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MASTER

Revision 1

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INPUT PARAMETERS TO CODES WHICH ANALYZE
LMFBR WIRE WRAPPED BUNDLES

(TO BE USED WITH THE ENERGY CODE MANUAL)
by

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ABSTRACT

This report provides a current summary of recommended values of key input parameters required for ENERGY code analysis of LMFBR wire wrapped bundles. This data is based on the interpretation of experimental results from the MIT and other available laboratory programs.

ACKNOWLEDGMENTS

Revision 1 was prepared based principally on the efforts of Kerry Basehore and Chong Chiu. Kerry was instrumental in clarifying the difference between continuum and subchannel notation and Chong is responsible for producing the new flow split and mixing parameter correlations. Thanks are also due to Mike Corradini who has labored through the preparation of Chong's reports and this one to insure the clarity and accuracy of the products.

NOMENCLATURE

Representation
(Continuum or Subchannel)

* → Normalized or Dimensionless (* or Blank)
 → P ψ A, I → Bundle region (I, II or Blank for overall Bundle) or Subchannel type (1, 2 or 3)
 ↑ Orientation (Axial, Normal or Swirl)

CONTINUUM

Velocities

$$U_A$$

$$U_{AI}$$

$$U_{AII}$$

$$U_{SII}$$

Volumetric Heat Sources

$$P^Q$$

$$P^Q_I$$

$$P^Q_I^* = \frac{P^Q_I}{P^Q}$$

$$P^Q_{II}$$

$$P^Q_{II}^* = \frac{P^Q_{II}}{P^Q}$$

Flow Splits

$$P^{X_I} = P^{X_1} = \frac{U_{AI}}{U_A} = \frac{U_{A1}}{U_A}$$

$$P^{X_{II}} = \frac{U_{AII}}{U_A}$$

$$P^{X_2} = \frac{U_{A2}}{U_A}$$

$$K_{P_{NI}}$$

$$K_{P_{NII}} = pK_{NI}$$

$$K_{P_{SII}}$$

SUBCHANNEL

$$V_A = U_A \left(\frac{1}{\lambda_A} \right)$$

$$V_{AI} = U_{AI} \left(\frac{1}{\lambda_{AI}} \right)$$

$$V_{AII} = U_{A2} \left(\frac{1}{\lambda_{AII}} \right)$$

$$V_{SII} = U_{S2} \left(\frac{1}{\lambda_{LII}} \right)$$

$$S^Q = P^Q \left(\frac{1}{\lambda_A} \right)$$

$$S^Q_I = P^Q_I \left(\frac{1}{\lambda_{AI}} \right)$$

$$S^Q_I^* = P^Q_I^* \left(\frac{\lambda_A}{\lambda_{AI}} \right)$$

$$S^Q_{II} = P^Q_{II} \left(\frac{1}{\lambda_{AII}} \right)$$

$$S^Q_{II}^* = P^Q_{II}^* \left(\frac{\lambda_A}{\lambda_{AII}} \right)$$

$$S^{X_I} = S^{X_1} = \frac{\lambda_A}{\lambda_{AI}} P^{X_I} = \frac{V_{AI}}{V_A}$$

$$S^{X_{II}} = \frac{\lambda_A}{\lambda_{AII}} P^{X_{II}} = \frac{V_{AII}}{V_A}$$

$$S^{X_2} = \frac{\lambda_A}{\lambda_{A2}} P^{X_2} = \frac{V_{A2}}{V_A}$$

$$K_{S_{NI}} = \frac{K_{P_{NI}}}{\lambda_{LI}}$$

$$K_{S_{NII}} = \frac{K_{P_{NII}}}{\lambda_{LI}} \equiv K_{S_{NI}}$$

$$K_{S_{SII}} = \frac{K_{P_{SII}}}{\lambda_{LI}}$$

v

Perimeter for Heat Transfer

$$P_I$$

$$S_I = \lambda_{LI} P_I$$

$$P_{SII}$$

$$S_{SII} = \lambda_{LII} P_{SII}$$

$$P_{NII} (\equiv P_I)$$

$$S_{NII} = \lambda_{NII} P_{NII} \equiv \lambda_{LI} P_I = S_I$$

Crosssectional Area for Flow

$$P_I^A$$

$$S_I^A = \lambda_{AI} P_I^A$$

$$P_{II}^A$$

$$S_{II}^A = \lambda_{AII} P_{II}^A$$

Key parameters which are the same for **continuum** and subchannel models

Enthalpy

$$h_{I \text{ or } II} = C_p T_{I \text{ or } II}$$

$$h^* = \frac{\rho C_p T U_A}{P Q_{De1}} = \frac{\rho C_p T V_A}{S Q_{De1}}$$

Axial Mass flow rate

$$m_A = \rho_P^A U_A = \rho_S^A V_A$$

Gradient

$$\nabla h = \frac{h_i - h_j}{\eta}$$

$$\nabla^* h = \frac{h_i^* - h_j^*}{\eta / D_{e1}}$$

ADDITIONAL NOMENCLATURE NOT FOLLOWING THE PREVIOUS CONVENTION

C, C_I, C_L, C_{IL}	Swirl velocity parameters - see TABLE 2
$\epsilon^*, \epsilon_I^*, \epsilon_L^*, \epsilon_{IL}^*$	Mixing parameters - see TABLE 2
$\alpha^*, \alpha_I^*, \alpha_{II}^*, \alpha_L^*, \alpha_{IL}^*$	Conduction parameters - see TABLE 3
$\epsilon_{HI}^{H_{NI}}, \epsilon_{HII}^{H_{NII}}, \epsilon_{HSII}^{H_{SII}}$	Directional eddy diffusivities - see TABLE 3 appearing directly in continuum equations.
$\lambda_A, \lambda_{AI}, \lambda_{AII}, \lambda_{LI}, \lambda_{LII}$	Porosity factors - see TABLE 1
A_r	Projected area of wires in subchannel (ft^2)
A_{r1}	Projected area of wires in subchannel 1 (ft^2)
A_{r2}	Projected area of wires in subchannel 2 (ft^2)
A_s	Subchannel area (ft^2)
A_{s1}	Axial average subchannel area for a single interior subchannel (ft^2)
A_{s2}	Axial average subchannel area for a single edge subchannel (ft^2)
A_{s3}	Axial average subchannel area for a single corner subchannel (ft^2)
A'_s	Subchannel area with no wire in the subchannel (ft^2)
A'_{s1}	Interior subchannel area with no wire in the subchannel (ft^2)
A'_{s2}	Edge subchannel area with no wire in the subchannel (ft^2)
D_e	Overall hydraulic diameter
D_{e1}	Equivalent hydraulic diameter for interior subchannels including wire
D_{e2}	Equivalent hydraulic diameter for edge subchannels including wire
D_F	Distance across flats of a hexagon
D	Rod diameter
D_w	Wire diameter
g	Gap between rod and duct
H	Wire wrap lead length

N_1	Number of interior subchannels in the bundle
N_2	Number of edge subchannels in the bundle
N_3	Number of corner subchannels in the bundle
n	Ratio of the gap between edge subchannels to the average edge subchannel width
N	Number of fuel pins in assembly
N_{RINGS}	Number of rings of rods in an array. The center pin itself is ring number 0
P	Fuel pin pitch
V_T	Average transverse velocity i.e. V_{SII}

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1.0 - INTRODUCTION

The purpose of this report is to provide an updated summary of recommended values of input data required by analysts for hexagonal wire wrapped bundles. This data is based on the interpretation of experimental results from the MIT and other available laboratory programs. The input data has been expressed specifically in the forms used in the ENERGY code family. It is intended that this report will be updated as data analysis yields (1) new input data for rod bundle geometries already surveyed or (2) input data for new rod bundle geometries.

The ENERGY family was originally formulated in both **continuum** and subchannel representation (1, 2 and 3)*. The **continuum** formulation reflected the physical model underlying our method of approach and gives great flexibility in the range of problems to which the model can be applied. The subchannel representation is simply a special case of the porous body formulation in which the relevant parameters are established on a subchannel basis for user ease in application of our model to this common way of discretizing a bundle. Our decision to retain both representations has resulted in some confusion regarding the proper form of the input parameters for each case due to (1) the complexity of the notation and (2) typographical and transformation errors in some of our own publications.

Because of the range of potential applications, we choose to retain both representations and strive to overcome the notational difficulties by a lucid presentation of definitions of parameters. The parameter definitions are presented in the next chapter and are followed by the recommended correlations for these parameters.

The correlations are based on physical models which should permit their extrapolation to some degree outside the range of the data base used in their development.

$$4 \leq H/D \leq 52$$

$$1.067 \leq P/D \leq 1.32$$

The bundles must have all wire wraps of the same diameter, wrapped in the same orientation from the same start position.

*Here we originally called our representations "porous body" and "subchannel". It is more proper to call them continuum and subchannel where the subchannel is a special case of a porous body representation.

