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PERFORMANCE AND FEASIBILITY OF FORCED GEOHEAT
RECOVERY FOR LOW TEMPERATURE APPLICATIONS

First Technical Status Report

MASTER

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Brief Restatement of Objectives

A basic evaluation of the technical performances and economic feasibility of providing low temperature heating by Forced Geoheat Recovery (FGR) from suitable geological structures including both thin sheet conductors with predominantly fracture flow and porous slab conductors with intergranular flow. Simple, heat-pump assisted and multi-source combined heating systems will be evaluated. Building and industrial heating applications with effective power requirements ranging upward from one MWt will be considered, with thermal water supply temperatures in the range of 25 C to 100+C. Emphasis is being placed on the adaption of the FGR technology to the user system, including both short and long term variations in the user system demand.

PROGRESS REPORT FOR THE PERIOD APRIL 1 to JUNE 30 1978

Personnel

Gunnar Bodvarsson has devoted three months at 1/3 time to the project. Gordon M. Reistad started working on the project on June 15 and has devoted 1/4 of his time during the period June 15 to June 30. Greg Mack, Graduate Research Assistant in Geology, has devoted three months at 1/2 time. One Research Associate and another Graduate Research Assistant have been hired on the project.

Progress Summary

Initial efforts have been directed at the following areas.

Gunnar Bodvarsson

(1) Forced Geoheat Recovery (FGR) resource system design and econometrics.

(2) Development of elastomechanical methods of testing fracture dimensions and inter-borehole fluid conductance.

(3) Investigations of conditions for FGR in the Willamette and the Lower Columbia Basins in Western Oregon.

Gordon M. Reistad

(4) Literature review in district heating, heat pumps, etc.

(5) Examination of residential heating systems as a function of available water temperature.

(6) Development of layouts for potential types of geothermal heating schemes.

(7) Selection and documentation of climatic areas to be considered in the evaluation of systems.

Results on item (1) were communicated in a paper entitled "Subsurface dimensions and exploration targets in forced geoheat recovery" given at the Nordic Symposium on Geothermal Energy, Gothenburg, Sweden, May 29 to 31, 1978. An extended summary of this paper is given in Appendix I. Moreover, a paper entitled "Energy Currents in Fractured Geothermal Systems" reviewing this subject from a somewhat different point of view was given at the Hot-Dry-Rock Workshop, Los Alamos, New Mexico, April 20 and 21, 1978. An abstract of this paper is furnished in Appendix II. Some of the material presented in these papers has been drawn from earlier reports of Gunnar Bodvarsson on related subjects.

Theoretical aspects of item (2) were discussed in some detail in a paper entitled "Hydroelastic oscillations in fracture systems" given at the same Hot-Dry-Rock Workshop. An abstract of this paper and copies of the slides displayed are given in Appendix III below. Some of the mathematical aspects of the theory of hydroelastic oscillations presented in this talk were derived during work on a theoretically oriented project entitled "Hydroelastic oscillations in liquid subsurface systems" which is being supported by the National Science Foundation.

Our work on item (3) has consisted in a search for pertinent stratigraphic and physical data on the Willamette and Lower Columbia Basins in the literature and from other sources. Detailed maps of geological sections with available data on porosities and permeabilities are being constructed. The results are being supplemented with available data on temperature gradients and heat flow in the region.

Plans for the period July 1 to September 30, 1978

During the second quarter, we intend to continue our work on items (1) to (7) above. Item (3) will be extended to other prospective areas in the Pacific Northwest. The main emphasis will, however, be on the more engineering aspects (4) to (7).

Given at the Nordic Symposium on Geothermal Energy, Gothenburg, Sweden,
May 29 to 31, 1978.

APPENDIX I

SUBSURFACE DIMENSIONS AND EXPLORATION TARGETS IN FORCED GEOHEAT RECOVERY

by G. Bodvarsson

School of Oceanography, Oregon State University, Corvallis, OR 97331, USA

(1) Free and stimulated geoheat production

Geoheat is now being recovered for electrical power generation from a number of resources in thermally active regions. The most notable examples are the Geysers, California; Larderello, Italy, and Wairakei, New Zealand. Common to all these cases is that the energy is being produced by thermo-artesian free flowing boreholes. This type of operation may be termed as free geoheat production.

