Annual Progress Report
CTR Plasma Engineering Studies*

for

October 1, 1978 - September 30, 1979

MASTER

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by

George H. Miley
Principal Investigator

Fusion Studies Laboratory
Nuclear Engineering Program
University of Illinois
Urbana, Illinois 61801

*Work supported by the U. S. Department of Energy under contract
EY-76-S-02-2218

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Abstract

The current plasma engineering studies report on three major areas of fusion reactor development. *Plasma engineering studies of the field-reversed mirror (FRM)* have focused on stability, start-up, and fusion product heating and leakage. A Monte Carlo technique has been developed to study high-energy fusion product transport in the FRM. The stability studies involve use of a perturbation theory applied to orbits calculated with the "SUPERLAYER" code. Studies of the *reversed-field pinches (RFP)* have centered around development of a 1-D dynamic MHD code which is designed to investigate enhanced transport, cold particle fueling, fusion product heating, and stability limits. Rotation effects in the *field-reversed theta pinch (FRTTP)* have been examined as a preliminary step in understanding its potential use in a reactor concept such as the *moving plasmoid heater (MPH)*, also briefly examined here. Studies of *fusion-product transport effects in tokamaks* include plasma heating, blister-induced first wall erosion, and "ash" buildup limitations on burn time. Finally, other *mirror systems studies* have been concerned with both first-wall bombardment and plasma buildup during neutral beam injection.

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I. Introduction

The objective of the CTR Plasma Engineering project at the University of Illinois is to help develop the plasma systems data base necessary for realistic conceptual fusion reactor studies. The effort this year has largely concentrated on plasma systems for field-reversed configurations including the field-reversed mirror (FRM), reversed-field pinch (RFP), and the field-reversed theta pinch (F RTP). As described in Section II, the FRM studies have involved the development of a state-of-the-art global systems zero-dimensional model. However, elaborate sub-sections of this model (actually involving separate calculations that provide input to the global model) include a Monte-Carlo study of fusion product heating, refueling calculations, and an evaluation of stability limits. Results from this work have been applied to an EPRI conceptual reactor study at Illinois (SAFFIRE) and some parts have also been used in FRM studies at LLL. Of particular significance are the discoveries that fusion product heating is more efficient than originally envisioned and that steady-state operation may potentially be achieved by a combination of diamagnetic currents and refueling.

The RFP and F RTP studies (Section III) are currently focused on specific aspects of the plasma systems of these concepts. A unique 1-D dynamic MHD model of the RFP has been developed to study burn extension by refueling and also to establish appropriate operating regimes within limits imposed by both global and local instabilities. Preliminary results have been used to supplement earlier RFP reactor studies at LASL and plans for applications to the ZT-40 experiment are underway.

The discussion of F RTP rotation mechanism is viewed as a first step in the development of a more comprehensive plasma systems model for this concept. Possible embodiment in a reactor of the moving plasmoid heater (MPH) has been considered and is briefly discussed.

The operation of TFTR will usher tokamaks into a new era where fusion products become an important component in plasmas. For the past renewal contract periods we have been considering ways to accurately include fusion products (fps) into tokamak plasma systems modeling. This includes high-energy (fp) transport, thermalization, and ash buildup. In Section IV, some recent results on high-energy modeling of importance to proposed experiments in TFTR are reviewed. Further, projections to potential wall bombardment effects, especially blistering, in reactors are discussed. Finally, the effect of electric fields on ash buildup is discussed. It is pointed out that such fields can play an important role in ash buildup, suggesting even the use of applied fields for partial control.

In Section V, studies of neutral beam injection into mirror systems are described. Two specific aspects are considered: wall bombardment due to charge-exchange and the dynamics of plasma buildup. A Monte-Carlo technique has been developed for the charge-exchange studies, and this code has been used for studies of first-wall effects in MFTF. The plasma buildup studies employ a new code that allows both spatial and energy resolution. Preliminary studies of 2X-IIB size plasmas confirm the importance of energy effects near the outer edge of the plasma, and further applications of this code to devices like MFTF and the end plugs of TMX are planned.

Finally, for the convenience of persons interested in possibly using one or more of the codes employed in the studies, brief descriptions of them are given in Section VI. In most cases, once a code is fully developed it will be available in package form (including a code description, input-output description and sample problem). Persons interested in obtaining code packages should contact the principal investigator.

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II. Plasma Engineering of the Field-Reversed Mirror (FRM)

The Field-Reversed Mirror (FRM) has attracted wide attention as a potentially attractive compact fusion reactor. Indeed conceptual studies at the U. of Illinois (resulting in the SAFFIRE design [1]*) and by a joint LLL-GA-PG&E team have confirmed that such a power plant would have many advantages. One of the most obvious, and potentially most important, would be the reduction in capital cost involved in construction of small plants. This could be particularly crucial during early stages of fusion power development since it would allow a wider number of utilities to become involved in fusion from the start. Still, the ability to stack FRM modules together would provide considerable flexibility in plant size as required for a larger central power station.

The main weakness in the FRM concept is the skimpiness of both the experimental and theoretical data base. The major thrust of the present work has been to provide a sounder basis for plasma systems analysis of FRM concepts.

As described in Sec. II.1, a comprehensive global code describing the plasma systems aspects of the FRM has been developed. (This code has been used extensively for the EPRI supported SAFFIRE reactor studies at Illinois and to a lesser extent for select calculations of FRM concepts at LLL). This global code uses an equilibrium magnetic configuration based on the Hill's vortex model. Within this framework it self-consistently calculates reversal current requirements, pressure balance, and plasma transport.

This global analysis relies upon auxiliary calculations to describe four important aspects of FRM plasma systems. These are: fusion-product

*NOTE: For convenience, in this and the following sections, reference listings appear at the end of each section. Thus, the numbering system starts over from one in each section. For example, see p. 4 for Ref. 1.

heating and ash buildup (Sec. II.2); refueling (Sec. II.3); stability considerations (Sec. II.4); and neutral-beam start-up requirements (Sec. II.5). The first three aspects -- fusion-product behavior, refueling, and stability -- represent unique characteristics of the FRM due to its combination of small size and important finite gyro-radius effects. Consequently, they have consumed major study efforts that have resulted in separate codes in these respective areas. The final area, namely, the start-up of the FRM, represents what is perhaps the most crucial aspect of FRM feasibility. Indeed current Beta-II experiments at LLL are designed to study this problem. For our part, a fairly elaborate hybrid particle-fluid code is under development to study the start-up. This is described here briefly. Also some highly simplified results, used for initial guidance in the study that assume buildup by conventional neutral-beam injection, are presented.

Reference

1. Fusion Studies Laboratory (University of Illinois), "Exploratory Studies of High-Efficiency Advanced-Fuel Fusion Reactors," EPRI Research Project 645-1, Annual Report (August, 1979). Also see G. H. Miley, et al., "Preliminary Design of a Self-Sustained Advanced-Fuel Field-Reversed Mirror Reactor - SAFFIRE," Trans. Am. Nuc. Soc., 30, (1978), p. 47.

II.1 The Zero-Dimensional Global Plasma Model - FRMOD

II.1.1. Introduction

As discussed earlier, the encouraging experimental results regarding field-reversal [1-5]* together with the fact that the FRM offers a small-size alternative to conventional magnetic fusion concepts (e.g. the tokamak) caused the FRM to come under intense investigation in order to determine its reactor potential. To facilitate this work at Illinois, a globally averaged point model of the steady-state FRM plasma was developed. This model (FRMOD) is based upon the Hill's vortex [6] representation of the FRM equilibrium and uses its simple analytical nature to reduce the balance equations for particles, energy, and pressure to volume averaged, zero-dimensional form. In accordance with the Hill's vortex analogy, then, the magnetic field $\vec{B}(r,z)$ is given by:

$$\vec{B}(r,z) = \begin{cases} -3/2 B_0 \left[\left(\frac{rz}{\kappa^2} \right) \hat{r} + (1-R^2 - r^2) \hat{z} \right], & R \leq 1 \\ -3/2 B_0 \left[\left(\frac{rz}{\kappa^2 R^5} \right) \hat{r} - \left(\frac{2}{3} - \frac{2}{3R^3} + \frac{r^2}{R^5} \right) \hat{z} \right], & R > 1 \end{cases}, \quad (\text{II.1.1})$$

and, from pressure balance, the self-consistent plasma density $n(r,z)$ is therefore, found to be:

$$n(r,z) = \begin{cases} n_0 r^2 (1-R^2) + n_c, & R \leq 1 \\ n_c, & R > 1 \end{cases}. \quad (\text{II.1.2})$$

Here, based on the good thermal conductivity of the plasma, a uniform temperature has been assumed, i.e.,

* See p. 37 for the references for this section.

$$T = \begin{cases} T_i, T_e & , & R \leq 1 \\ T_c & , & R > 1 \end{cases} \quad (\text{II.1.3})$$

In these equations, $R = (r^2 + z^2/\kappa^2)^{1/2}$; $\kappa = z_{\text{max}}/R_{\text{HV}}$ = elongation factor; r, z are dimensionless coordinates normalized to the radius of the separatrix, R_{HV} ; B_0 is the vacuum magnetic field strength; T_i, T_e are, respectively, the ion and electron temperatures in the closed-field region; and T_c is the temperature of the cold flowing plasma on the open field lines.

In the course of the reactor studies, many additional features were incorporated into the FRMOD code, and it is now one of the most complete FRM plasma models available. In addition to the consistent fp treatment, it also includes the following:

1. The effects of cold neutral gas (that refluxes from the wall or streams in through the beam ports) charge-exchanging and being ionized by the hot plasma, thus representing a power drain and a particle source to the closed field region.
2. Heat conduction losses to the cold, flowing plasma on the open field lines, that is introduced to reduce the cold neutral flux and to help "scrape off" the thermalized fps [7].
3. A model of neutral beam deposition that includes the effects of beam spreading from the source to the plasma and the beam shape, both of which are important in the FRM plasma due to its small size.
4. The ability to evaluate both near-term (D-T) and more advanced (D-³He, Cat-D) fuel cycles.
5. A variety of diffusion laws (including an "anomalous" diffusion rate) that allow different loss rates to be investigated, since the actual loss mechanism in the FRM is not yet known.
6. An optional ion cyclotron resonance heating source, that allows refueling via low energy neutral beams or pellet injection to be considered.

This section explains the details of the FRMOD code in order to illustrate the physics and some of the more crucial assumptions that went into its development, and it is organized in the following manner. Section II.1.2 discusses the major assumptions and their implications in the code. Section II.2.3 defines the terms in the balance equations and goes into their subsequent reduction to globally "averaged" form. Section II.2.4 presents the means of coupling the fp heating and ash buildup, predicted by the Monte Carlo code, into the FRMOD calculations, and section II.2.5 takes a general look at the zero-dimensional form of the balance equations in order to establish some expected scaling laws for the FRM system.

II.1.2 Physics Assumptions

As must be done in any fusion reactor study, a number of assumptions have been made concerning the scale-up of the FRM plasma parameters from current experimental values to fusion reactor conditions.

The first assumption regards the stability of the FRM configuration. FRM-related (e.g., the field-reversed theta pinch) experiments seem to exhibit gross MHD stability (attributed to finite Larmor radius, FLR, effects), but they are eventually disrupted by an $m = 2$ rotational instability that sets in after a long quiescent period [3, 5]. There are indications, however, that the rotation can be prevented by imposing a quadrupole barrier field on the plasma [1, 2, 4]. Therefore, the overall reactor designs presented here include a quadrupole field, but the effect of this azimuthal asymmetry is neglected in the plasma model. The code does, however, ensure that the necessary FLR stabilization is present by choosing the plasma size on the basis of the stability factor S , which is defined as the ratio of typical gradient lengths (i.e., $B/\vec{\nabla}B$, $n/\vec{\nabla}n$, etc.) to the

background ion gyroradius (ρ_i) in the vacuum field. In experiments, this ratio typically lies between five and fifteen; consequently, these limits on S are also chosen for this study. The conventional plasma radius is not, however, a convenient means of relating the plasma size to typical gradient lengths for the Hill's vortex model. Therefore, a more useful relation has been developed as follows. The magnetic field increases from the field null to $3/2$ of its vacuum value at a radial distance equal to $R_{HV}/3$, while the density is decreasing from its peak to the open field value. (Here R_{HV} is the radius of the closed Hill's vortex region). Thus, $R_{HV}/3$ provides the same estimate of the gradient lengths, for the vortex model, that the plasma radius gives in the experiments. For all of the results presented here, then, $S \equiv R_{HV}/3\rho_i$; and, to ensure FLR stability, the plasma size, which is subsequently set equal to $3\rho_i S$, is determined by choosing a value for S that lies in the stable range. This fact, together with the quadrupole field mentioned earlier, has led to the assumption that the plasmas considered in this study are stable.

The second general assumption involves the plasma energy and particle loss rates. Because FRM experiments give little indication as to the actual particle loss mechanism in the plasma, particle diffusion is normally assumed to be "near-classical" in the code, with the electrons being confined classically, and the ions being lost anomalously due to their large orbits. (Classically, of course, electrons and ions are lost at the same rate.) This sets up an electric field (found by equating the steady-state electron and ion currents perpendicular to the field lines) which is equal to:

$$\vec{E}_\perp = \frac{D_{\perp e} \vec{\nabla} n_e - Z D_{\perp i} \vec{\nabla} n_i}{n_e \mu_{\perp e} - Z n_i \mu_{\perp i}} \quad (II.1.4)$$

Here $n_i(e)$, $D_{\perp i}(e)$, and $\mu_{\perp i}(e)$ are, respectively, the ion (and electron) densities, classical diffusion coefficients, and mobilities; and $Z = n_e/n_i$ being the average ion charge. The cross field flux, $\vec{\Gamma}_i$, can, therefore, be expressed in terms of an "ambipolar" diffusion coefficient, $D_{\perp A}$, and is given by:

$$\vec{\Gamma}_i = n_i \mu_{\perp i} \vec{E}_{\perp} - D_{\perp i} \vec{\nabla} n_i = - D_{\perp A} \vec{\nabla} n_i, \quad (\text{II.1.5})$$

where the above form of the electric field can be used to show that:

$$D_{\perp A} = Z \left[\frac{n_i \mu_{\perp i} D_{\perp e} - n_e \mu_{\perp e} D_{\perp i}}{Z n_i \mu_{\perp i} - n_e \mu_{\perp e}} \right]. \quad (\text{II.1.6})$$

This expression can be simplified by remembering that for a strong magnetic field:

$$\mu_{\perp i} D_{\perp i} = - \frac{T_i}{Z T_e} \mu_{\perp i} D_{\perp e}, \quad (\text{II.1.7a})$$

$$\mu_{\perp e} \ll \mu_{\perp i}; \quad (\text{II.1.7b})$$

therefore, the ambipolar diffusion coefficient can finally be written in the following manner:

$$D_{\perp A} = D_{\perp e} \left(1 + \frac{T_i}{T_e} \right). \quad (\text{II.1.8})$$

This corresponds to the value of the diffusion coefficient that is used to evaluate the reference case, "near-classical" loss rates in the code, with the energy confinement time presently being set equal to the particle confinement time. (The latter assumption is used because thermal conduction losses are included in a separate calculation, based upon a model that is discussed later.) The results are consequently thought to provide a more optimistic look at the FRM than may later be justified by experiment;

therefore, a provision is also made in the code to allow the use of more pessimistic loss estimates, in order to determine how the reactor parameters would be affected if anomalous diffusion is found.

A final general assumption is that the steady-state FRM configuration can be sustained by simply balancing the particle and energy loss rates in such a way that the plasma pressure profile is maintained. This means that the reversal current must be supplied entirely by the plasma itself. If the configuration is a fluid equilibrium, this is automatically true due to diamagnetic effects; however, in practice, a fluid model does not apply over the whole FRM plasma because fluid theory breaks down near the field null. Fortunately, the large ion orbits should result in a fairly continuous ion drift motion (hence reversal current) across the null region, but studies have shown [8] that the electrons will tend to cancel this ion current in a small portion of the plasma, very near the field null, where they also begin to exhibit "large orbit" motion and are thus free to drag up to the ion drift velocity. This can be prevented by introducing additional ion species into the plasma with charges and velocities that differ from reversal current carriers. The added drag from these "secondary" ions then prevents the electrons from catching up with the reversal current carriers.

It is thought that fps will serve as the secondary ions in the present reactor designs; therefore, a steady-state equilibrium should result if the plasma pressure profile is maintained. Supplemental work has thus focused on identifying an effective means of refueling the plasma, without drastically disturbing the plasma pressure distribution. Indications are that this can be done either by using low energy neutral beams for refueling [9]

with additional energy being supplied by ion cyclotron resonance heating (ICRH), if necessary, or by using cold pellet injection to refuel [10] and balance energy losses completely with ICRH. The second option is very attractive for the present application because the pellets (since they contain virtually no energy) do not affect the plasma pressure profile (which drives the reversal current). Furthermore, due to the good cross field conductivity in the FRM, the ICRH should be redistributed across the hot region with a minimal change in the temperature distributions. Thus, the combination of ICRH and cold pellet refueling should allow the FRM plasma to be sustained without disrupting the equilibrium configuration.

On the basis of these general assumptions, FRMOD is limited to considering stable FRM equilibria, with diffusive loss rates, that are maintained by neutral beam, or cold pellet, injection (supplemented by ICRH when necessary). The details of this model, and the more specific assumptions, are discussed in the next section.

II.1.3 The Steady-State Balance Equations

II.1.3.1 Particle Balance.

In the FRM, the condition for steady-state particle balance can be expressed in the following way:

$$\left[\begin{array}{c} \text{Injection} \\ \text{source rate} \end{array} \right] + \left[\begin{array}{c} \text{Fusion} \\ \text{source rate} \end{array} \right] + \left[\begin{array}{c} \text{Cold neutral} \\ \text{source rate} \end{array} \right] = \left[\begin{array}{c} \text{Plasma} \\ \text{leakage rate} \end{array} \right] + \left[\begin{array}{c} \text{Fusion} \\ \text{burn-up rate} \end{array} \right]$$

Here the injection source is in the form of either low-energy neutral beams or cold pellets; the fusion source is due to thermal fp deposition in the closed field region; the cold neutrals are a result of streaming through the beam ports and cold gas reflux from the walls; the plasma leakage is calculated in accordance with diffusion theory, and the fusion burn-up is found from the fusion reaction rate. This type of balance must hold for each individual

species in a steady-state plasma; therefore, it not only specifies the required refueling rate but also allows the fusion ash concentrations to be determined. In the following discussion, then, the equilibrium density and temperature profiles in the closed field region,

$$n(r,z) = n_0 r^2 (1 - r^2 - z^2/\kappa^2) ,$$

$$T = T_i, T_e ,$$

are used to reduce these individual species balances to volume-averaged "point" form.

The first source term in the above balance, which only applies to fuel species, is written in the following manner:

$$\left[\begin{array}{l} \text{average injection} \\ \text{source rate} \end{array} \right] = \frac{1}{V} \int_V d^3r \dot{n}_j = \frac{4}{35} \dot{n}_{oj} \quad (II.1.9)$$

Here $V = (4/3)\pi\kappa R_{HV}^3$ is the plasma volume, and $\dot{n}_{oj}$ is the required refueling rate for the j -th injected ion, specified in terms of the "peak" density, n_0 . For the Hill's vortex case, the average density is, therefore, found to be 4/35 of the peak value. (This may seem small, but it is due to the fact that n_0 does not correspond to the physical maximum in the density, which is actually equal to $n_0/4$.) The injection rate necessary to match the required particle source is then found by evaluating the beam trapping efficiency and requiring that the trapped fraction be equal to the required refueling.

The second source term, due to the thermal fp deposition, generally applies only to ash species, but it can also represent a source of fuel (as in the case of the D-D reactions where the resulting fps include a T and a ^3He). It is, of course, related to the fusion reaction rate and is given by:

$$\left[\begin{array}{c} \text{average fusion} \\ \text{source rate} \end{array} \right] = \frac{1}{V} \int_V d^3r \sum_k f_{RP,k} n_1 n_2 \langle \sigma v \rangle_k \delta_{jk} = \frac{4}{35} \dot{n}_{oj,fp} \quad (\text{II.1.10a})$$

In this equation, the volume averaging only affects the density terms and thus introduces a factor of 64/3465. Therefore, the form which $\dot{n}_{oj,fp}$ takes on the code is:

$$\dot{n}_{oj,fp} = \frac{16}{99} \sum_k f_{RP,k} n_{o1} n_{o2} \langle \sigma v \rangle_k \delta_{jk} \quad , \quad (\text{II.1.10b})$$

where $n_{o1} n_{o2} \langle \sigma v \rangle_k$ is the k-th fp production rate; δ_{jk} is used to represent the fact that these fps are only included if they correspond to the species being considered; and $f_{RP,k}$ represents the fraction of these fps that are deposited in the closed field region. This fraction is estimated on the basis of the Monte Carlo calculation (using the method discussed in section II.1.4) and thus allows an accurate picture of the actual fp deposition to be obtained.

The last source term in the particle balance is written in the following manner:

$$\left[\begin{array}{c} \text{average cold neutral} \\ \text{source rate} \end{array} \right] = \left(\frac{n_g v_g}{4} \right)_j \frac{S_A}{V} \left(1 - \frac{\sigma_{cxj}}{2\sigma_{Tj}} \right) = \frac{4}{35} \dot{n}_{oj,cg} \quad (\text{II.1.11a})$$

Here $(n_g v_g / 4)_j$ is the incident flux of the j-th type of cold gas particles; S_A / V is the plasma surface to volume ratio which is equal to $(3/2R_{HV}) (\kappa^{-1} + \epsilon^{-1} \sin^{-1} \epsilon)$, where $\epsilon = \sqrt{1 - \kappa^{-2}}$; and the factor of $1 - \sigma_{cxj} / 2\sigma_{Tj}$ (where σ_{cxj} and σ_{Tj} are the charge exchange and total ionization cross sections for these cold neutrals, respectively) is included to correct for that fraction of the cold gas interactions where no net particles are deposited because the subsequent hot neutral is lost. The incident flux of these cold particles is then specified by assuming Franck-Condon dissociation (which results in

a velocity, v_g , corresponding to 3-eV) and allowing the background gas density to be chosen by the user. Hence, the form which $\dot{n}_{oj, cg}$ takes on in the code (in CGS units) is:

$$\dot{n}_{oj, cg} = \frac{7.895 \times 10^6}{R_{HV}} (\kappa^{-1} + \frac{\sin^{-1} \epsilon}{\epsilon}) \frac{n_{gj}}{\sqrt{m_j}} (1 - \frac{\sigma_{cxj}}{2\sigma_{Tj}}), \quad (II.1.11b)$$

where n_{gj} is thus the background density of the j -th type of cold neutrals, and m_j is their mass in AMU.

The first loss term, corresponding to leakage, is found by integrating the diffusion flux over the surface of the plasma and is, therefore, given by:

$$\left[\begin{array}{c} \text{average plasma} \\ \text{leakage rate} \end{array} \right] = \frac{1}{V} \int_{S_A} (-D_{\perp} \vec{\nabla} n) \cdot d\vec{s} = \frac{4}{35} \frac{n_{oj}}{\tau_p}. \quad (II.1.12a)$$

In this representation, τ_p is defined to be the average particle confinement time for the plasma, and it is, consequently, equal to:

$$\tau_p = \frac{R_{HV}^2}{7(4+\kappa^{-2})} D_{\perp}^{-1}. \quad (II.1.12b)$$

Furthermore, $D_{\perp}$ is the cross field diffusion coefficient, which is normally chosen to have the ambipolar value discussed in section II.1.2. Optional anomalous and completely classical values can also be chosen; however, for all of these cases $D_{\perp}$ is expressed in the following general form:

$$D_{\perp} \propto \rho_B^2 \nu_{cB}. \quad (II.1.13)$$

Here ρ_B is the characteristic step size (normally chosen to be the gyroradius in the vacuum field), and ν_{cB} is the corresponding collision frequency (evaluated using the average plasma density).

The various confinement options in the code result from choosing different values for the characteristic step size and collision frequency, and correspond to the following three cases:

- Classical, where the step size is the electron gyroradius in the vacuum field and the collision time is that for electron-ion interactions.
- Near-classical, where the classical diffusion coefficient is modified by the ambipolar factor $(1+T_i/T_e)$.
- Anomalous, where the step size is the ion gyroradius in the vacuum field, and the collision time is that for ion-ion interactions.

The corresponding values for the particle confinement times are thus given by

$$\text{Classical: } \tau_p = \frac{5.980 \times 10^{10}}{(4+\kappa^{-2})} \frac{R_{HV}^2 B_0^2}{Z^2} \frac{\sqrt{T_e}}{n_{oi}} \quad (\text{II.1.14})$$

$$\text{Near-classical: } \tau_p = \frac{5.980 \times 10^{10}}{(4+\kappa^{-2})} \frac{R_{HV}^2 B_0^2}{Z^2} \frac{\sqrt{T_e}}{n_{oi}(1+T_i/T_e)} \quad (\text{II.1.15})$$

$$\text{Anomalous: } \tau_p = \frac{1.981 \times 10^9}{(4+\kappa^{-2})} \frac{R_{HV}^2 B_0^2}{\sqrt{m_i} Z^2} \frac{\sqrt{T_i}}{n_{oi}} \quad (\text{II.1.16})$$

In these equations CGS units are used except for B_0 which is the vacuum field strength in kG, $T_i(e)$ which are the ion (and electron) temperatures in keV, and m_i which is the average ion mass in AMU.