2.0 - INPUT PARAMETER DEFINITIONS

The parameter definitions and their interrelationships are best presented in terms of the governing differential equations. For the ENERGY model these are the energy transport equations in the inner (region I) and outer (region II) regions of the bundle. Usually nomenclature is completely relegated to a table but we shall present our approach here now because it is complicated by our desire to present both the continuum and the subchannel approaches on a consistent basis. Therefore we have adopted the following approach. Let ψ represent any variable. Then

The key exceptions because of strong convention are for :

Velocities where instead of P or S we change ψ from U to V respectively.

Porosities λ where the $*$ is dropped since by definition they are dimensionless and L is used for the lateral or normal direction.

Subchannel areas where the orientation is dropped because they are all axial

2.1 - INNER BUNDLE REGION

2.1.1 - Continuum representation (ϵ^*)

The control volume for region I is an equilateral triangular prism, Δz units in length, which from physical considerations comprises the planar area of at least one subchannel and the sixth sector of each of three rods which border the subchannel i.e.

REGION I

CONTROL VOLUME

(for a planar area of one subchannel, the perimeter P numerically is equal to the pitch P .)

The energy transport equation for this control volume in our continuum representation is

$$P^A_I \rho C_P U_{AI} (T_{OUT} - T_{IN}) = -(\rho C_P \epsilon_{HNI} + \frac{K_{NI}}{P_{NI}} K) P_I \Delta z \nabla T + P^Q_I \Delta z P^A_I \quad (1 a)$$

$$\text{or} \quad \rho U_{AI} \frac{\partial h}{\partial z} = -\left(\frac{P}{A}\right)_I \rho (\epsilon_{HNI} + \frac{K_{NI}}{P_{NI}} \alpha) \nabla h + P^Q_I \quad (1 b)$$

where the effective conduction term with the enthalpy gradient is taken across each of the three lateral faces of the control volume.

We next bring equation (1 b) into dimensionless form utilizing the parameters U_A , De_I , and p^Q . The first two parameters are selected because they are convenient and unambiguous parameters. It may seem strange to adopt De_I since it is a subchannel parameter but recall that in the continuum model, the only characteristic dimensions available are those of the duct or the actual subchannels since the node size is arbitrary.

Utilizing these parameters, equation (1 b) becomes

$$\frac{U_{AI}}{U_A} \frac{\partial (h \rho U_A / P^Q D_{eI})}{\partial (z/D_{eI})} = -D_{eI} \left(\frac{P}{A} \right)_I \left(\frac{\epsilon_{HNI}}{U_A D_{eI}} + \frac{K_{NI}}{P_{NI}} \frac{\alpha}{U_A D_{eI}} \right) \frac{\nabla (h \rho U_A / P^Q D_{eI})}{1/D_{eI}} + \frac{P^Q_I}{P^Q} \quad (2)$$

$$\text{or } \frac{U_{AI}}{U_A} \frac{\partial h^*}{\partial z^*} = -D_{eI} \left(\frac{P}{A} \right)_I (\epsilon^* + \frac{K_{NI}}{P_{NI}} \alpha^*) \nabla^* h + \frac{P^Q_I}{P^Q} \quad (3)$$

which defines the continuum parameters ϵ^* and α^*

$$\epsilon^* \equiv \frac{\epsilon_{HNI}}{U_A D_{eI}} \quad \text{and} \quad \alpha^* \equiv \frac{\alpha}{U_A D_{eI}} \quad (4 \text{ a,b})$$

2.1.2 - Subchannel Representations ($\epsilon^*_I, \epsilon^*_{IL}$)

Now for convenience a subchannel based representation is desired in which all parameters appearing in the energy equation are the true values in the rod array. Hence we transform equation (3) into that form by introducing lateral and axial porosity factors defined in Table I. Note that the lateral porosity between interior rods is taken to be the ratio of the gap between rods to the pitch to be consistent with the change in the perimeter P for heat flow, i.e., $P-D$ versus P . For the outer region the same convention is adopted i.e., taking the gap between rod surface and duct as the relevant dimension.

Applying porosity factors to equation (3), we obtain :

$$\frac{\lambda_{AI}}{\lambda_A} \left(\frac{U_{AI}/\lambda_{AI}}{U_A/\lambda_A} \right) \frac{\partial h^*}{\partial z^*} = - \frac{\lambda_{LI} D_{eI}}{\lambda_{AI}} \left(\frac{P}{A} \right)_I \left(\frac{\lambda_{AI} \epsilon^*}{\lambda_{LI}} + \frac{\lambda_{AI}}{\lambda_{LI}} \frac{K_{NI}}{P_{NI}} \alpha^* \right) \nabla^* h + \frac{P^Q_I}{P^Q} \frac{\lambda_{AI}}{\lambda_{AI}} \quad (5)$$

Now the following relations exist between continuum and subchannel parameters :

$$\lambda_{LI} P P_I = S P_I \quad (6 a)$$

$$\lambda_{AI} P A_I = S A_I \quad (6 b)$$

$$\left(\frac{\lambda_A}{\lambda_{AI}} \right) P Q_I^* = \frac{P Q_I / \lambda_{AI}}{P Q / \lambda_A} = S Q_I^* \quad (6 c)$$

$$\frac{U_{AI} / \lambda_{AI}}{U_A / \lambda_A} = \frac{V_{AI}}{V_A} = S X_I = S X_I \quad (6 d)$$

Note that in the last relation we have introduced the definitions for flow split in the subchannel representation and since region I is comprised exclusively of interior subchannel (type I) we note $S X_I = S X_I$.

Utilizing these relations equation (5) becomes :

$$S X_I \frac{\partial h^*}{\partial z^*} = - D_{eI} \left(\frac{P}{A} \right)_I \left(\frac{\lambda_A}{\lambda_{LI}} \epsilon^* + \frac{\lambda_A}{\lambda_{LI}} \frac{K}{P_{NI}} \alpha^* \right) \nabla^* h + S Q_I^* \quad (7)$$

This is the origin of the subchannel based parameters ϵ_1^* and α_1^*

$$\text{i.e. } \epsilon_1^* = \frac{\lambda_A}{\lambda_{LI}} \epsilon^* ; \alpha_1^* = \lambda_A \alpha^* \text{ and } \frac{K}{S_{NI}} = \frac{P_{NI}}{\lambda_{LI}} \quad (8 a, b, c)$$

The application of porosity factors λ_A and λ_{LI} in equations 8 b and 8 c follows physically since α^* includes an axial velocity and $\frac{K}{P_{NI}}$ is a correction factor for lateral heat transfer. Note also that in previous reports the factor $\frac{K}{S_{NI}}$ was called "SF".

Analysis of LMFBR assemblies is based on equation (3) or (7) with parameters ϵ^* or ϵ_1^* , α^* or α_1^* , and $\frac{K}{P_{NI}}$ or $\frac{K}{S_{NI}}$ as input. The parameters ϵ^* and ϵ_1^* are determined from reduction of either heated pin bundle or

salt injection test data by the inverse use of equation (3) or (7). In this inverse use, the enthalpy or analogously the salt concentration, are known, α^* or α_1^* and K_{PNI} or K_{SNI} are input and ϵ^* or ϵ_1^* is determined by trial and error. From equations (3) or (7) we see that the resulting value of ϵ^* or ϵ_1^* is dependent on the flow split ($\frac{U_{AI}}{U_A}$ for porous body, $\frac{V_{AI}}{V_A}$ for subchannel) which is equivalent to saying they are functions of bundle size. This dependance of the interior region mixing parameter can be eliminated by redefinition of the parameter to incorporate the flow split. We will show this explicitly for the subchannel formulation by dividing equation (7) by S_{X1} yielding

$$\frac{\partial h^*}{\partial z^*} = -De_1 \left(\frac{P}{A} \right)_I \left(\frac{\lambda_A \epsilon^*}{S_{X1} \lambda_{LI}} + K_{PNI} \frac{\lambda_A \alpha^*}{\lambda_{LI} S_{X1}} \right) \nabla^* h + \frac{S_{QI}^*}{S_{X1}^*}$$

$$\text{or } \frac{\partial h^*}{\partial z^*} = -De_1 \left(\frac{P}{A} \right)_I (\epsilon_{IL}^* + K_{SNI} \alpha_{IL}^*) \nabla^* h + \frac{S_{QI}^*}{S_{X1}^*} \quad (9)$$

where :

$$\epsilon_{IL}^* = \frac{1}{S_{X1}} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* = \frac{\epsilon_1^*}{S_{X1}} ; \alpha_{IL}^* = \lambda_A \alpha^* = \frac{\alpha_1^*}{S_{X1}} \text{ and again } K_{SNI} = \frac{K_{PNI}}{\lambda_{LI}} \quad (10 \text{ a, b and 8 c})$$

For salt conductivity experiments the salt is input only at one plane and hence appears only as a boundary condition when salt data is used in the concentration equation analogous to (9) to deduce ϵ_{IL}^* . There is no source term. Therefore the deduced values of ϵ_{IL}^* are independent of bundle size i.e. S_{X1} for highly turbulent flow where $\epsilon_{IL}^* \gg K_{SNI} \alpha_{IL}^*$. On the other hand for heated bundle data reduction by equation (9), knowledge of S_{X1} is necessary to obtain the proper value of ϵ_{IL}^* . To use equation (9) for assembly temperature field prediction, accurate knowledge of S_{X1} is also needed to correctly express the source term. Therefore we formulate the results of salt mixing tests in terms of ϵ_{IL}^* to be independent of bundle size and we independently perform experiments to determine S_{X1} .