Large scale space heating by geoheat has been carried out in Iceland for more than three decades. The Reykjavik District Heating System, which now supplies energy for domestic heating for more than 100,000 people is supplied by thermal water from an extensive hydrothermal resource system in the vicinity of the city. The thermoartesian pressure is relatively low and large scale resource stimulation by borehole pumping is therefore being applied.

The free and stimulated production methods as described above are based on the presence of natural fluid conducting openings in the resource formations and on a natural convective recharge of the withdrawn fluid. Conditions favorable for the generation of such resources are, unfortunately, not widespread and appear, in fact, to be limited to a relatively few thermally active areas.

(2) Limitations on geoheat transport

Thermal waters and natural steam are bulky heat carriers which cannot be transported economically over long distances. In the case of power generation the limits are of the order of a few kilometers only. For direct uses such as space heating, the maximum distances may in extreme cases amount to a few tens of kilometers. At the present state-of-the-art where only natural convective/thermoartesian type sources are being harnessed, geoheat utilization, non-electrical uses, in particular, are therefore severely limited by the low transportability. The major convective sources are not favorably located with regard to the heat market.

(3) Forced geoheat recovery

The above mentioned limitations on the transportation of geoheat have prompted interest in new technologies based on the concept of forced geoheat recovery (FGR) (Bodvarsson, 1976), and which offer prospects of overcoming some of the difficulties stated above. The new methods are based on the creation of artificial recovery systems by the pumping of water through sufficiently hot rock and at relatively high pressures to maintain an adequate circulation for heat recovery. The injection is to be performed

through sufficiently deep boreholes into formations of suitable fluid conductivity and production is obtained through other boreholes located at a proper distance from the injection holes. For the operation of such systems to be economical, adequate contact areas or contact volumes between the circulating fluid and the hot rock have to be provided such that geoheat in commercial amounts can be recovered by the fluid.

There are mainly two possibilities of realizing FGR of the above type. First, by using artificial fluid conductors created entirely by hydraulic fracturing of solid crystalline rock. This type of FGR, shown in Figure 1, is now under thorough investigation in the U.S. by the Los Alamos Scientific Laboratory Dry-Hot-Rock Group in Los Alamos, New Mexico (NATO CCMS Report, 1977).

The second possibility involves the use of geological structures where an adequate fluid conductivity is either available, or can be induced with relative ease by the fluid pressure. Mainly two types of geological fluid conductors appear favorable for this type of FGR. First, the thin-sheet conductors with predominately fracture type conductivity such as fault/fracture zones, dikes and formation contacts. Typical examples involving vertical conductors with different flow directions are sketched in Figure 2 and 3. Horizontal conductors are shown in Figure 4. Second, the porous slabs with intergranular flow such as sedimentary horizons, as sketched in Figure 5. The various prospective conductors are listed in Table 1 below.

Table 1

Potential fluid conductors for forced geoheat recovery

<u>Type</u>	<u>Field observations on large scale fluid conductivity</u>	<u>Role in geoheat recovery</u>
<u>Thin-sheet conductors</u>		
(1) Fault zones	Many major geothermal systems are controlled by faults.	Borehole production obtained by intersecting fault zones.
(2) Basaltic dikes in flood-basalt areas.	Many geothermal systems in Iceland are controlled by dikes.	Boreholes in Central North Iceland produce by intersecting dikes.
(3) Other intrusions	Few data available, but columnar structure possibly indicative of conductivity.	Some production in Iceland appears to be obtained from thin basaltic sills.
(4) Formation contacts	Lava-bed contacts are major aquifers in the flood-basalt plateau of Iceland.	Major production in South-Western Iceland obtained from lava-bed contacts.
<u>Porous-slab conductors</u>		
(5) Sedimentary horizons	Many major sedimentary basins contain large volumes of hot permeable rock.	Large scale FGR from sedimentary basins in France (DGRST, 1976), and stimulated production in Hungary (Boldizsar, 1975)

Figure 1. FGR system with an artificial fluid conductor.