The other loss term in the particle balance, corresponding to burn-up, is evaluated by integrating the fusion reaction rate over the plasma volume and is found to be:

$$\left[\begin{array}{c} \text{average fusion} \\ \text{burn-up rate} \end{array} \right] = \frac{1}{V} \int_V d^3r \sum_{\ell} n_{\ell} n_j \langle \sigma v \rangle_{\ell j} = \frac{4}{35} \frac{n_{oj}}{\tau_{Fj}} \quad (\text{II.1.17})$$

Here τ_{Fj} represents the inverse of the sum of the various fusion reactivities involving the j -th species, $\langle\sigma v\rangle_{lj}$, and is therefore:

$$\tau_{Fj} = \frac{99}{16} \left[\sum_l n_{ol} \langle\sigma v\rangle_{lj} \right]^{-1} \quad (II.1.18)$$

The fractional burn-up, f_{Bj} , of a fuel species in the FRM can then be conveniently written, in terms of this average fusion time and the particle confinement time, as:

$$f_{Bj} = [1 + \tau_{Fj}/\tau_p]^{-1} \quad (II.1.19)$$

These individual terms are then recombined to produce the final form of the particle balance equation, which is written in terms of the "peak" density, and is, consequently, given by:

$$\dot{n}_{oj} + \dot{n}_{oj,fp} + \dot{n}_{oj,cg} = \frac{n_{oj}}{\tau_p} + \frac{n_{oj}}{\tau_{Fj}} \quad (II.1.20)$$

This is the form of the equation that is found in the code. It is then used either to evaluate the required refueling rate (for the case of injected particles) or to determine the steady-state ash density (for fps which are not reusable as fuel). In the latter case the injection, cold gas, and burn-up terms are all zero; thus, the steady-state ash concentration is found by setting the leakage rate equal to the thermal fp deposition rate and is, consequently, written as:

$$n_{oj} = \dot{n}_{oj,fp} \tau_p \quad (II.1.21)$$

II.1.3.2 Energy Balance

Separate energy balances are included in the code for ions and for electrons:

$$P_{Bi} + P_{Fi} + P_{IC} = P_{ie} + P_{Li} + P_{cx} + P_{HC} \quad (II.1.22)$$

$$P_{Be} + P_{Fe} + P_{ie} = P_{Le} + P_{Br} + P_{Cy} \quad (II.1.23)$$

Here $P_{Bi(e)}$ and $P_{Fi(e)}$ correspond to the injection and fusion power absorbed by the ions (and electrons), $P_{Li(e)}$ corresponds to the ion (and electron) leakage powers, P_{IC} represents the ICRH input, P_{ie} is the power exchanged between the ions and the electrons, P_{cx} represents the charge-exchange losses, P_{HC} the heat conduction losses, and P_{Br} and P_{Cy} the bremsstrahlung and cyclotron radiation. In the following discussion, these power terms are expressed as volume averaged power densities, by making use of the previous results for the average value of $n = 4n_0/35$ and $n = 64n_0^2/3465$, and they are specified in terms of CGS units, with the following exceptions: the energies and temperatures are in keV, the masses are in AMU, and the magnetic field strength is in kG.

The first two terms in these balance equations, $P_{Bi(e)}$, are evaluated by considering each injected species separately. The individual injection sources, $\dot{n}_{oj}$ (determined from Eq. II.1.20), and the specified injection energies, E_{oj} , allow the total trapped beam power, P_B , to be found; and it is given by the following equation:

$$P_B = 1.831 \times 10^{-10} \sum \dot{n}_{oj} E_{oj} \quad (II.1.24)$$

The fraction of this power that is subsequently absorbed by the electrons is then determined by integrating over the resulting slowing down distribution:

$$P_{Be} = \sum_j \int_{E_T}^{E_{oj}} f_j(E) \left\langle \frac{dE}{dt} \right\rangle_{e,j} dE \quad (II.1.25)$$

Here $f_j(E)$ is the steady-state slowing down distribution of the j -th injected species, $\langle dE/dt \rangle_{e,j}$ is the electron drag term for this species (cf. Sivukhin [11]), and the $E_{oj}(T)$ are the corresponding beam injection (and thermal) energies. In the code this energy integral is performed by defining a group structure, such that $\langle dE/dt \rangle_{e,j}$ is constant over a group, and representing the integration as a sum. Then, by further assuming that the "peak to average" ratio is the same for superthermal particle densities as for the background plasma, the following form of the trapped beam power absorbed by the electrons is found:

$$P_{Be} = 1.831 \times 10^{-10} \sum_j \sum_{g=1}^G n_{oj}^g \left\langle \frac{dE}{dt} \right\rangle_{e,j}^g \quad (II.1.26)$$

In this equation, n_{oj}^g is the "peak" density of the j -th particles in the g -th energy group, $\langle dE/dt \rangle_{e,j}^g$ is the electron drag for the particles in this group, and G is the total number of groups.

The value of the group density, n_{oj}^g , is determined by recognizing that, for continuous slowing down, the total energy lost by the particles in a group can also be written as the group width, ΔE_g , times the rate at which particles are being transferred to the next lower group. Then, in a steady-state situation, the group density will be

$$n_{oj}^g = \frac{\Delta E_g}{\left\langle \frac{dE}{dt} \right\rangle_j^g} \dot{n}_{oj} \quad (II.1.27)$$

where $\langle dE/dt \rangle_j^g$ is now the total drag (electron plus ion) for the j -th particles at the g -th group energy, and $\dot{n}_{oj}$ is again the j -th particle source rate. The remainder of the trapped energy must necessarily go to the ions; therefore:

$$P_{Bi} = P_B - P_{Be} \quad (II.1.28)$$

The next two terms considered are the fusion power inputs to the plasma P_{Fi} and P_{Fe} . They are also found by evaluating separate contributions from each k-th type (in this case charged fp) species. Therefore, the fusion power released in these particular fps must first be found by:

$$P_{F,k} = \int_V d^3r \left[n_1 n_2 \langle \sigma v \rangle_k + n_1 \sum_{g=1}^G n_2^g \langle \sigma v \rangle_{21k}^g + n_2 \sum_{g=1}^G n_1^g \langle \sigma v \rangle_{12k}^g \right] E_k. \quad (\text{II.1.29})$$

Here $n_1 n_2 \langle \sigma v \rangle_k$ is the k-th background reaction rate, and the superthermal group structure for the injected ions, together with the beam-background reaction rates $\langle \sigma v \rangle_{21k}$ and $\langle \sigma v \rangle_{12k}$ has been used to include the fusion during slow down. The plasma heating will, of course, only come from the fraction of this power that is deposited in the closed field region (which is written here as $f_{RE,k}$, the fraction of retained energy), and this fraction is estimated by using the method discussed in section II.1.4 (again based on a Monte Carlo calculation).

The specification of $P_{Fi(e)}$ is completed by splitting the resulting, retained fusion power, for each different charged fp, between the ions and electrons in the appropriate manner. This is done using a convenient formula given by Conn and Kesner [12], which describes the fraction of each fps energy going to ions during slow down, $f_{i,k}$, as:

$$f_{i,k} = \frac{1}{3} \frac{E_c}{E_0} \left\{ \ln \left[\frac{E_c - \sqrt{E_c E_0} + E_0}{E_c + 2\sqrt{E_c E_0} + E_0} \right] + 2\sqrt{3} \tan^{-1} \left[\frac{2\sqrt{E_0} - \sqrt{E_c}}{\sqrt{3}E_c} + \frac{\pi}{\sqrt{3}} \right] \right\}. \quad (\text{II.1.30})$$

In this equation E_0 is the fp birth energy, and E_c is the critical energy, where the ion drag is equal to the electron drag, which is given by:

$$E_c = 14.8 m_k T_e \left[\frac{\sum_j n_j Z_j^2 / m_j}{n_e} \right]^{2/3}. \quad (\text{II.1.30a})$$

Here $m_j(k)$ are the masses of the background ions (and fps), $n_j(e)$ are the background ion (and electron) densities, Z_j is the background ion charge, and T_e is the electron temperature. (Since the remainder of each fp's energy $(1-f_{i,k})$ goes to the ions, both $P_{Fi(e),k}$ have been specified and are found by multiplying the k-th charged fp production rate by the fraction of its retained energy that is given to the ions or electrons). The final form of the expressions for the total fp heating, $P_{Fi(e)}$, is determined by summing the individual contributions over all charged fps, giving:

$$P_{Fi} = 2.959 \times 10^{-11} \sum_k \dot{n}_{ok} f_{i,k} f_{RE,k} E_k, \quad (II.1.31)$$

where (for electrons) the $f_{i,k}$ factor is replaced by $(1-f_{i,k})$, and the $\dot{n}_{ok}$ term represents the total "peak" production rate for the k-th charged fps, namely,

$$\dot{n}_{ok} = n_{o1} n_{o2} \langle \sigma v \rangle_k + n_{o1} \sum_{g=1}^G n_{o2}^g \langle \sigma v \rangle_{21k}^g + n_{o2} \sum_{g=1}^G n_{o1}^g \langle \sigma v \rangle_{12k}^g. \quad (II.1.32)$$

The last two related terms are the plasma leakage powers P_{Li} and P_{Le} . For the ions, this power loss is defined in the following manner:

$$P_{Li} = 4.578 \times 10^{-10} \sum_j \left(\frac{n_{oj}}{\tau_p} + \frac{3}{5} f_{Lj} \frac{n_{oj}}{\tau_{Fj}} \right) T_i. \quad (II.1.33a)$$

In this equation, n_{oj}/τ_p represents the diffusive losses, where each particle transports $3kT_i/2$ out of the system and requires an additional kT_i to diffuse through; and $(3/5)f_{Lj}n_{oj}/\tau_{Fj}$ represents the fusion burn-up losses (which are not diffusive and thus remove only $3kT_i/2$). The loss factor f_{Lj} is then defined as:

$$f_{Lj} = 1 - \left[\frac{\sum_i n_i n_j \langle \sigma v \rangle_{ij} \sum_k f_{RE,k} / 2}{\sum_i n_i n_j \langle \sigma v \rangle_{ij}} \right] \quad (\text{II.1.33b})$$

Here the $f_{RE,k}$ are again the fraction of the k-th fps energy (resulting from the $\langle \sigma v \rangle_{ij}$ reaction) that is retained in the plasma, and f_{Lj} , therefore, accounts for the portion of the "fused" background energy that is carried out of the system by unconfined fps.

The electron leakage is similarly defined to be:

$$P_{Le} = 4.578 \times 10^{-10} \left[\frac{n_{oe}}{\tau_p} + \dot{n}_{oe} \right] T_e \quad (\text{II.1.34})$$

In this case, n_{oe}/τ_p again accounts for the diffusive losses (where n_{oe} is the "peak" electron density, but the second term, $\dot{n}_{oe}$, now represents the effect of fusion burn-up on the electron energy balance. It is equal to the number of electrons that must be lost to compensate for charge losses carried by escaping superthermal fps; therefore:

$$\dot{n}_{oe} = \frac{16}{99} \sum_k (1 - f_{RP,k}) Z_k n_{o1} n_{o2} \langle \sigma v \rangle_{12k} \quad (\text{II.1.35})$$

where $f_{RP,k}$ was defined earlier as the fraction of the k-th charged fps that are not lost, and Z_k is their charge. Finally, since both of their losses are diffusive in nature, the electrons each carry off $5kT_e/2$.

The remaining terms in the electron energy balance are straightforward. The first is the energy exchange term, P_{ie} , which is calculated in accordance with the standard formula for Maxwellian distributions (averaged using the appropriate density weighting):

$$P_{ie} = 8.952 \times 10^{-23} \frac{n_{oe}}{T_e^{3/2}} \sum_j \frac{n_{oj} Z_j^2}{m_j} (T_i - T_e) \quad (\text{II.1.36})$$

The others are the radiation terms. The bremsstrahlung losses, P_{Br} , are found using the standard expression (also appropriately averaged), namely,

$$P_{Br} = 9.884 \times 10^{-26} n_{oe} \sum_j n_{oj} Z_j^2 T_e^{1/2} \quad (II.1.37)$$

However, the cyclotron losses, P_{Cy} , are modified slightly (to account for the high β of the FRM) by introducing a $(1-\beta)/\beta$ factor, as suggested by Miley [13]. This factor is included in the average over the plasma volume by recognizing that $(1-\beta)$ represents the magnetic pressure in the plasma (given by the square of the Hill's vortex field strength), and that β represents the plasma pressure (which is also specified for the Hill's vortex case). The appropriate averaging factor for the cyclotron emission is thus found from:

$$\frac{1}{V} \int_V d^3r \left(\frac{1-\beta}{\beta} \right) n^2 = \frac{16}{3465} n^2, \quad (II.1.38)$$

and the final expression for the cyclotron radiation losses is, therefore:

$$P_{Cy} = 1.154 \times 10^{-27} K_c d n_{oe}^2 T_e^2 \quad (II.1.39)$$

Here $d = (1+n_i T_i / n_e T_e)(1+T_e/204)$ and K_c is the plasma reabsorption coefficient which is calculated in accordance with the Krajcik formula [14] assuming cylindrical geometry.

Two remaining terms involve the ion power balance. The first, P_{IC} , is the ICRH input which is set equal to the amount of power that is required to satisfy the ion energy balance. This can be done because ICRH replaces the ion temperature in the dependent variable set, when it is used; and, as a result, the ion temperature then becomes an independent value specified by the user, with the required ICRH input being calculated by the code. The details of ICRH coupling into the plasma do not enter into the ion

power balance but are, instead, included via an estimate of ICRH efficiency that is also specified by the user. The second simple term is the estimate of charge-exchange losses, P_{cx} , which corresponds to hot charge-exchange neutrals which are not reionized in the plasma. The power they carry out is thus equal to the average rate at which they leave the plasma (discussed when considering the cold neutral source rate) multiplied by their average energy, $3kT_i/2$. Therefore, this power loss is represented by the following expression in the code:

$$P_{cx} = \frac{2.168 \times 10^{-3}}{R_{HV}} (\kappa^{-1} + \frac{\sin^{-1} \epsilon}{\epsilon}) \sum_j \frac{n_{gj}}{\sqrt{m_j}} \frac{\sigma_{cxj}}{2\sigma_{Tj}} T_i \quad (II.1.40)$$

The final term in the ion power balance is P_{HC} , which represents the heat conduction losses. It is evaluated by using the boundary layer model that is shown in Fig. II.1.1. Here a uniform temperature profile, which is expected in the hot region, is assumed to be the case; except for a thin boundary layer (that is taken to be one hot ion gyroradius thick) where the density becomes uniform and the temperature profile takes on the shape of the Hill's vortex pressure distribution. The total conduction losses through this region are then given by:

$$Q = \int_{S_A} -k_{\perp} \vec{\nabla} T \cdot d\vec{S} = \frac{16\pi k_{\perp} R_{HV} \kappa (4 + \kappa^{-2})}{15 x_0 (2 - 5x_0 + 4x_0^2)} T_i \quad (II.1.41)$$

In this equation, x_0 is the thickness of the boundary layer (in units of the vortex radius, R_{HV}); T_i is the ion temperature in the hot region, and $k_{\perp}$ is the cross field thermal conductivity:

$$k_{\perp} = 6.310 \times 10^{-10} \frac{\sqrt{m_i} n_B n_c z^2}{B^2 \sqrt{T_B}} \quad (II.1.42)$$

Figure II.1.1. Boundary layer model used to calculate heat conduction losses.

Thus n_B , T_B , and B are the density, temperature, and magnetic field strength in the boundary layer; n_c is the density of the cold, open field plasma; and m_i and Z are the average ion mass and charge, respectively.

In the spirit of the zero-dimensional nature of the model, $k_\perp$ is evaluated for "average" properties of the boundary layer, which are taken to be those in the region of the steepest temperature gradient (where most of the conduction losses occur). For the Hill's vortex pressure profile, this corresponds to the region near $r = 1$, and $z = 0$; therefore, the "average" properties are found to be:

$$B^2 = \frac{3}{2} B_0^2 [1 - 4x_0 + \frac{26}{3} x_0^2] , \quad (\text{II.1.43a})$$

$$n_B = n_0 x_0 [2 - 5x_0 + 4x_0^2] , \quad (\text{II.1.43b})$$

with T_B left to be chosen by the user, since the results are fairly sensitive to its value. The conduction loss estimate is then completed by substituting these "average" boundary layer properties into the expression for the thermal conductivity and converting the losses to a volume averaged form. This results in the following expression (used in the code) for the heat conduction losses, namely:

$$P_{HC} = \frac{5.392 \times 10^{-19}}{(1 + 4x_0 + \frac{26}{3}x_0^2)} \frac{(4 + \kappa^{-2})}{R_{HV}^2 B_0^2} \sqrt{m_i} n_{oi} n_c Z^2 \frac{T_i}{\sqrt{T_B}} . \quad (\text{II.1.44})$$

II.1.3.3 The Remaining Balance Equations

The final three balance equations used in the code, are those for pressure, charge, and background composition. The pressure balance condition is satisfied by choosing the "peak" densities, n_{oj} , for each species and every group, such that the sum of the individual pressure contributions is equal to the total "peak" pressure, P_0 , which is given by:

$$P_o = \frac{9B_o^2}{32\pi} (4 + \kappa^{-2}) \quad (II.1.45)$$

In the code, the pressure balance then takes on the following form:

$$5.588 \times 10^{13} (4 + \kappa^{-2}) B_o^2 = n_{oe} T_e + \sum_j (n_{oj} T_i + \sum_{g=1}^G n_{oj}^g E_j^g). \quad (II.1.46)$$

Similarly, the charge balance requirement is written as:

$$n_{oe} = \sum_j Z_j \left(n_{oj} + \sum_{g=1}^G n_{oj}^g \right), \quad (II.1.47)$$

and the balance on the background ion composition is written as:

$$n_{oi} = \sum_j \left(n_{oj} + \sum_{g=1}^G n_{oj}^g \right). \quad (II.1.48)$$

This completes the equations necessary to specify the steady-state plasma parameters. They represent a system of five non-linear algebraic equations, which are solved by choosing values for the stability factor, S ; the elongation factor, κ ; the magnetic field strength, B_o ; the various injection energies, E_{oj} ; the open field density, n_c ; the boundary layer temperature, T_B ; and fuel species density ratios. The remaining plasma parameters are then calculated in the code by using a standard non-linear algebraic equation solver (QNWT) from the MSL subroutine library, which solves the equations by reducing the balance equation residuals to small values. The "solution" criterion is normally chosen to be when the square root of the sum of the squares of these residuals is less than 10^{-4} .

II.1.3.4 Neutral Beam Deposition

Because of the small size of the FRM plasma, the effects of beam spreading and the actual shape of the beam source are important in evaluating

the beam trapping efficiency in the FRM. Therefore, a detailed calculation of the beam trapping efficiency is included in FRMOD. This calculation begins by assuming that the beam has the following shape when it reaches the plasma:

$$J_0(y,z) = \frac{I_s}{\pi D^2 \tan \theta_y \tan \theta_z} \exp \left[- \left(\frac{y}{D \tan \theta_y} \right)^2 - \left(\frac{z}{D \tan \theta_z} \right)^2 \right] \quad (\text{II.1.49})$$

Here I_s is the total beam current (particle/sec), D is the distance from the source to the plasma, $\theta_y(z)$ are the beam divergence angles in the y (and z) directions, and the coordinate system is defined such that the beam is travelling in the positive x direction. Once the beam contacts the plasma surface it begins to be attenuated, and the beam current at any distance along the beam path x inside the plasma is, therefore, given by:

$$J(x,y,z) = J_0(y,z) \exp \left[- \langle \sigma \rangle_T \int_{-x_s}^x dx' n(x',y,z) \right] \quad (\text{II.1.50})$$

In this equation, $\langle \sigma \rangle_T$ is the total beam ionization cross section, $n(x,y,z)$ is the background density, and x_s is the value of x (which is a function of y and z for the present case) where the beam first makes contact with the plasma. The corresponding geometry is shown in Fig. II.1.2, for the beam coming in at an angle β relative to the reference coordinate frame which defines the plasma parameters.

The beam trapping efficiencies, for individual species, are then calculated by dividing the plane perpendicular to the beam direction (the y - z plane in this case) up into small unit areas, $\Delta y \Delta z$, and summing the transmitted beam for each area element over a rectangular region that encompasses the projection of the plasma in this plane. The result is

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Figure II.1.2. Beam injection geometry that matches the loss rate profile, thus maintaining the diamagnetic reversal current.

divided by the total injected current to find the trapping efficiency, which can thus be written as:

$$n_{BTj} = 1 - \frac{\Delta y \Delta z}{I_s} \sum_{i=1}^N J_{oi} \exp \left[- \langle \sigma \rangle_{Tj} \int_{x_{s1}}^{x_{s2}} dx' n_i(x') \right] . \quad (II.1.51)$$

In this form, J_{oi} represents the source current evaluated for the appropriate values of y and z (using Eq. II.1.49); $n(x')$ represents the background density along the path length (found from the same values of y and z , the vortex density profile, and the known rotation angle β); and the values of $x_{s1(2)}$ correspond, respectively, to the incident (and exit) values of x , for the beam, at this y and z location.

The total trapping cross sections, used in evaluating these trapping efficiencies, are made up of the individual components for the electron, and the ion, interactions and are thus given by:

$$\langle \sigma \rangle_T = \langle \sigma \rangle_{ii} + \langle \sigma \rangle_{ie} + \langle \sigma \rangle_{cx} , \quad (II.1.52)$$

where the $\langle \sigma \rangle_{ii}$ term represents ion-ionization, the $\langle \sigma \rangle_{ie}$ term represents electron-ionization, and the $\langle \sigma \rangle_{cx}$ represents charge exchange. Because the background particles are assumed to have Maxwellian distributions, the averages indicated by the angle brackets can be evaluated as:

$$\langle \sigma \rangle = \frac{2\beta_0}{\sqrt{\pi} v_0^2} e^{-\beta_0^2 v_0^2} \int_0^\infty e^{-\beta_0^2 v^2} \sigma(v) v^2 \sinh(2\beta_0^2 v_0 v) . \quad (II.1.53)$$

In this equation, $\beta_0 = \sqrt{m_B/2kT}$ is the inverse of the average background particle velocity, $v_0 = \sqrt{2E_0/m_0}$ is the velocity of the injected ion, $m_{B(0)}$ are their respective masses, and v is the relative velocity between the particles. The cross sections, $\sigma(v)$, are then taken from Ref. 15, and

the integral over the relative velocity is performed using 26 point Gauss-Laguerre quadrature.

II.1.4 Fusion Product Heating Estimates

The estimates of fp heating and ash deposition that are used in the FRMOD code are based upon actual MCFRM calculations for the fp behavior in typical FRM plasmas. Due to the method discussed here, these estimates not only cover a wide range of plasma parameters but are also quite accurate. This is made possible through the use of the closed-field absolute confinement limits discussed in Ref. 16. The actual fp energy and particle deposition in the closed field region must always lie between these two limits; therefore, the fp energy and particle deposition are both represented as fractions of the difference between the two confinement limits in the code. These fractions (referred to as f_{CDE} for energy and f_{CDP} for particles) are, in general, related to the ratio of the fp slowing down time in the hot, closed field region, τ_H , to that in the cold, open field region, τ_C . When τ_H/τ_C is large, the fp deposition should be near the lower, closed-field confinement limit (i.e., f_{CDE} and f_{CDP} will be near zero); but, as τ_H/τ_C becomes smaller, fps which are marginally confined (i.e., not completely confined within the closed field region) will also be able to deposit a larger fraction of their energy in the hot region, and fp energy deposition should, therefore, approach the upper, absolute confinement limit (i.e., f_{CDE} and f_{CDP} will be approaching one).

A similar result is expected as a function of the dimensionless birth energy, E_D , of the fps (where the dimensionless units are those noted earlier, namely, $(1/m) \cdot (qB_0 R_{HV}/C)^2$). This is because the dimensionless

energy is a measure of the time which fps spend in the hot (relative to the cold) region. For large dimensionless birth energies, the fps that are marginally confined spend most of their time outside the closed field region, and thus are expected to deposit little of their energy there (corresponding to f_{CDE} and f_{CDP} near zero); but, as the dimensionless birth energy becomes smaller, these marginally confined fps spend more time in the closed field region, and the values for f_{CDE} and f_{CDP} should approach unity.

The final parameter that is expected to influence f_{CDE} and f_{CDP} is the number of slowing down times required to reach thermal energies. This number is related to the parameter $\log(E_0/T_i)$, and if its value is large (corresponding to many slowing down times) the marginally confined fps will have more opportunities to interact with the cold plasma region than if its value is small. These fps are, therefore, expected to deposit more of their energy in the open field region because they will tend to be "dragged" outward as they slow down. Consequently, a large value of $\log(E_0/T_i)$ is thought to lead to smaller values of f_{CDE} and f_{CDP} .

The influence which these three parameters (τ_H/τ_C , E_D , and $\log(E_0/T_i)$) have on the fractions f_{CDE} and f_{CDP} has caused them to be combined to correlate the values for these fractions that are calculated using the MCFRM code. The results of the correlation are shown in Figs. II.1.3 and II.1.4, where, for both of these fractions, the correlation parameter turns out to be equal to $\log[(\tau_H/\tau_C)E_D/\log(E_0/T_i)]$. As is illustrated by the figures, this correlation provides an excellent approximation of the actual fp energy and particle deposition over a wide range of parameters. Using the fitted curves shown in the figures, the values for f_{CDE} and f_{CDP} are:

Figure II.1.3. Correlation of MCFRM results for fp energy deposition in the closed field region.

