shifted frequencies at the reference rational surfaces, x and x' measure the distance from the reference rational surfaces. The first term on the right hand side of Eq. (6) is the familiar term which is finite due to the dispersion of drift waves, and responsible for the transfer of fluctuation energy to lower $k_{\parallel}$ modes (i.e., poloidal coupling). We call this process "dispersion-induced ion Compton scattering" in this paper. The second term originates from the radial dependence of the equilibrium flow, and we call this "flow shear induced ion Compton scattering." It is crucial to note that unlike the dispersion induced ion Compton scattering, this term remains finite even for $k_{\parallel} \rho_s \rightarrow 0$, and

causes the mode coupling in the radial direction. This is one of the reasons we have to consider radially nonlocal analysis in the context of the nonlinear eigenmode equation, rather than dealing with the radial-mode-width-averaged spectral intensity alone. In this paper, we concentrate on the flow shear induced ion Compton scattering which is dominant for the longer wavelength side of the electron drift wave spectrum. Here, we have not renormalized the propagator $L_{\mathbf{k}}$ to include the broadening of the beat wave frequency caused by nonlinear effects. This weak turbulence approach limits the validity regime of the analysis to weak instability cases. The frequency broadening due to nonlinear effects, $\Delta\omega_{NL}$, can be evaluated straightforwardly.³⁰ In the present case, $\Delta\omega_{NL} \propto \Phi^2$ (rather than $\Delta\omega_{NL} \propto \Phi$, which is characteristic of strong turbulence). Therefore, the validity condition $\Delta\omega_{NL} < k_\theta w_{\mathbf{k}} r$ ($\partial V\phi/\partial r$)/ qR leads approximately to $\gamma < k_\theta w_{\mathbf{k}} r$ ($\partial V\phi/\partial r$)/ qR , where $w_{\mathbf{k}}$ is the radial linear mode width. Using the approximate expression of the linear susceptibilities, we get

$$\begin{aligned}
& \text{Im} \int d^3v J_0 g_{\mathbf{k}}^{(3)} \\
&= \text{Im} \sum_{\mathbf{k}'} (C_S \rho_S \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 (\omega - \mathbf{k} \cdot \mathbf{V}_0)^{-1} \int d^3v (\omega - \mathbf{k}'' \cdot \mathbf{V}_0(x'') - k_\theta x'' c_{||}/L_S)^{-1} \\
& \quad [\{ (\omega * \mathbf{e}/\omega_{\mathbf{k}}) - (\omega * \mathbf{e}'/\omega_{\mathbf{k}'}) \} + \{ (\omega * \mathbf{e} \mathbf{k} \cdot \mathbf{V}_0(x)/\omega_{\mathbf{k}}^2) - (\omega * \mathbf{e} \mathbf{k}' \cdot \mathbf{V}_0(x')/\omega_{\mathbf{k}'}^2) \}] \\
& J_0^2 J_0'^2 F_m |\Phi_{\mathbf{k}'}|^2 \Phi_{\mathbf{k}}.
\end{aligned} \tag{7}$$

In analytically performing the mode summation for various nonlinear interaction channels (i.e., the summation over all the background modes which satisfy $\mathbf{k}' = \mathbf{k} - \mathbf{k}''$), we adopt the following continuum approximation which takes the effect of magnetic shear on the distribution of mode rational surface into account.²⁶ By changing the independent variables from n', m' to k_ϕ' and x' , $\sum_{\mathbf{k}'} \equiv \int dn' dm' = \int |\partial(n', m')/\partial(k_\phi', x')| dk_\phi' dx' =$

