MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Report
of the
William H. Bates Linear Accelerator Laboratory
and the
Laboratory for Nuclear Science

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

PROCEEDINGS OF THE JUNE WORKSHOP IN INTERMEDIATE ENERGY
ELECTROMAGNETIC INTERACTIONS WITH NUCLEI

HELD AT MIT JUNE 13-24, 1977

by

A. M. BERNSTEIN, EDITOR

Department of Physics
Laboratory for Nuclear Science
and
Bates Linear Accelerator Laboratory

MASTER

Report No. COO-3069-677

Reproduction in whole or in part is permitted for any purpose by the U.S. Government. The data and the results that are presented in this report should not otherwise be published without prior consultation with the Laboratory for Nuclear Science.

Researches herein reported have been supported by the Department of Energy through Contract EY-76-C-02-3069.

This technical report is number 102 in a series of technical reports issued by the Laboratory for Nuclear Science since its inception.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

2 2 done
2 2 in all

TABLE OF CONTENTS

	Page
Preface - A. M. Bernstein	3
Introduction - A. M. Bernstein and E. J. Moniz	5
List of Participants	10
Workshop Reports	
I. Electromagnetic Coincidence Experiments - J. S. O'Connell and B. Schoch	11
II. Meson Current and Few Nucleon Systems - E. Hadjemichael, E. L. Lommon and W. Turchinets	46
III. Pion Production By Photons and Electrons- J. H. Koch and A. M. Bernstein, Co-chairmen	
1. Photoproduction of Pions at Low Energies - J. H. Koch	128
2. Single Nucleon Operators in Pion Photo- production From Nuclei - E. Dressler	147
3. Pion Production with Real and Virtual Photons on Nucleons in the Resonance Region - K. H. Althoff	155
4. Single Arm ($\gamma, \pi^{\pm}$) Reactions to Discrete Final States - A. M. Bernstein	179
5. High Energy Photonuclear Reactions and Iso-bar - Nuclear Dynamics - E. J. Moniz	212
6. Nuclear Physics Experiment at DESY- E. Rössle	229
7. Coherent Photo - and Electroproduction of Δ 's in Nuclei - M. Dillig and M. G. Huber	239
8. Charged Pion Electroproduction Near Threshold - T. W. Donnelly	250

	Page
IV. Knock-Out Reactions - J. L. Matthews and T. deForest, Co-chairpersons	
1. Knock-Out Reactions - T. deForest	259
2. Experimental Problems in Nuclear Knockout Reactions: Techniques For Producing Effective Monochromatic Photons Using Bremsstrahlung - J. L. Matthews	269
3. Experimental Problems in Nuclear Knockout Reactions: Triple Coincidence Experiments and a Very Large Solid Angle Spectrometer - C. P. Sargent	297
V. Electron Scattering as a Probe of Nuclear Structure J. Heisenberg and J. W. Negele, Cochairpersons	
1. Summary J. Heisenberg and J. W. Negele	305
2. Nuclear Electron Scattering - J. W. Negele	317
3. Reanalysis of Electron Scattering and Muonic X-Ray Data for ^{208}Pb - J. L. Friar, J. Heisenberg and J. W. Negele	325
4. High-Multipolarity Elastic Magnetic Electron Scattering - H. deVries, P. deWitt-Huberts, A.E.L. Dieperink and T. W. Donnelly	341
5. The Transition Density of the Octupole Vibration in ^{208}Pb Comparison of Theory and Experiment - B. Frois and J. Heisenberg	355
6. (a) Electroexcitation of High Spin States (b) Electroexcitation of Collective Magnetic Dipole Resonances W. Knupfer and M. G. Huber	363 368
7. On Electromagnetic Form Factors of Odd-Even Deformed Nuclei - E. Moya de Guerra and A. E. L. Dieperink	372
8. What Can We Learn With Polarized Targets in (e,e') ? - G. Rawitscher	393

PREFACE

A. M. BERNSTEIN

The field of experimental intermediate energy electromagnetic interactions with nuclei has been quite active and productive during the past decade. This is due in part to the emergence of a new generation of electron linear accelerators with energies above the pion production threshold, high intensities, excellent beam properties, and duty cycles greater than 1%. In addition the emergence of new detection techniques such as large scale wire chambers and associated electronic and computer developments has enabled a wider class of experiments to be performed. Because of these developments, and spurred on by the construction of a new experimental hall at the Bates-M.I.T. Linac, we felt that the time was right to convene a study group on Intermediate Energy Electromagnetic Interactions with Nuclei.

The goals of the workshop were to formulate the open questions in the field and to show how they can be clarified by electromagnetic experiments. To accomplish this goal we tried to bring together many of the prime movers in the field as well as interested experimentalists and theorists to study where the field is going and should go in the next decade. These considerations were not limited to the current state of the art but of course they in fact shaped the deliberations.

The ground for the workshop was broken by several previous meetings. In 1967 a Summer Study¹ was held at M.I.T. in this area of physics because at that time the new linacs at Saclay and M.I.T. were being constructed. Subsequently a workshop was held at Saclay.² More recently a Winter Workshop was held in Andover, Mass.³ to prepare the Bates Community for the experimental requirements of the proposed new experimental hall. At that latter meeting we decided that it would be more useful to study the physics in more depth, and the June Workshop grew out of that. The Workshop was organized about five working groups (see table of contents) each of which had several co-chairpersons whose responsi-

bility it was to convene the groups and to prepare the final reports.

In general, the Workshops were organized around physical topics, although the unity of physics always prevents a clean separation between subjects. Except for the Workshop on coincidence experiments we did not attempt to study the physics that will be performed with, say, 1 GeV accelerators, 100% duty cycle accelerators, or both combined. Neither time nor the small number of participants allowed a full coverage of all topics. We note that there are discussions of these topics, particularly in the Workshop groups on the few body problem, coincidence experiments, and pion production. However, a full study of what could be the physics of 1-2 GeV, 100% duty cycle accelerators, was not attempted and could be the subject of a future Workshop.

The June Workshop took place on the M.I.T. Campus from June 13 through 24, 1977. A Workshop preparation period was held from June 1 through 10 to set the stage for the working groups. The group was fairly small which enabled the participants to interact quite informally.

I would like to take this opportunity to thank the participants for their enthusiastic participation and diligent work, and to apologize for the delay in publication of this report. The Workshop was made possible by the enthusiastic cooperation of the personnel of the Laboratory of Nuclear Science and Physics Department of M.I.T. and by financial support from M.I.T.

1. Medium Energy Nuclear Physics with Electron Linear Accelerators, M.I.T. 1967 Summer Study, W. Bertozzi and S. Kowalski editors TID-24667.
2. Effets Mesoniques Dans Les Noyaux. Diffusion D'Electrons a Energie Intermediaire, Recontres de Saclay 12-16 Mai, 8-12 Septembre 1975, Saclay Report.
3. Report on the Bates Mid-Winter Workshop, Jan. 3-7, 1977. E.C. Booth, editor. Copies available from Bates Linear Accelerator, P.O. Box 95, Middleton, Mass. 01949.

INTRODUCTION

A. M. Bernstein and E. J. Moniz

M.I.T.

The central problem of nuclear physics is to understand the complicated structure of atomic nuclei and their interactions with external leptonic and hadronic probes. The starting point for "classical nuclear physics" is the view that the static properties and low frequency motion of nuclei can be well represented as a collection of non-relativistic nucleons (whose properties are assumed to be unaltered by the nuclear medium) interacting via static potentials. This approach has had considerable success in understanding a large body of experimental information, such as nuclear spectra, electromagnetic moments and transition rates, β -decay matrix elements, nuclear shapes, inelastic scattering, transfer reactions, electromagnetic form factors, etc. The unifying theoretical concept has been the nuclear mean field, in which nucleons move in well-defined "shell model" orbitals with comparatively weak residual interactions. The coupling between single particle and collective motion lies in the self consistent calculation of the shape and motion of this average field. In this language, "classical" nuclear structure physics aims at a microscopic description of the long range correlations (eg. shapes, configurations, vibrations) contained in the average field.

The electromagnetic probe has unquestionably made a large contribution to this understanding, the classic examples being the early measurements of the systematics of nuclear ground state charge distributions and of the giant dipole photoabsorption cross section. The new electron linacs, providing higher currents and duty cycles, larger momentum transfers, and significantly better energy resolution, have reached an unparalleled level of "microscopy" for testing and refining the theoretical picture. Examples of this are already abundant. Precision measurements of the high momentum transfer charge form factor of ^{208}Pb have presented a serious challenge to the most sophisticated Hartree-Fock calcula-

tions, and magnetic electron scattering from ^{17}O indicates a magnetization distribution considerably different from that expected in a simple shell model. In inelastic electron scattering, transition form factor measurements within a rotational band indicate a possible breakdown of the rigid rotator model, while precise extraction of the (octupole) vibrational transition charge in ^{208}Pb has demanded theoretical descriptions far more sophisticated than those incorporating only nuclear surface oscillations. These few specific examples indicate the richness of the field; much more extensive discussion of future directions forms the content of the workshop on electron scattering.

Another challenge in intermediate energy physics is that of going beyond the fundamental approximations made in "classical nuclear physics". First, the nucleon is itself polarizable (i.e. it has excited states) and this, coupled with the emission and absorption of pions, is understood to play a crucial role in the nuclear force. The resulting modifications of the nuclear electromagnetic current and charge densities can be large and fundamentally limit the validity of the conventional multi-nucleon description of nuclei. Further, the nuclear response to absorbing energies on the order of one to two pion masses fundamentally involves physics beyond that appropriate to a mean field description. For example, absorption of a pion involves at least two nucleons in close proximity and so is sensitive to short-range nuclear correlations. If the energy is to be absorbed by one nucleon, as might be required by appropriate restriction of the reaction kinematics, then momenta well beyond the Fermi level are probed. In either case, the absorption process probes short-range nuclear dynamics and is intrinsically connected to the microscopic origin of the nuclear force.

This interconnection between the nuclear force, pion reactions, and the nuclear electromagnetic current is illustrated in Fig. 1 for the one and two nucleon systems. The well-understood pion-nucleon coupling (proportional to $\vec{\sigma} \cdot \vec{k}$, where $\vec{k}$ is the pion momentum and $\vec{\sigma}$ the spin) explains both the long range nuclear force (OPEP,

depicted in Fig. 1a) and the occurrence of the $J=3/2$, $T=3/2$ πN resonance (the driving term is depicted in Fig. 1c). This resonance is conveniently viewed as the Δ -isobar (or first nucleon "excited state") and, as an intermediate state, is in turn responsible for a considerable part of the intermediate range attraction of the nuclear force (see Fig. 1b). The Δ also provides the dominant mechanism for absorption of pions by the deuteron (Fig. 1d), and it is clear that this process is closely related to the attractive NN interaction indicated in Fig. 1b. Finally, the exchange currents indicated in Figs. 1e and 1f are known to alter drastically the isovector magnetization distribution in the two-nucleon system, as seen in the near-threshold deuteron electrodisintegration. Clearly, these last diagrams resemble closely those shown as responsible for the nuclear force. All these processes are intimately related and a challenge in intermediate energy physics is to extend our understanding of these interrelationships to more complicated nuclear systems.

Electromagnetic interactions with appropriate energy transfers to the nucleus (i.e., ~140 to 300 MeV) must play an important role in complementing experimental information obtained at the meson factories. In particular, the same "entrance channel" dominates both pion and photon reactions in the energy region appropriate to the Δ -excitation. However, since the pion and photon have very different mean free paths in the nuclear medium, the different spatial dependence of the interactions can aid in unraveling the complicated reaction mechanisms. In some cases intermediate energy photo-induced reactions can supply unique information on the dynamics of pion and isobar propagation in nuclei. eg. in the production of near threshold pions or in coherent π^0 production at the resonance energy. There are clearly many opportunities for electron and photon induced reactions to contribute to a deeper understanding of nuclear dynamics (exchange currents, large momentum components, pion and isobar propagation, nuclear response to large energy transfer), and these will be discussed in more detail in many of the

workshop contributions.

It is perhaps worth mentioning that the evolution in experimental apparatus has made these studies possible and ultimately provides the rationale for holding the workshop at this time. This has been touched on in the preface and is discussed in some detail in several of the workshop reports.

The range of studies included in this workshop is too large and diversified to summarize in this introduction. We would like to stress the rapid evolution of ideas and experimental techniques in this field which means that no workshop could cover the entire field.

FIGURE 1

Pion-nucleon interaction in the one and two nucleon systems; (a) the NN one-pion-exchange potential; (b) one example of the Δ contribution to the NN interaction; (c) π N scattering: the Chen-Law driving term which, when iterated to all orders, generates the Δ ; (d) spin absorption inducted by the Δ ; (e) meson exchange currents in photonuclear interactions; (f) Δ contribution to nuclear electromagnetic exchange currents.

PARTICIPANTS

K. Althoff	Bonn
W. C. Barber	MIT
A. M. Bernstein	MIT
W. Bertozzi	MIT
G. Bertsch	Michigan State University
T. deForest	IKO
K. deJager	IKO
H. deVries	University of Massachusetts
P. deWitt-Huberts	IKO
A. E. L. Dieperink	IKO
T. W. Donnelly	Stanford University
E. Dressler	NBS
J. Friar	Los Alamos
B. Frois	Saclay
E. Moya deGuerra	MIT
E. Hadjimichael	Fairfield University
J. Heisenberg	MIT
M Huber	University of Erlangen
J. H. Koch	MIT
S. Kowalski	MIT
E. Lomon	MIT
J. Londergin	Indiana University
G. Luhrs	Mainz
J. Martin	Saclay
X. Maruyama	NBS
J. Matthews	MIT
E. J. Moniz	MIT
J. Negele	MIT
J. O'Connell	NBS
G. Rawitscher	University of Connecticut
D. Riska	Michigan State University
E. Rössle	University of Freiburg
C. P. Sargent	MIT
B. Schoch	MIT
G. Tamas	Saclay
W. Turchinetz	MIT
C. Williamson	MIT

I. ELECTROMAGNETIC COINCIDENCE EXPERIMENTS

J. S. O'Connell, National Bureau of Standards

B. Schoch, MIT and University of Mainz

Contribution to the June 1977 MIT Workshop on Intermediate
Energy Electromagnetic Interactions with Nuclei.

I. Introduction

Within the next few years electron beams with moderate (10%) and high (100%) duty factors will be available in the energy range 100-600 MeV. Such beam allows coincidence experiments not possible with existing linac duty factors (0.1 to 2%).

We will discuss three classes of coincidence measurements:

- (1) Electron scattering ($e, e'x$)
- (2) Tagged bremsstrahlung ($e, e'\gamma$) (γ, x) where e' and x are detected (henceforth denoted by (γ_t, x))
- (3) Correlated products (γ, xy) where the γ 's are from bremsstrahlung.

In the next section we discuss the nuclear physics motivation for doing coincidence measurements. Some common features of the reactions are mentioned as are factors affecting count rate limitations. The main part of this essay is the presentation and discussion of a variety of examples. We have either estimated counting rates for typical experimental arrangements or taken examples from the recent literature. Our purpose is a rough classification of types of measurements into feasibility classes. Three appendices show the expressions used for count rate estimation purposes.

II. General Features

The formalism of $A_T (e, e'x)$ A_R reactions in the one photon exchange approximation has been known for a long time.⁽¹⁾ Four form factors (as opposed to two for (e, e')) are required. In general, transverse and

longitudinal as well as electric and magnetic interference terms are possible. However, for electric transitions the Coulomb part of the operator dominates except at far forward and backwards electron scattering angles. Magnetic transitions are of course transverse at all angles and are dominated by the spin operator. In general multipole transitions (at the same energy) interfere.

Coincidence experiments are useful for determining the spin and parity of transitions and for shedding light on reaction mechanisms. Multipolarities are determined in $(e,e'x)$ by the angular distribution of the product about the direction of electron momentum transfer.⁽²⁾ Reaction mechanisms are delineated by the dependence of the coincident cross section on the excitation energy and momentum transfer or energy and angular correlations of the products in the case of (γ,xy) . The linear polarization of off-axis tagged bremsstrahlung offers an experimental method of distinguishing M1 and E1 transitions.

We will discuss experiments in three energy regions: giant resonance (20 MeV), knockout (100 MeV), and the photopion resonance (300 MeV) and give counting rate estimates for the reactions shown in Table I.

The location and characterization of the EL and ML multipole giant resonances has been difficult in (e,e') studies because of the approximate energy degeneracy and the similarity in momentum dependence of E0 and E2 form factors. A reaction like $(e,e'p)$ to residual final states of known spin and parity (and particle-hole character) would help in multipole assignments of (e,e') spectrum features. For example, the angular distribution of a proton (relative to the direction of the virtual photon direction) from a monopole transition would be isotropic while that of an electric quadrupole at the same excitation energy would not. Little has been done in the way of specific calculation of $(e,e'N)$ in the giant resonance region using particle-hole wave functions.^(3,4) This general area is pregnant with promise.

TABLE I

Reactions For Which Counting Rates Have Been Estimated

ω (MeV) Type	20 G.R.	100 Q.E.	300 π
(e,e'x)	$^{40}\text{Ca}(e,e'p)$	$^{12}\text{C}(e,e'p_n)$	$^{12}\text{C}(e,e'\pi)$
(γ_t ,x)	$^{40}\text{Ca}(\gamma_t,p)$		$^{12}\text{C}(\gamma_t,\pi)$
(γ ,xy)		$^{16}\text{O}(\gamma,np)$	$^{12}\text{C}(\gamma,p\pi)$

Quasielastic scattering ($e, e'p$) around $k_1 - k_2 = (k_1 - k_2)^2 / 2M$, on the other hand, has been well exploited experimentally and theoretically. Our knowledge of single particle binding energies and momentum distribution are derived in large part from this reaction. The high degree of energy and angular correlation between the ejected nucleon and the virtual photon permits this reaction to be observed with an electron beam of a few percent duty cycle. It would be interesting to repeat this measurement by detecting neutrons in coincidence with the scattered electron to compare the corresponding neutron distribution properties.

The quasi deuteron effect is thought to be the dominant absorption mechanism for real photons in the energy region between the multipole resonances and pion production threshold. However the systematics of (γ, np) as a function of atomic weight and relative np energy and angle have never been mapped. This measurement could be done with good resolution (< 1 MeV) with tagged bremsstrahlung and 100% d.f. or with coarse resolution (~ 10 MeV) with untagged bremsstrahlung with a few percent duty factor beam.

The main mechanisms of pion production by real and virtual pions in nuclei are just beginning to be understood. Coincidence measurements will be essential in verifying these ideas. The total absorption cross section in the delta region appears to be quasielastic, i.e., both the pion and nucleon are ejected from the residual nucleus. The free $\gamma N \rightarrow N\pi$ kinematics is modified in a nucleus by initial nucleon motion and final state interactions of both the nucleon and pion. However, it is the dynamic modification of the $\gamma N\pi$ vertex by the surrounding nuclear matter, as reflected in the four-fold differential cross section, that is the most interesting subject of speculation and study.

The pion production amplitude to specific final states in the residual nucleus is further modulated by the nuclear transition form factor. The resulting dependence on the energy of the initiating photon (real or virtual) often completely masks the characteristic delta resonance. The isoelastic transition ${}^3\text{He}(\gamma, \pi^+) {}^3\text{H}$ is a good illustration. Here the total cross section reaches its peak by $k_\gamma = 190$ MeV. The pion angular distribution takes on a diffractive shape since the momentum transfer to the nucleus $\vec{k}_\gamma - \vec{q}_\pi$ is a strong function of the emitted pion's direction.

Certain common elements emerge in the estimation of counting rates of the three types of reactions in the three energy regions mentioned above. These might be called rules-of-thumb for coincidence measurements. They are meant to be helpful guides to experimental feasibility, not theorems.

1. The true coincidence counting rate is usually limited by the accidental rate,

$$N_{\text{acc}} = \left(\frac{\tau}{\text{d.f.}}\right) N_1 N_2.$$

2. An $(e, e'x)$ measurement will have a high singles rate in the product (x) detector because most x 's are produced by electrons which scatter in the forward direction.
3. A tagged bremsstrahlung (γ_t, x) measurement will have a high singles rate in the e' counter. Items 2 or 3 then lead to item 1 and dead time corrections.
4. If the reaction of interest cannot be seen in a single arm experiment, it is unlikely a coincidence measurement will improve the signal to noise ratio dramatically. Accidental coincidences provide an unavoidable background.

III. Examples

The following examples are first order estimates of the singles and coincidence counting rates of selected nuclear reactions initiated by real or virtual photons. We have tried to be conservative in the choice of target thickness, solid angles, energy resolution, and coincidence resolving time to partially compensate for factors left out of the estimates, detection efficiencies, room background, etc. We have also ignored target heating problems with the 100 μ A beam. The kinematics were chosen somewhat arbitrarily, but at an interesting or typical set of conditions. Many possible improvements will occur to the practitioner of the Experimentalist's Art.

The single arm cross sections $\frac{d^2\sigma}{d\Omega_e dE_2}(e, e')$ and $\frac{d^2\sigma(\gamma, x)}{d\Omega_x dE_x}$ were either taken from the literature or estimated using virtual photon theory or sum rules. Simplifying assumptions were made about angular correlations. Counting rate expressions and bremsstrahlung formulae (real and virtual) are given in the appendix.

Although the tradeoff between acceptable singles rates and true-to-accidental coincidence ratio was not optimized, we did try to keep the $N_{\text{coin}}/N_{\text{acc}}$ ratio near unity. The counting rates may be scaled for other conditions of beam intensity or duty factor if appropriate allowance is made for the nonlinear dependence of the accidental rate.

$^{40}\text{Ca}(e,e'p)^{39}\text{K}$ in Giant Resonance

Current 50 μA

$\tau = 10 \text{ ns}$

Duty cycle 100%

$\theta_e = 45^\circ$

Target 20 mg/cm^2

$\theta_p = 90^\circ$

$\Delta \Omega_e = 5 \text{ msr}, \quad \Delta E_c = 1.4\%$

$\Delta \Omega_p = 40 \text{ msr}, \quad \Delta E_p = 100\%$

Estimated Rates

Electron 2000 s^{-1}

Proton $1.5 \times 10^5 \text{ s}^{-1}$

True coin. 2.4 s^{-1}

True/Acc 1

$^{40}\text{Ca}(\gamma_e, p)^{39}\text{K}$ in Giant Resonance (Tagged Bremsstrahlung)

Current 23. nA

$\tau = 10$ ns

d.f. 100%

Radiator = .001 r.l.

Ca target 20 mg/cm²

$\Delta \Omega_e = 100\%$

$\Delta E_e = 100$ keV

$\Delta \Omega_p = 40$ msr

$\Delta E_p = 100\%$

Estimated Rates (per electron channel)

Electron 10^6 s⁻¹

Proton 3 s⁻¹

True coin. $.06$ s⁻¹

True/Acc 2

Comments on $^{40}\text{Ca}(e,e'p)^{39}\text{K}$

This measurement looks quite feasible for a 100% d.f. electron beam. Note the high proton counting rate as explained in Section II. The momentum transfer $q = .74 \text{ fm}^{-1}$ is in a region where E1, E2, and E3 contribute more or less equally. This type of experiment will be feasible for $Z < 20$ but begins to become difficult at higher atomic numbers because of the radiation tail growing as Z^2 and the suppression of the proton yield by the Coulomb barrier. Also the density of levels in the residual nucleus becomes high for heavier nuclei.

The coincidence counting rate (for fixed true to accidental ratio) drops rapidly for smaller incident electron energies because of the elastic radiation tail. For $E_1 = 60 \text{ MeV}$ the giant resonance is only a few percent of the total electron counting rate and the coincidence rate drops by two orders of magnitude. This point was made by W. C. Barber⁵ in his 1962 review article on inelastic electron scattering.

Comments on $^{40}\text{Ca}(\gamma_t, p)^{39}\text{K}$

Tagged bremsstrahlung is also a feasible way to examine the decay modes of the giant multipole resonances, however the higher multipoles are not as readily excited by real photons as in the previous example. The high singles rate is now in the electron arm. Note the incident electron current is three orders of magnitude below that required by $(e,e'p)$. The low current suggests that this kind of apparatus be used in a parasitic mode.

A number of electron channels can be used at the same time to increase the true data rate without changing the true to accidental ratio.

$^{16}\text{O}(\gamma, np) ^{14}\text{N}$ (Bremsstrahlung)

E_γ (End point energy) 300 MeV

$E_p, E_n > 80 \text{ MeV}$

Current: 13 μA on 0.01 r.l.

Duty cycle: 100%

Target thickness: 2g/cm²

Solid angle p: 10 msr

Solid angle n: 10 msr

Acceptance ΔE_p : 20%

Acceptance ΔE_n : 100%

Coincidence resol.: 5 nsec

Detection eff. n: 0.2

With 5 cm lead in front: 0.1

Estimated Rates:

p-spectrometer: $4.8 \times 10^3/\text{sec}$

n-spectrometer: $8 \times 10^3/\text{sec}$

true coincidences: 7.7/sec (Fermi cone)

0.46/sec (4π)

true/accidental: 40 (fermi cone)

2.4 (4π)

Threshold n-counter 30 MeV

10 m flight path, 1m x 1m x 0.2 m array

$H(e, e' \pi^+)n$ (Bardin et al.)¹⁴

Current: $0.26 \mu\text{A}$

$\tau = 4 \text{ ns}$

Duty cycle: 0.8%

Target: $3 \text{ cm}^3 \text{ Liquid Hydrogen}$

$\Delta \theta_e \pm 23 \text{ mr}$

$\Delta E_e = 3.5\%$

$\Delta \theta_\pi \pm 30 \text{ mr}$

$\Delta E_\pi = 5\%$

Measured Rates:

Electron $1.5 \times 10^4 \text{ s}^{-1}$

Pion $5 \times 10^4 \text{ s}^{-1}$

True coin. 40 h^{-1}

True/Acc. 1

$^{12}\text{C}(e, e'p)$. Saclay (Mougey et al.)¹²

Current: 5-10 μA

Duty-cycle: 1%

Target thickness: 200 mg/cm^2

Solid angle e^- : 2.24 msr

Solid angle p : 3.36 msr

Acceptance ΔE_e : 6%

Acceptance ΔE_p : 20%

Resolution time: 4 nsec

Rates:

e^- -spectrometer: 278/sec

p -spectrometer: 833/sec

true-coincidence: 0.14/sec

true/accidental: 1.5

Estimate:

85/sec

474/sec

0.23 $\rightarrow$ 0.014

Fermi $\rightarrow 4\pi$

14 $\rightarrow$ 0.9

Comments on the (γ, np) Reaction With a Bremsstrahlung Beam

As various experiments (ref. 11) indicate, the quasideuteron photo-disintegration mechanism is the dominant one between giant dipole resonance and pion threshold. Even in the $(3,3)$ nuclear resonance region the quasideuteron model was successfully applied. An investigation of this reaction for different nuclei in combination with (γ, pp) and (γ, nn) may give valuable information about the reaction mechanism itself and the final state interaction of the outgoing hadrons with the residual nucleus. The experiment is feasible for moderate duty cycle machines.

Remarks on $(e, e'n)$ (with the same kinematics as $(e, e'p)$)

1. To get the same energy resolution:

$$\text{Flight path} \approx 15 \text{ m}$$

$$\Delta\Omega \approx 4.4 \text{ msr with a } 1 \text{ m} \times 1 \text{ m array}$$

2. The true coincidence signal is an order of magnitude smaller than the $(e, e'p)$ case.
3. The efficiency of the n-counter is ≈ 0.2 and gets smaller because of the lead filters needed to suppress the gamma background.
4. For a neutron detection threshold $> 30 \text{ MeV}$ the number of neutrons hitting the detector is a factor of ≈ 10 higher than the number of protons in the $(e, e'p)$ case.

Conclusion: The true rate is a factor ≈ 100 smaller than the proton case given the same background as in $(e, e'p)$.

Comments on $H(e, e' \pi^+) n$

This measurement¹⁴ was performed 100 MeV above threshold to measure the pion's form factor in the momentum transfer range $1-3 \text{ fm}^{-2}$. The result after six weeks of data taking was $r_\pi = 0.74 (\pm 16\%) \text{ fm}$. The kinematics were chosen to separate the longitudinal and transverse contributions to electropion production when the pion is detected in the direction of the virtual photon. This arrangement emphasizes the pion current diagram.

A similar measurement on a complex nucleus will be difficult (even for a 100% d.f.) for the following reasons:

1. The elastic radiation tail grows relative to the quasielastic electroproduction cross section as Z .
2. In certain kinematic regions the quasielastic peak and its radiation tail may contribute to the e' singles rate.
3. The energy correlation between π^+ and e^- were used to reject accidentals. This correlation is weaker in complex nuclei.
4. Near threshold the electroproduction cross section is further reduced by a nuclear form factor.
5. Measurements for a range of energy and momentum transfer is necessary to make an informative nuclear experiment. Six weeks were required to get one energy point in the Saclay-Clermont measurement at 1-2% d.f.

$$\frac{^{12}\text{C}(e, e' \pi^-)^{12}\text{N}}{\text{g.s.}}$$

Assumed cross-section: $10^{-36} \text{ cm}^2/\text{sr-MeV}^2$ (Ref. 8)

Electron energy : 350 MeV

Energy loss : 230 MeV

Current : 25 μA

Duty cycle : 100%

Target thickness: 200 mg/cm^2

Solid angle e^- : 10 msr

Solid angle π^- : 30 msr

Acceptance ΔE_e : 6%

Acceptance ΔE_π : >6%

Coincidence res.: 5 nsec

Estimated rate:

e - spectrometer : 450/sec

π^- - spectrometer : 1000/sec

true coincidences: $2.5 \times 10^{-3}/\text{sec}$

true/accidental : 1.

Background: e^- : Radiation tail
 π^-, π^+, π^0 product.

π^- : π^- - from forward
 scatt. e^-

tagged beam

$$\begin{aligned} E_\gamma &= 175 \text{ MeV} \\ E_\pi &= 20 \text{ MeV} \\ \theta_\pi &= 30^\circ \end{aligned}$$

Assumed cross section: $\frac{d\sigma}{d\Omega} = 10^{-30} \text{ cm}^2/\text{sr}$

e^- - energy : 230 MeV

current : 32 nA on 10^{-3} r.l.

duty cycle : 100%

target thickness: 200 mg/cm^2

solid angle π : 30 msr

acceptance : $\Delta E_\pi = 4 \text{ MeV} = \Delta E_e$

coincidence res.: 5 nsec

Estimated rates: (per electron channel)

e^- - counter: $10^7/\text{sec}$

π^- - counter: $1.32/\text{sec}$

true coinc.: $6 \times 10^{-3}/\text{sec}$

true/acc. : 0.1

Background: $(\gamma, \pi^- \text{P})$ $175 \rightarrow 230 \text{ MeV}$

Comments: A $(e, e\pi^-)$ to definite state in the residual nucleus.

The advantage of the $(e, e'\pi)$ reactions in comparison with (γ_t, π) reactions is the possibility of exciting longitudinal transitions and varying the momentum transfer independent of the energy transfer. To take advantage of this possibility, one has to measure many points. The estimated rate indicates that it is very difficult to get sufficient data in a reasonable runtime, even with a duty cycle 1 machine.

Comments: $^{12}\text{C}(\gamma_t, \pi^-)^{12}\text{N}_{\text{g.s.}}$

This kind of reaction (to the ground or first few excited state in the residual nucleus) is under investigation at a number of laboratories using the bremsstrahlung isochromat technique. For these cases, where the cross-section is of the order $1 \mu\text{b}/\text{sr}$, the tagged beam method doesn't seem competitive. For higher excited states in the residual nucleus e.g. excitation of spin-isospin resonances, where one expects cross-sections of the order $10 \mu\text{b}/\text{sr}$, the use of a tagged photon beam looks feasible and is superior to the isochromat method which is difficult to apply for broad higher lying states. The true/acc. can be improved considerably by running with a smaller incoming e^- energy. (For an energy of 195 MeV and detecting a 20 MeV electron, the ratio approaches 1).

$^{12}\text{C}(\gamma, \pi^- p) ^{11}\text{C}$ (Bremsstrahlung)

Current: 10 μA

Duty cycle: 100%

Target thickness: 200 mg/cm^2

Solid angle π : 30 msr

Solid angle P: 20 msr

Acceptance ΔE_π : 20%

Acceptance ΔE_p : 30%

Coincidence res.: 5 nsec

Estimated rates:

π^- - singles: $1.2 \cdot 10^2$

P - singles: $2.7 \cdot 10^3$

true : 1/sec (Fermi cone)

0.1/sec (4π)

true/accidental: $6 \cdot 10^2$ (Fermi cone)

$6 \cdot 10^1$ (4π)

Background:

P - spectrometer:

π^0 - production

P - quasideuteron prod.

Comments: $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}^*$

The measurement of this reaction for different nuclei looks very promising, even with the present moderate ($\sim 2\%$) duty cycle machines. To measure in the (3,3) nucleon resonance region moderate energy resolution is needed ($\sim 5\text{-}10\text{ MeV}$), which can be obtained using a bremsstrahlung beam with different end-point energies. Looking for the invariant mass of the (π^-, p) system as a function of the photon energy should give information on Δ -production and propagation in nuclei. If the quasifree production picture is valid, one may extract information on the momentum distribution of neutrons in nuclei, which is difficult to obtain by other reactions. The triple coincidence measurement with tagged photons $(A\gamma_t, \pi^- p)(A-1)$ does not seem feasible because of low counting rates.

Higher Order Coincidences

Phil Sargent made an interesting comment about triplet coincidence experiments, e.g. $(e, e' p \pi^-)$. Here one is often cross section limited (rather than limited by accidental coincidences) because the cross section is a six-fold differential and N_{coin} has three solid angles and three finite energy bites. In this situation the beam duty factor is no longer important. The d.f. need only be large enough to keep one out of the accidental limited region. Thus some triplet coincidence measurements are possible at low true coincidence counting rates on present 2% d.f. linacs.

$^{12}\text{C}(e, e'\pi^-)$ inclusive

Assumed cross-section: $1.3 \times 10^{-33} \text{ cm}^2/\text{sr}^2\text{-MeV}$

Electron energy: 500 MeV

Solid angle e^- : 10 msr

Energy loss: 300 MeV

Solid angle π^- : 30 msr

Current: 10 μA

Acceptance ΔE_e : 10%

Duty cycle: 100%

Acceptance ΔE_π : 10%

Target thickness: 200 mg/cm^2

Coincidence res.: 5 nsec

Estimated rate:

e - spectrometer: 750/sec

π^- - spectrometer: 1100/sec

true coincidences: 0.05/sec

true/accidental: 12.5

$^{12}\text{C}(\gamma, \pi^-)$ inclusive

tagged beam

$$E_{\gamma} = 300 \text{ MeV}$$

$$E_{\pi} = 130 \text{ MeV}$$

$$\theta_{\pi} = 30^{\circ}$$

Assumed cross section: $\frac{d\sigma}{d\Omega} = 30 \times 10^{-30} \text{ cm}^2/\text{sr}$

e^- - energy: 350 MeV

Current: 10 nA on 10^{-2} r.l.

Duty cycle: 100%

Target thickness: 200 mg/cm^2

Solid angle π : 30 msr

Acceptance: $\Delta E_{\pi} = 20\%$

$$\Delta E_{\gamma} = 5 \text{ MeV}$$

Coincidence res.: 5 nsec

Estimated rates: (per electron channel)

e - counter: $10^7/\text{sec}$

π - counter: 0.18/sec

true coincidence: 1.8×10^{-2}

true/accidental: 2

Comments: $^{12}\text{C}(e,e'\pi)$ and $^{12}\text{C}(\gamma,\pi)$

These inclusive reactions are probably dominated by the quasielastic reaction mechanism in which an undetected nucleon is ejected leaving the residual nucleus in some unknown excited state. Although the coincidence rates are low these reactions are important because they carry information about how the pions propagate in the nuclear medium. The tagged bremsstrahlung method probably does not compete with the method using the tip of the untagged bremsstrahlung spectrum unless one is interested in the low energy pion spectrum from high energy photons.

References

1. T. de Forest, Ann. Phys. (N. Y.) 45, 365 (1967).
2. T. A. Griffy and D. Whitehall, Phys. Rev. C 2, 441 (1970).
3. D. Drechsel and H. Überall, Phys. Rev. 181, 1383 (1969).
4. V. V. Balashov, S. J. Grishanova, N. M. Kabachnik, V. M. Kulikov and N. N. Titarenko, Nucl. Phys. A216, 574 (1973).
5. W. C. Barber, Ann. Rev. Nucl. Sci. 12, 1 (1962).
6. F. Cannata et. al., Can. J. Phys. 52, 1405 (1974).
7. P. Benz et. al., Nucl. Phys. B65, 158 (1973).
8. L. M. Saunders, Nucl. Phys. B7, 293 (1968).
9. E. J. Moniz, Phys. Rev. 184, 1154 (1969).
10. F. Partovi, Ann. of Phys. 27, 79 (1964).
11. B. Mecking et. al., Proceedings of the International Conf. on Photonuclear Reactions and Applications, Asilomar, 1973 (p. 967).
12. J. Mougey et. al., Nucl. Phys. A262, 461 (1976).
13. F. Borkowski et. al., Phys. Rev. Lett. 38, 742 (1977).
14. G. Bardin et. al., Nucl. Phys. B120, 45 (1977).

APPENDIX I

KINEMATICS

The reaction $A_T(e, e'x)A_R^*$ has the following four nonrelativistic conservation relations in the laboratory system:

$$\vec{q} = \vec{k}_1 - \vec{k}_2 = \vec{p}_x + \vec{p}_R$$

$$\omega = k_1 - k_2 = \frac{p_x^2}{2M_x} + \frac{p_R^2}{2M_R} + B_n$$

B_n is the separation energy to a specific final state in the residual nucleus A_R^* . Of the nine variables in the final state $(\vec{k}_2, \vec{p}_x, \vec{p}_R)$, $9 - 4 = 5$ must be measured to completely specify the reaction kinematics (e.g. k_2 , Ω_2 , and Ω_x) if B_n is known (otherwise a sixth variable p_x must be measured).

The angular distribution of inelastically scattered electrons (in Born approximation, but retaining the mass of the electron m to avoid the singularity in the forward direction) can be written as:

$$\frac{d^2\sigma}{d\Omega_e dE_2} = \sigma_c(\Theta) F_c^2(q, w) + \sigma_T(\Theta) F_T^2(q, w)$$

where the transition form factors are multiplied by "Coulomb" and "transverse Mott cross sections"

$$\sigma_c = \frac{2\alpha^2}{q_\mu} \frac{k_2}{k_1} \frac{q_\mu^4}{q^4} (E_1 E_2 + \vec{k}_1 \cdot \vec{k}_2 + m^2)$$

$$\sigma_T = \frac{2\alpha^2}{q_\mu} \frac{k_2}{k_1} (E_1 E_2 - (\vec{k}_1 \cdot \hat{q})(\vec{k}_2 \cdot \hat{q}) - m^2)$$

and $E = (k^2 + m^2)^{\frac{1}{2}}$, $q_\mu^2 = 2(E_1 E_2 - \vec{k}_1 \cdot \vec{k}_2 - m^2) = q^2 - w^2$

$$w = E_1 - E_2, \vec{q} = \vec{k}_1 - \vec{k}_2, k_\mu^2 = -m^2$$

when $m = 0$ we obtain the familiar forms

$$\sigma_c = \sigma_m \frac{q_\mu^4}{q^4}, \quad \sigma_T = \sigma_m \left(\frac{q_\mu^2}{2q^2} + \tan^2 \theta/2 \right), \quad \sigma_m = \frac{\alpha^2 \cos^2 \theta/2}{4k_1^2 \sin^4 \theta/2}$$

$$q_\mu^2 = 4k_1 k_2 \sin^2 \theta/2$$

The Coulomb and transverse Mott cross sections are plotted in Fig. 1 for $k_1 = 100$ MeV, $k_2 = 80$ MeV. We note the strong forward peaking of σ_T at

$$\theta_{\max}^{\oplus} \cong \frac{m\omega}{k_1 k_2}.$$

This peaking is responsible for the fact that the integrated over angle inelastic electron scattering cross section $\frac{d\sigma}{dE_2}$ comes mainly from the far forward transverse component (i.e. like a real photon). As a consequence most of the product x in a $(e, e'x)$ measurement is produced by electrons which are within one degree of the forward angle and therefore only contribute to the accidental coincidence rate.

For electric transitions $E1, E2$, etc. at angles away from the extreme forward and backward scattering angles the Coulomb transition dominate the current transition, both because $\sigma_c > \sigma_T$ and because, in general at low momentum transfer, $F_c > F_T$ as can be seen the relation

$$|F_T^L(q, w)|_{q \rightarrow 0}^2 = \left(\frac{w}{q} \right)^2 \frac{L+1}{L} |F_C^L(q, w)|^2.$$

Thus coincidence experiments $(e, e'N)$ in the giant multipole region are Coulomb dominated.

FIGURE 1

APPENDIX II

COUNTING RATES

If the incident electron current is I amperes the number of electrons per sec is $N_e = I/(1.6 \times 10^{-19} \text{ c/s})$. If the areal density of the target is D grams per square centimeter perpendicular to the incident electron beam, the number of nuclei in the beam is $n = D(\sqrt{A}/A)$ where A is the atomic weight and $\sqrt{A}$ is Avogadro's constant, 6×10^{23} molecules per gram-molecular-weight.

The counting rate in the electron spectrometer is then

$$N_{e'} = \frac{d^2\sigma}{d\Omega_e dE_2} n N_e \Delta\Omega_e \Delta E_e$$

For instantaneous rates larger than 10^7 s^{-1} deadtime corrections become important. The counting rate in the product detector N_x is of similar form. If all energy products are accepted the energy differentials dE_x and ΔE_x are replaced by unity.

The coincidence cross section is, in general, a four fold differential

$$\frac{d^4\sigma}{d\Omega_e dE_2 d\Omega_x dE_x}$$

and usually unknown.

A useful approximation for estimating counting rates is to assume the product angular distribution is uncorrelated with the electron's momentum transfer direction. This assumption gives the average coincidence rate. The coincidence cross section is then given by

$$\frac{d^4\sigma}{d\Omega_e dE_2 d\Omega_x dE_x} \approx \frac{1}{4\pi} \frac{d^2\sigma}{d\Omega_e dE_2} \frac{dn_x}{dw}$$

where $\frac{dn_x}{dw}$ is the density of final states in the residual nucleus,

$$w = E_1 - E_2.$$

The density is normalized to give the fraction of all decays of the state $A_T^*(w)$ to x 's.

$$\int_{\text{thres}}^w \frac{dn_x}{dw} dw = \frac{\Gamma_x}{\Gamma_{\text{tot}}}$$

Thus the coincidence cross section for x 's of all energies is

$$\frac{d^3\sigma_{\text{ex}}}{d\Omega_e dE_2 d\Omega_x} = \int \frac{d^4\sigma}{d\Omega_e dE_2 d\Omega_x dE_x} dE_x \approx \frac{\Gamma_x}{\Gamma_{\text{tot}}} \frac{1}{4\pi} \frac{d^2\sigma}{d\Omega_e dE_2}$$

The coincidence counting rate for x's of all energies assuming isotropy is then

$$N_{\text{coin}} = n_N N_e \frac{\Gamma_x}{\Gamma_{\text{tot}}} \frac{\Delta\Omega_x}{4\pi} \frac{d^2\sigma}{d\Omega_e dE_2} \Delta\Omega_e \Delta E_2$$

$$= \frac{\Gamma_x}{\Gamma_{\text{tot}}} \frac{\Delta\Omega_x}{4\pi} N_e$$

The accidental or chance counting rate is

$$N_{\text{acc}} = \tau N_e N_x / \text{d.f.}$$

where τ is the coincident circuit resolving time (typically 5 ns) and d.f. is the electron beam duty factor (≤ 1). Although accidental rates can be measured, a value of $N_{\text{coin}}/N_{\text{acc}} < .2$ severely compromises an accurate calculation of N_{coin} .

APPENDIX III

Bremsstrahlung (Real and Virtual)

Electron scattering experiments unavoidably involve photon emission in the Coulomb field of the nucleus. We give here some simplified spectral formulae for estimation purposes. More accurate expressions may be found in the references.

Notation:

E_1 incident electron energy

E_2 final electron energy

$k_\gamma = E_1 - E_2$, photon energy

θ electron scattering angle

$\epsilon = E_2/E_1$

$\alpha = 1/137$, $\hbar = c = 1$, $m_e = .511$ MeV

Forward Angle Bremsstrahlung

The completely screened Bethe-Heitler cross section is sufficient for $E_1 = 100 - 1000$ MeV on high Z radiators

$$\frac{d\sigma}{dk_\gamma}(E_1, k_\gamma) = Z^2 \left(\frac{4\alpha^3}{m_e^2} \right) \frac{1}{k_\gamma} \left\{ \left(1 + \epsilon^2 - \frac{2}{3}\epsilon \right) \ln \left[\frac{183}{Z^{1/3}} \right] + \frac{\epsilon}{9} \right\}$$

$.232 \text{ fm}^2$

The number of photons generated in a radiator with n_R nuclei per cm^2 is then

$$N_\gamma(k_\gamma) = n_R \frac{d\sigma}{dk_\gamma}$$

The radiator thickness is often expressed in units of radiation length, t_0 , the areal density through which an electron will convert $1/e$ of its energy to photons. Figure 2 shows t_0 as a function of Z .

$$n_R = t_0 \frac{t}{t_0 A}$$

$$A = 6 \times 10^{23} \text{ molecules per gram mol. wt.}$$

Figure 2. (from Koch and Motz)

Radiation lengths in grams per square centimeter for various materials.

As an example tungsten ($Z = 74$, $A = 184$) has $t_0 = 6 \text{ gm/cm}^2$. A 1% radiation length contains $2 \times 10^{20} \text{ nuclei/cm}^2$. A 100 MeV electron generates the following number of 50 MeV photons per MeV

$$\begin{aligned} N_Y(50 \text{ MeV}) &= (2 \times 10^{20}) (.87 \times 10^{-24}) \\ &\quad \text{cm}^{-2} \quad \text{cm}^2 \cdot \text{MeV}^{-1} \\ &= 1.7 \times 10^{-4} \text{ MeV}^{-1} \end{aligned}$$

Elastic Radiation Tail

A common source of background in the electron counter in an $(e, e'x)$ measurement are electrons which have elastically scattered from the target nucleus while undergoing bremsstrahlung in its Coulomb field. The cross section for this process, in the peaking approximation, is

$$\frac{d^2\sigma}{d\Omega_e dE_2} = \mathfrak{F}(E_1) \frac{d\sigma(E_2, \theta)}{d\Omega_e} + \mathfrak{F}(E_2) \frac{d\sigma(E_1, \theta)}{d\Omega_e}$$

$$\frac{d\sigma}{d\Omega_e} \approx \left[\frac{Z\alpha \cos \theta/2}{2E \sin^2 \theta/2} \right]^2 \exp(-q^2 R^2/3)$$

$$q \approx 2E \sin \theta/2, R \approx A^{1/3} \text{ fm}$$

where $d\sigma(E, \theta)/d\Omega_e$ is the elastic scattering cross section evaluated at the appropriate electron energy E_1 or E_2 and the radiation function is

$$\mathfrak{F}(E) = \frac{\alpha}{\pi k_\gamma} \left[(1 + \epsilon^2) \ln \frac{2E}{m_e} - \epsilon \right]$$

Virtual Photon Theory

The cross section for electroproduction $A_T(e, x)e'A_R$ can be estimated from the real photon reaction (γ, x) if the reaction is known to proceed with a definite parity and multipole. The total electroproduction cross section

section when the product x is detected is

$$\sigma_x(E_1) = \int_{k \text{ thres}}^{E_1} N^{L\pi}(E_1, k_Y) \sigma_Y(k_Y) dk_Y$$

where $N^{L\pi}(E_1, k_Y)$ is the virtual photon spectrum of multipole L and parity π .

For example the number of virtual photons per MeV in the electric dipole spectrum is

$$N^{1-}(E_1, k_Y) = \frac{\alpha}{\pi k_Y} \left[(1 + \epsilon^2) \ln \frac{2E_1 E_2}{m_e k_Y} - 2\epsilon \right].$$

Expressions for other multipoles can be found in the references. These spectra are computed assuming inelastic form factors follow $F(q) \propto q^{2L}$. They account for both longitudinal and transverse production.

Virtual photon theory can also be used to compute the inelastic electron scattering cross section at a given angle for a well-defined state

$$\frac{d\sigma}{d\Omega_e} = \frac{dN^{L\pi}(E_1, E_2, \theta)}{d\Omega_e} \sigma_Y(k_Y)$$

For electric dipole the longitudinal spectrum is

$$\frac{dN_L^{1-}}{d\Omega_e} = \frac{\alpha}{\pi^2} \frac{E_2^2}{q^2} (1 + \cos \theta)$$

while the transverse current (only important near $\theta = 0$ or 180°).

$$\frac{dN^{1-}}{d\Omega_e} = \frac{\alpha}{\pi^2} \frac{E_1^2 + E_2^2 + E_1 E_2 (1 - \cos \theta)}{4E_1^2 (1 - \cos \theta)} \left(\frac{k_Y}{q} \right)^2$$

Bremsstrahlung References

Forward Angle

H. W. Koch and J. W. Motz, Rev. Mod. Phys. 31, 920 (1959).

J. L. Matthews and R. O. Owens, Nuc. Instr. & Meth. III, 157 (1973).

Radiation Tail

L. W. Mo and Y. M. Tsai, Rev. Mod. Phys. 41, 204 (1969).

L. C. Maximon and D. B. Isabelle, Phys. Rev. 133, B1344 (1964).

Virtual Photon Theory

W. C. Barber, Am. Rev. Nucl. Sci. 12, 1 (1962).

B. F. Gibson and H. T. Williams, Nucl. Phys. A163, 193 (1971).

E. T. Dressler and E. L. Tomusiak, Nucl. Phys. A273, 383 (1976).

E. Wolyneć, et al., Phys. Rev. C11, 1083 (1975).

C. W. Soto Vargas, D. S. Onley and L. E. Wright, Nucl. Phys. A288,
45 (1977).

II. MESON CURRENTS & FEW NUCLEON SYSTEMS

E. Hadjimichael
Fairfield University
Fairfield, Connecticut 06430

E. L. Lomon
Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

W. Turchinets
Bates Linear Accelerator Center
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

(Including comments from discussions with J. Friar, E. Moniz
J. O'Connell and D. O. Riska)

CTP #728

MESON CURRENTS AND THE FEW-NUCLEON SYSTEMS

Several years of experimental and theoretical work (1-3) have established beyond doubt the important role that meson-exchange currents play in determining static properties and transition processes of bound nuclear systems.

A simple way to understand the necessity for meson currents is to invoke current conservation

$$i\nabla \cdot \vec{J} = [H, \rho]$$

and recall that the nuclear potential appearing in H is of non-local nature since it is mediated by the exchange of mesons. Hence $\vec{J}$ must also contain parts that depend on the coordinates of two or more nucleons, i.e. exchange parts.

We begin this report by discussing briefly the theoretical question of uniqueness and completeness of the results for meson-exchange currents, and end this first section by displaying a list of what we hope will constitute provisionally acceptable, forms of mesonic-exchange contributions to the nuclear current.

In the second section we focus on theoretical and experimental activities which appear to us to be compelling for a variety of reasons and suggest that these activities be undertaken in the near future.

A. Universality of Meson Currents

The primary question in deriving meson exchange current contributions is whether these currents can be derived in a

unique and unambiguous way or not. Additionally, one is interested in knowing whether the exchange currents can be derived completely^{*} or not. In other words, we are concerned with the uniqueness and completeness of the interaction of an external electromagnetic field (we omit discussion on the axial vector field here) with a bound nuclear system. The answer to those concerns would be a positive one if we had at our disposal a complete dynamical theory for the nuclear medium. We don't have that. One might think that charge conservation and gauge invariance might guarantee completeness for the derived nuclear current. This is not the case, however, There are subsets of exchange processes that are gauge invariant on their own, so that one may have a gauge invariant theory without having taken into account all the possible exchange processes. In the absence of a complete dynamical theory we are not able to say with certainty where the horizon lies as far as exchange processes are concerned. Certain kinematical regions are perhaps easier to deal with than others. For example, in the soft photon and soft meson limit ($k \sim 0$, $q \sim 0$), we get model-independent information on exchange processes by relating the amplitudes for these processes to non-radiative NN amplitudes.⁽⁴⁾ Away from

*i.e. "completely" in principle. However, if the interaction is only known to a given order, or down to a certain range, then a "complete" knowledge of the meson exchange currents will be limited in the same way.

the soft-photon limit, however, this is no longer possible, so that we have to rely on other devices.

