MASTER

K/QA-4175

CASCADE DESIGN CONSIDERATIONS FOR CASCADES
COMPOSED OF STAGES WITH LARGE SEPARATION FACTORS

E. Von Halle

Enrichment Planning Department
Operations Analysis and Planning Division

February 1978

UNION
CARBIDE

OAK RIDGE GASEOUS DIFFUSION PLANT
OAK RIDGE, TENNESSEE

*prepared for the U.S. DEPARTMENT OF ENERGY under
U.S. GOVERNMENT Contract W-7405 eng 26*

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

CASCADE DESIGN CONSIDERATIONS FOR CASCADES
COMPOSED OF STAGES WITH LARGE SEPARATION FACTORS

INTRODUCTION AND SUMMARY

It is well known that for the case of a close separation, such as the enrichment of U-235 by means of the gaseous diffusion process, in which the stage separation factors are very close to unity and a large number of stages connected in series is required to achieve the enrichment desired, the ideal cascade (one in which there is no mixing of streams of differing concentrations) requires the least amount of inter-stage flow. Thus, if the total cascade cost, taking into account both capital cost and operating cost, is directly proportional to the total inter-stage flow in the cascade (that is, to the sum of the stage throughputs of all of the stages in the cascade), it follows that the ideal cascade is a minimum cost cascade and that the ideal cascade operating at the optimum tails concentration is the cascade which minimizes the unit cost of product.

In the case of cascades composed of stages with large separation factors, only relatively few stages connected in series are required to achieve the desired degree of enrichment, and ideal cascades do not minimize the total amount of inter-stage flow required. Due also to the fact that ideal cascades composed of stages with large separation factors are capable of producing only certain discrete values of product concentration, none of which will, in general, coincide with the specific product concentration desired, and only certain discrete values of tails concentration, none of which will, in general, coincide with the optimum tails concentration, ideal cascades, in this case, do not minimize the unit cost of product.

In this paper the unit cost of product from ideal cascades and from non-ideal cascades are calculated and compared for a given separation task, that of producing reactor grade uranium containing 3.2% U-235 from normal feed containing 0.72% U-235, under the assumption that the total cascade costs are directly proportional to the total inter-stage flow of the cascade. For the examples chosen, in which the stage separation factors are in the range, $2 < \alpha < 5$, and the resulting cascades contain from four to six stages, the desired product can be obtained at a lower unit cost from a non-ideal cascade. However, the unit cost of product from the optimum non-ideal cascade is only about one per cent less than the unit cost of product from an ideal cascade. In general, this difference in the unit cost of product from non-ideal and ideal cascades will be larger for larger values of the stage separation factors and fewer stages in the cascades than considered here; on the other hand, the difference will be smaller for smaller values of the stage separation factors and larger numbers of stages in the cascades, becoming negligible in the case of a close separation.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

THE SEPARATION STAGE

In order to design the optimum cascade to carry out a specified separation task, it is necessary to know not only the separation parameters of the individual separating units or stages (that is, the stage separation factor, α , and the stage cut, θ), but also the dependence of the objective function (in this case, the unit cost of product), which is to be optimized, on these variables. For illustrative purposes it is satisfactory to use the following rather simple model of a stage in a two-step laser isotope separation process to generate the required data.

It is assumed that in the first step of the process (irradiation of the process gas by infra-red photons), one-half of the U-235 molecules present are raised to an excited energy state. In the second step of the process (irradiation of the process gas by ultra-violet photons), the fraction f_1 of the excited molecules present and the fraction f of all other molecules present are dissociated to form the product or enriched stream from the stage. The undissociated molecules make up the tails or depleted stream from the stage. Let σ^* represent the dissociation cross section of the excited molecules and σ the dissociation cross section of all other molecules. Then if ϕ is a measure of the u.-v. fluence (that is, a measure of the number of ultra-violet photons passing through the process gas), the fraction of the excited molecules decomposed, f_1 , can be assumed to be given by an expression of the form

$$f_1 = 1 - e^{-\sigma^*\phi}, \quad (1)$$

and the fraction of all other molecules decomposed, f , will then be given by

$$f = 1 - e^{-\sigma\phi}. \quad (2)$$

When the U-235 is present only in very dilute concentrations, the stage cut, θ , is essentially equal to the fraction f . For convenience, it will be assumed that this dilute approximation is valid for the range of concentrations of interest in this study.

