BEAM-HEAD EROSION IN A HIGHLY CONDUCTIVE BACKGROUND

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ABSTRACT

An analytic estimate of beam head erosion in a highly conductive background is presented and compared to numerical results. Model equations are derived and forced into a self-similar form from which parameter scalings are easily obtained. It is found that erosion in this limit is driven by scattering and is very rapid for high conductivities ($\geq 10^{11}$) due to the high fraction of current neutralization.

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I. Introduction

A propagating charged particle beam experiences competing forces in the transverse direction. A magnetic pinch force tends to constrict the beam while electrostatic repulsion of beam particles tends to expand the beam radius. An additional transverse force (expansive) arises if beam particles have transverse temperature (emittance). Repulsion always dominates by at least a factor of $1/\gamma^2$ for a beam in vacuum even if the beam is transversely cold.

For a beam propagating in a gas, however, substantial charge and current neutralization can occur as a result of beam generated ionization. This can lead to a net pinching force in the main body. Except at very low pressures one can regard charge neutralization as complete throughout the entire beam, leaving emittance to compete directly with the pinch force for control of beam radius. Current neutralization, on the other hand, is most complete near the beam head where the longitudinal electric field induced by current rise causes a maximum return current to flow in the beam generated plasma channel. The low net current results in a pinch force insufficient to overcome emittance pressure in this region and the head expands.

Away from the head, the induced return current dissipates resulting in a rise in net current and pinch force. If initial emittance is not too large the force of expansion may be eventually overcome and self-pinch propagation ensues. The transition between expanding and pinched propagation occurs in the so-called "neck" region. It is typically only one or two beam radii in length when the beam is propagating in initially unionized gas. This is because the induced electric field is shorted out by avalanche breakdown before plasma conductivity is high enough to "freeze in" the return current.

The rapid decay of the return current is accompanied by a rapid decrease of beam radius which together cause rapid turn-on of the pinch field. As the beam propagates, the transition region maintains its local character by moving back into the previously self-pinchcd main body. When the neck region passes through a section of previously pinched beam, the pinch force suddenly disappears and that section begins to expand. This is the phenomenon of beam head erosion which eventually consumes the entire pulse from head to tail. Computer simulation suggests that the rate of erosion becomes constant in a beam of uniform radius. The scaling laws for erosion have been examined in some detail by E. P. Lee.¹

It is of interest to examine the different situation of a beam injected into a medium of already high conductivity. For instance, one might be concerned with the propagation characteristics of a particle beam being used for pellet ignition in accelerator inertial fusion (AIF). In steady state power generation the gas within a confinement vessel may be highly ionized as an artifact of previous microexplosions. When a charged particle beam enters this plasma, a return current is induced which almost fully neutralizes the beam current. Since background conductivity is high, the time required for neutralization is very short and is independent of beam and avalanche ionization rates. As a result the pinch force in the neck region is reduced far below what is required to overcome emittance expansion and erosion is enhanced over what Lee would predict. In this report we estimate the erosion rate for the high conductivity case by assuming a self-similar development of beam quantities. It is found that the erosion rate is proportional to the initial conductivity and beam radius and inversely proportional to beam current and energy, γ . These predictions are compared to erosion rates observed in the axis-symmetric propagation code EPIC.

II. Model Equations of Beam Dynamics

For the purposes of this report we consider a beam composed of charged particles (electrons) propagating in the z direction at a velocity near the speed of light, c . Beam particles have kinetic energy characterized by the relativistic factor γ and beam current density is an azimuthally symmetric, self-similar function $J_b(r/R)$, of radius, R , a rms scale radius. The equation of motion for $R(z)$ has been previously derived by Lee and Cooper:³

$$R'' + \frac{U}{R} - \frac{E^2}{R^3} = 0 \quad (1)$$

where the prime indicates a derivative with respect to z , E is the beam emittance, and $U = I_{\text{eff}}/I_A$ is an effective current indicative of the spatially averaged magnetic pinch field scaled to the Alfvén current I_A . This equation describes the evolution of a thin longitudinal segment of beam under the opposing effects of the pinch force described by the second term and the emittance force described by the third. We can rewrite equation (1) in terms of R^2 as

$$R^2(R^2)'' - \frac{1}{2}(R^2')^2 + 2R^2U - 2E^2 = 0 \quad (2)$$

An equation for U can be derived beginning from Ampère's law assuming full charge neutralization

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} A - \frac{4\pi\sigma}{c} \frac{\partial A}{\partial x} = -\frac{4\pi}{c} J_b \quad (3)$$

where A is the magnetic vector potential and $x = ct - z$.

