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**INCREMENTAL STRESS-STRAIN LAW
FOR GRAPHITE UNDER MULTIAXIAL LOADINGS**

by
FU-KONG TZUNG

Prepared under
Contract DE-AT03-76ET35300
for the San Francisco Operations Office
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ABSTRACT

An incremental stress-strain law for describing the nonlinear, compressible and asymmetric behavior of graphite under tension and compression as well as complex loadings is derived based on a dry friction model in the theory of plasticity. Stress-strain relations are defined by longitudinal-lateral strain measurements for specimens under uniaxial tension-compression. Agreements with experimentally determined curves from biaxial loading experiments are shown. Agreements in finite element computations using the present model with strain measurements for diametral compression and 4-point bend tests of graphite are also obtained.

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SUMMARY

An incremental stress-strain law to describe the behavior of graphite under complex loadings has been derived and its predictions compared to a limited set of experimental data. The comparison is quite good and suggests that this theoretical approach should be pursued further.

The derivation is similar to that of the classical theory of plasticity inasmuch as a loading function and a hardening rule are established and in that kinematic hardening can be used to model hysteresis effects. It differs, however, in that the hardening function is determined by only a portion of, not the entire, loading function. This refinement, which is mathematically rigorous, adds the flexibility needed to treat the nonlinear, compressible and asymmetric deformation behavior of graphite under either tension or compression.

The theory has been tested by comparing its predictions to existing experimental data from diametral compression tests and 4-point bend tests for H-327 graphite. Material constants were determined from uniaxial tensile tests and then applied to the analysis of the experiments. The agreement with experimentally determined stress-strain curves from biaxial loading experiments was good and, as would be expected, superior to that achievable with a linear elastic model. Also, failure stresses were better predicted, i.e. they were closer to the measured ultimate tensile strength. Although more experimental data are needed before any firm conclusion can be drawn, it does appear that this theory might well provide a substantially improved framework for design analysis of graphite components.

INTRODUCTION

Since graphite is used extensively in the core and reactor internals of an HTGR, it is important that analytical models be available to predict the stress levels that result from the multi-axial thermal and mechanical loadings to which the graphite components are subjected. Current design techniques make use of a linear elastic model to describe the behavior of graphite. Although this approach, when coupled with suitable design conservatism, is adequate, past comparisons of its predictions with experimental data have not been wholly satisfactory. This report describes a study undertaken to see if a more sophisticated model of the behavior of graphite could improve the situation.

Classical plasticity assumes a yield stress and an incompressible plastic deformation — two assumptions that are inconsistent with experimental observations that the deformation of graphite is essentially non-linear and compressible from zero stress to failure stress as well as asymmetric in tension and compression. Therefore, this theory is not likely to provide satisfactory results for graphite.

Deformation type theories, such as those of Weng [2], Jones and Nelson [3] and Batdorf [4], have been proposed. Since these theories define strain as a unique function of stress, they are unsuitable for general design analyses which must model the hysteresis effects that will occur during operational cycling in a reactor.

Rate type theories were proposed by Greenstreet, Yahr, and Valanchovic [5], Greenstreet and Philips [6], and Chang, Reich and Suzuki [7]. In [5], the theory is applicable only in the tension quadrant; [6] only addresses uniaxial or proportional loadings; and [7] did not define the relationship among incremental strain, stress and plastic strain explicitly enough to permit practical applications in the absence of an extensive experimental program.

On the basis of this review, an incremental strain law based upon a dry friction model was formulated. The derivation of this model and the results of its application to several experiments are described below.

DERIVATION OF AN INCREMENTAL STRESS-STRAIN RELATION

To establish a material model to describe the response of graphite under stress, consider the dry friction model shown in Figure 1. Just prior to motion, the equality

$$\vec{f} + \mu \vec{N} = \mu \vec{W} , \quad (1)$$

must be satisfied.