FRACTION OF CONFINEMENT
DIFFERENCE

Figure II.1.4. Correlation of MCFRM results for fp particle deposition in the closed field region.

$$f_{CDE} = -0.42887 \log \left[\frac{\tau_H}{\tau_C} \frac{E_D}{\log(E_0/T_i)} \right] + 0.25044 \quad , \quad (II.1.54a)$$

$$f_{CDP} = -0.35011 \log \left[\frac{\tau_H}{\tau_C} \frac{E_D}{\log(E_0/T_i)} \right] + 0.34500 \quad . \quad (II.1.54b)$$

These results can subsequently be used to determine the fraction of fp energy and particles retained in the closed field region by making use of the closed-field confinement fraction f_{CF} and the absolute confinement fraction f_{AC} . The fraction of retained energy, f_{RE} , is then found from:

$$f_{RE} = f_{CF} + f_{CDE} (f_{AC} - f_{CF}) \quad , \quad (II.1.55)$$

and the fraction of retained particles, f_{RP} , is similarly given by:

$$f_{RP} = f_{CF} + f_{CDP} (f_{AC} - f_{CF}) \quad , \quad (II.1.56)$$

These fractions, f_{RE} and f_{RP} , will, of course, be different for each charged fp; however, because the confinement limits, f_{AC} and f_{CF} , are general (i.e., not restricted to a specific type of fp) the retained energy and particle fractions for fps other than those mentioned above can also be approximated using this method.

II.1.5 Scaling Laws

Because the size of the FRM is restricted due to the FLR stabilization, it has somewhat different parametric scaling than is normally found in a fusion reactor. It is therefore useful to consider how the plasma properties are related to the specified parameters: S , κ , and B_0 . From the expression for the peak plasma pressure, and the vacuum magnetic pressure ($B_0^2/8\pi$), an average value for β can be defined for the Hill's vortex FRM model, namely,

$$\beta = \frac{9}{35} (4 + \kappa^{-2}) \quad . \quad (II.1.57)$$

The spherical vortex is thus found to make the most efficient use of the applied magnetic field, with elongated plasmas having a β that is reduced by as much as 20% for large values of κ .

This means that the fusion power does not scale linearly with the plasma length, but instead takes on the following form:

$$P_f \propto \frac{(4+\kappa^{-2})^2 \kappa S^3 B_0 \langle \sigma v \rangle_F}{\sqrt{T_i} (1+f_e T_e/T_i)^2} \quad (II.1.58)$$

Here $\langle \sigma v \rangle_F$ is the total fusion cross section, $T_{i(e)}$ are the ion (and electron) temperatures, and f_e is the ratio of electrons to ions, n_e/n_i . Similarly, since leakage and bremsstrahlung are the dominant loss mechanisms, the plasma energy multiplication factor Q (defined as the fusion power divided by the total injection power) is given approximately by (for a plasma where fp heating is small):

$$Q \propto \frac{\langle \sigma v \rangle_F \sqrt{T_e}}{415 \sqrt{\frac{f_e}{m_i}} \frac{(4+\kappa^{-2})}{S^2} (1+\frac{T_i}{T_e}) (1+f_e \frac{T_e}{T_i}) + T_e} \quad (II.1.59)$$

Here the first term in the denominator represents the relative effect of plasma leakage, and the second term (T_e) corresponds to bremsstrahlung losses.

The optimum energy multiplication then occurs when the stability factor is as large as possible and (since $T_e \sim T_i$) falls near the temperature which corresponds to the maximum in the ratio $\langle \sigma v \rangle_F \sqrt{T_e}/(C_L + T_e)$, where C_L is now just a constant equal to $9845/S^2$ for a spherical D-³He plasma. Furthermore, as seen from Eq. II.1.58, larger output powers can be obtained by either

elongating the plasma or by increasing the vacuum magnetic field strength. The most attractive of these two alternatives is elongation; however, the plasma cannot be made too long because it becomes susceptible to tearing instabilities. To assure that this problem is avoided, the maximum value for the elongation factor is typically taken to be five.

These general scaling laws are useful, but the actual designs may vary from the predicted dependence due to the presence of fp heating and ash buildup. Their influence will tend to reduce the optimum temperature because as the system approaches ignition the Q value begins to peak at lower plasma temperatures where the fusion power input is just beginning to drive the system. It is clear, however, that the FRM is destined to be a small device (due to the limitation on both κ and S) with the only economy of scale coming from the possibility of stacking individual FRM plasmas together to form a larger power multiple-celled system.

II.1.6. Conclusion

The FRMOD code has been used extensively in design studies for SAFFIRE, a D-³He reactor study sponsored by the Electric Power Research Institute (EPRI). Results from this study, which serve to illustrate the capability of the FRMOD code, are detailed in the Annual Report for EPRI RP-645-1, prepared in August 1979, and will be available from EPRI. Some results are also outlined in Table 1 of the following section II.2 in the present report.

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II.2 Fusion Product Behavior in FRM.

Recent experimental results [1,2][†] have generated increased interest in the field-reversed mirror (FRM) concept. Field-reversed plasma layers have been observed to persist for times equivalent to a few hundred sound transit times across their major radii, confirming that phenomena such as finite-gyroradius effects provide stabilization to this configuration which would otherwise be predicted to be unstable based on classical MHD theory.[3] Because of these results, several conceptual FRM reactor studies were begun [4-6]. Initially these studies assumed that the FRM was strictly a beam driven system, with the plasma heating supplied by high-energy (>200-keV) neutral-beam injection. The small plasma size, which is dictated by stability requirements, was initially thought to rule out significant heating by fusion products (fps). However, recent work [7] has shown that this is not the case. Nearly 50% of the 3.5-MeV alphas produced in a D-T FRM plasma, which has a stability factor S (the ratio of the plasma radius to the fuel ion gyroradius) equal to 5, are absolutely confined. Physically this occurs because, in the FRM configuration, the fps do not have a conventional mirror "loss cone." Consequently, the fps are able to slow down and heat the closed-field plasma better than might be anticipated.

This observation is encouraging; however, the majority of the fps, while absolutely confined, still sample both the open and the closed-field plasma (see Fig. 1) making it impossible to evaluate their energy deposition via conventional Fokker-Planck techniques. Therefore, a Monte Carlo plasma model (MCFRM)* has been developed. It couples the Hill's spherical vortex representation of a field-reversed equilibrium (Figure 1) with a

[†] See p. 56 for the references of this section.

* A brief description of the MCFRM code and its availability appears in Sec. VI.

Table 1. Principle Plasma Parameters for Two SAFFIRE Reference Designs

	CASE A	CASE B
Stability Factor, $S \equiv \frac{R_{HV}}{2\rho_i}$	5	5
Vacuum Field (T)	6.0	6.0
Hill's Vortex Radius, R_{HV} (m)	0.236	0.236
Plasma Volume (ℓ)	55	55
Ion, Electron Temperature (keV)	120,99	120,92
Ion, Electron Density (10^{20} m^{-3})	3.6,5.4	3.5,5.3
Steady-State Ash Buildup (%)	0	12
Energy Multiplication (Q)	11.9	4.5
Fusion Power (MW)	1.4	1.1
Net Power Output* (MW)	0.70	0.45
Overall Efficiency (%)	50	40

* Assumes thermal, direct, injection, ion cyclotron efficiencies of 40, 60, 80, 80%.

Figure 1. Hill's Spherical Vortex Representation of Field-Reversal.

Monte Carlo treatment of Coulomb scattering, and thus provides a complete picture of fps from birth through their thermal diffusion. A related technique has previously been used in the case of neutral-beam injection into tokamak plasmas [8]; however, this is the first time such methods have been applied to the problem of fp behavior in FRMs. The incorporation of drag and scattering effects allows the code to address both energy deposition and ash buildup questions, with the latter being a crucial factor in considering steady-state FRM operation.

II.2.1 The Computer Model

II.2.1.1 The Background Plasma

As originally pointed out by Morse [9] and implemented by Wang and Miley [10], the steady-state FRM is expected to assume a configuration much like the Hill's spherical vortex, proposed by M. J. Hill [11] in 1894 while studying vortex formation in fluid flow. This simple analytical model (Figure 1) compares quite favorably with actual field-reversed equilibria predicted by the plasma simulation code SUPERLAYER [12], although in many cases a more elongated plasma layer is observed. Indeed, many of the experimentally produced field-reversed plasmas are also elongated; therefore, the Hill's vortex model has been modified in the present code to allow for such elongation by scaling the axial dependence of the vector potential by a factor κ . Self-consistent values for the plasma size, temperatures, densities, and the magnetic field strength are then chosen based on results obtained from a zero-dimensional steady-state plasma model [13] that incorporates appropriate energy and particle balances. The fps are treated as test particles in this background plasma; therefore, the effect of their

energy deposition (as well as contribution to ash buildup) on the background plasma is included via an iterative process. This involves using the fp heating results from MCFRM to obtain new background plasma parameters from the zero-dimensional model. These background parameters are then compared to those which were input to MCFRM, and the process is repeated until convergence is obtained.

Since the characteristic motion of the fp orbit is defined by a time interval corresponding to the gyro-period, the maximum timestep that can be used in the code is typically ~ 8 orders of magnitude less than the fp slowing-down time. Thus, in order to reduce the computation time to a tractable amount, the background densities are artificially enhanced by a factor of $\sim 10^5$ in the code. This should not affect the results because the scattering time is still long compared to a timestep, and the fps tend to orbit in well defined regions of configuration space over long time intervals. Therefore, as long as the fps are able to traverse this entire region before their energy has changed significantly, the calculated energy deposition profiles should be accurate.

II.2.1.2 The Particle Pusher

Because the ratio of the gyroradius of a typical fp to the plasma radius is near unity, a conventional guiding center model is not applicable to fp motion in the FRM. Instead, the particle equations of motion must be solved in detail. A modified leapfrog algorithm [14] is used for this, and the resulting difference equations are then:

$$x_i^{n+1} = x_i^n + v_i^{n+1/2} \Delta t, \quad (\text{II.2.1})$$

$$\vec{v}_i^{n+3/2} = \vec{v}_i^{n+1/2} + \frac{q\Delta t}{m} \left[\vec{E}_i^{n+1} + \frac{1}{2c} \left(\vec{v}_i^{n+3/2} + \vec{v}_i^{n+1/2} \right) \times \vec{B}^{n+1} \right]_i, \quad (\text{II.2.2})$$

where

X_i^n is the i^{th} position coordinate at the n^{th} timestep,

$\vec{v}_i^n$ is the i^{th} velocity component at the n^{th} timestep,

Δt is the timestep,

m is the test particle mass,

q is the test particle charge,

E_i is the i^{th} component of the electric field,

$\vec{B}^{n+1}$ is the magnetic field at the $n+1^{\text{th}}$ timestep, and

c is the speed of light.

These equations can be made explicit by solving for $\vec{v}_i^{n+3/2}$, and they also have the important property of exactly conserving energy. The calculation then proceeds by using the X_i^{n+1} in Equ. (II.2.1) to find the magnetic field $\vec{B}^{n+1}$, which is used to advance the velocity components. Currently, no electric field effects are included since fp energies are expected to be much larger than any plasma potentials which may develop. The particles also receive a random scatter in velocity space which is determined in accordance with the scattering model described in the next section. This "scattered" velocity vector is then used to advance the particle positions during the next timestep.

II.2.1.3 The Scattering Algorithm

A simple drag model is adequate to determine fp energy deposition; however, the desire to answer questions concerning ash buildup and removal

necessitate a full Coulomb scattering treatment. Because the conventional mirror loss cone is virtually eliminated, the background plasma is expected to assume a Maxwellian velocity distribution. Therefore fp thermalization is described by the well-known Spitzer model [15] for a test particle slowing down in a uniform background of Maxwellian field particles; i.e.,

$$\left\langle \frac{dv_{||}}{dt} \right\rangle = - \frac{\Lambda_D}{v_f^2} \left(1 + \frac{m}{m_f} \right) G \left(\frac{v}{v_f} \right), \quad (\text{II.2.3})$$

and the associated parallel and perpendicular diffusion coefficients

$$\left\langle \frac{dv_{||}^2}{dt} \right\rangle = \frac{\Lambda_D}{v} G \left(\frac{v}{v_f} \right), \quad (\text{II.2.4})$$

$$\left\langle \frac{dv_{\perp}^2}{dt} \right\rangle = \frac{\Lambda_D}{v} \left[\phi \left(\frac{v}{v_f} \right) - G \left(\frac{v}{v_f} \right) \right]. \quad (\text{II.2.5})$$

Here

$$\Lambda_D = \frac{8\pi e^4 n_f Z_f^2 Z^2 \ln \Lambda}{m^2},$$

$$v_f^2 = \frac{2kT}{m_f},$$

$$\phi(x) = \text{erf}(x),$$

$$G(x) = \frac{\text{erf}(x) - (2/\sqrt{\pi}) x \exp(-x^2)}{2x^2},$$

v_f, v = field and test particle velocities, respectively,

m_f, m = field and test particle masses, respectively

Z_f, Z = field and test particle charges, respectively,

n_f = field particle density, given by Equation II.1.2, and

$\ln \Lambda$ = the Coulomb logarithm.

Because scattering is dominated by small angle events, the Gaussian distributions which describe the random change in $v_{\perp}$ and $v_{\parallel}$ are approximated by triangular distributions with the same standard deviation as their Gaussian counterparts. The change in the local velocity components due to drag and scattering during a timestep is then:

$$\Delta v_{\perp 1} = \left[3 \left\langle \frac{dv_{\perp}^2}{dt} \right\rangle \Delta t \right]^{1/2} (\rho_1 - \rho_2) , \quad (\text{II.2.6})$$

$$\Delta v_{\perp 2} = \left[3 \left\langle \frac{dv_{\perp}^2}{dt} \right\rangle \Delta t \right]^{1/2} (\rho_3 - \rho_4) , \quad (\text{II.2.7})$$

$$\Delta v_{\parallel} = \left[6 \left\langle \frac{dv_{\parallel}^2}{dt} \right\rangle \Delta t \right]^{1/2} (\rho_5 - \rho_6) - \left\langle \frac{dv_{\parallel}}{dt} \right\rangle \Delta t . \quad (\text{II.2.8})$$

Here p_1 - p_6 are the random numbers used to generate the triangular distributions, and Δt is the timestep. These changes in the local velocity frame are then transformed back into the inertial frame of the particle pusher, resulting in the following expressions for the scattered velocity components:

$$v_{xs} = \left[\frac{\Delta v_{\perp 1} (v_x v_z / v) - \Delta v_{\perp 2} v_y}{v \sin \alpha} \right] + v_x \left[1 + \frac{\Delta v_{\parallel}}{v} \right], \quad (\text{II.2.9})$$

$$v_{ys} = \left[\frac{\Delta v_{\perp 1} (v_y v_z / v) + \Delta v_{\perp 2} v_x}{v \sin \alpha} \right] + v_y \left[1 + \frac{\Delta v_{\parallel}}{v} \right], \quad (\text{II.2.10})$$

$$v_{zs} = -\Delta v_{\perp 1} \sin \alpha + v_z \left[1 + \frac{\Delta v_{\parallel}}{v} \right], \quad (\text{II.2.11})$$

Here $v_x(s)$, $v_y(s)$ and $v_z(s)$ are the initial (scattered) velocity components and $\sin \alpha = (1 - v_z^2/v^2)^{1/2}$.

II.2.1.4 Particle Initialization and Termination

Because a representative picture of the overall fp energy and ash deposition in the FRM is desired, a starting grid is set up which chooses particles from all regions of phase space in the most efficient way possible. This is accomplished by forcing each MCFRM particle to have the same weight, i.e., represent the same number of fps. In configuration space, the fp birth distribution is proportional to $n^2 \langle \sigma v \rangle d^3r$ and a

corresponding spatial grid is chosen. Similarly in velocity space, since the fps are born isotropically, the equal weighting requirement leads to a special grid.

The initialization procedure thus ensures that all regions of phase space are represented equally, and the calculations are done in parallel. Histories are terminated when particles hit the wall, are carried out the mirror throat, or slow down to an energy less than $\sim 3T_i$. Below this cutoff energy, which typically corresponds to 300-keV, the fps are either designated as contributing to "ash" in the closed-field region, or as a component of the leakage plasma that originates in the open-field region and is subsequently diverted out the mirrors.

As the fps slow down they are energetically restricted to move in a region which is defined by their instantaneous canonical angular momentum, P_θ , and their energy, E . This region is defined by the following relation[10]:

$$E = \frac{1}{2} \left[\frac{P_\theta - \psi}{r} \right]^2, \quad (\text{II.2.12})$$

where ψ is the magnetic flux function given by

$$\psi = \begin{cases} -3/4 r^2 (1-R^2), & R \leq 1 \\ 1/2 r^2 (1-R^{-3}), & R > 1 \end{cases} \quad (\text{II.2.13})$$

In the above equations ψ , P_θ and E are dimensionless in units of $R^2_{HV}B_0$, $qB_0R_{HV}^2/C$, and $q^2B_0^2R^2_{HV}/mc^2$, respectively. Equations II.2.12, II.2.13 make it possible to determine where in configuration space an fp may reside, given a specific value for E and P_θ . This information is used in Figures 2a and 2b, where energetically allowed regions are shown for fps* at the 300-keV cutoff energy in a 23.6-cm, 6.0-T spherical plasma. These particular values of P_θ (-0.0557, -0.0016) are chosen because they represent, respectively, the greatest value fps can have and still be confined completely within the closed-field region for this particular plasma (Figure 2a), and the greatest value of P_θ which is absolutely confined for the same fps and plasma parameters (Figure 2b). As illustrated in the figures, the greater P_θ becomes, the less time the fps spend in the closed-field region. This allows ash buildup to be qualitatively related to the distribution of fps in P_θ as they reach the cutoff energy (Figure 6).

To be considered part of the ash buildup fraction, fps must have P_θ less than -0.0557, i.e., be closed-field confined. This limit is chosen because fps with $-0.0557 \leq P_\theta \leq -0.0016$, while still absolutely confined, sample both the open and closed-field regions (Figure 2b), and actually have very little effect on the total fusion power. In fact, nearly 80% of the fusion power is found to be produced in the cross-hatched region of Figure 2a, which is only accessible to fps having $P_\theta < -0.0557$. In addition, because these fps continually sample the open-field plasma, they are lost on

*Since some applications of this work have involved D-³He fueled systems the remaining references to fps in this section will imply a 3.67-MeV alpha or a 14.8-MeV proton. This should not be taken to indicate that the general technique is limited to these ions, however.

Figure 2a. Energetically Allowed Region for the Maximum P_θ Which is Closed-Field Confined. Nearly 80% of the Total Fusion Power Comes From Within its Inner Boundary. Positions are in Units of R_{HV} .

Figure 2b. Energetically Allowed Region for the Maximum P_θ Which is Absolutely Confined. Even Though These fps are Still Confined They Sample Only a Small Portion of the Closed-Field Region. Positions are Again in Units of R_{HV} .

a much shorter timescale than their closed-field confined counterparts. They are therefore neglected when calculating the average ash buildup fraction for us in the zero-dimensional model.

II.2.2 Model Verification

Several test cases were run to establish the validity of this method for modeling Coulomb scattering events, and two of them are presented here. The first was a much simplified case for which an analytical solution was possible. It involved turning off the energy exchange during scattering and watching an initially monoenergetic, unidirectional burst of test particles relax towards isotropy in an infinite background plasma composed of like particles with a Maxwellian distribution. Results from this test case are shown in Figure 3 where the Monte Carlo solution for 500 particles is plotted against the analytical solution. MCFRM not only approaches the correct asymptotic values for $E_{\perp}$ and $E_{\parallel}$, but also shows excellent agreement with the expected analytical time constant $\tau_{90^{\circ}}$.

A second crucial test case involved a direct comparison with Fokker-Planck code FOKN . [14] Since FOKN, like other available Fokker-Planck codes, cannot be applied to the full 3-D problem of interest, another simplified test case was devised. It involved the thermalization of a delta function source of alpha particles at 379-keV in an infinite background plasma of deuterium and electrons at 80-keV. The alpha distribution function calculated by MCFRM was then compared to that found by FOKN at several times prior to thermalization. The results of two of these comparisons are shown in Figure 4. The instantaneous distribution function

Figure 3. Comparison Between Analytical and MCFRM solutions for the Problem of a Unidirectional Monoenergetic 80-keV Deuterium Beam Approaching an Isotropic Distribution in a Maxwellian Background of Deuterium With Average Energy = 80-keV.

Figure 4. Particle Distribution Function. Fig. (a) Corresponds to a Time $\sim 0.03 \tau_{90^\circ}$ While by (b), Equal to $0.18 \tau_{90^\circ}$, the Distribution has Nearly Relaxed to a Maxwellian.

calculated by MCFRM is in excellent agreement with that predicted by FOKN at both times prior to thermalization, and it is also approaching the correct asymptotic Maxwellian distribution.

II.2.3 Typical Results

The primary use of MCFRM has been in conjunction with the SAFFIRE [13] reactor design study. In the SAFFIRE design a cold flowing plasma introduced on the open-field lines (see Figure 1) serves two purposes. It shields the hot closed-field region from Q limiting, cold neutral charge exchange interactions [17], and it aids in the removal of fp ash during steady-state operation of the device. This latter effect results from the FRM's small size which allows many fps to interact with both the open and closed-field plasma. Therefore, the fractional fp energy deposition and ash buildup in the closed-field region is dependent upon the ratio of the fp slowing down time in the hot closed-field region, τ_H , to that in the cold open-field region, τ_C .

Two limiting cases are immediately evident. The first occurs when $\tau_H \ll \tau_C$. Under these conditions, all fps that are absolutely confined, i.e., not in the prompt loss fraction, deposit their energy in the closed-field region. This optimistic limit corresponds to the absolutely confined curves shown in Figure 5, and represents an upper bound on the closed-field fp energy deposition. The second limiting case occurs when $\tau_H \gg \tau_C$. Under these conditions, only fps that are confined completely within the closed-field region deposit energy there. The remainder of the fps deposit their energy in the open-field region. This pessimistic limit corresponds to the closed-field confined curves shown in Figure 5, and represents a lower bound on the closed-field fp energy deposition.

Figure 5. Fraction of fp Closed-Field Energy Deposition Compared to the Closed-Field and Absolute Particle Confinement Limits.

Figure 6. Distribution of Thermal ash per Unit P_θ vs. P_θ . This Distribution is Accumulated as the fp Slow Down and Reach the $3T_i$ Cutoff. Note the Large Fraction Which are Absolutely Confined but not Closed-Field Confined.

The actual closed-field energy deposition then lies between these two limits, and has been evaluated using MCFRM. It is shown in Figure 5 for fps in a hot plasma, having density and temperature consistent with the reference SAFFIRE designs of Table 1; surrounded by a cold plasma, with density of $2 \times 10^{13} \text{cm}^{-3}$ and temperature of 50eV. As is illustrated in the figure, over 40% of the fp energy is deposited in the closed-field region of the reference ($S=5$, 6.0-T, 23.6-cm) SAFFIRE plasma. This results in a Q value of 12, and a net power production of 0.70-MW (see Table 1 Case A), which represent a substantial improvement over the Q of 2.0 and the 0.42-MW power output associated with the same system without fp heating. However, these Q values do not include the effect of ash buildup which will necessarily accompany the increased energy deposition.

In order to evaluate the buildup of ash associated with the fp closed-field energy deposition, MCFRM was modified to include Coulomb scattering effects so that fp behavior could be accurately modeled all the way down to thermal energies. The ash buildup was then estimated by calculating the fraction of the fps which were still absolutely confined when their energy had been reduced to $\sim 3T_i$. Below this cutoff, which was typically set at 300-keV, the fps were assumed to be part of the thermal background, and their leakage was found with the zero-dimensional code. In accordance with the absolute confinement criterion for fps at 300-keV in our reference 23.6-cm, 6.0-T plasma, fps with $P_0 < -.0016$ were assigned to the ash buildup fraction. This corresponded to over 70% of the non-prompt losses; however,

a substantial fraction of these fps had $-.0557 \leq P_\theta \leq -.0016$ and, therefore, they did not accumulate in the region where the fusion power density was the greatest (see Figure 2a). This is illustrated in Figure 6 where the alpha and proton ash distributions are plotted vs. P_θ for an open-field density of $2 \times 10^{13} \text{ cm}^{-3}$ and temperature of 50-eV. Because the effect of fp ash with $-.0557 \leq P_\theta \leq -.0016$ is minimal*, the ash buildup criterion was relaxed to include only those fps which have $P_\theta < -.00557$, i.e., are completely confined within the closed-field region at the 300-keV cutoff energy.

The effect of the buildup of ash on the reference SAFFIRE plasma parameters is shown under Case B of Table 1. The steady state ash buildup of 12% reduces the plasma Q to 4.5 and the net power production to 0.45-MW. This power output is small, but the Q value is still high enough to alleviate questions concerning the viability of advanced-fuel FRM systems. In addition, several avenues are open which can lead to higher power levels. The first rests on recent experimental observations which indicate that plasma layers with S as large as 10 (vs. the S of 5 cases discussed here) may be stable.[1] Increasing S by a factor of two potentially leads to an 8 fold increase in plasma volume and power output. There is, however, a disadvantage to this approach; namely, the larger S values correspond to increasing the quantity $R_{HV} B_0$. Therefore, as shown in Figure 5, the fractional fp energy and ash deposition in the closed-field region are both increased. The net effect of a larger plasma S factor then involves a trade off between the competing influences of fp energy and ash deposition, and MCFRM studies are underway to evaluate this trade off.

*See the discussion at the end of the Particle Initialization and Termination Section, II.2.1.4.

A second route to increased power lies in the elongation of the plasma. Preliminary MCFRM studies indicate that the fractional fp energy and ash deposition are unchanged if the plasma is simply elongated with its radial dimension held constant. Because the ash deposition does not change, the power density stays nearly constant in an elongated plasma, while the absolute power level increases in direct proportion to the amount of elongation. This suggests that the net power output associated with Case B could be raised to ~1.5-MW by a simple 3/1 elongation of the plasma. Long field--reversed layers, however, are subject to tearing instabilities which limit the amount of elongation to ~5/1.