Figure 2. An upflow FGR system with a natural fluid conductor.

Figure 3. A downflow FGR system with a natural fluid conductor.

Figure 4. FGR system with a natural horizontal fluid conductor.

(4) Thin-sheet or fracture type FGR systems

Minimum contact areas. The size of the fluid-rock contact area required to produce a sufficient amount of hot fluid to amortize a given system investment depends critically on the following factors:

- temperature of the source formations
- thermal properties " " "
- fluid conductivity " " "
- drilling and equipment costs
- pumping power required to provide the necessary penetration and contact area
- fluid losses, scaling.

The minimum economic contact area can be estimated on the basis of an idealized model. We assume that the circulating fluid is water absorbing heat from the rock in uniform and unidirectional flow through an infinitesimally thin fracture in a large volume of homogeneous rock which is isothermal at the initiation of the process. Using theoretical results by Bodvarsson (1974), the contact area as a function of plant investment and value of the energy produced can easily be calculated. The results for a single borehole-pair producing heat for building heating are shown in Figure 6 and the corresponding results for electrical power generation are shown in Figure 7. In both figures the useful life of the system is assumed to be 20 years, the total amortization, operational and maintenance costs are 18% of capital per annum. Other factors are given in the figures. In the electrical case, the required power per borehole pair amounts to a few MW. These results apply to both natural and artificial fluid conductors.

The following rock and fluid parameters were used in the computations underlying Figures 6 and 7; rock thermal conductivity $k_r = 2.1 \text{ W/m, } ^\circ\text{C}$, rock density $\rho = 2700 \text{ kg/m}^3$, specific heat of rock $c_r = 1000 \text{ J/kg, } ^\circ\text{C}$, and specific heat of water $c_w = 4200 \text{ J/kg, } ^\circ\text{C}$. The injection temperature is 30°C .

Factors affecting efficiency. The estimates given in Figures 6 and 7 are based on an idealized model. Deviations from the assumed conditions will in one way or other affect the results and will have to be considered carefully.

Of considerable concern are three heavily interrelated phenomena affecting the flow of thermal water in subsurface natural conductors viz., (i) natural flow channeling, (ii) thermoelastic effects and (iii) buoyancy and convective effects. The quantitative theory of these effects in the natural environment is basically difficult and poorly developed. By nature, these flow phenomena are non-linear effects.

Table 2 has been designed to furnish a very brief qualitative overview of the adverse influence of the above three flow effects on system performance.

Moreover, rock temperatures are generally not uniform. The water may therefore flow along rock surfaces where the temperature varies in the direction of flow. Finally, the load on FGR systems will vary considerably, in

Figure 5. FGR system with a horizontal porous slab conductor.

Figure 6. Minimum contact areas for thin-sheet FGR systems for building heating.

Figure 7. Minimum contact areas for thin-sheet FGR systems for power generation.

Figure 8. Example of a simple sweep process in porous rock.

Useful model and approximation
for thin-sheet systems

T_0 = resource temperature
 T_1 = production temperature
 $\Delta T = T_1 - T_2 - T_{\text{loss}}$
 t_0 = operational lifetime
 q = peak fluid circulation
 a = load factor
 $m = aqt_0$ = total mass of water recovered
 $P = sq\Delta T$ = effective peak power
 s = heat capacity of water

Q = thermal water recovery/ m^2 contact area

for $\tau = (T_1 - T_2)/(T_0 - T_2) = 0.8$

$Q \approx 5t_0^{\frac{1}{2}}$ metric tons/ m^2 (t_0 in years)

C = capital costs

linearized approximation

$C \approx c_1P + c_2D = c_3P + c_4T_0$

$v = b\Delta T$ = value of thermal water reduced
 by operational and maintenance costs