$R^2(dq/dr) \int dk_\phi |k_\phi| \int dx' \equiv (rS/q) \int dk_\theta |k_\theta| \int dx'$, where $S = d\ln q/d\ln r$ is the magnetic shear. Consequently, the right hand side can be evaluated explicitly at a radial position x after using an appropriate radially nonlocal expressions, $(\mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 |\Phi_{\mathbf{k}}|^2 \Phi_{\mathbf{k}} \equiv k_\theta^2 |\partial \Phi_{\mathbf{k}} / \partial x|^2 \Phi_{\mathbf{k}}$. Here, we consider a radially long wavelength test mode $\mathbf{k}$ and radially short wavelength background modes $\mathbf{k}'$ which can be treated by continuum approximation. Therefore, they satisfy $k_\theta'^2 k_r'^2 \ll k_\theta^2 k_r'^2$. The radial structure of the short wavelength background mode is given by $|\Phi_{\mathbf{k}}|^2 = I(k'_\theta) \exp(-(x' - \Delta_{\mathbf{k}}')^2 / w_{\mathbf{k}}'^2)$, where the radial shift $\Delta_{\mathbf{k}}'$ and the radial mode width are given by the straightforward linear analyses in the weak flow shear regime ($V_s < s$) where the effective potential becomes radially shifted quadratic well,

$$\Delta_{\mathbf{k}}' = -V_s \rho_s / 2s^2,$$

$$w_{\mathbf{k}}' = s^{-1/2} \rho_s.$$

We note that higher radial harmonics of electron drift waves are more strongly damped and subdominant to the gaussian-shaped fundamental radial harmonics considered here. For analytical progress, the beat wave-ion resonance is approximated by $\delta(\omega - \mathbf{k}'' \cdot \mathbf{V}_0(x''))$. Then, Eq. (7) simplifies to

$$\begin{aligned} \text{Im} \int d^3v J_0 g_{\mathbf{k}}^{(3)} = & \pi (C_s \rho_s)^2 (\omega - \mathbf{k} \cdot \mathbf{V}_0)^{-1} (rS/q) \int dk_\theta |k_\theta| \int dx' k_\theta^2 \{ (x' - \Delta_{\mathbf{k}}')^2 / w_{\mathbf{k}}'^4 \} I(k'_\theta) \\ & \exp(-(x' - \Delta_{\mathbf{k}}')^2 / w_{\mathbf{k}}'^2) \{ (\omega_{*e} \mathbf{k} \cdot \mathbf{V}_0(x) / \omega_{\mathbf{k}}'^2) - (\omega_{*e} \mathbf{k}' \cdot \mathbf{V}_0(x') / \omega_{\mathbf{k}}'^2) \} \delta(\omega - \mathbf{k}'' \cdot \mathbf{V}_0(x'')) \Phi_{\mathbf{k}}. \end{aligned} \quad (8)$$

Here, we have only kept the flow shear induced ion Compton scattering. For this nonlinear process, the dominant contribution comes from the close interactions between modes with similar frequencies and similar $\mathbf{k}$'s. This is because the distant interaction which involves high- $\mathbf{k}'$ contributions is reduced by the gyro-orbit average factor $J_0'^2$. For close interactions

between low k modes, we can approximate the Bessel functions by 1. We then perform the k_θ -integration using the following differential approximation for close interactions,²⁵ $I(k'_\theta) \equiv I(k_\theta) + (k'_\theta - k_\theta)\partial I(k_\theta)/\partial k_\theta$, to get

$$\text{Im} \int d^3 v J_0 g_k^{(3)} = \{N_A + N_B \omega_{*e}/(\omega_{*e} \omega_0'' - W_\perp x) + N_C \omega_{*e}^2/(\omega_{*e} \omega_0'' - W_\perp x)^2\} \Phi, \quad (9)$$

where the radial dependence of nonlinear response is indicated by defining

$$\begin{aligned} N_A &= \pi^{3/2} C_s \rho_s L_n (k_\theta^3/2 \omega_k w_k) (rS/q) \{I(k_\theta) + k_\theta \partial I(k_\theta)/\partial k_\theta\}, \\ N_B \omega_{*e} &= -\pi^{3/2} C_s \rho_s L_n (k_\theta^3/2 \omega_k w_k) (rS/q) W_\perp (x_v - \Delta_k) \{I(k_\theta) + 2k_\theta \partial I(k_\theta)/\partial k_\theta\}, \\ N_C \omega_{*e}^2 &= \pi^{3/2} C_s \rho_s L_n (k_\theta^3/2 \omega_k w_k) (rS/q) W_\perp^2 \{(x_v - \Delta_k)^2 + 3w_k^2/4\} k_\theta \partial I(k_\theta)/\partial k_\theta. \end{aligned}$$

