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Time-Dependent Field Equations For Paraxial Relativistic Electron Beams

W. M. Sharp, S. S. Yu, and E. P. Lee

Abstract

A simplified set of field equations for a paraxial relativistic electron beam is presented. These equations for the beam electrostatic potential ϕ and pinch potential $\chi \equiv A_z - \phi$ retain previously neglected time-dependent terms and for axisymmetric beams reduce exactly to Maxwell's equations.

I. Introduction

Most analytic and numerical work on relativistic electron beams in resistive media has used a time-independent set of field equations developed by E. P. Lee.¹ These equations are adequate for modeling beams that, in the particle rest frame, are evolving slowly over typical betatron oscillation times. However, they may be inappropriate for describing critical phases of recent AIA propagation experiments. For example, a beam experiences strong radial forces as it is launched from a foil and as it hits a beam dump, and even during propagation the eroding beam head changes on a betatron frequency time scale.

To model rapidly evolving beams, we have developed a new set of approximate field equations that retain time-variation terms neglected in the earlier Lee equations. In this paper, we first derive the new equations and then discuss hose-instability calculations using both sets of equations.

II. Derivation

Maxwell's equations may, in general, be written in terms of a vector potential $\mathbf{A}$ and electrostatic potential ϕ :

$$\nabla^2 \phi + \frac{1}{c} \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) = -4\pi\rho \quad (1)$$

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$$\nabla^2 \underline{A} - \frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} - \nabla (\nabla \cdot \underline{A} + \frac{1}{c} \frac{\partial \phi}{\partial t}) = - \frac{4\pi}{c} \underline{J} \quad (2)$$

Here, ρ and $\underline{J}$ are the charge and current densities, and the corresponding electric and magnetic fields are given by

$$\underline{E} = - \frac{1}{c} \frac{\partial \underline{A}}{\partial t} - \nabla \phi \quad (3)$$

$$\underline{B} = \nabla \times \underline{A} \quad (4)$$

For a paraxial beam propagating in the z direction through a collisional plasma with scalar conductivity σ , the current density in Eq. (2) is the sum of a beam contribution $\underline{J}_b$ and a plasma current contribution $\sigma \underline{E}$

$$\underline{J} = \underline{J}_b + \sigma \underline{E} \quad (5)$$

with $|\underline{J}_{bz}| \gg |\underline{J}_{b\perp}|$.

The fields $\underline{E}$ and $\underline{B}$ are, of course, invariant under gauge transformations of $\underline{A}$ and ϕ . These transformations are given in general by

$$\underline{A} \rightarrow \underline{A}' = \underline{A} + \nabla \Lambda \quad (6)$$

$$\phi \rightarrow \phi' = \phi + \frac{1}{c} \frac{\partial \Lambda}{\partial t} \quad (7)$$

where Λ is an arbitrary scalar field. We are therefore free to impose any gauge condition that is consistent with Eqs. (6) and (7). A particularly convenient choice is the condition.

$$\nabla_{\perp} \cdot \underline{A}_{\perp} = 0 \quad (8)$$

where the $\perp$ subscripts denote vector components perpendicular to the z direction. Combining Eqs. (6) and (8) gives a simple equation for Λ

$$\nabla_{\perp}^2 A = \nabla_{\perp} \cdot \underline{A}_{\perp} , \quad (9)$$

where $\underline{A}$ can be in any gauge. In the new gauge, Eq. (2) may be written as

$$\nabla_{\perp}^2 \underline{A} - \frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} - \nabla \left(\frac{\partial A_z}{\partial z} + \frac{1}{c} \frac{\partial \phi}{\partial t} \right) = - \frac{4\pi}{c} \underline{j} , \quad (10)$$

where we have used the relation $\nabla \cdot \underline{A} = \partial A_z / \partial z$. The z-component of Eq. (10) reduces to

$$\nabla_{\perp}^2 A_z + \frac{1}{c} \frac{\partial E_z}{\partial t} = - \frac{4\pi}{c} (j_{bz} + \sigma E_z) , \quad (11)$$

where, from Eq. (3),

$$E_z = - \frac{1}{c} \frac{\partial A_z}{\partial t} - \frac{\partial \phi}{\partial z} . \quad (12)$$