A = minimum required fracture contact area
 (idealized model)

r = interest rate

$$A \approx \frac{Crt_0^{\frac{1}{2}}}{5v[1 - \exp(-rt_0)]}$$

minimum values

$$P \approx \frac{srC}{ab[1 - \exp(-rt_0)]}$$

THIN-SHEET FORCED GEOHEAT RECOVERY SYSTEMS

Minimum dimensions for economic operation

C = Total invested capital

v = Net value of thermal water produced
(operational and maintenance costs
subtracted) in \$/metric ton

$b = v / T$

Assumptions:

t_0 = operational life = 20 years

r = interest rate (real) = 0.06/year

Minimum contact volumes for porous slab systems

Summary:

Minimum subsurface dimensions for
economic operation:

Assume:

T_1	=	60	90 °C
v		0.15	0.6 \$/mt
D		2	3 km
C		2	3 M\$ (2-hole)

Thin-sheet systems:

A		1.2	0.4 km ²
P		8	6
q		80	30 kg/s

Porous slab systems:

V		5×10^7	$2 \times 10^7 \text{ m}^3$
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Small forced geoheat recovery systems

D = production depth

P = power MWt

Capital cost $\sim C_1P + C_2D$

Annual capital + OM costs $\sim a_1P + a_2D$

Total annual income: rP

$$rP > a_1P + a_2D$$

$$P > \frac{a_2}{r-a_1} D$$

Table 2

Adverse flow phenomena in thin-sheet conductors

<u>Type of phenomena</u>	<u>Inefficient heat recovery</u>	<u>Potential effects</u>	
		<u>Pumping power</u>	<u>Water losses</u>
(1) Non-uniform conductivity, flow channeling	Potential major factor	High pumping pressure may be required to overcome non-uniformity.	Can be a major factor in channeling injected cold water out of the heating zone.
(2) Thermoelastic effects	Enhances channeling of water colder than the rock.	Narrowing of fractures carrying water hotter than the rock requires increasing pumping pressure.	May increase water losses by enhanced channeling.
(3) Buoyancy and convection	Enhanced channeling in down-flow systems.		Downward convective penetration of cold water may enhance losses.

particular, in cases where the heat is to be used for building heating. A varying production rate will usually be required in such cases.

(5) Porous slab FGR systems

The main concepts in geohat recovery from porous slabs such as sedimentary horizons are given by Bodvarsson (1974). The theory and operational characteristics are somewhat simpler than in the case of the thin-sheet systems discussed above. In particular, a uniform permeability allows a very efficient recovery of sensible heat from the formation sections permeated by the circulating fluid.

At sufficient mass flow rates in homogeneous porous slab conductors that are a few tens of meters thick, at least, the heat recovery takes place by a simple sweep process. There is practically a perfect thermal contact between the rock grains and the water. An elementary heat balance consideration (Bodvarsson, 1974) shows that the heat is simply exchanged between the rock and the fluid such that about two unit volumes of (wet) rock at a given temperature T produce about one volume of water at T independent of the initial temperature of the water. An example of a sweep process is indicated in Figure 8 where water at 30°C is injected in uniform flow through a volume of rock at 70°C . At mass flow rates that will usually be attained in practice, the temperature front simply sweeps through the rock, and every 2 m^3 of the rock produce about one m^3 of water at 70°C . In this case, the specific heat recovery is 0.1 GJ/m^3 of rock.

Some of the adverse effects listed in Table 2 may affect porous slab FGR operations. Of particular concern are flow channeling and dispersion that prevent significant fractions of the formations to come into contact with the circulating water.

(6) Design examples

To illustrate the order of magnitude of the subsurface dimensions of simple FGR systems of some economical significance, we have chosen to sketch hypothetical designs of the two types discussed above, each producing water at 70°C for building heating. The systems are taken to generate an effective power of 6 MWt, which at an availability of 0.5, is sufficient to meet the base heating demand (about 80% of total) for a community of about 5×10^3 inhabitants located in the Pacific Northwest of the United States of America. The systems operate in an environment where the average surface temperature is 10°C and the geotemperature gradient is assumed to be 31°C/km.