$\omega_0'' \omega_{*e} = k_\theta \partial \omega_k / \partial k_\theta$ is smaller than ω_k due to finite $k_\theta \rho_s$, $W_\perp = k_\theta \partial V_\phi / \partial r$ measures the perpendicular flow shear, $x_v = \omega_0'' \omega_{*e} / W_\perp$.

Finally with Eq. (9), the quasineutrality condition $\delta n_e/n_0 = \delta n_i^{(1)}/n_0 + \delta n_i^{(3)}/n_0$ yields the following nonlinear eigenmode equation in terms of dimensionless variables.

$$\partial^2 \Phi / \partial^2 x = [-1 - b_s + i\delta + 1/(\omega_0 - V_s x) + s^2 x^2/(\omega_0 - V_s x)^2] \Phi + i \text{Im} \int d^3 v J_0 g_k^{(3)}, \quad (10)$$

where $b_s = (k_\theta \rho_s)^2$, $s = SL_n/qR$. $\omega_0 = \omega_k$ is the Doppler shifted eigenfrequency of the test mode, $V_s = L_n W_\perp / k_\theta C_s$, x has been normalized to ρ_s . We note that $\text{Re} \int d^3 v J_0 g_k^{(3)}$ shifts the real frequency by an order of Φ^2 ($\ll 1$) and therefore can be ignored consistently with the weak turbulence ordering.²⁵

III. Nonlinear Eigenmode Analysis

In the preceding section, we have derived a nonlinear eigenmode equation. In this section, we solve that equation to obtain the fluctuation amplitude and its radial dependence at nonlinear saturation. The third and fourth terms on the right hand side of Eq. (10) describe the convection of the ion density and the sound wave contribution respectively with the radial dependence coming from the Doppler shift of the test mode frequency. To obtain the nonlinear dispersion relation in a closed form, we modify the linear response slightly in the following way.

$$\partial^2 \Phi / \partial^2 x = [-1 - b_s + i\delta + \omega_0'' / \omega_0 (\omega_0'' - V_s x) + s^2 x^2 / (\omega_0'' - V_s x)^2] \Phi + i \text{Im} \int d^3 v J_0 g_k^{(3)}. \quad (11)$$

The fourth term in the modified linear response still agrees with the corresponding term in Eq. (10) at small x (i.e., $\mathbf{E} \times \mathbf{B}$ convection of mean density), meanwhile the fifth term in the modified linear response still agrees with the corresponding term in Eq. (10) at large x . We have made this particular choice since the large- x asymptotics of the sound term has been found to be crucial in determining the radial mode structure in the linear regime. Then, Eq. (11) can be put into the following Whittaker's form³⁴ by defining $Y = 2i\alpha(\omega_0'' - V_s x)/V_s$, with $\alpha = [s^2 + V_s^2 \{-1 - b_s + i(\delta + N_A)\}]^{1/2}$.

$$\partial^2 \Phi / \partial^2 Y + \{-1/4 + \kappa/Y + (1/4 - \mu^2)/Y^2\} \Phi = 0, \quad (12)$$

where

$$\begin{aligned}\kappa &= i\{2(s/V_s)^2\omega_0'' - \omega_0''/\omega_0 - iN_B\}/2\alpha, \\ \mu^2 &= 1/4 - \{(s/V_s)^2\omega_0''^2 + iN_C\}/V_s^2.\end{aligned}$$