Let the mole fraction of the U-235 in the uranium in the process gas fed to a stage be denoted by z . The mole fraction of the U-235, y , in the enriched stream leaving the stage, consisting of the dissociated uranium molecules, can then be expressed

$$y = \frac{z}{2} \left(1 + \frac{1 - e^{-\sigma^*\phi}}{\theta} \right), \quad (3)$$

and the mole fraction of the U-235, x , in the depleted stream leaving the stage, consisting of the undissociated uranium molecules, can be expressed

$$x = \frac{z}{2} \left(1 + \frac{e^{-\sigma^*\phi}}{1 - \theta} \right). \quad (4)$$

Since Φ can be expressed in terms of θ , the stage cut, by

$$\Phi = \frac{1}{\sigma} \ln \frac{1}{1-\theta}, \quad (5)$$

the overall stage separation factor, α , can be written in the form

$$\alpha = \frac{y}{x} = \frac{1-\theta}{\theta} \frac{1+\theta-(1-\theta)^{\sigma^*/\sigma}}{1-\theta+(1-\theta)^{\sigma^*/\sigma}}, \quad (6)$$

and the "heads" stage separation factor, β , in the form

$$\beta = \frac{y}{z} = \frac{1+\theta-(1-\theta)^{\sigma^*/\sigma}}{2\theta} \quad (7)$$

The cross section ratio, σ^*/σ , which appears in the preceding two equations is presumed to be a physical constant of the system: its value depends on frequency characteristics of the irradiating photons.

The separative work produced by the stage per unit stage throughput is obtained from a value balance around the stage and in this case, in which the dilute approximation is assumed to be valid, is given by

$$\delta U = (1-\theta) \ln(\alpha) - \ln(\beta). \quad (8)$$

From the relationships given above one can determine the values of the stage separation factors, α and β , and of the separative work produced by the stage per unit of stage throughput as a function of the stage cut, θ , for a specified value of the cross section ratio σ^*/σ . The variation of the stage separation parameters for the case in which σ^*/σ is equal to 3.0 is shown in Figure 1 and for the case in which σ^*/σ is equal to 10.0 in Figure 2. It can be seen from these figures that the performance of a stage, as measured by its output of separative work per unit stage throughput, has a maximum value with respect to the stage cut in each case: for the case in which σ^*/σ is equal to 3.0, the maximum value of δU occurs when the stage cut is equal to about 0.47; for the case in which σ^*/σ is equal to 10.0, the maximum value of δU occurs when the stage cut is equal to about 0.21. It might therefore be expected that ideal cascades composed of stages operating at these maximum separative work outputs would accomplish a specified separation task with the minimum amount of cascade inter-stage flow being required. However, in general, it is not possible to design a completely ideal cascade to perform an arbitrarily selected separation task when the stage separation factors are appreciably greater than unity. There are two reasons for this. One is that in order to avoid internal mixing in the cascade (that is, the mixing of streams of differing concentrations at the inlet to a stage), it is necessary that the stage separation factors stand in the relationship

$$\alpha = \beta^{\left(\frac{k+j}{k}\right)},$$

Figure 1

VARIATION OF THE STAGE SEPARATION PARAMETERS
WITH THE STAGE CUT FOR THE CASE
IN WHICH σ^*/σ IS EQUAL TO 3.0

Figure 2

VARIATION OF THE STAGE SEPARATION PARAMETERS
WITH STAGE CUT FOR THE CASE IN
WHICH σ^*/σ IS EQUAL TO 10.0

where k and j are both small integers. The significance of k and j in the cascade design is that the enriched stream from the n^{th} stage is fed to the $(n+k)^{\text{th}}$ stage and the depleted stream from the n^{th} stage is fed to the $(n-j)^{\text{th}}$ stage. Clearly, when the stage separation factors are large and, consequently, relatively few stages comprise the cascade, both k and j must be small integers. The cascade index, k/j , is shown by the dashed curves in Figures 1 and 2 and the values which permit ideal cascade operation are indicated. The second reason is that an ideal cascade can only produce discrete values of the cascade product concentration and of the cascade tails concentration, depending on the number of stages in the enriching and stripping sections, and these concentrations will, in general, differ somewhat from the specified target concentrations.