Taking the perpendicular divergence of Eq. (2) yields a second equation

$$\nabla_{\perp}^2 \left(\frac{\partial A_z}{\partial z} + \frac{1}{c} \frac{\partial \phi}{\partial t} \right) = \frac{4\pi}{c} \nabla_{\perp} \cdot (j_{b\perp} + \sigma \underline{E}_{\perp}) \quad (13)$$

with

$$\underline{E}_{\perp} = - \frac{1}{c} \frac{\partial \underline{A}_{\perp}}{\partial t} - \nabla_{\perp} \phi . \quad (14)$$

The pinch force on a particle with charge q is given in general by

$$\underline{F}_{\perp} = q \left[\frac{1}{c} (\underline{v} \times \nabla \times \underline{A})_{\perp} - \frac{1}{c} \frac{\partial \underline{A}_{\perp}}{\partial t} - \nabla_{\perp} \phi \right] ,$$

which, for a paraxial beam, can be written to $\mathcal{O}(\frac{v_{\perp}}{c})$ as

$$\underline{F}_{\perp} \approx q \left[\nabla_{\perp} (A_z - \phi) - \frac{\partial \underline{A}_{\perp}}{\partial z} - \frac{1}{c} \frac{\partial \underline{A}_{\perp}}{\partial t} \right] . \quad (15)$$

Since a relativistic paraxial beam has $v_z = \beta c \approx c$, it is customary to rewrite the field equations in terms of the distance back from the beam head, $\zeta = ct - z$, and z , which serves as the time coordinate in the beam frame. To make this change of variables, we introduce the transformations

$$\frac{1}{c} \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \zeta} \quad (16)$$

$$\frac{\partial}{\partial z} \rightarrow \frac{\partial}{\partial z} - \frac{\partial}{\partial \zeta} \quad (17)$$

It is also convenient to replace A_z in the equations by the "pinch potential" $\psi \equiv A_z - \phi$, which makes the dominant contribution to $E_\perp$ in Eq. (15). With these transformations, Eqs. (11) through (15) are rewritten as

$$\nabla_\perp^2 (\psi + \phi) + \frac{\partial E_z}{\partial \zeta} = - \frac{4\pi}{c} (J_{bz} + \sigma E_z) \quad (18)$$

$$\nabla_\perp^2 \left(\frac{\partial \psi}{\partial z} - \frac{\partial \psi}{\partial \zeta} + \frac{\partial \phi}{\partial z} \right) = \frac{4\pi}{c} \nabla_\perp \cdot (J_{b\perp} + \sigma E_\perp) \quad (19)$$

$$E_z = - \left(\frac{\partial \psi}{\partial \zeta} + \frac{\partial \phi}{\partial z} \right) \quad (20)$$

$$E_\perp = - \left(\frac{\partial A_\perp}{\partial \zeta} + \nabla_\perp \phi \right) \quad (21)$$

$$E_\perp = q \left(\nabla_\perp \psi - \frac{\partial A_\perp}{\partial z} \right) \quad (22)$$

The $J_{b\perp}$ term in Eq. (19) can be put into a more usable form using the charge-conservation relation

$$\nabla_\perp \cdot J_{b\perp} = - \frac{\partial J_{bz}}{\partial z} - \frac{\partial \rho_b}{\partial t} \quad (23)$$