The thin-sheet system sketched in Figure 9 is supposed to recover heat from a quasi-vertical fault zone or dike of sufficient natural or artificially induced fluid conductivity. The system has two injection/production borehole pairs flowing 50 kg/s and the total effective contact area required for an operational period of 25 years is 1.2 km².

The porous-slab system sketched in Figure 10 recovers heat from a porous horizon of 0.2 km thickness embedded between the depths of 1.9 and 2.1 km. The total effective contact volume required for 25 years of operation is 5×10^7 m³. The flow is the same as above.

As a matter of course, we should mention that porous-slab FGR systems have been under development in France and a few such systems are now in operation in the Paris Basin (DGRST, 1976).

(7) Exploration targets and field methods

The two systems sketched above are helpful in giving some indications as to the necessary dimensions of the subsurface production sections of FGR systems of moderately significant output power. The exploration targets for geological/geophysical field work investigating the possibilities for building such FGR systems in given locations are thus defined. We conclude that in regions of fairly normal geotemperature conditions, we are confronted with the task of finding suitable fluid conductivity at depths of 2 to 3 km and of similar horizontal dimensions. The fluid conductivity or permeability is of primary interest, whereas an above normal temperature gradient would be favorable but not essential.

Exploration field methods to be applied in FGR projects depend to a considerable degree on the local situation. Obviously, geological mapping to detect fault zones, porous horizons at depth, and other fluid conducting structures is essential. Moreover, shallow drilling for temperature/heat flow data and deeper drilling for structure is equally important. Seismic reflection exploration may be essential in sedimentary basins. Recent investigations have indicated a useful relation between the concentration of radioactive material in basement rock and the local heat flow.

Figure 9. Experimental design of a thin-sheet FGR system of 6 MWt.

Figure 10. Experimental design of a porous slab FGR system of 6 MWt.

(8) Inter-borehole fluid conductance

Once a conductor structure of suitable dimensions has either been created or located, a matter of primary importance will be to investigate whether the fluid conductivity is of sufficient magnitude and sufficiently uniform to allow a successful FGR operation during a prescribed period of time. In most cases, this is a difficult task and reliable test methods have yet to be developed. Conductivity testing is, as a matter of fact, one of the primary unresolved problems in FGR technology.

As of now, the methods that appear to be useful in inter-borehole conductance testing are primarily of two types, viz., (1) pressure transient methods and (2) chemical/isotope methods.

Pressure buildup and/or drawdown test methods have been developed in the oil industry (Ramey, 1976) and have been used with considerable success to investigate the nature of borehole-formation contact conductances. Recently, quite sensitive pressure transducers that enhance considerably the resolution of this type of testing have been marketed. Pressure transducers that can resolve 1 to 10 Pa, that is, 0.1 to 1 millimeter of water, are now available for field work.

Using the jargon of the electrical engineer, the pressure buildup method is a driving point test method where the pressure signal is monitored at the system port where the flow transient is either imposed or controlled. Such methods have little sensitivity to conductance details away from the driving port. For inter-borehole tests, a transfer impedance procedure has to be carried out where the propagation of pressure transients between boreholes is being investigated. For example, a prescribed amount of fluid is pumped into one of the holes, and the resulting pressure transient in the other one is recorded. Theoretically, an apparent inter-borehole hydraulic diffusivity can be measured this way and the results interpreted in terms of the nature of the formation fluid conductivity. Some theoretical basis for such test methods is available, but the procedure has yet to be developed for reliable field application.

In principle, the chemical and/or isotopic methods are similar. A tracer is pumped into one of the holes, and its emergence in the other one monitored. Again, an apparent inter-borehole diffusivity is obtained which can yield useful information of the formation fluid conductivity. For various reasons, the chemical/isotopic methods tend to be less sensitive than the pressure methods.

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Given at the Hot-Dry-Rock Workshop, Los Alamos, New Mexico, April 20 and 21, 1978

APPENDIX II

Abstract

Dup

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Energy Currents in Fractured Geothermal Systems

by Gunnar Bodvarsson, School of Oceanography, Oregon State University,
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The success and economic feasibility of dry-hot-rock or forced-geoheat-recovery projects depends to a considerable degree on the available subsurface temperatures at depths which can be reached by routine drilling. Sections of natural geothermal systems having elevated temperatures, but low fluid conductivity, are therefore of particular interest as heat sources.