II.2.4 Conclusions

The MCFRM code is unique in that it combines the Hill's vortex model of a field-reversed equilibrium with a full Coulomb scattering treatment of the particles. Thus it is able to evaluate both fp energy deposition and their subsequent buildup as thermal ash. MCFRM has been used extensively in developing the SAFFIRE reactor design, and has made several important contributions. It initially identified the presence of a significant amount of fp heating of the closed-field plasma, and effect which greatly enhance the prospects for identifying a viable FRM reactor. Most recently it has been used to examine the buildup of fusion ash.

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II.3. Refueling of the FRM

A steady-state ("self-contained") field-reversed mirror (FRM) appears possible if: a) fresh fuel can be introduced such as to maintain the radial density profile, i.e., strong diamagnetic currents; b) a cold-plasma "shielding" layer can be maintained in the open-field line region; and c) fusion-product ash build-up can be limited. Self-contained operation appears essential in order to exploit the unique opportunity for a small-size (10-100 MWe) fusion power plant utilizing an FRM.^{(1-3)*} The present study considers use of simultaneous neutral beam and pellet fueling to achieve (a) and (b) above while other studies⁽⁴⁾ suggest that the cold-plasma shield may satisfy (c).

The equilibrium field configuration is modeled using a Hill's spherical vortex approximation (Fig. 1) while cross-field diffusion is assumed to be dominated by ion-electron collisions. For simplicity, a spatially averaged diffusion coefficient is employed. Under these assumptions, maintenance of the radial plasma density profile (consistent with pressure balance in the Hill's vortex) gives rise to a diamagnetic ("bootstrap") current adequate to supply >90% of the current necessary for field reversal.^(1,2) This reduces, or eliminates, high-energy injection requirements, significantly increasing the net energy production.

With this model, the closed field flux surfaces ψ are found to be

$$\psi = -3/4(r'^2)(1-R^2), \quad (1)$$

where $R^2 = r'^2 + Z'^2$, and r' and Z' are normalized cylindrical coordinates (Fig. 1). ψ may assume any value between $-3/16$ and 0 . From the continuity equation we find that the required fuel rate S (ions/cm³-sec) is:

$$S = (9r'^2 + 2Z'^2 - 2) \Gamma_0/a, \quad (2)$$

where Γ_0 is a constant (determined by the total plasma leakage rate) and "a" is the radius of the closed field region. Since cross-field diffusion is slow

* See p. 63 for references.

Figure 1. Hill's vortex representation of the FRM.

compared to parallel streaming, injected particles trapped on a surface quickly cover it. Consequently, Eq. (2) is integrated over the volume between flux surfaces to obtain the ideal trapping profile of Fig. 2. This result is for a reactor with a 60-kG vacuum field, $a = 10$ cm, $T_e = 60$ keV, and $T_i = 75$ keV.

Compared to a tokamak, fueling of the FRM is considerably simplified due to its smaller size and the accessibility due to the outward "squeezing" of flux surfaces at $Z = 0$ (Fig. 1). Consequently, Fig. 2 shows that fueling can be accomplished with a beam penetration of only ~ 3 cm, corresponding to the null of the nested surfaces of Fig. 1. Still, with the dense plasma in the present design, this represents a line density of $\sim 2 \times 10^{15}$ ion/cm².

Preliminary designs have been completed for neutral beam injectors to meet these requirements.⁽⁶⁾ A current of ≈ 0.2 amp at ~ 60 keV for D, injected perpendicular to the Z axis, is found to roughly match the desired source profile.

The shielding plasma (Fig. 1) appears essential to protect the closed-line region from background neutral-gas interaction.⁽⁴⁾ It will be partly formed by plasma particles and energy leaking from the closed region, but an additional particle source is necessary to obtain adequate densities. While injection along field lines from external sources is possible, current calculations suggest that ablation of injected pellets at $Z = 0$ may be most energy efficient. Assuming an ablation theory⁽⁷⁾ is applicable in the lower density - temperature "edge" plasma, we find that injection of 0.1-cm pellets at $\approx 5 \times 10^3$ cm/sec at a rate of ~ 30 /sec will establish an adequate shield plasma. Under these conditions, the pellets penetrate less than 1% of the distance to the null. Consequently, disruption of the closed region plasma is not anticipated, but this requires further study.

Fuel pellets of considerably higher velocities ($10^5 - 10^6$ cm/sec) may be used to refuel the closed-field region of the FRM. Even though these high-velocity pellets may still be only capable of penetrating 50% of the distance to the field null on the $Z = 0$ plane, calculations show that an

Figure 2. Trapping profile necessary to maintain an equilibrium density profile in the FRM. Depth is measured perpendicularly to the Z axis at Z=0; the closed-field null is at 3 cm.

inward diffusion resulting from the large density spikes formed by pellet ablation may distribute fuel ions all the way to the null. The requirement that fuel pellets penetrate to the null to provide adequate fueling appears to be conservative, and in this light, it appears that pellets containing between 10% and 30% of the plasma mass injected at velocities between 10^5 and 10^6 cm/sec will be sufficient to fuel the FRM and maintain its pressure profile if additional ion-cyclotron heating is employed (see Fig. 3).

In summary, fueling of a self-contained FRM appears feasible using a combination of low-energy neutral beams and pellets. Energies, currents, and velocities indicated are within reach of near-term technology, but steady-state injector operation will represent a major developmental challenge.

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Figure 3. Transient Density Profile After Pellet Injection ($v = 5 \times 10^5$ cm/sec).

II.4. Stability of FRM Plasma

In the field-reversed mirror, unlike the Astron concept, ^{(1,2)*} the ion gyroradii are smaller than the plasma radius (with the size factor $S \equiv B(\rho_i \nabla B)^{-1} \sim 1.5 \rightarrow 10$). The field-reversed mirror geometry (Fig. 1), which consists of closed magnetic field lines curved inward against the plasma, is unstable when the plasma is modeled by the MHD equations. However, the finite ion gyroradii are expected to cause different stability properties than MHD theory predicts. Thus it becomes necessary to study the stability of the FRM configuration by the use of the Vlasov equation.

i) Vlasov Stability Theory

An energy principle which can be used to treat FRM stability can be derived directly from the Vlasov equation by recognizing a constant of motion of an ensemble of particles and fields.⁽⁶⁾ This constant is the total energy U of a plasma and the electromagnetic field. It can be written:

$$U = \int d^3x \left\{ \frac{E^2}{8\pi} + \frac{B^2}{8\pi} + \sum_{\alpha} \int d^3v \frac{m_{\alpha} v^2}{2} f_{\alpha} \right\}. \quad (II.1)$$

Here α is the species index and E and B are written in cgs units. The requirement for perturbations $\delta \vec{E}$, $\delta \vec{B}$, and δf_{α} to be allowed on the system is that U is stationary to these perturbations, i.e., $\delta U = 0$ for all allowed perturbations. For collisionless interactions, δf is coupled to $\delta \vec{E}$ and $\delta \vec{B}$ through the perturbed Vlasov equation. The first order expression for δf yields:

* See p. 76 for references.

Figure 1. Magnetic field geometry and plasma current in the Field-Reversed Mirror. (From Ref. 1)

$$\delta^{(1)}_f = \int_{-\infty}^t dt' \frac{q}{m} (\delta \vec{E} + \frac{\vec{v}}{c} \times \delta \vec{B}) \Big|_{t'} \cdot \frac{\partial f_{0\alpha}}{\partial \vec{v}} \quad (II.2)$$

This integral is performed over the path of the particles from $t' = -\infty$ to the present. Substitution of (II.2) into the energy principle (II.1) results in an expression for the first order perturbation in energy $\delta^{(1)}_U$:

$$\delta^{(1)}_U = \int d^3x \left(\frac{\vec{B}_0 \cdot \delta \vec{B}}{4\pi} + \frac{\vec{E}_0 \cdot \delta \vec{E}}{4\pi} + \sum_{\alpha} \int \frac{m_{\alpha} v^2}{2} \delta^{(1)}_f d^3v \right) \quad (II.3)$$

In a field-reversed mirror equilibrium, with no electrostatic fields, $\vec{E}_0 = 0$ and then an integration by parts yields:

$$\delta^{(1)}_U = \int d^3x \vec{A} \cdot \left(\frac{\vec{\nabla} \times \vec{B}_0}{4\pi} - \frac{q}{c} \int f_0 \vec{v} d^3v \right) \quad (II.4)$$

Thus the first order perturbed energy is a functional of Maxwell's equation $\vec{\nabla} \times \vec{B} - \frac{4\pi}{c} \vec{J} = 0$. Hence it vanishes when the equilibrium possesses electromagnetic self-consistency. The second-order perturbation then represents the stability of the system by examination of the existence of roots to $\delta^{(2)}_U = 0$. This equation reads:

$$\delta^{(2)}_U = \int \left[\frac{\delta B^2}{8\pi} + \frac{\delta E^2}{8\pi} + \int q \delta^{(1)}_f \vec{v} \cdot \vec{E} dt' d^3v \right] d^3x \quad (II.5)$$

In the above equation, $\delta^{(1)}_f$ is for each species in the plasma. For a single ion component and electrons, (II.5) becomes explicitly:

$$\delta^{(2)}_U = \int \left(\frac{\delta B^2}{8\pi} + \frac{\delta E^2}{8\pi} + \int (\delta \vec{J}_i \cdot \delta \vec{E} + \delta \vec{J}_e \cdot \delta \vec{E}) dt' \right) d^3x \quad (II.6)$$

This is just Poynting's theorem. Thus, we have shown that the first order perturbation $\delta^{(1)}U$ vanishes as a result of Maxwell's equations, which are the equations establishing the equilibrium. The second-order variation $\delta^{(2)}U$ represents the energy flux out of the system, which, of course, corresponds to Poynting's theorem.

In a field-reversed mirror with $T_e \ll T_i$ (typically, $T_e \sim 150$ eV and $T_i \sim 100$ keV), the electron response can be modeled by treating the electrons as a cold fluid following the equation of motion

$$nq_e \left\{ \delta \vec{u}_e \times \vec{B} + \delta \vec{E} \right\} = 0, \quad (II.7)$$

and this leads to the equation $\delta \vec{J}_e \cdot \delta \vec{E} = 0$. Thus the cold electron fluid does not exchange energy with the perturbed field. This observation justifies the shorter energy principle of eq. (II.5) with only the ion component included.

The energy principle can be evaluated for a given distribution function representing an FRM equilibrium. One such equilibrium distribution which has been studied analytically and can be shown to fit computer simulations of FRM equilibria is the so-called rigid rotor distribution, which has the form:

$$f_0 = n_0 \exp[-(H - \Omega P_\theta)]. \quad (II.8)$$

For this particular distribution function, eq. (II.5) takes the form:

$$\begin{aligned} \delta^{(2)}U = \int d^3x \left(\frac{\delta B^2}{4\pi} - \frac{nq^2}{c^2} \left(\frac{m\Omega^2 R^2}{T} \right) \frac{\delta \Lambda_0^* \cdot \delta \Lambda_0}{T} \right. \\ \left. + \frac{q}{c^2} \left[\frac{(\omega\omega^* - \omega_0 \ell \Omega)}{T} \langle g^* g \rangle + \frac{i\gamma \ell \Omega}{T} \left\langle \int g^* \frac{dg}{dt} - g \frac{dg^*}{dt} \right\rangle \right] \right). \quad (II.9) \end{aligned}$$

This energy principle agrees with the thermodynamic energy principle of Rosenbluth and Sudar⁽⁵⁾ when the background density is zero. The development here, however, stresses that the electromagnetic perturbations are taken to have the nature of the variational principle. Perturbations are taken to have the form:

$$\begin{aligned}\delta B &= \vec{\nabla} \times \delta \vec{A}, \quad \delta \vec{E} = -\frac{1}{c} \frac{\partial \delta \vec{A}}{\partial t}, \\ \delta A &= \underline{\delta A} e^{-i\omega t} e^{i\ell\theta} + \underline{\delta A}^* e^{i\omega^* t} e^{-i\ell\theta}.\end{aligned}\tag{II.10}$$

In (II.9), g is an orbit integral of the perturbed lagrangian of each particle taken over its trajectory:

$$g = \int_{-\infty}^t dt' \vec{v} \cdot \delta \vec{A}(t'),$$

and $\langle X \rangle$ indicates a canonical average $\int d^3v f_0 X$. The complex frequency can be written as $\omega = \omega_0 + i\gamma$, where ω_0 and γ are real.

The stability of an FRM equilibrium which can be modeled by eq. (II.8) is then determined by examination of the roots to the RHS of eq. (II.9). It is sufficient for stability that no zeros to $\delta^{(2)}U$ exist for all perturbations. To prove that $\delta^{(2)}U = 0$ is a necessary and sufficient condition for instability requires that the operators contained in eq. (II.6) have a self-adjointness property. While this cannot be proved in general for a Vlasov operator, as shown later, special cases of self-adjointness can be found using a simplified model for the equilibrium system.

ii) Energy Principle Numerical Studies

The stability of the FRM is studied using the energy principle derived above with an equilibrium generated numerically by the SUPERLAYER code developed at Lawrence Livermore Laboratory. This code is a 2 1/2

dimensional orbit simulation code which self-consistently computes the trajectories of hot ions in a self-consistent electromagnetic field. The code allows for ionization of injected neutral beams through a Monte Carlo technique, and by calculation of the particle orbits after injection and ionization obtains a value of J_θ , the current in the azimuthal direction. The code ignores electrostatic effects, space charge, and all electron currents. Further, the currents in the transverse direction, J_R and J_Z , are assumed to be zero. This forces azimuthal symmetry and purely poloidal magnetic fields. This then leaves the following set of equations to be solved self-consistently:

$$\begin{aligned}
 -\frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial}{\partial R} (R A_\theta) \right) - \frac{\partial^2 A_\theta}{\partial Z^2} &= \frac{4\pi}{c} J_\theta, \quad A_R = 0, \quad A_Z = 0, \\
 E_\theta &= -\frac{1}{c} \frac{\partial A_\theta}{\partial t}, \quad E_R = 0, \quad E_Z = 0, \\
 \frac{d\vec{v}}{dt} &= \frac{q}{m} \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right).
 \end{aligned}
 \tag{III.1}$$

These equations can be solved by finite-difference methods, resulting in a distribution of particles which possess the necessary information to evaluate eq. (II.5).

A particular equilibrium studied using the SUPERLAYER code had an equilibrium distribution function which could be fitted to the rigid-rotor distribution of eq. (II.8). The parameters of this equilibrium were $R_0 = 12$ cm = injection mean radius, axial length = 30 cm, $T = 0.0415 m \omega_{ci}^2 R_0^2$, B_0 = external field = 5.0 kG, injection current = 25.3 kAmp, injection energy = 38 keV, ion component = 100% deuterium, maximum density $n = 9.5 \cdot 10^{13} \text{ cm}^{-3}$ at $R = 10$ cm, peak beta $B_0^{-2} 8\pi p = 3.36$, average rotation rate $\Omega = 0.651 \omega_{cio}$.

Using these parameters and the energy principle given earlier, one can determine the stability of the above computer simulation equilibrium state by the application of trial functions.

For $\ell = 0$ modes, representing axisymmetric perturbations to the equilibrium, the simulation code itself is a test of stability, since SUPERLAYER has freedom of motion in the (R,Z) directions. Since the code shows stable solutions for axisymmetric FRM equilibria, one expects that FRM configurations such as the one quoted above are stable to $\ell = 0$ modes. The energy principle was applied with $\ell = 0$ perturbations as an additional test. Here a sufficient condition for stability can be developed by dropping a positive definite spectral energy term, yielding:

$$\delta^{(2)}U \geq \int d^3x \left\{ \delta B^2 - \left(\frac{\omega_p}{c} \right)^2 \frac{2m\Omega_R^2}{T} \delta A_\theta^2 \right\} \geq 0. \quad (\text{III.2})$$

This condition was tested with a set of analytic trial functions $\delta \vec{A}$ and was found to hold for the equilibrium case cited above. This is an indication that the FRM can be made stable to tearing modes ($\ell = 0$) for "short, fat" equilibria such as the simulation model chosen.

For $\ell = 1, 2$ modes, the equilibrium code does not model the stability properties, since non-axisymmetric perturbations allowed are not to the equilibrium. A diagram of the bulk features of the instabilities is shown as Fig. 2. Here the energy principle is used directly in the manner described above. A number of trial functions were used to detect zeroes to $\delta^{(2)}U$ with variable parameters of frequency and growth rate.

Perturbations with negative energies have been found for the $\ell = 1, 2$ case. These trial functions have the form:

$$\delta \vec{A} = \vec{\xi} \times \vec{B}, \quad \vec{\xi} = (\xi_R \hat{R} + i\xi_\theta \hat{\theta}) e^{i\ell\theta}, \quad (\text{III.3})$$

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 $l=0$ (Tearing) Mode $l=1$ (Kinking) Mode $l=2$ (Fluting) Mode

Figure 2. Deformations accompanying instabilities. (In this figure, the top view is the view along the magnetic axis.)

where ξ_R and ξ_θ are real. An example of such a perturbation is shown as Fig. 3. With this trial function, zeros to $\delta^{(2)}U$ occurred at low growth rate ($\gamma \sim 0.07\omega_{ci}$) and at finite real frequency ($\omega_0 \sim 0.7\Omega$).

This is illustrated for the $\ell = 2$ case by the curve of Fig. 4.

The $\ell = 1$ growth rate for zero energy was slightly lower with an extrapolated growth rate of $\ell = 0.055\omega_{ci}$. These results suggest precessional instabilities for $\ell = 1$ and $\ell = 2$ modes at low growth rates. While the above trial functions have not been optimized, these do indicate failure of the sufficient condition for stability developed above. The requirements for strengthening the sufficient condition to a necessary and sufficient condition, indicating an actual instability in the FRM, are outlined in the next section.

iii) Extension of Variational Method and Simple Orbit Model

A. Extension to necessary and sufficient conditions.

To prove the existence of instability to the $\ell = 1$ and $\ell = 2$ modes outlined above, one must establish the adjointness properties of the operators involved in the variational principle. This is necessary in order to show that an extremum to $\delta^{(2)}U$ with zero eigenvalue represents a solution to the equations governing the instability. The energy principle equation can be written in the form

$$\delta^{(2)}U = \int dt' d^3x \delta \vec{E} \cdot \left\{ \vec{\nabla} \times \delta \vec{B} - \frac{4\pi}{c} \delta \vec{J} \right\}. \quad (IV.1)$$

Since $\delta^{(2)}U$ the time dependence $e^{2\gamma t}$, this equation can be simplified as

$$\delta^{(2)}U = \frac{1}{2\gamma} \int d^3x \delta \vec{E} \cdot \left\{ \vec{\nabla} \times \delta \vec{B} - \frac{4\pi}{c} \delta \vec{J} \right\}, \quad (IV.2)$$

Figure 3. Equivalent fluid perturbation ξ_r vs. radius at midplane for $\ell = 1, 2$ modes.

Figure 4. Graph of perturbed energy vs. real frequency for $l = 2$ trial mode.

In this equation $\delta\vec{J}$ is regarded as a functional of $\delta\vec{A}$, the perturbation. Alternatively, then, the above equation can be written as

$$\delta^{(2)}U = \int d^3x \delta\vec{A} \cdot (\vec{\nabla} \times \vec{\nabla} \times \delta\vec{A}) + \int dt' \delta\vec{E} \cdot \delta\vec{J}(\delta\vec{A}) \quad (IV.3)$$

The operator $\vec{\nabla} \times \vec{\nabla} \times$ is self-adjoint. If the form $\delta\vec{J} \cdot \delta\vec{E} dt'$ can be proved a self-adjoint functional of $\delta\vec{A}$, then (IV.3) represents a variational principle of Ritz form. This property can then be used to show that for a particular frequency and growth rate where $\delta^{(2)}U$ is stationary with eigenvalue zero, the operator involved solves the equation

$$\vec{\nabla} \times \vec{\nabla} \times \delta\vec{A} - \frac{4\pi}{c} \delta\vec{J} = 0 \quad (IV.4)$$

which is the definition of instability for this system. An example of a self-adjoint form for eq. (IV.3) is shown in the following section.

Alternately, $\delta^{(2)}U$ can be written in adjoint form as

$$\delta^{(2)}U = \frac{\int dt' d^3x \delta E^+ \cdot \{ \nabla \times \delta B - \frac{4\pi}{c} \delta\vec{J}(\delta\vec{A}) \}}{\int \delta E^{++} \cdot \delta\vec{E} d^3x} \quad (IV.5)$$

Then trial functions resulting in a minimum for the $\delta^{(2)}U$ for a given growth rate with $\delta^{(2)}U = 0$ represent a solution to the regular and adjoint Maxwell equations

$$\vec{\nabla} \times \vec{\nabla} \times \delta\vec{A} = \frac{4\pi}{c} \delta\vec{J}, \quad \vec{\nabla} \times \vec{\nabla} \times \delta\vec{A}^+ = \frac{4\pi}{c} \delta\vec{J}^+ \quad (IV.6)$$

where $\delta\vec{J}^+$ is the adjoint to $\delta\vec{J}$. Thus, a computational algorithm yielding necessary and sufficient conditions for stability could be constructed by utilizing an adjoint variational method.

B. Simple Model for $\ell = 1, 2$ Instability.

The orbit integrals appear in eq. (II.9) can be evaluated approximately by considering only the azimuthal terms, i.e., $v_\theta \delta A_\theta$ terms in the orbit integrals. The motion in the (R,Z) plane occurs at Betatron frequencies that are not coupled to the azimuthal rotation frequencies. Hence the terms in $v_R \delta A_R$ and $v_Z \delta A_Z$ are small under time integration.

A simple model can thus be constructed where only the θ motion is thus considered. The orbit of each particle can be approximated as a circle about the axis, i.e., $R = \text{constant}$, $Z = \text{constant}$, and $v_\theta = \Omega' R$. This results in orbit integrals of the form

$$g = \frac{v_\theta \delta A_\theta(t=0)}{i(\omega - \ell \Omega')} \quad (IV.7)$$

If the particle distribution is then given a finite spread of rotation frequencies Ω' given by a simple gaussian distribution

$$n(\Omega') \sim \exp [(\Omega' - \Omega)^2 / (2\Delta^2)] \quad (IV.8)$$

Then a simple model for the total energy results from the simple integration:

$$\delta^{(2)}U = \int d^3x (\delta \vec{A} \cdot (\vec{\nabla} \times \vec{\nabla} \times \delta \vec{A})) + \int n(\Omega') d\Omega' \delta \vec{A}^* \delta \vec{A}_\theta \frac{m \Omega'^2 R^2}{T} \cdot \left[-1 + \frac{\gamma^2 + (\omega_0 - \ell \Omega)^2 + \ell \Omega (\ell \Omega' - \ell \Omega)}{\gamma^2 + (\omega_0 - \ell \Omega')^2} \right] \left(\frac{\omega_p}{c} \right)^2 \quad (IV.9)$$

A graph of the energy from this analytic model is shown for a particular trial function on Fig. 4. The model roughly fits the output from the SUPERLAYER stability routine, which calculates exact orbit integrals.

It is also significant that eq. (IV.9) represents a self-adjoint model for the Vlasov operator. Thus the fitness of the analytic description of the system with this form shows that the operators in the exact energy analysis may not deviate from self-adjointness to such a degree that the failure of the sufficient condition does not represent instability in this FRM configuration.

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II.5. Start-up via Neutral Beam Injection

A key problem in the realization of the field-reversed mirror (FRM) as a power reactor is the development of a start-up technique. One method, which allows later scale-up to reactor operation is neutral-beam start-up. This method uses tangential neutral-beam injection into a background mirror plasma to form an ion "ring." As the "ring" grows, its field cancels out the mirror field, giving reversal.

The goals of the study on neutral-beam start-up are to find the neutral-beam energies and currents needed for start-up and to identify optimal start-up scenarios. The principle tools are two computer codes: 1) a simplified volume averaged 0-D code, and 2) a 1-D radial code which treats the radial profiles of the plasma parameters. The 0-D code, which was operational at the end of the previous contract period, predicts that start-up is possible with modest neutral-beam currents ($<300\text{A}$). Preliminary versions of two programs which together make up the 1-D radial code are currently operational separately. These two programs describe the background fluid and the high-energy ion ring, respectively. They are scheduled to be combined near the end of the current contract period.

The remainder of this section is organized as follows. Section a) describes the 0-D code and gives an example of its results. Section b) is a brief sketch of the 1-D radial code. Details of this code are given in Sections c) through g); while Section h) provides the current status of the code and talks about future work.

a) The Zero-Dimensional Code

The Zero-D code is a two-region finite-difference global model. [1]* The open- and closed-field volumes are represented as separate regions in the code. The open-field region is a low-density cold plasma with poor confinement (mirror confinement), and the closed-field region consists of a high-density hot plasma with good confinement (essentially classical confinement).

The neutral-beam current and energy per particle are given as input and rate equations are solved numerically for the time evolution of the number of ions in the ring current, and for the electron and ion temperatures in both the open- and closed-field regions. Since a global code is used, only the spatially averaged values of the variables mentioned above are computed.

The time evolution of the volume of the closed-field region (required for calculation of confinement times) is also computed. While the volume changes, the shape of the current profiles for the ion ring and background plasma are maintained as time independent. These currents are assumed to be only in the azimuthal direction and to have azimuthal symmetry. The amplitude of the ion-ring profile is determined by the number of trapped ring ions, and the background-plasma diamagnetic current is determined by the background-plasma pressure. The total magnetic field is a linear combination of self-fields of the ring and background plasma with a uniform external magnetic field representing the mirror field.