Since we are concerning ourselves with a background conductivity that is initially high we can drop the first term of equation (3) and differentiate with respect to r . Upon rearranging we have

$$\frac{\partial}{\partial x} \left(\frac{\partial A}{\partial r} \right) = \frac{\partial}{\partial r} \left(\frac{J_b}{\sigma} \right) \quad (4)$$

We now define an effective current, I_{eff} , in terms of the beam averaged pinch magnetic field

$$\left\langle \frac{1}{r} B_\theta \right\rangle = - \left\langle \frac{1}{r} \frac{\partial A}{\partial r} \right\rangle = \frac{1}{R} \left(\frac{2 I_{eff}}{R c} \right) \quad (5)$$

We select an averaging process that assumes that all beam particles are moving in a harmonic well and average over betatron orbits,

$$\langle f(r) \rangle = \frac{1}{R^2} \int_0^\infty 2\pi r dr r^2 f(r) J_b(r) / I_b \quad (6)$$

Assuming a Gaussian profile for beam current density

$$J_b = \frac{I_b}{\pi R^2} e^{-r^2/R^2} \quad (7)$$

and performing the averaging operation of equation (6) on both sides of equation (4), we obtain the equation

$$\frac{\partial}{\partial x} \left(\frac{I_{eff}}{R^2} \right) = \frac{c I_b}{4\pi \sigma R^4} \quad (8)$$

For convenience, we have assumed that the conductivity, σ , is uniform in the radial direction.

We now adopt a simple scaling in the x direction by assuming that since σ is initially high, no significant E_z can build. Conductivity is

generated only by direct deposition of beam energy into the channel but is moderated by the effects of recombination. We then define

$$\eta = \frac{4\pi\sigma R^2}{c} \frac{I_A}{I_b} \quad (9)$$

and assert that it is constant in x and z . That this is rigorously true is difficult to justify ab initio but it is necessary for what is to follow and is shown to be consistent with other numerical results. In any case, this assumption is not unreasonable, at least in an approximate sense. We expect R^2 to show a decrease with increasing x (at fixed z) while σ must increase. Since the beam current through the neck region is nearly constant, η should be relatively insensitive to x and z . Finally equation (8) becomes

$$\eta R^2 \frac{\partial}{\partial x} \left(\frac{U}{R^2} \right) = 1. \quad (10)$$

It remains to write an equation which describes the increase of emittance due to scattering. We shall assume that emittance increases primarily as the result of many small angle scattering of beam particles against molecules of the background gas. The governing equation is then

$$\frac{\partial E^2}{\partial z} = \frac{4 C_N c}{\lambda_R I_A} R^2 = \beta R^2 \quad (11)$$

where C_N is the Nordsieck constant (3.7×10^{12} watts) and λ_R is the bremsstrahlung radiation length in the background gas ($3 \times 10^4 / \hat{\rho}$ cm in air where $\hat{\rho}$ is gas density in units of normal atmospheric).

III. Self-similar Solution of Model Equations

Equations (2), (10), and (11) constitute a closed set of equations in the three variables R^2 , U , and E^2 and the two constants η and β . We write them here for convenience.

$$R^2(R^2'') - \frac{1}{2} (R^2')^2 + 2R^2U - 2E^2 = 0 \quad (2)$$

$$\eta R^2 \frac{\partial}{\partial x} \left(\frac{U}{R^2} \right) = 1 \quad (10)$$

$$E^2' = \beta R^2 \quad (11)$$

where again the prime denotes differentiation with respect to z , and η and β are defined in equation (9) and (11). These equations are quite non-linear making analytic solution difficult.