The two most talked-about methods for the derivation of meson currents are (a) the S-matrix scheme elaborated first by Chemtob and Rho⁽⁵⁾ for the $k^2=0$ case and since extended to $k^2 \neq 0$ by several people,⁽¹⁻³⁾ and (b) the "unified" method discussed most extensively by J. L. Friar.⁽⁶⁾ This latter is "unified" in the sense that the input to the theory is the nucleon + anything interaction and the output consists of the sought-after meson currents, as well as the N-N potential, so that the nuclear interaction and the meson currents are constructed in a consistent manner. In particular, in this method one begins with the equation of motion for a single nucleon interacting with external electromagnetic and mesonic fields, and reduces this to a non-relativistic form by a Foldy-Wouthuysen transformation, which produces strong and electromagnetic vertices ($\sim \frac{1}{M^3}$). In the following step, one uses these vertices and time-dependent perturbation theory to calculate the S-matrix for two-body processes (terms of order f^2 are kept in meson-nucleon coupling) and hence the two-body operators. Among the processes so generated (assuming that the initial interaction includes pions only) are the "pair" process, Figure (1a), the Recoil process, Figure (1b), and processes corresponding to disconnected graphs which generate the N-N potential, Figure (1c). Perturbation theory, of course, generates an energy dependent potential. In this case, the Recoil graph is needed

to assure orthonormality of the nuclear wavefunction.

One finds, unfortunately, that the two-body operators found by the unified scheme are not unique. The various sets are unitarily equivalent. The unitary transformations, however, change the non-static parts of the potential and also introduce non-local parts, so that one cannot easily see which representation corresponds to the usual OPEP-semiphenomenological potential. Further, different representations arising from different unitary transformations have different properties. One representation, for example, simplifies relativistic corrections. Another eliminates completely the isoscalar charge operator $(-\frac{1}{M^2})$, and a class of 3-body forces. A third agrees, as far as the two-body operators is concerned, with the results of the S-matrix approach to be described below.

The virtue of the method of course is that starting with the nucleon-photon and nucleon-meson interaction, in one grand sweep one generates the N-N interaction, the two-body currents, relativistic and off-shell effects, and three-body forces.

The major drawback is the unitary equivalency of a large number of representations for the two-body operators and potentials, all having different features. It is also noteworthy that changing the sequence of the non-relativistic and the unitary transformation will produce results which differ by a unitary transformation.

A second problem with the unified method is that several exchange processes do not enter into play. To be more specific,

these are processes where the external photon couples to two different hadrons, as in Figures (2a, 2b). We note that, in contrast to the processes in Figure (1), those in Figure (2) would be non-existent if the photon is removed, so that they do not contribute to the N-N potential.

Let us call the processes in Figure (1) the "potential" exchange processes, those in Figure (2) the "non-potential" exchange processes. It is the potential exchange processes that require a certain consistency in their treatment with that for the nuclear force, since they introduce non-nucleonic components in the wavefunction and thus raise questions of wavefunction renormalization. It can be deduced from the results and the spirit of the unified method that if a static N-N potential is used to evaluate the nuclear wavefunction, then the renormalization due to the non-nucleonic components cancels to a large extent the contribution from the Recoil process, Figure (1b).

It is important to note that the Unified method treats only the potential exchange processes. It is also interesting that the Pair graph, Figure (1a), can be considered either as a potential or as a non-potential exchange process depending on whether the nucleon and the anti-nucleon are taken to be in the same Fock space or in different spaces. These contributions are unambiguous in the latter case.

In the S-matrix approach to the problem of meson-exchange currents, we evaluate both potential and non-potential exchange processes by using Feynman rules to write down the covariant amplitude for two-body processes, and eventually three-body processes if it is so desired, and via a non-relativistic reduction where Dirac matrices are written in terms of Pauli matrices, we obtain the Hamiltonian density in the familiar Maxwell form

$$H \sim \vec{J} \cdot \vec{A}$$

thus identifying the two-body, or many-body current J . Theoretically we cannot rest until all the possible processes are evaluated.

The disconnected graphs are not treated here in the same sweep. Instead the usual static semi-phenomenological N-N potentials are used for the evaluation of the nuclear wavefunctions, which impose the necessity for renormalization, as discussed earlier. In contrast to J. Friar's method (where the renormalization scheme would split off the parts of the disconnected graphs which provide a cancellation between the bulk of the Recoil process contribution and the disconnected graphs, leaving intact a small non-static part only), the S-matrix perturbative approach⁽³⁾ reduces only partly the Recoil contribution by means of non-static contributions.

We stress that all ambiguities which appear in either J. Friar's approach or in the S-matrix approach are due to an incomplete field-theoretical description of the nucleon-nucleon force. Whenever a unitary transformation is made on the current operator, the same transformation on two (or more)-nucleon potentials will suitably alter the nuclear wavefunction so that the physical matrix element will be unaltered. The ambiguity of the Friar approach arises from the uncertainty of which representation corresponds to given two-nucleon forces, three-nucleon forces, etc. Similarly in the S-matrix method, the Recoil-re-normalization corrections depend on the non-static parts of the nuclear force which are usually not defined together with the prescribed static force.

With complete field-theoretical descriptions of realistic inter-nucleon interactions it is possible to formulate a "unified S-matrix" approach, which simultaneously prescribes the nuclear wavefunctions and the current operators. It has been shown⁽⁷⁾ that field-theoretical descriptions of the two-nucleon potential, with two-pion exchange and non-static effects included, lead to a realistic description for ranges greater than 0.7f., although some terms remain to be calculated. Thus only the multi-meson exchange, shorter range MEC effects, which are expected to be small, need remain ambiguous in the long run. At present, these shorter-range effects are not included in either the S-matrix or the Friar approach. Indeed even the two pion exchange current

terms have not been calculated although the two pion nucleon-nucleon force is known.

The two-body Recoil-Renormalization controversy remains a valid theoretical point. In practice, it may be less important than it is feared, because the Recoil process does not usually dominate in calculations of exchange current contributions. Still, there are instances where it is crucial to know the extent to which the Recoil contribution is reduced by renormalization effects. One such case is, for example, the magnetic moment of the Deuteron.⁽⁸⁾ We show in Figure 3 the values obtained in the Impulse Approximation, with meson pair currents added to the IA but no Recoil-Renormalization contributions and then with the Recoil contribution. In view of the large effect of the Recoil contribution alone, it is imperative to know by how much the Recoil contribution is reduced by the renormalization. The effects of relativistic corrections and of the minimal current arising from nucleon-nucleon spin-orbit operators have recently been calculated, and found to be important.⁽⁸⁾

Further, three-body Recoil graphs, Figure (4) are occasionally necessary in order to insure charge conservation (e.g., in ^3He). Connected graphs, however, of the type shown in Figure 5 introduce negligible contributions.

The question of Renormalization does not enter in the treatment of the non-potential exchange processes, Figure (2). The

evaluation of at least these processes by the S-matrix method may be considered unambiguous.

In the S-matrix approach, we introduce in separate steps, relativistic effects of kinematical or dynamical nature, three-body effects and off-shell corrections for both nucleons and mesons. This latter correction may be accomplished by the introduction of hadronic form factors.

With the exception of the two-body Recoil process, the two-body operators in at least one representation of Friar's unified method⁽⁶⁾ agree exactly with those derived by the S-matrix method for the potential exchange processes. Furthermore the recoil and renormalization correction is small. Hence, we begin to achieve a degree of confidence with regard to meson exchange currents and feel more certain about answering positively the question of their uniqueness. We still have to wrestle with questions of off-shell and relativistic effects, hadronic and nucleon electromagnetic form factors, meson-nucleon coupling constants, etc., that enter into the two-body operators, but these quantities are intimately connected to the exchange processes themselves and will eventually be determined in a theory that includes exchange effects, not one without them.

As far as the question of sufficient completeness is concerned, lacking basic principles, we can only rely on comparisons with experimental data for some guidance and treat every case on an individual basis.

At low q^2 an approach which may be fruitful is the parameterization of our ignorance of the short-range contributions (or the as-yet uncalculated medium-range contributions). The graphs of reference 9 show that each meson-exchange graph contribution to the deuteron quadrupole form factor $q^{-2}G_2(q^2)$ decreases as $\beta e^{-\alpha q^2}$ where α, β are nearly constant for $q^2 < 40 \text{ fm}^{-2}$, but α and β depend on the graph. Furthermore, the value of β decreases systematically with increasing exchange mass of the graph. Also α tends to decrease as the exchanged mass increases.

Similarly the charge form factor divided by q^2 , $q^{-2}G_0(q^2)$ can be seen⁽⁹⁾ to have contributions from each MEC graph that behave as $\beta' e^{-\alpha' q^2}$ for $q^2 < 40 \text{ fm}^{-2}$, and in fact $\alpha' \sim \alpha$. Again β' decreases with increasing exchange mass. It is expected that the MEC for the magnetic form factor G_1 will behave similarly, i.e., $\beta'' e^{-\alpha'' q^2}$.

Thus we may try to parametrize the leading non-computed MEC in the above form. Although this approach has six parameters, three of those, β' and β'' and β can be fitted to the quadrupole moment, the magnetic moment and low q^2 precision measurements of $A(q^2)$ respectively. If one can use $\alpha \sim \alpha' \sim \alpha''$ also, then only one parameter is left to fit to the electric and magnetic elastic scattering and the tensor polarization for $q^2 \leq 40 \text{ fm}^{-2}$. The α 's in any case are probably small for short-range contributions. Over this range of q^2 the impulse approximation has substantial structure and is sensitive to the nuclear model. Hence

detailed data on $A(q^2)$, $B(q^2)$ and $P(q^2)$ for $q^2 \leq 30 \text{ fm}^{-2}$ may be able to select nuclear models and simultaneously give an experimental estimate for the size of short-range MEC contributions.

If the unknown MEC contributions were smaller than any of those of reference 9 for $q^2 < 40 \text{ fm}^{-2}$, they would make negligible contributions. However, there is evidence from μ and Q (see Figure 3) that the known contributions are inadequate. This may be due to relativistic effects or contributions from the non-static part of the nucleon force, or they may come from the one type of medium-range graph which has been neglected so far, that involving two pion exchange. These may be expected to have a magnitude comparable to that of the $\rho\pi\gamma$ contribution or perhaps even larger. There are several two-pion exchange graphs, so that the phenomenological approach may be a convenience while waiting for them to be calculated.

We display in Appendix A canonical forms for meson-exchange currents for a variety of exchange processes.

B. Suggested Investigations on the Role of Meson Exchange Currents In Few-Nucleon Systems

I. Electron-Deuteron Scattering

a. Elastic Scattering

The cross-section for elastic scattering in the lab frame is

$$d\sigma = d\sigma_{\text{Mott}} (A(q^2) + B(q^2) \tan^2 \theta/2)$$

In terms of the monopole, quadrupole and magnetic dipole form factors G_0 , G_2 , G_1 , we have

$$A = G_0^2 + G_2^2 + 2/3 \eta G_1^2$$

$$B = 4/3 \eta (1+\eta) G_1^2$$

$$\eta = \frac{q^2}{4M_D^2}$$

Further, the tensor polarization of the recoil deuteron is

$$P = \frac{2G_0 G_2 + G_2^2/\sqrt{2}}{G_0^2 + G_2^2}$$

Measuring A , B , P could furnish information on the Deuteron wavefunction via G_0 , G_1 , G_2 , provided one knows the current operator $J = (\vec{J}, \rho)$ which receives contributions from mesonic-exchange processes.

A further difficulty arises from incomplete knowledge of nuclear electromagnetic form factors, in particular the neutron electric form factor G_E^n . Although the data for the neutron magnetic form factor G_M^n are better, one must carefully choose the model form factor fit to those data, as some popular models are not good fits to G_M^n .

Finally, relativistic effects, must be taken into account. Hence, there are four distinct areas of concern. (i) The two-body wavefunction or, equivalently, the N-N interaction; in

particular, the short-range part of the interaction and the strength of the tensor part that determines the D-state admixture. (ii) The current operator, in particular the non-Impulse part due to mesonic exchange. Included in this area are the meson-nucleon coupling constants and vertices (hadronic form factors), as well as the determination of the exchange processes. (iii) The electromagnetic structure of the neutron. (iv) Relativistic corrections.

As we shall see, these are, more or less, the same concerns we have in investigating other electron or photon interactions with the two- and three-body nuclear systems.

1. $q^2 = 0$; Quadrupole Moment and Magnetic Moment of the Deuteron.

In the absence of exchange currents, the D-state of the Deuteron is an observable via the magnetic moment

$$\mu_D \approx \mu_n + \mu_p - 3/2(\mu_n + \mu_p - 1/2)P_D = \mu_D^I$$

Where the "percentage D state" $P_D = \int W^2(r) dr / [\int (u^2 + w^2) dr]$ with u and W being the S and D state deuteron wave functions.

However, due to mesonic-exchange currents (MEC), we have

$$\mu_D = \mu_D^I + \mu_D^{MEC}$$

where MEC are functionals of the S- and D-state. Similarly for the quadrupole moment

$$Q = F_Q^I(q^2 \rightarrow 0) + F_Q^{MEC}(q^2 \rightarrow 0);$$

$$F_Q^I(q^2) = 6 \sqrt{2} \frac{G_E^S(q^2)}{q^2} \int dr j_2(qr/2) (UW - \frac{W^2}{\sqrt{8}})$$

where $G_E^S = 1/2(G_E^P + G_E^N)$, the neutron and proton electric form factors respectively.

The importance of Q rests in the fact that it is a measure of the tensor components in the N-N interaction. Experimentally, Q is found by molecular methods from the electric field gradient of a simple molecule, such as HD or D_2 . One measures $ef^Q/h^{(10)}$ from which Q is extracted after the field gradient f is evaluated theoretically by a variational calculation of the electronic wavefunction.⁽¹¹⁾ The latest value is $Q = 0.2875 \pm 0.002 \text{ fm}^2$.

This method leaves something to be desired, as the error on f is estimated from the convergence of the electronic energy in the variational calculation. MEC corrections to Q may give as much as 0.02 fm^2 .^(9,12,13) Obviously Q^I is sensitive to P_D while Q^{MEC} is less so. P_D varies from 4% to 7% in different models of the N-N force (and to as much as 11% if relativistic effects are taken into account). It appears that a systematic investigation of these models should be undertaken via accurate determination of Q^{MEC} and in conjunction with new experimental determinations of Q . Figure 3 shows results of some models and Q^{MEC} with and without Recoil-Renormalization effects.⁽⁸⁾

Similar comments hold true for μ_D . In this case μ^{MEC} is as sensitive as μ^I to the value of P_D . In Appendix B we display the MEC corrections to Q and μ_D , and give results in Figure 3. These corrections include relativistic effects and contributions from the $\Delta\Delta$ configuration.^(14,15) All of the model

values of $\mu^I + \mu^{MEC}$ are larger than μ indicating that significant contributions to μ^{MEC} have yet to be calculated. In the phenomenological description of $\Delta\mu^{MEC}$ discussed above $\alpha' \neq 0$. In addition Q determines an α'' which only vanishes for small values of P_D .

$$2. \quad 0 < q^2 \leq 1 \text{ fm}^{-2}$$

In this region the slope of the neutron electric form factor G_E^n is well known from the reflection of thermal neutrons.⁽¹⁶⁾ Relativistic effects are small and are only model dependent to less than 0.2% of the G_0 amplitude. Estimated contributions of MEC are less than 0.5%, therefore measurements of $A(q^2)$ in this range help determine the acceptable nuclear models. Accurate measurements by Berard et al⁽¹⁷⁾ determine the deuteron radius and provide a measure of the deuteron D state. The analysis of Reference (17) shows that the Feshbach-Lomon, FL, potential with 7.53% D state fits both of those parameters well, while 4.57% and 5.20% D state FL potentials are two standard deviations out. The latest G_E^n slope experiment⁽¹⁶⁾ fits the 7.53% case even more precisely than the older value used in Ref. (17).

This determination uses an experimental accuracy⁽¹⁷⁾ of 0.2% in $A(q^2)$, but was analyzed without including the model dependent relativistic effects and the MEC contributions. A re-analysis with these elements may alter the conclusions somewhat. An analysis of G_E^n by Schumacher and Bethe (unpublished)

also prefers the 7.53% FL potential to that of Reid, Hamada-Johnston and several other potentials when comparing with $G_E^{n'}(0)$. This analysis however, suffers from the same shortcomings as that of reference 17.

One may allow for uncalculated MEC and relativistic corrections by suitably adjusting α in the parametrization discussed above. Experiments for $1 < q^2 < 40 \text{ fm}^{-2}$ will then test the consistency of specific models and the value of α that they require.

$$3. \quad 1 < q^2 \leq 12 \text{ fm}^{-2}$$

The impulse approximation expressions for $G_i(q^2)$ are

$$G_0 = G_0^{\text{IA}} = 2G_E^S C_E$$

$$G_1 = \frac{M_d}{M_p} (2G_M^S C_S + G_E^S C_L) \quad G_2 = \frac{4\sqrt{2}}{3} \eta C_Q$$

where $G_E^S = 1/2 (G_E^P + G_E^n)$ and $G_M^S = 1/2 (G_M^P + G_M^n)$

$$C_E = \int (U^2 + W^2) j_0 \left(\frac{1}{2} qr \right) dr$$

$$C_Q = \frac{3}{\sqrt{2}\eta} \int W \left(U - \frac{1}{\sqrt{8}} W \right) j_2 \left(\frac{1}{2} qr \right) dr$$

$$C_L = \frac{3}{2} \int W^2 [j_0 \left(\frac{1}{2} qr \right) + j_2 \left(\frac{1}{2} qr \right)] dr$$

and

$$C_S = \int [(U^2 - 1/2 W^2) j_0 \left(\frac{1}{2} qr \right) + \frac{W}{\sqrt{2}} \left(U + \frac{W}{\sqrt{2}} \right) j_2 \left(\frac{1}{2} qr \right)] dr$$

$\frac{U}{r}$ and $\frac{W}{r}$ are the deuteron S and D wave-functions respectively, and the j_i are the Spherical Bessel functions.

From the work of Sprung and Rao⁽¹⁸⁾, Coester and Ostebee⁽¹⁹⁾ and J. L. Friar,⁽²⁰⁾ we find that in this region $A(q^2)$, $B(q^2)$ and $P(q^2)$ are not very sensitive to: (a) Relativistic effects, and (b) Wavefunction models (with $P_D \approx 6\%$) for $q^2 \leq 2.0 \text{ fm}^{-2}$. With regard to (b), however, E. Lomon⁽²¹⁾ finds that the D-state admixture can make a substantial difference. He finds a 10% change in P and 5% change in A at $q^2 = 5.0 \text{ fm}^{-2}$ as P_D changes from 4.7% to 7.55%. The Feshbach-Lomon, Reid Soft Core and Hamada-Johnston potentials have been used here.

Table I, as calculated by Lomon, illustrates the sensitivity of the experimental functions $A(q^2)$, $B(q^2)$ and $P(q^2)$ to nucleon force models, to nucleon electromagnetic form-factors and to MEC contributions at $q^2 = 5 \text{ fm}^{-2}$.

The form-factor dependence shows up in $B(q^2)$ because the $\sim 8\%$ model difference in G_M^n is amplified by its cancellation with G_M^p in G_M^s . The experimental value of B favors f-f I. The experimental value of G_E^n is determined to approximately $\pm .02$ by the slope of G_E^n at $q^2 = 0$. This would make $A(q^2)$ uncertain by $\pm 6\%$, an effect not indicated in Table I. Reducing the error in the experimental value of $A(q^2)$ from its present $\pm 6\%$ would proportionately reduce the error in G_E^n , if the nucleon force model could be determined from other data. We note that all f-f dependence in $P(q^2)$ cancels out as it depends only on the ratio G_2/G_0 .

In the impulse approximation, $A(q^2)$ has a 4% nucleon force dependence which does not depend only on the % D state. The H-J prediction is similar to the low % D state FL models, although the RSC prediction does fall as expected between the small and large % D state FL models. The $B(q^2)$ has a negligible model dependence at $q^2 = 5 \text{ fm}^{-2}$ compared to experimental accuracy. The important nucleon force model dependence in the IA is in $P(q^2)$, and that is largely controlled by the % D state.

This situation is altered when the MEC contributions are added, except for $B(q^2)$. The MEC corrections of reference 9 to $B(q^2)$ are negligible compared to the f-f dependence and the experimental error. The calculations of Gari and Hyuga⁽⁹⁾ show that the MEC corrections to G_0 and G_2 can substantially alter $A(q^2)$ and $P(q^2)$. However the isobar corrections^(14,15) to $B(q^2)$ ignored in ref. 9 are substantial and increase the predicted $B(q^2)$. This makes the fit with type II nucleon form factors even worse.

The uncertain size of the MEC corrections does not permit us to use the Gari-Hyuga, or any other available calculations directly. The calculations of Hadjimichael (Ref. 2 and Figure 3) show that the calculated graphs would already be very model selective on the basis of Q alone. If one allows for some uncalculated MEC contributions to make a given nucleon force model consistent with Q , one then has a prediction for MEC effects for $0 < q^2 < 40 \text{ fm}^{-2}$. (See the last part of Section A.)

We may get an approximate estimate of these predictions using the parameterization of Moniz⁽²²⁾ described in Table I, scaled to Q . The results in Table I show (if the Moniz scaling holds for the unknown MEC terms) that $A(5 \text{ fm}^{-2})$ varies by 13% with nucleon force models, and is in rough correspondence to the % D state. This exceeds the uncertainty due to G_E^n and the experimental error. Allowing for a reduction of $\approx 5\%$ in the predicted values of $A(q^2)$ due to relativistic effects, the present experiment⁽²³⁾ and the Moniz parameterization does not favor the 7.55% FL model. Increased experimental accuracy would further narrow down a combination of G_E^n and the range of models.

Using the scaled MEC corrections⁽²²⁾ for $P(q^2)$, the model dependence of $P(5 \text{ fm}^{-2})$ is nearly cancelled, except for the 7.55% D state FL model. That model predicts a tensor polarization 7% smaller than the other models. Model dependent relativistic effects are of the order of 2%. It is expected that experiments can determine $P(q^2)$ to an absolute accuracy of $\pm .03$ ⁽²⁴⁾ which may then eliminate at least one nucleon force model, in conjunction with the value of Q .

Experimental researchers should also note that the present uncertainties in A ⁽²³⁾ are of the order of 5% to 10% at $q^2 \leq 20 \text{ fm}^{-2}$. Reduction of these to the 3% level or less is needed to help distinguish model dependence and MEC effects.

4. The next region of interest in e-D elastic scattering is $12\text{fm}^{-2} \leq q^2 \leq 26\text{fm}^{-2}$. G_2 and MEC corrections are very important here because the point where $G_0 = 0$, i.e., the first diffraction minimum of the charge form factor, is somewhere in this region. Just where depends on the MEC contributions and relativistic effects, as well as the nuclear model.

At $G_0 = 0$ we have

$$P = 1/\sqrt{2}$$

$$A = G_2^2 + 2/3 \eta G_1^2$$

$$B = 4/3 \eta (1 + \eta) G_1^2$$

The zero point in the Impulse approximation is at $q^2 = 19.5 \text{ fm}^{-2}$ - 21 fm^{-2} . MEC corrections shift it to lower values by $\sim 4\text{-}5 \text{ fm}^{-2}$.

The slope of G_0 near the zero point and the size of G_2 at the same point should reveal further details in wavefunction models. From the theoretical point of view, relativistic effects must be taken into account. Complications do arise with respect to $A(q^2)$, however, from our inadequate models for the isoscalar charge form factor. Different models and experiments for G_{ES} (23,25,26) may differ by as much as 30% at $q^2 \sim 20 \text{ fm}^{-2}$. Tackling the problem of neutron electromagnetic form factors along different lines as we shall see later, may allow us to derive maximum information on wavefunction models near the zero point of G_0 .

Figure 6 illustrates the rapid variation of $P(q^2)$ in this region. It reaches its maximum value $\sqrt{2}$ when $(G_2/G_0) = \sqrt{2}$, is reduced to $1/\sqrt{2}$ when $G_0 = 0$, and vanishes when $(G_2/G_0) = -2\sqrt{2}$. It then becomes negative to reach its minimum value of $-1/\sqrt{2}$ as shown in Figure 7. Figures 6 and 7 illustrate the model dependence of $P(q^2)$ in the impulse approximation. This will be considerably modified by MEC and relativistic effects, but unaltered by the nucleon electromagnetic form-factors.

For all the three FL models and those of HJ and RSC, G_0 vanishes for $q^2 = 20-21 \text{ fm}^{-2}$ (see Table III). On the other hand, the Gari-Hyuga MEC contributions to G_0 causes it to vanish at $q^2 = 15.5 \text{ fm}^{-2}$, where the IA contributions still differ from each other by only 17%. It follows that the experimental determination of the value of q^2 at which $P = 1/\sqrt{2}$ provides an accurate determination of the magnitude of G_0^{MEC} at that q^2 by comparison with G_0^{IA} . The rapid variation of $P(q^2)$ in that vicinity facilitates the ability to determine q_0^2 accurately.

Having determined q_0^2 through the measurement of P , measurements of $B(q_0^2)$ and $A(q_0^2)$ will enable a separate determination of G_1 and G_2 at q_0^2 . If G_1^{MEC} and G_2^{MEC} have been extrapolated (as in the last of Section A) using μ and Q and any lower q^2 information (as discussed above), then these measurements will determine an allowed range of G_1^{IA} and G_2^{IA} . We must remember that the latter do not depend only on the nucleon force model, but also on the nucleon electromagnetic form factor. But G_1^{IA} is dominated

by G_M^S which is fairly unambiguous. Therefore, we can expect that G_0^{IA} and G_1^{IA} can be moderately well determined by the P and B experiments at q_0^2 . Then the models which predict adequate G_0^{IA} and G_1^{IA} can be used to compare with $G_2(q_0^2)$ and hence obtain a measure of G_E^n .

The rapid q dependence of the IA values of G_1 and the expected smoothness of MEC, f-f and relativistic contributions in this region imply that measurements of A, B, and P at $q^2 \approx q_0^2 + 10 \text{ fm}^{-2}$ will further filter out the nucleon force models with the correct momentum transfer dependence. Table II presents the IA predictions for A, B, and P at $q^2 = 15 \text{ fm}^{-2}$ and 26 fm^{-2} . It should be noted that, even at $q^2 = 26 \text{ fm}^{-2}$, A^{IA} and B^{IA} are adequate to account for half to all of the experimental values, (27,28,29,30) depending on the model.

5. High q^2 region, $26 \text{ fm}^{-2} < q^2 \leq 80 \text{ fm}^{-2}$

Only data on $A(q^2)$ exist in this region. (27,28) Nucleon force models, neutron electromagnetic form factors, meson exchange currents and relativistic effects all have large uncertainties in this range. Furthermore, for all of these contributions, extrapolation from lower q^2 information becomes very model dependent. Hence it is important that $B(q^2)$ and $P(q^2)$ also be measured in order to separate out the various effects.

The most likely source of unambiguous information in this region is $B(q^2)$. As shown by Figure 8, the IA of this measurable quantity varies by orders of magnitude either with the

nucleon force model or with the model value of G_M^n . The term in $B(q^2)$ proportional to G_E^n is not important in this range. The largest values of $B(q^2)$ go with the 4.57% D state FL model and f-f set I (which is much more consistent with quasi-elastic e-d data than f-f set II).⁽³¹⁾

Furthermore, the IA amplitude of B vanishes for q^2 between 31 fm^{-2} and 52 fm^{-2} depending on the model as illustrated in Figures 9 and 10. These IA results, together with the MEC contribution to G, will determine the position and depth of a dip or shoulder in $B(q^2)$. Figure 10 shows that for the RSC potential with the all-dipole, or all-IJL f-f's used by Gari-Hyuga,⁽⁹⁾ there is only a shoulder in the vicinity of $q^2 = 80-90 \text{ fm}^{-2}$. However, for the more likely f-f's (Case I), and especially for the lower % D state models, the IA will be much larger (after the zero of G_1). Therefore, if the MEC contributions are not much larger than calculated by Gari and Hyuga, one may expect a significant dip in $B(q^2)$ whose position and width will help determine both the nucleon force model and the magnitude of the MEC contributions.

As shown in Figures 11 and 12, $A(q^2)$ is also substantially affected by the model and f-f's in this region. It should be noted that almost all of the variation comes from the magnetic contribution ηG_1^2 to A, the electric scattering. This changes, at $q^2 = 78 \text{ fm}^{-2}$, from being negligible for HJ and the 7.55% D state FL potentials, to being equal to the electric contribution, $G_0^2 + G_2^2$, for the lower % D state FL potentials. As shown in

Figure 11, f-f I leads to a much slower decrease of $A(q^2)$ with q^2 than does f-f II, for the smaller % D states. The experimental data⁽²⁸⁾ indicate that the IA is still 25% - 50% of the whole cross section at $q^2 = 78 \text{ fm}^{-2}$, therefore this effect can show up if $A(q^2)$ is measured at a few more energies and with somewhat greater precision, for $40 \text{ fm}^{-2} < q^2 < 80 \text{ fm}^{-2}$.

$P(q^2)$ varies widely with model and MEC effects for $q^2 < 70 \text{ fm}^{-2}$ as indicated in Figure 7. A 5 - 10% measurement in this region will be especially powerful in determining both the model and the MEC if it can be combined with measurements of $B(q^2)$.

6. Very High q^2 region ($q^2 > 80 \text{ fm}^{-2}$)

An experiment has recently been performed⁽²⁸⁾ in this region which shows that there is ample opportunity for the investigation of: (a) meson currents and associated meson-nucleon vertices, (b) nucleon-electromagnetic form factors, (c) a quark-constituent picture of the Deuteron,⁽³²⁾ and (d) possibly even impulse approximation effects.

Recently, incomplete theoretical analyses of the impulse approximation^(28,33) have given the impression that the hadronic picture of the deuteron at high q^2 is not valid and that scaling to quark models sets in at about $q^2 = 0.7 \text{ GeV}^2$.⁽³²⁾ The rate of decrease of the electric form factor appears compatible with a quark law prediction,⁽³²⁾ although the dimensional scaling law is in doubt.⁽³⁴⁾ In particular, the measured point at highest $q^2 = 103 \text{ fm}^{-2}$ falls a little lower than the scaling law.

While it is possible to explore the quark degrees of freedom in the high q^2 region, we suggest that a theoretical treatment correctly incorporating relativistic effects and MEC corrections and using proper neutron form factors, in particular G_E^n , is also in order here. G_E^n is of crucial importance, and it would be better obtained from other lines of research, for example, quasi-elastic scattering.

It may not be impossible to determine the impulse approximation deuteron structure functions in this region for two reasons:

- a) There are several zeros of the nucleon and deuteron impulse form factors that may show up on a smooth MEC background as a dip or shoulder if the experimental accuracy is better than 10%, and if MEC is not more than 80% of the cross-section.
- b) $G_{0,2}^{MEC}$ may be very small for $150 \text{ fm}^{-2} < q^2 < 200 \text{ fm}^{-2}$, because of a cancellation between the $\rho\pi\gamma$ and π exchange contributions.⁽⁹⁾ Whether and where there is a complete cancellation depends on the nucleon form-factors.

In Table III we show where various deuteron and nucleon form factors vanish. It is seen that C_Q (see Appendix C) vanishes between $q^2 = 86$ and 125 fm^{-2} depending only on the $\% D$ state. This shows up correspondingly (see Figure 7) as the position where $P(q^2)$ vanishes. As $P(q^2)$ changes rapidly with q^2 , the zero should be relatively easy to locate with some precision, if $P(q^2)$ can be measured to ± 0.2 or better. A

shoulder or dip also appears in $A(q^2)$ when one uses set II for the f-f (see Figures 11 and 12), but is swamped by the magnetic contribution when set I is used.

Table III shows that C_E vanishes close to $q^2 = 160 \text{ fm}^{-2}$ for all the models. Thus all these models predict $P(160 \text{ fm}^{-2}) = 1/\sqrt{2}$, while if the MEC contributions dominate one expects a very small polarization (see Figure 7). Similarly at approximately 140 fm^{-2} the impulse approximation predicts $P^{IA} = P_{\text{max}} = \sqrt{2}$, when $P^{\text{MEC}} = 0$. Therefore, a measurement of P between $q^2 = 130$ and 160 fm^{-2} will give a strong indication of whether MEC corrections dominate, or whether they are small as predicted by Ref. 9. Where G_0 vanishes, one may also expect a dip in $A(q^2)$, but this effect is dominated by a much sharper dip which we discuss below.

C_L vanishes at $q^2 = 177 \text{ fm}^{-2}$ for FL 1, but does not vanish until $q^2 > 200 \text{ fm}^{-2}$ for the other models. On the other hand, C_S vanishes for q^2 between 170 fm^{-2} and 193 fm^{-2} for the various models, and G_E^S vanishes at $q^2 = 193 \text{ fm}^{-2}$ for both sets I and II. Therefore, G_0 and G_2 and the combination $G_E^S C_L + G_M^S C_S$ (and therefore G_1) vanish somewhere in this region. Just how rapidly G_1 vanishes depends on how close are the zeros of G_E^S and C_S . This explains the sharp dips in A and B for $q^2 = 180 \text{ fm}^{-2}$ for FL 1, FL 5 and HJ, and the broader, but deep, dip for FL 15, as shown in Figures 8, 11 and 12.

If any such narrow dip shows up in A or B in this region, it will be strong evidence that the deuteron structure is not completely swamped by either MEC or by the quark scaling limit.

We summarize e-D elastic scattering:

The static moments of the Deuteron could supply information on wavefunction models provided MEC corrections are taken into account. The difference between the experimental values and the impulse approximation results are small, but it would be difficult to find a nuclear model which eliminates this difference for both μ and Q without incorporating MEC corrections. Given uncertainties in MEC contributions, μ and Q can be used to normalize MEC contributions.

We suggest an experimental determination of the quadrupole moment with no more than 1% uncertainty, and one that is less dependent on complex electronic wavefunction calculation than the experiments on the HD or DD molecules.

The $0 < q^2 < 1 \text{ fm}^{-2}$ region can be useful in eliminating models of the N-N interaction that generate unacceptable $G_E^{n'}(0)$ and rms radii. Only $A(q^2)$ is relevant. The available data should be re-analyzed taking MEC and model dependent relativistic effects into account. A review of the low q data on G_E^D and G_M^D is also in order. A precise determination of P would provide an independent check of the static quadrupole moment.

For $1 \text{ fm}^{-2} < q^2 < 12 \text{ fm}^{-2}$ improved measurements of $B(q^2)$ would help determine G_M^n as this quantity B has little model dependence. The isobar corrections are substantial and ambiguous, but are fixed by the static magnetic moment.

Measurements of $P(q^2)$ together with increased precision on $A(q^2)$ will help determine consistency between nucleon force models and MEC contributions. This will be particularly fruitful if combined with a smooth extrapolation of MEC contributions from $q^2 = 0$.

The study of meson currents can begin in earnest at $q^2 = 12 \text{ fm}^{-2}$. Relativistic effects and differences in models for nucleon electromagnetic form factors do not obscure the essential role of MEC.

The first diffraction minimum in the charge form factor falls somewhere between $q^2 = 15.0$ and 19.0 fm^{-2} at which $P = 1/\sqrt{2}$ uniquely. Precise experimental localization of this point is important. The localization of this point combined with information on MEC from lower q^2 will yield unambiguous information on short-range correlations in the N-N interaction and will help to fix further the value of P_D and hence the tensor component in the N-N potential. This information is, of course, not independent of meson currents, but when combined with measurements of A , B and P throughout this region should provide many checks on each of the factors of nuclear model, MEC contributions and G_E^S .

The region $26 \text{ fm}^{-2} < q^2 < 80 \text{ fm}^{-2}$ is chiefly notable for

the orders of magnitude dependence of $B(q^2)$ on nuclear model and on nucleon electromagnetic form factors. Both the magnitude and an expected dip in $B(q^2)$ are very much affected. These sensitivities are unlikely to be wiped out by MEC contributions.

$P(q^2)$ is also very sensitive to nuclear model in this region and combined with $A(q^2)$ can give much information on both nuclear model and MEC effects. Present measurements of $A(q^2)$ indicate that MEC effects are important but not yet greater than IA effects in this range.

Should some structure be discovered in A , B or P at very high q^2 (say a rapid variation of $P(q^2)$ or a second minimum in $A(q^2)$ at $q^2 = 180 \text{ fm}^{-2}$), it will provide grounds for a nucleonic picture of the Deuteron cum meson degrees of freedom and repudiate the applicability of a simple quark-constituent picture. In addition, it is possible that cancellations make MEC relatively small. Form factors without structure, on the other hand, may motivate a quark-constituent analysis. This region of $q^2 > 80 \text{ fm}^{-2}$ will also provide information on G_M^n and the nucleon force model through $B(q^2)$, and on G_E^n through the sharpness of the high q^2 dip in $A(q^2)$.

These remarks on the high q^2 region have been in the spirit of the workshop where our instructions were to set aside for the moment the question: On what accelerator and with what apparatus could the proposed experiments be done? We are not unaware of the difficulties, both technical and financial, of the work discussed but are confident that our colleagues and sponsoring agencies can rise to the challenge.

7. Deuteron Photodisintegration

We consider four separate ranges of excitation in the np continuum: (1) The threshold region up to about $E_\gamma = 10$ MeV; (2) $10 \text{ MeV} < E_\gamma < 75 \text{ MeV}$; (3) $75 < E_\gamma < 140 \text{ MeV}$ and $E_\gamma > 140 \text{ MeV}$.

Since the early 1960's it has been known that regions (1) and (2) are dominated by the E1 amplitude with small but important contributions from M1 and E2 amplitudes. The early hope that this process might help distinguish between phase-shift-equivalent potentials has been frustrated by larger experimental uncertainties than theoretical differences.⁽³⁵⁾ In Region 1, in the impulse approximation, the effective range and scattering length largely determines the cross section. For the M1 amplitude connecting the 3S_1 and the 1S_0 states, a recent calculation of MEC⁽³⁶⁾ has removed the long-standing discrepancy of about 10% between theory of the thermal neutron capture cross-section and experiment. MEC corrections to the E1 amplitude, which dominates the differential cross section beyond 0.5 MeV above threshold, are thought to be negligible. Attempts to detect the influence of MEC on other observables such as the neutron polarization have not been conclusive owing to the small size of the effect and the comparatively large experimental uncertainties.⁽³⁷⁾ It will be very difficult to get more useful quantitative information in the near future from further work in this energy region since the experimental uncertainties are large compared with the MEC and they are near the limits imposed by available time and technology.

In Region 2, particularly at the higher energies, there is substantial sensitivity to the N-N force model.⁽²⁾ Differences are predicted of the order of 20% in the ratio of $\sigma(0^\circ)/\sigma(90^\circ)$ and for the polarization observables. Of course MEC must be taken into account, but estimates indicate that they are not as serious as in the 100-140 MeV region.

The range of variations in the value of observables produced by different models and MEC is however small compared with the discrepancy between theory and the experimental results of Hughes et al⁽³⁸⁾ for the cross section at $\theta = 0^\circ$, $30 \text{ MeV} < E_\gamma < 130 \text{ MeV}$. Theoretical calculations have been done by Arenhovel and Fabian⁽³⁹⁾ who have included isobar configurations and meson currents in their work. The results are shown in Figure 13. The solid and dashed lines are calculated with the Hamada-Johnston potential with and without the corrections of Ref. 39. E. Lomon⁽⁴⁰⁾ has calculated similar results with the Feshbach-Lomon potentials, varying P_D from 4.57% to 7.55% also shown in Figure 13. The lowest D state probability substantially improves agreement for $E_\gamma \geq 100 \text{ MeV}$, but has not much improved the agreement with the experimental results at lower energies. It is noted however that only MEC corrections to the current density $J(r)$ were considered in Ref. (39). This is because the assumption was made, on the basis of Siegert's theorem, that the charge density $\rho(r)$ remains unaffected. Nevertheless, at $E_\gamma = 50 \text{ MeV}$, the ρ -dependent contribution to the dominant E1 amplitude is approximately eight times larger than the J - dependent contribution so that corrections to the

usual form of $\rho \sim e\psi^*\psi$ will be of great importance. Hence, modifications of $\rho(r)$ of MEC origin should be considered, irrespective of whether Siegert's theorem is violated or not, provided current conservation remains valid, e.g. in the continuity equation

$$\nabla \cdot \underline{J}(r) = -i [H, \rho(\underline{v})]$$

$\underline{J}$ must reflect MEC contributions to $\rho(\underline{r})$ as well as those in the potential contained in H .

A review of the other existing data in this energy region shows that different experiments differ from each other by amounts that often exceed the stated experimental errors but that, on the average, they extrapolate smoothly to the Mainz data. This extrapolation with angle is not consistent with the guidance given by the theory as calculated say by Partovi.⁽³⁵⁾ The integral of these data yields total cross sections 15 to 20% smaller than predicted. This is reminiscent of the results of Baglin et al⁽⁴¹⁾ which a few years ago was the crisis of that day, and which have not yet been satisfactorily explained. Subsequent experiments, designed to check the Yale⁽⁴¹⁾ result, notably the electrodisintegration measurement at Saskatoon⁽⁴¹⁾ and the total absorption measurement at Mainz,⁽⁴¹⁾ failed to confirm it. Clearly a new effort to establish the experimental facts to a precision of 5% at a number of the new facilities is

warranted before theoretical work moves into exotica. It is also possible that higher partial waves in the theoretical calculation for high E_γ might help. For lower E_γ , it is hard to imagine that higher multipoles are important.

Towards the high energy end of Region 2, $E_\gamma \approx 70$ MeV and throughout Regions 3 and 4 there is modest (20%) model dependence.⁽⁴²⁾ Above 140 MeV one finds gross disagreement between impulse approximation calculations such as those of Partovi and the data;⁽⁴³⁾ presumably due to the increasing importance of meson degrees of freedom. Calculations have been made,⁽⁴⁴⁾ and are in process, in which intermediate excitation of various nucleon resonances and rescattering terms are considered explicitly. Schematic calculations of this kind were first done in the 1950's but only recently has it been possible to include in a systematic way enough diagrams and the associated vertex form factors to feel some confidence in the results.⁽⁴⁵⁾ Comparisons with data are hampered, as at lower energies, by widespread disagreement between different sets of data.

Further, the role of meson currents in the photodisintegration should be compared with that in p-p and n-p bremsstrahlung at comparable photon energies.⁽⁴⁶⁾

Finally, of great interest is the recent experimental finding⁽⁴⁷⁾ of an enormous peak in the 90° proton polarization

following deuteron photodisintegration at photon energy $E_\gamma \sim 500$ MeV. The cross section is decreasing smoothly in the same energy region. A relativistic treatment of the polarization does not reproduce the peak. The polarization peak may be due to a nucleon-nucleon resonance. Also MEC contributions are being calculated.⁽⁴⁸⁾ With the sole exception of the cross section at low energies^(35,49) we have no firm grasp on any of the important features of deuteron photodisintegration. We are really not much closer to understanding this reaction than we were fifteen years ago. Consequently, a systematic non-fragmented effort toward the resolution of some of the outstanding problems in $d(\gamma, n)p$ is recommended.

8. Deuteron Electrodisintegration

Inelastic e-D scattering offers a wide variety of kinematical conditions that allow the investigation of MEC effects, the role of Isobar configurations (IC), relativistic contributions, interaction models and the nucleon electromagnetic structure.

Near the electrodisintegration threshold and for relatively low values of the momentum transfer, theoretical calculations^(50,51) that have incorporated MEC and IC show promising qualitative agreement with experimental data. MEC effects are prominent for $q^2 \geq 5.0 \text{ fm}^{-2}$ in this region⁽⁵²⁾ and should be studied in detail. At higher excitation energies the interaction effects are seen best for $q^2 < 5.0 \text{ fm}^{-2}$.^(52,53)

The dominant contribution to MEC comes from processes of pionic range, predominantly from the Pair process. This fact brings in focus again the question of the choice of the πN coupling. The possibility for differentiating between the pseudovector and the pseudoscalar coupling was examined⁽⁵⁴⁾ in elastic e-D scattering but was not resolved there. Intimately related to the choice of the πN coupling is the nature of the nucleonic form factor that must be utilized in evaluating the amplitude for the Pair graph, e.g. F_1^V vs. the axial vector form factor g_A . This latter is suggested by considerations of soft pion constraints.⁽⁵⁵⁾ The choice between the two form factors is not clear at the moment but it is hoped that the investigation of the electrodisintegration process in conjunction with pion electroproduction may yield answers to this problem.

The experimental electrodisintegration data at backward angles should be reconsidered. Plotting experimental points and theoretical results from Ref. (50) on a linear scale, we realize that there is sufficient quantitative discrepancy to make further examination of this cross section quite compelling.

It would be desirable to have experimental data for excitation energies away from the disintegration threshold but below the pion production threshold, and for low values of q^2 ($< 5.0 \text{ fm}^{-2}$) for the purpose of studying MEC effects. A coincidence experiment which counts simultaneously the outgoing electrons

and protons at, say, $q^2 \approx 1 \text{ fm}^{-2}$ would be very useful because the cross section is sensitive to MEC+ IC effects.⁽⁵⁶⁾ In contrast, for $q^2 \geq 12 \text{ fm}^{-2}$, we may focus on the sensitivity of the single-arm cross section on N-N interaction models.

We must remark that in examining MEC effects, the mesonic exchange corrections to the charge density should not be overlooked in spite of apparent violations of Siegert's theorem. We have already made this point in discussing $D(\gamma, n)p$ in the previous subsection.

At low excitation energy but very high momentum transfer, e.g. $q^2 \geq 100 \text{ fm}^{-2}$, we have the opportunity to check the very short range behavior of the nuclear wave function. In addition, the possibility that the data may be explained in terms of a quark-constituent picture of the Deuteron should be pursued. The data of Schütz et al⁽⁵⁷⁾ in this kinematical region has still to be analyzed successfully.

In the nucleon-constituent picture of the Deuteron, MEC effects are damped by non-static mesonic form factors of the nucleons, whose momentum dependence is the subject of on-going investigations.⁽⁵⁸⁾ The MEC corrections show considerable sensitivity to these form factors and hence, this kinematical region affords the opportunity for a phenomenological study of these quantities.

Further, the neutron charge form factor plays a significant role in this region and for this reason its determination is necessary if the data are to yield correct information.

Obviously the quasielastic peak in e-D scattering offers the appropriate conditions to study G_E^n provided MEC and IC effects are taken into account. A recent MEC estimate⁽⁸⁴⁾ is strongly modified when the IC effects, which are of opposite sign, are taken into account.⁽⁸⁵⁾ Hence at present it is not known whether without MEC, the analysis of this cross section will yield erroneous results for G_E^n .

Moreover, the behavior of the inelastic cross section just above the quasielastic peak in several electron-nuclear experiments⁽⁵⁴⁾ appears to suggest the presence of Δ isobars in the final nuclear state. Alternative mechanisms for the explanation of this behavior are certainly possible, in particular via appropriate modifications of the short range correlations in the nuclear wave function. It is most important to examine this question in the case of e-D inelastic scattering since the nuclear aspects of this interaction can be handled here so much more reliably than is possible in larger nuclei.

II THE THREE-BODY SYSTEM

The three-body system is an important source of information for the N-N interaction which is in some respects complementary to that obtained from the bound and unbound two-body system. Such important aspects as the short and intermediate range radial dependence of the N-N interaction, the strength of the short-range repulsion, the relative strength of the tensor to the central components and the locality or non-locality of the interaction can be investigated fruitfully via the correlated

examination of the three-nucleon and two-nucleon systems. It is fairly clear at this time that the two-nucleon system by itself cannot furnish an unambiguous description of several important aspects of the N-N interaction.

The binding energy E_T of ^3H is found to have a sensitive dependence on the strength of the tensor potential as measured by, say, the D-state probability in the Deuteron, P_D . In fact E_T increases as P_D decreases as shown in the table below.

(The Yamaguchi potential is a separable non-realistic interaction, but it serves our purposes here.)

Potential	P_D	E_T
Yamaguchi	7%	-8.0 MeV
"	4%	-9.0 MeV
"	2%	-10.0 MeV

Further, the properties of the three-body system are studied via the fully off-the-energy shell T-matrix elements which are sensitive to the locality or non-locality of the two-body interaction. While this aspect can be investigated also via, say, N-N bremsstrahlung, theoretical uncertainties have not allowed an unambiguous answer to this problem so far. On the other hand, the introduction of a short-range non-locality in the N-N interaction, which does not affect measurably the two-body data, can change the theoretical estimate of E_t by as much as 80%, the rms charge radius by 5% and shift the location of the first diffraction minimum in the ^3H charge form factor by 8 fm^{-2} . (61,62) This extensive sensitivity must certainly be exploited.

The short-range correlations in the NN interaction manifest themselves sensitively in the radial form of the three-body wave function, $\psi_T(r, \rho)$. The dependence on the relative coordinate $\underline{r}$ can be made clear via the correlation

$$g_T(r) = \int d\Omega_r \langle \psi_T | \delta(|\vec{r} - (\vec{r}_1 - \vec{r}_2)|) | \psi_T \rangle$$

It is found that this function looks very much like the Deuteron correlation near the origin.

$$g_D(r) = \int d\Omega_v \psi_D^*(r) \psi_D(r)$$

e.g. the two-body subsystems in the three-body system behave very much like the Deuteron at short inter-particle distances. This behavior will show up in the evaluation of MEC effects in the three-body system.

The correlation function with respect to ρ manifests itself in the charge form factors of the three-body system. The diffraction minimum in this form factor in the neighborhood of $k^2 = 11.6 \text{ fm}^{-2}$ in ^3He ,⁽⁶⁴⁾ is related to the short-range repulsion in the NN interaction.⁽⁶⁵⁾

Uncertainties in the ω -meson experimental coupling constant and in field-theoretical treatments of the exchange processes that mediate the short-range NN interaction⁽⁶⁶⁾ are responsible for a fair degree of ambiguity with regard to the short-range repulsion. Hence the additional phenomenology provided by the

three-body system is most helpful in untangling this complicated situation.

The NN interaction effect has been documented to play a significant role in shaping the electromagnetic structure and influencing the electromagnetic processes of the three-body system. It is significant that the large isovector exchange operators can contribute here. These contributions are of opposite sign in ${}^3\text{He}$ and ${}^3\text{H}$ and a careful study of relevant effects in these two nuclei may sort out MEC effects from nuclear structure effects.

Unlike the case of the Deuteron, the spin-isospin selection rules allow Δ isobar to appear singly in intermediate states of mesonic-exchange processes in the three-body system. They are found to contribute significantly to MEC corrections. Further, isobar configurations in the three-body wave function can make non-negligible contributions.

Leaving aside purely hadronic reactions such as n-D scattering and $D(n,p)2n$, and weak interaction considerations like the β -decay of ${}^3\text{H}$, we would like to discuss in the following the response of the three-body system to a probing electromagnetic field, e.g. we shall focus on the magnetic moments, elastic electron scattering charge and magnetic form factors and rms radii, radiative n-D capture, photodisintegration and electrodisintegration of the three-body systems.

(i) Magnetic moments

Defining the isovector and isoscalar magnetic moments by

$$\mu_s = \frac{1}{2} (\mu_{3_H} + \mu_{3_{He}}) \quad \mu_v = \frac{1}{2} (\mu_{3_H} - \mu_{3_{He}})$$

we find the theoretical expressions to be

$$\mu_s = \frac{1}{2} (\mu_p + \mu_n) (P_s + P_{s'}) + \frac{1}{2} P_D + \delta\mu_s = \mu_s^{(1)} + \delta\mu_s$$

$$\mu_v = \frac{1}{2} (\mu_p - \mu_n) (P_s - \frac{1}{3}P_{s'} + \frac{1}{3}P_D) - \frac{1}{6} P_D + \delta\mu_v = \mu_v^{(1)} + \delta\mu_v$$

while the experimental values are

$$\mu_s^{\text{exp}} = 0.426 \text{ n.m.} \quad \text{and} \quad \mu_v^{\text{exp}} = 2.553 \text{ n.m.}$$

P_s , $P_{s'}$, P_D are the admixture probabilities for the dominant states in the three-body system, and $\delta\mu_s$, $\delta\mu_v$ are corrections, most likely to arise from mesonic-exchange effects. We neglect the trivially small P-states and $T=3/2$ state admixture. In calculations with realistic potentials⁽⁶⁰⁾ the probabilities are found to be of the order

$$P_s \sim 90\%, \quad P_{s'} \sim 1-2\%, \quad P_D \sim 9\%$$

We see in the table below however that there is no reasonable combination of P_s , $P_{s'}$, P_D which would reproduce μ_s^{exp} , μ_v^{exp} without the help of $\delta\mu_s$, $\delta\mu_v$.