2.1.3 - Continuum representation (ϵ_L^*)

To complete this discussion and illustrate the symmetry of our definitions we observe that the continuum mixing coefficient independent of bundle size can be obtained by dividing equation (3) by $\frac{U_{AI}}{U_A}$ which from equation (6 d) and the definition for flow split from the continuum model is

$$\frac{U_{AI}}{U_A} = S_{X1} \frac{\lambda_{AI}}{\lambda_A} \equiv P_{X1} \equiv P_{XI} \quad (11)$$

yielding

$$\begin{aligned} \frac{\partial h^*}{\partial z^*} &= -De_1 \left(\frac{P}{A} \right)_I \left(\frac{\epsilon^*}{P_{X1}} + \frac{K}{P_{NI}} \frac{\alpha^*}{P_{X1}} \right) \nabla^* h + \frac{P_{QI}^*}{P_{X1}} \\ \text{or} \quad \frac{\partial h^*}{\partial z^*} &= -De_1 \left(\frac{P}{A} \right)_I \left(\epsilon_L^* + \frac{K}{P_{NI}} \alpha_L^* \right) \nabla^* h + \frac{P_{QI}^*}{P_{X1}} \end{aligned} \quad (12)$$

$$\text{where } \epsilon_L^* = \left(\frac{1}{S_{X1}} \frac{\lambda_A}{\lambda_{AI}} \right) \epsilon^* = \frac{\epsilon^*}{P_{X1}} \quad \text{and} \quad \alpha_L^* = \left(\frac{1}{S_{X1}} \frac{\lambda_A}{\lambda_{AI}} \right) \alpha^* = \frac{\alpha^*}{P_{X1}} \quad (13 \text{ a, b})$$

2.1.4 - Summary of Inner Region Relations

The resulting forms of the mixing coefficients for the four representations, equations 4a, 8a, 10 a and 13a, are listed in Table 2. One can note that symmetry does not exist in passing from bundle size dependent parameters to the corresponding bundle size independent cases. Specifically, the parameters for the continuum formulations are related as

$$\epsilon_L^* = \frac{\epsilon^*}{S_{X1}} \left(\frac{\lambda_A}{\lambda_{AI}} \right)$$

where as the subchannel formulations are related as

$$\epsilon_{IL}^* = \frac{\epsilon^*}{S_{X1}}$$

This difference occurs because the subchannel based flow split S_{X1} is used in both cases. The ratio $\frac{\lambda_A}{\lambda_{AI}}$ could be eliminated if a continuum based flow split was used. However since experimentally determined flow splits are based on subchannel velocity ratios, we have chosen to use S_{X1} throughout and accept the resulting lack of symmetry in definitions.

2.2 - OUTER BUNDLE REGION

2.2.1 - Continuum representation (C)

The energy transport equation for region II in the continuum representation may be written as

REGION II
CONTROL VOLUME

Where $P_{NII} = P$
 $P_{SII} = \frac{D}{2} + g$

$$\begin{aligned} & \rho C_P U_{SII} P_{SII} \Delta z (T_i - T_j) + \rho C_P U_{AII} P_{AII} (T_{OUT} - T_{IN}) \\ &= - (\rho C_P \epsilon_{H_{NII}} + \frac{K}{P_{NII}} \kappa) P_{NII} \Delta z \nabla T \\ & \quad - (\rho C_P \epsilon_{H_{SII}} + \frac{K}{P_{SII}} \kappa) P_{SII} \Delta z \nabla T + P_{Q_{II}} \Delta z P_{AII} \quad (14 a) \end{aligned}$$

$$\begin{aligned} \text{or } & \rho U_{SII} \left(\frac{P_{SII} P_{NII}}{P_{AII}} \right) \frac{\partial h}{\partial s} + \rho U_{AII} \frac{\partial h}{\partial z} \\ &= - \left(\frac{P_{NII}}{P_{AII}} \right) \rho (\epsilon_{H_{NII}} + \frac{K}{P_{NII}} \alpha) \nabla h \\ & \quad - \left(\frac{P_{SII}}{P_{AII}} \right) \rho (\epsilon_{H_{SII}} + \frac{K}{P_{SII}} \alpha) \nabla h + P_{Q_{II}} \quad (14 b) \end{aligned}$$

Bringing the equation into dimensionless form utilizing again the parameters U_A , De_1 and pQ , we obtain :

$$\begin{aligned}
& \frac{U_{SII}}{U_A} \left(\frac{P_{SII}}{P_{AII}} \frac{P_{NII}}{P_{AII}} \right) \frac{\partial (\rho h U_A / P^Q D_{eI})}{\partial (S/D_{eI})} + \frac{U_{AII}}{U_A} \frac{\partial (\rho h U_A / P^Q D_{eI})}{\partial (Z/D_{eI})} = \frac{P_{LI}}{P^Q} \\
& - D_{eI} \left(\frac{P_{NII}}{P_{AII}} \right) \left(\frac{\epsilon_{HNII}}{U_A D_{eI}} + \frac{K_{NII}}{P_{NII}} \frac{\alpha}{U_A D_{eI}} \right) \frac{\nabla (\rho h U_A / P^Q D_{eI})}{1/D_{eI}} \\
& - D_{eI} \left(\frac{P_{SII}}{P_{AII}} \right) \left(\frac{\epsilon_{HSII}}{U_A D_{eI}} + \frac{K_{SII}}{P_{SII}} \frac{\alpha}{U_A D_{eI}} \right) \frac{\nabla (\rho h U_A / P^Q D_{eI})}{1/D_{eI}} \quad (15)
\end{aligned}$$

At this point rather than define various directionally and region dependant mixing parameters, we note that (1) the diffusivity effects of the RHS for turbulent flow and probably to some as yet unconfirmed level of laminar flow are much smaller than the convective effects (1st term LHS) and (2) the diffusivity effects are approximately equal for all regions in all directions of the bundle. Therefore take the eddy diffusivity as isotropic yielding

$$\epsilon^* \equiv \frac{\epsilon_{HNII}}{U_A D_{eI}} \cong \frac{\epsilon_{HSII}}{U_A D_{eI}} \cong \frac{\epsilon_{HNI}}{U_A D_{eI}} \quad (16)$$

In a consistent manner the molecular conduction could be approximated as equal in cases where conduction effects are not significant. However we will retain the regional and directional notation for generality.

Expressing equation 15 in dimensionless nomenclature we have :

$$\begin{aligned}
C \left(\frac{P_{SII}}{P_{AII}} \frac{P_{NII}}{P_{AII}} \right) \frac{\partial h^*}{\partial S^*} + \frac{U_{AII}}{U_A} \frac{\partial h^*}{\partial Z^*} = & - D_{eI} \left(\frac{P_{NII}}{P_{AII}} \right) (\epsilon^* + \frac{K_{NII}}{P_{NII}} \alpha^*) \nabla h^* \\
& - D_{eI} \left(\frac{P_{SII}}{P_{AII}} \right) (\epsilon^* + \frac{K_{SII}}{P_{SII}} \alpha^*) \nabla h^* \\
& + P^Q_{II} \quad (17)
\end{aligned}$$

Note that we have introduced a new parameter, the swirl velocity ratio as

$$C = \frac{U_{SII}}{U_A} \quad (18)$$

2.2.2 - Subchannel representation (C_1, C_{1L})

Proceeding to a subchannel representation we transform equation (17) to the following form by introducing porosity factors

$$\begin{aligned} & \left(\frac{P_{SII} P_{NII}}{P_{AII}^A} \right) \frac{\lambda_{LII}}{\lambda_A} \left(\frac{U_{SII}/\lambda_{LII}}{U_A/\lambda_A} \right) \frac{\partial h^*}{\partial S^*} + \frac{\lambda_{AII}}{\lambda_A} \left(\frac{U_{AII}/\lambda_{AII}}{U_A/\lambda_A} \right) \frac{\partial h^*}{\partial Z^*} = \\ & - De_1 \left(\frac{P_{NII}}{P_{AII}^A} \right) \frac{\lambda_{LI}}{\lambda_{AII}} \left(\frac{\lambda_{AII} \epsilon^*}{\lambda_{LI}} + \frac{K_{NII}}{\lambda_{LI}} \lambda_{AII} \alpha^* \right) \nabla h^* \\ & - De_1 \left(\frac{P_{SII}}{P_{AII}^A} \right) \frac{\lambda_{LII}}{\lambda_{AII}} \left(\frac{\lambda_{AII} \epsilon^*}{\lambda_{LII}} + \frac{K_{SII}}{\lambda_{LII}} \lambda_{AII} \alpha^* \right) \nabla h^* + P_{QII}^* \frac{\lambda_{AII}}{\lambda_{AII}} \quad (19) \end{aligned}$$

Next we introduce the subchannel flow split definition for the outer region

$$S_{XII}^* = \frac{V_{AII}}{V_A} = \frac{U_{AII}/\lambda_{AII}}{U_A/\lambda_A} \quad (20 a)$$

(Note carefully that this flow split definition is based on the condition in the entire outer region which includes both edge and corner channels. In our experimental reports we have separately identified the flow split in each of the three subchannel types as 1 interior, 2 edge and 3 corner. Therefore X_{II} is not strictly equal to X_2 because of the presence of corner channels in region II and we have therefore introduced the notation X_{II} to highlight this difference. In practical cases for large rod bundles, 37 pins or greater, X_{II} can be taken equal to X_2).