THE SEPARATION CASCADE

The separation task to be performed will be the production of enriched uranium containing 3.2% U-235 from normal uranium feed material containing 0.72% U-235. It will be assumed for the purposes of this illustration that the total cascade cost, including both the capital cost components and the operating cost components, can be expressed as a constant of proportionality times the total cascade inter-stage flow: thus, in this case, the cascade which minimizes the sum of the stage throughputs will also be the minimum cost cascade. It will be assumed in the economic optimizations which follow that this constant of proportionality is \$5.00 per unit of stage throughput, and that the cost of natural feed containing 0.72% U-235 is \$100.00 per unit. With this information one can determine the cascade design which produces the desired product at the lowest unit cost. It is shown in the examples that the unit cost of product from the optimum non-ideal cascade is only about one per cent lower than that from an ideal cascade. In all of the cascades considered, the stage cut and the stage separation factors are taken to have the same values for each stage in the cascade.

A. The Case in Which σ^*/σ Is Equal to 3.0.

The optimum stage (that is, the stage that produces the maximum separative work per unit throughput, and therefore, in this illustrative example, has the minimum unit cost of separative work) has the following characteristics (see Figure 1):

$$\theta = 0.46606 \quad \alpha = 2.19364$$

$$\delta U = 0.0761973 \quad \beta = 1.40952$$

$$K(\text{unit cost of separative work}) = \$65.6191$$

If it were possible to construct an ideal cascade using these stages to produce enriched material containing 3.2% U-235 and depleted material at the optimum tails concentration, with no internal or external mixing of streams of different concentrations, the optimum value of the tails concentration would be 0.185581% U-235 and the unit cost of product would be \$879.02. The unit cost of product is made up of a separative work cost

component, reflecting the amount of inter-stage flow required to produce a unit of product, and a feed cost component, reflecting the amount of natural feed required to produce a unit of product. For this case the cost breakdown is

Sep. Work Cost:	\$314.96
<u>Feed Cost</u>	<u>: 564.06</u>
Product Cost	: \$879.02

This figure represents a lower limit of the unit cost of product, unattainable in practice, since this stage cannot be used in a cascade to perform the specified separation task without some internal and external mixing of streams of different concentrations taking place.

An ideal cascade can however be constructed of stages with the following characteristics (see Figure 1):

θ	= 0.40393	α	= 2.17765
δU	= 0.0747674	β	= 1.47569
K(unit cost of separative work) = \$66.8740			

These stages have the property that $\alpha = \beta^2$, which produces a value of the cascade index k/j equal to 1.0, signifying that a "1-up, 1-down" cascade will have no internal mixing of streams of different concentrations. The unit cost of separative work from this ideal stage is only 1.91% greater than that from the optimum stage. If it were possible for an ideal cascade composed of these stages to produce enriched material containing 3.2% U-235 and depleted material at the optimum tails concentration, with no external mixing of streams of different concentrations, the optimum value of the tails concentration would be 0.187446% U-235 and the unit cost of product would be \$885.02, made up of the following cost components

Sep. Work Cost:	\$319.34
<u>Feed Cost</u>	<u>: 565.68</u>
Product Cost	: \$885.02

The ideal cascade composed of the stages described above, however, can only produce certain discrete tails concentrations, the values of which depend on the number of stages in the stripping section, and none of which will, in general, coincide with the optimum tails concentration. Assuming for yet another moment that the cascade produces 3.2% product, one can construct the following table showing how the unit cost of product varies with stripper length.

No. of Stages in Stripper	Tails	Sep. Work Cost	Feed Cost	Product Cost
	Concentration x_w			
0	0.00487908	178.31	1168.54	1346.85
1	0.00330631	231.73	736.93	968.66
2	0.00224052	290.62	600.05	890.67
3	0.00151829	354.58	536.49	891.07
4	0.00102887	423.13	501.87	925.00
5	0.00069722	495.70	481.38	977.08

It can be seen that the best stripping section for the ideal cascade, the one associated with the lowest unit cost of product, contains two stages. The ideal cascade also does not produce the desired product concentration directly, but must produce material at a somewhat higher than desired concentration which can then be mixed with a quantity of natural feed material to yield the desired product. In this case (that is, with the stages described above), an ideal enriching section consisting of four stages will produce material containing 3.41436% U-235. Mixing 0.920441 units of this cascade product with 0.079559 units of natural feed yields 1.0 unit of material containing 3.2% U-235. Accompanying this mixing, there is a loss of 0.0589943 units of separative work per unit of the desired 3.2% product formed. This amount of additional separative work per unit product must be supplied by the ideal cascade, adding \$3.95 to the unit cost of product. The unit cost of desired product containing 3.2% U-235 from a six-stage ideal cascade is therefore

Sep. Work Cost: \$294.57
Feed Cost : 600.05
 Product Cost : \$894.62

The preceding results are summarized in the following table.

