TITLE  SYMMETRIES IN EXOTIC NEUTRON-RICH NUCLEI

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SU(3) Symmetries in Exotic Neutron-Rich Nuclei

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Abstract

We examine the structure of the exotic neutron-rich nucleus $^{11}$Li with an emphasis on understanding the origin of the soft E1 resonance and the neutron halo. The similarities and differences between shell model and di-neutron cluster model descriptions of the system are displayed using the Hecht expansion techniques. We find that the ground state of $^{11}$Li as described in large shell model calculations is well approximated by the di-neutron cluster state. In contrast to the ground state, the soft E1 model of $^{11}$Li appears to have a more complicated structure and the E1 strength of this resonance is very sensitive to cancellations between p→s and p→d contributions to the dipole matrix elements.

Introduction.

Recently there has been renewed theoretical and experimental interest in the structure of nuclei far from stability. This is particularly true for p-shell nuclei where very large neutron to proton ratios are found, providing a testing ground for present models of nuclear structure under extreme conditions. For example, the largest ratio observed for particle stable nuclei occurs in $^8$He where N/Z=3, while for $^{11}$Li N/Z=2.667. As the atomic mass increases it becomes more difficult to make very neutron rich nuclei, and for comparison the most neutron rich Pb isotope, $^{214}$Pb, has: $^{214}$Pb N/Z=1.62. In this paper I discuss theoretical studies of the light neutron-rich nuclei and use the Hecht technique for expressing SU(3) cluster states in terms of non-spurious SU(3) shell model states to illuminate the underlying physics behind the interesting phenomena observed for these systems.

The increased availability of radioactive ion beams has greatly enhanced studies of nuclei far from stability and experiments using high and intermediate energy beams have yielded very surprising results. The measurements on neutron-rich systems which have been carried out at these facilities fall into two main classes, i.e., (1) measurements of total interaction cross-sections on various light targets to extract information on the size of the system, and (2) measurements of the Coulomb dissociation cross-sections to obtain information about the photo-absorption cross-section and hence E1 distribution in the nucleus.

This paper is dedicated to the memory of Helmut Baer who made invaluable contributions to the study of neutron rich nuclei using pion reactions. Some of his recent work on $^{11}$Li described herein represents just one aspect of his many contributions to nuclear physics. We will miss him greatly, both personally and professionally.
sets of experiments found anomalously large cross-sections and significant deviations from expectations based on systematics, and have led to the concepts of a neutron halo and a soft E1 resonance in some of the exotic neutron-rich systems. The largest effects have been seen in 11Li and most theoretical efforts to understand these nuclei have concentrated on this system. An additional reason for concentrating on 11Li is that recent 11B(π−,π+)11Li experiments at LAMPF directly observed the soft E1 resonance and determined the size of the halo.

Neutron Halos.

The interaction radius of neutron dripline nuclei can be extracted directly from high energy interaction cross-section measurements using

\[ \sigma_I = \pi [R_I(p) + R_I(t)]^2, \]

where \( R_I(p) \) and \( R_I(t) \) are the interaction nuclear radii of the projectile and target. This simple parameterization has been justified in high energy experiments where the extracted value of \( R_I(p) \) is found to be independent of the target used for p-shell nuclei. As expected as one moves across the shell \( R_I \) grows relatively smoothly with \( A^{1/3} \), i.e., the size of the systems grow with the number of nucleons. What was not so expected was the sudden jump from the \( A^{1/3} \) dependence seen for the neutron dripline nuclei (Fig. 1).

Fig. 1 Interaction radii (left fig.) for p-shell nuclei showing the large deviation from the smooth \( A^{1/3} \) dependence for nuclei close to the neutron dripline. A similar deviation from systematics is seen in the r.m.s. matter radii (right fig).