P_S	$P_{S'}$	P_D	$\mu_V^{(1)}$	$\mu_S^{(1)}$	$\delta\mu_V$	$\delta\mu_S$
%	%					
96	4	0	2.284	0.423	0.269	0.003
92.8	6	1.2	2.211	0.417	0.342	0.009
92	6	2	2.185	0.417	0.368	0.009
91	8	1	2.183	0.410	0.370	0.013
89	9	2	2.134	0.406	0.419	0.020

In Ref. 67 approximate three-body wavefunctions were used to calculate MEC effects of the order

$$\delta\mu_V = 0.193 \qquad \delta\mu_S = 0.009$$

In calculations with realistic wave functions⁽⁵⁷⁾ derived from variational calculations with admixtures $P_S = 90\%$, $P_{S'} = 1.6\%$, $P_D = 8.4\%$ so that

$$\mu_S^{(1)} = 0.408 \qquad \mu_V^{(1)} = 2.154$$

the MEC corrected magnetic moments were found to be

$$\mu_S = 0.407 \qquad \mu_V = 2.65$$

and in Ref. (68), with wavefunctions obtained from the Faddeev equations with $P_S = 90\%$, $P_{S'} = 1-2\%$, $P_D = 8-9\%$ it was found that $\delta\mu_V = 0.38$ n.m. and $\delta\mu_S = 0.010$ n.m.

One of the important features in these calculations is the large contribution from the N^* process via S-D matrix elements; hence we are furnished with the means to investigate tensor forces.

The 2π exchange processes have been examined in Ref. (69) but more work is needed in this area. Also, the contributions to the magnetic moments from the presence of nucleon isobars in the wavefunction have not been investigated yet. The tensor interaction enters here again as part of the transition potentials. Unfortunately the magnetic moment of the isobar itself is an experimentally unknown quantity while theoretically it is extremely model dependent. (71)

It is obvious from this review that while experimentally the measurements of $\mu(^3\text{H})$ and $\mu(^3\text{He})$ are excellent, the theoretical work remains incomplete. Nuclear structure ambiguities arising from both the NN interaction model and calculational techniques have to be eliminated. The magnetic moments can be sensitive probes in this area.

(b) Radiative n-d Capture

This reaction offers an excellent opportunity to study the small components of the three-body wave function, namely the S' and D-states, as well as MEC. Due to selection rules, only the mixed symmetry S' state enters into the Impulse approximation calculation. With a reasonable S' probability of 1-2%, the IA result is approximately one half [(0.29mb)] of the experimental value for the cross-section which is given as 0.65 ± 0.05 mb. (72) Calculated MEC with a wavefunction featuring $P_S = 90.4\%$, $P_{S'} = 1.6\%$, $P_D = 8.0\%$ raised the theoretical value of the cross-section to

.52 mb.⁽⁷³⁾ MEC depends heavily on P_D and on the radial dependence of the wavefunction. In spite of the dramatic improvement of the agreement between experiment and theory with MEC there is still room for investigating the three-body structure. Further, we know of only one experimental measurement⁽⁷²⁾ of the cross-section. It appears to us that a reliable repetition of the experiment is in order here.

(c) Elastic e - ^3He & e - ^3H form factors.

The charge and magnetic form factors of ^3He and ^3H have been the subject of several investigations recently.

The distinctive features of the form factors are their diffraction-like minima and maxima. A semi-log plot of the f.f. vs. k^2 shows the agreement between theory and experiment to be reasonably good for $k^2 \leq 3.0 \text{ fm}^2$. The evaluation of the rms radii however, from

$$F(k^2) \rightarrow 1 - \frac{1}{6} r_{\text{rms}}^2 k^2 \quad \text{as } k^2 \rightarrow 0$$

emphasizes the model dependence and other peculiarities of the wave function derived from calculational techniques and underlines again the importance of MEC.

The experimental charge rms radii are^(74,75)

$$r_{\text{ch}}(^3\text{He}) = 1.87 \pm 0.05 \text{ fm.}$$

$$r_{\text{ch}}(^3\text{H}) = 1.70 \pm 0.05 \text{ fm.}$$

In the impulse approximation, the difference between the two charge form factors is

$$F_{ch}({}^3\text{He}) - F_{ch}({}^3\text{H}) = -G_E^p(F_2 + \frac{1}{3}F_3) - G_E^n(3/2 F_1 - F_2 - 1/3F_3)$$

$$F_1 = \int d^3\rho d^3r e^{i2/3\mathbf{k}\cdot\boldsymbol{\rho}} U^2$$

$$F_2 = -3 \int d^3\rho d^3r e^{i2/3\mathbf{k}\cdot\boldsymbol{\rho}} U V_1$$

$$F_3 = -3 \int d^3\rho d^3r \rho e^{i2/3\mathbf{k}\cdot\boldsymbol{\rho}} U V_2$$

where U is the radial part of the S state and V_1, V_2 are the radial parts of the mixed symmetry S' state. We note that the corresponding difference in the charge rms. radii goes to zero for a fully space-symmetric three-body system (S -state only) since $G_E^n(k^2 = 0) = 0$. Hence, we note again the relevance of the S' state in calculating the quantity $\Delta r = r_{ch}({}^3\text{He}) - r_{ch}({}^3\text{H})$ which can be measured experimentally with good accuracy. We have then a sensitive means for assessing NN models and calculational techniques as they affect the three-body structure. Above $k^2 = 3 - 5\text{fm}^{-2}$, there is little hope that the data can be reproduced without MEC considerations.

The diffraction minimum in the f.f. is sensitively dependent on the short-range correlations in the wave function and on MEC contributions. The data near the minimum is understandably very poor for ${}^3\text{He}$, but is non-existent in the case of ${}^3\text{H}$. We cannot exaggerate the need for good data in this area.

The IA result seems to be strongly model dependent in the intermediate range for F_{ch} .⁽⁷⁶⁾ The model dependence of the MEC however cancels that of IA. The role of the short-range correlations in localizing correctly the diffraction minimum is not diminished however. No matter how the correlations are introduced e.g. via a repulsive core in the NN interaction or via three-body forces they improve the charge f.f. for ${}^3\text{He}$ and ${}^3\text{H}$.

The notion that some definite relation may be derived between the tri-nuclear binding energy and the localization of the diffraction minimum must be resisted until we are convinced that the short-range correlation and MEC cannot reproduce F_{ch} in a way that is compatible with the experimental binding energy.

The second diffraction maximum in the magnetic form factor is obviously a good place to do experimental measurements. In the case of the magnetic form factor, measurements in that region could lead to some insights into the pseudoscalar-pseudovector π -NN coupling and provide for limits on the axial mass.⁽⁵⁴⁾ This region is relatively free of ambiguities due to nucleon form factor models both electromagnetic and hadronic. Hence one can concentrate on meson currents, the π -NN coupling and wavefunction models. In the case of the charge form factor, the secondary maximum is again an excellent place to investigate meson currents and wavefunction models, in particular the latter,

in view of the fact that MEC corrections are dominated apparently by one graph^(64,76) (Pair) which is the one that is best understood. Further, the pseudoscalar-pseudovector ambiguity is not present here in an important way.⁽⁷³⁾

Determination of meson currents from the work at the secondary maximum will allow us to move on to higher $q^2 (> 25\text{fm}^{-2})$, focus on meson-nucleon hadronic form factors and particularly on nucleon electromagnetic form factors.

Further, Fourier-transforming the charge form factor, we get the charge distribution $\rho(r)$ in the three-body system. Unfolding the nucleon form factors, we get a central depression in $\rho(r)$ which is the combined effect of meson currents and short-range correlations.^(65,75) An accurate experimental description of this central depression will advance considerably our knowledge on short-range correlations. Meson currents are dominated by the well-understood Pair graph.

(d) Photodisintegration

The recent three-body experimental data for the two-body photodisintegration of ${}^3\text{He}$ is quite good. The theoretical analysis of this and other photodisintegration data however is

making slow progress due to the difficulty in procuring the three-body scattering states that are necessary for the calculation of the final state effect on the cross section. This final-state effect has been shown to be quite important.⁽⁷⁷⁾ Recent theoretical efforts⁽⁷⁸⁾ have been fruitful.

The low energy photodisintegration data is dominated by E1 transitions. As in the case of $D(\gamma, n)p$, a basic question is the extent to which MEC contribute here. Clearly, however, calculational techniques and NN models can be investigated via these processes.

Further the utilization of sum rules can be quite profitable. For example

$$J = \int_0^{\infty} \sigma_{E1}(E_{\gamma}) \frac{dE_{\gamma}}{E_{\gamma}} = \frac{4}{3} \pi^2 \alpha (2/3 R)^2$$

where α is the fine structure constant; for a spatially symmetric three-body system R is the rms radius of the charge distribution. Hence it may be compared with the results from electron scattering. The experimental data yield

$$R \approx 1.81 \pm 0.06 \text{ fm}$$

for a symmetric ${}^3\text{He}$ nucleus, which compares very well with $r_{\text{rms}} \approx 1.87 \pm 0.05 \text{ fm}$ found from $e-{}^3\text{He}$ scattering.

Incompatibilities among different sets of experimental data, in particular for $E_\gamma < 20.0$ MeV should be eliminated. As we make progress in securing computationally accurate three-body scattering wavefunctions, we expect to be able to derive information on NN models.

(e) Electrodisintegration:

etc.

Within the limitations of the Gibson-Lehman formalism,⁽⁷⁸⁾ a recent theoretical analysis of two-body threshold electrodisintegration⁽⁷⁹⁾ seems to have succeeded in explaining quite well $p + D$ breakup data at threshold. The rather unrealistic separable Yamaguchi interaction was employed in this work. Refinements will be obviously necessary in terms of more realistic NN interactions and MEC effects on the electromagnetic operators.

In principle we should be able to examine via these breakup processes other elements of concern such as nucleon form factors, relativistic effects, and short-range correlations. Special attention should be given to this last point in calculations that use separable approximations to realistic NN potentials. It has been observed that short-range correlations that are inherent in the realistic potential, are not always carried over in the separable approximation.

III Electro- and Photoproduction of Pions

There is a wealth of information to be gained from an analysis of electro- and photoproduction data. (80,81)

We are concerned here with reactions of the type

$$\gamma + D \rightarrow \pi^0 + D \quad \text{and} \quad \gamma + {}^3\text{He} \rightarrow \pi^0 + {}^3\text{He}$$

$$\gamma + D \rightarrow \pi^+ + n + n$$

$$\gamma + D \rightarrow \pi^0 + p + n$$

$$e + D \rightarrow e' + D + \pi^0 \quad \text{and} \quad e + {}^3\text{He} \rightarrow e' + {}^3\text{He} + \pi^0$$

$$e + D \rightarrow e' + \pi + n + n$$

$$e + D \rightarrow e' + \pi^0 + p + n$$

The cross section for π -meson production from a single nucleon is much larger for charged pions than for neutral pions. Nevertheless the π^0 production from nuclei in the near threshold region allows the probing of aspects of the production mechanism that are relevant to our understanding of exchange theories. The mechanism for $\pi^\pm$ production on the other hand seems to be fairly well understood, so that the study of the corresponding cross sections allows us to focus on other elements, such as the short range behavior of the N-N potential, the structure of the Deuteron (in terms of its D-state admixture) and ${}^3\text{He}$, and final state interaction effects, in particular the n-n scattering length.

With few exceptions, the production process has been treated in the impulse approximation (IA) only, where the elementary production amplitude

$$\gamma + N \rightarrow \pi + N'$$

is summed over all the nucleons in the nucleus. There are obviously corrections to this IA amplitude that arise from (a) transformations from the πN to the π -nucleus C.M. system (b) the Fermi motion of the nucleons in the nuclear system (c) the off-shell extrapolation of the elementary $\gamma + N \rightarrow \pi + N'$ amplitude. From the practical point of view, it is best if these effects are studied first in few-nucleon systems.

With respect to the π^0 electroproduction at threshold, there is some evidence⁽⁸²⁾ that a two-step (rescattering) process competes with the normal production from a single nucleon which, unlike that for $\pi^\pm$ production, is momentum dependent. This is an important element whose study will have some bearing on our understanding of exchange theories.

Further, the question of the choice of the πN coupling can be addressed again in terms of the electroproduction processes.⁽⁸³⁾ Of equal importance is the information that we may gain on the π -nucleus interaction which obviously contributes to the final state interaction effects in the photo- and electro-production processes.

Appendix A

Partial list of Canonical forms
for Meson-Exchange Currents

A. Charge Density

$\vec{k}$ = photon momentum

$\vec{Q} = \vec{p}_2' - \vec{p}_2$ $\approx$ momentum carried by the exchanged meson

$\vec{p}_2, \vec{p}_2'$ = initial and final nucleon momenta in the non-radiative leg of the two-body graph

1. $N\bar{N}$, π -exchange

$$\rho(k, Q) = i \left(\frac{G}{2M}\right)^2 \frac{e}{2M} [\vec{\tau}_1 \cdot \vec{\tau}_2 F_M^S(k^2) + \tau_{2z} F_M^V(k^2)] \\ \otimes \vec{\sigma}_1 \cdot \vec{k} \vec{\sigma}_2 \cdot \vec{Q} \cdot \frac{|F_{\pi NN}(Q^2)|^2}{Q^2 + \mu_\pi^2} + 1 \leftrightarrow 2$$

2. $N\bar{N}$, ρ -exchange

$$\rho(k, Q) = \frac{ie}{(2M)^3} g_{\rho NN}^2 (1 + Kv) \vec{\tau}_1 \cdot \vec{\tau}_2 F_M^S(k^2) [\vec{Q} \cdot \vec{k} \\ \text{isosc;} + (\vec{\sigma}_2 \times \vec{Q}) \cdot (\vec{\sigma}_1 \times \vec{k})] \frac{|F_{\rho NN}(Q^2)|^2}{Q^2 + \mu_\rho^2} + 1 \leftrightarrow 2 \\ \rho(k, Q) = \frac{ie}{(2M)^3} g_{\rho NN}^2 (1 + Kv) F_M^V(k^2) [\tau_{2z} (\vec{Q} \cdot \vec{k} + \\ (\vec{\sigma}_2 \times \vec{Q}) \cdot (\vec{\sigma}_1 \times \vec{k}) - (\vec{\tau}_1 \times \vec{\tau}_2)_z \vec{k} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \times \vec{Q}] \\ \frac{|F_{\rho NN}(Q^2)|^2}{Q^2 + \mu_\rho^2} + 1 \leftrightarrow 2$$

Appendix A (Cont'd)

3. $N\bar{N}$, ω -exchange

We obtain these results from those for $N\bar{N}$, ρ -exchange via substitutions

$$g_{\rho NN} \rightarrow g_{\omega NN} \quad F_{\rho NN}(Q^2) \rightarrow F_{\omega NN}(Q^2)$$

$$\kappa_V \rightarrow \kappa_S$$

$$M_\rho \rightarrow M_\omega$$

Further, in the isoscalar part

$$\vec{\tau}_1 \cdot \vec{\tau}_2 \rightarrow 1$$

$$(\tau_1 \times \tau_2)_z \rightarrow 0$$

$$g_{\rho NN} = 2.56, \quad g_{\omega NN} = 7.86$$

τ_{1z} and τ_{2z} are interchanged

4. $\pi\rho\gamma$

$$\rho(k, Q) = ie \frac{g_{\rho NN} g_{\rho\pi\gamma}(k^2)}{4 M^2 m_\rho} (1 + \kappa_V) (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$\otimes [Q^2 \vec{\sigma}_1 \cdot \vec{Q}' \vec{\sigma}_2 \cdot \vec{Q}' - \vec{Q} \cdot \vec{Q}' \vec{\sigma}_1 \cdot \vec{Q} \vec{\sigma}_2 \cdot \vec{Q}'] \frac{F_{\pi NN}(Q'^2) F_{\pi NN}(Q^2) F_{\rho NN}(Q^2)}{(Q^2 + \mu_\rho^2)(Q'^2 + \mu_\pi^2)}$$

$$g_{\rho\pi\gamma}(k^2) = g_{\rho\pi\gamma}(0) F_{\pi\rho\gamma}(k^2); \quad + \quad 1 \neq 2$$

5. $\pi\omega\gamma$

obtain results from those for $\pi\rho\gamma$ via $g_{\rho NN} \rightarrow g_{\omega NN}$, $m_\rho \rightarrow m_\omega$,

$$\kappa_V \rightarrow \kappa_S, (\vec{\tau}_1 \cdot \vec{\tau}_2) \rightarrow \tau_{2z}$$

$$g_{\pi\omega\gamma} = g_{\pi\omega\gamma}(0) F_{\pi\omega\gamma}(k^2)$$

$$g_{\rho\pi\gamma}(0) = 0.39$$

$$g_{\pi\omega\gamma}(0) = 1.17$$

$$F_{\pi\rho\gamma}(k^2) = \frac{1}{1 + \frac{k^2}{m_\rho^2}} \approx F_{\pi\omega\gamma}(k^2)$$

Appendix A (Cont'd)

6. Recoil, π -exchange

$$\rho(k, Q) = \frac{ie}{2} \frac{(G)^2}{2M} [\vec{\tau}_1 \cdot \vec{\tau}_2 G_E^S(k^2) + \tau_{2z} G_E^V(k^2)] \vec{\sigma}_1 \cdot \vec{Q} \vec{\sigma}_2 \cdot \vec{Q}$$

$$\otimes \frac{|F_{\pi NN}(Q^2)|^2}{(Q^2 + \mu_\pi^2)^{3/2}} + 1 \neq 2$$

7. N^* process, no contribution (purely non-local)8. π -current, no contribution in the static limit.B. Current Density1. $\bar{N}\bar{N}, \pi$ - exchange

$$J^{\text{isov}}(k, r_i) = -ef^2_{\pi NN} F_1^V(k^2) (\vec{\tau}_1 \times \vec{\tau}_2)_z (\vec{\sigma}_1 e^{ikr_1} \vec{\sigma}_2 \cdot \hat{r} + \vec{\sigma}_2 e^{ik \cdot r_2} \vec{\sigma}_1 \cdot \hat{r}) \phi_1(x_\pi)$$

$$\text{and } J^{\text{isosc}}(k, r_i) = -\frac{ie}{(2M)^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \mu_\pi f^2_{\pi NN} F_M^S[(\vec{k} \times \vec{\sigma}_1)$$

$$\otimes \frac{\phi_1(x_\pi)}{x_\pi} - \vec{\sigma}_1 \cdot \hat{r} (\vec{k} \times \hat{r}) \phi_2(x_\pi) e^{ik \cdot r_2} + ((\vec{k} \times \vec{\sigma}_2) \phi_1(x_\pi) / x_\pi$$

$$- \sigma_2 \cdot \hat{r} (\vec{k} \times \hat{r}) \phi_2(x_\pi) e^{i\vec{k} \cdot \vec{r}_1}]$$

$$\phi_1(x_\pi) = \left(\frac{1+x_\pi}{x_\pi}\right) e^{-x/x_\pi}; \quad x = \mu_\pi r; \quad r = r_1 - r_2$$

ϕ_2 is defined below in number 3.

$F_1^V(k^2)$ = isovector Dirac form factor.

2. π -Current

$$J(k, r_i) = e \frac{f_{\pi NN}^2}{\mu_\pi^2} F_\pi(k^2) (\tau_1 \times \tau_2)_z \vec{\sigma}_1 \cdot \vec{\nabla}_1 \sigma_2 \nabla_2 \int_{-1/2}^{1/2} dt \frac{x}{x} e^{-x} e^{ik\xi}$$

$$\xi = \vec{R} - t\vec{r}; \quad \vec{R} = 1/2 (\vec{r}_1 + \vec{r}_2); \quad \vec{x} = L_\pi \vec{r}; \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

$$L_\pi = \mu_\pi \left[1 - \frac{k_\lambda^2}{4\mu_\pi^2} (1 - 4t^2) \right]^{1/2}$$

k_λ = four momentum transfer.

Appendix A (Cont'd)

$$\begin{aligned}
3. \quad N^* \quad J(k, r_1) = & -A \{ -4h_2 [\phi_2(x_\pi) \hat{r} \cdot (\vec{\sigma}_2 \tau_{2z} e^{ikr_2} + \vec{\sigma}_1 \tau_{1z} e^{ikr_1}) \vec{k} \times \vec{r} \\
& + \phi_1(x_\pi) (\vec{\sigma}_2 \tau_{2z} e^{ik \cdot r_2} + \vec{\sigma}_1 \tau_{1z} e^{ik \cdot r_1}) \times \vec{k}] \\
& + h_1 (\vec{\tau}_1 \times \vec{\tau}_2)_z [(\vec{\sigma}_1 \cdot \vec{r} \vec{\sigma}_2 \times \vec{r} \times \vec{k} e^{ik \cdot r_2} - \vec{\sigma}_2 \cdot \vec{r} \vec{\sigma}_1 \times \vec{r} \times \vec{k} e^{ik \cdot r_1}) \\
& \phi_2(x) + (\vec{\sigma}_1 \times \vec{\sigma}_2) \times \vec{k} (e^{ik \cdot r_1} + e^{ik \cdot r_2}) \phi_1(x)] \}
\end{aligned}$$

$$A = \frac{i}{6} F_M^V(k^2) \frac{e}{2M}$$

$$\hat{r} = \frac{\vec{r}}{r} \quad h_1 = 0.074, \quad h_2 = 0.0658$$

$$\phi_2(x_\pi) = (1 + \frac{3}{x_\pi} + \frac{3}{x_\pi^2}) \frac{e^{-x_\pi}}{x_\pi}$$

4. Recoil graph, π -exchange

$$\begin{aligned}
J(k, r_1) = & -\frac{1}{\pi} \frac{ie}{2M} \frac{f_\pi^2}{\mu_\pi} \frac{NN}{2} \{ \tau_1 \cdot \tau_2 F_M^S(k^2) [\vec{\nabla}_r \times \vec{k} \vec{\sigma}_2 \cdot \\
& \vec{\nabla}_r e^{ik \cdot r_1} + \vec{\nabla}_r \times \vec{k} \vec{\sigma}_1 \cdot \vec{\nabla}_r e^{ik \cdot r_2}] K_0(x_\pi) \\
& + F_M^V(k^2) [\tau_{2z} \vec{\nabla}_r \times \vec{k} \vec{\sigma}_2 \cdot \vec{\nabla}_r e^{ik \cdot r_1} + \tau_{1z} \vec{\nabla}_r \times \vec{k} \vec{\sigma}_1 \cdot \vec{\nabla}_r e^{ik \cdot r_2}) \\
& + (\vec{\tau}_1 \times \vec{\tau}_2)_z (\vec{\sigma}_1 \times \vec{\nabla}_r \times \vec{k}) \vec{\sigma}_2 \cdot \vec{\nabla}_r e^{ik \cdot r_1} - (\vec{\sigma}_2 \times \vec{\nabla}_r \times \vec{k}) \vec{\sigma}_1 \cdot \vec{\nabla}_r \\
& e^{ik \cdot r_2}] K_0(x_\pi) \}
\end{aligned}$$

$K_0(x_\pi)$ = Bessel function of the second kind

The relations $\frac{d}{dx} K_0(x) = -K_1(x)$

and $\frac{dK_1}{dx}(x) = \frac{1}{x} K_1(x) - K_2(x)$ are useful here.

Appendix B

MEC Corrections to μ_D = Deuteron magnetic moment

$$1. \quad \bar{N}\bar{N}, \pi \quad \delta\mu_D = \mu_\pi (1+K_S) \frac{f^2_{\pi NN}}{M} \int dr [U^2 \phi_0(x) + \sqrt{2} UW \phi_2(x) + W^2 \otimes (\phi_2(x) - \frac{1}{2} \phi_0(x))] \quad K_S = -0.12$$

2. π current, no contribution3. Nucleon Recoil, π -exchange

$$\delta\mu_D = -f^2_{\pi NN} / \pi (1+K_S) \left[\int dr (2U^2 - W^2) (K_0(x) - \frac{K_1(x)}{x}) \right. \\ \left. + 2\sqrt{2} \int dr (UW + \frac{W^2}{\sqrt{2}}) K_2(x) \right]$$

$$f^2_{\pi NN} = 0.08$$

 $K_i(x)$ = Bessel functions of the 2nd kind.

$$x = \mu_\pi r$$

4. Renormalization (Chemtob-Rho prescription)

$$\delta\mu_D = \frac{-f^2_{\pi NN}}{\pi} \left[2 \int dr (U^2 + W^2) (K_0(x) - \frac{K(x)}{x}) \right. \\ \left. + \frac{4}{3} \int dr (2\sqrt{2} UW - W^2) K_2(x) \right] \mu_D^I \quad \mu_D^I = \text{Impulse approximation value for the magnetic moment}$$

5. $\pi\rho\gamma$

$$\delta\mu_D = \frac{g_{\rho NN} g_{\rho\pi\gamma}}{4\pi} G \left(\frac{m_\pi}{m_\rho} \right)^3 \left[\int dr (2U^2 - W^2) \phi_0(x_\pi) \right. \\ \left. + 2\sqrt{2} \int dr (UW + \frac{1}{\sqrt{2}} W^2) \phi_2(x_\pi) \right] \\ \phi_0(x) = \frac{e^{-x}}{x} ; \quad \phi_2(x) = (1 + \frac{3}{x} + \frac{3}{x^2}) \phi_0(x)$$

6. $\bar{N}\bar{N}, \rho$

$$\delta\mu_D = \frac{2M}{m_\rho} f^2_{\rho NN} (1+K_V) \int dr x_\rho \phi_1(x) (U^2 - W^2 - \frac{1}{\sqrt{2}} UW) \\ f^2_{\rho NN} = \frac{1}{4\pi} \left(\frac{g_{\rho NN} \mu_\rho}{2M} \right)^2 ; \quad x_\rho = \mu_\rho r \quad \phi_1(x) = \left(\frac{1+x}{x} \right) e^{-x/x}$$

Appendix B (Cont'd)

7. $N\bar{N}, \omega$

$$\delta\mu_D = -\frac{2}{3} \frac{M}{m_\omega} f_{\omega NN}^2 (1+K_S) \int dx_\omega \phi_1(x_\omega) (U^2 - W^2 - \frac{1}{\sqrt{2}} UW)$$

$$x_\omega = \mu_\omega r$$

8. $\pi\omega\gamma$, no contribution9. Recoil, ρ -exchange

$$\begin{aligned} \delta\mu_D = \frac{3}{2} \left(\frac{g_{\rho NN}}{4\pi} \right)^2 4(1+2\kappa_S) \int dx_\rho K_0(x_\rho) (2U^2 - W^2) \\ - \frac{1}{3} \int dx_\rho K_1(x_\rho) [4U^2 - 4W^2 - 2\sqrt{2}UW] \end{aligned}$$

10. Recoil, ω -exchange

$$\begin{aligned} \delta\mu_D = -\frac{1}{2} \left(\frac{g_{\omega NN}}{4\pi} \right)^2 \left[4(1+2\kappa_S) \int dx_\omega K_0(x_\omega) (2U^2 - W^2) \right. \\ \left. - \frac{1}{3} \int dx_\omega K_1(x_\omega) (4U^2 - 4W^2 - 2\sqrt{2}UW) \right] \end{aligned}$$

Appendix B (Cont'd)

MEC Corrections to Q_D = Deuteron Quadrupole Moment1. $N\bar{N}$, π -exchange

$$\delta_{Q_D} = \frac{f_{\pi NN}^2}{2M} (1+\kappa_S) \int r dr \phi_1(x_\pi) [4U^2 - 2\sqrt{2}UW - 2/5W^2]$$

2. $N\bar{N}$, ρ -exchange

$$\delta_{Q_D} = -2 \frac{f_{\rho NN}^2}{2M} (1+\kappa_V) (1+\kappa_S) \int r dr \phi_1(x_\rho) (2U^2 + W^2 - \frac{34}{5\sqrt{2}} UW)$$

3. $N\bar{N}$, ω -exchange

$$\delta_{Q_D} = 2/3 \frac{f_{\omega NN}^2}{2M} (1+\kappa_S)^2 \int r dr \phi_1(x_\omega) (2U^2 + W^2 - \frac{34}{5\sqrt{2}} UW)$$

4. $\pi\rho\gamma$

$$\delta_{Q_D} = -\frac{3}{4\pi} \frac{g_{\rho NN} g_{\rho\pi\gamma} G}{M^2 m_\rho} (1+\kappa_V) \int_{-1/2}^{1/2} v dv \left[\frac{4}{3} \int dr e^{-Lr} / r (2U^2 + \frac{1}{10}W^2 - \frac{1}{2\sqrt{2}} UW) \right.$$

$$\left. - (1/5)L^2 \int dr e^{-Lr} (U^2 - W^2 - \frac{1}{\sqrt{2}} UW) \right]$$

$$L = \left[(v + \frac{1}{2}) (\mu_\rho^2 - \mu_\pi^2) + \mu_\pi^2 \right]^{1/2}$$

5. Recoil, π -exchange

$$\delta_{Q_D} = \frac{\sqrt{2}}{5} \mu_\pi / \mu_\pi f_{\pi NN}^2 \int r^2 dr \left[(UW - \frac{1}{2\sqrt{2}} W^2) (K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right.$$

$$\left. + \sqrt{2} (U^2 - \sqrt{2}UW + (3/2)W^2) K_2(x_\pi) \right]$$

Appendix B (Cont'd)

6. Recoil, ρ -exchange

$$\delta_{Q_D} = \frac{3\sqrt{2}}{20\pi^2} f_{\rho NN}^2 \int r^2 dr K_0(x_\rho) \left[UW - \frac{1}{2\sqrt{2}} W^2 \right]$$

$$f_{\rho NN} = \frac{1}{4\pi} \left(\frac{g_{\rho NN} \mu_\rho}{2M} \right)^2$$

7. Recoil, ω -exchange

$$\delta_{Q_D} = \frac{\sqrt{2}}{20\pi^2} f_{\omega NN}^2 \int r^2 dr K_0(x_\omega) \left[UW - \frac{1}{2\sqrt{2}} W^2 \right]$$

$$f_{\omega NN}^2 = \frac{1}{4\pi} \frac{(g_{\omega NN} \mu_\omega)^2}{2M}$$

$K_0(x)$ = Bessel function of the second kind

8. Renormalization, π -exchange (prescription of Chemtob-Rho)

$$\delta_{Q_D} = -2 \frac{f_{\pi NN}^2}{\pi} \left[\int dr (U^2 + W^2) (K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi}) \right. \\ \left. + \frac{2}{3} \int dr K_2(x_\pi) (2\sqrt{2} UW - W^2) \right] Q_D^I$$

References

1. M. Chemtob, "Meson Theory of Nucleon Vector and Axial Vector Exchange Currents" chapter in Mesons in Nuclei edited by Rho and Wilkinson.
2. E. Hadjimichael, Effets Mesoniques dans les Voyeaux, Saclay 1975.
3. D. O. Riska, "Exchange Currents in Electron Scattering" chapter in Mesons in Nuclei, edited by Rho and Wilkinson.
4. S. Adler and Dothan, Phys. Rev. 157 (1966) 1267.
5. M. Chemtob and M. Rho, Nucl. Phys. A163 (1971) 1.
6. J. L. Friar, Phys. Lett. B59 (1975) 145, Phys. Rev. C15 (1977) 1783 and several other papers.
7. M. H. Partovi and E. L. Lomon, Phys. Rev. D2 (1970) 1999.
8. E. Hadjimichael, to be published.
9. M. Gari and H. Hyuga, Nucl. Phys. A264 (1976) 409; there are several typographical and some non-trivial errors in this paper.
10. W. E. Quinn et al, Phys. Rev. 112 (1958) 1929; R. F. Code and N. F. Ramsey, Phys. Rev. A4 (1971) 1954.
11. R. V. Reid and M. L. Vaida, Phys. Rev. Lett. 29 (1972) 494.
12. E. Hadjimichael, unpublished.
13. A. D. Jackson, A. Lande, D. O. Riska, Phys. Lett. 55B (1975) 23; there are discrepancies between this paper and Ref. 9.
14. W. Fabian, H. Arenhövel and H. G. Miller, Z. Phys. 271 (1974) 93.
15. P. Haapakoski, M. Saarela, Phys. Lett. 53B (1974) 333.
16. L. Koester et al, Phys. Rev. Lett. 36 (1976) 1021.
17. R. W. Berard et al, Phys. Lett. 47B (1973) 355.
18. D. N. L. Sprung and K. S. Rao, Phys. Lett. 53B (1975) 397.
19. F. Coester and A. Ostebee, Phys. Rev. C11 (1975) 1836.
20. J. L. Friar, Ann. Phys. 81 (1973) 332.

21. E. Lomon, private communication.
22. E. Moniz, private communication.
23. D. Benaksas et al, Phys. Rev. 148 (1966) 1327, S. Galster et al, Nucl. Phys. B32 (1971) 221.
24. D. Saylor and W. Turchinets, private communication.
25. F. Iachello et al, Phys. Lett. 43B (1973) 191.
26. K. M. Hanson et al, Phys. Rev. D8 (1973) 753.
27. Elias et al, Phys. Rev. 177 (1969) 2075.
28. R. G. Arnold et al, Phys. Rev. Lett. 35 (1975) 776.
29. R. E. Rand et al, Phys. Rev. D8 (1973) 3229.
30. F. Martin et al, Phys. Rev. Lett. 38 (1977) 1320.
31. W. Bartel et al, Nuc. Phys. B58 (1973) 429.
32. S. Brodsky and B. T. Chertok, Phys. Rev. Lett. 37 (1976) 269.
33. M. Chemtob et al, Phys. Rev. C10 (1974) 344.
34. G. West, Phys. Rev. Lett. 37 (1976) 1454.
35. F. Partovi, Ann. Phys. (N.Y.) 27 (1964) 79.
36. E. Hadjimichael, Phys. Lett. 46B (1973) 147, D. O. Riska and G. E. Brown, Phys. Lett. 38E (1972) 193.
37. W. Turchinets, Proceedings, Intern. Conf. on Low and Intermediate Energy Electron. Inter., Dubna, 1967; L. Drooks, thesis, Yale University, 1976.
38. Hughes et al, Nucl. Phys. A267 (1976) 392.
39. H. Arenhovel and W. Fabian, Nucl. Phys. 282 (1977) 397.
40. E. Lomon, Phys. Lett. 68B (1977) 419.
41. J. E. Baglin et al, Nucl. Phys. A201 (1973) 593; Y. M. Shin, Proc. Int'l. Conf. on Photonicuclear Reactions Asilomar VI (1973) 373; Jahrens et al, Phys. Lett. 52B (1974), 49.
42. E. Lomon and H. Feshbach, Ann. Phys. (N.Y.) (1963) 94.
43. P. Dougan et al, Z. Physik A276 (1976) 55, A280 (1977) 341.

44. H. Hasselman, DESY-68/38, unpublished report.
45. J. M. Laget, contribution to VII International Conf. on High Energy Phys. and Nuclear Structure (1977).
46. G. Bohannon, MIT preprint, 1977.
47. T. Kamae et al, Phys. Rev. Lett. 38 (1977) 469; also (private communication) new data has confirmed the 90° result and extended it to 70°
48. W. Fabian and W. Buck, (Stony Brook), J. M. Laget (Saclay) work in progress.
49. J. Abrams et al, Phys. Lett. 52B (1974) 49.
50. J. Hochert et al, Nucl. Phys. 217 (1973) 14.
51. J. A. Lock and L. L. Foldy, Phys. Lett. 51B (1974) 212 and Ann. Phys. (N.Y.) 93 (1975) 276.
52. W. Fabian and H. Arenhovel, Nuc. Phys. A258 (1976) 461.
53. G. G. Simon et al, Phys. Rev. Lett. 37 (1976) 739.
54. E. Hadjimichael, Phys. Lett. 63B (1976) 161.
55. M. Chemtob in "Mesons on Nuclei" ed D. H. Wilkinson and M. Rho, to be published.
56. W. Fabian, private communication.
57. W. P. Shütz et al, Phys. Rev. Lett. 38, 259 (1977).
58. A. D. Jackson et al, Nucl. Phys. A282 (1977) 404; W. T. Nutt and C. M. Shakin, Phys. Lett. 69B (1977) 290.
59. G. Do Dang, Phys. Lett. 69B (1977) 425.
60. E. Harms, "Selected topics in Phys., Astrophys. and Biophys." D. Reidel Publishing Co. Dordrecht-Holland (1973).
61. E. Hadjimichael and A. D. Jackson, Nucl. Phys. A180 (1972) 217.
62. M. I. Haffel and M. Tabakin, Phys. Rev. C3 (1971) 921.
63. E. Hadjimichael, S. N. Yang and G. E. Brown, Phys. Lett. 39B (1972) 594.
64. E. Hadjimichael, Nucl. Phys. A294 (1978) 513.
65. A. Barroso and E. Hadjimichael, Nucl. Phys. A238 (1975) 422.

66. A. D. Jackson, D. O. Riska and B. Verwest, Nucl. Phys. A249 (1975), 397.
67. M. Chemtob and M. Rho, Nucl. Phys. A163 (1970) 1.
68. E. P. Harper et al, Phys. Lett. 43B (1972) 533.
69. M. Ichimura et al, Nucl. Phys. A196 (1972) 17.
70. R. L. Burman, Proc. Int. Conf. on High Energy Phys. and Nuclear Structure, Los Alamos (1975) ed Nagle D.
71. M. A. Bey et al, Phys. Rev. Lett. 13 (1964) 514, Y. A. Chao and R. K. P. Zin, Nuovo Cimento 19A (1974) 651.
72. F. T. Turney and N. T. Metz, unpublished.
73. E. Hadjimichael, Phys. Rev. Lett. 31 (1973) 183.
74. H. Collard et al, Phys. Rev. B 138 (1964) 802.
75. J. S. McCarthy et al, Phys. Rev. C15 (1977) 1316.
76. M. I. Haftel and W. M. Kloeet, Phys. Rev. C15 (1977) 404.
77. A. C. Phillips, Nucl. Phys. A184 (1972) 337; I. M. Banboursy Phillips, Phys. Rev. C1 (1970) 165.
78. B. F. Gibson and D. R. Lehman, Phys. Rev. C11 (1975) 29; C13 (1976) 477.
79. C. R. Heimbach et al, Phys. Rev. C16 (1977) 2135.
80. C. Tzara, Nucl. Phys. A256, 381 (1976).
81. E. C. Booth et al, Phys. Lett. 66B, 236 (1977); J. V. Noble, Phys. Lett. 67B, 39 (1977); G. Audit et al, Phys. Rev. C16, 1567 (1977).
82. J. H. Koch and R. W. Woloshyn, Phys. Rev. 16C, 1968 (1977); E. Hadjimichael and J. Vergerdos, private communication; P. Bosted and J. M. Laget, Nuc. Phys. A296 (1978) 413.
83. N. Dombey and B. J. Reid, Nucl. Phys. B60, 65 (1973); M. Gluck, Phys. Rev. Lett. 28, 1486 (1972).
84. G. Kingma and A.E.L. Dieperink - preprint Gnönigen (1978).
85. H. Arenhövel et al - preprint Mainz (1978).

Table I Elastic e-d Scattering
Model and form-factor dependence at $q^2 = 5\text{fm}^{-2}$

Nuclear Model	D State %	ff Model	$A(q^2)$, IA	$A(q^2)$ IA + MEC (est) (a)	$B(q^2)$, IA	$P(q^2)$, IA	$P(q^2)$, IA+MEC (est.) (a)
F.L. 1	4.57	I ^(b)	0.0202	0.0187	1.07×10^{-3}	0.668	0.79
		II ^(c)	0.0203	0.0188	$1.29 \times "$	"	"
F.L. 5	5.20	I	0.0202	0.0192	$1.07 \times "$	0.693	0.77
		II	0.0203	0.0193	$1.28 \times "$	"	"
F.L. 15	7.55	I	0.0211	0.0213	$1.08 \times "$	0.735	0.72
		II	0.0211	0.0213	$1.27 \times "$	"	"
R.S.C.	6.47	I	0.0207	0.0200	$1.06 \times "$	0.722	0.78
		II	0.0208	0.0201	$1.26 \times "$	"	"
H.J.	6.97	I	0.0203	0.0200	$1.05 \times "$	0.738	0.77
		II	0.0204	0.0201	$1.24 \times "$	"	"
Expt			(d) .0180 ± .0011		(d) .0011 ± .00015		

(a) MEC estimated from E. Moniz's parameterization⁽²²⁾

$$\Delta G_0 = 2 \frac{q^2}{q_0^2} \Delta F \quad \Delta G_2 = \frac{q^2}{\sqrt{18}} \Delta F_Q$$

With $\Delta F_Q = \Delta Q \exp(-q^2/q^2)$, ΔQ from experiment and $q_0^2 = 15\text{m}_\pi^2$

(b) Form-factors chosen to fit neutron and proton data independently,

G_E^p and G_M^p are from IJL⁽²⁵⁾, G_M^n is the scaled dipole fit and G_E^n is taken from the best fit of S. Galster et al⁽²³⁾

(c) As in (b) except that G_M^n is replaced by the IJL model

(d) Benaksas et al, Ref. 23

Table II. Impulse Approximation Predictions with
"Best Fit" Form Factors I.

Nuclear Model	A(15fm ⁻²)	A(26fm ⁻²)	B(15fm ⁻²)	B(26fm ⁻²)	P(15fm ⁻²)	P(26fm ⁻²)
FL 1	38.5x10 ⁻⁵	4.06x10 ⁻⁵	48.0x10 ⁻⁶	0.952x10 ⁻⁶	1.36	.022
FL 5	42.7x10 ⁻⁵	5.12x10 ⁻⁵	51.4x10 ⁻⁶	1.54x10 ⁻⁶	1.33	.057
FL 15	55.3x10 ⁻⁵	7.68x10 ⁻⁵	67.0x10 ⁻⁶	5.04x10 ⁻⁶	1.30	.200
HJ	51.5x10 ⁻⁵	7.08x10 ⁻⁵	60.2x10 ⁻⁶	4.03x10 ⁻⁶	1.28	.162
Expt	(a) (44.2±4)x10 ⁻⁵	(b) (9.5±1.3)x10 ⁻⁵	(c) (80±25)x10 ⁻⁶	(d) (5.9±12.0)x10 ⁻⁶		

(a) Average of 14.72fm⁻² and 15.42fm⁻² values of Elias et al⁽²⁷⁾

(b) 25.68fm⁻² value of Arnold et al⁽²⁸⁾

(c) Extrapolation to 15fm⁻² of the 14fm⁻² data of Buchanan and Yearian reported in Ref 29

(d) 25.68fm⁻² value of F. Martin et al⁽³⁰⁾

Table III. Zeros of the Nucleon and Deuteron Form Factors, $0 < q^2 < 200 \text{ fm}^{-2}$
 (Entries are q^2 in units of fm^{-2})

deut-f-f	Model	FL 1	FL 5	FL 15	RSC	HJ
C_E		21,160	20.5, 158	21,160	19.5, 148	20,160
C_Q		86	103	125	118	125
C_L		45,177	57	88	?	88
C_S		30.5, 180	32,180	39.5, 170	?	39,193
Nucleon FF	Model	I		II		
G_E^S		193		193		
G_M^S		—		71		

Figure Captions

- Figure (1): "Potential" Exchange Processes.
- Figure (2): "Non Potential" exchange Processes.
- Figure (3): Deuteron Magnetic and Quadrupole moments calculated with various potentials:
1. Hamada-Johnson, 2. Yale, 3. Reid Soft Core
 4. BCM (4.57%), 5. BCM (5.25%), 6. BCM (7.52%)
 7. Paris, 8. Sprung-de Tourreil, 9. Stony Brook.
- x - Impulse + MEC (No recoil - no normalization)
 0 - Impulse + MEC (with recoil - no normalization)
 • Impulse alone.
- Figure (4): Three Body Recoil Graphs
- Figure (5): Connected Graphs
- Figure (6): $P(q^2)$, the low momentum-transfer $e d$ scattering tensor polarization predicted in impulse approximation by three FL potentials with different %D states (E. Lomon, to be published).
- Figure (7): $P(q^2)$, the $e d$ scattering tensor polarization predicted in impulse approximation by three FL potentials and the HJ potential. The result for MEC dominance is also shown, using the MEC amplitudes of Ref. 9. The HJ result is the same as that for the 7.55% D state FL potential, except at the highest momentum transfers (E. Lomon, to be published).
- Figure (8): $B(q^2)$, the magnetic $e d$ scattering predicted in impulse approximation for the 4.57% D state FL potential with a dipole G_M^N (—) or IJL pole fit G_M^N (---), and the HJ potential with the dipole G_M^N (---) (a) lower momentum-transfer. (b) higher momentum transfer (E. Lomon, to be published).
- Figure (9): The magnetic amplitude $F_M(q^2) = \frac{M_N}{M_D} \sqrt{1 + \eta} G_1(q^2)$ predicted for the Reid-soft core (RSC) potential and the nucleon electromagnetic form-factors and MEC contribution of Ref. 9. In addition the arrows show the positions of the zeros of $G_1(q^2)$, as calculated by E. Lomon, for three FL potentials, the HJ potential, and the RSC potential. For lower momentum-transfer.
- Figure (10): The magnetic amplitude $F_M(q^2)$ as calculated in Ref. 9 for higher momentum-transfer, using the R.S.C. potential and the nucleon electromagnetic form-factors and MEC correction of Ref. 9.

- Figure (11): $A(q^2)$ at high momentum transfer for three FL potentials and both choices of G_M^n (E. Lomon, to be published).
- Figure (12): $A(q^2)$ at high momentum transfer for the HJ potential and both choices of G_M^n (E. Lomon, to be published).
- Figure (13): The photodisintegration cross-section of the deuteron for forward going protons, from Ref. 40, for three FL potentials and the HJ potential. The MEC corrections and experimental data are as given in Ref. 38.

Fig. 1

(a)

(b)

Fig. 2

Fig. 3

Fig (4)

FIGURE 6

— MEC - Gari - Hyuga

FL Pot.

--- 4.57
• • • 5.20
— 7.55 } % D state

- HJ Pot.

FIGURE 7

FIGURE 8a.

FIGURE 8b

FIGURE 9

FIGURE 10

FIGURE 11

FL Pot..

FIGURE 13

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

III.1 PHOTOPRODUCTION OF PIONS AT LOW ENERGIES

J. H. Koch
 Center for Theoretical Physics
 Laboratory for Nuclear Science and Department of Physics
 Massachusetts Institute of Technology
 Cambridge, Massachusetts 02139

INTRODUCTION

The first pion photoproduction experiments on nuclei were done almost thirty years ago¹. However, it was not until recently with the advent of the high flux electron accelerators, that photoproduction became a competitive tool for nuclear physics research. Major experimental programs are now under way at facilities such as the MIT-Bates and Saclay machines.

The information that can be obtained from nuclear photoproduction can be grouped into three categories². First, we can learn about the electromagnetic production of a hadron on a free nucleon and then further investigate what modifications occur when the production takes place inside a nucleus. Second, pion photoproduction represents an excellent probe for the pion-nucleus interaction. In particular, pion photoproduction near threshold ($T_{\pi} \lesssim 30\text{MeV}$) can provide information in an interesting energy region which cannot be reached by pion scattering experiments. Thus, low energy production will be an important tool in arriving at a unified description (optical potential) of low energy π -nucleus interactions, ranging from π -mesic atoms to pion scattering. Finally, pion photoproduction is also a probe for the structure of the target and its isobaric analog states. Since the pion is a pseudoscalar

particle, we are dealing with the axial nuclear formfactor. While it will be difficult to extract new information about the target structure from photoproduction experiments alone, these reactions will serve as a valuable independent test for nuclear models derived, e.g., from electron scattering.

For each of these three physical aspects - production mechanism, pion-nucleus interaction and target structure - one has to use a model. In the following remarks I will discuss these aspects and some of the theoretical models in more detail by using as an example the photoproduction of pions at low energies, where most of the present experiments are done.

A discussion of photoproduction at higher energies, where the Δ_{33} resonance dominates both the production mechanism and the final state π -nucleus interaction, is contained in the workshop contributions by E.J. Moniz and M.G. Huber. The experimental situation of pion photoproduction, mainly at low pion energies, is covered in the paper of A. Bernstein.

2. PION PHOTOPRODUCTION AT LOW ENERGIES

2.1 Production mechanism

We first discuss the production from a single free nucleon. A variety of theoretical approaches exist for the description of low energy pion photoproduction: dispersion relations³, current algebra⁴ or chiral Lagrangians⁵. Below, we will use the latter approach to discuss some general features of the free $\gamma N \rightarrow \pi N$ process. This model uses an effective Lagrangian for the πN system

which includes an axial vector πN interaction and also a $\pi N \Delta$ coupling term. A gauge invariant Lagrangian appropriate for photoproduction is then obtained by means of the minimal substitution. Fig. 1 shows the relevant Born terms. (For a pseudoscalar πN coupling, the seagull term, Fig. 1c, would be absent.) This model is consistent with the current algebra approach. The use of a specific chiral Lagrangian corresponds to a particular choice of extrapolating the soft pion current algebra results to the on mass shell case⁵.

In the two-body CM frame, the photoproduction amplitude has the general structure³

$$M_{\gamma\pi} = \chi^\dagger \left[F_1 \vec{\sigma} \cdot \vec{\epsilon} + F_2 \vec{\sigma} \cdot \vec{k} \vec{\epsilon} \cdot \vec{q} + F_3 \vec{\sigma} \cdot \vec{q} \vec{\epsilon} \cdot \vec{q} + i F_4 \vec{q} \cdot (\vec{k} \times \vec{\epsilon}) \right] \chi \equiv \chi^\dagger \vec{\epsilon} \cdot \vec{J}^5 \chi. \quad (1)$$

where $\vec{q}$, $\vec{k}$ and $\vec{\epsilon}$ are the pion momentum photon momentum and polarization vector, respectively. $\vec{J}^5$ is the nucleon axial current. The transverse gauge, $\vec{\epsilon} \cdot \vec{k} = 0$, is used. The coefficients F_i , which contain all the information about the specific production model, depend on the invariants s , t , u and are operators in the nucleon isospin space. Near threshold, the F_1 term dominates and the cross section becomes

$$\sigma_T = \frac{q}{k} a_T,$$

where the constant a_T is the "reduced production cross section".

Photoproduction from a single nucleon is experimentally feasible only for a hydrogen target

For π^+ production, the measured value for the reduced cross section is⁶

$$a_T = 194 \pm 7 \mu b.$$

This agrees well with the prediction based on the Born terms, Fig. 1a-1f, which yield $a_T^+ = 210 \mu b$. For charged pions, the most important contribution to the s-wave production term, $F_1 \vec{\sigma} \cdot \vec{\epsilon}$ in Eq. (1), comes from the seagull diagram, Fig. 1c. In the limit of a static nucleon, $M \rightarrow \infty$, this yields the well known Kroll-Ruderman term⁷. The other diagrams yield only smaller, non-static corrections. The isobar, Fig. 1e, f, contributes only to order $(m_\pi/M)^2$ near threshold.

For $\gamma + p \rightarrow \pi^0 + p$, the measured reduced cross section is $a_T^0 = .9 \pm .2 \mu b$, two orders of magnitude smaller than the π^+ cross section⁸. In the theoretical approach chosen here, this large difference is easily understood: For neutral pions, there is no seagull term from the minimal substitution and the π^0 cross section is zero in the static limit. The (γ, π^0) amplitude therefore depends entirely on small, nonstatic correction terms to the Kroll-Ruderman interaction and is very sensitive to the details of the production

mechanism. The diagrams Fig. 1a-f yield $a_T^0 = 1.7 \mu\text{b}$, which does not agree with experiment. As a refinement, Berends et al.⁹ suggested that the exchange of an ω vector meson be included Fig. 1g. This affects only π^0 production and brings the reduced cross section down to $1.1 \mu\text{b}$ ¹⁰. The exchange of other vector mesons is suppressed, consistent with their smaller radiative width. The great sensitivity of π^0 production to details of the underlying dynamics is also reflected in by the large difference in the predicted cross section for pseudoscalar and pseudovector πN coupling¹¹: pseudoscalar theory yields a value for a_T^0 about ten times larger than pseudovector coupling, while the predictions for charged pion production differ by only 2%. The contrast between charged and neutral pion production disappears rapidly as the photon energy is increased. In the region where the $\Delta_{3,3}$ resonance dominates, the charged and neutral cross section are of the same magnitude ($\sigma \sim 200 \mu\text{b}$).

Experimental information for production from a single free nucleon (proton) is meager in the low energy region. In particular for the energies where the recent nuclear photoproduction experiments are done, $T_\pi < 15 \text{ MeV}$, only a few measurements exist with rather large errorbars. The available data show that the free production mechanism is reasonably well understood and no major discrepancies exist¹². However, for a reliable and conclusive analysis of nuclear photoproduction experiments, more single nucleon data with higher precision are definitely needed.

This is particularly necessary since many nuclear experiments are done relative to the proton, i.e. the ratio $\sigma_{\gamma,\pi}(A)/\sigma_{\gamma\pi}(p)$ is measured.