Numerical treatments have indicated that after an initial rapid expansion of the head region ($x \rightarrow 0$) the beam settles into a steady state erosion driven by scattering. For a background initially unionized, erosion is also driven by ohmic energy loss but the high conductivity condition of our model makes this feature negligible. Simulations also suggest that the evolution of beam variables becomes asymptotically self-similar.

These observations prompt us to assume the following functional forms for the three beam variables involving the similarity variable ξ ,

$$\begin{aligned} R^2(x,z) &= f(z) \psi(\xi) \\ U(x,z) &= g(z) \phi(\xi) \\ E^2(x,z) &= h(z) \rho(\xi) \end{aligned} \quad (12)$$

where in general $\xi = \xi(x, z)$. Assuming that a solution exists of the form of equations (12) we can predict some of the general features of the functions $\psi(\xi)$, $\phi(\xi)$, and $\rho(\xi)$. In particular $\psi(\xi)$ should have a finite value at $\xi \rightarrow 0$ and fall monotonically to zero at $\xi \rightarrow \infty$, characteristic of the trumpet bell shape seen in simulation. The specification of the "neck" is ambiguous but for convenience will be taken as the value ξ_0 for which

$$\psi(\xi = \xi_0) = \delta\psi(0) \quad (13)$$

where $\delta < 1$ ($1/e$ for instance). In principle the procedure is now quite straightforward. The assumed forms of equations (12) are substituted into equations (2), (10), and (11) and the functions f , g , and h are chosen so that the new set of equations involves ξ only. The form of $\xi(x, y)$ will be that which is required to make this possible and is termed the similarity variable. Three equations in the functions $\psi(\xi)$, $\phi(\xi)$, and $\rho(\xi)$ result and are solved, again in principle, for $\psi(\xi)$. Using the functional form $\xi = \xi(x, z)$ and the criterion (13) we solve for x as a function of z , β , and η . The erosion rate, α , is obtained by differentiation:

$$\left. \frac{dx}{dz} \right|_{\xi_0} = \alpha(x, z) \quad (14)$$

The observation that the erosion rate is steady in z suggests $\xi = x/z$ for the similarity variable in equations (12). The three beam variable equations then become

$$\begin{aligned} & \left[\frac{f}{gz^2} - \frac{\dot{f}}{2gz} \right] \xi \psi \psi' + \left[\frac{f}{2gz^2} \right] \xi^2 \psi \psi'' \\ & + \left[\frac{\ddot{f}}{2g} - \frac{\dot{f}^2}{4fg} \right] \psi^2 - \left[\frac{f}{4gz^2} \right] \xi^2 \psi'^2 \\ & + \phi \psi = \left[\frac{h}{fg} \right] \end{aligned} \quad (15)$$

$$\eta \left[\frac{g}{z} \right] \psi \left(\frac{\psi}{\phi} \right)' = 1 \quad (16)$$

$$- \left[\frac{h}{fz} \right] \xi \rho' + \left[\frac{\dot{h}}{f} \right] \rho = \beta \psi \quad (17)$$

where we have altered the notation slightly by representing a differentiation by z with a dot and a differentiation by ξ by a prime.

We have grouped all those terms which are functions of z within brackets. It is now necessary to find the functions f , g , and h such that all the bracket terms are dimensionless. This condition is satisfied by

$$\begin{aligned} f(z) &= z^3 \\ g(z) &= z \\ h(z) &= z^4 \end{aligned} \quad (18)$$

(verifying our assumption for the form of ξ .) Substitution yields the equations

$$- \frac{1}{2} \xi \psi \psi' + \frac{1}{2} \xi^2 \psi \psi'' + \frac{3}{4} \psi^2 - \frac{1}{4} \xi^2 \psi'^2 + \phi \psi = \rho \quad (19)$$

$$\eta \psi \left(\frac{\phi}{\psi} \right)' = 1 \quad (20)$$

$$- \xi \rho' + 4\rho = \beta \psi \quad (21)$$

This final equation set is still highly non-linear and requires numerical methods in general. We do find, however, that at $\xi = 0$ the allowable initial conditions are

$$\begin{aligned} \psi_0 &= \beta \\ \phi_0 &= -\beta/2 \\ \rho_0 &= \beta^2/4 \end{aligned} \quad (22)$$