Also we note the following relations between continuum and subchannel parameters

$$\lambda_{AII} P_{AII}^A = S_{AII}^A \quad (20 b)$$

$$\lambda_{LII} P_{SII}^S = S_{SII}^S \quad (20 c)$$

$$\lambda_{LI} P_{NII}^N = S_{NII}^N = S_{PI}^P \quad (20 d)$$

$$(\text{since } \lambda_{LI} \equiv \lambda_{NII} \text{ and } P_{NII} \equiv P_{PI})$$

$$\left(\frac{\lambda_A}{\lambda_{AII}}\right) P^{Q*}_{II} = \frac{P^{Q_{II}}/\lambda_{AII}}{P^Q/\lambda_A} = S^{Q*}_{II} \quad (20 \text{ e})$$

Applying relations (20 a) thru (20 e) to equation (19), we obtain :

$$\begin{aligned} & \left(\frac{S^{P_{SII}} P^{P_{NII}}}{S^{A_{II}}} \right) C_1 \frac{\partial h^*}{\partial S^*} + S^{X_{II}} \frac{\partial h^*}{\partial Z^*} \\ &= - D_{e1} \left(\frac{S^{P_{NII}}}{S^{A_{II}}} \right) \left(\frac{\lambda_A}{\lambda_{LI}} \epsilon^* + \frac{K_{NII}}{\lambda_{LI}} \lambda_A \alpha^* \right) \nabla h^* \\ & - D_{e1} \left(\frac{S^{P_{SII}}}{S^{A_{II}}} \right) \left(-\frac{\lambda_A \epsilon^*}{\lambda_{LII}} + \frac{K_{SII}}{\lambda_{LII}} \lambda_A \alpha^* \right) \nabla h^* + S^{Q*}_{II} \end{aligned} \quad (21)$$

Finally we usually make the simplification that

$$\frac{\lambda_A}{\lambda_{LI}} \epsilon^* = \frac{\lambda_A}{\lambda_{LII}} \epsilon^* \quad (22)$$

Equation (21) shows the origin of the subchannel based parameter C_1 , and equation (22) is the reason why we only have only one eddy parameter ϵ^*_1 versus two conduction parameters $\frac{K_{NII}}{S^{NII}}$ and $\frac{K_{SII}}{S^{SII}}$.

$$\text{i.e. } \epsilon^*_1 \equiv \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \cong \frac{\lambda_A}{\lambda_{LII}} \epsilon^* ; \alpha^*_1 = \lambda_A \alpha^* \quad (8 \text{ a, } 8 \text{ b})$$

$$C_1 = \frac{\lambda_A}{\lambda_{LII}} C \quad (23 \text{ a})$$

$$\text{and } \frac{K_{NII}}{S^{NII}} = \frac{K_{NII}}{\lambda_{LI}} = \frac{K_{NI}}{S^{NI}} \quad (8 \text{ c})$$

$$\frac{K_{SII}}{S^{SII}} = \frac{K_{SII}}{\lambda_{LII}} \quad (23 \text{ b})$$

Proceeding to eliminate the dependance of the subchannel parameter on flow split, we divide equation 21 by S_{II}^X yielding

$$\begin{aligned} \left(\frac{S_{SII}^P P_{NII}^P}{S_{AII}^A} \right) \frac{C_1}{S_{XII}^X} \frac{\partial h^*}{\partial S^*} + \frac{\partial h^*}{\partial z^*} = - De_1 \left(\frac{S_{NII}^P}{S_{AII}^A} \right) \left(\frac{1}{S_{XII}^X} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \right. \\ \left. + \frac{K_{NII}^P \lambda_A \alpha^*}{S_{XII}^X \lambda_{LI}} \right) \nabla h^* \\ - De_1 \left(\frac{S_{SII}^P}{S_{AII}^A} \right) \left(\frac{1}{S_{XII}^X} \frac{\lambda_A}{\lambda_{LII}} \epsilon^* \right. \\ \left. + \frac{K_{SII}^P \lambda_A \alpha^*}{S_{XII}^X \lambda_{LII}} \right) \nabla h^* + \frac{S_{OII}^Q}{S_{XII}^X} \end{aligned} \quad (24)$$

$$\text{where we now define } C_{1L} \equiv \frac{C_1}{S_{XII}^X} \quad (25)$$

Additional eddy and molecular diffusivity parameters could also be defined from (24) but for a consistent nomenclature one should return to our basic approach and redefine ϵ and α values from start with that system. Again in practice we have not yet found it necessary to retain such generality since C_{1L} has been deduced from salt conductivity data (i.e. source term zero) where the mixing parameters have been determined from the bundle inner channels. Hence the approximations have been made that

$$\begin{aligned} \epsilon_{1L}^* \equiv \frac{1}{S_{XII}^X} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \approx \frac{1}{S_{XII}^X} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \\ \approx \frac{1}{S_{XII}^X} \frac{\lambda_A}{\lambda_{LII}} \epsilon^* \end{aligned} \quad (26)$$

2.2.1 - Continuum representation (C_L)

Again for completeness of definitions we observe that the continuum swirl parameter independent of bundle size is obtained by dividing equation 17 by $\frac{U_{AII}}{U_A}$ which from equation (20 a) is itself equal to

$$\frac{U_{AII}}{U_A} = S_{XII} \frac{\lambda_{AII}}{\lambda_A} \quad (27)$$

yielding

$$\begin{aligned} & \left(\frac{P_{SII} P_{NII}}{P_{AII}} \right) \frac{1}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} C \frac{\partial h^*}{\partial S^*} + \frac{\partial h^*}{\partial Z^*} = - De_1 \left(\frac{P_{NII}}{P_{AII}} \right) \left(\frac{1}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} \epsilon^* + \right. \\ & \left. + \frac{P_{KII}}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \right) \nabla h^* - De_1 \left(\frac{P_{SII}}{P_{AII}} \right) \left(\frac{1}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} \epsilon^* + \frac{P_{KII}}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \right) \nabla h^* \\ & \quad + \frac{P_{QII}^*}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} \end{aligned} \quad (28)$$

where

$$C_L \equiv \frac{1}{S_{XII}} \frac{\lambda_A}{\lambda_{AII}} C \quad (29)$$

and we do not assign new definitions for the eddy and molecular diffusivity terms.

The results are summarized in Tables 2 and 3. Table 2 summarizes only those parameters needed for turbulent bundle analysis (i.e. the mixing factor of region I and the swirl factor of region II). Table 3 presents a complete tabulation of parameters from our foregoing development which maintains the distinctions between parameters which would become important as the mixed convection regime is entered.

2.3 - POROSITY FACTORS

For a wire wrapped (one per rod) bundle in a triangular array the porosity factors of TABLE I are defined in terms of the following geometric factors.

$$D_F = \text{duct flat to flat distance} = \left[\sqrt{3} P N_{\text{RING}} + D + 2 D_W \right]$$

N_{RING} = number of hexagonal rings of fuel rods in the bundle excluding the center pin which is counted as ring 0.