In choosing the right solution of Eq. (12), the $i(\delta+N_A)$ contribution to the definition of α should not be taken literally, since both the nonadiabatic electron response and the nonlinear ion response should decay as the distance from the reference mode rational surface increases. Then, for weak flow shear case with $s > V_s$, we have to choose a solution $W_{\kappa,\mu}(Y)$ with κ and μ satisfying the following dispersion relation for the dominant fundamental radial harmonics,

$$\kappa(1-\kappa) = 1/4 - \mu^2. \quad (13)$$

The asymptotic behavior of $W_{\kappa,\mu}(Y)$ for large x is given by

$$W_{\kappa,\mu}(Y) \approx Y^\kappa \exp(-Y/2),$$

showing algebraic decay, since $\alpha^2 > 0$, and $\text{Re}(\kappa) < 0$, for $\text{Im } \omega_0 > 0$. For weak flow shear, the nonlinear dispersion relation, Eq. (13) simplifies to

$$1/\omega_0 = 1 + b_s - i(\delta+N_A) + i(\alpha - N_B)/\omega_0'' - iN_C/\omega_0''^2. \quad (14)$$

At nonlinear saturation, $\text{Im } \omega_0 \rightarrow 0$, therefore, the imaginary part of Eq. (14) yields

$$\delta - \alpha/\omega_0'' = -N_A - N_B/\omega_0'' - N_C/\omega_0''^2. \quad (15)$$

This is a first-order inhomogeneous differential equation for the spectral intensity $I(k_\theta)$,

$$\partial I(k_\theta)/\partial k_\theta + T(k_\theta) I(k_\theta) = -S(k_\theta)/U(k_\theta), \quad (16)$$

where

$$T(k_\theta) = \Delta_k/x_v k_\theta \{ (\Delta_k/x_v)^2 + 3w_k^2/4x_v^2 \},$$

$$S(k_\theta) = \delta - \alpha/\omega_0'',$$

$$U(k_\theta) = \pi^{3/2} C_s \rho_s L_n(k_\theta^4/2\omega_k w_k) (rS/q) \{ (\Delta_k/x_v)^2 + 3w_k^2/4x_v^2 \}.$$

Equation (16) can be written in a more tractable form of

$$U(k_\theta) \exp(-R(k_\theta)) \partial \{ I(k_\theta) \exp(R(k_\theta)) \} / \partial k_\theta = -S(k_\theta), \quad (17)$$

with $R(k_\theta) = \int dk_\theta T(k_\theta)$.

Here, $U(k_\theta) \exp(-R(k_\theta))$ plays the role of convective velocity in k_θ space, while $S(k_\theta)$, being the linear drive (damping), plays the role of a source (sink). The positive definiteness of $U(k_\theta)$ indicates that flow shear induced ion Compton scattering transfers fluctuation energy to lower k_θ region. After integrating Eq. (17) from k_θ to k_H (high- k_θ where dispersion-induced ion Compton scattering becomes significant) we get

$$I(k_\theta)\exp(R(k_\theta)) = I(k_H)\exp(R(k_H)) + \int_{k_\theta}^{k_H} dk_\theta S(k_\theta)\exp(R(k_\theta)) / U(k_\theta). \quad (18)$$

We take $\omega_0'' \equiv 1$ such that

$$S(k_\theta) = (\delta(k_H)/k_H) k_\theta - \alpha. \quad (19)$$

Then, the linear instability occurs for $k_\theta > k_L = k_H \alpha / \delta(k_H)$. With this simplified $S(k_\theta)$, Eq.(18) reduces to

$$I(k_\theta) = (\delta(k_H)/k_H U_0(1+T_0)) k_\theta^{-1} - (s/U_0(2+T_0)) k_\theta^{-2}, \quad (20)$$

where $k_\theta \ll k_H$ has been used to ignore subdominant surface terms, and the following quantities are k_θ -independent within the approximations we have mentioned,

$$T_0 = -\Delta_{\mathbf{k}}/x_v \{ (\Delta_{\mathbf{k}}/x_v)^2 + 3w_{\mathbf{k}}^2/4x_v^2 \},$$

$$U_0 = \pi^{3/2} (L_n^2 r S / 2w_{\mathbf{k}} q) \{ (\Delta_{\mathbf{k}}/x_v)^2 + 3w_{\mathbf{k}}^2/4x_v^2 \}.$$