Consistent with these anomalously large interaction radii, the r.m.s. matter radii deduced from these experiments from Glauber analyses were also found to be unusually large. For example, the r.m.s. radius for stable nuclei are found to grow relatively slowly across the
shell, with typical vs ranging between 2.3 - 2.4 fm, to be compared with \(^{11}\text{Li}\) whose r.m.s. radius is \(3.2 \pm 0.17\) fm. It is these results and the sudden increase in radius in going from \(^9\text{Li}\) to \(^{11}\text{Li}\) which have led to the concept of a neutron halo. While a precise definition of the halo has not really been stated, it generally refers to the idea that the last two neutrons extent considerably beyond the core of \((Z,N-2)\) nucleons. Confirmation of the neutron halo comes projectile fragmentation experiments\(^2\) of \(^{11}\text{Li}\) on low Z targets where the observed transverse momentum distribution of the \(^9\text{Li}\) fragments is very narrow, reflecting the large size of the projectile. In addition, the angular distribution of the outgoing neutrons in fragmentation is very forward peaked\(^3\). These results suggest that it is indeed the last two neutron which are responsible for the large radius of \(^{11}\text{Li}\).

Although evidence for the halo is extensive, there is some uncertainty as to the magnitude of its radius, and results for \(^{11}\text{Li}\) range from a neutron r.m.s. radius\(^1\) of \(3.21 \pm 0.17\) fm to a neutron halo with a radius\(^3\) of 12 fm. However, an analysis\(^5\) of the sensitivity of the \(^{11}\text{B}(\pi^- , \pi^+)\text{Li}\) cross-section to the assumed neutron radius shows that the pion data\(^4\) can be used to provide an accurate determination of the radius of the last two neutrons. The strong sensitivity of the pion experiment arises because of the large difference between the binding of the protons of \(^{11}\text{B}\) and the neutrons of \(^{11}\text{Li}\) which are exchanged in the reaction. In particular, the cross-section for the reaction will be significant only if the overlap of the wave functions for the initial well bound protons and final loosely bound neutrons is sizeable.

\[
\begin{array}{ccc}
\text{\(^{11}\text{Li}: \text{very loosely bound:}\)} & 3/2^- & T=5/2 \\
\text{\(^9\text{Li} +n+n\)} & 0.2\text{MeV} & 0.2\text{MeV} \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{\(^{11}\text{Be} + p\)} & 11\text{MeV} \\
\text{\(^{11}\text{B}: \text{well bound:}\)} & 1/2^- & T=1/2 \\
\end{array}
\]

Fig 2 The large mismatch in the binding energy of the exchanged protons of \(^{11}\text{B}\) and neutrons of \(^{11}\text{Li}\) causes the pion double charge exchange reaction (DCX) to be very sensitive to the size of the neutron halo. The transition density is determined by the overlap of the initial and final wave functions so that the DCX cross-section directly probes their radial extent.

At a pion energy of \(T_\pi = 164\) MeV, the DCX reaction is dominated by two sequential single charge exchanges. We have performed calculations of the DCX reaction using finite range distorted waves and a closure approximation for the intermediate states. The calculational technique has already been described elsewhere\(^6\). The nuclear structure input was expressed in term of shell model two-body density matrix elements derived from a p-shell calculation. In a p shell model \(^{11}\text{Li}_{g.s.}\) is described as a single \(p_{3/2}\) proton with the neutrons forming a closed shell. The magnetic moment is then just the Schmidt \(p_{3/2}\) value, 3.79 \(\mu_n\), which is
in reasonable agreement with the measured value of 3.667μm. The wave function for 11Bz,.., obtained by diagonalizing the Cohen-Kurath (8-16)2BME effective interaction, reproduces the ground state magnetic and quadrupole moments reasonably well.

In calculating the DCX reaction the rms radius of the last two neutrons in 11Li (R2n) was varied in order to determine the value giving the best representation of the data. This was done by adjusting the size of the Woods-Saxon well used to obtain the single particle wave functions, while keeping the binding energy of the 11Li neutrons fixed at 200 keV. The rms radius of the two protons on which the reaction proceeds was held fixed at 2.65 fm, a value suggested by the difference in charge radius between 11B and 9Li. The 11B protons were bound by 11 MeV.

As the volume in which the exchanged neutrons are to be found increases the cross section decreases since the reaction only has significant strength when the overlap of the wave function of the final neutrons with the initial protons is large. In the limit as the radius of the two final neutrons becomes very large, it is the initial wave function alone which controls the volume over which the reaction takes place. When this limiting situation is reached the shape of the transition density no longer changes and the cross section scales as the inverse volume squared or as 1/R2n. The results of the calculation are shown in Figure 3 for a range of rms radii covering those that have been suggested in the literature. We note that a radius of 12 fm, as suggested by Anne et al.3, would imply a cross section 3 orders of magnitude smaller than that observed.