Fig. 1. Dry Friction Model

The direction of the motion is related to the shear force $\vec{f}$ and independent of the normal force $\vec{N}$. To generalize this concept for a relation between the stress σ_{ij} and the inelastic strain tensors ϵ_{ij}^P ($i, j=1, 2, 3$), we define a loading function

$$F(\sigma_{ij}, \epsilon_{ij}^P, k) \equiv f(\sigma_{ij}) + \mu N(\sigma_{ij}) - k(\epsilon_{ij}^P) , \quad (2)$$

and a hardening rule

$$d\epsilon_{ij}^P = \frac{\partial f}{\partial \sigma_{ij}} d\lambda . \quad (3)$$

The hardening rule defined above differs from the flow rule of classical plasticity theory by using $\partial f / \partial \sigma_{ij}$ instead of $\partial F / \partial \sigma_{ij}$ in Eq. (3). This is permissible for the same reason that the direction of the block motion in Fig. 1 depends on the force $\vec{f}$ but not on the force $\vec{N}$. Thermodynamic restrictions are clearly satisfied. Indeed mathematic proof of this statement can be found in [8] for a more general case. Since the cycling of graphite between zero stress and preload stress does not change the stress-strain hysteresis appreciably [1], the loading function has been assumed to be a function of the state variable σ_{ij} and ϵ_{ij}^p in Eq. (2). A simple expression for $f(\sigma_{ij})$, $N(\sigma_{ij})$ and $k(\epsilon_{ij}^p)$ that enables one to treat the different volume changes and the asymmetric behavior of graphite in tension and compression is given in the following:

$$f(\sigma_{ij}) = B \left\{ C_{ijkl}^* \sigma_{ij} \sigma_{kl} + \frac{1}{2} \gamma_{ij} \sigma_{ij} \left[\gamma_{kl} \sigma_{kl} + |\gamma_{kl} \sigma_{kl}| \right] \right\}^{n/2},$$

$$N(\sigma_{ij}) = B \sigma_{ii} |\sigma_{jj}|^{n-1}, \quad n > 0$$

$$k(\epsilon_{ij}^p) = (\epsilon_{ij}^p \epsilon_{ij}^p)^{1/2} \quad (4)$$

where $i, j, k, l = 1, 2, 3$, and a repeated subscript indicates summation.

The constants C_{ijkl}^* must satisfy the condition that $F(\sigma_{ij}, \epsilon_{ij}^p, k) \geq 0$ for all σ_{ij} and ϵ_{ij}^p , and $|\gamma_{kl} \sigma_{kl}|$ denotes the absolute value of the argument. The value of $d\lambda$ is obtained here as follows:

$$d\lambda = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} \left/ \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial k}{\partial \epsilon_{kl}^p} \right., \text{ for } F = 0 \text{ and } \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} \geq 0$$

$$= 0, \text{ for } \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} < 0 \text{ or } F < 0. \quad (5)$$

For monotonic proportional loadings, the inelastic incremental strain components ϵ_{ij}^p defined by (2), (3) and (4) are also proportional. The total plastic strain component ϵ_{ij}^p can therefore be obtained by an integration of (3),

$$\epsilon_{ij}^p = (f + \mu N) \frac{\partial f}{\partial \sigma_{ij}} \left/ \left(\frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}} \right)^{1/2} \right. . \quad (6)$$

The total strain components ϵ_{ij} are therefore related to the stress components σ_{ij} as,

$$\epsilon_{ij} = C_{ijkl} \sigma_{kl} + \epsilon_{ij}^p, \quad (7)$$

where C_{ijkl} are the elastic compliances.

To approximate the hysteresis behavior of graphite under cyclic loading, Prager's kinematic hardening rule [9] can be used, i.e.,

$$f \equiv f(\sigma_{ij} - c\epsilon_{ij}^p) \quad (8)$$

replaces $f \equiv f(\sigma_{ij})$ in (2) and (4). Although the precise description for the hysteresis, see [6], may not be attainable, the present hardening rule offers a practical and unambiguous treatment for a specimen subjected to complex three-dimensional loadings.

APPLICATION OF THE THEORY

To apply the theory, one must experimentally determine values for C_{ijkl}^* , C_{ijkl} and γ_{ij} .

For graphite, this task can be simplified by making use of the experimental observation that the transverse-to-longitudinal strain ratio remains constant in uniaxial compression tests but decreases with increasing stress in uniaxial tension tests. This suggests the following approximations:

$$C_{ijkl}^* = \frac{C_{ijkl}}{C_{1111}} ,$$

$$\gamma_{11} = \gamma_{22} = \sqrt{-C_{1122}^*} ,$$

$$\gamma_{33} = C_{1133}^* / \gamma_{11} .$$

With these approximations, Eq. (6), written for uniaxial monotonic loadings, reduces to

$$\epsilon_{11}^p = \left[\left(1 + \gamma_{11}^2 \right)^{\frac{n}{2}} + \mu \right] B \sigma_{11}^n , \text{ for } \sigma_{11} \text{ in tension} ,$$

$$= \left[1 - \mu \right] B \sigma_{11} |\sigma_{11}|^{n-1} , \text{ for } \sigma_{11} \text{ in compression} .$$

This provides a way to use relatively simple uniaxial tests to determine μ , B , and n for subsequent applications. Note that this also establishes the relation that the plastic strain is proportional to the n th power of the stress.