We note that the fluctuation exists ($I(k_\theta) > 0$) even for k_θ lower than k_L (the low- k_θ cut-off given by the linear marginal stability). Physically, flow shear induced ion Compton scattering transfers fluctuations towards lower k_θ causing over-spill beyond the linear cut-off. Equation (20) predicts the nonlinear low- k_θ cut-off at $(1+T_0)k_L/(2+T_0) < k_L$.

For $(1+T_0)k_L/(2+T_0) \ll k_\theta$, the first term of Eq. (20) dominates and we can write Eq. (20) in a more illuminating form,

$$I(k_\theta) = F_s \Im(k_H)(1/k_H k_\theta L_n^2), \quad (21)$$

where $F_s = (4/\pi^{3/2}) S^{1/2} q^{3/2} (R/L_n)^{1/2} (\rho_s C_s^2 / r^3) (\partial V \phi / \partial r)^{-2}$ for $s < 1$. We note that the saturated amplitude of the fluctuations is smaller than the usual mixing length estimates $I(k_\theta) \equiv 1/k_\theta^2 L_n^2$. The reduction contains a form factor F_s due to flow shear, in addition to the linear dependence on the normalized growth rate δ which is typical of a weak turbulence analysis result.²⁵ The spectral shape exhibits a k_θ -dependence which is flatter than the prediction²³ based upon dispersion induced ion Compton scattering which is applicable to the shorter wavelength region.

IV. Conclusions

In this paper, plasma flow shear effects on nonlinear saturation of electron drift waves have been analyzed in the weak turbulence regime. The strongly dissipative trapped electron instability has been considered in a sheared slab geometry with a focus on wave-ion nonlinear interaction. It has been shown that flow shear can enhance ion Compton scattering of moderately long wavelength (weakly dispersive) electron drift waves not only by modifying the beat wave-ion resonance condition, but also via the radial dependence of linear susceptibility. The weak turbulence expansion of the nonlinear gyrokinetic equation leads to a nonlinear eigenmode equation which can be put into the familiar Whittaker's form. A nonlinear dispersion relation is obtained from a radially nonlocal eigenmode analysis. At nonlinear saturation, the spectral intensity of the fluctuations scales with flow shear as $(\partial V_\phi / \partial r)^{-2}$ in addition to the linear dependence on the linear growth rate.

Acknowledgments

The author would like to thank W. M. Tang, P. H. Diamond, F. L. Waelbroeck, N. Bretz, and M. Artun for useful discussions.

This work was supported by U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