*See p. 91 for references.

As an example, the 0-D code indicates that reversal can be achieved with a target plasma of density $2.5 \times 10^{13} \text{ cm}^{-3}$, $T_i = 5 \text{ keV}$, and $T_e = 100 \text{ eV}$ confined by a 6-kG field. A 230A-10 keV neutral beam creates the ion ring. Incipient reversal occurs at 1.5 ms when $\approx 3 \text{ MA}$ of ring current is trapped ($\approx 0.03 \text{ C}$). The total injector energy required for incipient reversal for both beams is $\approx 2 \times 10^3 \text{ J}$. The above injector requirements appear achievable with present technology. However, due to the many simplifying assumptions in the 0-D code, these values probably represent a lower limit. Consequently, development of the more detailed 1-D code has been undertaken.

b) Brief Sketch of 1-D Radial Code

The zero-D code solved rate equations for the volume averaged plasma quantities and the amplitude of the ring current with its shape remaining fixed. To better model spatial effects in the background plasma current, transport, neutral-beam ionization, and the ring current a 1-D radial code has been written. In the 1-D code the plasma variables are treated as azimuthally symmetric and independent of the axial coordinate, Z . Only the variation of the plasma quantities with the radial coordinate r is treated. A hybrid plasma model is used: high energy ring ions are modeled using a detailed kinetic treatment while background (thermal) ions and electrons are modeled with fluid theory. Transport effects in the closed field region are presently ignored in this code, although they will be included before the end of the next contract period.

c) Kinetic Model for High Energy Ring Ions

Because the ratio of plasma size to ring ion gyroradius is not large (~ 2 to 20), the fluid equations are not valid for the ion ring. Instead, to

model these ions, the time development of their distribution function is followed over a grid in energy-canonical angular momentum (P_c) space. For simplicity, the ring ions are assumed to have no axial velocity ($v_z = 0$). At each time step radial current and density profiles are constructed from the distribution function via particle orbits. Slowing down effects (from inductive electric field E_ϕ or collisions with the background plasma) are not included at present, but will soon be added.

The high energy ion section of the 1-D code is currently operational, and it will not be discussed further here. A detailed explanation is given, through, of the background fluid part of the code, in the following sections.

d) Motivation for Fluid Section of 1-D Code

The background plasma section of the code is needed for three reasons. First of all, the diamagnetic plasma current must be included. Only then can one model the transition of a reversed state supported by high energy injected ions to one supported by the background plasma pressure profile. Secondly, knowing background temperatures and densities as a function of radial position allows accurate determination of the birth distribution and slowing down of the high energy ions. Finally, it has been suggested by Baldwin, et al. [2,3] that electrons at the field null, via collisions with ions, could cancel out the local ion current, making reversal impossible. Our code treats the field null region separately to allow the inclusion of various models for the electron current there.

e) Field Configuration

Figure 1 shows the field configuration implicit to the 1-D code, along side an example of the field shape in an "actual" FRM. The field lines in the code are all in the $\pm Z$ direction. In effect, two cuts have been made from the field null to the separatrix in the "actual" FRM field lines, and the closed field lines have been opened up to form the 1-D configuration. Hence the closed-field flux surfaces, which are tori, are each mapped into two concentric cylinders.

An important consequence of the mapping mentioned above is that any closed-field flux surface in the 1-D code has 2 radial positions. Since the background plasma particles can easily move along a flux surface, the background densities and temperatures are the same at each of these pairs of radii. To take this coupling into account implicitly, a Lagrangian coordinate system with the flux coordinate, ψ , as the independent variable is used. Details of this coordinate system is given in Section g).

f) The Radial Fluid Equations

The fluid equations used to model the background plasma, except in the region near the field null, are shown in Eulerian form in Figs. 2 and 3. These simplified equations are derived from the full set of fluid equation by first setting $\frac{\partial}{\partial Z}$ and $\frac{\partial}{\partial \psi}$ to zero. Then, inertial terms are dropped to eliminate unwanted MHD oscillations. Finally, the high energy ion density is assumed to be much smaller than the background plasma density, as was predicted by the 0-D code. This allows one to set the radial electric

Figure 1. Magnetic Field Configuration

Radial Fluid Equations – Eulerian Form

Closed Field Region

approximations : $m_e, m_i \rightarrow 0$

$$n_e \approx n_i \gg n_{ring} \quad E_r \rightarrow 0$$

**e⁻
continuity**

$$\frac{dn_e}{dt} + n_e \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = S_e$$

**ion
continuity**

$$\frac{dn_i}{dt} + n_i \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = S_i$$

**e⁻
energy**

$$\frac{dp_e}{dt} + \frac{5}{3} P_e \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = \frac{2}{3} \dot{\epsilon}_e$$

**ion
energy**

$$\frac{dp_i}{dt} + \frac{5}{3} P_i \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = \frac{2}{3} \dot{\epsilon}_i$$

**flux
conservation**

$$\frac{dB_z}{dt} + B_z \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = 0$$

**pressure
balance**

$$\frac{\partial}{\partial r} \left[\frac{B_z^2}{2\mu_0} + P_e + P_i \right] = -j_{ring} B_z$$

Figure 2.

Radial Fluid Equations—Eulerian Form

Open Field Region

$$m_e, m_i \rightarrow 0 \quad \text{"line tying"} \quad E_r \rightarrow 0$$

**e⁻
continuity**

$$n_e = n_i + n_{\text{ring}}$$

**ion
continuity**

$$\frac{dn_i}{dt} + n_i \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = -\frac{n_i}{\tau_i} + S_i$$

**e⁻
energy**

$$p_e = n_e k T_e \quad T_e = T_{\text{Stream}}$$

**ion
energy**

$$\frac{dp_{i\perp}}{dt} + (1 + f^*) p_{i\perp} \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = -\frac{p_{i\perp}}{\tau_i} + f[\epsilon_i - W_{ie}]$$

**flux
conservation**

$$\frac{dB_z}{dt} + B_z \frac{1}{r} \frac{\partial}{\partial r} (rU_r) = 0$$

**pressure
balance**

$$\frac{\partial}{\partial r} \left[\frac{B_z^2}{2\mu_0} + p_e + p_{i\perp} \right] = -j_{\text{ring}} B_z$$

f, f^* are factors characterizing the anisotropy of the ion velocity distribution.

$$f \equiv p_{i\perp}/(\text{ion energy density}) \quad 2/3 \leq f, f^* \leq 1$$

Figure 3.

field equal to zero in the closed field region. (The radial electric field in the open field region is zero because of "line tying.") The important features of the resulting set of equations are discussed in the next paragraph.

The two sets of equations in Figs. 2 and 3 consist of several "flow" equations along with a pressure balance condition. The "flow" equations for the magnetic fields imply flux conservation - a background plasma particle always moves so as to remain on the same flux surface. The pressure balance equation forms a constant between the magnetic field, the plasma pressure and the ion ring current. The ring current is, of course, supplied by the high energy ion section of the code. However, the fluid approximation does not apply near the field null, where the low magnetic field permits a large electron and ion gyroradius. In this region the background plasma diamagnetic current is no longer given by $j_{\text{plasma}} = \frac{1}{B_z} \frac{\partial p}{\partial r}$; a separate model for the plasma current density and temperatures must be supplied by the user. A principle objective of the code will be to examine the effects of various models for the plasma near the field null (esp. the electron current) on start-up.

g) Method of Solution - Lagrangian Mesh

The fluid equations of Figs. 2 and 3 are solved using a Lagrangian mesh with the flux coordinate ψ as the independent variable. Plasma densities and temperatures for electrons and ions as well as the values of the magnetic field are tabulated over a grid of flux surfaces. The radii of each flux surfaces are also stored. During each time step the changes in the background plasma values are found by solving the pressure balance

equation for the changes in the flux surface position.

The fluid equations used for the closed field region are shown in Fig. 4 in their Lagrangian form. (The corresponding equations for the open field region are not shown, but contain no essential differences.)

$\Psi, \psi \equiv \int_0^r r B_z r$, is the flux coordinate; the h 's are the two radii associated with each flux surface; the u_h 's are the corresponding velocities. The coupling between the inner and outer part of a flux surface, mentioned in Section e), is indicated by the angle brackets,

$$\langle f \rangle_\psi = \frac{h_a}{h_a + h_b} f_a + \frac{h_b}{h_a + h_b} f_b.$$

This averaging assumes that a particle is equally likely to be at any point on its flux surface. Thus the ratio of its time spent on the inner side of a flux surface to the time spent on the outer part is the ratio of their areas, or h_a/h_b .

The final Lagrangian equation in Fig. 4 is the time derivative of the pressure balance constraint. The equations for dp/dt and dB_z/dt are substituted into the pressure balance equation to yield the horrendous second order equation shown in Fig. 5. This equation plus its open field counterpart form a boundary value problem (BVP) in $u_h \Delta t \equiv h^1$, the radial shifts of each flux surface. The boundary conditions on $u_h \Delta t = h^1$ are:

- 1) $h^1(r=0) = 0$
- 2) $h^1(r=R_{\text{wall}}) = 0$, i.e., the vessel walls are infinitely conducting.
- 3) continuity of plasma plus magnetic pressure at the open field-closed field boundary.

Radial Fluid Equations – Lagrangian Form

Closed Field Region

**e⁻
continuity**

$$\frac{dn_e(\psi)}{dt} = - n_e(\psi) \left\langle \frac{1}{h} \frac{\partial}{\partial h} (hU_h) \right\rangle_{\psi} + \left\langle S_e \right\rangle_{\psi}$$

**ion
continuity**

$$\frac{dn_i(\psi)}{dt} = - n_i(\psi) \left\langle \frac{1}{h} \frac{\partial}{\partial h} (hU_h) \right\rangle_{\psi} + \left\langle S_i \right\rangle_{\psi}$$

**e⁻
energy**

$$\frac{dp_e(\psi)}{dt} = - \frac{5}{3} p_e(\psi) \left\langle \frac{1}{h} \frac{\partial}{\partial h} (hU_h) \right\rangle_{\psi} + \left\langle \frac{2}{3} \dot{\epsilon}_e \right\rangle_{\psi}$$

**ion
energy**

$$\frac{dp_i(\psi)}{dt} = - \frac{5}{3} p_i(\psi) \left\langle \frac{1}{h} \frac{\partial}{\partial h} (hU_h) \right\rangle_{\psi} + \left\langle \frac{2}{3} \dot{\epsilon}_i \right\rangle_{\psi}$$

**flux
conservation**

$$\frac{dB_z}{dt} = - B_z \frac{1}{h} \frac{\partial}{\partial h} (hU_h)$$

**pressure
balance**

$$\begin{aligned} \frac{B_z}{\mu_0} \frac{\partial}{\partial \psi} \left[\frac{dB_z}{dt} \right] + \frac{1}{\mu_0} \frac{dB_z}{dt} \frac{\partial B_z}{\partial \psi} + \frac{\partial}{\partial \psi} \left[\frac{dp(\psi)}{dt} \right] \\ = - \left[\frac{1}{h} \frac{\partial j_{\text{ring}}}{\partial t} + \frac{1}{h} \frac{dh}{dt} \frac{\partial j_{\text{ring}}}{\partial r} - \frac{j_{\text{ring}}}{h^2} \frac{dh}{d\psi} \right] \end{aligned}$$

$$\psi = \int_0^r r B_z dr$$

$$\langle f \rangle \equiv \frac{h_a}{h_a + h_b} f_a + \frac{h_b}{h_a + h_b} f_b$$

$h_a(h_b)$ $\equiv$ radial position of inner (outer) side of flux surface ψ .

Figure 4.

Equation for Motion of the Flux Surfaces Closed Field Region

$$\begin{aligned}
 & -B_z^a \left[\frac{(B_z^a)^2}{\mu_0} + \frac{5}{3} p \alpha_a \right] y_a'' - \left[\frac{3(B_z^a)^2}{\mu_0} (B_z^a)' + \left(\frac{5}{3} p B_z^a \alpha_a \right)' \right] y_a \\
 & - \frac{5}{3} p B_z^b \alpha_b y_b'' - \left(\frac{5}{3} p B_z^b \alpha_b \right)' y_b' + \frac{1}{(h^a)^2} \left[\frac{\partial j_{\text{ring}}}{\partial r} - \frac{j_{\text{ring}}}{h^a} \right] y_a \\
 & = - \frac{j_{\text{ring}}^1}{h^a} - \frac{2}{3} (\alpha_a \dot{\epsilon}_a + \alpha_b \dot{\epsilon}_b)' \tau
 \end{aligned}$$

along with the same equation with a and b interchanged

$$y \equiv h^0 h^1, \quad ' \equiv \frac{\partial}{\partial \psi}, \quad p = p_e + p_i, \quad \dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_i$$

Figure 5.

- 4) A jump condition across the region near the field null involving the integral of the local plasma plus ring current.

When the background section of the 1-D code is successfully merged with the ring ion section, each time step will follow this sequence:

- 1) Ion ring currents and densities are calculated by the high energy ion model.
- 2) Using the j -ring calculated in 1) the solution to the pressure balance BVP is found by solving the corresponding matrix equation over the Lagrangian mesh. Here, a gaussian elimination algorithm is used.
- 3) The values of $\langle \frac{\partial}{\partial \psi} (hu_h) \rangle_\psi$ are computed at each grid point and the plasma density, pressure and magnetic field are updated.
- 4) Increment time. Return to step 1) the recipe as outlined here is self-consistent.

h) Status of Code and Future Work

An initial skeletal version of the fluid section of the 1-D code has just become operational. The output of this code is currently being tested for numerical stability, convergence with decreasing Δt and $\Delta \psi$, and physical rasonableness. An analytic test case is also being devised. Early analysis of results reveal good convergence properties, and no instability is observed for reasonable Δt . However, in clear contradiction with experimental evidence, the code predicts inverted ion temperature profiles ($\frac{\partial T_i}{\partial r} > 0$) in the open field region. Therefore, the inclusion of a neutral gas species, which lowers the ion temperature near the wall, is also being investigated.

After the accuracy of the fluid part of the code has been verified, it will be merged with the high energy ion section of the code, using the algorithm of Sect. g). The numerical stability and convergence properties of the entire radial code must then be analyzed. If the convergence or stability properties of the combined codes are inadequate, the algorithm may have to be used in a "predictor-corrector" mode: the values generated in the first pass through a time step (predicted) will be used to generate much more accurate values in the second iteration (corrected).

Work scheduled for the next contract period is summed up in the accompanying report "Proposed Work." Briefly, preliminary results from operation of the initial version of the entire 1-D code will be compared with the β -II experiment. Appropriate models for the electron current at the field null will be searched for, and their effects on start-up found using the code. The code will be expanded to a 2-D code either self-consistently, or through some reasonable assumption for the shape of flux surfaces. Finally, the effects of a radial electric field may be added if initial runs with the code ($E_r = 0$) indicate a gross charge imbalance.

References

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III. Related Field Reversal Studies

In addition to the preceding analysis of the FRM, the project has been involved in studies of several related field-reversed configurations. A 1-D dynamic-type MHD code (*RFPBRN*) for Reversed-Field Pinch (RFP) simulations has been developed in close collaboration with workers at LASL. The initial motivation for this undertaking was to consider refueling techniques for burn extension; however, it is now clear that the code will also be valuable for predicting stable operating regimes for reactors and for analysis of experiments such as ZT-40 at LASL. Preliminary results from this code are described in Sec. III.1.

We have proposed a detailed plasma systems study of the Field-Reversed Theta Pinch (F RTP) for next contract period. Based on encouraging experimental results at LASL, the F RTP appears to be an important contender for a "compact tori" type reactor. Alternately, it might be viewed as a possible start-up approach for an FRM reactor. As a first step in this effort, Sec. III.2 describes an analysis of the mechanisms involved in the plasma rotation observed in F RTP experiments. An understanding of this effect is crucial for any attempts to scale-up the concept to a reactor-size device.

Finally, Sec. III.3 describes exploratory studies of a new reactor concept, the Moving Plasmoid Heater (MPH), that we have proposed for possible use with concepts such as the F RTP or Spheromak. While similar to the earlier moving ion ring concept studied at Cornell and Pacific Gas & Electric, the MPH uses plasmoid target plasmas for buildup rather than ring stabilized plasmas. This difference introduces new plasma considerations but retains many of the advantages of the moving ring concept.

III.1 Dynamic Simulation of the Reversed-Field Pinch (RFP) -- a 1-D MHD Model

The stability of a Reversed-Field Pinch (RFP) is strongly dependent on the plasma and the confining sheared magnetic field. Magnetic diffusion and thermal transport produce changing conditions of stability. Despite the limited understanding of RFP transport, modelling is important to predict general trends and to study possible field programming options.

To study the ZT-40 experiment and to predict the performance of future RFP reactors, a one-dimensional transport code has been developed. This code includes a linear, ideal MHD stability check based on an energy principle. The transport section integrates plasma profiles forward in time while the stability section periodically checks the stability of the evolving plasma profile.

Transport Model: To accommodate fusion plasmas, the code has three fluids: electrons, ions, and alpha particles. Confined alpha particles are assumed to instantaneously thermalize in-situ, joining the background ion group. The quasi-static, quasi-neutral approximation gives five working equations:

$$\frac{\partial n_i}{\partial t} + \vec{v} \cdot \vec{r}_i = S_i - 2f(1-f) \langle \sigma v \rangle n_i^2 \quad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + \vec{v} \cdot \vec{r}_\alpha = f(1-f) \langle \sigma v \rangle n_i^2 \cdot F \quad (2)$$

$$\frac{3}{2} n_j \frac{dT_j}{dt} + p_j \vec{v} \cdot \vec{v}_j = - \vec{v} \cdot \vec{q}_j + Q_j \quad (j=i+\alpha, e) \quad (3)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{v} \times (-\vec{v}_e \times \vec{B} + \frac{1}{en_e} \vec{R}_{th} + (n_\parallel J_\parallel) \hat{e}_\parallel + (n_\perp J_\perp) \hat{e}_\perp) \quad (4)$$

$$\vec{v} \cdot \vec{p} = \vec{J} \times \vec{B} \quad (5)$$

where i , e , and α , denote ion, electron, and alpha particle terms, respectively. S_i , and Q_j are particle and energy source terms; R_{th} is the thermal force; f is the tritium fraction; and F is the alpha particle confinement fraction.

The above equations are solved on a 1-D Lagrangian finite difference mesh using a 2-step process. First, Eqns. (1-4) are integrated forward in time. Next the mesh is rebalanced to satisfy Eq. (5) using flux conservation and ideal gas law.

Stability Model: Stability is monitored in two fashions. Suydam's criterion^{(1)*} is used to check for local modes (assumed to saturate) while a Rayleigh-Ritz type energy principle is used for global modes. Both checks are derived from the ideal MHD equations using a cylindrical approximation.

Assuming that the plasma extends to the conducting wall, the 1-D energy principle is expressed as⁽²⁾:

$$\delta_w = \frac{\pi}{2} \int_0^a dr \left[f \left(\frac{d\xi}{dr} \right)^2 + g \xi^2 \right] , \quad (6)$$

where

$$f = \frac{r(KrB_z + mB_\theta)^2}{(K^2r^2 + m^2)} , \quad (7)$$

$$g = \frac{2K^2r^2}{(K^2r^2 + m^2)} \frac{dp}{dr} + \frac{1}{r} (KrB_z + mB_\theta)^2 \frac{(K^2r^2 + m^2 - 1)}{(K^2r^2 + m^2)} \\ + \frac{2K^2r}{(K^2r^2 + m^2)^2} (K^2r^2B_z^2 - m^2B_\theta^2) ,$$

and $K \equiv$ axial wave number

$m \equiv$ mode number

$p \equiv$ pressure

$B_z \equiv$ toroidal field

$B_\theta \equiv$ poloidal field

$\xi \equiv$ radial displacement

Since ξ can be singular when $f \rightarrow 0$, ξ is transformed to a finite form⁽³⁾

$$\psi = \sqrt{f} \xi , \quad (8)$$

*See p. 117 for references.

which transforms (6) to

$$\delta_w = \frac{\pi}{2} \int_0^a dr \left[\left(\frac{d\psi}{dr} \right)^2 - \frac{1}{f} \frac{df}{dr} \psi \frac{d\psi}{dr} + \left(\frac{1}{4f^2} \left(\frac{df}{dr} \right)^2 + \frac{g}{f} \right) r^2 \right] \quad (9)$$

Using the Rayleigh-Ritz technique, ψ is expanded in terms of a finite sine series:

$$\psi = \sum_{i=0}^n c_i \sin(A_i r) ; \quad A_i = \frac{(i+1)}{a} \pi \quad (10)$$

Here C_0 is normalized to 0 and C_i ($i = 1, \dots, n$) are computed to minimize δ_w by requiring

$$\frac{\partial \delta_w}{\partial c_i} = 0, \quad i = 1, \dots, n \quad (11)$$

This yields a set of n -inhomogeneous equations in n -unknowns which can be inverted to solve for C_i , $i = 1 \dots n$. Once the C_i 's are known, δ_w is calculated. If $\delta_w < 0$ the plasma is unstable and if $\delta_w > 0$ it is stable. Stability is checked for $m=0$, $k \rightarrow 0$ and $m=1$ - $\frac{30}{R} < K < \frac{30}{R}$ where R is the major radius.

Suydam's criterion⁽¹⁾ checks for pressure driven interchange instabilities near rational surfaces and can be expressed as:

$$\frac{B_\theta^2}{8r} \left(\frac{dp}{dr} \right)^2 + \frac{dp}{dr} > 0, \quad (12)$$

for stability where $\underline{p} \equiv \frac{rB_z}{B_\theta}$.

Suydam's criterion is included in the more general energy principle (Eq. (6)). However, these modes are characterized by eigenfunctions that are asymptotically singular at a rational surface. Since the numerical integration mesh size is finite and determined by the transport section of the code, eigenfunctions that are more localized than the mesh size will not be detected by the global analysis. This is a positive feature in that Suydam modes are assumed to saturate and not disrupt the plasma. Consequently, the stability check is able to find gross modes which may occur

simultaneously with Suydam modes. The global analysis also checks the validity of the assumption that Suydam modes are localized in that it will detect these modes if their eigenfunctions extend over a macroscopic volume of the plasma.

Results: Verification: Simulations (Table 1) for conditions similar to the RFPR point model⁽⁴⁾ have shown good agreement (see Figures 1, 2, and 3). In order to match the point model conditions as closely as possible, the plasma was confined to a region interior to the reversal radius. The radius of the reversal point was assumed to vary as

$$r_{\text{rev}} = \frac{r_{\text{rev}}(0)}{\sqrt{1-\beta_\theta}} \quad , \quad (13)$$

consistent with the point model. Here, $r_{\text{rev}}(0)$ is the initial reversal radius and β_θ is the maximum poloidal beta. Simulations were also done keeping I_z constant, but these did not match the point model radius variation as closely.

Initially, the temperature distribution is constant and density is assumed to vary as:

$$n = n_0 J_0^2(\mu r); \quad \mu \equiv 2.405/(r_{\text{rev}}(0) + .1) \quad , \quad (14)$$

which is consistent with the point model. The toroidal field is initially:

$$B_z = B_{z0} J_0(\gamma r); \quad \gamma \equiv 2.405/r_{\text{rev}}(0) \quad , \quad (15)$$

and the poloidal field is calculated to satisfy pressure balance.

Results of RFPBRN and the point model have four principle differences which can qualitatively be attributed to differences in assumptions. First of all, the point model assumes constant temperature which results in constant reaction cross-sections across the plasma radius. RFPBRN allows

TABLE I

RFPR INTERIM DESIGN SUMMARY: AIR-CORE POLOIDAL
TRANSFORMER, SUPERCONDUCTING COILS

<u>Symbol</u>	<u>Definition</u>	<u>Value</u>
r_w	first-wall radius (m)	1.5
R	major radius (m)	12.7
I_c	poloidal coil current (MA)	34.8
I_p	plasma current (MA)	20.0
j_z	toroidal current density (MA/m ²)	5.4
P_A	filling pressure (mTorr)	2.25
f_B	burnup	0.54
$n_i \tau_E$	Lawson parameter (10 ²⁰ s/m ³) during burn	1.5
W_{TOR}	required toroidal coil energy (GJ)	4.67
W_{POL}	required poloidal coil energy (GJ)	6.62
τ_B	burn time (s)	21.6
τ_C	cycle time (s)	26.6
I_w	average 14.1-MeV wall loading (MW/m ²)	2.73
Q_E	1/recirculating power fraction	6.2
η_p	plant efficiency, $\eta_{TH}(1-1/Q_E)$	0.34
P_{TH}	total thermal power (MWt)	2980
η_{TH}	thermal power density (MWt/m ³)(a)	-0.50
P_{ET}	gross electric power (MWe)	1192
P_E	net electric (MWe)	998

(a) Based on volume enclosed by and including superconducting coils.

Figure 1. Comparison of RFPBRN and point model average temperatures (top), and poloidal beta and burnup fraction (bottom).

Figure 2. Comparison of RFPBRN and point model wall loadings.

Figure 3. Comparison of RFPBRN and point model toroidal currents.

the cross-sections to vary with the local plasma temperature. This causes enhanced reactivity in the central region which results in a higher peak temperature after ignition (see Fig. 1).

RFPBRN also burns out faster and has lower burn up than the point model. This can be attributed to differences in alpha deposition profiles and fuel density profiles as the burn evolves. The point model assumes that alpha deposition is uniform across the plasma radius and the fuel maintains its initial radial dependence (Eq.14). RFPBRN assumes alphas are deposited in-situ. This results in a comparatively higher alpha fraction and lower fuel fraction in the center of the plasma. Since the center is where most of the fusion occurs, RFPBRN burns out faster and has lower fractional burnup than the point model.