Unfortunately, not much is known about the temperature distribution at greater depths in natural systems. In particular, we have little data on the conditions in the main heat-source regions. To make an attempt at gaining further insight into the situation there, we have carried out some rather simplistic modeling of the source regions of the larger hydrothermal systems in the Great Basin.

We find that a system flowing about 40 kg/s at 180°C for a period of a few 10⁴ years and thus having a constant dissipation power of 30 MWt would require a fracture contact area of a few tens of km² for heat recovery. The contact area could be quasi-vertical and located at depths of 4 to 6 km within a master fault zone of the region. As a matter of course, other source configurations are possible.

Moreover, it is of some interest to compare the estimated contact area of the above natural system with the area required for heat recovery in small forced-geoheat recovery systems. An elementary economic consideration indicated that the output power of the smallest feasible forced geoheat recovery for direct use will probably not be less than a few MWt. We find that a system operating at 6 MWt for 25 years at an output temperature of 70°C would at fairly normal geoheat conditions require a fracture contact area of about 3 km².

Given at the Hot-Dry-Rock Workshop, Los Alamos, New Mexico, April 20 and 21, 1978.

APPENDIX III

Abstract

Hydroelastic oscillations in fracture systems

by Gunnar Bodvarsson, School of Oceanography, Oregon State University,
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Liquids embedded in openings in elastic spaces are capable of oscillations which we refer to as hydroelastic oscillations. The liquid provides the main inertia and the walls the restoring force. Two specific types of such systems are of practical interest, viz., the borehole-cavity system and the long, very narrow water-filled fracture. We shall briefly discuss the dynamics of such models.

Consider a borehole connected with a very narrow penny-shaped cavity at depth and opening into a basin at the top. The surface area of the basin is considerably larger than the cross section of the hole. The system is filled with static water. In rock with common elastic properties the basic oscillations frequency of this system expressed in MKS units is approximately $(700/R)(a/dR)^{1/2}$ where R is the radius of the cavity, d and a the depth and cross section of the hole, respectively.

Observations of the oscillations frequency can thus yield data on the cavity dimensions which may be difficult to obtain otherwise. These results are readily extended to more complex flowing systems.

In the case of narrow fractures, we are mainly interested in the propagation of pressure signals, for example, between boreholes. The dynamics of such systems is governed by a partial differential equation which includes an interesting cross section differential operator of fractional order. For fractures of about one millimeter width and at wave-lengths of 10^2 meters, the propagation velocity is of the order of several tens of meters/second.

Slides displayed at the Hot-Dry-Rock Workshop in Los Alamos, N.M. are shown on pages 22 through 31.

Acknowledgments

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The simple borehole-cavity resonator with a surface basin

large basin with constant fluid level

ρ = density of fluid

V = volume of cavity (depends on pressure)

p = pressure in cavity (constant)

$e = dV/dp$ = elastance of cavity

d = depth of borehole

a = cross section of borehole

u = oscillation velocity in borehole (small amplitude)

q = impressed mass flow source term

Linearized equation of small amplitude motion (attenuation neglected):

$$\rho dDu = V/e$$

Kinematic condition:

$$DV = -au + (q/\rho)$$

System equation:

$$\underline{D^2u + \omega_0^2u = (q/\rho ed)}$$

where $D = d/dt$

resonant frequency:

$$\underline{\omega_0 = (a/\rho ed)^{1/2}}$$

(2)

The simple open borehole-cavity resonator

h = oscillation amplitude of water level

$$Dh = u$$

Equation of motion:

$$\rho dDu = (V/e) - gph$$

hence

$$\omega_0 = [(a/\rho ed) + (g/d)]^{\frac{1}{2}}$$

(3)

Simple borehole-cavity resonator with a
surface basin and constant mass flow

mass flow $q_0 = \rho a v$ (constant)