Fig 3 The angular distribution of the cross section for the 11B(p−, p+)11Li reaction for several values of the rms radius of the last two neutrons in 11Li. A radius of 12 fm, as suggested from a recent projectile fragmentation experiment would imply a cross section 3 orders of magnitude smaller than observed.
We estimated the uncertainty in the calculated cross sections from systematic studies of nuclear structure and DCX for nuclei in this mass region. The use of p-shell wave functions may be too restrictive, particularly in the case of the $^{11}$Li nucleus where di-neutron clustering is important. The description of a di-neutron cluster state in a harmonic oscillator shell model basis would require a large multi-$\hbar\omega$ calculation. The inclusion of states of very high excitation is necessary in order to give a realistic description of the relative motion wave function. The problem is mitigated by our use of Woods-Saxon single-particle wave functions, and by the fact that our p-shell neutrons are strongly correlated (as discussed below). However, additional correlations can be introduced by increasing the model basis to allow sd-shell or higher excitations, and these have been shown\textsuperscript{8} to give up to a factor of two in the DCX cross-sections between isobaric analog states. In the present non-analog transition these effects will be reduced by roughly $\sqrt{2}$ since two nucleon clustering is important only in the final ($^{11}$Li) wave function.

The second source of possible error lies in the calculation of the distortions of the pion. We can estimate the accuracy of the calculated distortions by comparing with the analog state transition in $^{14}$C. Using wave functions obtained by diagonalizing the same Chen-Kurath interaction we find that the data for the $^{14}$C($\pi^+, \pi^-)^{14}$O reaction are correctly reproduced around 20° but are over estimated by almost a factor of 2 at 50°. Thus we assign a factor of $\sqrt{2}$ error from the distortion. The corrections from nuclear structure and pion dynamics have approximately equal but opposite effect, with the former increasing and the latter decreasing the calculated cross section. Combining the two we estimate that the uncertainty in the calculated cross sections has a one standard deviation error of 40%. Allowing this uncertainty we conclude that the measured cross section determines the radius of the last two neutrons in $^{11}$Li to be $5.1^{+0.6}_{-0.5}$ fm.

The Soft E1 Resonance

A second set of experiments measured\textsuperscript{2,3} the Coulomb fragmentation cross-sections in the ($^{11}$Li, $^9$Li) dissociation reaction on high Z targets. Coulomb fragmentation cross-sections measured in peripheral high energy collisions are well understood in terms of the corresponding photoabsorption cross-section and are dominated by electric dipole radiation. The electromagnetic dissociation cross-section for $^{11}$Li on $^{208}$Pb is observed to be anomalously large, about 20 times larger than for $^{12}$C. Such a large cross-section can only be explained by a ground state E1 distribution strength which is strongly enhanced at low excitation energies. An estimate of the enhancement needed suggests that a soft E1 mode exists in $^{11}$Li in which about 20% - 40% of the total ground-state dipole strength lies as low as 1-2 MeV of excitation.

E1 strengths between low-lying nuclear states are normally greatly suppressed compared to single-particle estimates and most of the dipole sum rule is accounted for by the giant dipole resonance (GDR) at higher excitation. Indeed, low-lying transition strengths are typically $10^{-6} \cdot 10^{-4}$ W.u., and correspond to $\lesssim 0.5\%$ of the strength seen in the GDR. This is explained within the shell model by the fact that low-lying transitions involve cancellation between the different single particle amplitudes contributing to the transition. In contrast
in the GDR region these amplitudes add constructively and exhaust most of the available E1 strength.

\[
\begin{array}{c}
\text{E1} \\
\text{g.s.}
\end{array}
\quad
\begin{array}{c}
(0s)^6 (0p)^{n-1} 1s + (0s)^6 (0p)^{n-1} 0d \\
(0s)^4 (0p)^3
\end{array}
\]

Fig. 4 The normal suppression of low-lying dipole transitions in p-shell nuclei arises because \( p\rightarrow s \) and \( p\rightarrow d \) E1 amplitudes are comparable in magnitude but opposite in sign. In contrast these two amplitudes add constructively in the giant resonance region.