Minimum Theoretical Unit Cost of Product	:	\$879.02
Penalty associated with the fact that the performance of the ideal stage is less than that of the optimum stage	:	6.00
Penalty associated with the fact that the ideal cascade does not produce the optimum tails concentration	:	5.65
Penalty associated with the fact that the ideal cascade does not produce directly the desired product concentration	:	3.95
Unit Cost of Product from the Ideal Cascade:		\$894.62

It can be seen that the unit cost of product from the ideal cascade is \$15.60, or 1.77 per cent, higher than the minimum theoretical unit cost

of product which is an unattainable lower limit of the product cost. A detailed investigation of non-ideal cascades will reveal what fraction of this difference is recoverable and what fraction is consumed by the mixing losses intrinsic in non-ideal cascades.

For convenience, only non-ideal cascades in which all stages in the cascade operate at the same value of the stage cut and, hence, have the same values for the stage separation factors, will be considered. The mathematical treatment for such cascades is presented in the Appendix. Typical cascade configurations are shown in Figures A.1 (1-up, 1-down) and B.1 (2-up, 1-down). It can be seen that provision is made for both product and tails reflux.

Calculation of the performance of non-ideal cascades shows that as the stage cut is increased from the value of 0.404, corresponding to that of the ideal cascade, the unit cost of the desired product decreases until a minimum unit cost of product of \$886.18 is attained at a value of the stage cut equal to 0.416. The results of these calculations are shown in Figure 3. The initial decrease in the unit cost of product, starting from the ideal cascade, is due to the improved stage performance (higher separative work per unit stage throughput, to reduced external mixing losses since the cascade product concentration drops as the stage cut increases, and to the fact that the cascade tails concentration move towards the optimum value as the cut increases. At a value of stage cut equal to about 0.411 the cascade product concentration, with no reflux, is equal to 3.2%; for higher cuts product reflux is needed to maintain the desired product concentration. The optimum cascade found has the following description:

θ	= 0.416	α	= 2.18116
δU	= 0.0752696	β	= 1.46253
No. of enricher stages	= 4,	f_e	= 0.856913
No. of stripper stages	= 2,	f_s	= 1.0
Cascade product concentration	= 0.032		
Cascade tails concentration	= 0.00204257		
Cascade efficiency	= 99.35%		
Sep. Work Cost:	\$305.32		
<u>Feed Cost</u>	:	<u>580.86</u>	
Product Cost	:	\$886.18	

The optimum non-ideal cascade, therefore, has a unit cost of product which is \$8.44, or 0.94%, less than that of the ideal cascade.

B. The Case in Which σ^*/σ Is Equal to 10.0.

The optimum stage, in this case, has the following characteristics (see Figure 2):

Figure 3

THE PERFORMANCE OF NON-IDEAL CASCADES FOR THE CASE IN WHICH σ^*/σ IS EQUAL TO 3.0. THE CASCADES CONSIST OF SIX STAGES, FOUR ENRICHING STAGES AND TWO STRIPPING STAGES

$$\theta = 0.20893 \quad \alpha = 4.75059$$

$$\delta U = 0.253071 \quad \beta = 2.66347$$

$$K(\text{unit cost of separative work}) = \$19.7573$$

If it were possible to construct an ideal cascade using these stages to produce enriched material containing 3.2% U-235 and depleted material at the optimum tails concentration, with no internal or external mixing of streams of different concentrations, the optimum value of the tails concentration would be 0.088227% U-235 and the unit cost of product would be \$625.89, made up of the following cost components

Sep. Work Cost: \$133.35

Feed Cost : 492.54

Product Cost : \$625.89

This product cost is the minimum theoretical cost, unattainable in practice, for the reasons given earlier.

A 1-up, 1-down ideal cascade can be constructed of stages with the following characteristics (see Figure 2):

$$\theta = 0.33867 \quad \alpha = 3.81322$$

$$\delta U = 0.215939 \quad \beta = 1.95275$$

$$K(\text{unit cost of separative work}) = \$23.1547$$

These stages have the property that $\alpha = \beta^2$. If it were possible for an ideal cascade composed of these stages to produce enriched material containing 3.2% U-235 and depleted material at the optimum tails concentration, with no external mixing of streams of different concentrations, the optimum value of the tails concentration would be 0.098526% U-235 and the unit cost of product would be \$648.29, made up of the following cost components

Sep. Work Cost: \$149.24

Feed Cost : 499.05

Product Cost : \$648.29

The ideal cascade, however, can only produce certain discrete values of tails concentration. The following table shows how the unit cost of product varies with the length of the stripping section (assuming, still, that the cascade produces 3.2% product).