References

- 1 R. J. Taylor, M. L. Brown, B. D. Fried, H. Grote, J. R. Liberati, G. J. Morales, P. Pribyl, D. Darrow, and M. Ono, Phys. Rev. Lett. **63**, 2365 (1989).
- 2 R. J. Groebner, K. H. Burrell, and R. P. Seraydian, Phys. Rev. Lett. **64**, 3015 (1990).
- 3 Ch. P. Ritz, R. D. Bengston, S. J. Levinson, and E. J. Powers, Phys. Fluids **27**, 2956 (1984).
- 4 K. Ida, S. Hidekuma, Y. Miura, T. Fujita, M. Mori, K. Hoshino, N. Suzuki, T. Yamauchi, and JFT-2M Group, Phys. Rev. Lett. **65**, 1364 (1990).
- 5 T. Chiueh, P. W. Terry, P. H. Diamond, and J. E. Sedlak, Phys. Fluids **29**, 231 (1986).
- 6 H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B2, 1 (1989).
- 7 K. C. Shaing and E. C. Crume, Jr., Phys. Rev. Lett. **63**, 2369 (1989).
- 8 S.-I. Itoh and K. Itoh, Nucl. Fusion **29**, 1031 (1989).
- 9 A. B. Hassam, T. M. Antonsen, Jr., A. M. Dimits, J. F. Drake, P. N. Guzdar, Y. T. Lau, C. S. Liu, K. C. Shaing, P. J. Christenson, W. A. Houlberg, and R. D. Hazeltine, Plasma Physics and Controlled Nuclear Fusion Research 1990, Proceedings of the 13th International Conference, IAEA-CN-53/D-4-11, Washington DC, USA (IAEA, Vienna)(1991)
- 10 S. Scott, P. Diamond, R. Fonck, R. Goldston, R. Howell, K. Jaenig, G. Schilling, E. Synakowski, M. Zarnstorff, C. Bush, E. Fredrickson, K. Hill, A. Janos, D. Mansfield, D. Owens, H. Park, G. Pautasso, A. Ramsey, J. Schivell, G. Tait, W. Tang, and G. Taylor, Phys. Rev. Lett. **64**, 531 (1990).
- 11 K. H. Burrell, R. J. Groebner, T. S. Kurki-Suonio, T. N. Carlstrom, R. R. Dominguez,

- P. Gohil, R. A. Jong, H. Matsumoto, J. M. Lohr, T. W. Petrie, G. D. Porter,
G. T. Sager, H. E. St. John, D. P. Schissel, S. M. Wolfe, and the DIII-D Research
Group, Plasma Physics and Controlled Nuclear Fusion Research 1990, Proceedings of
the 13th International Conference, IAEA-CN-53/A-2-3, Washington DC, USA (IAEA,
Vienna)(1991)
- 12 N. Mattor and P. H. Diamond, Phys. Fluids **31**, 1180 (1988).
 - 13 T. S. Hahm and W. M. Tang, in Proceedings of the Workshop of the US-Japan Joint
Institute for Fusion Theory Program on Structures in Confined Plasmas (National
Institute of Fusion Science, Nagoya, Japan, 1990), p.55
 - 14 M. Artun and W. M. Tang, to be published in Phys. Fluids B.
 - 15 X. H. Wang and P. H. Diamond, to be published in Phys. Fluids B.
 - 16 S. Hamaguchi and W. Horton, to be published in Phys. Fluids B.
 - 17 F. L. Waelbroeck, T. M. Antonsen, Jr., P. N. Guzdar, and A. B. Hassam, University of
Maryland Laboratory for Plasma Research Preprint 92-011, November 91.
 - 18 B. A. Carreras, K. Sidikman, P. H. Diamond, P. W. Terry, and L. Garcia, Submitted to
Phys. Fluids B (1992).
 - 19 P. H. Diamond, private communications (1990).
 - 20 C. S. Liu, M. N. Rosenbluth, and W. M. Tang, Phys. Fluids **19**, 1040 (1976).
 - 21 A. Rogister and G. Hasselberg, Phys. Fluids **26**, 1467 (1983).
 - 22 P. L. Similon and P. H. Diamond, Phys. Fluids **27**, 916 (1984).
 - 23 F. Y. Gang, P. H. Diamond, and M. N. Rosenbluth, Phys. Fluids B **3**, 68 (1991).
 - 24 T. S. Hahm, Phys. Fluids B **3**, 1445 (1991).
 - 25 R. Z. Sagdeev and A. A. Galeev, in Nonlinear Plasma Theory, edited by T. M. O'Neil