To understand the origin soft E1 resonance in \(^{11}\text{Li}\) we carried out a large \((0+2)\hbar\omega\) shell calculation\(^7\) for the ground state and a \((1+3)\hbar\omega\) calculation for the dipole states. The \(2\hbar\omega\) excitations in the g.s. play an important role in determining the dipole distribution. Also, two-neutron \(p^{-2}(sd)^2\) excitations lie low in the theoretical spectrum and can mix strongly into the g.s. Our basis states were labelled in the SU(3) harmonic-oscillator (HO) scheme, which allowed the exact elimination of spurious centre of mass states. However, to determine the dipole distribution requires more realistic single-particle wave functions. Replacing E1 single-particle matrix elements with those obtained using more realistic functions unavoidably introduces some spurious contributions. For the shell model two-body interaction we used the Cohen-Kurath (CK) p-shell, Chung-Wildenthal sd-shell, and Millener-Kurath (MK) particle-hole interactions\(^8\). The single particle energies were taken to be the CK and MK values.

Diagonalizing the \((0+2)\hbar\omega\) space yielded a \(3/2^-\) ground state which is dominated by the \(p_{3/2}\) proton configuration with a closed neutron shell. The \(2p-2h\) configuration are predicted to makeup 25\% of the g.s. wave function. The low-lying positive-parity states are dominantly \(p^{-1}(sd)\) neutron excitations, and \(3\hbar\omega\) configurations typically makeup \(\leq 10\%\) of the wave functions of states below 10 MeV. As expected the predicted dipole distribution in a HO basis shows no soft E1 resonance because of the strong destructive interference between the \(p\rightarrow s\) and \(p\rightarrow d\) amplitudes. In particular, the dominant structure of the low-lying positive parity states involves a neutron excitation to the \(1s_{1/2}\) level so that the \(p\rightarrow s\) one-body density matrix elements (OBDMEs) are large. Although the components in the wave functions corresponding to neutron excitations to the \(d_{5/2}\) level are relatively small, the single particle E1 \(p\rightarrow d\) matrix element (SPME) is large in a HO basis. This causes the two amplitudes to be comparable in magnitude, yielding a small total B(E1) value.

To include binding energies in evaluating the E1 matrix elements we replaced the usual shell-model OBDMEs by a sum over the parentage coefficients to all physical states of the \(A I\) system. Then, following Millener\(^9\), the E1 matrix elements using Woods-Saxon (WS) wave functions were evaluated for the binding relative to each core state \([A-1; J_C]\). Evaluating the SPME with WS wave functions led to very enhanced E1 transitions at low excitation energies. The main reason for the strong enhancement was the large increase of the \(p\rightarrow s\) SPME relative to the \(p\rightarrow d\) SPME, thus breaking the strong cancellation between the two contributions.
Fig. 5 The dipole distribution predicted in a $(0\rightarrow1)\hbar\omega$ model HO wave functions and a $(0+2\rightarrow1+3)\hbar\omega$ model using WS wave functions for states below 5.0 MeV of excitation. The soft E1 resonance arises when the weak binding energy of the last neutrons and corresponding low tail of the single-particle wave function reduces the cancellation between large contributions to the matrix elements.

Di-neutron cluster model versus shell model descriptions of $^{11}$Li.

The theoretical studies of the structure of $^{11}$Li fall into two main classes, namely, shell models and di-neutron cluster models. In the cluster model\textsuperscript{10} it is proposed that the low two-neutron separation energy (0.2 MeV) leads to a di-neutron cluster outside the $^9$Li core, giving rise to a large neutron radius. In addition, the soft E1 resonance arises naturally in the cluster model when the core and cluster are restricted to be in their respective ground states. In this case the E1 matrix elements are simply given in terms of single particle matrix elements of the vector connecting the centres of mass of the core and cluster. There are no cancellations similar to those seen in the shell model calculations, and cluster estimates for the Coulomb dissociation cross-section generally overestimate experiment.