Furthermore, our present attention is directed to evolution of the beam head ($4\xi \ll \beta\eta$). We therefore drop those terms in equations (19) and (21) with coefficients linear or higher in ξ . In this approximation the three equations are easily combined and solved

$$\begin{aligned}\psi(\xi) &= \beta e^{-4\xi/\beta\eta} \\ \phi(\xi) &= \beta \left(1 - 3e^{-4\xi/\beta\eta} \right) / 4 \\ \rho(\xi) &= \beta^2 e^{-4\xi/\beta\eta} / 4\end{aligned}\tag{23}$$

If we define the pinch point as in equation (13) then we can calculate the erosion rate α from equation (14)

$$\alpha = \beta\eta/4\tag{24}$$

and inserting the definitions of β and η

$$\alpha = 1.01 \left(\frac{\sigma}{10^{11} \text{ sec}^{-1}} \right) \left(\frac{R^*}{0.2 \text{ cm}} \right)^2 \left(\frac{10 \text{ kA}}{I_b} \right) \frac{\hat{\rho}}{\gamma}\tag{25}$$

Recall ρ is the density of the background gas in units of atmospheric density and R^* is the effective radius of the beam at the neck point.

A different criterion for specifying the pinch point is suggested when it is noticed that the approximate solution of equations (23) predicts that the function $\phi(\xi)$ changes sign as ξ is increased from 0. The implication that the return current changes direction is a result of violation of the ordering scheme at large ξ . We can then define the neck to occur at the point where $\phi(\xi_0) = 0$. The erosion rate of equation (24) is altered by less than ten percent.

IV. Discussion

Equation (25) is the central result of this work. The obvious difference between this and previous scalings is the linear dependence of the erosion rate on conductivity. At a conductivity of $3 \times 10^{11} \text{ sec}^{-1}$ the erosion rate is a factor of ten greater than that predicted with previous models.

In order to check equation (25) the code EPIC was used to calculate erosion rates with high initial conductivities in air. Beam parameters were consistent with the Advanced Test Accelerator (ATA) with $\gamma = 100$, $I_b = 10 \text{ kA}$, and $R = 0.2 \text{ cm}$. Erosion rates are plotted for various conductivities in Figure 1 as well as the predictions of equation (25) (with $R^*/R = 2$). It is seen that agreement with simulation is good (20%) with simulation clearly showing a linear dependence of erosion rate on background conductivity.

For comparison we repeat here the analytic and simulation results of Lee¹ and Chambers² for collision driven erosion (at high conductivity net current is small so that the ohmic contribution to erosion is small).

$$\alpha_{\text{scat}} \approx 2.22 \left(\frac{R^*}{1 \text{ cm}} \right)^{\frac{1}{2}} \frac{1}{\gamma^{3/4}} \frac{1 \text{ kA}}{I_b} \frac{1 \text{ kA}}{I^*}^{\frac{1}{2}} \quad (26)$$

Note that scaling with current and energy is similar between equations (26) and (25) but that dependence on R^* is much different. Using test case parameters in equation (26) we obtain an erosion rate much lower than the high conductivity theory or EPIC runs predict. At low conductivities ($\lesssim 3 \times 10^{10}$) the high conductivity model of equation (25) begins to fail due to an increasing level of ohmic contribution.

The analysis leading to equation (25) made several significant approximations and assumptions as were discussed below equation (9). These were necessary in general to force the problem to a self-similar form and were somewhat vindicated by the demonstrated agreement of Figure 1. Different formulations should be possible which incorporate more of the pertinent physics but which retain a self-similar character.

References

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2. F. W. Chambers, J. C. Clark, T. J. Fessenden, "Comparison of Initial ETA Gas Propagation Experiments with Theoretical Models," LNL UCID-19383, April 20, 1982.
3. E. P. Lee and R. K. Cooper, Particle Accelerators 7, 83 (1976).

Figure 1. Comparison of EPIC computer calculations of erosion rates in a high conductivity background to the prediction of equation (25). The prediction of the theory of E. P. Lee is shown for comparison.