Equation of motion:

$$\underline{D^2 u + (v/d)Du + \omega_0^2 u = (q/\rho e d)}$$

q = source term, impressed oscillatory mass flow

(4)

Forced oscillations of the simple
borehole-cavity system with a surface basin

$$\omega_0 = (a/\rho e d)^{\frac{1}{2}},$$

$$u = u_0 \exp(i\omega t)$$

$p = p_0 \exp(i\omega t)$ = pressure at the surface
of basin

p_0 = pressure amplitude

Equation of motion:

$$\rho d D u = (V/e) - p$$

hence

$$D^2 u + \omega_0^2 u = -(1/\rho d) D p = -(1/\rho d) i \omega p_0 \exp(i\omega t)$$

$$u = \frac{i \omega p_0}{\rho d (\omega^2 - \omega_0^2)}$$

(5)

Elastance of the penny-shaped cavity
(Sneddon, 1946)

$$w \ll R,$$

G = shear modulus

E = Young's modulus

$$\text{elastance } e = dV/dp = 16(1-\sigma^2)R^3/3E$$

$$\text{or for most rocks } e \approx 2R^3/G$$

hence for simple borehole-cavity system

$$\omega_0 = (aG/2\rho R^3 d)^{\frac{1}{2}}$$

and in common rock

$$\omega_0 = (700/R)(a/Rd)^{\frac{1}{2}}, \quad \text{MKS}$$

(6)

The lumped connector-cavity ladder

The mechanics of a ladder of this type can be discussed theoretically on the basis of the methods given by Bodvarsson in "Direct Interpretation Methods in Applied Geophysics" *Geoexploration*, 4, (1966), 113-138.

(7)

The Elastic Pipe

equilibrium

linearized small amplitude

$$\text{eq. of motion: } \rho \partial_t v = -\partial_x p - Rv$$

$$\text{cons. of mass: } -\rho A_0 \partial_x v = \rho \partial_t A$$

$$\text{hence } \underline{\rho \partial_{tt} p + R \partial_t p - S \partial_{xx} p = f}$$

 ρ = density p = pressure v = velocity

$$A_0 = \pi r_0^2, \quad A = \pi(r_0 + u)^2$$

 S = stiffness operator

$$p = Su$$

for membrane

$$S = a - b \partial_{xx}$$

 f = source term

(8)

Elasticity of Hookean half-space

u = vertical displacement

$p(x, y)$ = pressure distribution

G = shear modulus

σ = Poisson's ratio

$$u = \frac{(1-\sigma)}{2\pi G} \int_{\text{surface}} \frac{p dx^1 dy^1}{[(x-x^1)^2 + (y-y^1)^2]^{1/2}}$$

$$S = \frac{G}{1-\sigma} (-\nabla_2^2)^{1/2}$$

inverted

$$p = Su$$

$$\nabla_2^2 = \partial_{xx} + \partial_{yy}$$

(9)

The narrow flat fracture

μ = viscosity , f = source term

equilibrium position

$$R = 12\mu/u_0^2$$

$$p = u_0 S\left(\frac{u}{u_0}\right)$$

basic equation:

$$\rho \partial_{tt} p + \frac{12\mu}{u_0^2} \partial_t p - \frac{u_0 G}{2(1-\sigma)} (-\nabla^2)^{\frac{3}{2}} p = f$$

one - dimensional:

$$\rho \partial_{tt} p + \frac{12\mu}{u_0^2} \partial_t p - \frac{i u_0 G}{2(1-\sigma)} \partial_x^3 p = f$$

(10)

The narrow flat fracture

Skin depth:

$$p = \exp(i\omega t - \frac{x}{d})$$

$$\omega u_0^2 \ll 12\mu/\rho$$

$$d = \left(\frac{u_0^3 G}{24\mu\omega(1-\sigma)} \right)^{\frac{1}{3}}$$

Phase velocity

$$p = \exp[i(\omega t - kx)]$$

$$\omega u_0^2 \gg 12\mu/\rho$$

$$\alpha = \left(\frac{u_0 G k}{2\rho(1-\sigma)} \right)^{\frac{1}{2}}$$