BIAXIAL LOADING COMPARISONS

As a first test of the theory, its predictions were compared with experimental data for ATJ graphite under biaxial loadings [2].

For specimens subjected to monotonic-proportional-biaxial stresses σ_{11} and σ_{33} ($= m \sigma_{11}$), Eq. (7) reduces to

$$\epsilon_{11} = A_1 \sigma_{11} + B_1 \sigma_{11} |\sigma_{11}|^{n-1}$$

$$\epsilon_{33} = A_3 \sigma_{33} + B_3 \sigma_{33} |\sigma_{33}|^{n-1} ,$$

where

$$A_1 = C_{1111} + m C_{1133}$$

$$A_3 = C_{3333} + C_{3311}/m .$$

In the absence of uniaxial test data, the following assumptions were made:

$$\mu = 0.1$$

$$B = 5.5 \times 10^{-11} (\text{psi})^{-n}$$

$$n = 2 .$$

With these assumptions, Eqs. (4), (6), (7) and (9) were used to determine values of B_1 and B_3 for four different values of m , the stress ratio. The comparison with the experimental data is shown in Figures 2 and 3. The agreement is good throughout the range analyzed and suggests that the theory is indeed applicable to graphite.

Figure 2. Comparison of experimental data (X, ∇ for $\sigma_{33}/\sigma_{11} = 0.43/-1.$, $\triangle$, $\square$ for $\sigma_{33}/\sigma_{11} = -1.05/1.$) and theoretical curves.

Figure 3. Comparison of experimental data ($\square, \triangle$ for $\sigma_{33}/\sigma_{11} = 0.5/1$, $\nabla, \times$ for $\sigma_{33}/\sigma_{11} = -0.66/-1$.) and theoretical curves.

FINITE ELEMENT ANALYSES OF 4-POINT BEND AND DIAMETRAL COMPRESSION TESTS

To provide a tool for further testing, a finite element program, GTEPC-2D [11], was modified to include the incremental stress-strain relationships defined by Eqs. (2)-(6) within a solution scheme based upon the initial stress method [12] for modelling inelastic deformations.

This program was then used to analyze the 4-point bend test and the three diametral compression tests upon H-327 graphite described in [13]. Figures 4 and 5 show the finite element idealizations used in this comparison. The numerical treatment of contact phenomena followed the procedure discussed in [14].

The uniaxial tensile test data shown in Fig. 6 were used to generate the required material constants. From the figure, it is apparent that the data are well fit by the formula

$$\epsilon_{11} = 1.163 \times 10^{-6} \sigma_{11} + 2 \times 10^{-13} \sigma_{11}^3$$

where σ_{11} is in psi. This fit, together with the experimental observation that the transverse-to-longitudinal strain ratio observed during the test varied between 0.04 and 0.09 [15], suggests the following approximations

$$C_{1111}^* = C_{2222}^* = 1 \quad , \quad C_{3333}^* = 0.4444 \quad ,$$

$$C_{1122}^* = C_{2211}^* = -0.09 \quad ,$$

$$C_{1133}^* = C_{3311}^* = C_{2233}^* = C_{3322}^* = -0.04 \quad ,$$

$$\gamma_{11} = \gamma_{22} = 0.3 \quad , \quad \gamma_{33} = C_{1133}^* / \gamma_{11} \quad , \quad \mu = 0 \quad ,$$

$$n = 3 \quad , \quad B = 1.7575 \times 10^{-13} \text{ (psi)}^{-n} \quad . \quad (13)$$

Figure 4. Finite element idealization for 4-point bend of graphite beam.

Figure 5. Finite element idealization for diametral compression of graphite ring.

Figure 6. Uniaxial stress-strain relation for H-327 graphite

Furthermore, since the C_{ijkl}^* tensor is proportional to the elastic compliance C_{ijkl} , we have

$$C_{ijkl} = 1.163 \times 10^{-6} C_{ijkl}^*$$

which completes the selection of material constants.