2.3.1 - λ_A - Bundle average axial porosity

$$\lambda_A = \frac{\text{void area}}{\text{total area}} = \frac{\frac{\sqrt{3}}{2} D_F^2 - \frac{N\pi}{4} (D^2 + D_W^2)}{\frac{\sqrt{3}}{2} D_F^2} \quad (30)$$

assume that the fuel rod pitch P can be approximated by

$$P \cong D + D_W \quad (31)$$

then (30) becomes

$$\lambda_A = 1 - \frac{N\pi}{2\sqrt{3}} \frac{\left[\left(\frac{D}{P}\right)^2 + \left(1 - \frac{D}{P}\right)^2 \right]}{\left[\sqrt{3} N_{\text{RING}} + 2 - \frac{D}{P} \right]^2} \quad (32)$$

2.3.2 - λ_{AI} - Region I axial porosity

$$\lambda_{AI} = \frac{\frac{\sqrt{3}}{2} P \frac{1}{2} P - \frac{1}{2} \frac{\pi}{4} (D^2 + D_W^2)}{\frac{\sqrt{3}}{2} P \frac{1}{2} P} = 1 - \frac{\pi\sqrt{3}}{6} \left(\frac{D^2}{P^2} + \frac{D_W^2}{P^2} \right)$$

$$\frac{U_{AII}}{U_A} = S^{X_{II}} \frac{\lambda_{AII}}{\lambda_A} \quad (27)$$

yielding

$$\begin{aligned} & \left(\frac{P_{SII} P_{NII}}{P_{AII}} \right) \frac{1}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} C \frac{\partial h^*}{\partial S^*} + \frac{\partial h^*}{\partial Z^*} = - De_1 \left(\frac{P_{NII}}{P_{AII}} \right) \left(\frac{1}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} \epsilon^* + \right. \\ & \left. + \frac{P_{KII}}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \right) \nabla h^* - De_1 \left(\frac{P_{SII}}{P_{AII}} \right) \left(\frac{1}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} \epsilon^* + \frac{P_{KII}}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \right) \nabla h^* \\ & \quad + \frac{P_{QII}^*}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} \end{aligned} \quad (28)$$

where

$$C_L \equiv \frac{1}{S^{X_{II}}} \frac{\lambda_A}{\lambda_{AII}} C \quad (29)$$

and we do not assign new definitions for the eddy and molecular diffusivity terms.

The results are summarized in Tables 2 and 3. Table 2 summarizes only those parameters needed for turbulent bundle analysis (i.e. the mixing factor of region I and the swirl factor of region II). Table 3 presents a complete tabulation of parameters from our foregoing development which maintains the distinctions between parameters which would become important as the mixed convection regime is entered.

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2.3.1 - λ_A - Bundle average axial porosity

$$\lambda_A = \frac{\text{void area}}{\text{total area}} = \frac{\frac{\sqrt{3}}{2} D_F^2 - \frac{N\pi}{4} (D^2 + D_W^2)}{\frac{\sqrt{3}}{2} D_F^2} \quad (30)$$

assume that the fuel rod pitch P can be approximated by

$$P \cong D + D_W \quad (31)$$

then (30) becomes

$$\lambda_A = 1 - \frac{N\pi \left[\left(\frac{D}{P}\right)^2 + \left(1 - \frac{D}{P}\right)^2 \right]}{2\sqrt{3} \left[\sqrt{3} N_{\text{RING}} + 2 - \frac{D}{P} \right]^2} \quad (32)$$

2.3.2 - λ_{AI} - Region I axial porosity

$$\lambda_{AI} = \frac{\frac{\sqrt{3}}{2} P \frac{1}{2} P - \frac{1}{2} \frac{\pi}{4} (D^2 + D_W^2)}{\frac{\sqrt{3}}{2} P \frac{1}{2} P} = 1 - \frac{\pi\sqrt{3}}{6} \left(\frac{D^2}{P^2} + \frac{D_W^2}{P^2} \right)$$

2.3.3 - λ_{AII} - Region II axial porosity

$$\begin{aligned}\lambda_{AII} &= \frac{\text{Region II void area}}{\text{Region II total area}} = \frac{\text{Region II total area} - \text{Region II area for rods and wires}}{\text{Bundle total area} - \text{Region I total area}} \\ &= \frac{\frac{\sqrt{3}}{2} D_F^2 - \frac{\sqrt{3}}{2} (D_F - D - 2 D_W)^2 - \pi (D^2 + D_W^2) - \frac{3}{4} \pi (D^2 + D_W^2) (N_{RINGS} - 1)}{\frac{\sqrt{3}}{2} D_F^2 - \frac{\sqrt{3}}{2} (D_F - D - 2 D_W)^2} \\ &= 1 - \frac{(D^2 + D_W^2) (\pi + \frac{3}{4} \pi (N_{RINGS} - 3))}{\frac{\sqrt{3}}{2} [D_F^2 - (D_F - D - 2 D_W)^2]}\end{aligned}\quad (34)$$

2.3.4 - λ_{LI} - Minimum lateral porosity (region I)

$$\lambda_{LI} = \frac{\text{lateral cross flow area}}{\text{pitch length}} = \frac{P - D}{P} = 1 - \frac{D}{P} \quad (35)$$

2.3.5 - λ_{LII} - Minimum lateral porosity (region II)

$$\lambda_{LII} = \frac{g}{g + \frac{D}{2}} \quad (36)$$

assume that g can be approximated by

$$g = P - D \quad (37)$$

then equation (36) becomes

$$\lambda_{LII} = \frac{P - D}{P - D + \frac{D}{2}} = \frac{1 - \frac{D}{P}}{1 - \frac{1}{2} \frac{D}{P}} \quad (38)$$

2.4 - CONVERSION AMONG MIXING AND SWIRL PARAMETERS

It may often be necessary to convert between the parameter definitions of Table 2 to obtain input in the form required. Table 2 illustrates that the conversion process requires evaluation of ratios of porosities which in general vary with P/D and bundle size. Two cases have been evaluated.

2.4.1 - ϵ^* to ϵ_1^*

Equation 8 a relates these parameters as

$$\epsilon_1^* = \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (8 a)$$

The ration $\frac{\lambda_A}{\lambda_{LI}}$ is obtained from equations 32 and 35 as

$$\frac{\lambda_A}{\lambda_{LI}} = \left[1 - \frac{N\pi}{2\sqrt{3}} \frac{\left[\left(\frac{D}{P}\right)^2 + \left(1 - \frac{D}{P}\right)^2\right]}{\left[\sqrt{3} N_{RING} + 2 - \frac{D}{P}\right]^2} \right] \left[\frac{1}{\left[1 - \frac{D}{P}\right]} \right] \quad (39)$$

is presented graphically in Fig. 1 as a function of P/D and the number of fuel rods, N.

2.4.2 - C to C_1

Equation (23 c) relates these parameters as

$$C_1 = \frac{\lambda_A}{\lambda_{LII}} C \quad (23 c)$$

The ratio $\frac{\lambda_A}{\lambda_{LII}}$ is obtained from equations 32 and 38 as

$$\frac{\lambda_A}{\lambda_{LII}} = \left[1 - \frac{N\pi}{2\sqrt{3}} \frac{\left[\left(\frac{D}{P}\right)^2 + \left(1 - \frac{D}{P}\right)^2\right]}{\left[3 N_{RING} + 2 - \frac{D}{P}\right]^2} \right] \frac{\left[1 - \frac{1}{2} \frac{D}{P}\right]}{\left[1 - \frac{D}{P}\right]}$$

is presented graphically in Fig. 2 as a function of P/D and the number of fuel rods, N.

3 - FLOW-SPLIT

The division of flow at the bundle inlet between regions or between subchannel types is required for the ENERGY continuum and subchannel versions respectively. The relationship between region and subchannel velocity ratios is first described. Next a recent correlation for subchannel flows is presented. Finally the effect of tolerances on the results is discussed.

3.1 - Region and subchannel flow splits

<u>Region notation</u>	<u>Subchannel notation</u>
$S_I^X = \frac{V_{AI}}{V_A} \equiv \frac{V_{A1}}{V_A}$	S_1^X interior
$S_{II}^X \equiv \frac{V_{AII}}{V_A}$	S_2^X edge
	S_3^X corner

By definition region I contains only interior subchannels so that

$$S_I^X \equiv S_1^X \quad (41)$$

The relation between S_{II}^X and S_2^X can be derived as follows : From a mass balance assuming constant density

$$\begin{aligned}
 (N_2 A_{S2} + N_3 A_{S3}) V_{AII} &= N_2 A_{S2} V_{A2} + N_3 A_{S3} V_{A3} \\
 S_{II}^X &= S_2^X \frac{N_2 A_{S2}}{N_2 A_{S2} + N_3 A_{S3}} + S_3^X \frac{N_3 A_{S3}}{N_2 A_{S2} + N_3 A_{S3}} \\
 \frac{S_{II}^X}{S_2^X} &= \frac{1 + \frac{S_3^X N_3 A_{S3}}{S_2^X N_2 A_{S2}}}{1 + \frac{N_3 A_{S3}}{N_2 A_{S2}}} \quad (42)
 \end{aligned}$$

This relation is plotted in figure 3 which illustrates that for the plausible range of $\frac{S^X_3}{S^X_2}$ i.e. $(0.7 < \frac{S^X_3}{S^X_2} < 1.3)$, for bundles of 37 pins or more, S^X_{II} can be taken equal S^X_2 . An assumption of this sort is required because the value of S^X_3 has not yet been determined experimentally.