No. of Stages in Stripper	Tails Concentration	Sep. Work Cost	Feed Cost	Product Cost
	x_w			
0	0.00368711	74.86	805.97	880.83
1	0.00188817	110.16	566.88	677.04
2	0.00096693	150.43	497.88	648.30
3	0.00049516	194.73	469.88	664.61
4	0.00025357	242.08	457.02	699.10

It can be seen that the best stripping section for the ideal cascade contains two stages. The ideal cascade also does not produce the desired product concentration directly. An ideal enriching section consisting of three stages will produce material containing 5.36131% U-235. Mixing 0.534332 units of this cascade product with 0.465668 units of natural feed yields 1.0 unit of material containing 3.2% U-235. Accompanying this mixing, there is a loss of 0.418870 units of separative work per unit of the desired 3.2% product formed. This amount of additional separative work must be supplied by the cascade, adding \$9.70 to the unit cost of product. The unit cost of product from this five-stage ideal cascade is therefore

Sep. Work Cost: \$160.12

Feed Cost : 497.88

Product Cost : \$658.00

Figure 4 shows that reduction of the stage cut from $\theta = 0.339$ to $\theta = 0.320$ reduces the unit cost of product from a five stage cascade to \$656.53. A four stage, 1-up, 1-down cascade has a lower unit cost of product of \$643.15 at a cut θ equal to 0.316. Further improvement may be realized by utilizing a 2-up, 1-down cascade arrangement.

It is evident from Figure 2 that in this case in which σ^*/σ is equal to 10.0, stages consistent with a 2-up, 1-down ideal cascade have a separative work output per unit throughput better than that for the stage described above for which $\alpha = \beta^2$. A 2-up, 1-down ideal cascade can be constructed of stages with the following characteristics:

$$\theta = 0.177888 \quad \alpha = 4.97523$$

$$\delta U = 0.249407 \quad \beta = 2.91435$$

$$K(\text{unit cost of separative work}) = \$20.0475$$

An ideal cascade composed of these stages, producing the desired product and the optimum tails concentration of 0.0891398% U-235, with no internal or external mixing of streams of different concentrations, would have the following costs:

Sep. Work Cost: \$134.73

Feed Cost : 493.12

Product Cost : \$627.85

Figure 4

THE PERFORMANCE OF NON-IDEAL CASCADES FOR
THE CASE IN WHICH σ^*/σ IS EQUAL TO 10.0

The best stripping section for this ideal cascade contains three stages, produces a cascade tails concentration of 0.084771% U-235; it has a \$0.09 penalty associated with its inability to produce the optimum tails concentration. The best enriching section for this ideal cascade also consists of three stages and it produces material containing 4.50111% U-235; it has a \$6.25 penalty associated with its inability to produce the desired product concentration. The cost breakdown for this six-stage, 2-up, 1-down ideal cascade is

Sep. Work Cost:	\$143.79
<u>Feed Cost</u>	: 490.41
Product Cost	: \$634.20

Calculation of the performance of non-ideal, 2-up, 1-down cascades shows that as the stage cut is increased from the value of 0.178, corresponding to that of the ideal cascade, the unit cost of the desired product decreases until a minimum unit cost of product of \$632.20 is attained at a value of the stage cut equal to 0.194. The optimum cascade found has the following description:

θ	= 0.194	α	= 4.86048
δU	= 0.252266	β	= 2.77911
No. of enricher stages	= 3,	f_e	= 1.0
No. of stripper stages	= 3,	f_s	= 1.0
Cascade product concentration	= 0.037724		
Cascade tails concentration	= 0.00070950		
Cascade efficiency	= 99.15%		
Sep. Work Cost:	\$150.10		
<u>Feed Cost</u>	:	482.10	
Product Cost	:	\$632.20	

Thus in this case the best non-ideal cascade has a unit cost of product which is \$2.00 or about 0.3% less than that of a six-stage, 2-up, 1-down ideal cascade.

APPENDIX

THE MATHEMATICAL TREATMENT OF CONSTANT-CUT,
LARGE-ALPHA, NON-IDEAL CASCADES

A single separating unit, or stage, is shown schematically in the figure below. It is assumed, for convenience, that the dilute approximation is applicable. Since only concentrations in the range of 0 to 5% U-235 are considered, this approximation should be satisfactory.