Direct comparisons between shell model and cluster model wave functions are difficult because of the differences in the two philosophies for choosing the truncation of the model space. In the shell model one usually includes all cluster and non-cluster configurations up to $n\hbar\omega$ of excitation, where typically $n \leq 4$ for light nuclei. On the other hand the cluster model includes states of very high excitation through the relative motion wave function, which is a function of the vector connecting the centres of mass of the core and cluster. The truncation of the cluster space is made by assuming a simple structure for the core and cluster, e.g., they are usually restricted to be in their respective ground states. In this paper the term 'di-neutron cluster' is used to mean a pair of neutrons coupled to $S=0$ and $L=0$ and by default to have SU(4) symmetry \cite{f}. In principle a (non-orthogonal) cluster basis can be formed by including several excited states of the $^9$Li core. In addition, the pair of neutrons outside the $^9$Li core need not necessarily from a cluster in that they can be coupled to $S=1$. The relative importance of these different configurations in $^{11}$Li can be estimated from their shell model spectroscopic factors and from the expansion of the cluster wave function in terms of the shell model basis.

Hecht\textsuperscript{11} has derived an explicit method for expanding antisymmetric cluster functions, coupled in SU(3), into a non-spurious SU(3) shell model basis. The SU(3) cluster wave
function with $Q$ quanta associated with the relative motion wave function can be written as

$$\psi\left(\lambda, \mu, \nu \right) = N\left(\lambda, \mu, \nu \right) A \left[ \phi^{(\lambda, \mu, \nu)}_{2n} \phi^{(Q, 0)}_{c-2n} \phi\left(R_{cm}\right) \right] (\lambda, \mu) LSJ$$

Thus, the allowed SU(3) representations for the cluster state are those obtained from the product $(\lambda_c, \mu_c) \times (Q, 0)$, where $(\lambda_c, \mu_c)$ refers to the core wave function. The low-lying states of $^9$Li, obtained from the Cohen-Kurath interaction, are dominated by the single $s^4p^5$ SU(3) representation $(\lambda_c, \mu_c) = (1, 2) \equiv [f] = [432]$ in an SU(4) representation. Thus, to a good approximation, the cluster configurations in $^{11}$Li, corresponding to $n\hbar\omega$ of excitation, transform as $(\lambda, \mu) = (1, 2) \times (Q, 0)$, where $Q = n + 2$ is the number of excitation quanta between the clusters, and have good SU(4) symmetry $[432] \times [2] \rightarrow [4322]$.

If we start with a shell model basis coupled in SU(3) then the cluster state corresponds to that non-spurious linear combination of states which consumes all the di-neutron cluster spectroscopic strength; i.e.,

$$\psi\left(\lambda, \mu\right) = N \sum_i \theta_{2n_i} \phi^{(\lambda, \mu)}_{\text{shell}_i}, \quad N = \frac{1}{\sqrt{\sum_i \theta_{2n_i}^2}}$$

where $\phi^{(\lambda, \mu)}_{\text{shell}_i}$ are a set of non-spurious states with SU(4) symmetry $[f] = [4322]$ and two-neutron spectroscopic amplitude $\theta_{2n_i}$. A set of non-spurious states with good $[4322]$ symmetry was obtained by diagonalizing the operator

$$H_{cm} + \sum_{i<j} P_{ij},$$

in our SU(3) shell model basis, where $\sum_{i<j} P_{ij}$ is the space exchange operator.

We first consider the relation between the p-shell wave function, (which dominates the g.s. wave function of $^{11}$Li in complete $(0+2)\hbar\omega$ calculations), and the corresponding cluster state. In this case the cluster state carries $Q = 2$ quanta of relative excitation, which is the minimum allowed by the Pauli exclusion principle. There is only one $0\hbar\omega$ shell model state and it is one and the same as the $Q = 2$ cluster state. One could considers other $Q = 2$ configurations which involve coupling of the cluster to excited states of the core, but after antisymmetrization they do not differ from the state obtained by coupling a di-neutron to the ground state of $^9$Li.

The expansion of the $Q = 4$ cluster state in the $2\hbar\omega$ shell model basis was obtained using eqs (3), and the results are summarized in Tables 1 and 2. The SU(3) structure of the two model wave functions differ somewhat, reflecting the fact that the two-body interaction used in the shell model calculation tends to favour the higher symmetries. However, as is clear from the spectroscopic factors (Table 2) the interaction strongly favours two-neutron clustering in the g.s. of $^{11}$Li. The cluster parentage to the g.s. of $^9$Li dominates and the coupling to excited states of the core is weak. The relative orbital angular momentum between the core and the cluster is predicted to be dominantly $L_r = 0$. In addition, the
non-cluster \((\lambda, \mu)\) symmetries and the \(S=1\) di-neutron configurations are small. Since the dominant \(0\hbar\omega\) component of the shell model wave function is equivalent to the \(Q = 2\) cluster state, the full \((0 + 2)\hbar\omega\) wave function, in a harmonic basis, is well approximated by the \(2n \otimes ^9\text{Li} \) cluster state. In this sense the neutron halo of \(^{11}\text{Li}\), predicted when more realistic single particle wave functions are used, has a similar origin in the two models.