Another difference is that the point model specifies the pressure balance analytically and does not allow for toroidal field decay. RFPBRN includes toroidal field decay which requires the toroidal current also to decay to keep the radius comparable to the point model (see Fig. 3). This results in less ohmic heating for RFPBRN, however, the effect is small due to the elevated temperatures.

Figure 4 shows typical radial dependences of the temperature, field, density and pitch profiles. These profiles maintain their qualitative shape throughout the burn. Unlike previous simulations,⁽⁵⁾ the temperature and pressure peak on centerline. This is due to the assumed Bohm-like thermal conductivities. Figure 4 also indicates that Suydam violation will occur over half the plasma radius. The implications of this will be discussed in the next section.

Table II shows energy parameters for RFPBRN. Although RFPBRN does not contain a detailed system energy balance, comparison of Table I and

Figure 4. Typical radial dependences of temperature (top, a), density, (c, next page), magnetic field (bottom, b), pitch and Suydam factor (d, next page) for RFPBRN at 12.0 seconds into the simulation.

Table II

RFPR Energy Parameters

<u>Symbol</u>	<u>Definition</u>	<u>Value</u>
w_B	Peak magnetic energy (GJ)	3.13
w_f	Total fusion energy (GJ)	46.8
Q	Fusion energy/Ohmic heating	57.6
w_n	Neutron energy (GJ)	32.7
w_r	Radiation energy (GJ)	1.14
w_c	Thermal conduction energy (GJ)	7.42

Table II indicates that power output from RFPBRN is about 2/3 that of the point model. This is consistent with the fractional burnup differences.

Anomalous transport: Due to the large plasma regions with Suydam instabilities, simulations were done to incorporate anomalous transport in these regions. Similar to previous studies,⁽⁶⁾ particle transport in Suydam unstable regions was assumed to scale like Bohm diffusion. To do this, the perpendicular resistivity was modified:

$$\eta_{\perp} = \eta_{\perp c} + f_T S_u C_s B^2 D_B / n_e , \quad (16)$$

where

$$S_u = \begin{cases} \frac{\Sigma-1}{\Sigma} , & \Sigma > 1 , \\ 0 , & \Sigma < 1 , \end{cases} \quad (17)$$

and

$$\Sigma = - \frac{dp}{dr} / \left(\frac{B_{\theta}^2}{8r} \left(\frac{dp}{dr} \right)^2 \right) ; \quad \underline{p} \equiv \frac{r B_z}{B_{\theta}} . \quad (18)$$

Here C_s is a constant scaling factor taken as 1×10^{-4} , f_T is a triangular function which scales the transport over five mesh points adjacent to the point where the Suydam violation occurs. This corresponds to a width of approximately 10 ion Larmor radii for the turbulence.

For the simulations the plasma was assumed to extend to the wall with the fields programmed to keep it there. Initially the temperature profile was assumed to be flat and the density was assumed to scale as

$$n(r) = n_0 J_0(\mu r) , \quad (19)$$

where

$$\mu \equiv 2.405 / (r_w + .1) .$$

Figures 5-7 are a comparison of a simulation without anomalous transport and one with anomalous transport in Suydam unstable regions. The principle difference is that increasing the perpendicular resistivity

Figure 5. Comparison of average temperatures (top), poloidal beta and burnup fraction (bottom) for cases with and without turbulent transport.

Figure 6. Comparison of radial dependences of temperature (a) and magnetic field (b) without turbulent transport. Figures (c) and (d) on next page are for the case with turbulent transport.

Figure 7: Comparison of radial dependences of density (a) pitch and Suydam factor (b) for cases without turbulent transport. Figures (c) and (d) on next page are for the case with turbulent transport.

flattens the density profile in the central region of the plasma. This causes a similar leveling of the pressure profile which pushes the plasma towards the marginal Suydam stability limit.

Comparison of the Suydam violation factor S_u (Fig. 7) indicates that turbulent transport shifts the plasma from uniform Suydam violation to pointwise violation. This is also indicative of marginal stability. As a plasma region self-stabilizes by transport flattening the local pressure, pressure gradients in adjacent regions are steepened causing them to become unstable.⁽¹⁾ The result is randomized Suydam violation over regions of weak shear.

Another interesting feature of these simulations is that the magnetic fields maintain their topological structure despite the turbulent transport. The aberrations in the field and pitch profiles in figures 6 and 7 are small compared to the density differences. This is a result of scaling the turbulence through the perpendicular resistivity rather than both the perpendicular and parallel components. Doing this implicitly assumes that turbulence results in transport rather than increased collisionality.

Parametric variations of the scaling factor C_s show very little change in the plasma behavior as long as the turbulent resistivity is significantly larger than the classical one. This is also indicative of marginal stability. The value of C_s chosen was done for numerical convenience. The only effect of increasing C_s is that the plasma heats a little faster and computer execution time is greatly increased.

MHD Stability: The global stability check indicates that the RFPR reactor will probably remain MHD stable when the fields are programmed to keep the plasma at the wall. This assumes that Suydam modes will remain localized and saturate.

Comparisons between the anomalous transport and classical cases (Figs. 5-7) indicate that anomalous transport stabilizes Suydam modes. In the classical case, Suydam violation was strong enough that these modes were detected by the global stability analysis. This did not occur for the anomalous transport case. Neither of these cases exhibited gross instability.

However, for the classical case (Figs. 8-10) gross instability did occur shortly after ignition. In this instance, the large poloidal field necessary to confine the high- β plasma caused a current driven instability. The eigenfunction for this mode is in Fig. 11.

Figure 8. Fig. 5 with classical thermal conductivities.

Figure 9. Fig. 6 with classical thermal conductivities.

Figure 10. Fig. 7 with classical thermal conductivities.

Figure 11. Radial eigenfunction for $m = 1$, $R = 8/R$ current driven gross kink mode.

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III.2 Rotation Effects in Field-Reversed Theta Pinch (F RTP)

The Field-Reversed Theta Pinch (F RTP) has emerged as an important concept for a compact reactor, either using D-T or advanced fuel.^{(1)*} However, an improved understanding of plasma rotational effect is necessary when considering scale-up to reactor size devices. Up to now the experiments have achieved a confinement time of only $\sim 40 \mu\text{sec}$ due to rotational instabilities.⁽²⁻⁴⁾ Therefore, the rotation mechanism is intimately related to confinement scaling, but to date a satisfactory analysis has not been available. A new model is proposed here that is in qualitative agreement with the angular acceleration measurements in the experiment, FRX-A.⁽³⁾

Two factors are responsible for the angular acceleration (deceleration). One is the Hall effect, i.e., $j_r \times B_z$ force producing a torque on ions and electrons, respectively, where j_r is the radial diffusion current for ions or electrons. The other is the increase of the moment of inertia that results from radial particle motion. The main feature of this new model is that it incorporates the transport properties through the radial diffusion, which turns out to depend sensitively on the electric field.

Equilibrium Configuration and Assumptions

The equilibrium configuration for the F RTP is shown in Fig. 1. It consists of a quasi-neutral plasma confined radially by an externally applied magnetic field B_∞ . A cylindrical polar coordinate system (r, θ, z) is introduced, with the z -axis along the axis of symmetry. The ions and electrons rotate about the axis of symmetry with angular frequencies ω_i and ω_e , respectively.

To make the theoretical analysis more manageable, the following assumptions are made in describing the equilibrium by the steady-state Vlasov-Maxwell equations:

*See p. 130 for references.

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Figure 1. F RTP Configuration and the Spatial Coordinate System.

(a) Equilibrium properties are azimuthally symmetric and independent of z-coordinate.

(b) The plasma is collisionless and has a Maxwellian distribution with uniform and constant temperature.

(c) The equilibrium distribution function is independent of the axial velocity, i.e., there is no motion along the z-axis and end effect is neglected.

(d) The plasma is in a quasi-steady state and in rigid rotor equilibrium throughout the evolution.

(e) The magnetic flux inside the conducting cylinder is conserved, and magnetic field diffusion is neglected.

(f) The growth rate of the rotation frequency is assumed small such that $ct \ll 1$, where c is the growth rate and t is the time of evolution, and thus the constants of motion are adiabatic invariant during this period.

Equilibrium

It is well known that any distribution function that is a function of the constants of particle motion satisfies the steady-state Vlasov equation. In an azimuthally symmetric system the constants of motion are identified as the Hamiltonian and the canonical angular momentum, which are given as

$$\bar{\epsilon} = 1/2 m_{\alpha} v_{\alpha}^2 + q_{\alpha} \phi(r) , \quad (1)$$

and

$$\bar{p}_{\theta} = m_{\alpha} r v_{\theta\alpha} + q_{\alpha} r A_{\theta}(r) , \quad (2)$$

where $\alpha=e,i$ represents electron or ion, ϕ and A_{θ} are electric and vector potential, respectively.

A Maxwellian-like form of rigid rotor distribution is assumed and is given by⁽⁵⁾

$$f_{\alpha} = \frac{n_{\alpha 0}}{(2\pi T_{\alpha}/m_{\alpha})^{3/2}} \exp[-(\epsilon - \omega_{\alpha} P_{\theta})/T_{\alpha}] \delta(\epsilon - \bar{\epsilon}) \delta(P_{\theta} - \bar{P}_{\theta}) \quad (3)$$

By substituting Eqs. (1) and (2) into Eq. (3) and making the integration in velocity space of Eq. (3) we obtain

$$n_{\alpha} = n_{\alpha 0} \exp[(-2q_{\alpha} \phi + 2\omega_{\alpha} r e A_{\theta} + m_{\alpha} \omega_{\alpha}^2 r^2)/2T_{\alpha}] \quad (4)$$

If we assume charge quasi-neutrality everywhere in space, we obtain a relation between ϕ and A_{θ} , i.e.,

$$e\phi = \frac{T_i T_e}{T_i + Z T_e} \left[\left(\frac{Z\omega_i}{T_i} + \frac{\omega_e}{T_e} \right) \frac{r e A_{\theta}}{c} + \left(\frac{m_i \omega_i^2}{2T_i} - \frac{m_e \omega_e^2}{2T_e} \right) r^2 + \ln Z \right] \quad (5)$$

where Z is the ion charge with $n_e(r) = Z n_i(r)$. Note that we only consider two species here.

Now the Ampere's law requires

$$\nabla \times B_Z(r) = [\nabla \times (\nabla \times A)]_{\theta} = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(r A_{\theta})}{\partial r} \right) = \mu_0 J_{\theta}(r) \quad (6)$$

where μ_0 is the magnetic permeability and J_{θ} is the total azimuthal current which is given by

$$J_{\theta}(r) = n_i(r) e r Z (\omega_i - \omega_e) \quad (7)$$

Combining Eqs. (4) - (7) and following a same procedure as in Ref. (5), we obtain a self-consistent solution of density, magnetic field and electric field distribution as

$$\begin{aligned}
n_i(r) &= n_e(r)/Z \\
&= n_{i0} \operatorname{sech}^2 K \left(\frac{r^2}{R^2} - 1 \right) / \operatorname{sech}^2 K, \quad (8)
\end{aligned}$$

$$\begin{aligned}
B_z(r) &= (2\mu_0 n_{i0} (T_i + ZT_e) / \operatorname{sech}^2 K)^{1/2} \tanh K \left(\frac{r^2}{R^2} - 1 \right) - \frac{m_i \omega_i^2 + m_e \omega_e^2}{eZ(\omega_i - \omega_e)} \\
&\equiv B_\infty \tanh K \left(\frac{r^2}{R^2} - 1 \right) - \frac{m_i \omega_i^2 + Z m_e \omega_e^2}{eZ(\omega_i - \omega_e)} (1 - \tanh K \left(\frac{r^2 - R^2}{R^2} \right)). \quad (9)
\end{aligned}$$

Here n_{i0} is ion density at center, B_∞ the external magnetic field, and

$$\begin{aligned}
E(r) &= - \frac{\partial \phi}{\partial r} = \frac{4KT_i T_e}{eZ(\omega_i - \omega_e) R^2} \left(\frac{Z\omega_i}{T_i} + \frac{\omega_e}{T_e} \right) r \tanh K \left(\frac{r^2}{R^2} - 1 \right) \\
&+ \frac{\omega_i \omega_e (m_i \omega_i + Z m_e \omega_e)}{eZ(\omega_i - \omega_e)} r. \quad (10)
\end{aligned}$$

An additional relation is automatically obtained, namely:

$$\frac{n_{i0} R^4}{k^2 \operatorname{sech}^2 K} = \frac{8 (T_i + ZT_e)}{Z^2 e^2 (\omega_i - \omega_e)^2 \mu_0}, \quad (11)$$

here R and k are two constants introduced in solving Eqs. (4) - (7); the former corresponds to the radius at which density is maximum, i.e., $n_{\max} = n_0 / \operatorname{sech}^2 K$, while the latter is related to the degree of field reversal as

$$K = - \tanh^{-1} [(B(r=0) + B_c) / (B_\infty + B_c)], \quad (12)$$

where $B_c = (m_i \omega_i^2 + Z m_e \omega_e^2) / eZ(\omega_i - \omega_e)$. It is interesting to note that the second terms of Eqs. (9) and (10) represent the field associated with a centrifugal effect, and, as will be shown later, they are much smaller than the first term. A typical plot of the self-consistent density and field profiles is shown in Fig. 2.

Model of Angular Rotation

From Eqs. (9) and (10) we see that ω_i and ω_e are unknown, and we have to determine them from other equations. A model based on conservation of the canonical angular momentum is proposed to calculate these two quantities.

The definition of the canonical angular momentum is defined as

$$\vec{L}_z = \int m n \vec{r} \times \vec{v}_\theta dV + \int n q \vec{r} \times \vec{A}_\theta dV, \quad (13)$$

and integration is taken over total plasma volume. The conservation of canonical angular momentum gives

$$\begin{aligned} \frac{d\vec{L}_z}{dt} = 0 = & \int m n \vec{r} \times \left(\frac{d\vec{v}_\theta}{dt} \right) dV + \int m n \vec{v} \times \vec{v}_\theta dV \\ & + \int n q \vec{r} \times \left(\frac{d\vec{A}_\theta}{dt} \right) dV + \int n q \vec{v} \times \vec{A}_\theta dV. \end{aligned} \quad (14)$$

Here we define $\vec{v} = d\vec{r}/dt = v_r \hat{r} + \vec{v}_\theta \hat{\theta}$, and $v_\theta = r\omega$ is the total drift velocity, ω is the corresponding angular velocity. From basic mathematical manipulation we obtain

$$\begin{aligned} \vec{r} \times \frac{d\vec{v}_\theta}{dt} + \vec{v} \times \vec{v}_\theta &= \vec{r} \times \left(\frac{\partial \vec{v}_\theta}{\partial t} + v_r \omega \hat{\theta} \right) + \vec{v} \times \vec{v}_\theta \\ &= (r^2 \frac{\partial \omega}{\partial t} + 2 r v_r \omega) \hat{z}, \end{aligned} \quad (15)$$

Figure 2: A Typical Plot of the Self-consistent Density, Electric Field, and Magnetic Field.

and

$$\begin{aligned}
 \vec{r} \times \frac{d\vec{A}_\theta}{dt} + \vec{v} \times \vec{A}_\theta &= \vec{r} \times \left(\frac{\partial \vec{A}_\theta}{\partial t} + v_r \frac{\partial A_\theta}{\partial r} \hat{\theta} \right) + v_r A_\theta \hat{z} \\
 &= r v_r \left(\frac{\partial A_\theta}{\partial r} + \frac{A_\theta}{r} \right) \hat{z} = r v_r \frac{\partial}{\partial r} (r A_\theta) \hat{z} \\
 &= r v_r B_z \hat{z} .
 \end{aligned} \tag{16}$$

Here we have assumed that the external field is fixed, i.e., $\frac{\partial A_\theta}{\partial t} = 0$.

Inserting Eqs. (15) and (16) into (14) and rearranging, we obtain

$$\frac{\partial \omega_d}{\partial t} = \frac{-1}{I} \left[\int j_z B_z r dV + 2\omega \int m r \Gamma_z dV \right] , \tag{17}$$

where $I = \int m r^2 dV$ is the moment of inertia, and $j_z = n q v_r = q \Gamma_z$ is the radial diffusion current density.

The above formalism is either for ions or electron, thus from Eq. (17) we can solve for their guiding center drift angular velocities. Note that in writing Eq. (14) we have assumed that the density profile does not change. This is not true because from Eq. (8) the profile depends on the major radius and on the parameter K which depends on ω_i and ω_e through Eq. (8). However, to a first approximation we will assume the density profile to be fixed and retain the work on improvement.

In evaluating Eq. (17) it is required to know the diffusion flux.

We further assume classical diffusion with the flux Γ_α given by

$$\Gamma_\alpha = -D_\perp \nabla n + \frac{q}{|q|} n \mu_\perp \vec{E} , \tag{18}$$

where $D_\perp$ and $\mu_\perp$ are the transverse diffusion coefficient and mobility, respectively. These coefficients are related by Einstein relation as

$$D_\perp = \mu_\perp \frac{T_\alpha}{Ze} . \tag{19}$$

The diffusion coefficients for ion and electron are given by

$$D_{\perp i} = \frac{T_i}{m_i(\nu_{in} + \nu_{ie})} \left(1 + \left(\frac{\omega_{ci}}{\nu_{in} + \nu_{ie}} \right)^2 \right)^{-1}, \quad (20)$$

$$D_{\perp e} = \frac{T_e}{m_e(\nu_{en} + \nu_{ei})} \left(1 + \left(\frac{\omega_{ce}}{\nu_{en} + \nu_{ei}} \right)^2 \right)^{-1}, \quad (21)$$

where ν_{in} and ν_{en} are the ion-neutral and electron-neutral collision frequency, respectively; ω_{ci} and ω_{ce} are the Larmor gyrofrequency for ion and electron, respectively.

If we neglect the second term in Eq. (10) and insert Eqs. (8), (10) into Eq. (18) it gives

$$\Gamma_{\perp i} = -D'_{\perp i} \nabla n_i, \quad (22)$$

$$\Gamma_{\perp e} = -D'_{\perp e} \nabla n_e, \quad (23)$$

here

$$D'_{\perp i} = D_{\perp i} \left(1 + \frac{T_e}{(\omega_i - \omega_e)} \left(\frac{Z\omega_i}{T_i} + \frac{\omega_e}{T_e} \right) \right), \quad (24)$$

$$D'_{\perp e} = D_{\perp e} \left(1 - \frac{T_i}{(\omega_i - \omega_e)} \left(\frac{\omega_i}{T_i} + \frac{\omega_e}{ZT_e} \right) \right), \quad (25)$$

From above there also exists an intimate relation with the ambipolar diffusion, as will be discussed later.

If the initial angular frequency is defined as ω_0 , Eq. (17) then has a solution

$$\omega_d(t) = (\omega_0 + c_1/c_2) e^{-c_2 t} - c_1/c_2, \quad (26)$$

here c_1 and c_2 are constants; the former has a dimension sec^{-2} which can be shown to be proportional to $eB_{\omega} D_{\perp i} / m_i R^2$ and accounts for the angular acceleration due to the Hall effect, and the latter has a dimension sec^{-1} which

can be shown to be proportional to the particle confinement time $D_{\perp i}/2R^2$ and thus accounts for damping due to increase of moment of inertia when particles move outwards. Here the acceleration is used in the sense that if the direction is the same as that of the diamagnetic drift $\omega_{\nabla p}$.

In Eq. (26), initial angular drift frequencies of the guiding center of both ions and electrons will be taken to be the ExB drift frequency. Since the electric field is initially unknown, the initial angular drift frequencies cannot be determined this way. However, from the experiment FRX-A⁽⁵⁾ the initial ($t \lesssim 7\mu\text{sec}$) total drift frequency of ions is almost zero, i.e.,

$$\omega_i = \omega_{\nabla p_i} + \omega_{\text{ExB}} \approx 0 \quad (27)$$

We then inferred that the electrons have a finite initial angular drift

$$\omega_{eo} = \omega_{\text{ExB}} \times \omega_{\nabla p_e} \quad (28)$$

A diagram is given in Fig. 3 to illustrate these relations.

Discussion

As stated in last section, the electric and magnetic fields depend on the angular frequency which itself is time dependent. Thus, the assumption of a steady-state in Maxwell equation is not consistent. However, it is found that the second terms of Eqs. (9) and (10) are lower than their first terms by at least one order of magnitude for $|\omega_i| < 10^7 \text{sec}^{-1}$ as observed experimentally, and thus the assumption of steady state is still valid.

On the other hand the results show that ω_e is greater than $|\omega_i|$ by

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Figure 3. Plot of the Drift Frequencies of Ions and Electrons.

one or two order of magnitude. Under this condition if we assume $\omega_e \gg |\omega_i|$, then Eq. (10) with second term neglected becomes

$$\begin{aligned} E(r) &\cong \frac{-4KT}{eR^2} r \tanh k\left(\frac{r^2}{R^2} - 1\right) \\ &= \frac{T_i}{en} \frac{\partial n(r)}{\partial r} \end{aligned} \quad (29)$$

This gives the same result as in the case of ambipolar diffusion in the collisionless limit. As seen in Eq. (24) the second term is quite close to -1 depending on the ratio $|\omega_i/\omega_e|$, consequently, a slight change in $|\omega_i/\omega_e|$ may cause a significant change in the effective ion transverse diffusion coefficient $D_{\perp i}$ and the rotational growth rate c_2 .

On the other hand this model can also explain the effect of a partially shorted-out electric field. To see this we can check Eq. (18).

If we put $E \approx 0$ in Eq. (18), then the second term in Eq. (24), which is originally close to -1, is now negligible with respect to the first term. This would lead to a substantial increase in the ion radial diffusion and the associated growth rate of rotation.

Based on this model some suggestions are made to improve the instability control. These are:

- 1) A highly collisionless and fully ionized plasma is essential in order to reduce the radial leakage and the growth rate of rotation, if classical diffusion is really obeyed.
- 2) Refueling the system after implosion phase may help maintain the moment of inertia, this would reduce the growth rate of rotation significantly.
- 3) The end shorting, which is quite detrimental to the stability of the system, must be avoided or reduced.

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III.3. The Moving Plasmoid Heater (MPH) Reactor Concept

Interest in the MPH concept stems from recent preliminary computer studies in which we predict that presently attainable, mobile, high-density ($10^{21-22} \text{ m}^{-3}$) field-reversed theta-pinch (F RTP) plasmoids will provide excellent targets for high-energy ($\sim 150\text{-keV}$) neutral-beam heating injectors.

The MPH is visualized as a near-term, energy-multiplying, steady-state fusion device. The overall system utilizes plasma rings similar to the moving compressor scheme,^{(1)*} the startup techniques of the F RTP,⁽²⁻⁵⁾ the toroidal $\vec{B}$ -field component of the spheromak,⁽⁶⁾ and neutral beam heating similar to that in use on current devices such as PLT. However, the combination of these concepts is unique and draws on individual strengths of each: high density-rings (F RTP), MHD stability (spheromak), reduced steady-state ash buildup and direct conversion (compressor), and near-term technology (neutral beams). It is envisioned that the thermal conversion systems and neutral-beam injectors will operate on a steady-state basis.

Mobile, high-density field-reversed theta pinch plasmoids have been formed in several experiments^(2,5). Plasmoid densities of $10^{21-22} \text{ m}^{-3}$ and temperatures to $\sim 100 \text{ eV}$ have been typical in these experiments⁽²⁻⁵⁾. However, experimental confinement times ($\sim 100 \mu\text{s}$) have been ultimately limited by an MHD rotational ($m=2$) instability. Thus, possible stabilizing techniques being examined include the application of appropriate toroidal plasma fields⁽⁶⁾. Such toroidal fields might be generated through the use of axial (Z-direction in Figure 1) currents⁽⁷⁾ or pulsed conical theta-pinch coils.^(2,8)

*See p. 134 for references.

Figure 1. The Moving Plasmoid Heater (MPH) Concept.

A finite-difference global code has been developed to study heating of these plasmoids via neutral beams. Beam current and energy, average density and temperature of the target, and dimensions of these plasmoids are given as input. While classical diffusion across the closed $\vec{B}$ -field line was assumed for the results cited here, enhanced diffusion losses have been considered as a limiting case. Rate equations are then numerically solved for the time evolution of the temperature and density of the confined plasmoids.

These high-density reversed-field plasmoids make excellent targets for trapping of neutral beams (i.e., 80-90% efficient). As an example (Table 1), we consider injecting a neutral beam of current, 150 A, and energy, 150 keV, diagonally through a plasma target of density, 10^{21} m^{-3} , ion temperature, 100 eV, length, 0.40 m, and radius, 0.20 m. Ion temperatures of 25 keV were computed after 1 msec.

After heating, the plasmoid is moved on to the burn chamber using small "kicker" magnetic field coils. Burn times must be 1 - 10 sec for $n\tau \sim 10^{20} \text{ m}^{-3} \text{ sec}$. Efficient energy collection is expected, since there will be few wall penetrations in the burn chamber. Blanket engineering and material requirements are eased due to the essentially steady-state electromagnetic, charged particle, and neutron wall loadings.

In conclusion, the MPH concept, as a new fusion power system, potentially offers advantages previously found only in separate fusion schemes. Generation of the closed-field plasmoids has been demonstrated experimentally and the present results show that they would be excellent targets for neutral-beam heaters. Engineering aspects of the concept are especially attractive due to the steady-state and linear modular nature of the system.