Since $\rho_b = J_{bz}/c + \mathcal{O}(v_\perp^2/c^2)$ for paraxial beams, Eq. (23) can be written to $\mathcal{O}(v_\perp/c)$ as

$$\nabla_\perp \cdot J_{b\perp} \approx - \frac{\partial J_{bz}}{\partial z} - \frac{1}{c} \frac{\partial J_{bz}}{\partial t} \rightarrow - \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \zeta} \right) J_{bz} - \frac{\partial J_{bz}}{\partial \zeta} \approx - \frac{\partial J_{bz}}{\partial z} \quad (24)$$

To simplify the plasma current term in Eq. (19), we assume that inductive contribution is small compared with the electrostatic part:

$$\nabla_{\perp} \cdot (\sigma \underline{E}_{\perp}) = - \nabla_{\perp} \cdot \left[\sigma \left(\frac{\partial \underline{A}_{\perp}}{\partial \zeta} + \nabla_{\perp} \phi \right) \right] \approx - \nabla_{\perp} \cdot (\sigma \nabla_{\perp} \phi) \quad (25)$$

This relation should be acceptably accurate when B_z is small, and it is exact either when $\nabla_{\perp} \sigma = 0$ or when the system is axisymmetric. The approximation is obviously poor for cases, such as a large-amplitude hose instability, in which $|J_{b\perp}|/|J_{bz}|$ is appreciable.

When the $\underline{A}_{\perp}$ terms are discarded from $\nabla_{\perp} \cdot (\sigma \underline{E}_{\perp})$ and from $\underline{E}_{\perp}$, Eqs. (18) through (22) reduce to

$$\nabla_{\perp}^2 (\omega + \phi) + \frac{\partial E_z}{\partial \zeta} = - \frac{4\pi}{c} (J_{bz} + \sigma E_z) \quad (26)$$

$$\nabla_{\perp}^2 \left(\frac{\partial \omega}{\partial z} - \frac{\partial \omega}{\partial \zeta} + \frac{\partial \phi}{\partial z} \right) = - \frac{4\pi}{c} [\nabla_{\perp} \cdot (\sigma \nabla_{\perp} \phi) + \frac{\partial J_{bz}}{\partial z}] \quad (27)$$

$$E_z = - \left(\frac{\partial \omega}{\partial \zeta} + \frac{\partial \phi}{\partial z} \right) \quad (28)$$

$$\underline{E}_{\perp} = - q \nabla_{\perp} \omega \quad (29)$$

This set is similar in form to the equations derived earlier by Lee¹ using the Lorentz gauge and the further assumption that $\partial/\partial z \approx -\partial/\partial \zeta$. However, the $\partial E_z/\partial \zeta$ term in Eq. (26) and the $\partial/\partial z$ terms in Eqs. (27) and (28) account for displacement current and time-variation contributions that were previously neglected, and the definitions of ω and ϕ are altered by the new gauge in Eq. (8).

II. Effect on Hose Growth

The new terms in Eqs. (26) through (29) have been neglected in most earlier treatments of the resistive hose instability. To determine whether this previous work is seriously incorrect, we have calculated small-amplitude hose growth rates for ATA-like beams using the time-independent equations of Lee and the new set. This work uses numerically-generated axisymmetric equilibria with a realistic radial and axial structure and calculates the axial growth of single-frequency modes by using the multicomponent beam dynamics model of Sharp, Lampe, and Uhm.² As expected, the time-dependent terms make effectively no change in wave growth in the charge-neutral tail

region of the beam. However, growth of perturbations in the non-neutral beam head for the new equations can be as much as a factor of ten higher than in comparable cases using the time-independent set. While not insignificant, this difference is not great enough to invalidate previous hose predictions, and there is no plan to implement the new equations in the LLNL propagation code EMPULSE.

References

1. E. P. Lee, The New Field Equations, Lawrence Livermore National Laboratory, Livermore, CA, UCID-17826 (1978).
2. W. M. Sharp, M. Lampe, and H. S. Uhm, "Multicomponent model of the resistive hose instability," Phys. Fluids **25**, 1456 (1982).