3.2 - Correlations for X_1 and X_2

The results are based on a model (4) which correlates experimental data for edge and interior channels. Corner data was not available so that S^X_3 was taken as equal to S^X_2 . The resulting uncertainty in S^X_1 and S^X_2 is $\pm 1\%$ provided

- a) $N > 37$
- b) $0.6 < S^X_3 \text{ actual} < 1.4$
- c) $1.0 < P/D < 1.4$

The recommended correlations given in equations 43 and 44 below and plotted in figures 4 and 5 for special cases of interest.

$$X_1 = (A_{s1} N_1 + A_{s2} N_2 + A_{s3} N_3) / \left\{ A_{s1} N_1 + \frac{2200 \left(\frac{De_1}{H}\right) \left(\frac{A_{r1}}{A_{s1}}\right) \left[\frac{P}{\sqrt{\pi^2 P^2 + H^2}} \right]^2 + 1}{1.2 \left[1 + \left[1.9 n \frac{V_T}{V_A} \right]_{\text{gap}} \right]^2} \right\}^{0.571} \times \left(\frac{De_2}{De_1}\right)^{0.714} (A_{s2} N_2 + A_{s3} N_3) \quad (43)$$

and

$$X_2 = (A_{s1} N_1 + A_{s2} N_2 + A_{s3} N_3) / \left\{ A_{s1} N_1 + \frac{1.2 \left[1 + \left[1.9 n \frac{V_T}{V_A} \right]_{\text{gap}} \right]^2}{2200 \left(\frac{De_1}{H}\right) \left(\frac{A_{r1}}{A_{s1}}\right) \left[\frac{P}{\sqrt{\pi^2 P^2 + H^2}} \right]^2 + 1} \right\}^{0.571} \times \left(\frac{De_1}{De_2}\right)^{0.714} (A_{s2} N_2 + A_{s3} N_3) \quad (44)$$

where

$$n = \frac{(P - D) \frac{P}{2}}{(P - \frac{D}{2}) \frac{P}{2} - \frac{\pi D^2}{16}} \quad (\text{this function is plotted in figure 6}) \quad (45)$$

$$\left. \frac{V_T}{V_A} \right|_{\text{gap}} = 10.5 \left(\frac{P-D}{P} \right)^{0.35} \frac{(D + D_w)}{\sqrt{\pi^2 (D + D_w)^2 + H^2}} \left(\frac{A_{r2}}{A_{s2}} \right)^{0.5} \quad (46)$$

$$A_{r2} = \frac{(\frac{D}{2} + D_w)^2 \pi - \frac{\pi D^2}{4}}{4}, \text{ for edge subchannels} \quad (47)$$

$$A'_{s2} = P \left(\frac{D}{2} + D_w \right) - \frac{\pi D^2}{8}, \text{ for edge subchannels} \quad (48)$$

$$A_{r1} = (\pi (P - \frac{D}{2})^2 - \pi D^2/4) / 6.0, \text{ for interior subchannels} \quad (49)$$

$$A'_{s1} = \frac{\sqrt{3}}{4} P^2 - \frac{\pi D^2}{8}, \text{ for interior subchannels} \quad (50a)$$

$$A_{s1} = A_{s1} - \pi D_w^2/8 \quad (50b)$$

$$A_{s2} = A_{s2} - \pi D_w^2/8 \quad (50c)$$

$$A_{s3} = \frac{1}{\sqrt{3}} \left(\frac{D}{2} + D_w \right)^2 - \frac{\pi}{24} D^2 - \frac{\pi}{24} D_w^2 \quad (50d)$$

3.3 - Effect of tolerances

The correlations proposed in this report are based on data from test bundles. All bundles except the MIT blanket bundles were represented by nominal dimensions. For the MIT blanket bundles, the actual geometry of these bundles was used to evaluate flow splits and reduce mixing data, i.e., actual average and tolerance values of pin and wire diameters and duct dimensions were used and the looseness was distributed according to a value of $F = 0.82$ (see figure 7).

The bounding effects of tolerances have been evaluated for some cases at MIT by means of assuming all the looseness allowed by the tolerances is concentrated in the edge region ($F=0$) and alternately that it is concentrated in the interior region ($F=1$). See Figure 7. For the specific case of the blanket geometry, the analysis of Ref. 8 demonstrates that the hot spot temperature differences between $F=1$ and $F=0$ can be as much as 10-15% of the total axial temperature rise. However, the assessment of the effects of tolerances and heated rod alignment modes on bundle temperatures must be considered an unresolved area.

4.0 - WIRE SWEEPING CORRELATIONS

Recent results based on a model (5) which correlates experimental data are presented here in the form of parameter used in the ENERGY code,

4.1 - Energy code parameters

New correlations have replaced those used in the original issue of this report which were taken from ref. (1). The original correlations were based purely on the experimental results, most of which pertain to large $\frac{P}{D}$ ratio bundles ($\frac{P}{D} > 1.20$). These correlations reflected the conclusions that ϵ^*_{1L} increases as the $\frac{P}{D}$ ratio increases and that C_{1L} is not a function of the $\frac{P}{D}$ ratio. However, the new correlations for ϵ^*_{1L} and C_{1L} (which are numerically very close to ϵ^*_{1L} and C_{1L}) suggest that both parameters are functions of the $\frac{P}{D}$ ratio and that ϵ^*_{1L} decreases as the $\frac{P}{D}$ ratio increases. The discrepancy between the original and the new correlations of the $\frac{P}{D}$ ratio effect on the parameter ϵ^*_{1L} (or ϵ^*_{1L}) is mainly due to the fact that the low $\frac{P}{D}$ ratio data (M.I.T. $\frac{P}{D} = 1.067$) were not previously available. Therefore, Skok's (Cadache's) 7-pin experimental data ($\frac{P}{D} = 1.14$) was originally included in the ϵ^*_{1L} correlation to make it valid for $\frac{P}{D} \geq 1.14$. Since the sweeping flow of the interior subchannels in a 7-pin bundle does not represent the sweeping flow in a "true" interior subchannel (as discussed in Chapter 2, ref. 5), the trend originally thought to exist for the mixing parameter ϵ^*_{1L} may not be correct even though the original correlation predicts all the interior subchannel mixing data for bundles of $1.14 \leq \frac{P}{D} < 1.32$ within $\pm 35\%$. Comparisons between the original and the new correlations for ϵ^*_{1L} and C_{1L} are illustrated in Figs. 8 and 9, respectively.

Interior-to-interior subchannels

$$\epsilon^*_{1L} = 0.128 \left(\frac{P-D}{\eta} \right)^{-0.50} \left(\frac{A_{r1}}{A'_{s1}} \right)^{0.5} \frac{(D + D_w) P^2}{\sqrt{\pi^2 (D + D_w)^2 + H^2}} \left(\frac{1}{A'_{s1}} \right) \quad (51)$$

where

$$A'_{s1} = \frac{\sqrt{3}}{4} P^2 - \frac{\pi D^2}{8} \quad (52)$$

$$A_{r1} = \frac{\pi (P - \frac{D}{2})^2 - \frac{\pi D^2}{4}}{6} \quad (53)$$

$$\eta = \frac{\sqrt{3}}{3} P \quad (54)$$

Interior-to-edge subchannels

$$\left(\frac{1}{s_{II}} \frac{\lambda_A}{\lambda_{LI}} \frac{\epsilon_{H_{NII}}}{U_A De_1} \right) = 0.128 \left(\frac{P-D}{\eta} \right)^{-0.50} \left(\frac{A_{r1}}{A'_{s1}} \right)^{0.5} \frac{(D + D_w) P^2}{\sqrt{\pi^2 (D + D_w)^2 + H^2}} \left(\frac{1}{A'_{s1}} \right) \quad (55)$$

Note that the LHS parameter appears in Table 3 and has not been given a special ϵ type symbol.

$$\text{Where } \eta = \frac{\sqrt{3}}{3.46} P + \frac{\sqrt{3}}{2} (P - D/2) \quad (56)$$

$$\text{and } s_{II}^{X_{II}} \cong s_{X_2}$$

Edge-to-edge subchannels (donor edge subchannel is not next to corner pin)

$$C_{IL} = 10.5 \left(\frac{P-D}{\eta} \right)^{0.35} \left(\frac{A_{r2}}{A'_{s2}} \right)^{0.5} \frac{D + D_w}{\sqrt{\pi^2 (D + D_w)^2 + H^2}} \quad (57)$$

$$\text{where } A_{r2} = \frac{(\frac{D}{2} + D_w)^2 \pi - \frac{\pi D^2}{4}}{4} \quad (58)$$

$$A'_{s2} = P \left(\frac{D}{2} + D_w \right) - \frac{\pi D^2}{8} \quad (59)$$

$$\eta = P \quad (60)$$

Edge-to-edge subchannels (donor edge subchannel located next to a corner pin)

$$C_{1L} = 10.5 \left(\frac{P-D}{\eta} \right)^{0.35} \left(\frac{A_{r2}}{A_{s2}''} \right)^{0.5} \frac{D + D_w}{\sqrt{\pi^2 (D+D_w)^2 + H^2}} \quad (61)$$

where

$$A_{s2}'' = P(P - \frac{D}{2}) - \frac{\pi D^2}{8} + (\frac{1}{2} (P - \frac{D}{2})^2 \tan 30^\circ - \frac{\pi D^2}{48}) \quad (62)$$

which represents the edge plus corner subchannel areas

$$A_{r2} = ((\frac{D}{2} + D_w)^2 - \frac{D^2 \pi}{4}) \frac{1}{3} \quad (63)$$

$$\eta = ((\frac{D}{2} + \frac{D_w}{2}) \tan 30^\circ + \frac{P}{2})^2 \quad (64)$$

Plots of ϵ_{1L}^* and C_{1L} (donor edge subchannel not next to a corner pin) are presented in figures 9 and 10.