We now turn to a discussion of the nuclear production mechanism. There are two different approaches: the elementary particle approach of Kim and Primakoff¹³ and the microscopic description using the impulse approximation. A discussion of the elementary particle approach can be found in Refs. 13-16. We will use here the standard impulse approximation where the free single nucleon amplitude, Eq. (1), is used for production from each of the target nucleons. Many body effects such as binding corrections are clearly neglected by this approach. Before the single nucleon operator, obtained for example from the Born terms in Fig. 1, can be used for nuclear calculations, a necessary practical step is to cast this relativistic expression into a more manageable form suitable for use with non-relativistic nuclear wavefunctions. This is done in a general kinematical frame and the amplitude will depend on the nucleon momentum in this frame, allowing for a proper description of the nuclear Fermi motion^{10,12,17}. The choice of the dynamical model for the single nucleon production provides the necessary prescription how to extend the mechanism to a bound, off-shell nucleon.

A logical first step in studying the mechanism for nuclear production is to look at photoproduction from few body targets ($A=2,3,4$). A simple nucleus such as the deuteron has a well

known wavefunction and we can, in principle exactly solve the full three-body problem. Therefore, the deuteron provides a simple test for the impulse approximation and its inherent uncertainties. A low energy experiment for $\gamma d \rightarrow nn\pi^+$ was performed at MIT¹⁸ and later at Saclay¹⁹. An analysis^{18,20,21} in terms of the impulse approximation explained these measurements very well.

It is possible to focus on individual aspects of the production mechanism by choosing different kinematical conditions for a $\gamma d \rightarrow NN\pi$ experiment²¹. Another way to project out different pieces in the production amplitude, mainly for heavier targets is to select specific final nuclear states²².

The reaction mechanism in the $\gamma d \rightarrow \pi^0 d$ reaction near threshold is somewhat unusual¹⁰. Since the direct π^0 production on one nucleon, Fig. 2a, is so small, production proceeding through an intermediate charged pion, Fig. 2b, dominates. The situation is similar for other light targets²³.

The data available at present for low energy photoproduction from the proton and light targets (mainly total cross sections), which we need for a detailed test of the production mechanism, are sparse and often not very accurate. The new data now coming from the high flux electron accelerators will be extremely valuable for a strict and detailed test of the existing theoretical concepts.

2.2 Pion Nucleus Interaction

Using the distorted wave impulse approximation, the production amplitude from a nuclear target becomes

$$M = \langle f | \int d\vec{x} \varphi_{\pi}^*(\vec{x}) \vec{\varepsilon} \cdot \vec{J}^{\pi}(\vec{x}) e^{i\vec{k} \cdot \vec{x}} | i \rangle. \quad (2)$$

$\vec{\varepsilon} \cdot \vec{J}_5$ is the elementary production operator, Eq. (1), in the nuclear kinematical frame and $|i\rangle$, $|f\rangle$ denote the initial and final states of the target. ϕ_{π} is the wavefunction of the produced pion. At low energies, the π -nucleus interaction is most conveniently described by an optical potential. Thus, ϕ_{π} is a solution of the Schrödinger equation with the potential

$V_{\text{opt}} = V_c + V_s$, where V_c is the Coulomb potential and the strong π -nucleus interaction, V_s , is given by

$$V_s = - \frac{4\pi}{2\mu} [a_0 g + a_1 (g_n - g_p) + A g^2 + \vec{\nabla} \alpha(r) \cdot \vec{\nabla}],$$

where

$$\alpha(r) = b_0 g + b_1 (g_n - g_p) + B g^2. \quad (3)$$

For a low energy pion the nucleus is rather transparent, in contrast to the "black sphere" a pion sees in the resonance region. The pion can therefore exist in regions of large density and higher order corrections such as the Lorentz-Lorenz effect might become important. Unfortunately, this low energy region is not

readily accessible with pion scattering experiments, which use pions with $T_\pi \gtrsim 30$ MeV. Some information on V_S can also be obtained from the shifts and width of the energy levels in π -mesic atoms. Pion photoproduction, on the other hand, can easily sweep the entire energy region from threshold up to the (3,3) resonance - with two further advantages: (1) The incident photon reaches the entire nuclear volume, essentially without distortion or disturbing the target. This is in contrast to hadronic production. (2) With pion photoproduction we can study the π -nucleus interaction for different spin-isospin configurations of the target. For example, in elastic scattering on ^{12}C the pion interacts with the target in its ground state, $J^\pi T = 0^+0$, and only the ρ -terms in V_S , Eq. 3, are involved. In a photoproduction reaction on the same target $\gamma + ^{12}\text{C}(0^+0) \rightarrow \pi^- + ^{12}\text{N}(1^+1)$, also the $(\rho_n - \rho_p)$ parts of V_S contribute to the final state interaction.

To illustrate the effects of the distortion of the produced pion²⁴, we discuss below the near threshold production of π^+ and π^- mesons on ^{12}C . Fig. 3 shows the total production cross section for $\gamma + ^{12}\text{C} \rightarrow \pi^- + ^{12}\text{N}(1^+1)$, calculated with the $\vec{\sigma} \cdot \vec{e}$ term and keeping the pion s-wave only. Curve a shows the cross section for an undistorted pion, $\phi_\pi \rightarrow j_0(qr)$. Turning on the nuclear Coulomb potential, V_C , the pion wavefunction gets pulled into the nuclear interior. As a result the threshold cross section increases to a roughly constant step-function, curve b. With the wavefunction large inside the nucleus, the strong interaction, V_S , then causes

sizeable effect on ϕ_π and reduces the step height of the threshold cross section considerably. As the incident photon energy is increased and new final nuclear states become accessible, new steps will appear in σ_T .

The situation is quite different for threshold π^+ production, $\gamma + {}^{12}\text{C} \rightarrow \pi^+ + {}^{12}\text{B}(1^+1)$. The repulsive Coulomb potential pushes the wavefunction out of the nucleus, causing a decrease of the cross section, curve b in Fig. 4. With the wavefunction already small, the strong interaction then yields only a small additional suppression (curve c). Fig. 5 further illustrates these qualitative remarks by showing the pion wavefunctions involved in the (γ, π^-) and (γ, π^+) reactions of Figs. 3 and 4. The curvature of the fully distorted π^- wavefunction shows that the momentum dependent terms in the production operator, Eq. (1), can be important - even at threshold where the asymptotic pion momentum is zero. It is thus clear that low energy π^- production is much more sensitive to the strong final state interaction than π^+ production, where V_s plays only a minor role.

Fig. 5 also shows the nuclear transition density (dashed) which peaks in the diffuse nuclear surface. In charged pion production a nucleon - typically in one of the outer shells - undergoes a transition. This diminishes the possibility of studying the π -nucleus interaction in the nuclear interior. Coherent production of neutral pions, which leaves the target in its ground state does not have this problem and the entire nuclear volume contributes. However, as was pointed out in section 2.1, at low energies the

charge-exchange production mode for π^0 has to be taken into consideration and this makes the interpretation of (γ, π^0) experiments more difficult.

2.3 Nuclear Structure

If we assume that the produced pion leaves the nucleus without any final state interaction, the nuclear production amplitude, Eq. 2, becomes with $\phi_\pi = e^{i\vec{q} \cdot \vec{x}}$

$$M = \vec{\epsilon} \cdot \int d\vec{x} \langle f | e^{i\vec{x} \cdot (\vec{k} - \vec{q})} \vec{J}_5(x) | i \rangle$$

$$= \vec{\epsilon} \cdot \vec{J}_5^{fi}(\vec{Q}).$$

This means that we can use photoproduction to directly map out the nuclear axial formfactor, $J_5^{fi}(\vec{Q})$, by varying the momentum transfer $\vec{Q} = \vec{k} - \vec{q}$. (With electroproduction we can go one step further and also independently vary the energy transfer.) In this fashion, (γ, π) or $(e, e' \pi)$ experiments would provide information about a component of the nuclear current which cannot be obtained from electron scattering, where only the vector part contributes. However, due to the pion-nucleus interaction, a distorted pion wavefunction, $\phi_\pi(\vec{x})$, is folded into the current. This makes the analysis more difficult and the interpretation more ambiguous: What information about the nuclear formfactor one extracts from an experiment will depend on the optical potential model that was chosen to calculate ϕ_π . In order to minimize these complications it is advantageous to deal with low energy photo-

production of positive pions. Only a few partial waves of the pion wavefunction contribute and - as was shown in the previous section - the strong π -nucleus interaction plays only a minor role for π^+ production.

To illustrate these remarks, we discuss below the reaction $\gamma + {}^6\text{Li}(1^+0) \rightarrow \pi^+ + {}^6\text{He}(0^+1)$ near threshold²⁵. Fig. 6 shows schematically other sources of information for the nuclear wavefunctions $|i\rangle$ and $|f\rangle$ in Eq. 2: elastic and inelastic electron scattering, μ capture and β -decay. Fig. 7 shows several theoretical predictions for the photoproduction cross section. Curve a is obtained¹⁶ by using the shell model nuclear wavefunction of Donnelly and Walecka²⁶, who restrict the model space to the harmonic oscillator p-shell and then use the electromagnetic properties of ${}^6\text{Li}$ to fit the free parameters. Using the same configuration of states, but replacing the harmonic oscillator by Woods-Saxon wavefunctions yields curve c. Curves b and d correspond to the phenomenological transition densities of Bergstrom²⁷. While all these parametrizations fit the (e, e') data very well they differ in their predictions for the (γ, π^+) cross section. For electron scattering the M1 formfactor for the $1^+0 \rightarrow 0^+1$ transition has two parts, a spin and a convection current component. The photoproduction amplitude is dependent mainly on spin $(\sim \vec{\sigma} \cdot \vec{\epsilon})$. Thus, wavefunctions which give the same fit for electron scattering data will yield different predictions for low energy photoproduction according to how much of the total M1 transition strength is concentrated in the spin part¹⁶. The

experiment²⁵ is best described by the models used for curves b and c. Thus pion photoproduction serves as an independent test for nuclear wavefunctions. The other reactions in Fig. 6, β decay and μ^- capture, which involve the same nuclear levels, also probe the axial current but at a fixed momentum transfer. In pion photoproduction, we can vary the momentum transfer and thus obtain a more stringent test for the nuclear model wavefunctions. It is clear, however, that the best way to construct model wavefunctions for the target states remains the analysis of electron scattering data. A necessary criterion for such wavefunctions, $|i\rangle$ and $|f\rangle$ in Eq. 2, is that they give a good fit to the (e,e') formfactors involving $|i\rangle$ and $|f\rangle$ (or their isobaric analogs) around momentum transfers of $Q=140$ MeV/c.

3. SUMMARY

A theoretical description of nuclear photoproduction needs three model inputs: production mechanism, pion-nucleus potential and target structure. It can be difficult to sort out the dependence on each of these aspects and to unambiguously extract new information. However, as was shown in the above remarks, one can choose the experimental conditions - target, kinematics, charge of the produced pion etc. - such that each aspect comes in on a different level and one focuses on a particular physical detail. The π^0 production is clearly the most sensitive probe for the low energy production mechanism itself - both for the nucleon and the nucleus. For charged production, the well known Kroll-Ruderman term dominates the low energy production and we

thus can study the π -nucleus interaction and target structure by using (γ, π^-) and (γ, π^+) reactions. Still, as was seen for example in section 2.3, a conclusive analysis often requires an even wider scope and other processes such as nuclear electron or pion scattering have to be considered simultaneously with pion photoproduction.

REFERENCES

1. J. Steinberger and A.S. Bishop, Phys. Rev. 78, 494 (1950).
2. J.H. Koch, in "Meson-Nuclear Physics-1976" ed. by P.D. Barnes et al. (AIP, New York, 1976) p. 591.
3. G.F. Chew, M. Goldberger, F. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1959).
4. M.S. Bhatia and P. Narayanaswamy, Phys. Rev. 172, 1742 (1968).
5. R.D. Peccei, Phys. Rev. 181, 1902 (1969).
6. M. Adamovich et al., Yad. Fiz. 9, 848 (1969) [Sov. J. Nucl. Phys. 8, 865 (1969)].
7. N.M. Kroll and M.A. Ruderman, Phys. Rev. 93, 233 (1954).
8. B.B. Govorkov and E.V. Minarik, Yad. Fiz. 14, 437 (1971) [Sov. J. Nucl. Phys. 14, 245 (1972)].
9. F.A. Berends, A. Donnachie, and D.L. Weaver, Nucl. Phys. B4, 103 (1967).
10. J.H. Koch and R.M. Woloshyn, Phys. Lett. 60B, 221 (1976) and Phys. Rev. C (1977), to be published.
11. J.L. Friar and B.F. Gibson, Phys. Rev. C15, 1779 (1977).
12. I. Blomqvist and J.M. Laget, Nucl. Phys. A280, 405 (1977).
13. C.W. Kim and H. Primakoff, Phys. Rev. 140, B566 (1965).

14. J. Delorme, Nucl. Phys. B19, 573 (1970).
15. M. Ericson and M. Rho, Phys. Reports 5, 58 (1972).
16. J. B. Cammarata and T.W. Donnelly, Nucl. Phys. A267, 365 (1976).
17. L.M. Saunders, Nucl. Phys. B7, 293 (1968) and Thesis, MIT (1967), unpublished.
18. E.C. Booth et al. Phys. Lett. 66B, 236 (1977).
19. G. Audit et al, Phys. Rev. C (to be published).
20. J. Noble, Phys. Lett. 67B, 39 (1977).
21. J.M. Laget, Saclay Preprint (1977).
22. A. Figureau and N.C. Mukhopadhyay, in "Meson-Nuclear Physics-1976" (see Ref. 2), p. 616.
23. J.D. Vergados and R.M. Woloshyn, Phys. Rev. C16, 292 (1977).
24. J.H. Koch and T.W. Donnelly, Nucl. Phys. B64, 478 (1973).
25. G. Audit et al, Phys. Rev. C (to be published).
26. T.W. Donnelly and J.D. Walecka, Phys. Lett. 44B, 330 (1973).
27. J.C. Bergstrom, I. Auer and R. Hicks, Nucl. Phys. A251, 401 (1975).

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

III.2 SINGLE NUCLEON OPERATORS IN PION PHOTOPRODUCTION FROM NUCLEI

Edward Dressler
 Center for Radiation Research
 National Bureau of Standards
 Washington, D.C. 20234

There are three questions about the pion production process in nuclei which one hopes to answer:

- 1) What is the pion production mechanism for nucleons in a nucleus?
- 2) How does the pion interact with the nucleus after being produced?
- 3) What new features of nuclear structure can be seen?

In this paper we¹ investigate question 1) using the model proposed by Blomqvist and Laget² which is the sum of pseudovector Born diagrams plus the contribution from the S-channel $\Delta(1236)$ (Fig. 1). To use these amplitudes in a nuclear physics calculation, Blomqvist and Laget make a nonrelativistic reduction of the amplitude associated with each of these diagrams keeping terms of $O(P^2/M^2)$ without specifying a particular reference frame. The advantages are the following:

- 1) No transformation from the γN center-of-momentum system to the nuclear reference frame is needed.
- 2) The Fermi momentum dependence is explicit.
- 3) Double counting is avoided by having a model which tells which diagrams have been taken into consideration.

Consider the process $p(\gamma, \pi^+)n$. In the C. M. system, the amplitude can be written

$$T_{fi} \sim i \left[F_1 \vec{\sigma} \cdot \hat{\epsilon} + F_2 \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \hat{\epsilon} + F_3 \vec{\sigma} \cdot \vec{q} \vec{q} \cdot \hat{\epsilon} \right] + F_4 \hat{\epsilon} \cdot (\vec{q} \times \vec{k}) \quad (1)$$

where we have separated the amplitude into those terms which can flip the spin (in brackets) and the pure non-spin-flip term. After performing the spin and polarization sums we obtain

$$\sum_{i,f} |T_{fi}|^2 \sim |F_1|^2 + \frac{\vec{q}^2}{2} \sin^2 \theta_\pi \{ \vec{k}^2 |F_2|^2 + \vec{q}^2 |F_3|^2 + 2R_e(F_1^* F_3) + 2(\vec{k} \cdot \vec{q}) R_e(F_2^* F_3) + \vec{k}^2 |F_4|^2 \} \quad (2)$$

Fig. 2 shows that up to 60 MeV above threshold, the cross section can be represented by the Born parts of the $\vec{\sigma} \cdot \hat{\epsilon}$ term in $|F_1|^2$. The interesting point is that the cross section is dominated by this term not because the other terms are individually small, but because of a surprising cancellation that occurs between the other terms. The cancellation is among those terms which depend on θ_π , and, therefore, makes the cross section essentially isotropic in this region. In the region 60 - 100 MeV, the real part of the resonance contribution upsets this cancellation, and the cross section exhibits strong angular dependence. The implication for nuclear physics calculations is that if we can approximate the production amplitude by the $\vec{\sigma} \cdot \hat{\epsilon}$ Born term in the energy region from 0 - 60 MeV above threshold, we could greatly simplify the calculations and concentrate on the pion final state interactions and nuclear structure (questions 2 and 3).

To investigate further the possibility of using this approximation, consider the process ${}^2\text{H}(\gamma, \pi^+)2n$. This cross section can be separated into two parts: the transition to the singlet 2 neutron final state and the

triplet final state. The singlet transition neglecting Fermi momentum corrections and pion final state interactions is

$$\sum_{i,f} |T_{fi}|^2_{\text{sing.}} \sim |\Psi_{\text{sing.}} \Psi_{\text{gnd.}}|^2 \left\{ |F_1|^2 + \frac{q^2}{2} \sin^2 \Theta_\pi \left[\vec{k}^2 |F_2|^2 + \vec{q}^2 |F_3|^2 + 2R_e(F_1^* F_3) + 2(\vec{k} \cdot \vec{q}) R_e(F_2^* F_3) \right] \right\}. \quad (3)$$

For the triplet final state,

$$\sum_{i,f} |T_{fi}|^2_{\text{trip.}} \sim 2 |\Psi_{\text{trip.}}^* \Psi_{\text{gnd.}}|^2 \left\{ |F_1|^2 + \frac{q^2}{2} \sin^2 \Theta_\pi \left[\vec{k}^2 |F_2|^2 + \vec{q}^2 |F_3|^2 + 2R_e(F_1^* F_3) + 2(\vec{k} \cdot \vec{q}) R_e(F_2^* F_3) + \frac{3}{2} \vec{k}^2 |F_4|^2 \right] \right\}. \quad (4)$$

The singlet transition has no $|F_4|^2$ term, and the cancellation of the $\sin^2 \Theta_\pi$ terms that took place in the proton cross section might not occur in the deuteron case giving a nonisotropic pion angular distribution in the threshold region where

$$|\Psi_{\text{sing.}}^* \Psi_{\text{gnd.}}|^2 \gg |\Psi_{\text{trip.}}^* \Psi_{\text{gnd.}}|^2.$$

If the singlet and triplet overlap integrals are comparable in size, Eqs. (3) and (4) add to give the same form as the proton matrix element squared. However, even for the deuteron π^+ production, the Born part of the $\vec{\sigma} \cdot \hat{\epsilon}$ operator dominates the cross section up to 60 MeV above threshold, and the cancellation of the $\sin^2 \Theta_\pi$ terms occurs for the singlet and triplet terms individually. We are led to conjecture that approximating the production operator by the $\vec{\sigma} \cdot \hat{\epsilon}$ Born term should be valid for most nuclei even well above threshold, and a measurement of the pion angular distribution from deuterium would be a useful test of the Blomqvist-Laget model and the above approximations.

For pion electroproduction from nuclei, one faces again the problem of choosing a model for the single nucleon production amplitude. The problem is complicated by the fact that these amplitudes now contain form factors. In the Dombey and Read model³, for instance, the amplitudes require knowledge of the nucleon axial vector form factor F_A , the pion form factor F_π , and the nucleon electromagnetic form factors $F_{1,2}^{S,V}$. The form factors $F_{1,2}^{S,V}$ of course are determined independent of the model, but F_A and F_π are determined from parameterized fits to the $p(e,e'\pi^+)n$ data, and their form is tied to this model. Also, there is the problem of making the amplitude gauge invariant. Should the sum of the diagrams be gauge invariant or should each diagram be gauge invariant?

Although one might hope to investigate these models by studying nuclear processes as in the (γ,π) case, the experimental problems in measuring $(e,e'\pi)$ cross sections⁴ will make this much more difficult.

REFERENCES

- [1] This work was done in collaboration with J. S. O'Connell and W. M. MacDonald.
- [2] I. Blomqvist and J. M. Laget, Nucl. Phys. B280 (1977) 405.
- [3] N. Dombey and B. J. Read, Nucl. Phys. B60 (1973) 65.
- [4] See J. S. O'Connell and B. Schoch in this publication for a discussion of coincidence experiments.

Figure Captions

Fig. 1. Diagrams A - D represent the Born contributions to pion photoproduction using pseudovector pion-nucleon coupling.

Diagram E is the s-channel $\Delta(1236)$ contribution.

Fig. 2. $\vec{\sigma} \cdot \hat{\epsilon}$ BORN is the contributions to the $p(\gamma, \pi^+)n$ cross section from the parts of the Born diagrams which have a $\vec{\sigma} \cdot \hat{\epsilon}$ form.

The other curves show the contributions from the other Born terms and the importance of the $\Delta(1236)$ terms.

FIGURE 1

FIGURE 2

III.3 Pion Production with Real and Virtual Photons on Nucleons in the Resonance Region

K.H. Althoff, Bonn University

In pion photo- and electroproduction the hadronic structure of the photon (via vector mesons) and the electromagnetic structure of hadrons can be studied. Of special interest is the region of the nucleon resonances below a total energy W of the hadronic system of 2 GeV. In fig.1 the known resonances are listed: Δ -resonances with isospin $I = 3/2$ and N^* -resonances with isospin $I = 1/2$. The dominating resonances excited by photons are: $P_{33}(1232)$, $D_{13}(1520)$, $F_{15}(1688)$ and $F_{37}(1950)$. Already the total cross section distributions for the reaction $\gamma N \rightarrow \pi N$ and $eN \rightarrow e'\pi N$ (which is equivalent to the pion production with virtual photons $\gamma_V N \rightarrow \pi N$) show four bumps at these energies.

Experiments with real photons γ_R and virtual photons γ_V give different information:

Real photons: Determination of magnitude and sign of radiative decay amplitudes of resonances and nonresonant background amplitudes.

Virtual photons: Probing of internal structure of hadrons in the groundstate $W=m_p$ (elastic electron scattering, formfactors G_E and G_M) and excited states $W = m_{N^*}, \dots$ (inelastic scattering, transition formfactors $G_{\gamma N \Delta}, G_{\gamma N N^*}$)

	Mass	Helicities	electr. magn. Field Components	Helicity Amplitudes
γ_R	0	$\lambda = \pm 1$	transvers	$H_1 H_2 H_3 H_4$
γ_V	q^2	$\lambda = \pm 1, 0$	transvers + longitudinal	$H_1 H_2 H_3 H_4 H_5 H_6$

I. Real Photons

Common synchrotron photon sources are:

1) Bremsstrahlung:

large photon flux, continuous spectrum, not polarized.

2) Coherent Bremsstrahlung

large photon flux, partly monochromatic, partly polarized

3) Tagged Photons:

small photon flux, monochromatic, not polarized

At electron linacs monochromatic photons have been produced using e^+e^- -annihilation and compton scattering of laser photons.

I.1 First resonance region $\Delta(1232)$

In the region of the first resonance a large number of precise cross section data on proton targets exist for the reaction $\gamma p \rightarrow \pi^+ n$ and $\gamma p \rightarrow \pi^0 p$. Various polarization experiments have been performed. They give information about smaller amplitudes and are necessary to disentangle overlapping contributions. The agreement between data of different laboratories is good.

As an example some angular distributions for the reactions $\gamma p \rightarrow \pi^+ n$ and $\gamma p \rightarrow \pi^0 p$ around 350 MeV are shown (fig 2). The π^0 -data are symmetric around 90° , which is consistent with the excitation of one single resonance (in this case a magnetic excitation via the M_{1+} multipole amplitude). The background contribution is very small ($< 2\%$). (1)

For charged pion production the following graphs have to be taken into account:

Resonance term

"Born" terms

The strong electric dipole amplitude E_{0+} (due to the electric charge of the pion) can not be neglected anymore.

The photoproduction on neutron targets $\gamma n \rightarrow \pi^- p$ and $\gamma n \rightarrow \pi^0 n$ has been studied not so extensively. Also the accuracy of the data and the agreement between different laboratories is by far not as good as for proton targets. This is mainly due to the difficulties connected to the nuclear targets (mostly deuterium).

In fig. 3, 4, 5 data from different laboratories, are plotted for the reaction $\gamma n \rightarrow \pi^- p$ around the first resonance (2, 3).

In fig. 6 a comparison of the CERN negative pion capture data $\pi^- p \rightarrow \gamma n$ with photoproduction data $\gamma n \rightarrow \pi^- p$ is shown (2). In this case there are no nuclear target problems, but the evaluation is model dependent.

This reaction has also been used to test the time-reversal invariance. No violation has been seen within a phase angle of 5° , the experimental uncertainty (4), (fig. 7).

The question whether the photon has isoscalar, isovector and isotensor components has been discussed extensively during the last years. Recent measurements on the ratio

$$R_{\pm} = \frac{\sigma(\pi^{\pm} p)}{\sigma(\pi^{\mp} p)}$$

and

$$R_0 = \frac{\sigma(\pi^0 p)}{\sigma(\pi^0 n)}$$

showed that the contribution of an isotensor part is less than 2% of the isovector part (5, 6) (fig. 8).

I.2 Conclusion

The experimental and theoretical situation for photon pion production on proton targets is in a good shape. New measurements will not change the proton amplitude appreciably.

For neutron targets, however, more extensive and precise data are highly desired. Corrections due to nuclear target effects might be unpleasant for an elementary particle physicist, for a nuclear physicist the physics involved should be very interesting.

At high intensity linacs with high duty cycles these problems should be studied. As an example the π^+ -photoproduction on protons could be compared to the π^+ -production from few nucleon targets like D , ^3He and ^4He . If the corrections due to nuclear effects are understood the π^+ -cross section, obtained from these targets, must be the same.

As a result of these studies also the pion photoproduction data on neutrons could be improved. At the same time one learns about contributions of pion- and nucleon resonances to the nuclear potential.

Since the accuracy of experimental data on time reversal violation and admixtures of isotensor contributions is limited by nuclear target effects, a new activity could be started.

I.3 Higher Nucleon Resonances

As shown in fig.1 there are many overlapping resonances (the width is typically 150 MeV). To disentangle the different contributions one needs polarisation experiments of various types (7) (defined in fig.9) beside exact cross section measurements. As examples some cross section and polarized target data for the reaction $\gamma p \rightarrow \pi^+ n$ are shown in fig.10, 11, 12 (7,8). Better experimental equipment and a continuous improvement of the data evaluation (on-line computer e t c) in the last years has led to a considerable improvement of the data. But still there is a lot of work to be done in this energy region. Predictions based on quark

model calculations showed remarkable agreement with the data for magnitudes and signs of the dominating radiative decay amplitudes. To test the limits of this successful model, which even works at very high energies, the study of pion photoproduction in the resonance region is a very good tool.

II. Virtual photons

The interaction of electrons with hadrons can be described approximately by the exchange of one virtual photon.

q = momentum transfer

q^2 = "mass" of virtual γ_v

Due to the additional longitudinal part of the photon the electroproduction of pions gives additional information for the electromagnetic interaction with hadrons. Unfortunately the cross section is very small and decreases rapidly with the momentum transfer q . ($\sigma \sim \frac{\alpha}{q^4}$). One needs large solid angle equipment.

First single arm experiments $ep \rightarrow e X$ have been performed, detecting only the scattered electron. In coincidence experiments the reaction $ep \rightarrow e \pi^+ n$ and $ep \rightarrow e \pi^0 p$ have mostly been studied.

II.1 Kinematic

For the following discussion we need some kinematical definitions:

Four momenta: $e(E, \vec{e}), e'(E', \vec{e}'), p(m_p, 0)$

Mass squared of virtual photon: $q^2 = (e - e')^2 \approx -4EE' \sin^2 \phi_e/2$

Energy of virtual photon: $\nu = E - E'$

Invariant mass squared of hadrons: $W^2 = m_p^2 + 2m_p\nu - |q^2|$

The differential cross section can be written as:

$$\frac{d^3\sigma}{d\Omega_e dE' d\Omega_\pi} = \Gamma_t \cdot \frac{d\sigma}{d\Omega_\pi}; \quad \Gamma_t = \text{flux of } \gamma_V / d\Omega_e dE'$$

$$\frac{d\sigma}{d\Omega_\pi} = \sigma_u + \epsilon\sigma_L + \epsilon\sigma_p \sin^2\theta_\pi \cos^2\phi_\pi + \sqrt{\epsilon(\epsilon+1)}\sigma_I \sin^2\theta_\pi \cos\phi_\pi$$

ϵ = transverse polarization parameter

$0 < \epsilon < 1$: For $\phi_e \rightarrow 0$; $\epsilon \rightarrow 1$

σ_u and σ_p correspond to the transvers component of the virtual photons (unpolarized and polarized); σ_L is the longitudinal part and σ_I describes the interference part. ($\sigma_I = f(q^2, W, \theta_\pi)$).

II.2 Total Cross Section ($ep \rightarrow e X$)

If only the scattered electron is detected (single arm) the total cross section is measured (9) (fig.13).

$$\sigma^{\text{tot}}(q^2, W) = \sigma_u^{\text{tot}} + \epsilon\sigma_L^{\text{tot}}$$

Again the resonance structure is clearly seen. The first resonance becomes less prominent with increasing q^2 . The second resonance stays almost constant in magnitude and the third resonance moves its position.

The contribution of σ_L is very small ($\frac{\sigma_L}{\sigma_u} \leq 0.1$) and stems mainly from electrons scattered in the forward direction.

II.3 Threshold region

In coincidence experiments the reactions

have been studied near threshold to determine the Axial Vector Formfactor of the nucleon. A good fit to the data could be obtained using a simple dipole fit (9) (fig.15):

$$\frac{F_A(q^2)}{F_A(0)} = \frac{1}{(1 + q^2/m_A^2)^2}$$

with $m_A = 0.96 \pm 0.03$ GeV

compared to $m_A = 0.89 \pm 0.08$ GeV obtained in neutrino scattering experiments.

II.4 $\Delta(1232)$ Region

In the reaction $ep \rightarrow e \pi^0 p$ it has also been shown that the Δ -resonance is excited mainly by the M_{1+} -multipole amplitude. (Angular distribution is symmetric to $\theta_\pi = 90^\circ$). The transition formfactor $G_{\gamma N \Delta}$ decreases more rapidly with q^2 than G_E of the proton.

The reaction $ep \rightarrow e \pi^+ n$ has been studied extensively in the last years. The pion exchange graph is dominating at small θ_π . As a result the longitudinal cross section σ_L is strongly enhanced and can be twice as large as σ_u (10). From these measurements the pion formfactor F_π can be extracted. The agreement with experimental results based on different methods ($\pi^- p \rightarrow e^+ e^- n$ and νe -scattering) is quite good (9).

II.5 Higher Resonances

In both reactions $ep \rightarrow e \pi^0 p$ and $e \pi^+ n$ the dominating resonances $D_{13}(1520)$ and $F_{15}(1688)$ have been identified. At forward direction the distribution of the cross section is substantially different from the photoproduction ($q^2=0$). This again is *certainly* due to the large σ_L contribution (9) (fig.14).

II.6 π^0 -Dip

The photoproduction data for the reaction $\gamma p \rightarrow \pi^0 p$ show a dip at a momentum transfer $|t| = 0.5(\frac{\text{GeV}}{c})^2$. This can be explained assuming the exchange of a Regge-trajectory or simply as the first diffraction minimum.

In the electroproduction data, however, this dip is missing even at low values of q^2 (9) (fig.16). New experiments, designed for very low q^2 , are in progress.

II.7 Conclusion

In spite of the experimental difficulties there exist a large number of pion-electroproduction data. Compared to the situation in photoproduction, however, we are still at the beginning. There are a lot of open questions. The use of polarized electrons and targets has just been started. High intensity electron linacs with large duty cycle are especially suited to operate at small ϵ and large q^2 , a kinematic region, where new and interesting phenomena can be expected.

References

- 1) G.v. Holtey: Spring Tracks 59,3(1971)
H. Genzel et al.: Z.f.Physik 268,43 (1974)
- 2) H. Fischer: Proc.Electron and Photon Interaction
Bonn 1973, page 77
- 3) P. Benz et al. (ABHHM-Collaboration) Nucl.Phys.B65,158(1973)
- 4) J.C. Comiso et al.: UCLA-10-P25-26 and Phys.Rev.D12,719(1974)
- 5) A. Donnachie, Proc. Lepton and Photon Interaction, Stanford
1975, page 473
- 6) H. Rollnik: Acta Physica Austriaca, Suppl. XIV, 5 (1975)
- 7) K.H. Althoff: a) Proc.INS Symposium, Tokyo 1975
page 35 and Bonn HE-76-3 (1976)
b) Proc.Electromagnetic Interaction
Moscow 1975, page 312 and Bonn HE-76-6 (1976)
c) Proc. AIP Conference Nr.35, Argonne 1976
page 330 and Bonn HE-76-22 (1976)
- 8) H. Fischer: Proc. Lepton and Photon Interaction, Stanford 1975
page 413
- 9) J. Gayler: Proc. Baryon Resonances, Oxford 1976, page 219
K. Heinloth: All Union School, Yerevan 1975,
Bonn HE-75-17 (1975)
- 10) G. Bardin et al.:Lettre al Nuov.Cim. 13, 485 (1975)
and Saclay Preprint 1976

Figure Captions

- Fig.1 Summary of known pion-nucleon resonances (Ref.2)
- Fig.2 Angular distributions for $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ around the first resonance.
- Fig.3 Angular distributions for $\gamma n \rightarrow \pi^- p$ (Tokyo, ABHMM, Bonn) (Ref.2)
- Fig.4 Angular distribution for $\gamma n \rightarrow \pi^- p$ (Orsay and others) (Ref.2)
- Fig.5 Total cross section $\gamma n \rightarrow \pi^- p$ (Ref.2,3)
- Fig.6 Comparison of CERN data $\pi^- p \rightarrow \gamma n$ with data from the revers reaction $\gamma n \rightarrow \pi^- p$ (Ref.2)
- Fig.7 Check for time reversal invariance $\gamma n \leftrightarrow \pi^- p$ (Ref.4)
- Fig.8 Ratio $\sigma(\pi^0 n)/\sigma(\pi^0 p)$. Test of isotensor contribution (Ref.5)
- Fig.9 Decomposition of helicity amplitudes for: $d\sigma/d\Omega$, p , Σ and T . (For double polarization coefficients G and H , see Ref.8).
- Fig.10 Diff. cross section $\gamma p \rightarrow \pi^+ n$ at $\theta_\pi = 180^\circ$. Data up to 1970, Ref. H. Fischer: Bonn Univ PI 1-122 (1970)
- Fig.11 Diff. cross section $\gamma p \rightarrow \pi^+ n$ at $\theta_\pi = 180^\circ$. Bonn data (1975) (Ref.8)
- Fig.12 Angular distribution of polarized target asymmetry T for $\gamma p \rightarrow \pi^+ n$ at $E_\gamma = 700$ MeV. (Ref.7)
- Fig.13 $\sigma_\pi + \epsilon\sigma_L$ ($\epsilon = 0.9$) in the resonance region (Ref.9)
- Fig.15 Nuclear axial vector formfactor (Ref.9)
- Fig.14 Diff. cross section $ep \rightarrow e \pi^+ n$ at forward angles. The dashed band are photoproduction data ($q^2=0$) (Ref.9)
- Fig.16 Diff. cross section versus t for $\gamma_V p \rightarrow \pi^0 p$ and $\gamma_R p \rightarrow \pi^0 p$ (Ref.9).

Fig. 1

Fig. 2

Fig. 3

Fig. 5

Fig. 6

Fig. 7

Fig. 8

$\frac{d\sigma}{d\Omega}$		Photon Beam	unpolarized
		Target	unpolarized
		Helicity Comb.	$(H_1^2 + H_2^2 + H_3^2 + H_4^2)$
P		Photon Beam	unpolarized
$\frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$		Target	unpolarized
		Helicity Comb.	$\text{Im} (H_1 H_3^* + H_2 H_4^*)$
Σ		Photon Beam	linearly polarized
$\frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}}$		Target	unpolarized
		Helicity Comb.	$\text{Re} (H_1 H_4^* - H_2 H_3^*)$
T		Photon Beam	unpolarized
$\frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$		Target	polarized
		Helicity Comb.	$\text{Im} (H_1 H_2^* + H_3 H_4^*)$

Fig. 9

Fig.10

Fig.11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16

III.4 $(\gamma, \pi^{\pm})$ EXPERIMENTS TO DISCRETE NUCLEAR STATES

A. M. Bernstein

Massachusetts Institute of Technology.
Cambridge, Massachusetts 02139

I. Introduction

The purpose of this article is to indicate how the (γ, π) reaction mechanism in complex nuclei can be tested by single arm experiments. In this connection I will briefly review the recent work which has been performed to discrete final nuclear states and indicate the likely direction of work in the near future. Recommendations for future directions will also be made. / The theoretical aspects of the subject are covered in a companion article by J. H. Koch.

An extremely interesting aspect of photo-pion experiments which have been made possible by the new generation of electron linacs is the measurement of total cross sections in the region just above threshold (0.1 to 10 MeV). This regime is interesting because the mean free path of pions is long (see Fig. 1) and therefore the final state interactions are minimal. This originally led many theorists to try soft pion theorems for complex nuclei¹. However, when one tries to incorporate final state interactions into the calculation, one is usually led back to the impulse approximation², and most calculations are performed in this manner. From an experimental point of view the energy region between pionic atoms and pion scattering will only be studied by the (γ, π) process.

When a low energy pion is produced from a nucleus, the Coulomb final state interaction plays a dominant role^{3,1}. As a concrete example, consider the $C^{12}(\gamma, \pi^+)$ reaction (see Fig. 2). Calculations for the total cross sections for the $C^{12}(\gamma, \pi^+)$ reactions just above threshold³ are presented in Fig. 3. For the (γ, π^+) case it can be seen that since the pion wave is pushed away from the nucleus that the cross section is reduced for pion kinetic energies below the Coulomb barrier (≈ 2.6 MeV); in addition this effect drastically reduces the sensitivity of the cross section to the final state strong interaction. In contrast the (γ, π^-) cross section is enhanced by the effect of the attractive final state Coulomb interaction, and in addition, its sensitivity to the final state strong interaction is greatly increased by the increased overlap

between the outgoing pion wave and the final nucleus. We conclude that the best way to test the reaction mechanism in the region just above threshold is via the (γ, π^+) reaction; this has the additional pleasant feature that the experiment can be performed relative to the elementary $p(\gamma, \pi^+)n$ reaction. On the other hand the (γ, π^-) reaction is more sensitive to the pion wave function, and one hopes to learn something about this from these reactions. These experiments will be discussed in Sec. III.

II. The Experimental Status of the Elementary Interaction in the Low Energy Region

One of the most interesting aspects of the (γ, π) reaction is that the theory of the elementary process^{4,5} is well understood and is in good agreement with experiment. The energy region that we are presently discussing is from threshold through the Δ region, i.e. lab photon energies from about 150 to 400 MeV. Here the total cross section shows the familiar, and large, Δ peak (see Fig. 4). In fact, as we shall show below, the agreement between theory and experiment is as good as the experiments.

The most well studied reaction is $\gamma p \rightarrow n\pi^+$. The best experiments quote absolute errors of ≈ 3 to 5%. However the discrepancies between different measurements are often several times the quoted errors, so that they have been underestimated in general. Fig. 5 shows the data⁶ for photon energies from 165 to 185 MeV. It can be seen that the discrepancies are often larger than the quoted errors; however in general terms the quality of the data is reasonable. In the Δ region the data^{7,8} is better as is seen in Fig. 6. This is typical of the Bonn data which is of high quality. They have studied both the π^+ and π^0 production for energies above 200 MeV.

The situation on the neutron is naturally not as clean as for the proton. The main problem is that one has to deal with deuteron targets or to study the inverse reaction $\pi^- p \rightarrow \gamma n$ which is experimentally more difficult, although theoretically cleaner. In

dealing with deuteron targets one usually employs the spectator model. A technique to minimize errors is to measure the $\gamma d \rightarrow pp\pi^-$ and $\gamma d \rightarrow pn\pi^+$ reactions and then to take the π^-/π^+ ratio. To first approximation the deuteron structure cancels out. Generally a small correction is applied which takes into account the difference of final state Coulomb interaction. Some more detailed experiments have also been performed^{9,10}. At DESY the ABHMM collaboration⁹ used a bubble chamber to study this reaction with photon energies from 200 MeV to 2 GeV. They could use bremsstrahlung because they observed all the three charged particles in the π^+pp final state. A careful job was done to study the spectator model and to extract the final cross sections. A sample of the results is shown in Fig. 7. It can be seen that there is good agreement with the Bonn and Tokyo results which were obtained with single arm experiments using the π^-/π^+ ratio technique. On the other hand there is poor agreement with the inverse reaction data^{11,12}.

More recently a group at the Saclay linac have also performed a coincidence experiment¹³ in the Δ region and have obtained the results shown in Fig. 8 which is in reasonable agreement with data from the inverse reaction. This implies a disagreement with the Orsay and DESY results although no discussion of this point is made in the paper.

It is clear that at least a review of this subject is required, and it is possible the new experiments are also required in the Δ region.

For the $\gamma n \rightarrow p\pi^-$ process below 200 MeV the data are even more skimpy. These data will be discussed below.

The low energy region is of considerable interest because of soft pion theorems. The experimental situation has been summarized by Adamovich⁶ who has done the most work in the field. Fig. 9 shows the data at different angles for the $\gamma p \rightarrow n\pi^+$ reaction and the extrapolation to zero kinetic energy. From this data one

obtains the value¹⁴ of the intercept $|E_0^+|^2 = (16.0 \pm 0.6) \mu\text{b}$.

In Fig. 10 the rather less accurate results for the $\gamma n \rightarrow p\pi^-$ reaction are shown which leads to an intercept¹⁴ of $|E_0^+|^2 = 20.2 \pm 2.6 \mu\text{b}$.

The results are tabulated in Table I and compared to various predictions including soft pion theorems, and one can see that the agreement is as good as the experiments. Although we have not discussed the (γ, π^0) reaction experiments we note several points. First, in the case of π^0 production the amplitudes are smaller and are far more sensitive to the details of the theory, and are therefore more interesting. Secondly there is no experimental result for π^0 production on the neutron. Because of the small predicted amplitude this experiment is extremely difficult. Finally we note that it would be difficult for the theory to predict a much different result. From this last point we conclude that although the experiments on the low energy regime, particularly on the neutron, are not very good, it is not likely that a surprising new result will be found.

For future reference we note that the total cross section σ just above threshold can be written as:

$$\sigma = a(p/k)$$

where p is the pion momentum and k is the photon momentum. The constant $a = 4\pi|E_0^+|^2$. The values quoted above give:

$$a_p = 201 \pm 7 \mu\text{b}$$

and

$$a_n = 254 \pm 32 \mu\text{b}$$

$$a_n/a_p = 1.26 \pm 0.16$$

We conclude this section by observing that in general the experiments on the elementary in photo-pion production from the nucleon are in reasonable shape although the situation could be improved at threshold and most particularly for the neutron. Further experiments on the inverse reaction are needed and a better

theoretical treatment of the deuteron and further tests of this theory by measurements of the pion spectra can be made. Nevertheless it is unlikely that large surprises are in store for us in this area.

III. Total Cross Sections for $(\gamma, \pi^{\pm})$ Reactions Just Above Threshold

A. Experimental Techniques for (γ, π^+) Experiments

The technique for these measurements is relatively simple and is indicated schematically in Fig. 11. π^+ mesons are photoproduced just above threshold so that they stop in the target and subsequently decay via the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay chain. The last step occurs with the mean life of the μ meson of $2.2 \mu\text{sec}$; detection of the resulting positrons (with an end point energy of 53 MeV) takes place after the beam burst which means that the background problems are minimized. Targets containing hydrogen, such as CH_2 , are used to calibrate the apparatus in terms of the reasonably well known elementary cross section (note that the threshold for π^+ production from C^{12} is 3.07 MeV higher than for the proton).

The quantity that is directly measured is not the cross section $\sigma(k)$, where k is the photon energy, but the bremsstrahlung yield, $Y(E_0)$;

$$Y(E_0) = \int_{E_T}^{E_0} \sigma(k) \phi(E_0, k) dk \quad (1)$$

where $\phi(E_0, k)$ is the bremsstrahlung shape function, E_T is the threshold energy and E_0 is the end point energy. These quantities are plotted in Fig. 12. It can be seen that even for a (γ, π^-) cross section, which has a sharp rise at threshold, that $Y(E_0)$ rises quite slowly from zero as E_0 is increased above threshold. This feature makes these measurements difficult just above threshold and is one of the reasons that these measurements were not possible until the advent of high intensity, high beam quality linacs. Another consequence of Eq. (1) is that $\sigma(k)$ has to be unfolded from the experimental values of $Y(E_0)$; this point will be discussed

below in connection with the results of specific experiments.

B. The $d(\gamma, \pi^+)nn$ Reaction

This is the simplest pion production reaction for a complex target. Therefore all of the possible complications that occur in complex nuclei come in to play, e.g. Fermi motion, off mass shell amplitudes, and the effect of the final state interaction. This simple system gives us the opportunity to test our ideas in a place where we have the most knowledge about the nuclear wave functions. It was quickly realized that since at low energies the pion-nucleon scattering length is small compared to the nucleon-nucleon scattering length, it is possible to neglect the final state pion nucleon interaction compared to the final state neutron neutron interaction¹⁵. In fact the cross section is quite sensitive to this latter interaction¹⁵.

The experiment was first performed at Bates¹⁶ and independently at Saclay¹⁷. As can be seen in Fig. 13 the results for the experimental yield curves are in good agreement. This is gratifying because it is the only case which has been done in both laboratories. Both experiments have been performed relative to the proton, whose yield curve is shown in Fig. 14.

To obtain the cross section $\sigma(k)$ from the yield, one must invert Eq. (1). The Bates group used polynomials to express the cross section as a function of the energy above threshold. The best fit coefficients and their errors were obtained by solving Eq. (1). The error envelope is plotted in Fig. 15. The method used at Saclay was to assume a functional form for the cross section and make a fit. The cross sections obtained in this manner are in agreement with those obtained at Bates but with smaller errors because the Bates analysis contains a contribution to the error caused by the uncertainty in the functional form. The method used at Saclay is specialized to the low energy regime (up to ≈ 3 MeV above threshold). In Fig. 15 we present the cross section based on a polynomial fit to all of the data. This is the first time that a unified consistent analysis of both sets of

data has been presented. We note that each set of data alone gives a cross section which is in very good agreement with the other. It should be noted that the most precise agreement comparison between theory and experiment should be made by comparing the yield curve calculated using the theoretical cross section.

The cross sections and yield curves are compared with various theoretical calculations in Figs. 13 and 15. As was originally pointed out by Tzara¹⁵ the omission of the final state neutron-neutron interaction makes the predicted cross section very small. Therefore all of the theoretical predictions have been made using the experimental value of the neutron-neutron scattering length, $a_{nn} = -16.7$ fm. The calculation of Koch¹⁶ used the $\vec{\sigma} \cdot \vec{e}$ interaction with s wave nucleons; it agrees with experiment in the low energy region but deviated above 3 MeV which indicates the need for higher nucleon partial waves. Calculations performed with higher partial waves (this calculation is not shown) by Noble¹⁸, Laget¹⁹, and O'Connell and Dressler²⁰ are in reasonable agreement with experiment. The differences between these calculations indicates the sensitivity of the cross section to the wave functions. Noble used a separable interaction and only the S state part of the deuteron wave function. O'Connell and Dressler used the Hamada-Johnston potential for both the deuteron bound state and the outgoing neutron-neutron wave functions.

In order to make a more stringent test of pion production in the two nucleon system, experiments which extend the energy range are required. The Bates group has extended the experiment to 20 MeV above threshold and the results should be available in the near future. This is about the maximum energy for the $\pi\mu e$ technique since above that energy the outgoing pions escape from the target.

Additional experiments of the differential cross section which measure the energy of the outgoing pion would be more specific and would tend to better discriminate between different calculations. These experiments are presently being planned at Bates using the pion spectrometer.

C. The ${}^6\text{Li}(\gamma, \pi^+) {}^6\text{He}$ Reaction

The first measurement using the π pe technique was for this reaction performed at Saclay²¹ and was 40% higher, lower than theory³. This caused an extensive re-examination of the calculation to take place. It was shown that the cross section was indeed insensitive to the final state strong interaction of the pion. There has also been a careful investigation of the nuclear physics part of the transition density. The original idea was to use the magnetic inelastic electron scattering from the ground state of ${}^6\text{Li}$ to $(1^-, T = 0)$ to the state which is the analogue of the ${}^6\text{He}$ ground state $(0^+, T = 1)$. A shell model basis was assumed and the coefficients were determined by fitting to the electromagnetic properties of the ${}^6\text{Li}$ ground state, the magnetic form factor, and the ft value of the ${}^6\text{He}$ β^+ decay. In this way the nuclear physics uncertainties were minimized but not completely eliminated. For example the M1 transition operator has both an orbital and a spin contribution, while the (γ, π) operator has just a spin term. It is therefore possible to obtain agreement with the M1 form factor with differing amounts of spin and orbital contributions and this difference will lead to different predictions for the (γ, π) cross section. The first calculation of this effect was performed by Bergstrom²² and then repeated in more detail by Camerata and Donnelly²³. In addition some experimental error involving the background evaluation was found and the new results²⁴ are in much better agreement with the theoretical predictions. A detailed comparison is presented in Table 2. It is concluded that there is no serious discrepancy between theory and experiment.

D. The ${}^{12}\text{C}(\gamma, \pi^-) {}^{12}\text{N}$ Reaction

The (γ, π^-) reaction is of interest because it reflects the elementary $\gamma n \rightarrow p \pi^-$ process in a complex system. Experimentally this particular reaction is of interest for two reasons; first is that the $A = 12$ system has been studied by electron scattering and the wave functions are reasonably well known; second is the

fact that ^{12}N has only one particle stable state. This enables us to perform an experiment²⁵ in which the residual radioactivity of ^{12}N is detected and the resulting cross section is for the one stable state. The advantage of such an experiment is that one can in principle go very close to threshold. The practical disadvantage is that secondary processes in the target such as $^{12}\text{C}(\gamma, n)$ followed by $^{12}\text{C}(p, n)^{12}\text{N}$ produce the same final state and therefore limit the accuracy of the cross section measurement close to threshold. Another experimental problem is that one cannot normalize relative to the elementary $\gamma n \rightarrow p\pi^-$ reaction but must make an absolute measurement. For these reasons (γ, π^-) reactions are more difficult to study than their (γ, π^+) counterparts.

The $^{12}\text{C}(\gamma, \pi^-)^{12}\text{N}$ reaction cross section just above threshold has been measured²⁵ and the results for the yield and cross section are presented in Fig. 16. We note that the results are consistent with a sharp rise of the cross section at threshold as was predicted. A simple parameterization of σ which has a step at threshold and a linear slope above that energy was made and the results are presented as the "best fit" in Fig. 16. The results of several calculations are also shown. The calculation of Koch, which is an improved version of an earlier version³, employs the $\vec{\sigma} \cdot \vec{\epsilon}$ interaction and s wave pions. This calculation was only meant to be accurate in the near threshold region, and it clearly does not have the correct slope above threshold. The calculations of Nagl and Uberall²⁶ employ the full Hamiltonian and use several pion partial waves. The nuclear transition density was taken from a Helm model fit to the magnetic inelastic electron form factor to the 15.1 MeV analogue of the ^{12}N ground state in ^{12}C . Curve one replaces the pion momenta appearing in the interaction Hamiltonian by a gradient operating on the distorted pion wave function. Curve two does that and it additionally uses the local pion momenta in the amplitudes. It is clear that the Nagl and Uberall calculation represents an improvement, but it is neither a completely satisfactory theoretical calculation, nor is it in agreement with the experiment.

A recent careful calculation²⁷ has resolved the discrepancy between theory and experiment. The main ingredients of this calculation are: 1) the same nuclear wave functions, obtained from the electromagnetic properties of the $A = 12$ system as used by Koch and Donnelly; 2) the $\vec{\sigma} \cdot \vec{e}$ interaction but with higher partial pion waves; 3) a careful treatment of the pion optical potential. In Fig. 17 we show the calculation using the interaction with s wave pions and with higher pion partial waves which are seen to be important. In Fig. 17 the effect of normalizing the elementary amplitude to the experimental data are shown. It is clear that the agreement between the present experiment and theory is as good as can be expected at the present state of knowledge of the elementary cross section.

We conclude that in order to make more progress on (γ, π^-) experiments in complex nuclei, we will have to improve the experimental situation for the elementary amplitude.

E. Other Reactions

The $^{11}\text{B}(\gamma, \pi^-)^{11}\text{C}$ reaction has been measured at Bates,²⁸ and the cross section is consistent with a step function rise for each final state reached in the process. The magnitude of the step is consistent with theory although the errors are not small.