- and D. L. Book (Benjamin, New York, 1969).
- 26 J. A. Krommes, Phys. Fluids **23**, 736 (1980).
 - 27 P. H. Diamond and M. N. Rosenbluth, Phys. Fluids **24**, 1641 (1981).
 - 28 T. S. Hahm, "Weak Turbulence Analysis of Drift Waves in the Presence of Plasma Flows" presented at US-Japan Workshop on Edge Turbulence and the Physics of H-mode (Madison, Wisconsin, 1991).
 - 29 L. Chen, R. L. Berger, J. G. Lominadze, M. N. Rosenbluth, P. H. Rutherford, Phys. Rev. Lett. **39**, 754 (1977).
 - 30 P. H. Diamond, P. L. Similon, P. W. Terry, W. Horton, S. M. Mahajan, J. D. Meiss, M. N. Rosenbluth, K. Swartz, T. Tajima, R. D. Hazeltine, and D. W. Ross, in Plasma Physics and Controlled Nuclear Fusion Research 1982 (IAEA, Vienna, 1983), Vol. I, p. 259.
 - 31 T. S. Hahm and W. M. Tang, Phys. Fluids B **3**, 989 (1991).
 - 32 E. A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).
 - 33 P. J. Catto, M. N. Rosenbluth, and C. S. Liu, Phys. Fluids **16**, 1719 (1973).
 - 34 M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions (Dover, New York, 1972).