It should be noted that since the underlying uniaxial test data were only available up to a maximum stress level of ~1500 psi, the results that follow should be interpreted with this implicit limitation in mind.

Figures 7 and 8 show the comparison for the four point bend test. Deflection is well predicted over the entire range of loadings but the strain is overpredicted for higher loadings. Since the stresses at these loadings are outside the data base, it is not clear whether the material data or the theory are at fault or even if the results lie outside the experimental error bounds.

Figures 9, 10, and 11 display the comparison for a compression test on a graphite ring with 2.2 inch ID and a 2.75 inch OD. Here the strain is well predicted over the entire range of loadings but the deflection is underpredicted at the higher loadings. The strain comparisons are especially interesting because they indicate a substantial experimental uncertainty.

Figures 12, 13 and 14 show a similar comparison for a thicker ring — the ID is 1.375". The deflection is now well predicted as is the strain.

Table I contains a comparison of the predictions and the measured data at failure. Also included are the maximum tensile stresses predicted by the linear model currently in use. Both models calculate a failure stress higher than the measured ultimate tensile strength, which is typically 1500 psi, but the new model is ~30% closer to this value. Thus it represents a substantial improvement.

The last two rows summarize two other comparisons. One is a very thick ring, an I.D. of 0.5". Here the agreement is worsened although the new model retains its superiority. The other is an experiment similar to that summarized in row 2 but which was performed without a strain gage. Note that the failure load is decreased by ~15%. This suggests that the presence of a strain gage can alter the experimental results significantly. The predicted results, which were obtained using consistent material property data, are in better agreement than in the cases where a strain gage was used. It appears possible that this might be because this experiment was more homogeneous than the others but there is not enough data to support this statistically.

Figure 7. Comparison of deflection measurements ($\square$) and predictions for H-327 graphite under 4-point bend.

Figure 8. Comparison of strain measurements on the top (+) and the bottom (Δ) surfaces at the mid-span of the beam with predictions for H-327 graphite under 4-point bend.

Figure 9. Comparison of deflection measurements ($\square$) and predictions for graphite ring of 2.2 inches inside and 2.75 inches outside diameters under diametral compression.

Figure 10. Comparison of strain measurements ($\square, \circ$) at location A and predictions for graphite ring of 2.2 inches inside diameter and 2.75 inches outside diameter under diametral compression.

Figure 11. Comparison of strain measurements at location B ($\square$ and $\circ$) and C (+ and X) with predictions for graphite ring of 2.2 inches inside and 2.75 inches outside diameters under diametral compression.

Figure 12. Comparison of deflection measurements ($\square$) and predictions for graphite ring of 1.375 inches inside and 2.75 inches outside diameters under diametral compression.

Figure 13. Comparison of strain measurements ($\square$) at location A and predictions for graphite ring of 1.375 inches inside diameter and 2.75 inches outside diameters under diametral compression.

Figure 14. Comparison of strain measurements at location B ($\bigcirc$) and C ($\square$) with predictions for graphite ring of 1.375 inches inside and 2.75 inches outside diameters under diametral compression.

Table I

Comparison of Predictions and Measurements for 4-Point Bend
and Diametral Tests

Specimen	Measured			Present Theory			Linear Elastic Theory
	Failure Load lbs	Deflection mils	Strain at A %	Max Tensile Stress psi	Deflection mils	Strain at A %	Max Tensile Stress psi
4-Point Bend	32.9	16.1	0.35	2163	15.3	0.44	2654
Diametral Compression $d_i^* = 2.2"$	64.2	26.3	0.45	2167	20.0	0.45	2745
Diametral Compression $d_i = 1.375"$	434.7	12.8	0.45	2219	12.6	0.48	2725
Diametral Compression $d_i = 0.5"$	1633.0	36.0	0.68	2715	36.0	0.82	3642
Diametral Compression $d_i = 2.2"$ Without Strain Gage	55.0	22.0	--	1845	19.3	0.43	2175

* d_i = inside diameter of ring specimen.

CONCLUSION

The present study indicates that a modified classical plasticity theory can be used to predict the response of graphite with reasonable accuracy. This simply confirms the result in Ref. [5]. The maximum principal stress failure criterion appears to be consistent for the three cases studied. It may require more analyses as well as careful data measurement to answer the question whether classical theory is indeed applicable to graphite [16]. This theory certainly offers an attractive alternative to linear elastic theory in design analyses for graphite components.

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