There are several (γ, π^+) cross section measurements which have been performed and which are presently being written up for publication. These include the $^9\text{B}(\gamma, \pi^+)^9\text{Be}$, $^{12}\text{C}(\gamma, \pi^+)^{12}\text{B}$, and $^{16}\text{O}(\gamma, \pi^+)^{16}\text{N}$ reaction studied at Bates and the $^3\text{He}(\gamma, \pi^+)^3\text{H}$ studied at Saclay. In a recent preprint agreement it is shown with an impulse model calculation.²⁹ On the other hand a study of the $^3\text{He}(e, e' \pi^+)^3\text{H}$ reaction at Saskatchewan in which the recoil ^3H was detected³⁰ showed deviations from the simple impulse approximation.

IV. Future Trends and Conclusions

The previous discussion concentrated on work that has been completed and was primarily on charged pion photo-production total cross sections just above threshold. At the present time

this sphere of activity is declining in favor of several different activities. One is π^0 photoproduction just above threshold. At Saclay the thrust is to study light nuclei with the motivation to get at the elementary neutron amplitude. At Bates the motivation is more towards nuclear physics and towards complex nuclei, both just above threshold and in the Δ region. At Bates and at Sendai programs are being carried out in the low energy charged pion reactions with magnetic spectrometers. Both laboratories have plans to construct second generation spectrometers. One can expect that cross differential cross section measurements will be carried out to discrete nuclear final states, both in the near threshold region and through the Δ region. At Bates plans are being made to perform (γ, π^0) reactions also.

Another line of activity is to measure total cross sections for inclusive hadron production. At Bonn this is proceeding with tagged photon beams and at Saclay plans are being made in this direction using the monochromatic photon beam facility which is being developed with the positron beam. These experiments aim at detecting all of the produced hadrons, both in singles and in coincidence measurements. At Bates plans are being made to participate in the field but with a somewhat different emphasis. At Saclay the very beautiful work on $(\gamma, p\pi^-)$ coincidence measurements will be extended.

One can conclude that the field of photo- and electro- pion physics will undergo a quantum jump in the next few years.

References

1. M. Ericson and M. Rho, Phys. Reports 5C, 59 (1972).
2. L.M. Saunders, Nucl. Phys. B7, 293 (1968).
3. J.E. Koch and T.W. Donnelly, Nucl. Phys. B64, 478 (1973), and Phys. Rev. 10C, 2618 (1974).
4. G.F. Chew, M.L. Goldberger, F.E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).
5. K.I. Blomqvist and J.M. Laget, Nucl. Phys. A280, 405 (1977).
6. M.I. Adamavich, Proceedings (Trudy) of the P.N. Lebedev Physics Inst., Vol. 71, Photonuclear and Photomesic Process (1976).
7. G. Von Holtey, Springer Tracts in Modern Physics Vol. 59, G. Hohler, editor, 1971.
8. K.H. Althoff, Article in these proceedings.
9. P. Benz et al., Nucl. Phys. B65, 158 (1973) ABHHM collaboration at DESY.
10. J. Boucrot et al., Nuovo Cimento 18, 635 (1973).
11. P.A. Berardo et al., Phys. Rev. Lett. 26, 20 (1971).
12. J. Favier et al., Phys. Lett. 31B, 609 (1970).
13. P.E. Aragan et al., to be published.
14. Compilation of Coupling Constants and Low Energy Parameters, M.M. Nagels et al., Nucl. Phys. B, 1 (1976).
15. C. Tzara, Nucl. Phys. A256, 381 (1976).
16. E.C. Booth et al., Phys. Lett. 66B, 236 (1977).
17. G. Audit et al., Phys. Rev. 16C, 1517 (1977).
18. J. Noble, Phys. Lett. 67B, 39 (1977).
19. J.M. Laget, Nucl. Phys. A296, 388 (1978).
20. E.T. Dressler, W.M. MacDonald, and J.S. O'Connell, to be published.
21. J. Deutsch et al., Phys. Rev. Lett. 33, 316 (1974).
22. J.C. Bergstrom, I.P. Auer, and R.S. Hicks, Nucl. Phys. A251, 401 (1972).

References - Cont'd

23. J.B. Cammarata and T.W. Donnelly, Nucl. Phys. A267, 365 (1976).
24. G. Audit et al., Phys. Rev. 15C, 1415 (1977).
25. A.M. Bernstein et al., Phys. Rev. Lett., 37, 819 (1976).
26. A. Nagl and H. Uberall, Phys. Lett. 63B, 291 (1976).
27. G.W. Epstein, M.K. Singham and F. Tabakin, Phys. Rev. C17, 702 (1978).
28. K. Min et al., Phys. Rev. 14C, 807 (1976).
29. P. Aragan et al., to be published.
30. D.M. Skopic et al., to be published.
31. C. Tzara, Nucl. Phys. B18, 246 (1970).

Fig. 1 Pion mean free path λ versus pion kinetic energy. ρ_0 is the central nuclear density and $\bar{\sigma}_{Tot}$ is the average π^+p and π^-p total cross section. The contribution from pion absorption have not been taken into account.

Fig. 2 The $C^{12}(\gamma, \pi)$ reaction and other reactions to the same final states.

Fig. 3 Total cross section calculated for the $C^{12}(\gamma, \pi^{\pm})$ reaction just above threshold³ as a function of k_{π} , the outgoing pion momentum. The momentum of 30 MeV/c $_{\pi}$ corresponds to approximately 3 MeV pion kinetic energy, which is just above the Coulomb barrier.

Fig. 4 Total (γ, π) cross section on the proton versus photon energy.

Fig. 11. Angular dependence of the differential cross sections of π^+ mesons generated by photons with the energies 165 (a), 170 (b), and 175 MeV (c). 1) Our data; 2) data of [97]; 3) data of [74]; 4) data of [87]. The dashed curves denote the approximation B + I(33); the dash-dot curves indicate the approximation B + I(33) + I(13); the solid curves indicate versions A and B of the approximation B + I(33) + I(13) + I(SP).

Fig. 12. Angular dependence of the differential cross sections of the production of π^+ mesons by photons with the energies 180 (a), 185 (b), and 190 MeV (c). 1) Our data; 2) data of [97]; 3) data of [86]; 4) data of [87]. The labeling of the curves is the same as in Fig. 11.

Fig. 5 Differential cross sections for the $\gamma p \rightarrow n\pi^+$ reaction just above threshold (Figs. 11 and 12 from Adamovich⁶ with references to the original data given there).

Fig. 6 Differential cross sections for the $\gamma p \rightarrow n \pi^+$ reaction in the Δ region (Bonn data, Ref. 7)

Fig. 13. CMS differential cross sections $d\sigma/d\Omega^*$ of the reaction $\gamma n \rightarrow \pi^- p$ at $E_{\gamma}^{\text{eff}} = 350$ MeV and $E_{\gamma}^{\text{eff}} = 500$ MeV. The black points are our data in the closure approximation. In addition to the statistical errors shown, there is an overall normalization uncertainty of 5%. Also shown are the photoproduction measurements of the Tokyo (ref. [3], squares) and Bonn groups (ref. [4], triangles; $E_{\gamma} = 360$ MeV). This is compared with the results of Berardo et al. (ref. [12], diamonds; $E_{\gamma} = 354$ MeV, and $E_{\gamma} = 480$ MeV and 518 MeV respectively) and Favier et al. (refs. [51, 52], crosses) as predicted by detailed balance from measurements of the inverse reaction $\pi^- p \rightarrow \gamma n$.

Fig. 7 Differential cross sections for the $\gamma n \rightarrow p \pi^-$ reaction. Data from the $\gamma D \rightarrow \pi^- p p$ reaction⁹ (see original for references).

Fig. 8 Total cross section for the $\gamma n \rightarrow p\pi^-$ reaction versus energy from the Saclay $\gamma D \rightarrow pp\pi^-$ coincidence experiment¹³. The shaded zone represents the photon data. The points represent the results of the inverse reaction (see original for references but note ABBHHM⁹, NFPR¹², CERN¹¹).

Fig. 36. Approximation of the dependence of $(k/q) \times (d\sigma/d\Omega)$ on q^2 for $\theta = 33^\circ$ (a), 50° (b), and $\arccos[(k\omega - k_0)/kq]$ (c). 1) Our data; 2) data of [97]; 3) data of [87]; 4) data of [86]; 5) data of [74].

Fig. 37. Approximation of the dependence of $(k/q) \times (d\sigma/d\Omega)$ on q^2 for various angles θ . 6) Data of [90]; 7) data of [88]; the rest of the notation is the same as in Fig. 36.

Fig. 9 Extrapolation of $\gamma p \rightarrow n\pi^+$ data to the limit of pion momentum $q \rightarrow 0$. Note that q is given in units of $m_\pi c$. Figure from Adomovich⁶.

Fig. 10 Extrapolation of $\gamma n \rightarrow P\pi$ data ⁶ to limit of pion momentum $q \rightarrow 0$. The units of q are in $m\pi c$. The quantity $|T|^2 = (k/q)\sigma(0)$ which is the same as is plotted in Fig. 9.

Fig. 11 Schematic representation of the apparatus to measure (γ, π^+) total cross sections just above threshold. Part (a) shows the physical layout of the target and detectors. Four are indicated, the thick ones labeled C_1 and C_2 are Cerenkov counters and the thin ones are scintillation counters. Thin absorbers between the detectors are not indicated. Part (b) shows the time sequence of beam and counting times to observe positions from the $\pi^+ \rightarrow \mu^+ \rightarrow \beta^+$ decay chain.

$$Y(E) = \int_{E_T}^E \Phi(E'; E) \sigma(E') dE'$$

Fig. 12 Schematic relationship of the (γ, π) total cross section $\sigma(E)$, the Bremsstrahlung flux function $\Phi(E, E_0)$ and the yield $Y(E)$.

Fig. 13 Experimental yield Y (per electron) versus electron energy.

Fig. 14 Experimental yields¹⁶ for the $\gamma p \rightarrow \pi^+ n$ and $\gamma D \rightarrow \pi^+ nn$ reactions versus ΔE , the energy above threshold.

Fig. 15 Total cross section as obtained from the combined Saclay¹⁷ and Bates¹⁶ yield curves versus ΔE , the energy above threshold.

Fig. 16 Results for the $C^{12}(\gamma, \pi^-)N^{12}$ total cross section just above threshold²⁵. Parts (a) has the yield curve and (b) the total cross section shown by the cross hatched area. For a description of the calculations see the text and the original article²⁵.

Fig. 17 Recent theoretical calculations for the $C^{12}(\gamma, \pi^-)N^{12}$ reaction just above threshold.

TABLE 1

The experimental and theoretical values of the dipole pion photoproduction amplitudes E_{0+} in units $10^{-3} m_{\pi}^{-1}$

Channel	Experiment ^a	Born PV ^b	Born FS ^c	Current ^d Algebra
$\gamma p \rightarrow n\pi^+$	28.6 ± 0.14	28.	28.1	29.0
$\gamma n \rightarrow p\pi^-$	-31.5 ± 1.0	-32.2	-31.7	-33.2
$\gamma p \rightarrow p\pi^0$	-1.8 ± 0.6	-2.5	-8.1	-2.4
$\gamma n \rightarrow n\pi^0$	0.3 ± 0.9^e	0.4	5.6	0.3

- a) Ref. 6
- b) P. Bosted and J. M. Laget, Nucl. Phys. A296, 413(1978)
- c) P. deBaenst, Nucl. Phys. B24, 633(1970)
- d) R. D. Pecci, Phys. Rev. 181, 1902(1969)
- e) From isospin using the other three experimental values in the table.

This table was prepared by M. deBotton

TABLE II

Comparison of theory and experiment for the $\text{Li}^6(\gamma, \pi^+)\text{He}^6$ reaction²⁴.

AUTHORS	OPERATOR	PION-WAVE DISTORSION	NUCLEAR WAVE-FUNCTION		$\frac{a(^6\text{Li} \rightarrow ^6\text{He})}{a(p \rightarrow n)}$ thres.
			Configuration of ^6He g.s.	Radial behaviour	
J.C. Bergström et al. 12)	$\vec{\sigma} \cdot \vec{\epsilon}$	Mean optical potential (Krell-Ericson) ²¹⁾ "	SASK A*	Phenomeno- logical	.091
			SASK B**		.101
J.B. Cammarata and T.W. 20) Donnelly	$\vec{\sigma} \cdot \vec{\epsilon}$	"	STAN**	Harm.Oscil.	.113
	"	"	STAN	Saxon-Wood	.108
	"	"	SASK-A	Phenomeno- logical	.093
	"	"	SASK-B	"	.109
	Elementary particle treatment		.106		
J. Delorme and A. Figureau 18)	$\vec{\sigma} \cdot \vec{\epsilon}$	Optical potential adjusted to mesic X-rays of light nuclei 22)	SASK A	Phenomeno- logical	.097
	"		SASK B	"	.108
	momentum dependent terms included	"	SASK A	"	.105
		"	SASK B	"	.117
THIS EXPERIMENT (0 < ω < 2 MeV)			$a(^6\text{Li} \rightarrow ^6\text{He})/a(p \rightarrow n) = .098 \pm .04$ ***		

*) Strong L=1 admixture

**) Weak L=1 admixture

***) Comparing this result to the theoretical predictions, one should recall that it was deduced assuming $a(\text{}^6\text{Li} \rightarrow \text{}^6\text{He})/a(p \rightarrow n) = \text{cte}$ in the energy-region of interest [$0 < \omega < 2$ MeV]. As discussed in 3.3., this approximation has a negligible influence for reasonable variations of $a(\text{}^6\text{Li} \rightarrow \text{}^6\text{He})$ and $a(p \rightarrow n)$.

III.5 HIGH ENERGY PHOTONUCLEAR REACTIONS AND ISOBAR-NUCLEAR DYNAMICS*

E. J. Moniz

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

*This work is supported in part through funds provided by
ERDA under Contract EY-76-C-02-3069.*000.

This is intended to serve as a discussion paper for the M.I.T.
Workshop on Nuclear Physics with Electron Accelerators
(Cambridge, June, 1977)

June 1977

CTP #646

A fundamental challenge in nuclear physics has been the investigation both of exceedingly diverse and complicated many-body phenomena and simultaneously of the underlying two-body force. Considerable progress has been made in recent years on both fronts. The nucleon-nucleon force can be understood qualitatively in terms of meson exchange and, for the medium- and long-range parts of the force, quantitative agreement has been obtained between successful phenomenological potentials and interactions constructed from the fundamental pion-nucleon dynamics. Key ingredients in obtaining this agreement are the long-range one-pion-exchange potential (OPEP) and the medium-range potential arising from two-pion exchange with intermediate (virtual) excitation of the nucleon isobar $\Delta(1232)$. On the other hand, significant advances have been made also in microscopic theories of single-particle and collective motion in nuclei (the unifying concept being that of the average field). For example, modern high precision, high momentum transfer electron scattering experiments are revealing the details of nuclear ground state and transition charge densities, and theoretical calculations have yielded impressive agreement with many of these data. Perhaps, surprisingly, this agreement has been achieved largely with very simple effective forces which, while retaining some of the general features known to be important in the nuclear force (such as spin and isospin dependence), reflect only little of our evolving understanding of the two-

nucleon interaction. Obviously, it is hoped that this situation will change. A deeper insight into and more precise description of nuclear dynamics will require quantitative understanding of the role of the fundamental pion and isobar degrees of freedom in the nuclear medium.

The electromagnetic interaction, being weak and well understood, is well suited to revealing the pionic and isobaric presence in nuclei. For subthreshold pions and isobars, backward electrodisintegration of the deuteron has unambiguously demonstrated the importance of exchange currents. Photopion production close to threshold is providing new information on the propagation of low energy pions in nuclei (this region is inaccessible with pion beams). Both the sub- and near-threshold regions will be explored in much more detail and will benefit greatly from improvements in electron scattering facilities (eg., coincidence measurements and tagged photons will be very important for exchange current and low energy pion studies, respectively). In this workshop discussion paper, I will discuss the higher energy region in which photon-nucleon interactions are dominated by Δ -excitation (i.e., ~ 300 MeV photon energy). Photoreactions will be emphasized, although electron scattering (i.e., virtual photon induced reactions) will be discussed also.

Isobar-nucleus physics

What are some of the things we hope to learn from pion and photon induced reactions in the resonance region? Of course, we hope to learn about nuclear structure with pion scattering; for example, double charge exchange is possible only with pions. Beyond this, we hope to extract information on the dynamics of isobar (and therefore pion) propagation in the nuclear medium; this information is directly related to the fundamental strong interaction. We hope to learn about isobar-nuclear collective "states"; these reflect the full nuclear many-body dynamics and play the role of doorway states for pion reactions. We hope to learn quantitatively about the pion absorption mechanism. It is this feature which provides a direct link between pion reactions and the nuclear force and it is known that absorption plays an important part in describing pion-nuclear interactions (for example, half the inelastic cross section for π - ^{12}C scattering at 130 MeV is given by absorption). This list could be extended but perhaps this is a good time to ask another question instead. Why study pion/isobar physics with photons? To the extent that Δ -excitation acts as the "entrance channel" for pion and photon reactions, basically the same physics is being studied. However, because of the weakness of the electromagnetic interaction, the photon illuminates the entire nucleus, whereas pion induced reactions take place primarily in the surface. Since much of the interesting isobar physics is presumably enhanced at higher nuclear

densities, the photon and pion can give complementary information. Further, we shall see that some experiments are uniquely suited to the electromagnetic probe.

We now go on to discuss more specifically some of the relevant photonuclear experiments. It should be clear that the list discussed here is in no sense intended to be exhaustive.

Photoreactions in the resonance region

1. No hadrons detected.

We discuss first completely inclusive experiments in which no hadrons are detected in the final state; this means measuring the total photoabsorption cross section or the (e,e') inelastic cross section in the resonance region. The main point here is that the inclusive reaction gives the most direct measure of single particle properties in the nucleus. This is well known in the quasielastic region. Nuclear Fermi momenta have been extracted from high energy quasielastic scattering¹ across the Periodic Table, and at lower energies an effective mass for nucleons can be extracted² (see Figure 1). The same should be possible for the Δ if good experiments are carried out in that region. In principle, photoabsorption on large nuclei is the most interesting theoretically (real photons deliver less momentum for the same energy than spacelike photons). Nevertheless, some older data taken by Titov et al³ for (e,e') on tungsten show an interesting narrowing effect in the isobar region (see Figure 2). Such experiments should be done systematically on a variety of nuclei.

Another interesting feature in the (e,e') inelastic spectrum is the excessively large cross section in the region between the quasielastic and quasifree Δ -excitation peaks¹. This may be related to exchange current effects⁴. A re-examination of this region might prove very interesting, especially if the transverse response functions were separated and if a pion or nucleon were detected in coincidence.

We note that Compton scattering (γ, γ') in the resonance region would also be extremely interesting, although presumably it is difficult to measure. With intermediate isobar excitation, this reaction is analogous to pion scattering but without the limitation to the nuclear surface region. Compton scattering would provide an interesting probe of collective isobar-hole doorway states⁵.

2. One hadron detected

We consider next experiments in which one hadron is detected. We start with a discussion of incoherent processes, which are best for isolating the reaction mechanism apart from uncertainties due to nuclear structure.

Inclusive (γ, N) experiments (with reasonable energy resolution for the photon) in the resonance region provide a good test of the pion absorption mechanism (assuming that the photon produces a Δ). The high energy inclusive proton spectrum (but still far from the kinematical end point; i.e., in the range 50-150 MeV) should be dominated by events in which the pion is absorbed ("radiationless damping" of the Δ). The analogous result for incident pions has been confirmed with intranuclear cascade calculations⁶ which include the Δ absorption mechanism and do fairly well in reproducing the data. However, for incoming pions, pion absorption takes place mostly in the nuclear surface region "facing" the incoming pion beam (see Figure 3)⁶. This would not be the case for photoreactions and the absorption mechanism could be tested in the high density region of the nucleus.

The inclusive $(\gamma, \pi^\pm)$ reaction is analogous to quasielastic pion scattering. A thorough understanding of the quasielastic and absorption mechanisms is essential to understanding the pion optical potential, since these channels carry almost all the inelasticity in pion-nucleus reactions and therefore determine directly the imaginary part of the optical potential. Quasielastic pion scattering remains virtually unexamined experimentally. Again, the photoreaction carries the possibility of being extremely interesting by being more sensitive to the quasifree process occurring in the nuclear high density region. Further, it may be very instructive to examine the (γ, π) spectrum down to very low pion energies (for high photon energies). This can give us an idea how the pion "thermalizes" and, by seeing how the spectrum cuts off at low pion energies, how the absorption takes place through multistep processes. Because of the differing absorption probability distributions in the pion and photon cases, it may be especially important here to have both sets of data. As an example, Figure 4 shows the spectrum of pions back scattered from ^{27}Al ⁶; the incident energy was 220 MeV. The error bar on the histogram indicates the statistical accuracy of the Monte Carlo calculations⁶. Also shown (dashed line) is the free Fermi gas spectrum with arbitrary normalization. The results are obviously quite different, the impression being that of a significant narrowing of the quasifree peak. At the upper end of the spectrum (final $T_\pi \gtrsim 120$ MeV), we can probably explain

this in terms of multistep processes "degrading" the pion energy, thereby shifting the spectrum to lower energies. However, the low energy part of the spectrum is entirely suppressed and this is presumably due to absorption. Consequently, the spectrum contains information on the multistep nature of the scattering and absorption process; this is fundamental for understanding the success of the first order optical potential and the second order absorption term.

Exclusive single-arm photoreactions are also of considerable interest. The high energy (γ, p) reaction, leaving the residual nucleus in the ground state, has been discussed extensively⁷. Here, we only point out that understanding the absorption mechanism in the (γ, N) inclusive reaction is important for separating the effects of isobar excitation and high momentum components in high energy exclusive (γ, p) reactions.

Coherent (γ, π^0) studies can be an especially important source of information on the pion optical potential in the resonance region. Equivalently, this is a good probe of the collective isobar-hole doorway states which dominate pion elastic scattering. First, elastic pion scattering can be used only for charged pions. Second, charge exchange reactions $(\pi^\pm, \pi^0)$ or $(\gamma, \pi^\pm)$ to the neighboring nuclear ground state are sensitive mostly to the nuclear surface region⁸ because that is where the nuclear transition density is peaked. Consequently, the (γ, π^0) reaction offers a source of supplementary information. It is perhaps worth noting that "old" data available for this reaction

indicate surprisingly little absorption for the outgoing π^0 and it is important that these experiments be repeated.

3. Two hadrons detected

It is obvious that for examining the properties of the Δ in the nucleus, an important experiment is the coincidence measurement ($\gamma, \pi N$). Again, because the photon penetrates into the nucleus, the Δ will be formed uniformly throughout the nuclear volume, and the invariant mass distribution for the πN pair shows directly collision effects on the Δ "line shape". It is interesting to speculate on the consequences of having strong pion absorption. Presumably, those isobars which have a long path length through the nucleus (and therefore pass through the high density region) will suffer strong distortions, presumably broadening, of the mass distribution. However, if the pions are absorbed in these cases, the results of the ($\gamma, \pi N$) measurements may then be a modestly altered mass distribution but with considerably diminished magnitude. In fact, the A -dependence should then be $A^{2/3}$, whereas the analogous pion reaction ($\pi, \pi N$) would give $A^{1/3}$. Consequently, even the qualitative behavior (general characteristics of the line shape and A -dependence) can tell us a great deal about the role of the absorption mechanism.

4. Other reactions

This section is primarily intended to remind us that the above list is simply representative and not in any sense exhaustive. First, all the photoreactions could be studied with

electrons, which in principle can yield more information.

Further, polarization studies could be interesting; for example, one may be able to separate different components of the $NN \rightarrow N\Delta$ "transition potential". Also, we have not discussed high energy photopion excitation of nuclear collective states. It is clear that high energy electromagnetic interactions can make important and, in some cases, unique contributions in the study of pion- and isobar-nucleus dynamics.

References and Footnotes

1. E. J. Moniz et al., Phys. Rev. Lett. 26 (1971) 445.
2. E. J. Moniz, Phys. Rev. 184 (1969) 1154.
3. Yu. I. Titov et al., Soviet Journal of Nuclear Physics 13 (1971) 660.
4. T. deForest and T. W. Donnelly, private communication.
5. M. Hirata et al., papers to be published; preliminary results can be found in the talks given by F. Lenz and E. J. Moniz, Proceedings of the International Conference on Meson-Nuclear Physics, P.D. Barnes et al., editors (Carnegie-Mellon, 1976).
6. J. Ginocchio, to be published.
7. J. L. Matthews et al., Phys. Rev. Lett. 38 (1977) 8.
8. A particular exclusive charge exchange reaction which would be interesting is the $^{13}\text{C}(\gamma, \pi^-)^{13}\text{N}(\text{g.s.})$. This reaction has been studied with (π^+, π^0) and is quite sensitive to the pion optical potential.
9. H. Crannell, talk given at Bates Midwinter Workshop (January 1977).

Figure Captions

- Figure 1: Inelastic electron scattering from ^{12}C :
 $\epsilon_1=148.5$ MeV, $\theta=135^\circ$, $k_F=235$ MeV/C. Full and dashed curves show the calculation with an effective mass $M^*=M/1.4$ and $M^*=M$, respectively, while the dotted line indicates the effect of removing the Pauli restriction on the struck nucleon. Data are those of Leiss and Taylor (unpublished); calculations given in Ref. 2.
- Figure 2: Inelastic electron scattering on tungsten.
 Data points from Ref. 3; calculations following Ref. 2.
- Figure 3: Absorption probability per volume for pions incident on nickel at 220 MeV. Calculations are based upon the intranuclear cascade model and are taken from Ref. 6.
- Figure 4: Backward inelastic scattering of pions from aluminum at 220 MeV. The histogram represents the Monte Carlo results of Ref. 6; the error bar indicates the size of the statistical uncertainty in the calculation. The dashed curve comes from the free Fermi gas model (Ref. 2), arbitrarily normalized.

Fig. 1

$^{184}\text{W}_{74}$

$k_F = 263 \text{ MeV}$

$\bar{\epsilon} = 42 \text{ MeV}$

$\epsilon_1 = 1.18 \text{ GeV}$

$\theta = 35^\circ$

220 MeV π^+ On ^{62}Ni — Absorption Probability Per Volume
In Units Of F^{-3}

— Nuclear Density Contour Lines

III.6 Nuclear Physics Experiments at DESY =====

E. Rössle, Fakultät für Physik, Freiburg, West Germany

A collaboration of the Universities of Bonn - Freiburg - Karlsruhe - Lund and DESY is performing nuclear physics experiments at DESY [1 - 4]. In the following I will confine myself to two topics of single arm experiments: The total cross section of virtual photons (1) in the quasielastic and resonance region and (2) in the deep inelastic region (shadowing of virtual photons).

(1) Quasielastic Scattering and Production of the Δ_{33} -Resonance in Nuclei

The electron scattering from nuclei at high energies is dominated by the quasielastic scattering and the production of resonances. Particularly the excitation of the Δ_{33} -resonance is pronounced at energies in the GeV-region. This is demonstrated in the energy spectrum of scattered electrons, measured in a single arm experiment which is equivalent to the measurement of the total cross section of virtual photons at the corresponding invariant mass. As an example the electron spectrum from ^{12}C at 15° for an incident energy at 2 GeV is shown in fig. 1. The measured spectrum has been analyzed with the assumption that three processes are contributing, namely the quasielastic scattering, the Δ_{33} -production, and the second resonance at 1520 MeV. The shapes of the first two contributions have been calculated in the impulse approximation using shell model wave functions and elementary cross sections for free nucleons, i.e. the electromagnetic form factors for the quasielastic part and the model of Gutbrod and Simon for Δ_{33} contribution. The second resonance has been approximated by a Breit-Wigner shape. The amount of these contributions has been obtained from the best fit to the experiment.

For quasifree processes one expects that the nuclear cross section (σ_A) is given by the incoherent sum of the corresponding nucleon cross sections (σ_p, σ_n), i.e. that the ratio of

effective (A_{eff}) and real mass number (A)

$$\frac{\sigma_A}{Z\sigma_p + N\sigma_n} = \frac{A_{\text{eff}}}{A}$$

is equal to one.

The results for the quasielastic scattering on ${}^6\text{Li}$ and ${}^{12}\text{C}$ are given in fig. 2 together with CEA data [5] on ${}^{12}\text{C}$. At high momentum transfers the ratio is as expected. At low $|q^2|$ the ratio drops drastically and seems to approach zero for vanishing momentum transfer. The solid line represents a calculation of Bernabeu [6] for ${}^{12}\text{C}$ which takes into account the binding effect. The form factor of bound nucleons has to vanish for $|q^2| = 0$ because of the Pauli principle.

The results for Δ_{33} -production are given in fig. 3. There is no marked q^2 -dependence. The average value of A_{eff}/A is about 0.6 for ${}^6\text{Li}$ and ${}^{12}\text{C}$. Phase space reduction for the decay nucleon of the resonance is estimated to be a few percent only. There is no explanation so far for the suppression of the Δ_{33} -production in nuclei.

The data have been taken with good statistics in order to search for a fine structure in the resonance due to the nucleon- Δ interaction. No statistically significant structure has been found in the nuclei ${}^6\text{Li}$, ${}^{12}\text{C}$ and ${}^{27}\text{Al}$. The quasifree production is therefore the dominant process.

(2) Shadowing of Virtual Photons

The total cross section of real photons on nuclei does not increase linearly with mass number as would be expected from the size of the elementary cross section on nucleons which corresponds to a mean free path in nuclei of about 700 fm. Instead the mass number dependence follows the relation $A_{\text{eff}} \propto A^{0.9}$. The interpretation of this experimental result is based on the model of vector meson dominance (VMD). In the model of VMD it is assumed that the photon-hadron interaction

is acting via an intermediate vector meson. This is equivalent to say that the photon has a hadronic component which increases with rising photon energy, and it can be perceived at photon energies exceeding several GeV.

The application of VMD to the total cross section of virtual photons predicts also a shadowing effect with decreasing magnitude as the momentum transfer increases. The scale of this variation with momentum transfer is given by the vector meson masses, particularly by that of the ρ -meson.

The experimental situation is not yet settled. Early experiments have failed to detect shadowing even in a region where it was predicted by VMD, whereas recent results reveal shadowing but of different magnitudes.

At least part of the inconsistencies can be ascribed to differences in the application of radiation corrections which have to take into account binding effects as well as in the use of the elementary nucleon cross sections, particularly the neutron cross section.

Our results on ^{12}C and ^{27}Al are given in fig. 4 and 5, where the ratios A_{eff}/A are plotted versus the scaling variable $x = \frac{q^2}{2M_N}$. In fig. 5 we have included results of Stein et al. [8] at 13 GeV. For these data the same nucleon cross section has been taken as we have applied. The solid lines are the predictions of VMD by Schildknecht [7].

Our results are in qualitative agreement with the predictions of VMD for the shadowing of virtual photons.

References

- 1) F. Heimlich et.al. Nucl. Phys. A228 (1974), 478 74/20
DESY Report 74/20
- 2) G. Huber Dissertation Freiburg 1976
S. Hartwig et.al. DESY Report 77/55
- 3) S. Hartwig et.al. {Lett.N.Cim. 12, (1975), 30;
Lett.N.Cim. 15, (1976), 429;
DESY Report 77/40
- 4) F. Heimlich et.al. Nucl. Phys. A267 (1976), 493
- 5) K. C. Sternfield et.al. Phys. Rev. C3 (1971), 1448
- 6) J. Bernabeu Nucl. Phys. 849 (1972), 186
- 7) D. Schildknecht Nucl. Phys. B66 (1973), 398
- 8) S. Stein et.al. Phys. Rev. D12 (1975), 1884

Figure Captions

1. Energy spectrum of electrons scattered from ^{12}C at 2 GeV and 15° . The curves drawn are (a) quasielastic contribution, (b) Δ_{33} -production, (c) second resonance contribution, (d) sum to fit the experimental data.
2. The ratio A_{eff}/A for the integrated quasielastic peak as function of momentum transfer.
3. The ratio A_{eff}/A for the integrated production cross section of the Δ_{33} -resonance as function of momentum transfer.
4. The ratio A_{eff}/A as function of the scaling variable $x = \frac{q^2}{2M_V}$ for ^{12}C . Solid line: VMD calculation [7]
5. The ratio A_{eff}/A as function of the scaling variable $x = \frac{q^2}{2M_V}$ for ^{27}Al . Included are SLAC data [8] and VMD calculation [7].

FIG.1.

QUASIELASTIC PEAK

FIG.2.

$\Delta(1236)$ -RESONANCE

FIG.3.

FIG. 4.

FIG.5.

III.7 Coherent Photo- and Electroproduction of Δ 's in Nuclei

Manfred Dillig and Max G. Huber
Institute for Theoretical Physics

University of Erlangen-Nürnberg, Erlangen, West Germany

Abstract

In this note we point out some of the reasons for investigating nuclei by real or virtual photons at excitation energies between 200 and 400 MeV; particular emphasis is put on the coherent photoproduction of pions where the final nucleus is left in a bound (ground or excited) state. The notion of collective A^* -resonances (as opposed to the well known elementary N^* -resonances) is exemplified for the deuteron and for ^{12}C in a OBEP-model.

I. Introduction

The investigation of the $\Delta(3,3)$ -excitation of a nucleon which is bound in a nucleus is one of the most interesting problems in intermediate energy physics for various reasons:

- (i) Considering the Δ as an elementary particle it is interesting to learn something about the ΔN -interaction. Obviously the appropriate way to do this is to produce the Δ inside of a nucleus where it interacts with the surrounding nucleons. In view of the complexity of such an investigation the deuteron seems to be the most promising target;
- (ii) with equal justification, however, one might visualize the free Δ as a resonating pion-nucleon system (i. e.

a p-wave resonance) and pursue the obvious question whether there are also resonating pion-nucleus systems of definite quantum numbers where the pion interacts coherently with the whole nucleus rather than with one particular target nucleon^{1,2,3,4)}. Again for the case of the deuteron: the pion might interact with the two nucleon system coherently; this (NN)-system can be either in a triplet ($S_{NN} = 1$) or in a singlet ($S_{NN} = 0$) state. The corresponding resonance energies are expected to be different (see Fig. 1);

Fig. 1

(iii) for heavier nuclei the situation becomes more complex, but at the same time - most likely - more interesting for the following reasons: the average nucleon-nucleon distance in a complex nucleus is reduced compared to the deuteron; consequently the effects of the Δ N-interaction ought to be more pronounced. Furthermore, from the presence of many strongly correlated nucleons, with which the Δ interacts dominantly via the exchange of a real pion we expect a coherent enhancement of those effects which we might find already in the deuteron. Such a collective resonance of the whole nucleus will reflect the interplay between the internal nucleonic degrees of freedom of an individual nucleon and its external degrees of freedom inside of the nucleus.

For those reasons it is interesting to see whether such nuclear excitations do exist and - if so - to find out their properties. To do this one carefully has to investigate the excitation spectrum of a given nucleus in the resonance region, i. e. between 200 and 400 MeV⁵⁾. Obviously, photoinduced reactions

(such as exclusive (γ, π) or $(e, e'\pi)$ processes which lead to a bound state of the final nucleus) are particularly suited for such an investigation since there are no distortion effects in the entrance channel; the exclusive $(e, e'\pi)$ reaction has the additional advantage that the momentum transfer to the nucleus can be varied independently.

Unfortunately those collective phenomena are not easy to disentangle: presently many of the available experimental data do indicate that the most simple reaction mechanism, namely the quasifree excitation of the Δ dominates the cross sections, in forward directions. However, it should be added, that the data which are available now do not cover the whole kinematical range of interest; they more or less are taken in the region of low momentum transfer where the quasielastic reaction mechanism should dominate. Unfortunately systematic, good and complete data sets are in general not yet available particularly not in the high momentum transfer region where dramatic deviations from the predictions of the quasielastic model are expected.

As a consequence one should be looking for interference effects between the various contributions. Those interferences are most pronounced if in a given experiment the final state is completely determined, i. e. in an exclusive reaction. The more inclusively a reaction is measured the more the interference effects are washed out, the better the reaction is described by a semi classical picture.

In the following we shall demonstrate for two different examples the effects of the ΔN interaction to be expected and point out how such features could be detected in an exclusive reaction.

II. Coherent π^0 Photoproduction from the Deuteron

The excitation of a nucleon which is bound in the deuteron leads to a ΔN -system in different quantum mechanical states which we consider as elementary excitations of the target, i. e. as D^* -resonances. The properties of those D^* -resonances depend on the ΔN -interaction and thus on the quantum numbers of the ΔN -system. For example there are two ($L = 0$)-configurations with $S_{\Delta N} = 2$ and 1, respectively; the central part of the ΔN interaction is shown in Fig. 2 together with the corresponding eigenenergies.⁺) The calculated splitting with realistic

Fig. 2: The D^* spectrum (a) calculated with a One Boson Exchange potential (b) is shown together with the corresponding central interactions for the 2^+ and the 1^+ configurations.

⁺) The two configurations contain also a ($L = 2$) admixture of about 20 %, due to the tensor part of the ΔN -interaction.

assumptions about the ΔN -interaction (OBEP with π , ρ , σ and ω -exchange, see Fig. 2) is about 50 MeV⁶). In Fig. 3 the total cross section, $\sigma_{\text{tot}}(\pi)$ is shown, for elastic pion deuteron scattering; furthermore, σ_{tot} is decomposed into contributions arising from only one resonance of a given spin and parity, J^π .

Fig. 3: The contributions of the various D^* -resonances to the total cross section for elastic (πd) -scattering (a) and for the coherent photoproduction (b).

As a result we are led to the conclusion that the deuteron exhibits a number of different strongly overlapping resonances of different spin and parity at an excitation energy between 200 MeV and 400 MeV. Thus by investigating reactions which are strongly influenced or dominated by those D^* -resonances we can obtain information about the ΔN -interaction. To be more specific the transition amplitude $T(\underline{k}_\gamma, \underline{k}_\pi)$ for the coherent (γ, π^0) reaction is given by:

$$T(\underline{k}_\gamma, \underline{k}_\pi) = \sum_{\mu} \frac{\langle \underline{k}_\pi D(g.s.) | h_\pi | D_\mu^* \rangle \langle D_\mu^* | h_\gamma | D(g.s.) \rangle}{E_\gamma - E_\mu}$$

where h_γ and h_π denote the $(\gamma N \rightarrow \Delta)$ and the $(\Delta \rightarrow N\pi)$ transition operators, respectively; $D(g.s.)$ and D_μ^* stand for the deuteron ground state and for the D^* resonances:

$$|D_\mu^*\rangle = |[\Delta N]^{STJ^\pi}\rangle$$

In order to disentangle the contributions to the reaction amplitude arising from different resonances it is necessary to measure the angular distributions at different energies or the excitation functions at different angles. An example is shown in Fig. 4 for the (γ, π^0) reaction assuming Δ -dominance, i. e. neglecting all background terms. By carefully measuring those interference effects it should be possible to unravel the D^* -spectrum and to obtain information about the ΔN -interaction which cannot be obtained directly from ΔN -scattering experiments. In accordance with the intention of this paper only the characteristic features to be expected in photo-pion production on the deuteron are discussed; a detailed description of this process is in progress⁶⁾.

III. Coherent Pion Photoproduction from Complex Nuclei

Similar phenomena as discussed in the previous Chapter occur when a nucleon bound in a complex nucleus gets internally excited, for example by absorbing a photon. The elementary process (see Fig. 5) leads to the creation of a Δ particle

and of a nucleonic hole in the target ground state. Such a particle hole excitation can decay by emitting a (real) pion which has a large probability for being reabsorbed by another nucleon before leaving the nucleus; via this mechanism the various Δ particle-N hole configurations of a given multipolarity are strongly coupled with each other; the new eigenmodes resulting from this coupling correspond to collective excitations of the whole nucleus, localized around 300 MeV; this mechanism of creating collective eigenmodes ("Giant (33) Resonances") has been pointed out recently^{2,3}.

Fig. 4: Calculated angular distributions of the photopions in $d(\gamma, \pi^0)d$ for two different photon energies; the individual contributions of the various D^* resonances are shown together with the complete cross section ("all $J\pi$ ").

Those collective resonances should manifest themselves more or less directly in all reactions dominated by the elementary excitation of a nucleon for example in elastic pion-scattering. For the case of ^{12}C the excitation strength for elastic pion scattering is shown in Fig. 6 as a function of the pion energy⁷⁾. Evidently - as in the case of the deuteron - the internal Δ excitation of a nucleon leads to a number of strong, overlapping resonances of different multipolarities.

A similar excitation pattern is expected for the coherent photo-productions of pions in $^{12}\text{C}(\gamma, \pi)$; it should be mentioned that a photon also is able to excite electric resonances (like $1^-, 2^+$ etc.) in contrast to pion induced reactions where only magnetic transitions into $0^-, 1^+$ etc. resonances are possible. Those resonances decay preferentially by emitting pions; by observing the angular distribution of those pions which leave the residual nucleus (^{12}B for (γ, π^+) and ^{12}N for (γ, π^-)) in the ground or an excited, although bound state one can disentangle the nuclear excitation pattern in the region of the (33) resonance, provided the energy dependence of those angular distributions (excitation functions) is experimentally known.

Fig. 5: Schematic picture of $h\gamma$ and $h\pi$ (a), of the ph-components (b) and of the particle-hole matrixelement of the exchange interaction (c).

It is obvious that electromagnetic probes are particularly suited to investigate such a phenomenon for essentially two reasons: (i) the photon experiences no distortion, it can penetrate easily into the interior of the nucleus (in contrast to the pion which gets "absorbed" already at the nuclear surface - at least in the resonance region); (ii) it can excite both electric and magnetic resonances whereas a pion impinging on a O^+ -target nucleus can only excite magnetic resonances ($O^-, 1^+$ etc.). (iii) The coherent $(e, e'\pi)$ reaction exhibits the additional advantage that the momentum transfer can be varied independently of the energy transfer which allows to map out the nuclear response function in the whole energy-momentumplane.

A word of caution has to be added: the forward production process are in general well described by some quasifree reaction model. Therefore, only a careful and systematic investigation of the angular distribution - particularly under backward angles - and of its energy dependence in the whole resonance region ($180 \text{ MeV} < E_Y < 400 \text{ MeV}$) can lead to reliable information about those collective A^* -resonances, discussed in the previous chapters.

On the other hand it is extremely interesting to find out how the internal degrees of freedom of an individual nucleon are coupled to the nuclear modes of excitation. To see how a resonating pion interacts with a strongly correlated nuclear system seems to be a prerequisite for a better description of the virtual pion fields which supposedly hold the nucleus together. Thus from an investigation of those medium energy phenomena a better understanding of the general baryon-baryon interaction and also of low energy nuclear physics may finally emerge.

We thank K. Klingenberg and R. Händel for numerous discussions and for informing us about their results prior to publication.

IV. Conclusions

The nuclear excitation spectrum in the region of the elementary (33) excitation is particularly interesting since it reflects the properties of the Δ N-interaction and the many body response of the nuclear system to the internal excitation of an individual nucleon. In all model calculations performed so far it turns out that the various excitation modes of a given multipolarity are strongly coupled with each other, thus leading to a collective resonance which - formally - is rather similar to the well known phenomena of the Giant Resonances in the 20 MeV region. The physical properties of those two types of resonances, however, are completely different: whereas the low energy resonances correspond essentially to charge and/or volume vibrations the new Giant (33) Resonances can be visualized as a resonating pion current inside of the nucleus.

Fig. 6: Calculated total cross section for elastic pion scattering on ^{12}C : the contributions of the individual $^{12}\text{C}^*$ resonances are shown separately.

References

- 1) L. S. Kisslinger, W. L. Wang - Phys. Rev. Lett. 30 (1973) 1071 and Ann. Phys. 99 (1976) 374
- 2) M. Dillig, M. G. Huber, Phys. Lett. 48 B (1974)
 Proc. of the Conference on "Mesonic Effects in Nuclear Structure", 80 (Ed. K. Bleuler et al, Bibl. Institut, Mannheim, 1975)
 Proc. of the Int. Symp. on "Interaction Studies in Nuclei", 781 (Ed. H. Jochim, B. Ziegler, North Holland Publ. Comp., Amsterdam, 1975)
- 3) F. Lenz - Ann. Phys. 95 (1975) 348;
 M. Hirata, F. Lenz, K. Yazaki - preprint, SIN (1976);
 M. Hirata, J. H. Koch, F. Lenz, E.J. Moniz - preprint (1977)
- 4) G. E. Brown, W. Weise - Phys. Rep. 22 C (1975) 281;
 W. Weise - Nucl. Phys. A 278 (1977) 402
- 5) D. Bachelier et al - Phys. Lett. 44 B (1973) 44
 G. N. Dudkin et al - Sov. Journ. of Nucl. Phys. 19 (1974) 153
 V. D. Epaneschnikov et al - Sov. Journ. of Nucl. Phys. 19 (1974) 242
- 6) M. Dillig, R. Händel, M. G. Huber - Contribution to the "Conference on High Energy and Nuclear Structure", Zürich (1977)
 R. Händel - in preparation
- 7) K. Klingenberg, M. Dillig, M. G. Huber - Contribution to the "Conference on High Energy and Nuclear Structure", Zürich (1977)
 K. Klingenberg - in preparation

III.8 Charged Pion Electroproduction Near Threshold*

T. W. Donnelly

Institute of Theoretical Physics, Department of Physics
Stanford University, Stanford, CA 94305

Here we are concerned with the reaction ${}_Z A(e, e' \pi^\pm) {}_{Z \mp 1} A$, where the energy of the pion in the final state center-of-mass system is near threshold (typically within a few MeV of threshold). Under these conditions only non-resonant s-wave pions are necessary and consequently we have the greatest chance of providing an adequate description of the process. Our present work (T. W. Donnelly and J. B. Cammarata) is built on previous studies⁽¹⁻²⁾ of the real photon reaction ${}_Z A(\gamma, \pi^\pm) {}_{Z \mp 1} A$ also for threshold pions and here we shall concentrate only on the new aspects which are introduced when virtual photons are used. Some examples of interest are:

- (1.) ${}^3\text{He}(e, e' \pi^+) {}^3\text{H}$
- (2.) ${}^3\text{H}(e, e' \pi^-) {}^3\text{He}$
- (3.) ${}^6\text{Li}(e, e' \pi^+) {}^6\text{He}$
- (4.) ${}^{11}\text{B}(e, e' \pi^-) {}^{11}\text{C}$
- (5.) ${}^{12}\text{C}(e, e' \pi^+) {}^{12}\text{B}, {}^{12}\text{C}(e, e' \pi^-) {}^{12}\text{N}$
- (6.) ${}^{16}\text{O}(e, e' \pi^+) {}^{16}\text{N}, {}^{16}\text{O}(e, e' \pi^-) {}^{16}\text{F}$

In (1.-3.) it may be feasible to detect either the pion or the recoil nucleus in coincidence with the scattered electron; in (4.-6.) we assume that the pion and electron are detected. Comparisons of π^+ and π^- reactions such as (5.) and (6.) may be interesting, since the Coulomb barrier pushes the π^+ out from the nucleus and draws the π^- into the nucleus, diminishing the strong interaction effects in the former case and enhancing

*Research supported by the National Science Foundation, Grant NSF PHY 77-16188

them in the latter. A comparison of reactions (1.) and (2.) might be interesting for the same reasons plus the fact that good three-body nuclear wave functions exist to describe the nuclear states involved. In fact to a large extent the nuclear aspects cancel out in comparing these processes.

The cross section appears in the form $d^4\sigma/d\Omega_e dE_e d\Omega_\pi^* dE_\pi^*$, where Ω_e and E_e refer to the scattered electron solid angle and energy in the laboratory system and where Ω_π^* and E_π^* are the analogous quantities for the pion, but in the final state center-of-mass system. We consider only s-wave pions in that system and so the integral over Ω_π^* is trivial. Furthermore, we consider a given nuclear transition energy ω_{fi} , and for fixed electron energies and fixed electron scattering angle we have a unique pion energy (i.e., there is an energy-conserving delta function relating the pion energy to the other variables). Thus we integrate over pion energies and write

$$\int d\Omega_\pi^* \int dE_\pi^* \frac{d^4\sigma}{d\Omega_e dE_e d\Omega_\pi^* dE_\pi^*} \equiv \sigma_{(e,e'\pi)} = 4\pi\sigma_M/M_T [(k_\mu^2/k^2)^2 S_L(k, k_0) + (k_\mu^2/2k^2 + \tan^2\frac{\theta}{2}) S_T(k, k_0)] \quad (1)$$

where $k_\mu = (\underline{k}, k_0)$ is the 4-momentum transferred from the electron to the hadronic system, $k=|\underline{k}|$ is the 3-momentum transfer, θ is the electron scattering angle, σ_M is the Mott cross section and M_T is the target mass. S_L and S_T are dimensionless hadronic response functions or form factors (longitudinal and transverse respectively). Photopion production is just a special limiting case of this more general virtual photon expression; thus we have

$$\sigma_{(\gamma,\pi)} = \frac{(2\pi)^3 \alpha}{k M_T} S_T(k, k_0=k) \quad (2)$$

For the process (e, e') we have the familiar expression analogous^(3,4) to Eq. (1), but with form factors F instead of S . For (e, e') the form factors may be decomposed in the usual manner into nuclear matrix elements of the vector multipole operators⁽³⁻⁵⁾ $\hat{M}_{JM}$, $\hat{T}_{JM}^{el}$ and $\hat{T}_{JM}^{mag}$ and these operators in turn involve the forms $j_L(kr)Y_L^M(\Omega_r)$, $j_L(kr)\underline{y}_{JL1}^M(\Omega_r) \cdot \underline{\nabla}$ and $j_L(kr)\underline{y}_{JL1}^M(\Omega_r) \cdot \underline{\sigma}$. In a like fashion the process $(e, e'\pi^\pm)$ requires matrix elements of the axial-vector multipole operators $\hat{M}_{JM}^5$, $\hat{L}_{JM}^5$, $\hat{T}_{JM}^{el5}$ and $\hat{T}_{JM}^{mag5}$. Again the forms involving spherical Bessel functions and spherical harmonics enter, but now the radial parts have as an addition factor the s-wave pion wave function $\phi_\pi^*(r)$. Except for the presence of the pion wave function and for the specific nature of the axial coupling constants used these axial-vector multipole operators are the same as the ones used in the description of semi-leptonic weak interactions⁽⁴⁻⁸⁾ (details of the formalism will be given in ref. 9).

At least two reasons for studying the process $(e, e'\pi^\pm)$ come to mind:

(1) The amplitude for the process is of interest in its own right, just as it is in photoproduction⁽¹⁻²⁾. We may test the assumptions which go into the calculation (PCAC, current algebra, soft pion theorems) and so further explore the role of pions-with-nuclei. Here the low-energy s-wave aspect of the problem is useful: it permits us to study pion-nuclear physics in a regime where the concepts involved are the clearest and most testable. Of special interest in such coincidence experiments will be a measurement of the angular distribution of the pions (either directly or via measurement of another related variable such as an energy distribution). This will test the s-wave assumption and signal the entrance of p-wave contributions (the tail of the 33-resonance for instance) via anisotropy in the cross section. The process $(e, e'\pi^\pm)$ has some advan-

tages over $(\gamma, \pi^\pm)$ in that the 3-momentum transfer may be varied, whilst the energy transfer is adjusted to keep the pion energy fixed near threshold. In particular for large momentum transfers the final hadronic state (for light nuclei such as cases (1.-3.) above) recoils with significant energy enhancing the chances of detection. Also with high momentum transfer high multipolarity transitions are not hindered as they are in threshold photoproduction where $k=k_0 \sim m_\pi$ and one has more nuclear transitions available to study the production process. On practical grounds, some of these high-spin transitions are the ones with the simplest nuclear configurations and hence clearest interpretations (Cf. electron scattering, ref. 4).

(2) Perhaps the most unusual possibility that arises with the process $(e, e' \pi^\pm)$ is the ability to measure the spatial distribution of the pion wave function. Given high quality (e, e') data with an attendant one-body density matrix decomposition, ^(4,8) one may make a high quality prediction of $(e, e' \pi^\pm)$, where this prediction involves the pion wave function in the radial parts of the matrix elements. Since in general $\phi_\pi(r)$ is spatially varying, the response functions $S_L(k, k_0)$ and $S_T(k, k_0)$ vary with k in ways which depend on the particular Fourier components of ϕ_π which are present. It should be emphasized that only in such virtual photon processes, $(e, e' \pi)$, is it possible to measure the pion form factor. For example it is not possible in the real photon process, (γ, π) , for here as one varies the momentum transfer (to map out a response function or form factor) one is forced to vary the energy transfer as well (since $k=k_0$ for real photons) and hence each value of k requires a different pion wave function. Various optical model calculations of the pion wave functions show different spatial forms (e.g., some with large depressions of $\phi_\pi(r)$ for small r ,

some without): these effects should be reflected in the k -dependence of S_L and S_T .