Table 1: Operating Parameters for Moving Plasmoid Heater (MPH) Reactor Concept

Parameters	Formation Chamber	Heating Chamber	Burn Chamber
$n \text{ (m}^{-3}\text{)}$	10^{21}	$10^{21}-10^{22}$	10^{22}
$T_i \text{ (keV)}$	10^{-1}	$10^{-1}-10^2$	10^2
$\tau \text{ (sec)}$	10^{-6}	10^{-2}	1-10
$B_{vac} \text{ (T)}$	1.0	1.0-6.0	8.0
$I_{inj} \text{ (A)}$	-	150	-
$E_{inj} \text{ (keV)}$	-	150	-

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IV. Fusion Product Transport in Tokamaks

As fusion development moves towards reactor-grade plasmas, fusion products (e.g., alpha particles in D-T fueled devices) will become, for the first time, a significant new component in the plasma. Indeed, alphas will play a key role in any D-T plasma; namely, they should provide sufficient heating to achieve ignition or nearly-ignited conditions. Indeed, such operation is essential to avoid the need for excessive external heating which is both expensive and comparatively inefficient. This benefit is not without its penalty, however. Some of the high energy alphas escape the plasma, bombarding the wall and eroding it. Those that thermalize in the plasma are called "ash" since they dilute the fusing fuel, causing a decrease in power density.

Since its inception, the current project has been concerned with these various aspects of alpha particle transport, particularly for tokamaks where reactor grade plasmas are expected first. Earlier work concentrated on escape-fractions and wall bombardment. Related effects such as internal electric field buildup and induced plasma rotation were also considered. To test our basic understanding of alpha transport, measurements of alphas hitting the wall in PLT and TFTR experiments were proposed and supporting calculations carried out.

The work described here in Sec. IV.1 represents studies of ways to extend previously developed techniques to non-circular tokamaks operating at higher β . This will be important for certain operating regimes of TFTR and for later reactors. The next section (IV.2) discusses an application of the earlier wall bombardment calculations to an estimate of impurity effects caused by the resultant blistering and sputtering.

Sec. IV.3 is concerned with the "last" phase of alpha transport, namely, ash buildup. In particular, the effect of an electric field (due either to high-energy alpha losses or to improved fields) on ash buildup is considered. This is considered important since some fields will always be present. In fact, however, it is shown that the use of externally programmed fields might be a desirable approach to aid in control of ash buildup.

IV.1. Prompt Fusion Product Loss Analysis

A) Regarding the modeling of charged-fusion product effects, we proposed to complete the prompt loss analysis, begin work on non-prompt loss detection, and do sample calculations of these losses. However, this effort was predicated on definitive implementation of a near-term experiment on PLT. Due to conflicts with high priority experiments on PLT, implementation was postponed to TFTR. Consequently, additional work on this problem was also postponed.

B) Regarding α -slowing down, we proposed to include large-banana width effects and complete sample calculations. Our method consists of bounce-averaging the Fokker-Planck equation after its variables have been transformed to the constants-of-motion space [1]*which characterize the orbit. This approach allows a solution for the distribution function, and since it solves a 3-D (+ time) equation, it is possible to derive scaling laws. This is in contrast to the Monte Carlo technique of Mikkelsen and Post [2], whereby it is easy to obtain moments of the distribution function (e.g., heating profiles), but prohibitively consuming of computer time to find the distribution function itself (for comparison with charge-exchange or direct detection data). Our source function code determines the flux-surface-

*Sec p. 140 for references.

averaged α -particle source rate, $\langle \dot{n}_\alpha(\psi) \rangle$, and the rate of increase of the net α -current, $\langle \dot{I}_\alpha(\psi) \rangle$. The net current arises because some α birth orbits intersect the wall and also due to large banana effects. As shown in Fig. 1, the difference between $\langle \dot{n}_\alpha(\psi) \rangle$ and the point-wise source rate, $\dot{n}_\alpha = n_0 n_T \langle \sigma v \rangle_{DT}$, is $\lesssim 5\%$ in the central 40% of the plasma. This result implies that the local heating ("in-situ" deposition) model [3] is adequate when only interior heating is important. Alpha source current rates have also been obtained, yielding $\langle \dot{I}_\alpha \rangle = 0.81$ MA/s for a D-shaped TNS equilibrium ($R_0 = 5.4$ m, $\bar{\beta} = 7.5\%$, $T_{i0} = 15$ keV, and $I_{p1} = 6$ MA) and giving $\langle \dot{I}_\alpha \rangle = 4.1$ MA/s for a denser D-shaped equilibrium ($R_0 = 5.3$ m, $\bar{\beta} = 14.8\%$, $T_{i0} = 15$ keV, and $I_{p1} = 10.6$ MA). The background plasma response is not included in this calculation; electron trapping and boot-strap effects may serve to enhance the resulting alpha-driven current by up to ten-fold. This is significant because the α -current might drive the plasma current, reducing the volt-sec requirements for a reactor. Our slowing-down code is still under development and currently calculates the slowing-down rates and drift velocities in the constants-of-motion space. We have shown analytically that the resulting slowing-down drifts are inward for co-going ions and outward for counter-going ions. This is because like currents attract and opposite currents repel, and is consistent with the small-banana width analysis by Cordey [4], Monte-Carlo calculations by Killeen, et al. [5], and Petrie's work [6]. Another analytical result of this model is that μ/E is constant without pitch-angle scattering, consistent with Meka's work [7]. The slowing-down code will form the core of the α -heating model, with the source function code as input for the initial conditions.

Figure 1. Comparison of $\langle \dot{n}_\alpha \rangle$ and $\dot{n}_\alpha$ for Several Non-circular TNS Designs.

C) In summary, a large-banana-width Fokker-Planck model of alpha slowing in non-circular tokamaks has been developed. Computational results indicate that in-situ alpha heating models are accurate when only interior heating is important. Alpha source current rates appear sufficiently large enough that the α -driven plasma current, including boot-strap effects, may significantly reduce ohmic heating volt-sec requirements for a tokamak reactor. A code to calculate slowing-down rates has been written and will be incorporated into an alpha particle heating model.

The accomplishments of this effort are:

- formulation of the large-banana-width bounce-averaged Fokker-Planck model,
- calculation of large banana width properties including bounce times,
- determination of bounce-averaged values of $v_{||}$ and fast ion source rate,
- numerical evaluation of flux-surface-averaged values of particle and current source rates,
- analytical equations for slowing down drifts,
- $\mu/E = \text{constant}$ obtained without pitch-angle scattering.

IV.1.1 Publications during this period included:

- 1) G. Fenske, L. Hively, G. Miley, and M. Kaminsky, "Effects of Alpha Incident Angle-Energy Distribution on First Wall Erosion," Fusion Reactor Materials Mtg., Miami, FL (29-31 Jan. 1979).
- 2) L. M. Hively and G. H. Miley, "Prompt and Non-Prompt Fusion Product Bombardment of a Tokamak First Wall," UI Rpt #C00-2218-515 (June 1979), submitted for publication in Nuclear Fusion.

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2. D. R. Mikkelsen and D. E. Post, "Alpha Particle Heating in Tokamaks," paper 1B21 of 1979 Sherwood Theory Conf.
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4. J. G. Cordey, "Effects of Particle Trapping on the Slowing-Down of Fast Ions in a Toroidal Plasma," Nuclear Fusion, 16, (1976) 499.
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IV.2. Wall Bombardment Effects

First-wall erosion and corresponding plasma contamination in future fusion reactors are projected to arise from two major sources: blistering due to bombardment by high-energy fusion alphas and sputtering due to bombardment by lower energy ions leaking from the D-T plasma itself. It may be possible to effectively control sputtering by use of a divertor to scrape off much of the leaking plasma before it comes in contact with the wall. However, control of alpha bombardment is more difficult since the high energy of these particles will carry them across the scrape-off region. Thus, an improved understanding of such blistering effects represents an important goal in reactor development.

In the present study, we apply an alpha transport model previously developed for axisymmetrical tokamaks, to determine the detailed velocity-angle distribution of the alpha-flux incident on the wall. The competition between D-T sputtering and α blistering is then examined and the effect on impurity buildup in the plasma considered. While the methods employed are quite general, to illustrate the effects, the conceptual Experimental Power Reactor (EPR) designed by Oak Ridge National Laboratory (ORNL) [1]* is used.

The alpha-particle bombardment of the wall is calculated using an extension of the transport model developed by Hively and Miley [2-3], described in last year's annual report.

Previous studies [3] have shown that prompt losses, i.e., alphas that escape the plasma during initial orbits before significant slowing down occurs, may be the major contributor to helium-induced blistering. Thus, the present study is restricted to prompt losses. Since blistering is

*See p. 144 for references.

dependent on the incident velocity direction, as well as the fp flux magnitude, we use the angular flux in all calculations. The analysis includes all the relevant particle drifts (neoclassical motion), streaming along the magnetic field, and random gyrophase direction. We find the resulting prompt flux versus incident angle, α (relative to the local wall normal), rising sinusoidally, from near zero at 90° , to a maximum at α_c (some critical angle), then falling rapidly to zero for $\alpha < \alpha_c$. Plots of the alpha flux (per steradian) as functions of the angle incidence, for given poloidal angles are shown in Fig. 1 for the specific case of the ORNL-EPR. Previous studies [2] of this design, which assumed temperature, density, and current profiles of the form $[1-(r/a)^2]^N$ (where r/a is the dimensionless plasma radius, and N is one and one-half, and one, respectively), indicated the average fp flux to the first wall is 1.5×10^{11} ions/(cm²sec). This study assumed centerline temperatures and densities of 22.5 keV and 8×10^{13} ions/cm³, and a plasma current of 7.2 MA.

The effect on first wall has been modeled assuming that blister formation is theoretically possible if a critical helium concentration is exceeded. Due to a non-uniform alpha flux as a function of the poloidal angle, surfaces near $\theta \sim 55^\circ$ exceed critical helium concentrations faster than regions at other angles. A uniform surface erosion rate due to D-T sputtering was assumed in the model to determine its influence on the time required to reach a critical concentration for blister exfoliation and the change in the skin thickness at the time of sputtering. With an average alpha flux of 1.5×10^{11} /cm²sec, a total D-T flux of 5×10^{13} /(cm²sec), and an average D-T sputtering yield of 0.006 atoms/ion, the time to reach a critical concentration of 30 at. % in a stainless steel wall is not significantly increased

Figure 1. Alpha particle flux-per-steradian vs. incident angle (α) for prompt losses at various poloidal positions (θ) on the ORNL-EPR wall. Values indicated on curves give θ in degrees.

due to sputtering. However, the average skin thickness is reduced by $\sim 50\%$. The relative effects of blistering on the impurity concentration in a plasma were calculated as a function of time. If blistering does occur, the impurity level in the plasma may be increased by as much as 80% above that due to sputtering alone. After the first layer is removed, the impurity level decreases; then, increases when the critical dose is reached for the second generation of blisters.

As noted earlier, these results depend strongly on the D-T flux assumed and the sputtering yields chosen. These will vary significantly depending on the device parameters and the presence (and type) of impurity controls. For example, a cold gas blanket could reduce the average energy of the D-T particles incident on the first wall. This would have an overall effect of reducing the sputtering yield. In contrast, the use of a divertor would reduce the D-T flux to the first wall without significantly changing the incident energy of the particles. Furthermore, the effect of other surface erosion processes, and synergistic effects on blister formation should be considered, but it is difficult to assess the influence of such effects at the present time due to a lack of both theoretical and experimental data.

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IV.3. Fusion Product Impurity Buildup

As the problems of wall-generated impurities in a fusing plasma are satisfactorily resolved, the question of burn quenching from internally generated impurities becomes increasingly crucial. Such fusion ash, uncontrollable by conventional techniques (e.g., divertors), may limit burn times of future tokamak reactors.

The source profile for thermal alpha particles (thermalization profile) has previously been studied as it influences the equilibrium ash profile^{(1)*} and as it influences buildup rates toward that equilibrium⁽²⁾. The loss of high-energy alpha particles can generate a radial electric field, leading, among other things, to a plasma rotation⁽³⁾. The present study considers the effect of radial electric fields, whether internally generated or externally modified, on thermal transport of both ash and background plasma. Intuitively, one might expect an inward-directed field, characteristic of alpha losses, to retard the leakage of the higher Z ash. This would be a deleterious effect, possibly shortening burn times. Fortunately, however, as shown in the present study the effect is more complex and in fact, when changes in fuel leakage are also considered, burn times may be extended somewhat under the influence of alpha generated fields.

The electric fields considered have boundary conditions of maximum magnitude at the plasma edge (to observe the case of global influence on the plasma) or of zero field at the plasma edge (to isolate the internal effects on transport). The buildup of ash fraction (alpha/fuel ion density) is considered in a TFTR-size plasma, and significant lengthening of time-to-buildup is observed both at the centerline (where buildup may be most important) and globally.

*See p. 151 for references.

In order to isolate the effects of radial-electric-field-induced transport on alpha ash fractional buildup, a TFTR-size plasma is assumed (major radius, 250 cm.; minor radius, 50 cm.; total plasma current, 1 MA). The temperature profile is taken to be fixed in time and to be equal for all species. A parabolic shape is chosen for the temperature profile while the current profile is assumed to be flat. The fuel is 50-50 D-T. Since the cross-field conductivity of such a plasma is dominantly determined by toroidal momentum loss due to charge-exchange losses⁽³⁾, the conductivity will depend strongly on the neutral particle density profile assumed. The neutral density is arbitrarily assumed to decay exponentially from an edge value of 10^{11} cm^{-3} to a centerline value of 10^7 cm^{-3} .

At time $t=0$, the fuel-ion density profile is set to be parabolic, with centerline density $5.0 \times 10^{13} \text{ cm}^{-3}$. The alpha density is set to zero. Then, a 1-D explicit forward-time differencing code with variable timestep and fixed spacial mesh is used to find the time-evolution of the density profiles. At the plasma outer edge, the reflux fraction of particles (negative partial current/positive partial current) is set to zero for fuel and for alphas. Since only thermal transport is included, a modified in-situ source term for alpha particles is included (in-situ deposition multiplied by the global fraction of alphas which are actually deposited within the plasma). This code then follows the evolution of the density profiles assuming neoclassical diffusion with the particle fluxes induced by the electric field added.

Two electric field models are considered. The first takes the electric field to be proportional to $(r/a)^3$ and sets the maximum magnitude of the field as 1kV/m or 5kV/m. The direction of the field is varied. The second

electric field model used is that of Fig. 1 taken from Ref. 3. This field shape, characteristic of a TFTR-type plasma, has a maximum magnitude of about 1kV/m, and vanishes at the plasma boundary.

Figs. 2 and 3 illustrate the results obtained for the case of an electric field proportional to $(r/a)^3$. The decay of the total number of fuel ions inside the plasma is shown to be significantly slowed by inward-directed electric fields. Decay from $3.3 \times 10^{20} \text{ cm}^{-3}$ to $1.0 \times 10^{20} \text{ cm}^{-3}$ is delayed from 7.5 sec (with a 5 kV/m outward-directed field) to 89.6 sec (with an inward-directed field of this magnitude). Also note that for the times shown, even a 5 kV/m-maximum inward field is not sufficient to completely halt the outward diffusion of fuel. On the other hand, the result of this retention of fuel and the lesser effect on alpha buildup combined (Fig. 3) show lengthened time-to-buildup for reasonable ash fractions. Buildup to 10% alphas varies from 33 sec. (5kV/m maximum field outward-directed) to 96 sec. (the same field inward-directed). Similar effects are noted with respect to the centerline fuel density and alpha fraction.

These results can be understood by noting that the cross-field conductivity is proportional to toroidal momentum loss, and, since the charge-exchange cross section for D or T is much larger than the ion-neutral collision cross sections for either fuel or alphas (with the alpha double-charge-exchange cross section being even smaller), fuel ions experience a higher cross-field conductivity than alphas. Thus, the electric field is more effective at retaining (or purging) fuel than ash. Further, at these times, the alpha ash fraction is still rising fairly quickly, so the alphas are not diffusing outward very quickly. The electric field, then, does not modify the global alpha loss rate much by slowing, or even stopping it at these low densities.

Figure 1. Electric Field Due to Alpha Losses Calculated by T. Petrie [3] for TFTR-type Plasma.

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Figure 2. Decay of the total number of fuel ions in the presence of an electric field proportional to $(r/a)^3$. The edge field is specified.

Figure 3. Global alpha ash fraction vs time in the presence of an electric field proportional to $(r/a)^3$. The edge field is specified.

Qualitatively similar results have been obtained for the electric field profile of Fig. 1⁽⁴⁾ where the field is zero at the plasma boundary. Here, because the field is entirely internal, the effects on fuel retention and ash time-to-buildup result from changes in the outward diffusion rates resulting from density gradients. Then, for example, buildup to 10% changes from 55 sec. without the field to 60 sec. and with the field (compared to 68 sec. with an electric field proportional to $(r/a)^3$).

Note these results are for the case of no refuelling. Longer extensions of expected times-to-buildup may be realized if refuelling increases the edge fuel density, allowing the electric field to pull this new fuel in toward the plasma center.

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V. Other Mirror Systems Studies

Plasma buildup, both in density and temperature during neutral beam injection, remains a key aspect of the development of future mirror experiments like the end-plug sections of TMX and the MFTF device. The best opportunity to compare previous plasma-buildup models and experiments came during the operation of 2X-IIB. Then a dynamic code developed by B. Stallard at LLL was shown to provide reasonable overall results, but some discrepancies remained apparent, particularly in the outer regions of the plasma near the interface with cold gas entering via beam lines. These discrepancies were thought to be due to the lack of energy structure in the code. Consequently, a new buildup code, described in Sec. V.1, has been developed that employs an energy group technique to provide energy resolution without requiring excessive computer time. Preliminary results presented in Sec. V.1 confirm that energy effects can be significant. Applications to MFTF and TMX situations are planned after further checking of details in the code and incorporation of new subroutines to handle enhanced losses due to DCLC effects, etc.

A second aspect of neutral beam injection into mirrors, described in Sec. V.2, that has been studied for several contract periods involves losses and wall bombardment due to charge-exchange between the beam and target plasma ions. The effect can be quite significant to both the plasma density evolution and the thermal loading on the first wall in MFTF-size devices. The contorted plasma shape created by Yin-Yang magnets becomes crucial in calculating these effects. To handle the resultant 3-D geometry most efficiently, a Monte Carlo code NUBIN, was developed. This code and some results using it are briefly described in Sec. V.2.

V.1. Mirror Plasma Buildup Simulations

Among the various problems being examined for injection-heated fusion devices, one that is not being adequately treated by present models is the buildup of plasma from target to steady-state conditions in mirror devices. There presently exist two basic approaches to mirror plasma simulation that are capable of yielding accurate data: Monte Carlo methods and numerical solution of the separable Fokker-Planck equation. Advanced Monte Carlo codes such as the one developed by R. Miller at the UIUC [1]*, contain exacting spatial treatments but cannot be iterated because of the large amount of computer time that would be required. On the other hand, current Fokker-Planck codes are generally iterative but use only very simplified geometries [2,3]. Again, expansion of such a code to full geometry would result in greatly increased computational costs (it is important to note that many dynamics calculations are now done with codes that use averaged quantities obtained from Fokker-Planck calculations and are designed to handle only a specific type of problem, e.g., single beam injection[4]). No codes of either type are presently available that take into account the effects of finite Larmor radius of plasma ions, or the effects of the local magnetic field on trapping, density, etc.

The starting point for the present dynamic model originates with R. W. Moir, who began work on a spatial shell model code (which has since been expanded by B. Stallard at LLL)[5] that includes effects of a finite ion Larmor radius (as outlined in a paper by Futch, et al.)[6] on charge-exchange losses, etc. This code breaks the plasma confinement region into concentric shells, with the actual density in each shell being determined by

*See p. 164 for references.

the contribution to that shell from all particles whose gyro-orbits intercept it. The reason for doing this is that, in a device whose size is comparable to an ion gyroradius, removal of an ion at a given point in the plasma affects the (time) averaged density at all points bounded by its gyroradius, i.e., the density at points far removed from the loss point (typically about 1-2 cm) will change. This can result in a charge-exchange diffusion effect that tends to flatten the plasma density profile.

However, a major approximation made in the LLL code is that all ions are considered to be at the mean plasma energy, making it impossible to determine the energy (temperature) profile of the plasma. The energy dependence is closely coupled to finite Larmor radius effects (because of cold neutral charge-exchange events near the plasma surface that tend to replace hot ions having large gyroradii with cold ions having smaller gyroradii). In adding an energy dependence, it was decided to first develop a "single shell" energy dependent dynamic model that could be adapted for later use in a multishell code. Such a model is operational and has already produced useful results. [7] This sub-unit is basically a finite differencing code for a uniform plasma which uses a one species Fokker-Planck relaxation of the ion distribution and Sivukhin-type formulations for ion-electron and alpha-background interactions. [8] The code is reasonably fast and yields results that agree surprisingly well with those of Futch, et al. [3]

Most recently, this code has been adapted to a Moir-Stallard type spatial code treatment. The model presently being developed involves using the finite-orbit modified "single shell" procedure to calculate an energy dependent time-averaged ion density for each shell and then uses this information to obtain fusion, trapping and loss rates in each radial shell. At the end

of each time step the averaged density is inverse-transformed back into a density of gyrocenters. The ion energy distribution is allowed to relax in each shell and then the cycle is repeated.

The resulting code is called FOREMD (Finite Orbit Radial, Energy and Magnetic-field Dependent) code. The FOREMD model supplants earlier energy-independent models with a 2-1/2-D (1-energy, 1 1/2-spatial) dynamic treatment which extends the finite gyroradius [FGR] treatment to a distribution of ion energies and includes local magnetic field effects on ion orbits. A more detailed description of how this code is structured can be found in earlier annual reports and will not be restated here. The primary advantage of modeling a plasma using FOREMD is its ability to generate the FGR radial temperature profile, which earlier models could not.

[5]

Some preliminary results for a small 2X-IIB-like plasma are shown in Figs. 1-4. Figs. 1 and 2 give the radial density and temperature profiles, respectively, at various times during buildup for a case with input parameters in Table I. For this case a high energy (18 keV) beam of D^0 is injected, and centered on a warm target (1 keV) with an initial density of D_2^0 in the vacuum vessel equal to 10^{10} cm^{-3} ($t = 0.0 \text{ sec}$) rising to about 10^{12} cm^{-3} near equilibrium ($t = 1.2 \text{ ms}$). The beam is treated as having a Gaussian distribution in energy and spatial widths with half-maximum widths (HMW) given in the Table I. Two thirds of the neutral beam passed (untrapped) fraction is trapped in a beam dump. The third that is not trapped in the dump is recycled, as are the charge-exchange losses, back into the vacuum region as cold D_2^0 .

Figure 1. Plasma density vs. radius at various buildup stages for a small plasma similar to 2X-IIB. See Table I.

Table I. Plasma Parameters

I) TARGET:

W_I	= 1 keV
W_E	= 100 eV
N_I	= $7.5 \times 10^{13} \text{ cm}^{-3}$
R_{VAC}	= 2.
RADIUS	$\cong 8 \text{ cm}$
B_{VAC}	= 1.0 T
Vac. Cham.	= $1 \times 10^5 \text{ cm}^3$

II) BEAM:

W_{INJ}	= 18 keV
Gaussian HMW	= $\pm 2 \text{ keV}$
I_{INJ}	= 400 A
Gaussian HMW	= $\pm 8 \text{ cm}$

III) CODE:	ENERGY RANGE:	1 eV-60 keV
	ENERGY GROUPS:	40
	SHELL SIZE:	1 cm
	SHELLS	20

Figure 2. Ion temperature vs. radius at various buildup stages for the same plasma as in Fig. 1.

Figure 3. Plasma density vs. radius at time 0.4 ms for the case of Table I comparing coarse and fine spatial grid mesh results.

Figure 4. Ion temperature vs. radius profiles compared for same cases as in Fig. 3.

Figure 1 ($t = 0.8$ ms, 1.2 ms) shows the steep density profile that can result from cold D_2^0 ionization and charge-exchange effects at the plasma boundary (if all D_2^+ ions Franck-Condon dissociate to D^+ and D^0 before being lost, the density drop-off is even sharper). At the surface, the e^{-1} attenuation length is less than twice the mean ion gyro-radius ($\bar{\rho}_i \sim 3$ cm). The interior profile rises gradually from the edge to the center (the central dip may be caused by beam attenuation in the outer region). The near-equilibrium profile ($t = 1.2$ ms) is in general agreement with Livermore's 1-D code for similar input parameters.

Figure 2 predicts the plasma radial temperature profile at an early buildup stage ($t = 0.04$ ms) and later ($t = 0.8$ ms) near equilibrium. The former shows that beam trapping near the target plasma surface causes a low-density, hot region to form there due to hot-ion orbits extending out into this vacuum region (this unusual effect may aggravate Drift Cyclotron Loss Cone (DCLC) losses). As target ions replace beam ions, the interior temperature rises ($t = 0.8$ ms) and the surface acquires an equilibrium cold ion density from ionization of the incident gas (this has been used in 2X-IIB to stabilize against DCLC models). The electron temperature (T_e) (not shown) tracks the ion temperature (T_i) during buildup ($T_e \sim 0.05 - 0.1 T_i$) and falls off rapidly in the boundary region as expected. The results in Fig. 2 further illustrate the importance of this technique in that previous simulations of 2X-IIB could not predict the dynamic variation in edge temperature during build-up.