Table 1
Decomposition of SU(3) intensities in the model wave function of \(^{11}\text{Li}_{as}\).

<table>
<thead>
<tr>
<th>((\lambda, \mu))</th>
<th>((0, 0))</th>
<th>((2, 2))</th>
<th>((0, 3))</th>
<th>((3, 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>cluster ((L = 0, Q = 2))</td>
<td>100</td>
<td>40.0</td>
<td>11.4</td>
<td>0.5</td>
</tr>
<tr>
<td>cluster ((L = 0, Q = 4))</td>
<td>4.9</td>
<td>2.2</td>
<td>2.7</td>
<td>4.4</td>
</tr>
<tr>
<td>shell ((0 + 2\hbar\omega))</td>
<td>9.5</td>
<td>0.3</td>
<td>5.6</td>
<td>3.4</td>
</tr>
<tr>
<td>shell (pure (2\hbar\omega))</td>
<td>16.5</td>
<td>4.2</td>
<td>3.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\(^9\text{Li} \otimes ^{11}\text{Li}_{as}, Q=4, J=L\)

<table>
<thead>
<tr>
<th>(S=0, L=0)</th>
<th>(S=0, L=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{2}^+)</td>
<td>0.3194</td>
</tr>
<tr>
<td>(\frac{1}{2}^-)</td>
<td>-0.026</td>
</tr>
<tr>
<td>(\frac{3}{2}^+)</td>
<td>0.002</td>
</tr>
<tr>
<td>(\frac{3}{2}^-)</td>
<td>0.04</td>
</tr>
<tr>
<td>(\frac{5}{2}^-)</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The spectroscopic amplitude for coupling to the g.s. of \(^9\text{Li}\) is 98% of the total strength.

We next turn to a comparison between the cluster model and shell model wave functions for the soft E1 resonance. The shell model calculations predict that the first excited \(\frac{3}{2}^+\) state accounts for a large fraction of the low-lying E1 strength, and that this state has a dominant \(1\hbar\omega\) structure. The positive parity cluster states, corresponding to \(1\hbar\omega\) of excitation, transform as \((\lambda, \mu) = (1, 2) \times (3, 0) \rightarrow (3, 1), (2, 0), (1, 2), (0, 1)\) under SU(3). A comparison of the SU(3) structure predicted in the two model for this state is given in table 3.

<table>
<thead>
<tr>
<th>((\lambda, \mu))</th>
<th>((3, 1))</th>
<th>((2, 0))</th>
<th>((0, 1))</th>
<th>((1, 2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>cluster ((Q=3))</td>
<td>49.3</td>
<td>7.47</td>
<td>32.8</td>
<td>10.58</td>
</tr>
<tr>
<td>shell ((\hbar\omega=1))</td>
<td>77.3</td>
<td>1.3</td>
<td>3.3</td>
<td>18.8</td>
</tr>
</tbody>
</table>

The difference in percentage of \((\lambda, \mu) = (2, 0)\) and \((0, 1)\) in the two cases is most significant for the E1 strength, since in a harmonic oscillator \((0 \rightarrow 1)\hbar\omega\) model, these are the only components in the \(\frac{3}{2}^+\) wave function which can contribute to the dipole matrix elements.
to be very strongly enhanced, $B(E1) = 0.87$ W.u. for an oscillator parameter $b_{\text{shell}} = 1.75$ fm. The transition is dominated by the p→1s amplitude, and does not involve any strong cancellations. Similar results were obtained for the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states. Thus, a soft $E1$ resonance is predicted in the cluster model even when HO single particle wave functions are used. This is in strong contrast to the shell model predictions where the soft $E1$ resonance only arises when the loose binding of the last neutron is taken into account.