EXTERNAL DISTRIBUTION IN ADDITION TO UC-420

Dr. F. Paoloni, Univ. of Wollongong, AUSTRALIA
 Prof. M.H. Brennan, Univ. of Sydney, AUSTRALIA
 Plasma Research Lab., Australian Nat. Univ., AUSTRALIA
 Prof. I.R. Jones, Flinders Univ, AUSTRALIA
 Prof. F. Cap, Inst. for Theoretical Physics, AUSTRIA
 Prof. M. Heindler, Institut für Theoretische Physik, AUSTRIA
 Prof. M. Goossens, Astronomisch Instituut, BELGIUM
 Ecole Royale Militaire, Lab. de Phy. Plasmas, BELGIUM
 Commission-Europeen, DG. XII-Fusion Prog., BELGIUM
 Prof. R. Bouciqué, Rijksuniversiteit Gent, BELGIUM
 Dr. P.H. Sakemaka, Instituto Fisica, BRAZIL
 Instituto Nacional De Pesquisas Espaciais-INPE, BRAZIL
 Documents Office, Atomic Energy of Canada Ltd., CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Dr. H.M. Skarsgard, Univ. of Saskatchewan, CANADA
 Prof. J. Telchmzin, Univ. of Montreal, CANADA
 Prof. S.F. Sreenivasan, Univ. of Calgary, CANADA
 Prof. T.W. Johnston, INRS-Energie, CANADA
 Dr. R. Bolton, Centre canadien de fusion magnétique, CANADA
 Dr. C.R. James,, Univ. of Alberta, CANADA
 Dr. P. Lukác, Komenského Univerzita, CZECHO-SLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Library, R81, Rutherford Appleton Laboratory, ENGLAND
 Mrs. S.A. Hutchinson, JET Library, ENGLAND
 Dr. S.C. Sharma, Univ. of South Pacific, FIJI ISLANDS
 P. Mähönen, Univ. of Helsinki, FINLAND
 Prof. M.N. Bussac, Ecole Polytechnique,, FRANCE
 C. Mouttet, Lab. de Physique des Milieux Ionisés, FRANCE
 J. Radet, CEN/CADARACHE - Bat 506, FRANCE
 Prof. E. Economou, Univ. of Crete, GREECE
 Ms. C. Rinni, Univ. of Ioannina, GREECE
 Dr. T. Mui, Academy Bibliographic Ser., HONG KONG
 Preprint Library, Hungarian Academy of Sci., HUNGARY
 Dr. B. DasGupta, Saha Inst. of Nuclear Physics, INDIA
 Dr. P. Kaw, Inst. for Plasma Research, INDIA
 Dr. P. Rosenau, Israel Inst. of Technology, ISRAEL
 Librarian, International Center for Theo Physics, ITALY
 Miss C. De Palo, Associazione EURATOM-ENEA , ITALY
 Dr. G. Grosso, Istituto di Fisica del Plasma, ITALY
 Prof. G. Rostangni, Istituto Gas Ionizzati Del Cnr, ITALY
 Dr. H. Yamato, Toshiba Res & Devel Center, JAPAN
 Prof. I. Kawakami, Hiroshima Univ., JAPAN
 Prof. K. Nishikawa, Hiroshima Univ., JAPAN
 Director, Japan Atomic Energy Research Inst., JAPAN
 Prof. S. Itoh, Kyushu Univ., JAPAN
 Research Info. Ctr., National Inst. for Fusion Science, JAPAN
 Prof. S. Tanaka, Kyoto Univ., JAPAN
 Library, Kyoto Univ., JAPAN
 Prof. N. Inoue, Univ. of Tokyo, JAPAN
 Secretary, Plasma Section, Electrotechnical Lab., JAPAN
 S. Mori, Technical Advisor, JAERI, JAPAN
 Dr. O. Mitarai, Kumamoto Inst. of Technology, JAPAN
 J. Hyeon-Sook, Korea Atomic Energy Research Inst., KOREA
 D.I. Choi, The Korea Adv. Inst. of Sci. & Tech., KOREA
 Prof. B.S. Liley, Univ. of Waikato, NEW ZEALAND
 Inst of Physics, Chinese Acad Sci PEOPLE'S REP. OF CHINA
 Library, Inst. of Plasma Physics, PEOPLE'S REP. OF CHINA
 Tsinghua Univ. Library, PEOPLE'S REPUBLIC OF CHINA
 Z. Li, S.W. Inst Physics, PEOPLE'S REPUBLIC OF CHINA
 Prof. J.A.C. Cabral, Instituto Superior Tecnico, PORTUGAL
 Dr. O. Petrus, AL I CUZA Univ., ROMANIA
 Dr. J. de Villiers, Fusion Studies, AEC, S. AFRICA
 Prof. M.A. Hellberg, Univ. of Natal, S. AFRICA
 Prof. D.E. Kim, Pohang Inst. of Sci. & Tech., SO. KOREA
 Prof. C.I.E.M.A.T, Fusion Division Library, SPAIN
 Dr. L. Stenflo, Univ. of UMEA, SWEDEN
 Library, Royal Inst. of Technology, SWEDEN
 Prof. H. Wilhelmson, Chalmers Univ. of Tech., SWEDEN
 Centre Phys. Des Plasmas, Ecole Polytech, SWITZERLAND
 Bibliotheek, Inst. Voor Plasma-Fysica, THE NETHERLANDS
 Asst. Prof. Dr. S. Cakir, Middle East Tech. Univ., TURKEY
 Dr. V.A. Glukhikh, Sci. Res. Inst. Electrophys. Apparatus, USSR
 Dr. D.D. Ryutov, Siberian Branch of Academy of Sci., USSR
 Dr. G.A. Eliseev, I.V. Kurchatov Inst., USSR
 Librarian, The Ukr.SSR Academy of Sciences, USSR
 Dr. L.M. Kovrizhnykh, Inst. of General Physics, USSR
 Kernforschungsanlage GmbH, Zentralbibliothek, W. GERMANY
 Bibliothek, Inst. Für Plasmaforschung, W. GERMANY
 Prof. K. Schindler, Ruhr-Universität Bochum, W. GERMANY
 Dr. F. Wagner, (ASDEX), Max-Planck-Institut, W. GERMANY
 Librarian, Max-Planck-Institut, W. GERMANY
 Prof. R.K. Janev, Inst. of Physics, YUGOSLAVIA

END

DATE
FILMED

4 / 01 / 92

I