It is useful also to define the cut of the desired component effected by the stage; this quantity is given by

$$\theta^* = \frac{Ly}{Gz} = \theta\beta = \frac{\alpha-\beta}{\alpha-1} \quad . \quad (5)$$

A. 1-Up, 1-Down Cascades

Consider first a simple separation cascade consisting of separating units connected in series as shown in Figure A.1. This 1-up, 1-down cascade configuration is the standard cascade arrangement used in the gaseous diffusion and gas centrifuge processes. The number of separating units, or stages, in the enriching section (including the cascade feed stage) is

Figure A.1

THE STAGE ARRANGEMENT IN A 1-UP, 1-DOWN CASCADE
WITH PRODUCT AND WASTE-END REFLUX

denoted by N . An overall material balance around the top of the cascade, taken above the n^{th} stage, can be written

$$\theta G_n = (1-\theta)G_{n+1} + P, \quad \text{or} \quad G_{n+1} - \frac{\theta}{1-\theta} G_n = -\frac{P}{1-\theta}, \quad (\text{A.1})$$

where all stages in the cascade are presumed to operate at the same stage cut and, hence, with the same stage separation factors. Denoting the quantity $\theta/(1-\theta)$ by ϕ , it follows that $\theta = \phi/(1+\phi)$, and the preceding difference equation can be written in the form

$$G_{n+1} - \phi G_n = -(1+\phi)P. \quad (\text{A.2})$$

The solution of this first-order linear difference equation is

$$G_n = P \frac{1+\phi}{1-\phi} \{C\phi^{n-1}\}, \quad (\text{A.3})$$

where C is a constant of integration. The boundary condition to be satisfied at the top of the cascade can be expressed

$$P = f_e L_N = f_e \theta G_N, \quad \text{where} \quad 0 < f_e \leq 1. \quad (\text{A.4})$$

It follows that the constant of integration, C , is given by

$$C = \frac{1}{\phi^{N+1}} \frac{1-\phi+\phi f_e}{f_e}. \quad (\text{A.5})$$

The number of separating units, or stages, in the stripping section is denoted by M . An overall material balance around the bottom of the cascade, taken above the m^{th} stage, can be written

$$(1-\theta)G_{m+1} = \theta G_m + W, \quad \text{or} \quad G_{m+1} - \frac{\theta}{1-\theta} G_m = \frac{W}{1-\theta} \quad (\text{A.6})$$

The difference equation for the stripping section, analogous to equation (A.2), therefore has the form

$$G_{m+1} - \phi G_m = (1+\phi)W. \quad (\text{A.7})$$

The solution of this first-order linear difference equation is

$$G_m = W \frac{1+\phi}{1-\phi} \{D\phi^{m+1}\}, \quad (\text{A.8})$$

where D is a constant of integration. The boundary condition to be satisfied at the bottom of the cascade can be expressed

$$W = f_s (G_1 - L_1) = f_s (1-\theta)G_1, \quad \text{where} \quad 0 < f_s \leq 1. \quad (\text{A.9})$$

It follows that the constant of integration, D , is given by

$$D = \frac{1-\phi-f_s}{\phi f_s} . \quad (\text{A.10})$$

The two stage flow rate equations, equation (A.3) for the enricher flows and equation (A.8) for the stripper flows, should predict the same stage throughput for the cascade feed stage. This requirement yields a relationship between P and W, which when combined with the material balance relationship, $F = P+W$, allows one to calculate P and W, given the cascade feed rate. Setting $G_1(\text{enr})$ equal to $G_{M+1}(\text{str})$, one obtains

$$P(C\phi-1) = W(D\phi^{M+1}+1) \quad (\text{A.11})$$

from which it follows that

$$\frac{P}{F} = \frac{1+D\phi^{M+1}}{C\phi+D\phi^{M+1}} . \quad (\text{A.12})$$

The total interstage flow in the cascade (that is, the sum of all of the stage throughputs) is obtained from the finite integration of equations (A.3) and (A.8). The sum of the G_n 's in the enriching section is given by

$$\sum_{n=1}^N G_n = P \frac{1+\phi}{1-\phi} \left\{ C \frac{\phi-\phi^{N+1}}{1-\phi} - N \right\} \quad (\text{A.13})$$

and the sum of the G_m 's in the stripping section is given by

$$\sum_{m=1}^M G_m = W \frac{1+\phi}{1-\phi} \left\{ D \frac{\phi-\phi^{M+1}}{1-\phi} + M \right\} . \quad (\text{A.14})$$

The total cascade interstage flow is just the sum of these two expressions.