To conclude this brief report on our efforts in these directions, results are shown in Figs. 1 and 2 for the reactions ${}^3\text{He}(e,e'\pi^+){}^3\text{H}$ and ${}^6\text{Li}(e,e'\pi^+){}^6\text{He}$ respectively. The nuclear wave functions used have been discussed in refs. 2, 7 and 8.

Fig. 1 The longitudinal and transverse response functions $S_L(k, k_0)$ and $S_T(k, k_0)$ are shown as functions of three-momentum transfer k for pion electroproduction with final state center-of-mass pion energy $E_\pi^* = 3.2$ MeV. The real photon point is indicated by γ .

Fig. 2 As for Figure 1.

References

1. J. H. Koch and T. W. Donnelly, Phys. Rev. C10 (1974) 2618.
2. J. B. Cammarata and T. W. Donnelly, Nucl. Phys. A267 (1976) 365.
3. T. deForest, Jr. and J. D. Walecka, Adv. in Phys. 15 (1966) 1.
4. T. W. Donnelly and J. D. Walecka, Ann. Rev. Nucl. Sci. 25 (1975) 329.
5. J. D. Walecka, in Muon Physics, vol. 2 ed. V. W. Hughes and C. S. Wu (Academic Press, New York, 1975) p. 113.
6. J. S. O'Connell, T. W. Donnelly and J. D. Walecka, Phys. Rev. C6 (1972) 719.
7. T. W. Donnelly and J. D. Walecka, Phys. Lett. B44 (1973) 330.
8. T. W. Donnelly and J. D. Walecka, Nucl. Phys. A274 (1976) 368.
9. T. W. Donnelly and J. B. Cammarata (to be published).

FIGURE 1

FIGURE 2

$$\epsilon_{\pi}^* = 3.2 \text{ MeV}$$

IV.1 Knock-out Reactions

T. de Forest, Jr.
I.K.O., Amsterdam

1. Introduction

The primary motivation for performing high energy single nucleon knock-out reactions, such as $(p,2p)$, $(e,e'p)$ and (γ,p) , is based on the concept of quasi-elastic scattering^{1,2)}. That is, the scattering takes place on a single nucleon which, due to the recoil, is subsequently knocked out of the nucleus as a free particle. By detecting this nucleon (energy ϵ_p , momentum $\vec{p}$) one can determine in which state it was initially, energy $E = \epsilon_p - \omega$, momentum $\vec{k} = \vec{p} - \vec{q}$, where ω and $\vec{q}$ are the energy and momentum transferred to the nucleus by the scattering or absorption process. As a consequence the cross section, in the plane wave impulse approximation (PWIA), is proportional to the probability, $S(k,E)$, for finding a nucleon in this state in the target nucleus,

$$\sigma = \text{kin. factors} \times \sigma_N S(k,E)$$

$$S(k,E) = \langle 0 | a_k^\dagger \delta(E - E_0 + H) a_k | 0 \rangle$$

where σ_N is the (off shell) cross section for scattering or absorption on a single nucleon, and E_0 is the energy of the ground state of the target nucleus.

The spectral function $S(k,E)$ provides one with detailed information about the single-particle aspects of nuclear structure. This is most easily appreciated in the independent-particle model where S has the form (neglecting center of mass effects)

$$S(k,E) = \sum_{\alpha} N_{\alpha} |\phi_{\alpha}(k)|^2 \delta(E - \epsilon_{\alpha})$$

Thus a measurement of S provides one with the single-particle occupation numbers, N_α , energies, ϵ_α , and momentum distributions $|\phi_\alpha(k)|^2$. Provided the orbital angular momentum, which to some extent can be determined from the experimental data, is known, the latter can be Fourier transformed to obtain the single-particle wave function. In real nuclei this basic picture still holds but the single-particle strength is spread out over various eigenstates of the residual nucleus, and one expects contributions at very large missing energies $E_m = -E$ (note: $E_m > 0$) from short range correlations. This latter effect has not yet been seen.

Rather than analyzing $S(k, E)$ into the contributions of the various orbitals α , one can also integrate over E to obtain global nuclear properties. For example the total momentum distribution and kinetic energy are given by

$$M(k) = \int dE S(k, E)$$

$$T = \int dE \int d\vec{k} \frac{k^2}{2m} S(k, E)$$

Another interesting sum rule is that for the binding energy³⁾

$$E_0 = 1/2 \int dE \int d\vec{k} \left(\frac{k^2}{2m} + E \right) S(k, E)$$

which is exact, aside from the assumption that only two-body forces are present. Experimentally it has been found that this sum rule is not satisfied for most nuclei.⁴⁾ It is however too early to consider this as evidence for the existence of three-body forces. A more likely explanation is that, since the expected strength at large E_m has not been seen experimentally, the sum rule has not yet converged.²⁾

2. Validity of and corrections to PWIA

The usefulness of analyzing knock-out experiments in the way described in the previous section depends upon how good the PWIA is and how accurately one can calculate the corrections to it. Also one must extrapolate σ_N off shell. The importance of these correction effects depends strongly on the type of process considered and also on the kinematics of the reaction.

In the region where the quasi-elastic cross section is large (relatively small k and E) the most important corrections are those for the distortion and absorption of the nucleons by the nuclear optical potential. In the analysis of $(e, e'p)$ experiments with proton energies on the order of 100 MeV⁴⁾ it has typically been found that distortion effects do not modify the momentum distributions very much and that these distributions agree with those expected theoretically, e.g. those obtained from Elton-Swift type wave functions. Absorption effects on the other hand are quite strong. Thus since the optical potential is not known very well, the occupation numbers obtained by such analyses should not be taken too literally. Since both distortion and absorption effects decrease with increasing proton energy, it is clearly preferable to work at higher energies. We also note that the problem of extrapolating σ_N off shell is not very serious for small k and E since in this region one is almost on shell ($E = \frac{k^2}{2m}$ for free scattering).

In addition there exist other types of PWIA violating processes whose contributions are usually quite small compared with the main quasi-elastic process but, relatively, can become very important in regions where the contribution of the latter is small, such as at high k or E_m . For example: a) The knocked-out nucleons which are 'absorbed' do not really disappear. Instead they are emitted in another

channel, usually with a lower energy. This gives a background in the region of large E_m .⁵⁾ b) The detected nucleon may not be the one upon which the scattering took place. For example if two nucleons are strongly correlated and one is knocked out, the other is also likely to escape. Such events are expected to be important in the backward region, that is antiparallel to $\vec{q}$.⁶⁾ c) Exchange current interactions, since they involve two or more nucleons, basically violate the impulse approximation. These effects are expected to be important at high k . By allowing some of this momentum to be shared with another nucleon this process is sensitive not only to the value of the momentum distribution at k but also in a region, roughly determined by the nuclear form factor, around k . If the momentum distribution decreases very rapidly, this effect can produce a large enhancement. Recent high energy (γ, p_0) measurements⁷⁾ are in qualitative agreement with theoretical predictions⁸⁾ based on this picture of the reaction mechanism for $k \gtrsim 450$ MeV/c. Also, in analogy with the effect of short range correlations, exchange current interactions naturally give rise to two nucleon emission processes and thus contribute strongly at large E_m .

For the purposes of extracting $S(k, E)$ these processes give unwanted contributions and thus represent corrections which one must calculate and/or try to minimize by optimizing the kinematics. On the other hand some of these processes, in particular those involving short range correlations or exchange effects, are of interest in their own right. Unfortunately it is difficult to isolate these contributions from those due to direct knockout, since almost nothing is known about the behavior of $S(k, E)$ at high k and at high E . The fact that the PWIA cross section obeys a kinematical invariance whereas the contribution of PWIA violating processes in general do not, however can be

used to help unravel these contributions. This is discussed in more detail in the next section.

Since the corrections to the PWIA are quite different for various types of knock-out reactions, some reactions are definitely preferable to others. In particular electromagnetic probes are particularly clean since the basic scattering process is not affected by strong interaction effects.¹⁾ Also, since the physics of electromagnetism is well understood, one has at least some idea of how to extrapolate σ_N off shell. Between $(e, e'p)$ and (γ, p) there is basically no fundamental difference. In fact in the limit of forward scattering $(e, e'p)$ is in essence identical to (γ, p) . However, there are important kinematical and experimental differences between the two reactions. Due to the restriction $q=\omega$ one is only capable of measuring rather high momentum components in (γ, p) . In $(e, e'p)$, on the other hand, one can measure down to $k=0$, but is restricted, due to the requirement of detecting a coincidence, to rather large cross sections and thus relatively low momentum components ($k_{\max} \sim 300 \text{ MeV/c}$)⁴⁾. Thus (γ, p) and $(e, e'p)$ complement each other very nicely. Recent (γ, p) measurements have in essence extended the results obtained from $(e, e'p)$ up to $k_{\max} \sim 800 \text{ MeV/c}$.⁷⁾

3. Kinematical Invariance of Knock-out Reactions and Tests of the Reaction Mechanism.

Since the spectral function depends on only two variables it can be measured in many different ways, provided of course that the PWIA is valid. Conversely such redundant measurements provide a means of checking the validity of the PWIA, or more usefully of the methods used to correct for PWIA violating effects. This is very similar to the way that redundant measurements on the Rosenbluth plot provide a check on the validity of Born approximation in electron scattering.

In order to illustrate how this approach can be employed we consider a schematic model for taking distortion effects into account, namely we replace the nucleon's asymptotic momentum $\vec{p}$ by an effective momentum^{9,10)}

$$\vec{p}' = \vec{p} \sqrt{1 - \frac{V(\epsilon_p)}{\epsilon_p}}$$

in the nucleus. Here V is some average value of the real part of the optical potential. In order to keep the model simple we neglect absorption and the Perey effect (which should be taken into account when an energy dependent potential is used). In this model the cross section is proportional to $S(k', E)$ where $\vec{k}' = \vec{p}' - \vec{q}$ and redundant measurements provide a check on the choice of $V(\epsilon_p)$ used in the analysis.

One immediate result of this distortion is that the classical method used to measure momentum distributions is not really optimal. The classical experimental (e,e'p) setup is sketched in fig. 1. One chooses $\vec{q}$ fixed and $|\vec{p}| = |\vec{q}|$ (so as not to have to worry about the energy dependence of V) and varies the angle between $\vec{p}$ and $\vec{q}$. In this way one varies the initial momentum $\vec{k} = \vec{p} - \vec{q}$, and in particular

one reaches the minimum value, $k=0$. One, however, misses the smaller values of the appropriate momentum k' . The result is that the dip at $k=0$ for orbits with $\ell \geq 1$ is filled in. This is normally ascribed to absorptive effects, but as we see it is also due to distortion. This effect can be eliminated by using a different experimental setup (see fig. 1), namely choosing $\vec{p}$ parallel to $\vec{q}$ with $|\vec{p}|$ fixed and varying $|\vec{q}|$. In this case one is guaranteed of reaching $k'=0$. The corresponding momentum distributions, plotted as a function of k , are shown in fig. 2. We note that the parallel setup actually increases the distortion effects, but it also makes it much easier to eliminate them. The whole momentum distribution is simply shifted by a constant amount, $k'-k = p'-p$, from the PWIA results. It is thus not necessary to "analyze" the results since one directly has the momentum distribution. If one wants one can also determine V from the size of the shift. With a full DWIA calculation the situation is of course more complicated, but such parallel measurements still provide a check on the choice of the optical potential used and will essentially determine the average depth, thus allowing one to improve the analysis. Following these ideas a parallel kinematics type of measurement has been recently carried out at Saclay.¹¹⁾

In (γ, p) , due to the requirement $q=\omega$, one is more restricted. Although one can measure a given initial momentum, k , in various ways, this requires working at different proton energies, ϵ_p . Thus, since the ϵ_p dependence of V plays a crucial role, one cannot directly test whether the correct value of V has been used. The proper choice of $V(\epsilon_p)$ should however give a unique momentum distribution, thus providing an important consistency

check of the analysis. This test has been applied in the analysis of (γ, p_0) data in terms of momentum distributions.¹⁰⁾

In a similar way such 'redundant' measurements provide a check on whether the high k and E_m behavior of cross sections for knock-out processes is due to the intrinsic behavior of $S(k, E)$ itself or to some more exotic process such as exchange current interactions. In this respect we would like to mention that further investigations of high energy (γ, p) would be very interesting. Measuring the extent to which the cross sections obey the DWIA kinematical invariance (i.e. the PWIA invariance, but with the appropriate corrections made for distortion and absorption) would help clarify the origin of the large cross sections seen at high k in the experiments of Matthews et al,⁷⁾ that is whether these reflect exchange current effects or are due to the existence of high momentum components in the nuclear wave function. In order to answer this question quantitatively a realistic calculation of the exchange effects must be carried out, in particular to see to what extent they violate the kinematical invariance of the 'pure' knock-out cross section.

References

1. G. Jacob and Th. A. J. Maris, Rev. Mod. Phys. 38, 121 (1966); 45, 6 (1973).
2. A. E. L. Dieperink and T. deForest, Jr., Am. Rev. Nucl. Sci. 25, 1 (1975).
3. D. Koltun, Phys. Rev. Lett. 28, 182 (1972).
4. J. Mougey et al., Nucl. Phys. A262, 461 (1976).
5. H. Hiramatsu et al., Phys. Lett. 44B, 50 (1973).
6. T. de Forest, Jr., Ann. Phys. 45, 365 (1967).
7. J. L. Matthews et al., Phys. Rev. Lett. 38, 8 (1977).
8. J. T. Londergan, G.D. Nixon and G.E. Walker, Phys. Lett. 65B, 427 (1976).
9. T. de Forest, Jr., Nucl. Phys. A132, 305 (1969).
10. D.J.S. Findlay and R. O. Owens, Phys. Rev. Letts 37, 674 (1976); Nucl. Phys., A292, 53 (1977).
11. M. Bernheim et al., preprint.

Fig. 1 Normal (a) and parallel (b) kinematics for knock-out experiments.

Fig. 2 Effect of distortions on the momentum distribution in the effective momentum approximation for $\epsilon_p = 80$ MeV and $V = -20$ MeV. The dot-dashed and dashed curves represent the (uncorrected) momentum distributions seen with normal and parallel kinematics respectively, and the solid curves are the undistorted distributions.

IV.2 EXPERIMENTAL PROBLEMS IN NUCLEON KNOCKOUT
REACTIONS: TECHNIQUES FOR PRODUCING EFFECTIVE
MONOCHROMATIC PHOTONS USING BREMSSTRAHLUNG.

J. L. MATTHEWS

DEPARTMENT OF PHYSICS AND LABORATORY FOR NUCLEAR SCIENCE -
MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASSACHUSETTS

I. INTRODUCTION

Nucleon knockout reactions have been studied for several decades using bremsstrahlung beams from electron synchrotrons. The low beam intensities available precluded detailed studies with any approximation of monochromaticity of the incident photon (except in a few special cases). With the advent of the new generation of high current and/or high duty factor accelerators, experiments which "differentiate" the bremsstrahlung spectrum and which in addition provide final state energy discrimination become practicable. It is expected that much valuable nuclear structure information will come out of these studies.

The purpose of this report is to review briefly the well known methods of producing effective monochromatic photons using the bremsstrahlung spectrum, indicate which techniques are most appropriate for particular experiments in the medium energy domain, and to suggest possible extensions of these techniques. Certain specialized methods will not be considered, e.g. annihilation of positrons in flight, coherent bremsstrahlung in crystals, or Compton scattering of laser light by storage-ring electrons. Tagged photon experiments will not be discussed in any detail, nor will electrodisintegration reactions (i.e. virtual photons), but both will be mentioned in certain contexts.

II. SURVEY OF TYPES OF PHOTONUCLEAR MEASUREMENTS AND SUITABLE MONOCHROMATIC PHOTON TECHNIQUES

A. Two-body Reactions, e.g. $d(\gamma, p)n$, $d(\gamma, d)\pi^0$, ${}^4\text{He}(\gamma, {}^3\text{H})p$, ${}^4\text{He}(\gamma, {}^3\text{He})n$, ${}^7\text{Li}(\gamma, {}^6\text{Li})n$, etc. Photoreactions in which only two particles are produced in the final state provide monochromatic photons in a special way, in that the measurement of the energy and angle of one of the reaction products determines the incident photon energy. Here the bremsstrahlung spectrum is an advantage rather than a drawback, allowing cross sections to be determined over a large energy range in a single measurement. The range of the measurement will be limited either by the acceptance (dynamic range, momentum bite, etc.) of the particle detector (e.g. counter telescope, magnetic spectrometer) or by the reaction kinematics. That is, in reactions such as $d(\gamma, p)n$ one is limited to photon energies in the bremsstrahlung spectrum above $(E_0 - 140)$ MeV if nucleons associated with pion production are not to be inadvertently detected.

We note that detection of the recoiling heavy particle (e.g. ${}^3\text{He}$, ${}^6\text{Li}$) provides a method of observing the (γ, n) reaction without the difficulties arising from either monochromatic photons or neutron spectroscopy. However, one pays a high price in counting rate: the targets must be thin ($\lesssim 100 \mu\text{g}/\text{cm}^2$) in order for the recoil ions to escape, and these ions are generally distributed over several charge states, appearing at different positions in the focal plane of a magnetic spectrometer. Resolution and background suppression must be excellent in order to identify the masses and charges of interest. A detailed comparison of the experimental problems and counting rates in a given photon energy range should be made between, for instance, the ${}^7\text{Li}(\gamma, {}^6\text{Li})n$ and the ${}^7\text{Li}(\gamma, n){}^6\text{Li}$ reactions before either type of measurement is attempted.

B. Photoreaction Cross Sections $\sigma(E_\gamma)$ which are Rapidly Changing with Photon Energy. The classical example of these processes is of course the giant dipole resonance where structure of width less than 1 MeV is seen. Although it is unlikely that

similar narrow features exist at higher energy, we will discuss these measurements briefly for the sake of completeness. Here the most efficient method of generating effective monochromatic photons is that of "yield curve unfolding," where one measures the yield for the reaction of interest as a function of E_0 , the bremsstrahlung endpoint (i.e. electron) energy.¹⁾

The term "yield" may be taken in either a broad or a narrow sense: the yield of induced radioactivity or the total number of neutrons, or the number of protons in a specific energy and angular range defined by a magnetic spectrometer or range telescope. The problem is then to solve empirically the set of integral equations

$$Y_i(E_{0i}) = \int_0^{E_{0i}} \sigma(E_\gamma) N(E_\gamma; E_{0i}) dE_\gamma \quad (1)$$

for the cross section of interest $\sigma(E_\gamma)$, where $Y_i(E_{0i})$ are the measured quantities and the bremsstrahlung spectrum $N(E_\gamma; E_{0i})$ is assumed known. The difficulty with this method is that uncertainties accumulate, and meaningless fluctuations in the derived cross section can result.

C. Specific Photoreaction Cross Sections (above the GDR) which Vary Smoothly with Energy. In the situation where one wants to know a specific cross section (e.g., (γ, p) to a certain final state) at a set of widely spaced (> 20 MeV) photon energies, a yield curve is not the most efficient method of measurement, although it has been used.²⁾ Previous experiments in the intermediate energy domain have concentrated on measuring the differential cross section for the (γ, p) reaction³⁻⁵⁾, for cases in which modest resolution (1-2 MeV) is sufficient to resolve ground from excited states of the residual nucleus. Two techniques have been employed, the two-radiator difference method^{3,6)} and the bremsstrahlung tip method.^{4,5)} Both methods require only two runs to produce equivalent monochromatic-photon data, compared with the five or more which would be necessary using the yield curve method (see ref. 6). Since these two techniques and variations thereof are expected to be the most useful in future knockout experiments at intermediate energies, we will devote Sections III and IV

to discussing their applications and possible extensions in more detail.

D. "Inclusive" and Multi-particle Knockout Cross Sections.

The single-particle knockout processes considered above constitute only a small fraction of the total photoabsorption cross section at intermediate energies. Multi-particle emission (e.g. $(\gamma, 2n)$, (γ, pn) , $(\gamma, p\alpha)$, etc.) becomes increasingly probable as the photon energy increases. Such reactions have been investigated in cloud chamber work using 170 MeV bremsstrahlung⁷⁾ which suffers from poor statistics and poor energy resolution. It is expected that future measurements of these processes, which should be very valuable in elucidating details of the photoabsorption reaction mechanism, will be performed using arrays of counter telescopes and/or time-of-flight detectors, or large-aperture magnets with the particle masses and energies determined by tracing the trajectories through a series of wire chambers⁸⁾. In either case it is likely that only weak, "clean" photon beams can be tolerated, and in our opinion this class of experiments are most suited to the low-intensity monochromatic photon beams obtained by tagging⁹⁾ or by in-flight positron annihilation.⁸⁾

III. THE TWO-RADIATOR DIFFERENCE AND BREMSSTRAHLUNG TIP METHODS

Our aim in the development and application of these two methods may be stated simply: to make use of the energy-analyzed electron beam from a high-intensity linear accelerator and a high-resolution magnetic spectrometer system to do as much photonuclear spectroscopy, in the energy region $\gtrsim 40$ MeV, as efficiently as possible. Since above the giant resonance photonuclear cross sections are decreasing rapidly, we begin by maximizing the number of photons hitting the nuclear target by placing a fairly thick radiator (.02-.05 radiation lengths) just upstream (~ 10 cm) of the target. This provides a factor of at least one hundred more intensity than in the conventional system in which the electron beam is swept out of the way after striking the radiator and the photon beam is collimated to produce an acceptable spot size at the nuclear target. The former scheme has the disadvantage, however, that the residual electron beam (as well as the bremsstrahlung photons) passes through the nuclear target, presenting two problems: i) background and ii) yield from the electron-induced reactions, which must be separated from the photon-induced yield of interest. The first problem has been solved by using a well-shielded magnetic spectrometer to detect the reaction products. We suspect that this is essential: a magnet must be present to remove the electrons scattered by the nuclear target; a counter telescope, for instance, would be swamped by this background. As for the second problem, we must either directly subtract the electroproduction yield, or contrive for it to cancel.* The two-radiator difference technique follows the latter approach, the tip method the former, as well be discussed below.

Given that these two problems are solved, the close-radiator scheme provides two additional advantages which are perhaps not quite as important as the factor of 100 in intensity but are useful nonetheless: i) in analyzing the photoreaction data we can use the integrated-over-angles bremsstrahlung spectrum formulae which are thought to be more accurate than those for the angular distribution of the spectrum, with cutoffs imposed by collimators; ii) the experiments can use a beam line and spectrometer setup

*

There are in fact two other alternatives: ignore the electron-induced yield, or use it (see Appendix II).

very similar to that employed in electron scattering experiments - one need only install a radiator and perhaps alter the focal plane detector and shielding, which is far less effort than setting up an electron dump magnet, dump, shielding, γ -ray collimators, etc.

A. The Two-radiator Difference Method. As in conventional, single-radiator difference measurements, one takes data at two electron energies, separated by ~ 2 MeV (in the work done so far^{3,10}), and subtracts, in order to obtain the yield from the photons in the uppermost ~ 2 MeV of the bremsstrahlung spectrum. If the two radiators used are the same thickness in radiation lengths, the intensity at photon energies below $(E_0 - 2)$ MeV will cancel, in the difference, almost as well as if a single radiator were used. In addition, one would like the electron-induced ("virtual photon") yield to cancel altogether in the difference. This will happen (exactly, neglecting straggling) if the mean energy of the electrons emerging from the radiator is the same in the two runs. For this to occur, the radiators must be of different thicknesses in g cm^{-2} . That these two criteria (cancellation of low energy real photons and of all virtual photons) can be satisfied simultaneously may be seen as follows: We write the bremsstrahlung intensity per g cm^{-2} as *

$$I \propto \frac{Z(Z+1)}{A_2} \times \left\{ \text{weakly } (E_0, k, Z) \text{ dependent terms} \right\} \quad (2)$$

Ignoring the curly bracket, we find $I_1 = I_2$ if we choose thicknesses t_1 and t_2 (in g cm^{-2}) such that

$$\frac{Z_2(Z_2+1)}{A_2} \cdot t_2 / \frac{Z_1(Z_1+1)}{A_1} \cdot t_1 = 1 \quad (3)$$

(or, approximately, $Z_2/Z_1 \approx t_1/t_2$). Now we need

$$E_{o1} - \left(\frac{dE}{dx} \right)_1 t_1 = E_{o2} - \left(\frac{dE}{dx} \right)_2 t_2, \quad (4)$$

and

$$E_{o1} - E_{o2} = \Delta k, \quad (5)$$

the width of the photon difference peak. If we let

$$\frac{dE}{dx}_1 = \frac{dE}{dx}_2 = \epsilon$$

*The form of Eq. (2) assumes that the electron-electron bremsstrahlung ($\propto Z$) spectrum has the same shape as the electron-nuclear bremsstrahlung ($\propto Z^2$). See reference 11 for further discussion.

eqs. (4) and (5) may be combined to give

$$\epsilon(t_1 - t_2) = \Delta k. \quad (6)$$

For given Z_1, A_1, Z_2, A_2 , and Δk , eqs. (3) and (6) may easily be solved for t_1 and t_2 . Using $\epsilon = 1.8 \text{ MeV/g cm}^{-2}$ (from dE/dx tables) and choosing $Z_1 = 4$ (Be), $Z_2 = 13$ (Al), and $\Delta k = 2 \text{ MeV}$, we find

$$t_1 = 1.67 \text{ g cm}^{-2} (\text{Be})$$

$$t_2 = 0.56 \text{ g cm}^{-2} (\text{Al}),$$

both approximately .02 radiation lengths. A photon difference spectrum obtained using the method outlined above is shown in fig. 1. The energies E_{o1} and E_{o2} and thicknesses t_1 and t_2 have been adjusted to give optimum cancellation of low energy (real) photons and of virtual photons. (The approximations used for the sake of clarity in the description above were not used in the actual calculations.) The different energy-loss straggling of the electrons in the different thickness radiators prevents an exact cancellation of the virtual photon intensity, but the net effect is small, as shown by the dashed curve in fig. 1.

The calculated width and shape of the peak in fig. 1 may be verified by comparison with the subtracted proton spectrum measured in the $^{12}\text{C}(\gamma, p)^{11}\text{B}$ reaction.¹²⁾ As seen in fig. 2, the resolution is just barely sufficient to resolve transitions to the first excited state (at 2.12 MeV) of ^{11}B . The next three states, at 4.44, 5.02, and 6.74 MeV, are clearly seen as a group but are not separately resolved. The region of cross section consistent with zero, between around 8 and around 13 MeV excitation, where there are of course many states in ^{11}B , supports both the calculated zero photon intensity in this region and the hypothesis of a direct knockout reaction mechanism.

The contribution of the low-energy tail seen in fig. 1 must be unfolded from the measured proton spectra in order to obtain the yield due only to the high energy peak. This is illustrated for the case of the $^6\text{Li}(\gamma, p)$ reaction in fig. 3 (see also ref. 3). The proton yield in the $\sim 20 \text{ MeV}$ excitation energy region remaining after the tail is unfolded may be evidence for the knockout of protons from the 1s-shell. However, fig. 3 also illustrates the difficulties due to statistics (taking small differences between

Fig. 1 Photon difference spectrum (solid curve) obtained from Be-Al subtraction. For Be, $E = 102.83$ MeV, $t = 1.700$ g cm $^{-2}$. For Al, $E = 101.00$ MeV, $t = 0.588$ g cm $^{-2}$. The ordinate scale (photon intensity) is the number of photons per MeV per incident electron (for the given radiator thickness), multiplied by the photon energy in MeV. The dashed curve represents the virtual photon spectrum.

Fig. 2 The proton spectrum from the $^{12}\text{C}(\gamma, p)$ reaction for $E_\gamma = 60$ MeV, $\theta_p = 45^\circ$ obtained using the Be-Al subtraction technique. The excitation energy in the residual nucleus, ^{11}B , is shown, with arrows indicating the positions of the low-lying states ($E = 2.12, 4.44, 5.02$, and 6.74 MeV).

Fig. 3 (a) Proton spectrum from ^6Li obtained using the Be-Al subtraction technique. The solid line represents the contribution of the tail below the peak in the photon difference spectrum, which is shown in (b).

large numbers), the magnitude of the tail, and uncertainties in its unfolding which one encounters in applying this method to (γ, p) reactions in which the residual nucleus is left in a highly excited state.

The two-radiator difference method has also been used¹⁰⁾ at fairly low energies (40-80 MeV) to study (γ, p) reactions on heavier nuclei: ^{27}Al , ^{40}Ca , ^{59}Co , ^{93}Nb . Although in these cases one cannot in general resolve the various final states, it is hoped that useful information about the systematics of the (γ, p) process may result from measuring cross sections to groups of states. The analysis of these data has demonstrated a disadvantage of the two-radiator method: simply that the resolution is fixed at ~ 2 MeV. Once the data are taken, not only is there no way to improve the resolution, but there is no direct way to worsen it, i.e. to sum over larger regions of excitation energy to obtain a statistically more significant result. We shall return to this point later in the discussion of the bremsstrahlung tip method which is somewhat more flexible.

In summary, figs. 1-3 illustrate the basic features of the two-radiator difference technique: moderately good (~ 2 MeV) photon energy resolution, and moderately good cancellation for photon energies below the peak. Clearly, better resolution (allowing more detailed nuclear spectroscopy) and better cancellation (allowing measurements at higher excitation energies (knockout from inner shells?)) would both be desirable. The problems associated with seeking these obvious improvements will be considered in Section IV.

B. The Bremsstrahlung tip method. This method may be described as the "inverse" of yield-curve unfolding: for a constant bremsstrahlung endpoint energy one "varies" the emitted particle energy, i.e. by measuring a proton spectrum with a counter telescope¹³⁾ or magnetic spectrometer^{4,5)}. Cross sections are obtained by fitting a calculated proton spectrum to the bremsstrahlung-produced data. The calculated spectrum contains the theoretical bremsstrahlung spectrum¹¹⁾ (in ab-

solute units*). with the electron energy loss and straggling in the radiator taken into account, plus the smearing effects due to proton energy loss in the nuclear target. Providing that the proton detector has adequate resolution (as is usually the case with a high-quality magnetic spectrometer) and that the radiator and target are sufficiently thin so as not to smear out the photon and proton spectra, the energy resolution of this method is in principle limited only by statistics. For a "simple" final state nucleus such as ^{15}N (see fig. 4a), the cross section for the $^{16}\text{O}(\gamma, p)$ reactions to the ground, 5.3 MeV doublet, and 6.3 MeV state of ^{15}N can be readily obtained from moderate-statistics data (see fig. 4b).** For the $^{40}\text{Ca}(\gamma, p)^{39}\text{K}$ reaction, however, only the ground state cross section may be obtained separately from data such as are shown in fig. 5b. For the reactions populating the excited states of ^{39}K (see fig. 5a), the cross section to regions of excitation energy (2.5 - 3.5 MeV, 3.5 - 8 MeV) is all that can be determined with statistical significance.¹⁰⁾

So far in this discussion we have ignored the contribution of the electro-disintegration process to the measured spectra. In principle, every run with the radiator in the electron beam should be accompanied by a radiator-out run, and the latter proton data subtracted from the former*** to obtain the (γ, p) yield. Certain

* The electron beam charge must be accurately measured using a non-intercepting toroid monitor placed upstream of the radiator and nuclear target.

** The photon energy dependence of the (γ, p) cross sections must also be taken into account in the fits to the data illustrated in figs. 4 and 5. Since these cross sections at intermediate energies are smoothly varying with energy, a simple iterative procedure using the measurements at different endpoint energies (e.g. $E_0 = 60, 80, 100, 120$ MeV...) is followed. One "guesses" a form for $\sigma(E_\gamma)$ from energy-independent fits, uses it in the fits to obtain a better $\sigma(E_\gamma)$, etc. Usually only one or two iterations are necessary.

*** The virtual photons contribute a yield approximately equivalent to that from a .02 radiation length radiator.

8.576	$3/2^+$
8.313	$1/2^+$
7.566	$7/2^+$
7.301	$3/2^+$
7.155	$5/2^+$
6.324	$3/2^-$
5.299	$1/2^+$
5.271	$5/2^+$

$^{15}\text{N} \quad J^\pi = 1/2^-$

Fig. 4 (a) Low-lying levels of ^{15}N

(b) Proton endpoint spectrum observed in the $^{16}\text{O}(\gamma, p)$ reaction for $E_\gamma = 160$ MeV and $\theta_p = 90^\circ$. Excitation of the $3/2^-$ state in ^{15}N at 6.3 MeV is clearly seen, in addition to the ground state. The positive parity doublet at ~ 5.3 MeV may also be populated to some extent. The cross sections are determined by fitting a calculated spectrum shape (the solid curve) to the data, as described in the text.

Fig. 5 (a) Low-lying energy levels of ^{39}K

(b) Proton endpoint spectrum observed in the $^{40}\text{Ca}(\gamma, p)^{39}\text{K}$ reaction for $E_0 = 60 \text{ MeV}$, $\theta_p = 45^\circ$. The dashed curves represent the contributions of the ground state, the first excited state at $E_x = 2.5 \text{ MeV}$, and four fictitious states each 1 MeV wide at $E_x = 4.5, 5.5, 6.5, \text{ and } 7.5 \text{ MeV}$. The fit to the latter "states" yields a cross section for the region of excitation $E_x = 3.5\text{--}8 \text{ MeV}$. (See ref. 10)

"short-cuts" have been attempted¹⁰⁾ such as deriving an E_γ -, E_p -, θ_p -, Z - dependent "fudge factor" from a few radiator-in/out comparison runs by which to multiply the rest of the radiator-in data. Even if one is willing to double the running time, by doing radiator-in and out runs at every energy and angle, there is a complication when relatively thick radiators are used: the electron straggling in the radiator when present significantly affects the emergent electron spectrum so that the radiator-in electrodisintegration contribution cannot be obtained exactly from the radiator-out data¹⁰⁾. These and other details are beyond the scope of this report and are mentioned only to indicate the problems one encounters, which become increasingly severe as regions of higher final-state excitation are examined.

IV. EXTENSIONS OF TWO-RADIATOR DIFFERENCE AND BREMSSTRAHLUNG TIP TECHNIQUES

A. Application to higher photon energies. The bremsstrahlung tip method has been used successfully over a photon energy range extending from 40 to 400 MeV (refs. 4 and 5). Although the two-radiator method has so far been used only up to 100 MeV, there is no essential reason why it, as well as the tip method, cannot be extended. All the cross sections involved (bremsstrahlung, electron collision loss, straggling, etc.) vary smoothly with energy. Fig. 6 shows a calculated two-radiator difference spectrum for $E_\gamma \approx 300$ MeV.

B. Application of two-radiator technique to other resolution widths. In principle one can obtain either a wider or a narrow photon difference spectrum by varying the radiator thicknesses (see eq. (6)). We find, approximately, that

$$\Delta k \text{ (MeV)} = t \text{ (\% } \ell_r \text{)}$$

However, for small Δk one pays dearly in intensity, as is seen in fig. 7, in which case the non-cancellation below the endpoint will make a bigger contribution, as will the virtual photons.

C. Use of higher Z radiators in the two-radiator method. In its first conception, the two-radiator method was to utilize a difference such as $I_{Ta}(k) - I_{Be}(k)$, with the same E_0 , and rely on the Z-dependence of the bremsstrahlung endpoint (see fig. 8) to produce the photon difference peak. Such a method would have the advantage of simplicity: the accelerator beam energy, tuning, etc. would not have to be changed, and one might expect greater stability as well as greater efficiency of operation. The virtual photons might cancel to a sufficient extent if the electron energy loss in the radiator(s) was small. However, as is seen in fig. 8, the bremsstrahlung cross section is Z-dependent not only at the endpoint but throughout the spectrum, due to the screening and Coulomb corrections. This Z-dependence prevents an adequate cancellation below the high energy spike, as is illustrated in fig. 9. Here the radiator thicknesses have been adjusted for the best overall cancellation over a large energy

Fig. 6 Be-Al difference spectrum
for $E_\gamma \approx 300$ MeV.

Fig. 7 Be-Al difference spectra for $E_\gamma = 100$ MeV, for two radiator thickness, $.02l_r$ and $.05l_r$. Each spectrum has been normalized to $.01l_r$. The ratio of the intensities in the peaks is seen to be ~ 10 . (The dashed curves represent the Bremsstrahlung spectrum from the Be radiator in each case.)

Fig. 8 Bremsstrahlung intensity spectra for $E_0 = 60$ MeV, $Z=4, 13, 47$, and 92 . The ordinate is the number of photons per MeV per radiation length multiplied by the photon energy (in MeV). The endpoint region is illustrated in the inset.

Fig. 9 Photon difference spectrum $W-.945$ Cu for $E_0 = 100$ MeV (solid curve) where the symbols W and Cu represent the intensity from .02 radiation lengths of tungsten and copper, respectively. The dashed curve is the virtual photon spectrum.

range below the endpoint. One does observe a narrow (0.4 MeV) photon difference peak, and it is possible that by also varying E_0 for each radiator, an acceptable spectrum could be produced, but the method is then no longer simple. In any case, the virtual photons present a problem. Since one runs with the thinner radiator at the same (or possible higher) E_0 , the mean residual electron energies cannot be made equal in the two runs, and the resulting virtual photon difference spectrum contains in fact more intensity than does the real photon spectrum (see fig. 9).

D. Measurements at very high nuclear excitation energies

A type of experiment which has recently aroused considerable theoretical interest is a measurement of the "inclusive" (γ, p) cross section in the $\Delta(1232)$ resonance region. For instance, one would measure the energy spectrum and angular distribution of 50 - 150 MeV protons produced by photons of energy 300 ± 5 MeV. For this one needs a photon difference spectrum of only moderate resolution, but with very good cancellation over a large range of lower photon energies. A two-radiator difference spectrum is not appropriate; the Z-dependence prevents adequate cancellation. A single-radiator, "conventional" difference will work, as is shown in fig. 10, but of course now the virtual photons no longer cancel. In principle one must do four runs for every data point:

$$[\text{Rad.in} - \text{Rad.out}]_{E_1} - [\text{Rad.in} - \text{Rad.out}]_{E_2},$$

unless one is willing to "neglect" the contribution ($\approx .02\%$) of the electrodisintegration process, i.e. make the assumption of a virtual photon spectrum which has the same shape as the real bremsstrahlung spectrum. (Four runs, however, still make this method compare favorably with a "dumped" photon beam, in which one loses a factor of at least 100 in possible intensity.) We should point out that the difficulties described at the end of Sec. III.B will exist in this type of measurement, and their importance should be evaluated carefully for a given set of experimental parameters (radiator and target thicknesses, energies, etc.)

The main problem associated with measurements at high excitation energies is expected to be that of "small-differences-between-large-numbers." In addition to statistics, excellent beam

Fig. 10 (a) Bremsstrahlung difference spectrum (solid curve) using a single (W) radiator, with $E_1=300$ MeV; $E_2=295$ MeV. The dashed curve represents the virtual photon spectrum.

(b) The same spectrum plotted logarithmically, to show the excellent cancellation down to low photon energies.

monitoring and stability are obviously required and detector efficiencies, dead time corrections, etc. must be known accurately. Let us examine these problems in more detail.

We first estimate the proton count rate due to the incident bremsstrahlung spectrum. Previous data¹⁴⁾ on the $^{12}\text{C}(\gamma, p)$ process when the sample is irradiated with ~ 300 MeV bremsstrahlung yield a cross section per equivalent quantum

$$\frac{d^2\sigma}{dE d\Omega Q} \approx 1 \frac{\mu\text{b}}{\text{MeVsrQ}}.$$

Using a 50 μA electron beam and a 3% radiator, one has an incident flux of $\sim 10^{13} \text{Q/sec}$. The Bates energy loss spectrometer has a solid angular acceptance of $\sim 3 \text{msr}$. With a 200 mg cm^{-2} carbon target, one finds a proton rate

$$\frac{dN}{dE_p dt} = 3 \times 10^2 \frac{\text{counts}}{\text{MeV sec}}.$$

The spectrometer has a momentum acceptance of 6% (or $\Delta E/E = 12\%$); at $E_p = 100$ MeV one finds a total rate in the focal plane detector of $N_b \approx 3 \times 10^3/\text{sec}$. With a 1% duty accelerator, this represents an instantaneous rate of $3 \times 10^5/\text{sec} = 0.3/\mu\text{sec} \approx 3$ per beam burst. This is acceptable provided that the data acquisition electronics and computer system can handle such a rate, and that dead time corrections can be made with sufficient accuracy.

Now let us estimate the count rate due to the effect we are attempting to observe, i.e. the yield of ~ 100 MeV protons from 300 MeV (monochromatic) photons. Previous, albeit crude, data¹⁴⁾ show the cross section to be

$$\frac{d^2\sigma}{dE d\Omega} \approx 1 \frac{\mu\text{b}}{\text{MeVsr}}.$$

Using the same beam current, radiator, and target, and assuming a bremsstrahlung difference spectrum of the form shown in fig. 11 which contains $\sim 3.3 \times 10^{-4}$ photons per electron (for $.03\lambda_r$), we find

$$\frac{dN}{dE_p dt} = 3 \frac{\text{counts}}{\text{MeVsec}},$$

or a total rate in the detector at $E_p = 100$ MeV of $N_m = 30 \text{ sec}^{-1}$. Comparing this figure for monochromatic photons with that for

bremsstrahlung, we find

$$\frac{N_m}{N_b} = \frac{3 \times 10^1}{3 \times 10^3} = 10^{-2}.$$

Since one is looking for a 1% effect in the difference between the proton spectra produced in two bremsstrahlung runs, 0.1% statistics, or 10^6 counts/"channel" (say 10^6 counts/3 MeV) in each run are needed. This implies a running time of 10^3 sec. $\times 4 \approx 1$ hour (two "photodisintegration" + two "electrodisintegration" runs), assuming, always, that 50 μ A is an acceptable beam current. It should be emphasized that the beam current monitoring and the detector dead time corrections must be known to $\sim 0.1\%$. It is quite likely that, on account of the latter effect, the beam current must be turned down, by perhaps as much as a factor of 10.

In order to cover the proton energy range from 50 to 150 MeV using a spectrometer with acceptance $\Delta E/E \approx 10\%$, one requires 14 runs. One would like to do these measurements at at least three photon energies spanning the $\Delta(1232)$ resonance: say 250, 300, and 350 MeV, on at least three nuclei (light, medium, heavy), and possibly three angles (e.g. 45° , 90° , 135°). One is therefore talking about a minimum of 400 hours of running.

The question arises whether the technique described above (photon difference, intense electron beam, magnetic spectrometer with limited solid angle and momentum acceptance) is the most appropriate for this type of experiment, which now appears to be costly in running time and difficult in terms of statistical and systematic accuracy required. In fact, some data have recently appeared¹⁵⁾ on inclusive (γ, p) reactions using an almost exactly opposite approach: a weak beam from a synchrotron, photon tagging, and a large aperture, broad range magnetic spectrometer. The tagging method requires a coincidence between a counter which defines the energy of the electron following bremsstrahlung (and thus the photon energy) and the proton detector. Owing to the high rate in the electron detector, and the problem of accidental coincidences, the usable beam currents are far smaller, typically < 1 nA. However, since no differences are required, 0.1% statistics (10^6 events) is no longer necessary; 3% (10^3 events) is probably

adequate. Moreover, one also gains by using a spectrometer with acceptance ~ 0.1 sr and a large enough dynamic range to cover the entire 50 - 150 MeV range at once. Finally, one can run with a single electron energy (say 400 MeV) since the tagging counters will give the photon energy information (e.g. $E_\gamma = 250, 300, 350$ MeV). The comparison between the bremsstrahlung difference and the tagged photon experiment is summarized in Table 1. It is seen that, given the above assumptions, the bremsstrahlung different method competes favorably with tagged photons.

TABLE I

	<u>Bremsstrahlung Difference</u>	<u>Tagged Photons</u>
Counts per 3 MeV required in spectrum, for each E_0	10^6	10^3
Runs per spectrum, each E_0	4	1
Number of electron energies E_0	3	1
Total events required ($E_p = 50-150$ MeV)	4×10^8	3.3×10^4
Beam current	$5 \times 10^{-5} \text{ A}$	10^{-9} A
Radiator thickness	$3 \times 10^{-2} \ell_r$	$10^{-4} \ell_r$
Spectrometer $\Delta\Omega$	$3 \times 10^{-3} \text{ sr}$	10^{-1} sr
Spectrometer $\Delta E / (\text{required range})$	7×10^{-2}	1
Events per unit time	$\alpha 3 \times 10^{-10}$	$\alpha 10^{-14}$
Running time	$\alpha 1.3 \times 10^{18}$	$\alpha 3.3 \times 10^{18}$
Relative running time	1	2.5

APPENDIX I

Accuracy of Bremsstrahlung Difference Techniques

In order to derive nuclear cross sections from photo-nuclear measurements one must make use of a theoretical form for the bremsstrahlung spectrum. The accuracy of these spectra is discussed in refs. 11 and 16, together with comments on the (meager) experimental data available. Some more recent experimental measurements made at Mainz¹⁷⁾ also confirm the basic Bethe-Heitler formulae. A few specific points relevant to the present paper are listed below:

1. Two-radiator difference method using Be, Al radiators.

For these low Z-radiators the Bethe-Heitler theory should be valid; the effect of the finite (Z-dependent) cross section at the endpoint is negligible, and the screening and Coulomb corrections are also very small. However, there are two difficulties:

- i) Electron-electron bremsstrahlung, which makes a fractional contribution of $\sim 1/Z$, becomes important. These spectra are difficult to calculate and not very well known^{11,18)}.
- ii) Low Z-radiators are relatively thick, in g cm^{-2} , for a given thickness in radiation lengths, so that the shape of the bremsstrahlung spectrum is strongly influenced by the energy loss and straggling in the radiator. The shape of the "straggled" electron spectrum, in particular its low-energy tail, is not very well known, experimentally or theoretically (see ref. 19).

2. Bremsstrahlung tip method using high Z-radiator.

In this case straggling and electron-electron bremsstrahlung are not important, but the finite Z-dependent endpoint becomes crucial. The available calculations of this quantity are discussed in ref. 11; and it should be pointed out that there are no calculations of the spectrum in the region $[E_0 - (\sim 2 \text{ MeV})] < k < E_0$, i.e. between the finite endpoint and where Bethe-Heitler is expected to be valid. Screening and Coulomb corrections are also moderately important

(a ~ 20% effect).

3. Consistency checks. We note that the shape of the bremsstrahlung spectra for low and high Z radiators, the effect of straggling, and the whole procedure for "unfolding" a cross section from an endpoint proton spectrum can be checked to some extent by measuring the same nuclear process, e.g. $^{16}\text{O}(\gamma, p_0)^{15}\text{N}$ with both methods. Such consistency checks have been performed, with favorable results within the ~ 20% systematic uncertainties (from other sources) in the data.²⁰⁾

4. Absolute intensity. We point out that the absolute intensity, and not just the shape of the bremsstrahlung spectrum must be known, since only the electrons are monitored. Few of the existing measurements of the spectrum have attempted to make an absolute comparison between theory and experiment. (This is also true for the electron straggling measurements; the "peak height" is not measured absolutely, but only its position and width, so that the area in the low energy tail cannot be determined accurately.)

Finally, we point out in passing that the commonly used, convenient Schiff formula is wrong, in both shape and magnitude, as illustrated by fig. 6 of ref. 11.

APPENDIX II

Electrodisintegration

In this report the contribution of the electrodisintegration process to the measured yield has been treated as a background which must be subtracted. Another approach which has been followed by various workers is to simply use this yield, and not measure photodisintegration at all. The equivalent photodisintegration cross section is then derived from the data with the aid of a theoretical virtual photon spectrum. A variety of forms of such spectra have been published²¹⁻²⁵⁾, containing different approximations (e.g. $m_e = 0$), some algebraic errors, and many unexplained discrepancies. A comparison or evaluation of the published expressions is beyond the scope of this report, but a few comments might be useful. The crudest assumptions, which lead to the simplest formula, are those of the Born approximation, forward scattering of the electron, a point nucleus, and transverse electric dipole radiation only. However, there is no essential difficulty in integrating over the scattered electron angles, (see eqs. 1.5 of ref. 22), or in deriving formulae for higher multipoles. The finite nuclear size can be taken into account with the introduction of nuclear form factors.²³⁾ For a high Z (≈ 50) target distortion of the outgoing electrons has been shown to be important.²⁶⁾

The basic problem with the use of virtual photons, as compared to real photons, is that in the former case the incident spectrum is not independent of the properties of the target (Z, A) or of the energy and angle of the emitted particle, so that one cannot simply "measure" the virtual photon spectrum in one experimental situation (as one could in principle with the bremsstrahlung spectrum) and apply it to other experimental situations.

However, if one is willing to accept this difficulty and the resulting uncertainties in the derived cross sections, virtual photons do offer some advantages. There is no radiator to smear out the electron energy or to multiply scatter the electrons. Thus the "endpoint" of the virtual photon spectrum is as sharp as the theoretical prediction and the beam spot on the target may

be adjusted to equalize the incident electron and outgoing particle energy losses. These three effects allow the possibility of improved resolution. But one returns, as with the bremsstrahlung tip method, to the problem of statistics. Even if one can resolve the uncertainties in the shape of the virtual photon spectrum, and obtain a form which may be fitted to the measured yields in order to derive cross sections (for the excitation of specific final nuclear states), there must be enough counts in the spectra to produce statistically significant results. As a comparison, we note that for the (γ, p) measurements made at Bates for $E_\gamma \geq 100$ MeV using the tip method, a $.05\lambda_r$ radiator and a $\sim 200 \text{ mg cm}^{-2}$ target were used. The virtual photon intensity is roughly equivalent to a $.02\lambda_r$ radiator, and in order to produce 50 keV resolution (at 100 MeV) a target thickness of $\lesssim 100 \text{ mg cm}^{-2}$ in the beam direction is required.²⁷⁾

REFERENCES

- 1) A. S. Penfold and J. E. Leiss, Phys. Rev. 114(1959)1332;
B. C. Cook, Nucl. Instr. and Meth. 24(1963)256.
- 2) G. Manuzio et al., Nucl. Phys. A133(1969)225;
M. Sanzone et al., Nucl. Phys. A153(1970)401;
E. Mancini et al., Nuovo Cim. 15A(1973)705.
- 3) J. L. Matthews et al., Nucl. Phys. A267(1976)51.
- 4) D.J.S. Findlay and R. O. Owens, Nucl. Phys. A279(1977)385.
- 5) J. L. Matthews et al., Phys. Rev. Lett. 38(1977)8.
- 6) J. L. Matthews and R.O. Owens, Nucl. Instr. and Meth. 91
(1971)37.
- 7) G. G. Taran and A. N. Gorbunov, Sov. J. Nucl. Phys. 6(1968)816.
- 8) R. Bergere, private communication.
- 9) J. Arends et al., preprint.
- 10) D. J. Gibson, Ph.D. thesis, Glasgow University (1978).
- 11) J. L. Matthews and R. O. Owens, Nucl. Inst. and Meth. 14(1973)
157.
- 12) D.J.S. Findlay et al., J. Phys. A7(1974)L157.
- 13) J. L. Matthews et al., Nucl. Phys. A112(1968)654.
- 14) P. Dougan and W. Stiefler, Z. Phys. 269(1974)97.
- 14a) J. W. Weil and B. D. McDaniel, Phys. Rev. 92(1953)391;
R. J. Cence and B. J. Moyer, Phys. Rev. 122(1961)1634;
G. Anderson, P. Dougan, and W. Stiefler, Z. Phys. A272
(1975)263.
- 15) B. Mecking, private communication.
- 16) H. W. Koch and J. W. Motz, Rev. Mod. Phys. 31(1959)920.
- 17) J. Ahrens et al., Nucl. Instr. and Meth. 108(1973)517.
- 18) V. N. Baier, V. S. Fadin, and V. A. Khoze, Sov. Phys.
JETP 24(1967)760; 26(1968)1238;
E. Haug, Z. Naturforsch, 30a(1975)1099.
- 19) J. L. Matthews and R. O. Owens, to be published.
- 20) D.J.S. Findlay, private communication.

21. C. J. Mullin and E. Guth, Phys. Rev. 82(1951)141;
J. A. Thie, C. J. Mullin and E. Guth, Phys. Rev. 87(1952)962.
22. R. H. Dalitz and D. R. Yennie, Phys. Rev. 105(1957)1598.
23. W. C. Barber, Phys. Rev. 111(1958)1642; W. C. Barber and
T. Wiedling, Nucl. Phys.. 18(1960)575.
24. R. Rodenberg, Z. Phys. 158(1960)44.
25. B. F. Gibson and H. T. Williams, Nucl. Phys. A163(1971)193.
26. C. W. SotoVargas, D. S. Onley and L. E. Wright, Nucl. Phys.
A288(1977)45
27. I. Blomqvist, private communication.