Figures 3 and 4 show a comparison between density and temperature profiles for a case run with two different spatial grids. In the first the shell thickness is 1 cm and the number of shells used is 20 and in the second the width is 0.5 cm and the number of shells used is increased to 40 to cover the same region. Both Figs. 3 and 4 show close agreement except that the small dip in the central density disappears with the smaller shell width. This may be due to anomalously high trapping in the 1 cm shells or to a small inaccuracy in the code and is now under investigation to discover the exact cause. The slightly steeper boundary density profile for the 0.5 cm case is a result of the much better spatial resolution and shows that the larger (1 cm) grid spacing yields very nearly the correct solution. This agrees well with results obtained previously by the author using Livermore's FGR code. [9]

The temperature profiles in Fig. 4 are in even closer agreement with the 0.5 cm case profile being steeper again because of much finer resolution. It is important to note that the "knee" of the profiles occurs within about a radial shell width (~ 1 cm) of each other as expected for a convergent solution. This profile shows how extraordinarily flat the temperature profile is in the plasma interior, a prediction which may be somewhat dependent on the absence of a streaming plasma (for D.C.L.C. stabilization) in the cases run.

These preliminary results are encouraging and in general agreement with predictions from Livermore's 1-D code using similar input parameters. Exact agreement is difficult to achieve in comparisons between these codes because the Livermore model inputs global values for nT , ion energy, etc. whereas the FOREMD model calculates these parameters self-consistently in each

spatial shell. Further work is underway to find comparable test cases for both codes.

Once the code is fully tested, it will be very useful for treating not only standard mirror problems, but also effects due to impurities from the walls and cold gas impinging upon the plasma surface. Other applications will include studies of MFTF, end-plug section for TMX, and the "twin beam" mirror concept.

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V.2. Simulation of Neutral-Beam Injection in Mirror Fusion Reactors

Computer simulation techniques using the Monte Carlo method have been developed for application to the modeling of neutral-beam injection into mirror-confined plasmas of interest to controlled thermonuclear research. The energetic (10-300 keV) neutral-beam particles interact with the target plasma ($T_i \sim 10$ -100 keV) through electron-atom and ion-atom collisional ionization as well as ion-atom charge-transfer (charge-exchange) collisions to give a fractional trapping of the neutral beam and a loss of charge-transfer-produced neutrals which escape to bombard the reactor first wall. Appropriate interaction cross sections for these processes are calculated for the assumed anisotropic, non-Maxwellian plasma ion phase-space distributions. The target plasma is assumed to be confined by a Yin-Yang mirror-coil configuration and has distinct radial and axial density profiles as modeled in steady-state and three spatial dimensions. The energy and angular distributions of a population of neutral test particles are computed to give statistical estimates of neutral-beam trapping efficiency, non-uniform first-wall surface heating rates and first-wall sputtering-erosion for use in engineering-design calculations. The reference parameters of the Mirror Fusion Test Facility (MFTF) and the mirror Fusion Engineering Research Facility (FERF) are used as particular applications of the simulation. Benchmark calculations of idealized configurations are summarized. A guide to the NUBIN computer code developed for this study is included.

Monte Carlo simulation techniques have been developed which permit the modeling of neutral-beam trapping in realistic mirror-machine plasma configurations and in addition provide information on neutral-particle first-wall surface heating and sputtering erosion of interest to

engineering-design studies. Application of these techniques to idealized benchmark calculations and to the MFTF and FERF systems confirms the qualitative conclusions of previously developed simple models insofar as the global beam trapping efficiency results are concerned only in a broad sense. The restrictions inherent in the simpler models tend to dictate their results. Only by generally relaxing those restrictions, as is allowed by the MC method, can the conflicting influences of these restrictions be sorted out. Beyond this, however, new aspects of the model, including the effect of higher-generation neutral particles, anisotropic distributions in three-dimensional target-plasmas, self-consistent interaction cross sections, and the resolution of neutral-particle energy spectra, provide information and insight not previously available. For the first time the anisotropic phase space distribution function of the mirror-confined plasma ions has been used to calculate self-consistent charge transfer and ionization rate coefficients. For the first time the tracking of charge-transfer-produced neutral particles through more than one generation has been accomplished. For the first time the radial and axial plasma density profiles of Yin-Yang mirror systems have been used in beam trapping calculations. For the first time the availability of the angular and energy distributions of the neutral particle bombardment of the first wall has enabled the calculation of nonuniform surface heating and sputtering erosion rates under the condition of the superposition of multiple beam sources. The angular distribution of the charge-transfer-produced neutral particles has been shown to be determined more by the anisotropic plasma ion distribution than by the original injection orientation. The computational framework developed for this study can accommodate the incorporation of more realistic physics as it becomes available from either theoretical work or experimental observation of mirror devices.

VI. Description of Computer Codes

Since readers may be interested in using codes developed as a part of this project, a brief description of major codes developed during the current year follows. Potential users can obtain more detailed information and instructions directly from the code author.

VI.1 Fusion Product Transport Codes

a) Tokamak Codes.

Codes for high energy transport in tokamaks are summarized in Table 1.

Table 1. Summary of High Energy Fusion Product Transport Codes*

CODE NAME	Description	Availability	Potential Users	Usage to Date*
1) WALLOADING	Calculation of fusion-product losses to tokamak first wall from DT, DD, D ³ He plasmas, assuming concentric circular flux surfaces. Output includes loss fraction, flux versus poloidal wall angle and incidence direction, non-prompt loss flux vs energy	MFECC: CDC 7600 CRAY-1	<ul style="list-style-type: none"> • ANL: M. Kaminsky, Fenske,... • Sandia: W. Bauer, et al. • INTOR • ETF (ORNL) 	<ul style="list-style-type: none"> • Fenske @ ANL • INTOR • Univ. of Ill.
2) FP SOURCE	Fusion-product particle and current rates for large banana-width orbits in non-circular cross section tokamaks. Output includes bounce time, bounce-averaged $v_{ }$, flux-surfaced averaged particle and current source rates	MFECC: CDC 7600 CRAY-1	<ul style="list-style-type: none"> • ETF (ORNL) • TFTR (PPPL) • GA • MIT • INTOR 	<ul style="list-style-type: none"> • Univ. of Ill. • ORNL
3) FP SLOWING	Fusion-product slowing-down and drifts for large banana-width orbits in non-circular cross section tokamaks. (not production-experimental code only). Output includes drift velocities, phase space boundaries in constants-of-motion space.	MFECC: CDC 7600 CRAY-1	<ul style="list-style-type: none"> • ETF (ORNL) • TFTR (PPPL) • GA • MIT • INTOR 	<ul style="list-style-type: none"> • Univ. of Ill.

*Author: L. Hively

b) Ash Dynamics

Code Name: IMOVE

Author: Wesley Downum

Availability: CTR 7600

Use: Simulates alpha particle buildup from zero in the presence of a decaying neoclassical fuel-ion profile. D-T fuel assumed. No temperature profile evolution. Includes cross-field conductivity effects for a given electric field profile.

Method: Forward time-differencing

Fixed spacial mesh

Variable timestep

Cross-field conductivity induced by ion-neutral collisions and charge-exchange collisions

Fixed neutral density profile

c) Fusion Products in FRMs

The Monte Carlo Particle Code - MCFRM

Author: D. Driemeyer

The code MCFRM provides a model of particle behavior in field-reversed mirror (FRM) plasmas. It not only calculates slowing down at high energies, but also includes a Monte Carlo treatment of Coulomb scattering, which enables particle motion to be followed even at lower energies where pitch-angle scattering becomes important. The background plasma is described in accordance with the Hill's spherical vortex model of a field-reversed equilibrium, and scattering effects are calculated using the Spitzer description of test particle interaction in a background plasma. MCFRM has been used extensively in the development of the D-³He SAFFIRE reactor design, and has led to an accurate evaluation of both fusion product (fp) heating and associated "ash" buildup in the closed-field region. The fps were found to have a significant effect on the closed-field energy and particle balances, despite the relative small size of the plasma (radius equal only a few ion gyroradii).

The code can be adapted to study particle behavior in any related geometry, thus potential users would include:

- 1) FRM reactor design group such as those at Lawrence Livermore Lab.
- 2) Continuing reactor design studies going on at Illinois.
- 3) Field-Reversed Theta Pinch studies such as those underway at Los Alamos, Illinois, and Math. Sciences, NW.
- 4) Studies related to the SPHEROMAK concept at Princeton.

The code has already been applied in the following studies:

- 1) The LLL-FRM reactor design.
- 2) The Moving Ring FRM design at LLL.
- 3) The SAFFIRE studies at Illinois.

For further study and examples of applications see:

1. D. Driemeyer, G. H. Miley, and W. C. Condit, Proc. ANS Topical Meeting on Computational Methods in Nuclear Engineering, Vol. 2, 7-37 (1979).
2. D. Driemeyer, G. H. Miley, M. Y. Wang, and W. C. Condit, Proc. Annual Controlled Fusion Theory Conf., p. 3 (1978).
3. D. Driemeyer, M. Ohnishi, and G. H. Miley, "Studies of Fusion Product Energy Distributions in Field-Reversed Mirrors," Submitted ANS Winter Meeting (1979).

VI.2 Reversed Field Configurations

a) Field-Reversed Mirror

Name: FROST: Field-Reversal One-dimensional Start-up Code

Authors: R. Stark and T. Haill

Description and Status: FROST is a one-dimensional radial code to simulate the start-up of the field-reversed mirror by neutral-beam injection. A hybrid plasma is used:

- The time development of the distribution function of the high energy ions (from the neutral beam) is followed over a grid in energy-canonical angular momentum (P_c) space. At each time step current and density profiles with radius are constructed from this distribution by averaging over particle orbits.
- The background electrons and ions are modeled with fluid equations. The field null region is treated separately to allow the inclusion of various models for the electron current.

Separate codes for the ring ions and the background plasma are now operational, in simplified form. The merging of these two codes to form FROST is targeted in Oct. 1979. During the next contract period, the code will be modified to include a radial electric field, thermal conduction effects and axial(Z) dependence of the plasma, field and ring. Models for the electron current near the field null will also be explored.

Projected Availability Date: October 1979

We will make requested runs.

Potential Users:

β -II experiment, LLL-FRM group,

EPRI SAFFIRE group

b) Moving Plasmoid Heater

1) Name: PHEBUS (Plasma HEating and Burn Simulation)

Authors: J. Galambos and R. Olson

- 2) PHEBUS is a finite difference, time dependent, 0-D code modeling the heating (via neutral beams) and burn of a compact torus plasma. It has capabilities for plasma species of electrons, protons, D, T, ^3He , and(or) ^4He . Neutral beam species are D, T, and/or ^3He . There are capabilities for n-energy groups per specie; a Maxwellian thermal distribution and n-1 delta distribution superthermal groups (presently n=2 is being used). Options include Coulomb or anomalous slowing down, and classical vs. anomalous particle diffusion.
- 3) It is now on the A-Machine (CTR 7600).
- 4) Anyone interested in startup and burn of small closed-field devices could use it (i.e., CTR-12 Group at LASL).
- 5) It is now being used to evaluate heating and burn of an MPH.
- 6) Publications: "Moving Plasmoid Heater Concept," ANS Transactions, June (1979), Atlanta, GA, and "Moving Plasmoid Heater Concept," IEEE Trans., Montreal Canada, June (1979).

c) 1-D MHD Code for the Reversed-Field Pinch (RFPBRN)

Author: R. Nebel

- RFPBRN is a one-dimensional, quasi-static, Lagrangian, resistive MHD code. It models the time evolution of the field, density, and temperature profiles of an RFP discharge. It includes an energy principle check for ideal MHD stability using an approximate Rayleigh-Ritz technique. The code also monitors local stability using Suydam's criterion which is assumed to result in anomalous (Bohm-like) transport. The code could be used to model any linear or large aspect ratio axisymmetric device. Boundary conditions have the option of field programming or flux conservation.
- RFPBRN is available on both the CTR 7600 and the CRAY.
- It should be of interest to both reactor studies groups and experimental groups working on RFP concepts.
- RFPBRN is currently being used in collaboration with division CTR-12 at LASL to study the RFPR reactor. Work will soon be expanded to include the ZT-40 experiment parameters.
- Further details and example applications are presented in the following references.

R.A. Nebel, G.H. Miley, and R.W. Moses, "1-D Transport Model for a Reversed Field Pinch," APS, 23, 4S11, p. 811, Colorado Springs, Colorado, (1978).

R.A. Nebel, G.H. Miley, and R.W. Moses, "1-D Reversed Field Pinch Burn Simulations," 1979 Sherwood Meeting, Mount Pocono, PA, April 18-20, (1979).

R.A. Nebel, G.H. Miley, and R.W. Moses, "Reversed Field Pinch Burn Dynamics," IEEE International Conference on Plasma Science, Montreal, Canada, 1B2, P. 6, (1979).

G.H. Miley, R.A. Nebel, and R.W. Moses, "Reversed-Field Pinch Plasma Model," 9th European Conference on Controlled Fusion and Plasma Physics, Oxford, UK, 17-21 Sept. (1979).

d) Rotation Effects in Field-Reversed Theta Pinch (F RTP)

Name of Code: RMF RTP (Rotational Model for Field Reversed Theta Pinch)

Author: Que-Tsang Fang

Purpose of Code: To calculate the time evolution of the guiding center and total drift angular frequency, and equilibrium radial electric field in F RTP.

Possible External User: LASL

Description of Code:

Input Parameters

MI	ion mass (kg)
ME	electron mass (kg)
T	ion temperature (eV)
TX	electron temperature (eV)
TN	neutral temperature (eV)
SSI	ion-neutral collision cross-section ($\times 10^{-15} \text{cm}^2$)
XX	impurity concentration (% of ion density)
Z	average charge of ions
ZEF	effective charge of plasma
NM	peak ion density (m^{-3})
R	plasma major radius (m)
K	field reversal parameter
B	initial external field (Tesla)
TM	time limit of run ($\times 10^{-6} \text{sec}$)
HH	time step increment of computation ($\times 10^{-6} \text{sc}$)
M	ratio of plasma outer radius to major radius.
WI	total ion drift freq. (sec^{-1})
WE	total electron drift freq. (sec^{-1})
AF	ratio of total ion drift to diamagnetic drift freq.
F, G	$E_r = F_r \tanh K \left(\frac{r^2}{R^2} - 1 \right) + G_r, \text{ V/m}^2$

VI.3 Mirror Codes

a) Mirror Plasma Dynamics

Name: FOREMD

Author: M. Campbell

The FOREMD (Finite Orbital Radial, Energy and Magnetic-field Dependent) code is a time dependent, modified Fokker-Planck model which treats finite ion gyro-orbit effects on plasma buildup in a small mirror fusion device. The plasma is radially subdivided into concentric shells with a "time-averaged" density in each shell found as the sum of contributions from ions whose orbits pass through it. Particle interactions (scattering, slowing, fusion, cold neutral charge-exchange, beam trapping, etc.) are calculated using this summed, energy-dependent density distribution in each shell. Since high energy (large orbit) ions contribute to the density in more than one shell, the gains or losses of a given shell will affect the densities in adjacent shells, and this is taken into account. Local magnetic field effects on ion orbits and beam trapping are also treated. The FOREMD code is a marked improvement over earlier finite orbit models (by R. Moir, B. Stallard, S. DeVoto at L.L.L.) because of the inclusion of a self-consistent energy and magnetics treatment (both of which were inputted averaged values in earlier models) and an improved treatment of neutral beam trapping.

The FOREMD code is in the final testing stages and has produced some preliminary results which are currently under examination. It is available on both the CDC-7600 and the CRAY-I MFEC machines at Livermore, however, the CRAY version is much faster and will likely be the only one to be used extensively. This code is useful for accurately simulating plasma buildup in

any small mirror device where finite-ion orbit effects are important. Potential users include MFTF and TMX groups at Livermore. Simulations of 2X-IIB and MFTF are currently underway and results will be made available to parties at Livermore (e.g., T. Rognlien, B. Stallard, R. Moir, A. Futch, S. DeVoto) and others who have expressed great interest in the work at various universities and research centers in the U.S.

Publications of methodology and preliminary results include previous progress reports, papers presented at IEEE and ANS meetings (1978 and 1979) and at the 1978 Sherwood Meeting. A related paper is now under review for publication in Nuclear Fusion.

b) Neutral Beam Injection

Name: NUBIN

Author: R. Miller

The Neutral Beam Injection (NUBIN) Monte Carlo (MC) simulation code is schematically illustrated in Fig. 1. The code begins with the specification of the plasma model. The user supplied input parameters are then obtained to specify the particular problem being considered. The total number of neutral particle histories tracked ($\sim 5(10)^4$) is divided into subgroups (cycles) of convenient length ($\sim 5(10)^3$) so that intermediate results can be obtained and convergence tests can be applied. For any particular test particle the simulation loop is followed until one of two termination conditions is satisfied: a) the test particle crosses the outer plasma boundary whereupon it is extrapolated to the first wall for tabulation of the bombardment, or b) the test-particle weight-value falls below a critical threshold and the particle is "killed" in a supplementary Russian Roulette procedure.

MONTE CARLO SIMULATION SCHEME

Figure 1. Computation flow diagram for the MC simulation code NUBIN.

At the end of the final particle cycle code results are processed for output and archival storage in the event the particular simulation should be restarted and continued in order to perhaps improve the convergence. An alternative procedure is to optionally readjust plasma conditions in view of the MC results and thus obtain a more self-consistent simulation. This latter option has not been implemented.

Code output consists of tables and graphs describing various key aspects of the beam trapping and first-wall bombardment results.

VII. Staff Contributors and Acknowledgements

In addition to Professor G. H. Miley, who served as principal investigator, Professors Chan Choi and John Gilligan were instrumental contributors to this project. Research assistants and their areas of involvement were: Dan Driemeyer (Global Code and Fusion Product Transport in FRM); Jack DeVeaux (Refueling of FRM); Ed Morse (FRM Stability); John Galambos and Robert Stark (Start-up of FRM); Rick Nebel (Dynamic Stimulation of RFP); Q-T Fang (Rotation in FRTP); John Galambos and Rick Olson (MPH Concept); Lee Hively (Fusion Product Loss Analysis); George Fenske and Lee Hively (Wall Bombardment Effects); Wesley Downum (Impurity Buildup); Mark Campbell (Mirror Plasma Buildup); Ron Miller (Injection in Mirrors).

Various parts of the research were carried out in close collaboration with researchers at National Laboratories including: Dr. J. Rome (ORNL), Drs. G. Carlson and W. Condit (LLL), Drs. R. Krakowski, R. Moses, R. Hagenson and R. Linford (LASL), Dr. M. Kaminsky (ANL), Drs. H. Furth, A. Todd, and H. Handel (PPPL), and Dr. L. Steinhauer (MSNW). Their vital contributions are gratefully acknowledged.

The expert typing of Mrs. Chris Stalker, Secretary for the Fusion Studies Laboratory, is also gratefully acknowledged.

This work was suggested by the Development and Technology Division of the Office of Magnetic Fusion Energy. Mr. Michael Murphy provided excellent encouragement and leadership in his role as contract monitor.

VIII. Report Listing

Prior progress reports in this series were issued as follows:

C00-2218-41 October 1, 1975 - Sept. 30, 1976

C00-2218-91 October 1, 1976 - Sept. 30, 1977

C00-2218-118 October 1, 1977 - Sept. 30, 1978

Individual reports issued prior to October 1978 are summarized in the above reports. Those issued during the present contract period are included in the following listing. Copies of any of these reports can be obtained by writing the principal investigator.

C00-2218 Report Listing, October 1978-September 1979

- C00-2218-124 "A Monte Carlo Method for Calculating Fusion Product Behavior in Field-Reversed Mirrors"
(D. Driemeyer, G. Miley, and W. Condit, *ANS Topical Meeting on Computational Methods in Nuclear Engineering*, Williamsburg, VA, 1979) (ABSTRACT)
- C00-2218-125 "Proposed Experimental Study of Fusion Product Losses from PLT and TFTR Plasmas"
(L. Hively and G. H. Miley, internal report, October 1978)
- C00-2218-126 "Transport of Alpha-Particle Ash in Tokamak with Radial Electric Field"
(G. H. Miley, W. B. Downum, and C. K. Choi, *Conference on Atomic Processes in High Temperature Plasmas*, Boulder, Colorado, January 17-19, 1979)
- C00-2218-127 "A Monte Carlo Method for Calculating Fusion Product Behavior in Field-Reversed Mirrors"
(D. Driemeyer, G. H. Miley, and W. C. Condit, *ANS Topical Meeting Computational Methods in Nuclear Engineering*, Vol. 2, 7-37, 1979)
- C00-2218-128 "Reversed Field Pinch Burn Dynamics"
(R. A. Nebel, G. H. Miley, and R. W. Moses, *IEEE International Conference on Plasma Science*, Montreal, Canada, 5Q1, p. 146, 1979)
- C00-2218-129 "Deposition and Buildup of Alpha Particles in Tokamak"
(W. B. Downum, C. K. Choi, and G. H. Miley, *IEEE International Conference on Plasma Science*, Montreal, Canada, 2B11, p. 40, 1979)
- C00-2218-130 "A Finite Orbit Treatment of Plasma Buildup in Small Mirrors"
(M. Campbell and G. H. Miley, *IEEE International Conference on Plasma Science*, Montreal, Canada, 1B2, p. 6, 1979)
- C00-2218-131 "1-D Reverse-Field Pinch Burn Simulations"
(R. A. Nebel, G. H. Miley, and R. W. Moses, *1979 Sherwood Meeting*, Mount Pocono, PA, April 18-20, 1979)

- C00-2218-132 "A Monte Carlo Model of Particle Motion in Field-Reversed Mirrors - MCFRM"
(D. E. Driemeyer, G. H. Miley, and W. C. Condit, 1979 *Sherwood Meeting*, Mount Pocono, PA, April 18-20, 1979)
- C00-2218-133 "Monte Carlo Simulation of Neutral-Beam Injection for Mirror Fusion Reactors"
(Ronald Lee Miller, Ph.D. Thesis, Nuclear Engineering, University of Illinois, Urbana, Illinois, 1979)
- C00-2218-134 "Reversed-Field Pinch Plasma Model"
(G. H. Miley, R. A. Nebel, and R. W. Moses, 9th *European Conference on Controlled Fusion and Plasma Physics*, Oxford, UK, 17-21 Sept., 1979) (ABSTRACT)
- C00-2218-135 "Studies of Finite Gyro-Radius Effects in a Field-Reversed Mirror Configuration"
(G. H. Miley, J. Gilligan, D. Driemeyer, and W. C. Condit, 9th *European Conference on Controlled Fusion and Plasma Physics*, Oxford, UK, 17-21 Sept., 1979) (ABSTRACT)
- C00-2218-136 "Stability of the Field-Reversed Mirror"
(Edward Clemens Morse, Ph.D. Thesis, Nuclear Engineering, University of Illinois, Urbana, Illinois, 1979)
- C00-2218-137 "Studies of Finite Gyro-Radius Effects in a Field-Reversed Mirror Configuration"
(G. H. Miley, J. G. Gilligan, D. E. Driemeyer, E. C. Morse, and W. C. Condit, 9th *European Conference on Controlled Fusion and Plasma Physics*, Oxford, UK, 17-21 Sept. 1979)
- C00-2218-138 "Reversed-Field Pinch Plasma Model"
(G. H. Miley, R. A. Nebel, and R. W. Moses, 9th *European Conference on Controlled Fusion and Plasma Physics*, Oxford, UK, 17-21 Sept. 1979)
- C00-2218-139 "A Finite Orbit Treatment of Plasma Buildup in Small Mirrors"
(M. Campbell, G. H. Miley, *Trans. Am. Nuc. Soc.*, San Francisco, CA, Nov. 11-16, 1979)
- C00-2218-140 "Fusion-Product Ash Buildup in Tokamak with Radial Electric Field"
(W. B. Downum, C. K. Choi, G. H. Miley, *Trans. Am. Nuc. Soc.*, San Francisco, CA, Nov. 11-16, 1979)

- C00-2218-519 "Depth Distribution of Bubbles in 20-keV $^4\text{He}^+$ Irradiated Nickel"
(G. Fenske, S. K. Das, M. Kaminsky, and G. H. Miley, *Trans. Am. Nucl. Soc.*, 28, 196 (1978))
- C00-2218-520 "Toroidal Plasma Rotation Driven by Fusion Product Losses"
(Thomas W. Petrie and George H. Miley, submitted to *Nuclear Fusion*, November 1978)
- C00-2218-521 "Application of a Unified Slowing-Down Theory to Fusion Plasmas"
(C. Choi., C. Bathke, and G. Miley, *APS*, Chicago, February 1977)
- C00-2218-522 "Effect of Hot-Beam Injection-Angle in a Twin-Beam Mirror"
(M. Campbell and G. H. Miley, submitted to *Nuclear Fusion* January 1979)
- C00-2218-523 "The Effect of Dose on the Evolution of Cavities in 500-keV $^4\text{He}^+$ Ion Irradiated Nickel"
(G. Fenske, S. K. Das, M. Kaminsky, and G. H. Miley, *Proceedings of the First Topical Meeting on Fusion Reactor Materials*, Miami Beach, Florida, January 29-31, 1979) (ABSTRACT and PREPRINT)
- C00-2218-524 "The Effect of Alpha Incident- and Poloidal-Angle Distributions on Blister-Induced First-Wall Erosion"
(G. Fenske, L. Hively, G. Miley, and M. Kaminsky, *Proceedings of the First Topical Meeting on Fusion Reactor Materials*, Miami Beach, Florida, January 29-31, 1979)
- C00-2218-525 "Comparison of Calculations of Fusion Product Losses From A Tokamak"
(L. M. Hively and G. H. Miley, submitted as *Letter to Nuclear Fusion*, June 1, 1979)
- C00-2218-526 "Deposition and Thermalization of Fusion Products in a Tokamak Plasma"
(Thomas W. Petrie and George H. Miley, submitted to *IEEE Transactions on Plasma Science*, July 1979)

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