The desired component material balance around the top of the cascade, taken above the n^{th} stage, can be written

$$\theta^* G_n z_n = (1-\theta^*) G_{n+1} z_{n+1} + P y_p , \quad \text{or} \quad [Gz]_{n+1} - \frac{\theta^*}{1-\theta^*} [Gz]_n = \frac{[P y_p]}{1-\theta^*} , \quad (\text{A.15})$$

where θ^* is the stage cut of the desired component. It is evident that the difference equation for the stage flow rates of the desired component is identical in form to equation (A.1), but with θ^* in place of θ and the desired component transport in place of the total material transport. It follows that the solution to the difference equation will have the same form as presented above and one can deduce, analogous to equation (A.12), that

$$\frac{P y_p}{F x_F} = \frac{1 + D^* \phi^{*M+1}}{C^* \phi^* + D^* \phi^{*M+1}} , \quad (\text{A.16})$$

where $\phi^* = \theta^*/(1-\theta^*)$, $C^* = \frac{1}{\phi^{*N+1}} \frac{1-\phi^*+\phi^* f_e}{f_e}$, and $D^* = \frac{1-\phi^*-f_s}{\phi^* f_s}$.

Dividing equation (A.16) by equation (A.12) yields an expression for y_p/x_F from which the cascade product concentration can be determined presuming that the value of the feed concentration, x_F , is known.

B. Two-Up, One-Down, Constant Cut Cascades

Consider now the separation cascade consisting of separating units connected in series as shown in Figure B.1. This cascade arrangement is conveniently called a two-up, one-down cascade. The number of separating units, or stages, in the enriching section (including the cascade feed stage) is denoted by N . An overall material balance around the top of the cascade, taken above the $(n+1)^{th}$ stage, can be written

$$\theta G_n + \theta G_{n+1} = (1-\theta)G_{n+2} + P, \text{ or } G_{n+2} - \frac{\theta}{1-\theta} G_{n+1} - \frac{\theta}{1-\theta} G_n = -\frac{P}{1-\theta} \quad (B.1)$$

Denoting the quantity $\theta/(1-\theta)$ by ϕ , it follows that $\theta = \phi/(1+\phi)$, and the preceding difference equation can be written in the form

$$G_{n+2} - \phi G_{n+1} - \phi G_n = -(1+\phi)P. \quad (B.2)$$

The solution of this second-order linear difference equation is

$$G_n = P \frac{1+\phi}{1-2\phi} \{C_1 R_1^n + C_2 R_2^n - 1\}, \quad (B.3)$$

where
$$R_1, R_2 = \frac{\phi \pm \sqrt{\phi^2 + 4\phi}}{2}, \quad (B.4)$$

and C_1 and C_2 are constants of integration. The boundary condition which applies at the top of the enriching section can be expressed

$$\theta G_N + \theta G_{N-1} = \frac{P}{f_e} \text{ or } G_{N+1} = P(1+\phi) \frac{1-f_e}{f_e}, \quad (B.5)$$

which yields the relationship

$$C_1 R_1^{N+1} + C_2 R_2^{N+1} = \frac{1-2\phi + 2\phi f_e}{f_e} \quad (B.6)$$

The number of separating units, or stages, in the stripping section is denoted by M . An overall material balance around the bottom of the cascade, taken above the $(m+1)^{th}$ stage, can be written

$$(1-\theta)G_{m+2} = \theta G_{m+1} + \theta G_m + W, \text{ or } G_{m+2} - \frac{\theta}{1-\theta} G_{m+1} - \frac{\theta}{1-\theta} G_m = \frac{W}{1-\theta}. \quad (B.7)$$

The difference equation for the stripping section can therefore be written in the form

$$G_{m+2} - \phi G_{m+1} - \phi G_m = (1+\phi)W. \quad (B.8)$$

Figure B.1

THE STAGE ARRANGEMENT IN A 2-UP, 1-DOWN CASCADE
WITH PRODUCT AND WASTE-END REFLUX

The solution of this second-order linear difference equation is

$$G_m = W \frac{1+\phi}{1-2\phi} \{D_1 R_1^m + D_2 R_2^m + 1\}, \quad (\text{B.9})$$

where D_1 and D_2 are constants of integration. The following boundary conditions apply at the bottom of the stripping section.