IV.3 EXPERIMENTAL PROBLEMS IN NUCLEON KNOCKOUT
REACTIONS: TRIPLE COINCIDENCE EXPERIMENTS
AND A VERY LARGE SOLID ANGLE SPECTROMETER

C. P. SARGENT

I. INTRODUCTION

Two of the most important problems in intermediate energy nuclear physics concern the production and propagation of the $\Delta(1232)$ and the role of exchange currents in general in producing two-nucleon correlations in nuclear matter. Deep inelastic electron scattering can provide a uniquely precise tool in studying these effects, and an efficient one as well. For example, considering the scattering of 500 MeV electrons at 60° in carbon, about two-thirds of the cross-section at 350 MeV inelasticity results in Δ production, and recent estimates indicate that most of the other third can produce two-nucleon correlated emission processes from exchange-current absorption of the virtual photon. The two nucleon processes can only be studied by measuring two nucleons in coincidence with the scattered electron, and the Δ processes can be most effectively studied by measuring a π^- and a proton in coincidence with the scattered electron. That is, such a program requires triple coincidence experiments, with three spectrometers mounted about a common pivot point.

Triple coincidence experiments of this type at intermediate energies in light nuclei have an experimental feature which should be recognized. Namely, as will be shown in the next section, they tend to be "cross-section" (or "beam current") limited rather than "accidental" (or "duty-ratio") limited, and one does not expect that improvements in duty-ratio above the 2% now available at Bates will result in increased efficiency in doing such experiments. When one deploys the largest state-of-the-art counting

arrays (in this case magnetic spectrometers) and puts as much beam on the thickest target that can be tolerated (for inelastic electron-scattering, say 100 microamperes on a few thousandths of a radiation length at most) and finds that the true rate is greater than the accidental rate, then improved duty ratio will not improve experimental efficiency, for to do so would require yet more current on the target. Two-arm (double) coincidence experiments at Bates' 2% duty are almost invariably "accidental" limited, and one must reduce beam or stop down solid angle or range of spectrometers. However, when one adds a third arm, beam current can usually be increased or acceptance increased. Instead of triple coincidence experiments being two or three orders more difficult than doubles, they are only one or two. The information obtained at 30 counts per hour in this way is often either unique or more valuable than the information obtained at 30 counts per minute in the associated double coincidence experiments.

II. A "TYPICAL" TRIPLE COINCIDENCE EXPERIMENT: $(e, e' \pi^- p)$

The deep inelastic electron scattering cross section from carbon has been measured¹⁾ at 60° and 500 MeV incident electron energy. It is about $4 \times 10^{-33} \text{ cm}^2 \text{ MeV}^{-1} \text{ sr}^{-1}$ at 350 MeV inelasticity, the region of the $\Delta(1232)$ resonance, having increased from a lower value at lower inelasticity. This behavior would indicate that about half of this cross section results in the production of a Δ , in reasonable agreement with theoretical estimates. If an electron spectrometer with 100 msr solid angle is placed at 60° to cover the range from 350 to 400 MeV inelasticity (100 to 150 MeV scattered electron energy), the counting rate will be $1.6 \times 10^4 \text{ sec}^{-1}$ with a full 100 μA of 500 MeV electrons incident on a 2×10^{-3} radiation length (100 mg/cm^2) carbon target. (A thicker target would have difficulty withstanding the beam, would begin to produce significant numbers of counts from

real bremsstrahlung made in the target, and would make it difficult to dump the beam without producing background.) Some fraction, f , probably about 10%, of the 10^4 sec^{-1} Δ 's produced by the electrons detected in the spectrometer will yield a π^- - proton pair with kinematics appropriate to production on a free neutron, modified by the Fermi motion of the neutron within the nucleus and by some refractive and binding effects produced by the other eleven nucleons.

The first goal of a coincidence experiment would be to measure f . This could be done by detecting either protons or pions in coincidence with the scattered electron and with the kinematics in momentum space appropriate to the postulated reaction mechanism. The signature would be more convincing if both proton and pion were detected in a triple coincidence experiment. Secondly, if f is sufficiently large, the nuclear production dynamics within the nucleus can be studied and compared to the free production dynamics. It is then particularly important to observe both proton and pion, since the dynamics has several facets and the proton and pion are

differently sensitive to them. The lighter pion acquires most of its energy-momentum from the Δ decay and is more sensitive to the features of that process. The heavier proton, on the other hand, gets most of its energy-momentum in the transformation from Δ rest frame to laboratory, and is more sensitive, for example, to the initial momentum of the neutron which absorbs the virtual quantum to produce the Δ .

The pion spectrometer covering 30 msr is placed at 80° relative to the incident electron (67° relative to the momentum transfer $\vec{q}$) to cover the range from 106 to 190 MeV. The proton spectrometer, also 30 msr, is placed at 20° relative to the incident electron (33° with respect to $\vec{q}$) to cover the range from 78 to 104 MeV. The above kinematics are chosen to detect π^- -p pairs produced at 90° relative to $\vec{q}$ in the Δ rest frame when the initiating neutron is at rest in the laboratory. The pion and proton

spectrometers are well matched to the very large solid angle electron spectrometer, covering the range of $\vec{q}$ and ω defined by that spectrometer. This range, expressed in Δ mass, and still for the initiating neutron at rest, is 1200 to 1250 MeV, covering the free Δ resonance region.

Double Coincidence Experiments - Singles rates in the proton and pion detectors are estimated to be $2 \times 10^4 \text{ sec}^{-1}$ and $1 \times 10^4 \text{ sec}^{-1}$. The estimates are from bremsstrahlung data and a 1% radiation length equivalent radiator. With modern magnetic spectrometers, which accurately define angles within their acceptance range, and with computer-assisted timing analysis, coincidence time resolution of 10^{-9} sec for charged particles is available. Dividing this time by the 2% duty ratio, and taking the singles rate for all three spectrometers to be $1.6 \times 10^4 \text{ sec}^{-1}$, the accidental rate for both the $(e, e'p)$ and $(e, e'\pi^-)$ double coincidence experiments is $A = (5 \times 10^{-8}) (1.6 \times 10^{-8})^2 = 13 \text{ sec}^{-1}$.

The estimated true rate is the fraction of the 10^3 sec^{-1} (assuming f is 10%) Δ 's defined by the electron spectrometer which are detected by the pion counter for the $(e, e'\pi^-)$ experiment or the proton counter for the $(e, e'p)$ experiment. This fraction is influenced by two factors. The first of these is the solid angle of the detectors when transformed to the Δ rest frame (the angular distribution will be assumed isotropic in that frame). The pion detector transforms to 40 msr and the proton detector to 180 msr. The detectors have adequate range for these solid angles. The second factor, allowing the initiating neutron to have its Fermi motion, is the volume fraction of the 160 MeV/c radius Fermi sphere which the counters can see. This fraction is about .08 for the pion counter and .02 for the proton counter. For the double coincidence experiments, the separation of the two factors is an approximation resulting in an underestimate of the rate by about 3. Including this

factor of three, the estimated true coincidence rates are then:

$$T_{\pi e} = 3 \times 10^3 \times .04 \times .08 (4\pi)^{-1} \approx .75 \text{ sec}^{-1}$$

$$T_p = 3 \times 10^3 \times .18 \times .02 (4\pi)^{-1} \approx .9 \text{ sec}^{-1}$$

The expected rates thus calculated are somewhat less than 1 sec^{-1} , about 5-10% of the accidental rate, and the beam current must therefore be reduced by a factor 10 or 20 below the $100 \mu\text{A}$ used for the calculation.

For both of the double coincidence experiments, therefore, one expects to be counting at about 250 hr^{-1} , accidental limited, with 5-10 μA on target. Such experiments are not particularly attractive with a 2% duty ratio accelerator.

Triple Coincidence Experiment - At $100 \mu\text{A}$ beam current, the true doubles rates between π^- and e^+ and between p and e^+ are much smaller than the $\approx 10 \text{ sec}^{-1}$ accidental rates. The true π^- - p doubles rate is difficult to estimate, but the kinematic region is remote from that allowed for meson production by real photons on nucleons, and it is probable that this rate also is smaller than 10 sec^{-1} . In this case, the accidental rate for the three-fold coincidence experiment is

$$A = (5 \times 10^{-8})^2 (1.6 \times 10^4)^3 = 1 \times 10^{-2} \text{ sec}^{-1}$$

The true triples rate, in this case not underestimated, is determined by the smaller of the two solid angle and fermi sphere fractions:

$$T_{\pi pe} = 10^3 \times .04 \times .02 (4\pi)^{-1} = 6 \times 10^{-2} \text{ sec}^{-1}$$

which is considerably larger than the accidental rate.

For the triple experiment one thus expects to be counting at about 200 hr^{-1} , beam current limited with $100 \mu\text{A}$ on target, which is an attractive experiment. A duty ratio higher than 2% would be of little value.

III. ELECTRON SPECTROMETER DESIGN

Two spectrometers with moderately large solid angular acceptance

(~30 msr), appropriate for the detection of pions and of protons, will be mounted about the same pivot point in the new experimental area at Bates. These instruments are currently being designed and will not be discussed further here. We will, however, address ourselves to the problem of the specifications and design of a third, very large solid angle, spectrometer for the detection of electrons.

An appropriate spectrometer should have at least $\pm 20\%$ range and the largest possible solid angle consistent with at least 1% momentum resolution. Angular resolution for both angles within the acceptance should be at least 20 mrad, and maximum momentum should be at least 700 MeV/c. The spectrometer should be mounted about the same pivot point as the other spectrometers; and, in order to exploit the polarization of the virtual photons, it should have more than a single possible position in the azimuthal angle relative to the scattering plane defined by the other spectrometers. A wide range of positions in the polar electron-scattering angle is less important. The other requirements may well in fact be incompatible with operation at forward electron angles. A spectrometer fixed at 90 degrees might be acceptable, at least at the beginning, but one with some variation capability is to be desired. The spectrometer should be located in the opposite scattering hemisphere to the proton spectrometer.

Preliminary design of the third spectrometer has centered about the use of a superconducting quadrupole as the only magnetic element. A quadrupole with 2 kG/cm gradient, 0.5 m by 0.25 m rectangular aperture, and 0.45 m effective length forms the basis of the design. The long dimension of the aperture represents the focusing dimension and the short the defocusing dimension. The short dimension is bridged at the quadrupole center by 0.25 m by 0.50 m by 0.12 m of thick heavy shielding between the target and the focal plane, allowing two "holes" with centers separated

by about 0.3 m for passage of the scattered beam through the quadrupole field to the detectors.

The central design momentum at maximum gradient is 500 MeV/c, with a distance of 0.2 m from the target to the effective entrance field boundary. Under these conditions the solid angle acceptance is about 150 msr, the momentum range is from 400 to 600 MeV/c, and the central dispersion about 1 cm/%. The focal plane is a trapezoid with bases 0.5 and 0.8 m, and height (along the quadrupole axis) 0.5 m. A central momentum of 700 MeV/c can be reached by increasing the target - quadrupole distance to 0.3 m, with a decrease of solid angle to about 100 msr.

The focal plane instrumentation (for electrons), consisting of a five-layer sandwich, should provide background-free coincidence triggering, submillimeter position measurement in the dispersion direction, measurement of both scattering angles with about 15 mr resolution, and some redundancy for noise suppression. The outer layers are vertical drift chambers with sense wires normal to the quadrupole axis, the intermediate layers are plastic Cerenkov counters, run in coincidence for fast triggering, and the fifth inner layer is a chamber with sense wires parallel to the quadrupole axis. Incidence angles on the chambers are very oblique, ranging from 54 to 82 degrees, so that relatively few active wires are required to provide the necessary information. About 50 wires over the 0.5 m should suffice with one-inch deep chambers. Whether one amplifier-TDC channel per wire is required or whether delay-line coding can be used, as for the present high-resolution spectrometer, will depend on background levels in the chambers. These levels must be no more than about one count per two microseconds per channel. Experience with the present photopion spectrometer (in the 14° area) at 90 degrees and 150-250 MeV incident energy in a roughly similar source-shielding-detector geometry suggests that background requirements for chamber

operation will be met at this angle or greater for low Z 100 mg/cm² targets and with currents of at least 40 μ A. At a given angle the situation will improve with increasing energy, provided that effective beam dumping is accomplished.

The momentum resolution which will be achieved is not certain. Dispersion, beam size, and multiple scattering will certainly limit this to about 1 part in 10^3 . However, it is unlikely that the field quality in the superconducting quadrupole will allow this limit to be reached. On the other hand, it seems reasonably certain that 1 part in 10^2 can be reached, since the angular measurements require acceptable field quality only over a much smaller area than the total quadrupole aperture. The spectrometer can be calibrated via electron scattering on hydrogen using the proton spectrometer in coincidence. A principal goal of more detailed study will be to resolve the field quality-resolution question, the aim being to achieve, if possible, at least 3×10^{-3} .

REFERENCE

1. R. R. Whitney et al., Phys. Rev. C. 9, 2230(1974)

V.1 ELECTRON SCATTERING AS A PROBE OF NUCLEAR STRUCTURE

J. HEISENBERG AND J. W. NEGELE

I. INTRODUCTION

Photons and charged leptons provide the most precise, quantitative probes of the nucleus available and a recent review of the present status of the field, has been reproduced as Article 2. One obvious conclusion from this review of the data arising from the new generation of accelerators is that electromagnetic probes are now sensitive to details of the full relativistic many body system of mesons and baryons and thus the description in the traditional nuclear structure language of non-relativistic quantum mechanics is inadequate.

Nevertheless, for organizational purposes, nuclear structure has been separated from other topics treated in this workshop. Such a distinction is at best, only approximate and at worst, outright misleading. Until one understands the effects of meson exchange currents, for example, one will never be able to extract nuclear distributions from one experimentally measured charge, current, or spin distributions. Similarly, until the high momentum components of the nuclear wave function are thoroughly understood, one will never succeed in removing this nuclear structure "background" from the isobar propagation one might like to investigate in $(e,e'p)$ or (γ,p) experiments.

One obvious corollary to our attempt to discuss nuclear structure in isolation from relativistic and meson effects is the restriction of our attention to low and intermediate momentum transfer, where relativistic and exchange effects are sufficiently

small to be ignored or to be treated as perturbative corrections. With this restriction we obviously cannot argue for new higher energy machines since the low and intermediate momentum transfer range can be covered by a machine like the Saday accelerator and higher energy would at most reduce somewhat the time for an experiment but not add qualitative new physics. Certainly, as discussed in Article 2, many exciting and fundamental questions can only be explored at high momentum transfer, and these issues will be emphasized in other sections of the workshop report.

A second limitation in scope for organizational convenience is the restrictions of our attention to single-armed electron scattering. Again, this rules out considerations of many significant coincidence experiments requiring a higher duty cycle than presently available. The primary justification for higher duty cycle, as for higher energy, thus properly belongs elsewhere in this report.

Before embarking on a review of the topics discussed in the workshop, it is useful to emphasize the role of electron scattering as a precision probe. Electron scattering is not just another probe like the α -particle or the proton with slightly different selection rules, useful to measure spin and parity of states. People who treat electron scattering this way do not understand what electron scattering is about. Electron scattering is a matter of precision. People claiming perfect agreement of their predictions within 20% should rather talk about a serious discrepancy of 20%. With respect to the collective modes of the nucleus the role of electron scattering is not to establish these modes,

which can be done as well with the help of other probes, but to measure the details with such precision that the only interpretation which can make sense is a microscopic interpretation. This way the hope is to learn on the one side about nuclear forces and on the other side about the precision of our microscopic approximations.

Although the neutron form factor was not explicitly discussed in the workshop, our ignorance concerning its detailed structure has impact on the quantitative precision presently available in analyzing experiments, so some comment is in order. At the present time most of the available information comes from scattering off the deuteron. If we take a nucleus like ^{238}U we find that the effects of the neutrons as given by the number of neutrons divided by the number of protons is larger in U than in the deuteron. Still nobody would claim that U is a better nucleus to study the neutron form factor because here everything is clouded by the uncertainty of the nuclear structure. The same is basically also true for exchange currents or other possible corrections. Electron scattering to discrete nuclear states is sensitive to these effects and in a comparison of theoretical predictions with experimental data these effects should definitely be included. Nevertheless scattering off these levels seems at the present time not a very suitable way to learn about these effects.

II. ELASTIC SCATTERING

An ideal case for the discussion of elastic charge scattering is provided by ^{208}Pb , which is the subject of Article 3. This is

a nucleus which provides an optimal comparison with mean-field theory and for which extensive data is available.

There are 272 different elastic cross section points measured and 6 independent X-ray transitions in muonic Pb. Parameterizing the charge distribution in the Fourier-Bessel expansion presents a particular transparent way of finding the requirements for the experimental work.

$$4\pi\rho(r) = \sum_{n=1}^N a_n j_0\left(\frac{n\pi r}{R}\right)$$

In this parametrization the formfactor at the momentum transfer $q_n = \frac{n\pi}{R}$ is given by

$$F(q_n) = \frac{1}{2} a_n \frac{R^3}{n^2 \pi^2} = \frac{1}{2} a_n \frac{R}{q_n^2}$$

Let us assume we want to determine the density at the origin as precisely as possible. With the density in the center given by

$$4\pi\rho(0) = \sum a_n$$

and, assuming that the coefficients a_n have been determined in an uncorrelated way, the uncertainty is given by

$$[4\pi\Delta\rho(0)]^2 = \sum \Delta a_n^2$$

In Article 3 a graph of $\frac{2F(q)q^2}{R}$ and the accumulated error

$$\left[\sum_{i=1}^n \Delta a_i^2 \right]^{1/2}$$

is presented. From such a graph it is obvious that we do not improve the total accumulated error much if we measure the form-factor in a region where $\frac{2Fq^2}{R}$ is already and stays smaller than the accumulated error up to this region. In the case of ^{208}Pb the cross section at low and intermediate momentum transfer has been measured so precisely that extending the data out to a momentum transfer of 3.5 fm^{-1} still helped a lot of reducing the uncertainty of $\rho(0)$. This exercise demonstrates clearly that two conditions are needed to obtain good information about the origin: Precise data at low and intermediate momentum transfer and data extending to high momentum transfer.

A related problem is that of normalization. As is evident from Article 3, in which the total χ^2 decrease from 630 to 443 when the normalization of each of the 14 data sets is treated as a free parameter, such uncertainties may increase the actual error in the central density well beyond the statistical estimate given by $\Sigma \Delta a_n^2$. So once again the issue of commensurability of the magnitude of high q data and the absolute accuracy of low q data is crucial in the design of experiments.

In trying to include all the muonic X-ray transitions sensitive to the nuclear charge distribution, an inconsistency arises between electron data and the muonic transitions. This inconsistency should be blamed most likely on the inadequacy of theoretical nuclear polarization corrections and constitutes another fundamental uncertainty in the extraction of charge distributions.

Although this possible discrepancy between theory and experiment in ^{208}Pb is still unresolved, this case study does highlight the problems concerning absolute normalization, accumulated errors, exchange currents, dispersion corrections, and nuclear polarization corrections which are relevant to all investigations of charge distributions.

On the other hand, there are examples where knowing the charge distribution to the level of precision as in ^{208}Pb is useful. B. Frois demonstrated that in predicting pp' experiments on ^{58}Ni the inclusion of the high q data from Saclay still had an effect on the prediction. Thus knowing the charge distribution that will at least remove one of the many uncertainties in predicting pp' data. With the precise pion and proton scattering data now arising from meson factories, there should be many applications in which properly conceived electron scattering experiment can remove certain nuclear structure uncertainties and thereby significantly contribute to the study of strong interaction effects.

Elastic magnetic scattering offers the potential for revealing the ground state current and spin distribution. In order to focus concretely on the practical issues involved in extracting accurate nuclear structure information from such experiments, the workshop considered in detail recent Saclay experiments on ^{87}Sr and ^{93}Nb which attempt to extract the $g_{9/2}$ single partial neutron and proton wave functions. Undoubtedly, since these are magnetic transitions serious consideration should be given to exchange currents, but such questions fell outside

our purview. Rather, as summarized in Article 4, we addressed the question of whether particle-hole excitations could introduce significant ambiguity in the analysis. Although M9 transitions are relatively immune to particle-hole contributions there exists the possibility that M7 transitions (for which the presence of a node in the transition density causes peaking at higher q) are non-negligible. Clearly quantitative control over such effects as well as exchange contributions is required before one can believe the nuclear structure inference presented with the original Saclay results.

III. INELASTIC SCATTERING

Inelastic scattering to the 3^- octupole state in ^{208}Pb is discussed in Article 5. Experimentally the situation here is much worse than in the elastic scattering case. Here we have two problems. At low momentum transfer the inelastic cross sections are usually not very accurate because the inelastic peak is a very small peak on a huge radiation tail from the elastic scattering. At high momentum transfer because of limited resolution and poor statistics again the data are not existing or poor. Therefore, a reconstruction of the transition density can be done only with larger errors than in the elastic case. Thus a direct comparison to the theory seems to be more appropriate.

B. Frois and J. Heisenberg prepared a comparison of theoretical predictions with some existing data. All the predictions were made using RPA with different forms of forces. The best overall agreement is obtained with the semi-empirical Migdal force. The p-h wave functions were generated using the DME

Hartree Fock code of J. Negele. The ground state charge distribution generated with these wave functions describes the elastic scattering rather well up to a momentum transfer of about 2 fm^{-1} . This is the same region where also for the inelastic prediction the larger deviations start to occur. It therefore seems plausible that the reason for the discrepancy is in the ground state wave functions. In the collective model the transition density is related to the ground state density, or in q space the inelastic form factor is related to the elastic form factor. Thus if the elastic form factor is incorrect beyond $q = 2 \text{ fm}^{-1}$ the inelastic has to be wrong too.

Such a collective level is mainly sensitive to the wave function and to the method of solving the many body problem but it is much less sensitive to the details of the nuclear force as long as sum rule approximations hold (such as the Fallieros sum rules). Most cases of inelastic scattering are, however, much more selective, and we can arrange the experiment as to whether we want to probe a particular part of the nuclear interaction or whether we want to check the quality of the approximation method. Let us give some examples:

When we measure the electric form factor of a neutron or a neutron hole transition such as in ^{207}Pb or ^{17}O we measure the effective charge or the core polarization induced by the single neutron. That means we know from stripping or pick-reactions the main component of the wave function. We measure, however, the small part of the wave function that arises from the interaction of the neutron with the protons in the core. Thus we

sample very specifically the neutron-proton interaction. Since the core polarization is only a small admixture we can calculate it almost in perturbation theory which means we have a very good handle on how to treat the many body problem meaning we really probe the interaction. Measuring the effective charge of a single proton we look at the proton-proton residual interaction. On the other hand if we measure the M6 transition to the 6^- state in ^{24}Mg which recently has been done by T. Drake, et. al., at the Bates Linac we again have a dominant configuration which is $d_{5/2}^{-1}f_{7/2}$. Because of the different selection rule of the M6, however, we sample a completely different part of the residual interaction than in the previous cases.

Interpreting the results on the first 2^+ or 4^+ states in the sd-shell region is a completely different situation. The shell model calculations that have been done by Wildenthal or others yield transition probabilities too small by a factor four. Obviously, with electron scattering we are probing the question on how to include the quite important corrections due to core polarization. This is a case where we ask for the precision of the approximation method much more than for the particular form of the interaction. M. Huber reported a first attempt to solve this problem by coupling to the many body sd-shell wave function the ^{16}O core excitations that are treated using the RPA.

Other subjects in inelastic electron scattering included excitations of high spin states and collective magnetic dipole resonances, which are discussed in Articles 6a and 6b.

IV. DEFORMED NUCLEI

The separate discussion of elastic and inelastic scattering is inappropriate for deformed nuclei since, at least in the rotational model, they yield complementary information concerning the same intrinsic state. As discussed in Article 2, low multipole transitions for even-even deformed nuclei are reasonably well understood in terms of the mean-field theory. Hence, it is important to understand odd-even rotators to at least the same level of sophistication, and this topic is discussed in detail in Article 7. For odd-even rotators, the magnetization has two contributions, one from the odd particle and a contribution from the convection current generated by the rotation. The part that can be calculated much more directly is the single particle contribution. The $M\lambda$ matrix element can be transformed into the intrinsic frame. For $K \neq 0$ states this gives two contributions

$$\begin{aligned} \langle J_f K | M\lambda | J_i K \rangle &= \langle J_i K \lambda 0 | J_f K \rangle \langle K | M\lambda | K \rangle \\ &+ \langle J_i -K \lambda 2K | J_f K \rangle \langle K | M\lambda | -K \rangle \end{aligned}$$

The second term vanishes for all multipolarities $\lambda < 2K$ and the Clebsch Gordan coefficient in front of the first term is rather small. As a general rule, assuming that the single particle contribution is the dominant part, we find that all magnetic scattering except from the largest multipolarity is strongly reduced, a result that qualitatively agrees with the experiments on ^{25}Mg . Again in our discussion it became apparent that the weakest part in our understanding is in the transition region where neither the strongly deformed nor the spherical picture holds.

Finally, we discussed the usefulness of aligning nuclei for elastic or inelastic scattering for which the reader is referred to Article 8. The effect of aligning nuclei can partly be understood by looking at the emission of real photons from aligned nuclei as a counterpart to the absorption of virtual photons which we have in electron scattering.

Assume the nucleus is completely aligned. Then, the γ -radiation becomes asymmetric. The asymmetry depends on the spin in the initial state and in the final state as well as on the multipolarity of the radiation. For electron scattering, that means we look at the angular distribution of the cross section with respect to the direction of alignment. Again we can from the angular distribution figure out the relative strength of the various multipole contributions as well as their relative phase. It is clear that separating the multipolarities requires rather precise measurements which at the present time require substantial efforts.

For aligned nuclei in elastic scattering we basically can determine the static quadrupole distribution of the nucleus. It turns out that most of the nuclei that can be aligned or polarized just by cooling are those that are strongly deformed. Therefore, one will be testing mainly the relation between the static and the dynamic quadrupole moment which certainly is of interest. Realistically in an experiment it will be extremely hard to measure this to a level of precision where one would start believing any discrepancies found.

However, there seems to be a very useful application in those cases where separating the multipolarities even to the 10% level is a useful task. The main region of application then appears to be in inelastic scattering in the giant resonance region where lines are not sharp and different multipoles strongly overlap. Using the angular distribution of the cross section with respect to the direction of alignment seems to be the only unambiguous way of separating those multipoles.

CONCLUSION

Clearly, the workshop discussions we have summarized in no way constitute an exhaustive review of the future possibilities for nuclear structure research with electron scattering. As befits the maturity of the field, the open questions require more in the way of physical insight and theoretical developments than new experimental facilities. Higher energy would, of course, be useful in increasing cross sections, and a 180° spectrometer is crucial in isolating transverse multipoles. The most essential ingredient of all however, remains the vision to select wisely from the multitude of nuclear states throughout the periodic table these special cases which isolate as clearly and unambiguously as possible the crucial questions of nuclear structure.

NUCLEAR ELECTRON SCATTERING

J. W. Negele⁺

Center for Theoretical Physics
Department of Physics and Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

*This work is supported in part through funds provided by ERDA under Contract EY-76-C-02-3069.*000.

⁺Alfred P. Sloan Foundation Research Fellow.

Talk given at VII International
Conference on High Energy Physics
and Nuclear Structure, Zurich,
Switzerland..

August, 1977
CTP # 664

NUCLEAR ELECTRON SCATTERING*

J. W. Negele⁺

Center for Theoretical Physics
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 U.S.A.

Abstract

The status of theoretical efforts to evaluate electromagnetic charge, magnetization and current densities in finite nuclei is briefly surveyed. Selected elastic Coulomb and magnetic electron scattering results, inelastic scattering data, and coincidence experiments are discussed and shown to raise significant questions well beyond the level of existing theory.

1. Introduction

Because of our understanding of electromagnetic interactions and the simplifications in reaction theory arising from the weakness of the electromagnetic coupling constant, photons and charged leptons provide the most precise probes of the nuclear wave function presently available. Whereas many high energy nuclear experiments manifest sensitivity to interesting aspects of nuclear structure, the electromagnetic interaction is unique in the degree of quantitative accuracy it affords.

Historically, electron scattering originally provided only limited information about nuclear shapes, specifying essentially the half-density radius and the surface thickness. Nuclear theory eventually caught up with experiment, with the mean field (or Hartree-Fock) approximation yielding an excellent understanding of the systematics of nuclear shapes.¹⁾ With the new generation of experimental electron scattering facilities now available, however, the experimentalists have again surged far ahead of the theorists. Present measurements of charge densities, current distributions and magnetization densities not only probe details of nuclear states for which existing theory is inadequate, but also evidence the unmistakable signature of mesonic contributions, for which no fundamental systematic theory exists.

To place recent experiments in proper perspective, I shall begin with a brief review of the present status of nuclear many-body theory. Due to the limitations of time and the emphasis of this conference, the ensuing review of experimental data will be grossly incomplete. With sincere apologies to all who have provided data that is not included, it has been necessary to select only a few representative examples which, I hope, have particu-

lar impact on our present theoretical understanding and relevance to the special interests of this conference.

2. Nuclear Many-Body Theory

In principle, the nucleus presents an exceedingly complicated relativistic many-body problem involving nucleons, anti-nucleons, isobars, mesons and ultimately even quarks. In practice, no relativistic many-body formalism has ever been implemented which is capable of treating the two-body correlations already found to be important in the non-relativistic problem. Hence, it is necessary at the outset to suppress mesonic degrees of freedom, first addressing the much simpler non-relativistic many-body problem with static interactions and then perturbatively estimating mesonic corrections.

2.1 Non-relativistic Ground States

The Schrodinger equation may be solved essentially exactly with realistic interactions for the three-body problem.^{2,3)} For ³He, ¹⁶O, and ⁴⁰Ca, the e^S formalism of Coester and Kümmel^{4,5)} has been implemented by Zabolitzky^{6,7)}. These results, which are equivalent to summing G-matrix ladder diagrams, three-body Faddeev diagrams, and the RPA ring diagrams, appear to have converged adequately in energy and density and are believed to be quite reliable.

At a less fundamental level, closed shell spherical and well-deformed nuclei throughout the rest of the Periodic Table are presently described in the mean-field approximation with density-dependent effective interactions. Intuitively, the mean field approximation, in which the mean field is defined to precisely cancel the most important low order diagrams occurring in the expansion of a one-body operator, is expected to be quite accurate, and explicit comparison with more complete calculations in light nuclei bears out this expectation.¹⁾ Density-dependent effective interactions, of which there are now legion,⁸⁾ can provide a reasonably accurate representation of a realistic G-matrix, and, when phenomenologically adjusted to the proper nuclear matter binding energy and saturation density, effectively compensate for the shortcomings of the original nuclear interaction and the omission of relativistic, mesonic and many-body correlation corrections. Thus, when restricted to its region of validity, the mean-field theory with an appropriate effective interaction yields a systematic, if not completely fundamental description of

Figure 1. Energy of deformation versus mass quadrupole moment.

much of the gross structure of nuclear energies and density distributions.

One obvious criterion for the validity of the mean-field theory is the degree of shell closure. For poorly closed shells, it would be surprising indeed if such a crude caricature of the residual interaction as the pairing approximation yielded more than qualitative results. Thus, pairing or Hartree-Fock-Bogoliubov calculations in open or poorly closed shells are theoretically far less reliable than mean-field results for well closed shells.

A second criterion requires that the deformation energy surface have a sharp, deep minimum at the equilibrium configuration. Otherwise, zero-point motion in the deformation coordinate will produce significant admixtures of additional components into the nuclear wave function, substantially altering it from the mean-field prediction. As a specific example, Fig. 1 demonstrates the softness of nuclei in the nickel region. Clearly, mean-field theory is of little use here, and we will subse-

quently see several examples of interesting nuclear structure in this region which is not at all understood in terms of present theory.

2.2 Excited States

If theorist's talents are limited in the microscopic treatment of ground states, they are paltry indeed when it comes to excited states. The most systematic theory for our applications is the random phase (RPA) approximation to the linear response function,¹¹⁾ which describes the particle-hole excitations induced by the action of an external potential on the ground state wave function and is applicable to a very limited set of collective excited states. The rotational and vibrational models yield reasonable zeroth order approximations for specific states in special nuclei, but no systematic hierarchy of corrections in terms of the nuclear interaction has been applied to these models. Given the precision and specificity of present experiments, transition nuclei are completely beyond our theoretical understanding.

2.3 Mesonic Corrections

Lacking a systematic formulation of the relativistic many-body problem, mesonic effects are treated by constructing effective operators to be used in the subspace of N -nucleons which reproduce the physical effects of mesons in the full Hilbert space. Several frequently calculated low-order contributions to the effective electromagnetic current operator are shown in Eq. 1.

$$\begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \quad | \\ \text{---} \text{---} \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \quad | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \quad | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \quad | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \quad | \\ \text{---} \text{---} \text{---} \end{array} \quad (1)$$

(a) (b) (c) (d)

Term (a) denotes the interaction of the photon with an exchanged meson and (b) represents the so-called "pair" term in which the meson and photon create and annihilate an antinucleon. These terms represent the unambiguous isovector exchange current arising from current conservation, $\nabla \cdot \mathbf{j} = -i[H, \rho]$, where H includes the one-pion exchange potential. Diagram (c) shows the "recoil" term and (d) displays the creation of a virtual isobar. Grave theoretical uncertainties persist in this approach, arising both from the problem of disentangling mesonic contributions to the nuclear interaction, which has been described in terms of a static potential, and from our general ignorance of the interactions between hadrons and mesons.

3. Elastic Electron Scattering

The contributions of intermediate excited nuclear states are sufficiently weak⁹⁾

Figure 2. Charge density, normalized to unit total charge, calculated from the impulse approximation form factor (IMP) and including the exchange current pair term (EXCH).

that electron scattering yields extremely accurate values of Coulomb, transverse electric, and transverse magnetic matrix elements of the electromagnetic current operator between initial and final nuclear states.¹³⁾ In the case of elastic scattering, given data over a sufficient range of momentum transfer one can thereby reconstruct in detail the charge density, current distribution and magnetization density produced by all the constituents in the nuclear ground state.

3.1 Charge Distributions

Due to the significant role that visualization of shapes of objects seems to play in human cognitive processes, determination of charge distributions from forward angle Coulomb scattering has always been of special interest. Using a variety of techniques,⁹⁾ the high momentum transfer data produced by contemporary accelerators may be inverted to obtain a precise determination of the charge density with almost negligible uncertainties arising from model assumptions.

To relate charge densities to proton distributions requires not only accurate knowledge of nucleon form factors, but also a quantitative understanding of exchange currents. Although Siegert's theorem¹⁴⁾ assures us that exchange current contributions vanish at low momentum transfer, these corrections are certainly significant in understanding present experiments. The largest contribution arises from the pair term, Eq. 1b, and its effect in ⁴He and ⁴⁰Ca is displayed in Fig. 2.¹⁵⁾

Starting at the beginning of the Periodic Table, the only new surprise concerning the proton is that its rms radius appears to be 0.87±0.02 fm, about 10% larger than the generally accepted value.¹⁶⁾ Since this results from a slight curvature of the

Figure 3. Deuteron charge form factor.

form factor at low q , old form factors yield rather negligible errors over most of the region for which they are used in analyzing electron scattering. However, for very precise muonic X-ray analysis, the result is non-negligible. Lamentably, I know of nothing new to report on the neutron form factor, the ignorance of which continues to introduce significant uncertainties.

The two-, three-, and four-body systems have been the subjects of exciting recent experiments at SLAC which impel us, as never before, to stretch our theoretical horizon beyond the narrow confines of non-relativistic quantum mechanics. The experimental deuteron charge form factor¹⁷⁾ and several theoretical predictions are shown in Fig. 3. Although it would be silly to do so at high q , phenomenological non-relativistic nucleon-nucleon potentials may always be defined to exactly reproduce the observed deuteron form factor. Thus, the Reid¹⁸⁾ (solid line) and Lomon-Feshbach¹⁹⁾ (long dashes) predictions are merely presented to set the scale expected from present potentials. Estimates of the effects of ρ, ω (short dashes)²⁰⁾ and ρ, π (dotted line)²¹⁾ exchange currents are interesting primarily because of the degree to which they dominate the nucleon contributions. The apparent disagreement with experiment is inconsequential, being easily resolved by the introduction of reasonable meson-nucleon form factors. In the asymptotic limit of arbitrarily high q , one might hope to describe the deuteron in terms of 6 quarks, in which case elastic scattering necessitates the distribution of roughly $q/6$ to each quark.²²⁾ The resulting q^{-10} asymptotic behavior (dash-dot curve) clearly has not yet been attained.

Figure 4. Absolute value of charge form factor of ^3He .

in the region accessible to experiment. Although various approximations below the onset of scaling happen to be roughly consistent with experiment, this agreement constitutes no more proof that one has seen quark effects in the deuteron than agreement for some non-relativistic potential would prove that relativistic and mesonic corrections are negligible.^{23,24)}

Comparable experiments at SLAC on ^3He have just been completed,²⁵⁾ and the results for ^3He are shown in Fig. 4 with older data²⁶⁾ and several theoretical calculations. The Reid potential result²⁷⁾ (short dashes) again falls significantly below experiment. Adding exchange corrections, the most important of which is the pair term, Eq. 1b, increases the form factor significantly (solid line). The additional inclusion of the recoil contribution, Eq. 1c, yields agreement which is probably fortuitously good.²⁸⁾ Finally, the dash-dot curve shows the results of pre-scaling quark model arguments²³⁾ which were consistent with the deuteron data, emphasizing the limitations of such descriptions below the asymptotic region.

Densities for ^3He obtained from inverting the experimental data are denoted by the shaded regions in Fig. 5, where the right portion shows the total charge density and the left section presents the result obtained after unfolding the effect of nuclear form factors.²⁹⁾ The dashed curve in each case is essentially the Reid potential result with no exchange corrections and the solid line shows the density including the contribution of exchange currents. As one expects intuitively, exchange corrections are comparable to the

Figure 5. Charge density for ^3He .

effect of form factors. It would be just as serious a mistake to think that the left hand solid curve, instead of the dashed curve, represented the effect of short range correlations on the nuclear density in ^3He as to forget to remove the effect of nucleon form factors.

Since exchange currents are so important in light nuclei, experimentalists should clearly try to perform experiments which focus as completely as possible on these effects with as little contamination from other uncertainties as possible. From

Figure 6. Absolute value of charge form factor for ^3He without exchange corrections (a), and including exchange corrections (b). The exchange contribution is given by curve (c).

Figure 7. Absolute value of charge form factor for ^3He without exchange corrections (solid line), and including exchange corrections (dashed line). The exchange contribution is given by the dot-dashed line.

the viewpoint of a theorist, who doesn't have to worry about the unpleasant properties of tritium, an experiment which simply cries out is a comparison of ^3H and ^3He form factors up to at least 25 fm^{-2} . Since the dominant exchange correction is isovector, it contributes with opposite sign in the two nuclei, as shown in Fig. 6 for an old calculation with S-state wave functions calculated with the Reid potential. The effect is quite dramatic, offering an unusually stringent test of any exchange current theory.

The case of ^4He is very similar to that of ^3He . The result of including exchange corrections with e^s wave functions incorporating both two- and 3-body correlations for the Reid potential is shown in Fig. 7.³⁰⁾ Again, the minimum is shifted inward and the high q form factor strongly enhanced. It is also interesting that the exchange corrections of Fig. 7 are very similar to the much simpler calculations of Ref. 15 which used the pair graph with a determinantal wave function, raising the hope that one may at least estimate such effects with relative ease.

High momentum transfer experiments on ^{28}Si , ^{32}S , and ^{56}Ni were reviewed at the Santa Fe conference.³¹⁾ Since none of these nuclei satisfy the criteria for the validity of the mean-field approximation very well, the case of ^{208}Pb is of particular interest to theorists as an optimal test of mean-field theory. A year ago, as shown in Fig. 8, the experimental situation was very confused.³¹⁾ The upper curves show the interior density reconstructed from the original Stanford experiment³¹⁾ both ex-

Figure 8. Interior charge density of ^{208}Pb . Note expanded scale.

cluding and including muonic X-ray data.³²⁾ The statistical inconsistency between muonic and electron data gave some hope that the discrepancy with theory, of which the density matrix expansion (DME) result³²⁾ denoted by the solid line is a representative example, was of experimental origin. This hope was further reinforced by new Mainz data.³³⁾ The density obtained from these results, shown in the lower portion of Fig. 8, not only agreed spectacularly with theory, but also was apparently consistent with muonic data.

As much as I might have preferred to end the discussion here, the exceedingly impressive data spanning 11 decades in cross section shown in Fig. 9 was subsequently taken at Saclay. Although the DME prediction, denoted by the solid line, appears excellent by conventional nuclear physics standards, in the region of 2 fm^{-1} , the Saclay data agrees with the Stanford data and definitely disagrees with the Mainz results, yielding an experimental density similar to the upper curve in Fig. 8. Careful remeasurements at Mainz²⁵⁾ confirm the Saclay result, and simultaneous analysis of all the old data used in Ref. 36, as well as the original³¹⁾ and remeasured³⁵⁾ Mainz data, Saclay data,³⁴⁾ and Tohoku data³⁷⁾ yields the result shown in Fig. 10. If one takes the barely perceptible statistical error envelope seriously

Figure 9. Cross section for elastic scattering from ^{208}Pb at 502 MeV compared with DME prediction (solid line).

Figure 10. Charge density of ^{208}Pb obtained from simultaneous analysis of all electron scattering and muonic data. The thickness of the solid curve denotes the statistical error envelope, and the dashed curve shows the DME prediction.

Figure 11. Differences in equivalent radii for adjacent odd-even nuclei in fm, with a typical experimental uncertainty of 0.7 fm.

and exchange current and dispersion corrections turn out to be negligible, there is an interesting discrepancy with theory. The theoretical S-state lump inside 1.5 fm doesn't worry me, because it is completely plausible that residual interactions smooth out quantum density fluctuations. Rather, the fact that the average theoretical density inside 4 fm is too high indicates that there may be a basic error in the overall balance of volume, Coulomb, symmetry, and surface energy which determines the gross shape. Before one takes the statistical error estimate as a realistic indication of the total error, however, there are a number of inconsistencies to be understood in the data. The total χ^2 of 629 for 265 degrees of freedom drops to 443 if the absolute normalization for each set of data is treated as a free parameter. In this minimization, the normalizations change by as much as 8.5%, and much smaller changes have been shown to completely remove an apparent central depression. Furthermore, the muonic P-state energies still remain in strong contradiction with the combined electron scattering data.

Density differences between isotones and isotopes give valuable information about the spatial distribution of valence protons and the polarization of the core, and several interesting examples have been reviewed by Sick.³⁰⁾ A particularly vivid example of how such results emphasize the inadequacy of present theory is provided by recent systematic electron scattering and muonic X-ray experiments in the Ni region.³¹⁾ Fig. 11 displays the pronounced odd-even staggering of charge radii in this region, and subsequent unpublished data³²⁾ display it in additional neighboring nuclei. Although Bertsch³³⁾ explains this effect by the enhanced core polarization (and resulting radius increase) due to the coherent effect of two paired nucleons, no meaningful quantitative calculations exist in this region because the softness displayed in

Figure 12. Magnetic cross sections from nuclei with odd $g_{9/2}$ nucleons. The left panel shows M9 cross sections as a function of q for ^{93}Nb and cross sections for ^{87}Sr on a scale compressed by the factor β . The right panel shows a mean field theory prediction (dashed line), a Woods-Saxon fit (solid line) and experimental data as a function of effective momentum transfer.

Fig. 1 invalidates the mean-field approximation.

3.2 Magnetic Scattering

In elastic scattering, parity and time reversal restrict the contributions of the transverse electromagnetic current to odd magnetic multipoles,³⁴⁾ which provide detailed information concerning the spatial distribution of ground state current and magnetization distributions.³⁵⁾ Since the full exploitation of magnetic scattering awaits the completion of 180° spectrometers, we will restrict our attention to a single example which exhibits the physics potentially available in carefully selected experiments.

Backward angle high q elastic scattering at Saclay on ^{87}Sr and ^{93}Nb has explored the $g_{9/2}$ single-particle wave function for a neutron hole³⁶⁾ and a proton particle.³⁷⁾ In the single-particle model for a state with $j = l + 1/2$, the orbital current does not contribute to the highest multipole, so the pentakosiadodekapole transition directly measures the j_0 Bessel transform of the spin density of the $g_{9/2}$ nucleon in each case. Since knowledge of the form factor in the limited region for which M9 dominates is insufficient to reconstruct the shape of the radial wave function, the most model-independent comparison of the two $g_{9/2}$ wave functions is obtained by assuming that the proton and neutron orbitals have the same basic shape

Figure 13. Transition density for excitation of the 3^- state in ^{208}Pb .

and differ only slightly in radial scale. Assuming $R_p(r) = R_n(r)\beta^{3/2}$ yields the excellent agreement shown in the left-hand portion of Fig. 12 for the compression factor $\beta = 0.954 \pm .006$.³²⁾ When corrected for binding energy effects, the inferred ratio of neutron radius to proton radius of $\beta^* = 0.962$ is reasonably consistent with mean-field values of 0.98 to 1.00, and grossly inconsistent with the degree to which neutron radii should be systematically decreased below mean-field predictions to resolve the Nolen-Schiffer Coulomb energy anomaly.³⁴⁾

The right panel of Fig. 12 shows that the direct comparison with experiment of the single-particle mean-field prediction for neutrons is unsatisfactory. Although few low-lying particle-hole admixtures couple to M9, configurations such as $g_{9/2}^{-1}d_{5/2}$ couple to M7. Since the resulting transition density has one node, its Bessel transform peaks at much higher q than for the $g_{9/2}$ orbital and may well fall into the range of the single particle M9 peak. In addition, one needs to determine whether ^{87}Sr is sufficiently soft to undergo significant shape polarization by the $g_{9/2}$ hole and to investigate mesonic exchange currents. Until these theoretical uncertainties are resolved, the extent to which these provocative experiments may be interpreted at the single-particle level will remain unclear.

4. Inelastic Electron Scattering

The high resolution and relatively large momentum transfer presently available offers unprecedented opportunities to explore the structure of excited states.

Figure 14. Contour plots of theoretical proton and neutron density distributions. Density contours are in increments of 0.005 fm^{-3} , the labels denote the density multiplied by 100, and r and z are in fm.

4.1 Discrete Excited States

A limited range of vibrational excited states is reasonably well described by the RPA. Thinking in terms of the linear response of a system to a weak external potential of frequency ω and wave number k , the RPA simply sums chains of particle-hole excitations relative to the mean-field ground state. A particularly favorable case is ^{208}Pb , for which the RPA with a very simple effective interaction^{11,45} is compared with the transition density obtained from Mainz data^{46,47} in Fig. 13. Other interactions yield similar agreement, and very modest modification of the effective force yields an excellent fit.⁴⁷

Inelastic scattering within the ground state band of rotational nuclei is presently described by a single intrinsic state calculated with mean-field theory. In this approximation, the transition density for a $0^+ \rightarrow J^+$ transition is simply given by the J th multiple in a Legendre expansion of the charge density of the intrinsic state, and typical examples¹⁰ of theoretical deformed density distributions are shown in Fig. 14. Interesting qualitative features are evident in the intrinsic density itself, such as, the decrease in hexadecapole moment in the actinides which is also observed in the rare earths. Although one might worry that projection of the deformed mean-field wave function, which actually represents a wave

Figure 15. Experimental values of q/q_{Mott} for the ground state rotational band in ^{152}Sm .

packet of various angular momentum states, yields significantly different transition densities than Legendre expansion of the intrinsic density, it has been shown that up to 6^+ states in nuclei of interest, the projection corrections are negligible.⁴⁸ Of somewhat greater concern is the possible effect of including $K \neq 0$ admixtures into the wave function, and these corrections⁴⁹ should definitely be evaluated.

Inelastic transitions up to 6^+ states in ^{152}Sm from Saclay⁵⁰ are compared with mean-field predictions⁵¹, denoted by dashed curves, in Fig. 15. Serious discrepancies arise in 4^+ and 6^+ excited states, and are most plausibly attributed to the fact that ^{152}Sm is not a very good rotator.⁵² Fig. 16 shows MIT data⁵³ for two better rotators, as well as predictions from various mean-field theories. The most realistic of the effective interactions (DME) yields good agreement up to 4^+ transitions in the rare earths and yields cross sections which are systematically too low for 6^+ transitions. In ^{238}U , where the rotational model should be even more accurate, the DME predictions are in reasonable agreement for all the transitions. The Skyrme V and VI curves bracket the full range of results obtained with Skyrme forces and suggest that the 6^+ discrepancies in the rare earths are not to be resolved by fiddling with the effective interaction. Clearly, before rotational nuclei may be further exploited to learn about details of nuclear shapes, theoretical uncertain-

Figure 16. Experimental values of q/q_{Mott} for the ground state rotational band in ^{166}Er and ^{238}U , compared with several mean field predictions.

ties such as dispersion corrections and the effect of $K \neq 0$ admixtures must be resolved.

To avoid giving the impression that the preceding carefully selected examples of rotational and vibrational nuclei are representative of our general level of theoretical understanding of transition densities, a last example is taken from the region of soft, and thus intractable, nuclei shown in Fig. 1. Transition densities to the second excited 2^+ state in Zn isotopes reconstructed from Mainz data^{54,55} are presented in Fig. 17. Although the excited state spectra do not change dramatically between ^{64}Zn and ^{70}Zn , it is clear from the transition densities that the structure of the second 2^+ states in ^{68}Zn and ^{70}Zn differ significantly from each other and from ^{64}Zn and ^{66}Zn . Although one might expect qualitative differences from the fact that ^{64}Zn and ^{66}Zn have double minima, ^{68}Zn has a rather shape-independent energy, and closed shell ^{70}Zn has a deep single minimum in Fig. 1, one may hope that systematic precision data of the quality displayed in Fig. 17 will stimulate efforts toward a quantitatively viable theory.

Figure 17. Transition densities for excitation of the second 2^+ state in Zn isotopes. Error envelopes are indicated by shaded areas. Note the different scale used for ^{70}Zn .

4.2 Quasi-elastic scattering

Although, in principle, the linear response function depending on frequency ω and wave number q determined from inelastic electron scattering could measure short-range correlations in the charge distribution, the accuracy required and uncertainties associated with meson production render such detailed information inaccessible at present.⁵⁰ In practice, instead of measuring the full q, ω dependence of the response function, the cross section as a function of energy loss is measured at fixed angle, as shown in Fig. 18, thereby providing the linear response for a particular line in the q, ω plane.

The most complete theoretical analysis has been performed⁵⁷ for ^3He . Using a Faddeev solution for the three-body wave

Figure 18. Quasi-elastic cross sections for ^3He , ^{12}C and ^{20}Pb .

function with the Reid potential and explicitly integrating over appropriate two- and three-body final states yields the solid curve which is compared with experimental data⁵⁷⁾ in the top panel of Fig. 18. For heavier nuclei, a much simpler theory⁵⁸⁾ in which one only considers particle-hole excitations in a simple Fermi gas wave function appears adequate, as shown by the solid curves for carbon and lead in the lower two sections of Fig. 18.⁵⁹⁾ The width of the quasi-elastic peak specifies the range of the dynamical Pauli correlations, yielding an average value of k_F , and the peak position specifies an average nuclear interaction energy $\bar{\epsilon}$ contributing to the particle-hole propagator.

At higher energy in the region of the Δ_{33} resonance, the response function is dominated by isobar-hole rather than particle-hole excitations. The predictions of a simple zero-width isobar model⁶⁰⁾ are denoted by dashed lines and compared with data on several nuclei in Fig. 19.⁶⁰⁾ The agreement in the second peak is generally worse than the particle-hole result (solid line) for the quasi-elastic peak, and it is clear from both Figs. 18 and 19 that

Figure 19. Quasi-elastic cross sections extending into the Δ_{33} resonance region.

much remains to be understood concerning pion production and isobar propagation above the quasi-elastic peak.

4.3 Deuteron Electrodissintegration

The deuteron merits separate discussion from that in the last subsection due to its unique sensitivity to exchange currents. Near breakup threshold, magnetic dipole transitions between the 3S_1 - 3D_1 groundstate and the 1S_0 continuum state, which are sensitive to isovector exchange currents, are unmasked by electric dipole transitions which must connect to a P-state. Back angle scattering with kinematics chosen to impart large q and very small ω is greatly enhanced by exchange currents, which provide a natural mechanism to share momentum equally between the two nucleons and thus produce a final state with low relative momentum. Particularly striking data⁶¹⁾ for a range of q at fixed relative energy of the proton and neutron, $E_{cm} = 1.5$ MeV, is shown in Fig. 20. Although impulse approximation using the Reid potential disagrees with experiment by up to an order of magnitude (dashed curve), inclusion of the pion and pair terms, graphs (a) and (b) of Eq. 1, raises the cross section into essential agreement (curve A).⁶¹⁾ Addition of the isobar term, graph (d), yields a small further increase (curve B). A similarly dramatic 60%

Figure 20. Deuteron electrodisintegration cross sections at cm energy 1.5 MeV.

threshold enhancement over the impulse approximation prediction for recent Mainz electrodisintegration data is well explained by exchange currents.⁶⁴⁾

5. Coincidence Experiments

At the expense of introducing considerable ambiguity arising from final state interactions, more specific nuclear structure information is attainable by measuring final hadrons in coincidence with the scattered electron. Due to the limitations imposed by present duty cycles, $(e, e'N)$ data, where N denotes any nucleon, is very sparse. Whereas I shall relegate discussion of meson production to a more appropriate session, I have taken the liberty of including selected (γ, N) results because of the intimate physical connection of the on-shell limit to the more general virtual photon exchange in $(e, e'N)$ reactions. (Purists who are upset at such a flagrant digression from the assigned topic may perhaps console themselves in the fact that the γ 's are produced by electron scattering, generally even at the same accelerators.)