The large difference between the oscillator cluster model and shell model predictions for the $E1$ transition strengths mainly results from our restriction in the former case to the $^9\text{Li}_{g.s.}$ core. Table 4 lists the two-neutron spectroscopic amplitudes for the $\frac{3}{2}^+$ state in $^{11}\text{Li}$ to several states of $^9\text{Li}$. In contrast to the g.s., the two-neutron spectroscopic strength in this case is fragmented over many states of the $^9\text{Li}$ core. The cluster strength to the $^9\text{Li}_{g.s.}$ only consumes 50% of the total strength for coupling to $^9\text{Li} \ j = \frac{3}{2}^-$ states, and coupling to other $j$ states, particularly the $j = \frac{5}{2}^-$ state, is important. Furthermore, the non-cluster $S=1$ two-neutron spectroscopic factors are large.

<table>
<thead>
<tr>
<th>$^{11}\text{Li}(\frac{3}{2}^+)\ Q=3, J=1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$S=0, L=1$</td>
<td>$S=1, L=1$</td>
</tr>
<tr>
<td>$-0.4987$ (50%)</td>
<td>0.191</td>
</tr>
<tr>
<td>$-0.2279$ (12%)</td>
<td>0.1304</td>
</tr>
<tr>
<td>$-0.2389$ (13%)</td>
<td>$-0.3343$</td>
</tr>
<tr>
<td>0.7643 (98%)</td>
<td>0.056</td>
</tr>
<tr>
<td>$-0.1569$ (54%)</td>
<td>0.004</td>
</tr>
</tbody>
</table>

When the di-neutron cluster is allowed to couple to excited states of $^9\text{Li}$ the predicted $E1$ strength becomes very sensitive to the structure of the core. This can be demonstrated rather schematically by the following two state mixing model in which the di-neutron is allowed to couple to the $^9\text{Li}_{g.s.}$ and $^9\text{Li}(\frac{5}{2}^-)$ state. We write

$$| \frac{3}{2}^+ \rangle_{\text{cluster}} = \cos \theta | 2n \otimes ^9 \text{Li}_{g.s.} \rangle + \sin \theta | 2n \otimes ^9 \text{Li}(\frac{5}{2}^-) \rangle, \quad (5)$$

and treat $\theta$ as a parameter. It must be noted that after antisymmetrization cluster states based on different states of $^9\text{Li}$ are no longer orthogonal. The non-orthogonality requires that the normalization of the cluster state in (5) be a function of the mixing angle $\theta$. In calculating the $E1$ matrix elements we assume that $^{11}\text{Li}_{g.s.}$ is described by the $Q = 2L_r = 0$ cluster state. Fig. 6 shows the $E1$ strength as a function of $\theta$, and it is clear that allowing coupling to the $\frac{5}{2}^-$ state in $^9\text{Li}$ can have a dramatic effect on the predicted $B(E1)$ value. The destructive interference seen for large mixing of the two $^9\text{Li}$ states corresponds to the
introduction of cancellations between the $p\rightarrow 1s$ and $p\rightarrow d$ amplitudes.

Fig. 6 The $B(E1)$ value predicted for the soft $E1$ resonance as a function of the mixing between the coupling of the cluster state to the ground state versus $\frac{5}{2}^-$ state of the $^9Li$ core (see eq. (5)).

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Summary

The physics behind the neutron halo and soft $E1$ resonance of $^{11}Li$ can be displayed clearly using the Hecht technique for expressing SU(3) cluster model states in terms of non-spurious shell model states. Such an analysis shows that the effective two-body interaction favours di-neutron clustering in the shell model wave function for the g.s. of $^{11}Li$. The large radius predicted in the two models when the loose binding energy of the last neutrons is taken into account has similar origins. The size of the halo radius is most accurately determined by $^{11}B(\pi^-,\pi^+)^{11}Li$ reaction measurement. In the case of the soft $E1$ resonance the predictions of the two models differ. No soft $E1$ resonance is predicted in the shell model unless the realistic single particle wave functions are used. In contrast, the cluster model predicts a resonance even in a HO basis. A comparison between the two models shows the need to include coupling of the di-neutron to excited states of the $^9Li$ core to obtain a realistic description of the soft $E1$ mode of $^{11}Li$.

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References