$$1) \quad G_0 = 0, \quad \text{or} \quad D_1 + D_2 = -1. \quad (\text{B.10})$$

$$2) \quad G_1 = \frac{W}{f_s(1-\theta)}, \quad \text{or} \quad D_1 R_1 + D_2 R_2 = \frac{1-2\phi-f_s}{f_s}. \quad (\text{B.11})$$

It follows that

$$D_1 = \frac{1}{R_1 - R_2} \frac{1-2\phi-f_s+R_2 f_s}{f_s} \quad \text{and} \quad D_2 = -\frac{1}{R_1 - R_2} \frac{1-2\phi-f_s+R_1 f_s}{f_s} \quad (\text{B.12})$$

At the interface of the enriching and stripping sections, the following conditions must be satisfied.

$$1) \quad G_n(n=0) = G_m(m=M), \quad \text{or} \quad P\{C_1 + C_2 - 1\} = W\{D_1 R_1^M + D_2 R_2^M + 1\}. \quad (\text{B.13})$$

$$2) \quad G_n(n=1) = G_m(m=M+1), \quad \text{or} \quad P\{C_1 R_1 + C_2 R_2 - 1\} = W\{D_1 R_1^{M+1} + D_2 R_2^{M+1} + 1\}. \quad (\text{B.14})$$

Solving this pair of equations for C_1 and C_2 , one obtains

$$C_1 = \frac{W}{P} D_1 R_1^M + \frac{F}{P} \frac{1-R_2}{R_1 - R_2} \quad \text{and} \quad C_2 = \frac{W}{P} D_2 R_2^M - \frac{F}{P} \frac{1-R_1}{R_1 - R_2}. \quad (\text{B.15})$$

Substituting these expressions for the constants C_1 and C_2 into equation (B.6), one obtains the relationship from which the cascade product rate, P , can be determined, presuming that the cascade feed rate, F , is known:

$$P\{D_1 R_1^{N+M+1} + D_2 R_2^{N+M+1} + \frac{1-2\phi+2\phi f_e}{f_e}\} = F\{D_1 R_1^{N+M+1} + D_2 R_2^{N+M+1} + \frac{(1-R_2)R_1^{N+1} - (1-R_1)R_2^{N+1}}{R_2 - R_1}\}. \quad (\text{B.16})$$

The sum of the stage throughputs in the enriching section can now be computed from

$$\sum_{n=1}^N G_n = P \frac{1+\phi}{1-2\phi} \left\{ C_1 \frac{R_1 - R_1^{N+1}}{1 - R_1} + C_2 \frac{R_2 - R_2^{N+1}}{1 - R_2} - N \right\}, \quad (\text{B.17})$$

and the sum of the stage throughputs in the stripping section from

$$\sum_{m=1}^M G_m = W \frac{1+\phi}{1-2\phi} \left\{ D_1 \frac{R_1 - R_1^{M+1}}{1 - R_1} + D_2 \frac{R_2 - R_2^{M+1}}{1 - R_2} + M \right\}. \quad (\text{B.18})$$

The total cascade interstage flow is the sum of these two expressions.

As in the preceding case, equations of the same form apply also for the computation of the interstage flows of the desired isotope. Thus equation (B.16) can be used to calculate the quantity P_{yp} if F_{x_F} is substituted for F and the cut of the desired isotope, θ^* , is used in place of θ in the calculation of ϕ , R_1 , and R_2 .

DISTRIBUTION

<p>1. <u>Engineering</u>
R. C. Green</p> <p>2-3. <u>Gaseous Diffusion
Development</u>
G. J. Kidd, Jr.
J. R. Merriman</p> <p>4-14. <u>Operations Analysis
and Planning</u>
J. T. Bradbury
S. Blumkin
A. de la Garza
M. Hanig
R. L. Hoglund
S. C. Jacobs
S. A. Levin
D. F. Starr
E. Von Halle
K. A. Williams
H. G. Wood</p> <p>15. ORGDP Records Dept. (RC)</p> | <p>16-21. <u>DOE:ORO</u>
T. H. Scott (6)</p> <p>22-23. <u>Lawrence Livermore
Laboratory</u>
J. I. Davis
J. Z. Holtz</p> <p>24-26. <u>Los Alamos Scientific
Laboratory</u>
G. Emanuel
F. T. Finch
C. P. Robinson</p> <p>27. <u>Technical Director</u>
W. J. Wilcox, Jr.</p> <p>28-29. <u>Y-12 Plant</u>
W. J. Hulsey
P. R. Vanstrum</p> |
|---|--|