5.0 - CRITERION TO DETERMINE WHEN BUOYANCY EFFECTS ARE IMPORTANT

In reference 7 a criterion has been derived to determine when buoyancy effects are important in a wire wrapped fuel or blanket assembly operating in steady state mixed convection flow.

Buoyancy effects were determined to be important when $G_r^* > G_{rc}^*$. The critical value of this dimensionless group, called G_{rc}^* , has been found to be approximately 0.025 (Ref. 7).

5.1 - Skewed Radial Power Profile

For an assembly operating in steady state conditions, the definition of G_r^* was determined to be as follows: (Errata to ref. 7 given in C00-2245-30 are included here plus necessary change in the γ definition to terms involving ϵ_{1L}^* vs ϵ_1^* and α_{1L}^* vs α_1^*)

$$G_r^* = \frac{Gr}{f R_e^2} \chi \quad (65)$$

where

$$\chi = \frac{1}{M} \frac{1}{\gamma_L} \frac{\hat{Q}_{p-1}}{2} \quad (66)$$

$$G_r = \frac{g_o \beta (T_M - T_{IN}) D_{e1}^3}{\nu^2} \quad (67)$$

$$\gamma = \frac{(P/D-1)}{\pi} \frac{16}{D_F} (\epsilon_{1L}^* + S_{NI}^K \alpha_{1L}^*) \quad (68)$$

$(T_M - T_{IN})$ = average axial temperature rise across the bundle of height L

$\hat{Q}_p$ = radial power skew (peak/avg)

Re = Reynolds number in bundle based on De_1 and interior subchannel axial velocity and bundle mean axial temperature

f = friction factor determined from smooth tube conditions at Re based on definition

$$f = \frac{\Delta p}{\frac{L}{De_1} \frac{\rho v_{A1}^2}{2}}$$

L = bundle axial length

ν = kinematic viscosity evaluated at mean bundle temperature $(T_M + T_{IN})/2$

β = volume expansion coefficient, evaluated at mean bundle temperature

M = Novendstern's multiplication factor for wire wrapped bundle

$$= \left\{ \frac{1.034}{(P/D)^{0.124}} + \frac{29.7 (P/D)^{6.94} Re^{0.086}}{(H/D)^{2.239}} \right\} 0.885 \quad (69)$$

g_o = acceleration due to gravity

5.2 - Flat Radial Power Profile (From reference 8)

For the flat radial power case, $\hat{Q}_p = 1$ and $G_r^* = 0$. This leads to a conclusion that buoyancy effects are negligible. However, this is not the case for a flat radial power assembly operated at low Raynolds

number. The difference between the flow rates of the different types of subchannels can result in a temperature difference between edge and interior subchannels. The radial temperature difference thus may cause a buoyancy effect.

The basic definition of the G_r^* reported in reference 7 is

$$G_r^* = \frac{g_o \beta [T(Z) - \bar{T}(z)]}{F_D} \quad (70)$$

where

$$F_D = \frac{Mf}{De_1}$$

$T(Z)$ = fluid temperature in the bundle

$\bar{T}(Z)$ = mean fluid temperature

In order to predict buoyancy effects in a conservative way, the $[T(Z) - \bar{T}(Z)]$ in the above equation is replaced by $[T_{\max}(L) - \bar{T}(L)]$, where $T_{\max}(L)$ denotes the maximum temperature at bundle exit where the $[T(Z) - \bar{T}(Z)]$ has a maximum value. Therefore, the G_r^* becomes

$$G_r^* = \frac{g_o \beta [T_{\max}(L) - \bar{T}(L)]}{F_D} \quad (71)$$

The $[T_{\max}(L) - \bar{T}(L)]$ can be obtained by solving the temperature profile of a two region assembly at uniform radial power. One region contains all the interior subchannels and the other region contains all the peripheral subchannels. The governing equations for the fluid enthalpy of these two regions are:

$$m_{AI} \frac{dh_I}{dz} = P \rho (\epsilon_{H_{NI}} + p_{NI}^K \alpha) \frac{h_{II} - h_I}{\eta} + p^{(QA)}_I \quad (72)$$

$$m_{AII} \frac{dh_{II}}{dz} = P \rho (\epsilon_{H_{NI}} + p_{NI}^K \alpha) \frac{h_I - h_{II}}{\eta} + p^{(QA)}_{II} \quad (73)$$

Where η = centroid-to-centroid distance between region I and region II

$p^{(QA)} = p^Q_p A$ = heat added to the region I or region II per unit length

Note $p^{(QA)}_I = s^{(QA)}_I = (QA)_I$

$p^{(QA)}_{II} = s^{(QA)}_{II} = (QA)_{II}$

The solution for $h_I(L)$ and $h_{II}(L)$ in equation (8) and (9) are as follows:

$$h_I(L) = h_{IN} + \frac{[(QA)_I + (QA)_{II}] L}{m_{AI} + m_{AII}} + \left(\frac{(QA)_I m_{AII}}{(m_{AI} + m_{AII}) \Theta} - \frac{[(QA)_I + (QA)_{II}]}{(m_{AI} + m_{AII}) n \Theta} \right) \left(1 - e^{-n \Theta L} \right) \quad (74)$$

$$h_{II}(L) = h_{IN} + \frac{[(QA)_I + (QA)_{II}] L}{m_{AI} + m_{AII}} + \left(\frac{(QA)_{II} m_{AI}}{(m_{AI} + m_{AII}) \Theta} - \frac{[(QA)_I + (QA)_{II}]}{(m_{AI} + m_{AII}) n \Theta} \right) \left(1 - e^{-n \Theta L} \right) \quad (75)$$

where

$$\theta = \frac{P\rho(\epsilon_{H_{NI}} + K_{P_{NI}}\alpha)}{\eta} \quad (76)$$

$$n = \frac{\frac{m_{AI}}{m_{AI}} + \frac{m_{AII}}{m_{AII}}}{\frac{m_{AI}}{m_{AI}} + \frac{m_{AII}}{m_{AII}}} \quad (77)$$

Subtracting eq. (75) from (74) we obtain

$$h_I(L) - h_{II}(L) = \frac{(QA)_{I AII}^m - (QA)_{II AI}^m}{(m_{AI} + m_{AII})\theta} (1 - e^{-n\theta L}) \quad (78)$$

Because $T_{\max}(L) - \bar{T}(L)$ is approximately equal to the temperature of the interior region minus the average fluid temperature which can be expressed as

$$\bar{T}(L) = \frac{N_1^m T_I(L) + N_2^m T_{II}(L)}{N_1^m + N_2^m} \quad (79)$$

where

N_1 = number of interior subchannels

N_2 = number of edge subchannels

$$\begin{aligned} T_{\max}(L) - \bar{T}(L) &= T_I(L) - \frac{N_1^m T_I(L) + N_2^m T_{II}(L)}{N_1^m + N_2^m} \\ &= \frac{N_2^m}{N_1^m + N_2^m} \left(T_I(L) - T_{II}(L) \right) \end{aligned} \quad (80)$$

Inserting equation (78) into above equation, we obtain

$$\begin{aligned} T_{\max}(L) - \bar{T}(L) &= \left(\frac{N_2^m}{N_1^m + N_2^m} \right) \left(\frac{(QA)_{I AII}^m - (QA)_{II AI}^m}{m_{AI} + m_{AII}} \right) \\ &\quad \left(\frac{1}{\theta C_p} \right) (1 - e^{-n\theta L}) \end{aligned} \quad (81)$$

Then equation (71) becomes

$$G_r^* = \frac{g_0 \beta \left\{ \frac{N_2^m}{N_1^m + N_2^m} \right\} \left(\frac{(QA)_I^m A_{II} - (QA)_{II}^m A_I}{m_{AI} + m_{AII}} \right) \left(\frac{1}{\theta_{Cp}} \right) (1 - e^{-n\theta L})}{F_D} \quad (82)$$

In addition, since we know

$$R_e = \frac{\rho V_{AI}^D e_1}{\mu} = \frac{V_{AI}^D e_1}{\nu}$$

$$m_{AI} = \rho A_{sI} V_{AI} = \rho A_{sI} V_{AI} = \rho A_{sI} V_{sI} X_I = \rho A_{sI} V_{sI} X_I$$

$$m_{AII} = \rho A_{sII} V_{AII} = \rho A_{sII} V_{sII} X_{II}$$

Equation (82) can be reformed as

$$G_r^* = \frac{g_0 \beta \left\{ \frac{N_2 s_{II} s_{XII}}{N_1 s_{IsI} X_I + N_2 s_{II} s_{XII}} \right\} \left(\frac{(QA)_{IsII} s_{XII} A_{II} - (QA)_{IIsI} s_{XII} A_I}{s_{IsI} X_I + s_{IIsII} A_{II}} \right) \left(\frac{1}{\theta_{Cp}} \right) (1 - e^{-n\theta L})}{Mf Re_v^2} e_1^3 \quad (83)$$

The above modified Grashof number is derived from the definition employed in Reference 7. However, whether the criteria stated in (Ref. 7) is applicable or not with respect to the above formula (i.e., a flat radial power case) has not yet been studied. In any case, if the above G_r^* number for flat radial power case is much larger than 0.025, it is reasonable to assume that the mixed convection condition will be experienced.

6.0 - IMPORTANT NOTE ON ENERGY CODE LISTING

The listing of the ENERGY codes uses "D" the rod diameter as the non-dimensionalizing parameter. However, it was later decided to use De_1 the hydraulic diameter of the interior subchannel as the characteristic dimension (as opposed to D) in order to maintain the same definition of Reynolds number as commonly used in literature. The code requires $\epsilon^* = \frac{\epsilon_{HNI}}{U_A De_1}$ as input but converts it to $\frac{\epsilon_{HNI}}{U_A D}$ for use in the ENERGY code. Revision I to the SUPERENERGY document (20TR) records the fact that SUPERENERGY was revised to utilize De_1 directly through the input parameter ϵ_1^* . The input for ENERGY code series are listed as follows for convenience.

Code	ϵ	C
ENERGY I, II, III (-18TR)	ϵ^*	C
SUPERENERGY (-20TR, Rev. 1)	ϵ_1^*	C_1
TRANSENERGY-S (Glazer's Thesis)	ϵ_1^*	C_1

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$$\epsilon_1^* = \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (\text{Eq. 39})$$

FIGURE 1. Porosity Ratio for ϵ_1^* to ϵ^* Conversion

FIGURE 2. Porosity Ratio for C_1 to C Conversion

FIGURE 3. Ratio of Flow Splits for Region II and Type 2 Subchannels

FIGURE 4. Predicted Interior Flow Split Parameter X_1 (Eq. 43)

FIGURE 5. Predicted Edge Flow Split Parameter X_2 (Eq. 44)

Figure 6. Parameter n in Equation (45) versus P/D Ratio.

$$\text{Flat Tolerance} \equiv D_F - S$$

$$F = \frac{\sqrt{3} (P-D-D_w) N_{\text{ring}}}{\text{Flat Tolerance}}$$

Figure 7. Definition of Flat Tolerance and Rod Alignment Modes of $F = 1.0$ and $F = 0$

$$\epsilon_{1L}^* = \frac{\sqrt{\Pi^2 P^2 + H^2}}{P}$$

Figure 8. Comparison Between Original and New Correlations for ϵ_{1L}^*

Figure 9. Comparison between Original and New Correlations for C_{1L} ($N > 19$)

FIGURE 10. ϵ_{1L}^* Predicted Behavior

FIGURE 11. C_{1L} Predicted Behavior

TABLE 1 - POROSITY FACTORS

REGION	AXIAL POROSITY	MINIMUM LATERAL POROSITY
I	λ_{AI}	λ_{LI}
II	λ_{AII}	λ_{LII}
BUNDLE AVERAGE	λ_A	
ONLY THOSE POROSITIES NEEDED IN THE ANALYSIS ARE DEFINED		

TABLE 2 - PARAMETERS FOR TURBULENT BUNDLE ANALYSIS

	REPRESENTATION	
	POROUS BODY	SUBCHANNEL
BUNDLE SIZE DEPENDENT PARAMETERS	$\epsilon^* = \frac{\epsilon_{HNI}}{U_A De_1} \quad (4 \text{ a})$ $C = \frac{U_{SII}}{U_A} \quad (18)$	$\epsilon^*_1 = \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (8 \text{ a})$ $C_1 = \frac{\lambda_A}{\lambda_{LII}} C \quad (23 \text{ c})$
BUNDLE SIZE INDEPENDENT PARAMETERS	$\epsilon^*_{L} = \frac{1}{S^X_1} \frac{\lambda_A}{\lambda_{AI}} \epsilon^* \quad (13 \text{ a})$ $C_L = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} C \quad (29)$	$\epsilon^*_{1L} = \frac{1}{S^X_1} \epsilon^*_1 \quad (10 \text{ a})$ $C_{1L} = \frac{C_1}{S^X_{II}} \quad (25)$

TABLE 3 - COMPLETE TABLE OF PARAMETERS FOR BUNDLE ANALYSIS

		REPRESENTATION					
		POROUS BODY			SUBCHANNEL		
BUNDLE SIZE DEPENDENT PARAMETERS	REGION I	$\epsilon^* = \frac{\epsilon_{HNI}}{U_A D_{e1}} \quad (4a)$	$\alpha^* = \frac{\alpha}{U_A D_{e1}} \quad (4b)$	$K_{PNI} \quad (3)$	$\epsilon^*_I = \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (8a)$	$\alpha^*_I = \lambda_A \alpha^* \quad (8b)$	$K_{SNI} = \frac{K_{PNI}}{\lambda_{LI}} \quad (8c)$
	REGION II	$\square = \frac{\epsilon_{HNII}}{U_A D_{e1}} \quad (15)$ $\square = \frac{\epsilon_{HSII}}{U_A D_{e1}} \quad (15)$ $C = \frac{U_{SII}}{U_A} \quad (18)$	$\alpha^* = \frac{\alpha}{U_A D_{e1}} \quad (4b)$ $\alpha^* = \frac{\alpha}{U_A D_{e1}} \quad (4b)$	$K_{PNI} \quad (17)$ $K_{PSII} \quad (17)$	$\square = \frac{\lambda_A \epsilon_{HNII}}{\lambda_{LI} U_A D_{e1}} \quad (21 \text{ without approx of } 16)$ $\square = \frac{\lambda_A \epsilon_{HSII}}{\lambda_{LII} U_A D_{e1}} \quad (21 \text{ without approx of } 16)$ $C_I = \frac{\lambda_A}{\lambda_{LII}} C \quad (23c)$	$\alpha^*_I = \lambda_A \alpha^* \quad (8b)$ $\alpha^*_I = \lambda_A \alpha^* \quad (8b)$	$K_{SNI} = \frac{K_{PNI}}{\lambda_{LI}} = \frac{K}{S_{NI}} \quad (8c)$ $K_{SII} = \frac{K_{PSII}}{\lambda_{LII}} \quad (23b)$
BUNDLE SIZE INDEPENDENT PARAMETERS	REGION I	$\epsilon^*_L = \frac{1}{S^X_I} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (13a)$ $= \frac{\epsilon^*}{P^X_I}$	$\alpha^*_L = \frac{1}{S^X_I} \frac{\lambda_A}{\lambda_{LI}} \alpha^* \quad (13b)$	$K_{PNI} \quad (12)$	$\epsilon^*_{IL} = \frac{1}{S^X_I} \frac{\lambda_A}{\lambda_{LI}} \epsilon^* \quad (10a)$ $= \frac{\epsilon^*}{S^X_I}$	$\alpha^*_{IL} = \lambda_A \alpha^* = \frac{\alpha^*_I}{S^X_I} \quad (10b)$	$K_{SNI} = \frac{K_{PNI}}{\lambda_{LI}} \quad (8c)$
	REGION II	$\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} \frac{\epsilon_{HNII}}{U_A D_{e1}} \quad (28 \text{ without approx of } 16)$ $\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} \frac{\epsilon_{HSII}}{U_A D_{e1}} \quad (28 \text{ without approx of } 16)$ $C_L = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} C \quad (29)$	$\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \quad (28)$ $\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{AII}} \alpha^* \quad (28)$	$K_{PNI} \quad (26)$ $K_{PNI} \quad (28)$	$\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{LII}} \frac{\epsilon_{HNII}}{U_A D_{e1}} \quad (24 \text{ without approx of } 16)$ $\square = \frac{1}{S^X_{II}} \frac{\lambda_A}{\lambda_{LII}} \frac{\epsilon_{HSII}}{U_A D_{e1}} \quad (24 \text{ without approx of } 16)$ $C_{IL} = \frac{C_I}{S^X_{II}} \quad (25)$	$\square = \frac{\lambda_A}{S^X_{II}} \alpha^* \quad (24)$ $\square = \frac{\lambda_A}{S^X_{II}} \alpha^* \quad (24)$	$K_{SNI} = \frac{K_{PNI}}{\lambda_{LI}} = \frac{K}{S_{NI}} \quad (8c)$ $K_{SII} = \frac{K_{PSII}}{\lambda_{LII}} \quad (23b)$

where $\square$ indicates that no defining symbol has been assigned.