5.1 Deuteron Photodisintegration

In the context of non-relativistic quantum mechanics with a specified potential, the deuteron offers the unique opportunity to completely eliminate uncertainties in the initial and final state wave functions. Although experimental photodisintegration data are in rough agreement with theoretical predictions for total cross sections, a recent Mainz measurement⁶⁵⁾ of $^2\text{H}(\gamma, p)$ for protons at 0° is quite incon-

Figure 21. Deuteron photodisintegration cross sections for protons at 0° .

sistent with present theory as shown in Fig. 21. In spite of the fact that calculations⁶⁶⁾ with various Feshbach-Lomon (FL) potentials indicate reasonable sensitivity to the D-state probability, neither inclusion of exchange currents and isobar contributions (MEC&IR) nor variation over a reasonable range of D-state probability can resolve the discrepancy. Due to the difficulty of the experiment, theorists should probably wait for independent confirmation before expending too much effort seeking exotic explanations.

5.2 The Spectral Function, $S(k, E)$

A quantity of particular theoretical interest is the spectral function

$$S(k, E) = \sum \langle (A-1) | a_k | A \rangle^2 \delta(E - E_{A-1}^* + E_A) \quad (2)$$

which specifies the probability of finding a nucleon in the ground state of nucleus A with momentum k and removal energy E . The most complete calculation⁵⁷⁾ of $S(k, E)$ has been carried out for the three-body problem, yielding the results shown in Fig. 22. The dashed curve corresponds to two-body break-up (in units of fm^{-3}) and the solid contours indicate that the strong repulsive core of the Reid potential introduces significant probability at high k and large E . A careful measurement of the details of $S(k, E)$ for ^3He would be exceedingly interesting and should be pursued with high priority. At present, however, we must be content with the inclusive (e, e') cross section shown in Fig. 18 in which the spectral function has been integrated with respect to the nucleon momentum.

Saclay $(e, e'p)$ measurements⁶⁷⁾ on ^{12}C , ^{28}Si , ^{40}Ca and ^{58}Ni provide our first detailed glimpse of $S(k, E)$, and a sketch⁶⁸⁾

Figure 22. The spectral function, $S(k, E)$, for ${}^3\text{He}$.

Figure 24. Cross sections for ${}^{12}\text{C}(\gamma, p_0)$ and ${}^{12}\text{C}(e, e'p)$ reactions.

constructed by slightly smoothing the experimental ${}^{40}\text{Ca}$ results is shown in Fig. 23. In addition to peaks at energies appropriate to $0d_{3/2}$, $1s_{1/2}$ and $0d_{5/2}$ single-particle wave functions which agree semi-quantitatively with the Fourier transforms of single-particle wave functions, interesting widths and strength at high q are evident which probe beyond the simple single-particle model. Significant uncertainties in final state interactions introduce errors in occupation number of the order of 40% in ${}^{12}\text{C}$ and 20% in ${}^{40}\text{Ca}$, so $(e, e'p)$ experiments are still far from yielding the degree of precision required for significant impact on theory. Perhaps until the reaction theory problems are resolved, the most convincing case experimentalists can make for the reliability of their extraction of $S(k, E)$ would be to measure sets of significantly different kinematics which correspond to the same values of k and E .

5.3 Momentum distributions and (γ, p_0) experiments

Although intrinsically (γ, p) experiments are kinematically more limited than $(e, e'p)$ reactions and suffer from difficulties in obtaining an adequately monochromatic source, in light nuclei (γ, p_0) reactions (leading to the ground state of the residual nucleus) do yield information pertaining to the momentum distribution. In Fig. 24, ${}^{12}\text{C}(\gamma, p)$ data⁶⁹ analyzed in distorted wave impulse approximation⁷⁰

(solid circles) is compared with the $(e, e'p)$ data of Ref.⁶⁷ (open circles), and the consistency in the region of overlap lends some confidence in the treatment of distortion. In the single-particle model, the impulse approximation yields the Fourier transform of the single-particle wave function, and the solid line shows that an Elton-Swift⁷¹) Op wave function is quite consistent with the data.

Figure 25 displays comparable Glasgow data⁷²) (solid circles) and high momentum data from Bates⁷³) for the ${}^{16}\text{O}(\gamma, p)$ reaction. The solid curve again denotes the momentum distribution predicted by an Elton Swift wave function, and should not be taken very seriously beyond the first minimum, where the single particle picture breaks down and where slightly different realistic wave functions yield results differing by as much as several orders of magnitude.⁷⁴)

Unfortunately, interpretation of (γ, p) measurements in impulse approximation solely in terms of the nuclear momentum distribution is much too naive. If this description were correct, for example, (γ, n) cross sections would be negligible whereas, in fact, they are comparable to those for (γ, p) . So, at the least, there is a very large "effective charge" contribution, in which the photon creates a proton particle-hole pair, which in turn knocks out the

ejected nucleon, and a reliable quantitative description of such processes is well beyond the state of the art. Another potentially interesting contribution in the region of 500 MeV/c arises from the production and propagation of a virtual isobar. Although initial calculations seem qualitatively reasonable,⁷⁵ one needs an accurate, well-tested theory of the high momentum component "background" before useful quantitative information about isobar propagation can be deduced.

6. Conclusion

The most obvious conclusion in the light of the wide range of superb experimental data being produced by the new generation of accelerators is that the ball is definitely in the theorist's court, and with rather high momentum at that. The mean field-theory has been pushed just about as far as is sensible, and there is very little to be learned by continuing to invent more and more new effective interactions or by taking too seriously minor discrepancies between predictions of different brands of reasonable interactions. The real issue is the relativistic many-body problem, involving substantive theoretical questions as fundamental, as difficult, and as challenging as are to be found anywhere in theoretical physics. If electromagnetic probes of nuclei serve to nudge us closer to a solution, they will have served us well.

Acknowledgments

The author is grateful to Ramond Arnold, Peter Demos, Bernard Frois, G. Lührs, R. O. Owens, S. Penner, A. Richter, and Ingo Sick, for helping to bring him up to date on recent activities at the major electron accelerators.

References

- * This work is supported in part through funds provided by ERDA under Contract EY-76-C-02-3069.*000.
- + Alfred P. Sloan Foundation Research Fellow.
1. J.W. Negele, Proc. of Conf. on Radial Shapes of Nuclei, Cracow, A. Budzanowski ed. (1976) 79.
2. A.D. Jackson, A. Lande, and P. Sauer, Nucl. Phys. **A156** (1970) 1.
3. M.R. Strayer and P.U. Sauer, Nucl. Phys. **A231** (1971) 1.
4. R. Brandenburg, Y. Kim and A. Tubis, Phys. Rev. **C12** (1975) 1368.

5. F. Coester, Lectures in Theoretical Physics, vol. XIB, Gordon and Breach, NY (1969).
6. H. Kümmel, K.H. Lührmann, & J.G. Zabolitzky, Physics Reports, to be published.
7. J.G. Zabolitzky, Nucl. Phys. A228 (1974) 272; Nucl. Phys. A228 (1979) 285.
8. M. Gari, H. Hyuga and J.G. Zabolitzky, Nucl. Phys. A271 (1976) 365.
9. J.L. Friar and J.W. Negele, Advances in Nuclear Physics, vol. 8, Plenum Press, N.Y. (1975) 219.
10. J.W. Negele and G. Rinker, Phys. Rev. C 15 (1977) 1499.
11. G.F. Bertsch and S.F. Tsai, Phys. Rep. 18 (1975) 125.
12. J.L. Friar, Phys. Lett. B59 (1975) 145; Phys. Rev. C 15 (1977) 1783; Ann. Phys. (N.Y.) 104 (1977) 380.
13. T. deForest and J.D. Walecka, Adv. Phys. 15 (1966) 1.
14. A.J.P. Siegert, Phys. Rev. 52 (1937) 787.
15. M. Radomski and D.O. Riska, Nucl. Phys. A274 (1976) 428.
16. F. Borkowski, G.G. Simon, V.H. Walther, and R.D. Wendling, Nucl. Phys. B93 (1975) 461; Z. Physik A275 (1975) 29.
17. R.G. Arnold et al., Phys. Rev. Lett. 35 (1975) 776.
18. R. Reid, Ann. Phys. (N.Y.) 50, (1968) 411.
19. E.L. Lomon and H. Feshbach, Ann. Phys. (N.Y.) 48 (1968) 94.
20. R. Blankenbecker and J. Gunion, Phys. Rev. D4 (1971) 718.
21. M. Chemtob, E.J. Moniz, and M. Rho, Phys. Rev. C 10, 344 (1974); this result was modified using a more recent ρ - $\pi\pi$ coupling constant.
22. S.J. Brodsky and G.R. Farrar, Phys. Rev. D11 (1975) 1309.
23. S. Brodsky and B. Chertok, Phys. Rev. D14 (1976) 3003.
24. G. West, Phys. Rev. Lett. 37 (1976) 464.
25. R.G. Arnold, proceedings of this conference.
26. J.S. McCarthy, I. Sick, and R.R. Whitney, Phys. Rev. C 15 (1977) 1396.
27. R. Brandenburg, unpublished.
28. E. Hadjimichael, preprint (June 1977). The Reid form factor used by Hadjimichael from Ref. 2 disagreed with the more accurate results of Brandenburg (Refs. 4,28) above $q^2 = 35 \text{ fm}^{-2}$. Hence, the Reid result from Ref. 28 is shown and results including exchange π currents are not plotted beyond 35 fm^{-2} .
29. W.M. Kloet and J.A. Tjon, Phys. Lett. 49B (1974) 419.
30. I. Sick, Proc. of 6th International Conf. on High Energy Physics and Nuclear Structure, Santa Fe, D. Nagel et al. ed. (1975) 388.
31. J. Heisenberg, private communication; J. Heisenberg, R. Hofstadter, J.S. McCarthy; I. Sick, B.C. Clark, R. Herman, and D.G. Ravenhall, Phys. Rev. Lett. 23 (1969) 1402.
32. J.W. Negele and D. Vautherin, Phys. Rev. C 5 (1972) 1472.
33. H. Euteneuer, J. Friedrich and N. Voegler, Phys. Rev. Lett. 36 (1976) 129.
34. B. Frois, J.B. Bellicard, J.M. Cavedon, M. Huet, P. Leconte, P. Ludeau, A. Nakada, Phan Xuan Ho, and I. Sick, Phys. Rev. Lett. 38 (1977) 152.
35. J. Heisenberg, private communication.
36. J.L. Friar and J.W. Negele, Nucl. Phys. A212 (1973) 93.
37. M. Nagao and Y. Torizuka, Phys. Lett. 37B (1971) 383.
38. E.B. Shera, E.T. Ritter, R.B. Perkins, G.A. Rinker, L. K. Wagner, H.D. Wohlfahrt, G. Fricke, and R.M. Steffen, Phys. Rev. 14 (1976) 731.
39. G. Lührs, private communication.
40. G.F. Bertsch, The Practitioner's Shell Model, North Holland (1972) p. 155.
41. T.W. Donnelly and J.D. Walecka, Nucl. Phys. A201 (1973) 81.
42. I. Sick, J.B. Bellicard, J.M. Cavedon, B. Frois, M. Huet, P. Leconte, A. Nakada, Phan Xuan Ho, S. Platchkov, P.K.A. deWitt Huberts and L. Lapikas, Phys. Rev. Lett. 38 (1977) 1259.
43. P.K.A. deWitt Huberts et al., Phys. Lett. 60B (1971) 157.
44. J.A. Nolen and J.P. Schiffer, Ann. Rev. Nucl. Sci. 19 (1969) 471.
45. G.F. Bertsch, private communication.
46. H. Rothhaas, J. Friedrich, K. Merle, and B. Dreher, Phys. Lett. 51B (1974) 23.
47. J. Heisenberg, Proceedings of 1977 Bates LINAC Summer Study, A. Bernstein, ed.
48. A. Zaringhalam and J.W. Negele, Nucl. Phys., in press.
49. F. Villars and N. Scheming-Rogerson, Ann. Phys. (N.Y.) 63 (1971) 443.
50. A. Nakada, N. Haik, J. Alster, J.B. Bellicard, S. Cochavi, B. Frois, M. Huet, P. Leconte, P. Ludeau, M.A. Moinester, Phan Xuan Ho, and S. Turck, Phys. Rev. Lett. 38 (1977) 584.
51. J. Decharge, M. Girod, and D. Gogny, Phys. Lett. 55B (1975) 361.
52. See Fig. 1 and Table 1 of Ref. 10.
53. C.W. Creswell, Ph.D. dissertation, MIT (1977).
54. R. Neuhausen, Nucl. Phys. A282 (1977) 125.
55. Hans-Eberhard Fuhr, University of Mainz, dissertation (1977).
56. G. Bertsch and J. Borysowicz, J. Phys. G: Nucl. Phys. 2 (1976) L121.
57. A.E.L. Dieperink, T. DeForest, I. Sick and R.A. Brandenburg, Phys. Lett. 63B (1976) 261.
58. E.J. Moniz, Phys. Rev. 184 (1969) 1154.
59. E.J. Moniz, I. Sick, R.R. Whitney, J.R. Plicenc, R.D. Kephart, and W.P. Trower, Phys. Rev. Lett. 26 (1971) 445.
60. Yu I. Titov, E.V. Stepula, N.G. Afanas'ev, R.V. Akhmerov, S.A. Byvalin, N.F. Severin and E.M. Smelov, Sov. Jour. Nucl. Phys. 13 (1971) 660.
61. R.E. Rand et al., Phys. Rev. Lett. 18 (1967) 469.
62. D. Ganichot, D. Grossetete and D.B. Isabelle, Nucl. Phys. A178 (1972) 545.
63. J. Hockert, D.O. Riska, M. Gari, and A. Huffman, Nucl. Phys. A217 (1973) 14.
64. G.G. Simon, F. Borkowski, Ch. Schmitt, V.H. Walther, H. Arenhövel and W. Fabian, Phys. Rev. Lett. 37 (1976) 739.
65. R.J. Hughes, A. Zieger, H. Wäffler, and B. Ziegler, Nucl. Phys. A267 (1976) 329.
66. H. Arenhövel and W. Fabian, to be published. Fig. 21 is taken from Proceedings of 1977 Bates LINAC Summer Study, A. Bernstein ed.
67. J. Mougey, M. Bernheim, A. Bussiere, A. Gillebert, Phan Xuan Ho, M. Priou, D. Royer, I. Sick, and G.J. Wagner, Nucl. Phys. A262 (1976) 461.
68. W. Bertozzi and J.W. Negele, Comments in Nuclear and Particle Physics, 6 (1976) 59.
69. J.L. Matthews, D.J.S. Findlay, S.N. Gardiner, and R.O. Owens, Nucl. Phys. A267 (1976) 51.
70. D.J.S. Findlay and R.O. Owens, preprint (1977).
71. L.R.B. Elton and A. Swift, Nucl. Phys. A94 (1967) 52.
72. D.J.S. Findlay and R.O. Owens, Nucl. Phys. A279 9L977) 385.
73. J.L. Matthews, W. Bertozzi, M.J. Leitch, C.A. Peridier, B.L. Roberts, C.P. Sargent, W. Turchinets, D.J.S. Findlay and R.O. Owens, Phys. Rev. Lett. 38 (1977) 8.
74. D.J.S. Findlay, R.O. Owens, M.J. Leitch, J.L. Matthews, C.A. Peridier, B.L. Roberts, and C.P. Sargent, preprint (1977).
75. G.D. Nixon, J.T. Londergan, and G.E. Walker, to be published.

III. REANALYSIS OF ELECTRON SCATTERING AND MUONIC X-RAY DATA FOR ^{208}Pb

J. L. Friar, J. Heisenberg and J. W. Negele

INTRODUCTION

Experimental determination of the charge distribution of ^{208}Pb is of particular interest because this nucleus constitutes an optimal test of the mean field theory. As reported a year ago, the experimental status was very confused, as is evident from Fig. 1. The upper curves show the interior density reconstructed from the original Stanford experiment² both excluding and including muonic x-ray data.³ The statistical inconsistency between muonic and electron data gave some indication that the discrepancy with theory, of which the density matrix expansion (DME) result⁴ denoted by the solid line is a representative example, was of experimental origin. This possibility was further reinforced by new Mainz data.⁵ The density obtained from these results, shown in the lower portion of Fig. 1 not only agreed spectacularly with theory, but also was apparently consistent with muonic data.

As much as theorists might have preferred to end the discussion here, the exceedingly impressive data spanning 11 decades in cross section shown in Fig. 2 was subsequently taken at Saclay.⁶ Although the DME prediction, denoted by the solid line, appears excellent by conventional nuclear physics standards, in the region of 2 fm^{-1} , the Saclay data agrees with the Stanford data and definitely disagrees with the Mainz results, yielding an experimental density similar to the upper curve in Fig. 1.

As a result of this apparent experimental discrepancy, one of us (JH) took part at Mainz in a careful remeasurement of the crucial region of momentum transfer around $q = 1.9 \text{ fm}^{-1}$. Given this additional data, it appeared worthwhile to perform a simultaneous analysis of the world's supply of data on ^{208}Pb during the summer workshop.

EXPERIMENTAL DATA

A total of 14 sets of electron scattering data were analyzed along with 6 independent muonic transition energies.) Five sets of electron scattering data (one from Darmstadt, two from Mainz and two from Stanford) were precisely as tabulated in Ref. 7. The conflicting data by Euteneuer, et. al.,⁵ at 119.7 MeV, 199.5 MeV and 289.0 MeV and from Saclay⁶ at 502.0 MeV and 501.6 MeV comprised five additional sets of data. Finally, two sets of data from Tohoku⁸ at 183.0 MeV and 248.2 MeV and the new remeasurements from Mainz at 199.9 and 290.0 MeV completed the electron scattering data.

The muonic data was as presented in Ref. 7, with slightly improved values for the QED corrections. Although it is clear that the 3S muonic state contains significant nuclear information and some experimental data exists, results for the 3S transitions have not yet been made available by the experimentalists.

ANALYSIS

The Fourier Bessel expansion program of Ref. 7 was expanded to include the full set of available data.) The fit was done starting with the DME density distribution⁴ and using 13 expansion

coefficients with a cutoff radius of 11 fm. The best fit density is compared with theory in Fig. 4. The total χ^2 of 2.4 per degree of freedom indicates some disturbing inconsistencies in the data. Part of these inconsistencies are already demonstrated in Fig. 1 for which the scatter of the data is responsible, which is significantly larger than the quoted error bars in the region of 1.9 fm^{-1} . The final result of the χ^2 fit is somewhat closer to the Saclay data, which is also reflected in the density in Fig. 4 that agrees reasonably well with that of Ref. 6.

An additional source of uncertainty has to do with the absolute normalization of the electron scattering data. Most of the data sets namely those from Mainz and Saclay were taken as relative angular distributions and subsequently normalized relative to previously taken data. The Stanford data, however, were taken relative to the proton cross section with an uncertainty of $\pm 3\%$. In view of this a second fit was done where the normalizations were allowed to vary within the quoted error bars in order to minimize χ^2 . For the Mainz or Saclay data sets it means essentially normalizing them with respect to a combined best fit rather than to a selected subset such as the Stanford data.

A second inconsistency is the fact that even with free norms the χ^2 per degree of freedom in the most recent Mainz data is considerably greater than one. This indicates that below the level of 1% other error sources than counting rate statistics not yet accounted for contribute to the total uncertainty in this experiment. Therefore, in order to set the statistical weight of these two data sets into more realistic proportion to the

rest of the data a 1% error was added in quadrature to each of the data points of the most recent Mainz measurements. The result of this second fit is listed in Table I and gives a χ^2 per degree of freedom of 1.4. For this fit the 502 MeV Stanford and Saclay data had to be normalized down by 5.1%. The deviation from this fit for the various data sets in the region of 1.9 fm^{-1} is shown in Fig. 3. Obviously, the combined fit is dominated in this region by the Saclay data while the Mainz data still shows a scatter much larger than the quoted error bars.

The final inconsistency concerns the incompatibility of electron scattering and muonic x-ray results. As in previous analyses,⁷ the absolute energies of the 2p states (obtained from $3d \rightarrow 2p$ transitions) disagree with the best fit result by 1 Kev, which is many times larger than the experimental error. The uncertainty of .5 Kev assigned in Table I represents a generous estimate of errors in the theoretical nuclear polarizations corrections. Thus, it remains an open question whether the discrepancy arises from an error in the nuclear polarization, some as yet undiagnosed error in the electron scattering or muonic experiments, or an additional physical correction which has yet to be considered.

In order to demonstrate the contribution of various regions of momentum transfer to the final calculated central density, it is instructive to write the density in terms of the expansion coefficients:

$$4\pi\rho(r) = \sum_{n=1}^N a_n J_0\left(\frac{n\pi r}{R}\right)$$

where $a_n = \frac{n\pi}{R} c_n$ with c_n given in Table I. With this parameterization, the form factor at $q_n = \frac{n\pi}{R}$ is given by $F(q_n) = \frac{a_n}{2} \frac{R}{q_n^2}$. Thus, with this expansion the sum over a_n determines $\rho(o)$ and effectively replaces the integral over $F(q)$ from Ref. 6 to determine $\rho(o)$.

In Fig. 5 $\sum_{i=1}^n a_i$ is plotted versus $q(n) = \frac{n\pi}{R}$ for $n \geq 5$. At sufficiently high q the sum converges to the value of the central density and one may observe the quantitative importance of each region of momentum transfer. As seen from Fig. 5, only for $q(n) \geq 3 \text{ fm}^{-1}$ becomes the magnitude of oscillation less than the statistical uncertainty of the sum. This demonstrates that the high momentum transfer data from Saclay are essential to reduce the uncertainty in $\rho(o)$. Fig. 5 also shows that the main contribution to the uncertainty in $\sum a_i$ comes from the data points below $q = 2 \text{ fm}^{-1}$ even though for ^{208}Pb these data were already taken with remarkable precision. This shows the need for high precision low and intermediate q -data.

Given the differences between different experiments in the region of $q = 1.9 \text{ fm}^{-1}$ one may assign an uncertainty of about 10% to the a_6 -coefficient, in addition to the statistical uncertainty which translates into a 1% uncertainty in $\rho(o)$. We find that even though experimental inconsistencies remain the charge distribution is known with remarkable precision.

In conclusion, significant questions, both theoretical and experimental, remain. If the density in Fig. 4 turns out to be correct, and dispersion and exchange current effects can be shown to be negligible, then indeed a very interesting and unexpected

deficiency of the mean field theory will have been revealed. The problem is not so much the fact that the S-state lump is much higher than experiment, since it is quite reasonable that the residual interactions will smooth out oscillations exaggerated by the single-particle approximation. Rather, the fact that the interior density is systematically too high over a region of several quantum density fluctuations suggests that there may be an error in the overall balance of volume surface, symmetry and Coulomb energy in the mean field theory. Thus, resolution of the present inconsistencies is essential.

REFERENCES

1. J. W. Negele, Proc. of Conf. on Radial Shapes of Nuclei, Cracow, A. Budzanowski ed. (1976) 79.
2. J. Heisenberg, R. Hofstadter, J. S. McCarthy; I. Sick, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Lett 23 (1969) 1402.
3. J. L. Friar and J. W. Negele, Advances in Nuclear Physics, Vol. 8, Plenum Press, N. Y. (1975) 219.
4. J. W. Negele and D. Vautherin, Phys. Rev. C5 (1972) 1472.
5. H. Euteneuer, J. Friedrich and N. Voegler, Phys. Rev. Lett 36 (1976) 129.
6. B. Frois, J. B. Bellicard, J. M. Cavedon, M. Huet, P. Laconte, P. Ludeau, A. Nakada, Phan. Xuan Ho, and I. Sick, Phys. Rev. Lett. 38 (1977) 152.
7. J. L. Friar and J. W. Negele, Nucl. Phys. A212 (1973) 93.
8. M. Nagao and Y. Torizuka, Phys. Lett. 37B (1971) 383.

TABLE CAPTION

Table I Results of simultaneous fit to 14 sets of electron scattering data and 6 muonic transitions. The expansion coefficient c_n is given by CFFIT and the error is CFERR. The final best fit density is RHOFIT with statistical error RHOERR. The DME density is RHOIST and all densities are multiplied by 4π . Details of the experimental analysis are given in Ref. 7.

FINAL RESULTS FOR ITERATION 5
Z= 82,A= 208,RAD= 11.0

	CFFIT	CF1ST	COEFF	DCOEFF	CFERR
1	2.7598942D 00	2.7649295D 00	-5.0352588E-03	-1.0341963D-04	2.00D-04
2	8.4783154D-01	8.6707510D-01	-1.9243546E-02	1.0051978D-03	1.2D-03
3	-8.0811033D-01	-7.9657726D-01	-1.1533059E-02	7.5760295D-04	1.4D-03
4	-2.9686088D-02	-2.4814286D-02	-4.8717931E-03	2.1613166D-04	1.7D-03
5	2.7291020D-01	2.7874331D-01	-5.8330894E-03	8.6704645D-04	5.4D-04
6	-7.2947873D-02	-7.8800415D-02	5.8525354E-03	4.1157646D-04	5.5D-04
7	-5.8457856D-02	-5.7452102D-02	-1.0057525E-03	9.6932834D-04	5.2D-04
8	4.2154232D-02	5.0240453D-02	-8.0862120E-03	-1.0256602D-03	3.3D-04
9	1.0209333D-02	1.9577483D-02	-9.3681440E-03	-6.6340302D-05	4.8D-04
10	-7.8486575D-03	-2.7147553D-03	-5.1338933E-03	1.1041062D-04	2.7D-04
11	2.9947125D-04	1.1123900D-03	-8.1291818E-04	7.8723209D-05	3.9D-04
12	1.1833945D-03	-8.1106050D-05	1.2645000E-03	-1.8840672D-04	3.0D-04
13	-3.8421460D-04	-6.6542145D-04	2.8120656E-04	4.1048661D-04	7.6D-04

DATA AND RESULTS FOR MUONIC ENERGY LEVELS

	TRANSITION	EMUEXP	EMUFIT	DELEMU	EMUERR	CHISQ	VPOL	LSHFT	NPOL
273	2P3/2--1S1/2	5963.297	5963.359	-0.06	0.50	0.02	37.54	-2.30	2.39
274	2P1/2--1S1/2	5778.500	5778.113	0.39	0.50	0.60	34.98	-2.64	2.26
275	2S1/2--2P1/2	1216.150	1216.419	-0.27	0.78	0.12	12.95	0.37	0.91
276	2S1/2--2P3/2	1031.180	1031.185	-0.00	0.46	0.00	10.39	0.03	0.79

DATA AND RESULTS FOR MUONIC ENERGY LEVELS

	ENERGY LVL	EMUEXP	EMUFIT	DELEMU	EMUERR	CHISQ	VPOL	LSHFT	NPOL
277	2P1/2	4815.109	4816.258	-1.15	0.50	5.28	32.19	-0.35	1.64
278	2P3/2	4630.309	4631.031	-0.72	0.50	2.09	29.63	-0.69	1.52

CHISQ= 8.10

THE RMS RADIUS IS 5.4989 (0.0007) FM.

TOTAL CHISQ= 354.58

CHISQE= 346.48

RADIUS	RHO FIT	RHO 1ST	DEL RHO	RHO ERR	PCERR
0.00	7.993E-01	8.819E-01	8.3E-02	5.0E-03	0.62
0.20	7.990E-01	8.793E-01	8.0E-02	4.8E-03	0.60
0.40	7.952E-01	8.689E-01	7.4E-02	4.2E-03	0.53
0.60	7.897E-01	8.534E-01	6.4E-02	3.5E-03	0.45
0.80	7.835E-01	8.351E-01	5.2E-02	3.0E-03	0.38
1.00	7.775E-01	8.165E-01	3.9E-02	2.7E-03	0.35
1.20	7.725E-01	7.999E-01	2.7E-02	2.5E-03	0.32
1.40	7.689E-01	7.870E-01	1.8E-02	2.2E-03	0.29
1.60	7.671E-01	7.790E-01	1.2E-02	1.9E-03	0.25
1.80	7.669E-01	7.758E-01	8.9E-03	1.6E-03	0.21
2.00	7.683E-01	7.771E-01	8.8E-03	1.4E-03	0.19
2.20	7.710E-01	7.816E-01	1.1E-02	1.2E-03	0.16
2.40	7.745E-01	7.877E-01	1.3E-02	1.0E-03	0.13
2.60	7.784E-01	7.938E-01	1.5E-02	8.3E-04	0.11
2.80	7.823E-01	7.985E-01	1.6E-02	7.4E-04	0.09
3.00	7.857E-01	8.011E-01	1.5E-02	7.0E-04	0.09
3.20	7.886E-01	8.015E-01	1.3E-02	7.0E-04	0.09
3.40	7.907E-01	8.000E-01	9.3E-03	7.2E-04	0.09
3.60	7.922E-01	7.974E-01	5.2E-03	7.4E-04	0.09
3.80	7.933E-01	7.947E-01	1.5E-03	7.2E-04	0.09
4.00	7.939E-01	7.926E-01	-1.3E-03	6.6E-04	0.08
4.20	7.939E-01	7.911E-01	-2.8E-03	5.9E-04	0.07
4.40	7.929E-01	7.898E-01	-3.1E-03	5.3E-04	0.07
4.60	7.899E-01	7.874E-01	-2.5E-03	4.8E-04	0.06
4.80	7.836E-01	7.821E-01	-1.5E-03	4.2E-04	0.05
5.00	7.722E-01	7.718E-01	-3.2E-04	3.6E-04	0.05
5.20	7.539E-01	7.545E-01	5.9E-04	3.2E-04	0.04
5.40	7.274E-01	7.285E-01	1.1E-03	3.1E-04	0.04
5.60	6.916E-01	6.928E-01	1.2E-03	2.9E-04	0.04
5.80	6.464E-01	6.474E-01	1.0E-03	2.9E-04	0.04
6.00	5.924E-01	5.931E-01	6.2E-04	3.2E-04	0.05
6.20	5.314E-01	5.315E-01	1.6E-04	3.7E-04	0.07
6.40	4.655E-01	4.652E-01	-3.0E-04	3.9E-04	0.08
6.60	3.976E-01	3.968E-01	-7.6E-04	3.7E-04	0.09
6.80	3.307E-01	3.295E-01	-1.2E-03	3.4E-04	0.10
7.00	2.677E-01	2.661E-01	-1.6E-03	3.2E-04	0.12
7.20	2.108E-01	2.089E-01	-2.0E-03	3.0E-04	0.14
7.40	1.616E-01	1.594E-01	-2.2E-03	2.8E-04	0.17
7.60	1.206E-01	1.182E-01	-2.4E-03	2.7E-04	0.22
7.80	8.776E-02	8.535E-02	-2.4E-03	2.6E-04	0.30
8.00	6.228E-02	6.000E-02	-2.3E-03	2.4E-04	0.39
8.20	4.314E-02	4.113E-02	-2.0E-03	2.0E-04	0.45
8.40	2.920E-02	2.753E-02	-1.7E-03	1.4E-04	0.49
8.60	1.932E-02	1.803E-02	-1.3E-03	1.3E-04	0.65
8.80	1.249E-02	1.158E-02	-9.1E-04	1.3E-04	1.08
9.00	7.883E-03	7.313E-03	-5.7E-04	1.3E-04	1.67
9.20	4.820E-03	4.553E-03	-2.7E-04	1.3E-04	2.61
9.40	2.807E-03	2.801E-03	-6.1E-06	1.4E-04	4.91
9.60	1.491E-03	1.706E-03	2.1E-04	1.5E-04	10.17
9.80	6.398E-04	1.030E-03	3.9E-04	1.5E-04	22.92
10.00	1.088E-04	6.172E-04	5.1E-04	1.4E-04	128.99
10.20	-1.867E-04	3.669E-04	5.5E-04	1.6E-04	-85.73
10.40	-2.993E-04	2.163E-04	5.2E-04	1.8E-04	-60.40
10.60	-2.702E-04	1.263E-04	4.0E-04	1.6E-04	-60.47
10.80	-1.412E-04	7.274E-05	2.1E-04	9.7E-05	-68.73
11.00	4.114E-05	4.117E-05	2.5E-08	6.1E-09	0.01
11.20	2.271E-05	2.271E-05	0.0	0.0	0.0
11.40	1.206E-05	1.206E-05	0.0	0.0	0.0
11.60	6.022E-06	6.022E-06	0.0	0.0	0.0
11.80	2.675E-06	2.675E-06	0.0	0.0	0.0
12.00	8.849E-07	8.849E-07	0.0	0.0	0.0

FIGURE CAPTIONS

Fig. 1 Interior charge densities of ^{208}Pb based on the original Stanford experiment (I) and the Mainz data (II). Note the expanded side.

Fig. 2 Differential cross sections for ^{208}Pb measured at Saclay.

Fig. 3 Percent deviation between experiment and the best simultaneous fit of Table I in the region of $q - 1.9 \text{ fm}^{-1}$.

Fig. 4 Charge density of ^{208}Pb obtained from simultaneous analysis of all electron scattering and muonic data. The thickness of the solid curve denotes the statistical error envelope, and the dashed curve shows the DME prediction.

Fig. 5 Graphs of $\sum_{i=1}^n a_n$ versus $q(n) = \frac{n\pi}{R}$ as described in the text.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 5

HIGH-~~q~~MULTIPOLARITY ELASTIC MAGNETIC ELECTRON SCATTERING

H. DeVries, P. DeWitt-Huberts,
A. E. L. Dieperink and T. W. Donnelly

Elastic magnetic electron scattering provides unique information on the structure of the nucleus.¹ The transverse form factor basically involves the Fourier transform of the nuclear current density distribution, as opposed to the longitudinal form factor which involves the charge density distribution. Moreover, at high momentum transfer q , contributions from the magnetization current are dominant over convection current contributions. In odd high-spin nuclei the highest allowed multipole, which is the most important at high q , arises principally from the magnetization current provided by the last unpaired nucleon. Consequently, (ee') measurements at high q give us a means of studying directly the radial distributions of valence nucleons in nuclei. In particular, since neutrons can contribute to the magnetization current on an equal footing with protons (in contrast to the situation which exists for the longitudinal form factor), measurements on odd-neutron nuclei yield important information, namely, the radial distributions of valence neutrons. Such information is very difficult to obtain in any other way.

A particularly interesting application of these ideas involves a comparison between proton and neutron radial distributions for a given shell obtained by measurements on odd-proton and odd-neutron nuclei in the same region of the periodic table. Recently new high-precision data of this type were obtained at high momentum transfer by the Saclay group.^{2,3} The nuclei studied

were ^{93}Nb and ^{87}Sr ; both have a $J^\pi = \frac{9}{2}^+$ ground state and can be assigned $1g_{9/2}$ - proton and $1g_{9/2}^{-1}$ - neutron configurations in the extreme single-particle limit respectively. Similar data on the $1f_{7/2}$ nuclei ^{51}V and ^{59}Co have also been obtained and the results of the analyses in these cases have been submitted for publication. Figs. 1 and 2 show some of the results from these investigations.

For an unpolarized experiment the elastic magnetic electron scattering cross section for a spin- J target nucleus can be expressed¹ in terms of the transverse form factor $F_T^2(q)$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} / \left(\frac{d\sigma}{d\Omega}\right)_M \frac{1}{1/2 + \frac{\tan^2 \theta}{2}} = F_T^2(q) = \frac{1}{2J+1} \sum_{\lambda=1, \text{odd}}^{2J} |\langle \Psi_J || T_\lambda^{\text{mag}}(q) || \Psi_J \rangle|^2 \quad (1)$$

where $T_{\lambda M}^{\text{mag}}$ is the magnetic transition operator of multipolarity λ and Ψ_J is the target ground-state wave function (which in first approximation for an odd nucleus consists of a single unpaired nucleon in orbit $j = J$ outside a spin-0 core). Each many-body reduced matrix element in eq. (1) can be decomposed into a linear combination of single-particle reduced matrix elements:

$$\langle \Psi_J || T_\lambda^{\text{mag}} || \Psi_J \rangle = \sum_{\alpha\beta} \rho_{\alpha\beta}^\lambda \langle \alpha || T_\lambda^{\text{mag}} || \beta \rangle, \quad (2)$$

where $\rho_{\alpha\beta}^\lambda$ are one-body density matrix elements,³

$$\rho_{\alpha\beta}^\lambda = \langle \Psi_J || [a_\alpha^\dagger a_\beta]^\lambda || \Psi_J \rangle / \sqrt{2\lambda+1}, \quad (3)$$

with labels α and β denoting complete sets of single-particle quantum numbers $\{n(l\frac{1}{2})j, m_j; t = \frac{1}{2}, m_t\}$. If the ground-state configuration is restricted to one major shell and has the largest j in that shell, and if the highest multipole $\lambda = 2J$ is considered, then the sum in eq. (2) is restricted to just one term, $\alpha = \beta = j$ (see Ref. 1). In that case, configuration mixing manifest itself only through a modification in the density matrix element ρ_{jj}^{2J} . The shape of the cross section at high q , even in the presence of configuration mixing will then still be characteristic of the single matrix element $\langle j || T_{2j}^{\text{mag}} || j \rangle$ and so permit us to extract the radial distribution of the j -shell valance nucleon. This is true with one proviso: the lower multipoles (predominantly the $\lambda = 2J-2$ multipole), which in general involve several reduced matrix elements with unknown density matrix coefficients, must not significantly contribute. In the pure single-particle model, for example, this condition is satisfied.

In fact, the nuclei considered here can in first approximation be described in the spherical shell model within an active configuration space of one main oscillator shell. This being the case, the data were analyzed under the assumption that the following condition is met: In the momentum transfer range $2 \text{ fm}^{-1} \leq q \leq 3 \text{ fm}^{-1}$, the highest allowed multipole dominates over the lower multipoles. That means, for example, that (2p-1h) or (1p-2h) admixtures in the ground states do not significantly modify the shape of the form factor at high q from the shape obtained for pure 1p or 1h configurations. It is our aim here to test the validity of this assumption by considering various admixtures

of (lp-2h) configurations in the ground state of a predominantly 1 h nucleus. We consider the recently studied case of ^{87}Sr with, in first approximation, a $1g_{9/2}$ - neutron hole configuration. We allow for admixtures as illustrated in fig. 3. Perhaps the most satisfactory way to proceed would be to use wave functions from a large-scale shell-model calculation, but such information is unavailable at present. Therefore, in our present study, we shall restrict our attention to a very specific model ground state:

$$\begin{aligned}
 |9^{+}/2> = & \beta_9 |1g_{9/2}^{-1}> + \beta_5 |[1g_{9/2}^{-1}[1g_{9/2}^{-1}2d_{5/2}]_7^{+}]_{9/2}^{+}> \\
 & + \beta_7 |[1g_{9/2}^{-1}[1g_{9/2}^{-1}1g_{7/2}]_7^{+}]_{9/2}^{+}>,
 \end{aligned}
 \tag{4}$$

where the $1g_{9/2}^{-1}$ amplitude was chosen to agree with the previous analysis,³ $|\beta_9| = \sqrt{\alpha_9} = .92$, where we write $\beta_K = |\beta_K| \zeta_K$, $K = 5, 7, 9$ with $\zeta_9 = +1$, $\zeta_5 = \pm 1$, $\zeta_7 = \pm 1$ to account for all relative sign possibilities and where, for lack of any better information, we take $|\beta_5| = |\beta_7| = \sqrt{\frac{1}{2}(1-\alpha_9)} = .28$ as a starting point. Of course, this is far from the most general form and should only be regarded as a plausible extension beyond the pure single-hole model. Our purpose here is simply to see whether any re-interpretation of the previous analysis is suggested by employing a (hopefully) reasonable wave function with generous (lp-2h) admixtures included.

The magnetic form factors were calculated with the code "microdens"⁵ using harmonic oscillator wave functions having

oscillator parameter $b = 2.0$ fm with the four different sign combinations $(\zeta_5 \zeta_7) = (++)$, $(+-)$, $(-+)$, $(--)$. The results are shown in figs. 4 and 5 along with the experimental error band. It can be seen that the combinations with $\zeta_5 = 01$ (fig. 5) lead to results that are incompatible with the data, whereas the shapes of the $\zeta_5 = +1$ form factors (fig. 4) are fairly close to experiment. Subsequently, best fits were obtained with our particular model wave function by allowing the overall normalization α_9 and the oscillator parameter b to vary. Acceptable values of χ^2 were found for the cases $\zeta_5 = +1$, $\zeta = \pm 1$ only. The best-fit values of b for these cases are $\sim 1\%$ and $\sim 2\%$ smaller respectively than the value used in the pure $1h$ analysis. Therefore, the extracted rms-radius of the $1g_{9/2}$ orbit obtained with these model wave functions would be smaller by the same amount. We conclude that, while the $(1p-2h)$ admixtures were chosen to enhance the contribution of the $M7$ multipole, the change of the radial scale is less than 2% and with a sign such that the observed discrepancy with the Hartree-Fock calculations is increased. Investigations of a wider class of configurations are in progress.

Finally, we make the following remarks:

(1) Although, for a more definitive test of possible $M7$ interference in the analysis of the $M9$ multipole, model wave functions of a less ad hoc character than the ones used here are required, we feel that the above considerations show that it is unlikely that the observed discrepancy between DDHF and experiment can be resolved in terms of $0 \hbar\omega$ configuration mixing.

(2) Upon examining fig. 4 it is clear that the momentum transfer region $1.5 \text{ fm}^{-1} \leq q \leq 2.0 \text{ fm}^{-1}$ may help in sorting out the relative importance of the various multipole matrix elements. In this region the M7 multipole is important and knowledge of whether it is quenched or not at these lower values of q has a direct bearing on whether or not it provides much uncertainty in the analysis of the sought after M9 at higher q .

(3) Possible interference effects may be expected to cancel to a large extent when comparing the radial size of valence orbits in two nearby nuclei, such as ^{93}Nb and ^{87}Sr .

(4) There is an interesting relationship between the one-body densities in high-multipolarity magnetic electron scattering and spectroscopic factors obtained in single-neutron transfer reactions. One can relate $\rho_{\alpha\beta}^{\lambda}$ to pick-up spectroscopic factors by making an intermediate-state expansion of the one-body densities, resulting in the sum rule,

$$\rho_{\alpha\beta}^{\lambda} = \sqrt{2\lambda+1} (2J_f+1) \sum_f (-1)^{J_f+\beta+J+\lambda} \left\{ \begin{matrix} J & J & \lambda \\ \alpha & \beta & J_f \end{matrix} \right\} \quad (5)$$

$$\times (S_{\alpha f}^{-1})^{1/2} (S_{\beta f}^{-1})^{1/2},$$

where $(S_{\alpha f}^{-1})^{1/2} = \langle \Psi_{J_f}^{A-1} || a_{\alpha} || \Psi_J^A \rangle$ is a spectroscopic amplitude. The $\rho_{\alpha\beta}^{\lambda}$ element most relevant for the M7 interference test, $\rho_{9/2,5/2}^7$, could in principle be determined experimentally by locating possible $\ell = 2$ strength in the reaction $^{87}\text{Sr}(p,d)^{86}\text{Sr}$ to final states $J_f = 2, 4, 6$. At present experimental data are scarce and such components have not been measured.⁴ More

accurate experiments designed to locate contributions which stem from the $2d_{5/2}$ components in the ^{87}Sr ground state would be very valuable.

A final comment: for $\lambda = \lambda_{\text{max}} = 2J = 2j$ only the diagonal ρ_{jj}^{2j} matrix element has to be determined, providing an interesting consistency check between spectroscopic factors and the magnetic scattering data. For ^{51}V , where there exist accurate spectroscopic factors from the $(d, ^3\text{He})$ reaction, application of the sum rule (eq. (5)) leads to $\rho_{7/2, 7/2}^7 = .96 \pm .05$, in good agreement with the value obtained from electron scattering (1.02 ± 0.05) . In the case of ^{87}Sr only a lower limit can be obtained from the $^{88}\text{Sr}(p, d)^{87}\text{Sr}(\text{gs})$ reaction: $\rho_{9/2, 9/2}^9 \geq .84$.

REFERENCES

1. T. W. Donnelly and J. D. Walecka, Nucl. Phys. A201 (1973) 81.
2. P. K. A. deWitt Huberts, et al., Phys. Lett. 60B (1976) 157.
3. I. Sick, et al., Phys. Rev. Lett. 38 (1977) 1259.
4. J. M. Morton, et al., Nucl. Phys. A161 (1971) 228.
5. H. C. Lee, "Nuclear Charge, Convection Current and Magnetization Current Densities," AECL-4839, Chalk River, Ontario, 1975.

FIGURE CAPTIONS

- Fig. 1 Elastic magnetic electron scattering from ^{87}Sr and ^{93}Nb as functions of momentum transfer q (see refs. 2,3).
- Fig. 2 Elastic magnetic electron scattering from ^{51}V and ^{59}Co as functions of momentum transfer q (see refs. 2,3).
- Fig. 3 lp-2h admixtures allowed for in ^{87}Sr .
- Fig. 4 Elastic magnetic electron scattering from ^{87}Sr as functions of momentum transfer q . The shaded region shows the experimental error band (ref. 3) and the dashed and dash-dot curves show results of calculations where lp-2h admixtures have been included in the predominantly lh ground state wave function.
- Fig. 5 As for fig. 4.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

THE TRANSITION DENSITY OF THE OCTUPOLE VIBRATION
IN $^{208}_{82}\text{Pb}$. COMPARISON OF THEORY AND EXPERIMENT

B. Frois

DPh N/HE, Centre d'Etudes Nucleaires
de Saclay, France

J. Heisenberg

Bates Linear Accelerator Center
and
Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139

Over the last five years we have considerably increased our knowledge of the nuclear ground state; very significant improvements have been achieved in both theory and experiments. Elastic electron scattering has been an essential component of this progress providing data of increased accuracy and higher momentum transfer.¹ One begins now to have a certain amount of confidence in the accuracy of the charge densities deduced by a combined analysis of muonic x-rays and electron scattering data. The case of ^{208}Pb discussed at this workshop, is a good example of the results that can now be obtained.

For excited states the situation is quite different. The range of momentum transfer for the present data is not as large as for the ground state and the resulting transition charge densities are not as well measured. This can be understood easily: Towards low momentum transfer the inelastic cross section becomes smaller while the elastic is increasing rapidly.

Thus, the inelastic cross section appears only as a small hump on a huge radiation tail and the extraction of cross sections necessarily contains large uncertainties. In addition, there are usually no equivalent data to the μ -mesic transitions available. Towards large momentum transfer because of the higher energy, resolution is often a problem. The combination of poor statistics and poor resolution again make the separation of levels much more difficult. In addition, the approximations made in the DWBA description of the electron scattering process leads to larger uncertainties in the case of inelastic electron scattering than it does in the elastic case. On the other side the nuclear structure calculations for nuclear ground states with Hartree-Fock calculations have been well established while for excited nuclear states only less precise methods were available.

This situation is changing now and considerable progress has been achieved recently in the theory of elementary excitations in closed shell nuclei.^{3,4,5} It is now urgent to measure the corresponding transition charge densities accurately to get critical tests of these calculations.

The best example is the lowest lying octupole vibrational state in ^{208}Pb ($E^* = 2.615$ MeV). It is a very collective state with a transition probability of about 38 Weisskopf units. There has been an extensive set of measurements^{6,7} in the range $0.5 \leq q \leq 2.8 \text{ fm}^{-1}$. The results span 7 decades and the lowest measured cross section is $\frac{d\sigma}{d\Omega} = 5 \times 10^{-8} \text{ mb/sr}$.

A combined fit of all the existing data was made by Rothaas et al.⁸ using similar to ground state analyses^{9,10} a Fourier

THE TRANSITION DENSITY OF THE OCTUPOLE VIBRATION
IN ^{208}Pb . COMPARISON OF THEORY AND EXPERIMENT

B. Frois

DPh N/HE, Centre d'Etudes Nucleaires
de Saclay, France

J. Heisenberg

Bates Linear Accelerator Center
and
Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139

Over the last five years we have considerably increased our knowledge of the nuclear ground state; very significant improvements have been achieved in both theory and experiments. Elastic electron scattering has been an essential component of this progress providing data of increased accuracy and higher momentum transfer.¹ One begins now to have a certain amount of confidence in the accuracy of the charge densities deduced by a combined analysis of muonic x-rays and electron scattering data. The case of ^{208}Pb discussed at this workshop, is a good example of the results that can now be obtained.

For excited states the situation is quite different. The range of momentum transfer for the present data is not as large as for the ground state and the resulting transition charge densities are not as well measured. This can be understood easily: Towards low momentum transfer the inelastic cross section becomes smaller while the elastic is increasing rapidly.

Thus, the inelastic cross section appears only as a small hump on a huge radiation tail and the extraction of cross sections necessarily contains large uncertainties. In addition, there are usually no equivalent data to the μ -mesic transitions available. Towards large momentum transfer because of the higher energy, resolution is often a problem. The combination of poor statistics and poor resolution again make the separation of levels much more difficult. In addition, the approximations made in the DWBA description of the electron scattering process leads to larger uncertainties in the case of inelastic electron scattering than it does in the elastic case. On the other side the nuclear structure calculations for nuclear ground states with Hartree-Fock calculations have been well established while for excited nuclear states only less precise methods were available.

This situation is changing now and considerable progress has been achieved recently in the theory of elementary excitations in closed shell nuclei.^{3,4,5} It is now urgent to measure the corresponding transition charge densities accurately to get critical tests of these calculations.)

The best example is the lowest lying octupole vibrational state in ^{208}Pb ($E^* = 2.615$ MeV). It is a very collective state with a transition probability of about 38 Weisskopf units. There has been an extensive set of measurements^{6,7} in the range $0.5 \leq q \leq 2.8 \text{ fm}^{-1}$. The results span 7 decades and the lowest measured cross section is $\frac{d\sigma}{d\Omega} = 5 \times 10^{-8} \text{ mb/sr}$.

A combined fit of all the existing data was made by Rothaas et al.⁸ using similar to ground state analyses^{9,10} a Fourier

Bessel expansion for the transition density:

$$\rho_{tr}(r) = \begin{cases} 0 & r \geq R \\ \sum_v a_v j_3(q_v r) & r \leq R \end{cases}$$

Here R is a cutoff radius so that all charge is contained within this radius that was chosen to be 11 fm. In order to estimate the uncertainty of $\rho_{tr}(r)$ the behavior of the form factor for $q > q_{max}$ has been estimated by the assumption:

$$|F(q)| \leq c F_{proton}(q) q^{-4}$$

c being a constant taken from matching this estimate to the last measured diffraction maximum of the form factor. This is a very strong assumption. Even though an upper limit of this analytic form may be justified the choice of the parameter c is completely ad hoc and needs to be confirmed by experiments. For example, in the case of the elastic scattering on ^{208}Pb it was possible to show that this q^{-4} assumption leads to a significant underestimation of the error band on the charge density.¹¹

The resulting transition charge density is shown in fig. 1 with two RPA calculations. It has a large peak centered at the nuclear surface at about $r = 7\text{fm}$ and pronounced oscillations in the interior of the nucleus for $0 \leq r \leq 5\text{ fm}$. With the hydrodynamic or the Tassie model the transition density can be related to the ground state charge density. However, the fluctuations in the ground state charge density are not at all large enough to explain the observed oscillations and the microscopic shell

structure of the octupole phonon has to be calculated. Almost all the RPA calculations produce a large peak at the surface that determines the overall strength of the transition but they differ strongly in the center. It is in this region $r \leq 4$ fm that a more accurate determination of ρ_{tr} is needed since the detailed form of these oscillations is much more sensitive to the details of the ph-ph force. At the surface all ph-components are in phase and any improper treatment can be compensated for by renormalizing the force. This is not the case for the interior structure. Here complicated cancellations take place which are strongly affected if the force is not appropriate or if the basis is chosen too small. Fig. 2 compares directly to the present data a few of the existing RPA results. The cross sections were calculated from the listed transition densities using the DWBA code HEINEL.

At this level of precision it is necessary to include in the calculations the effects of the neutron form factor. This correction was only included in the calculation using the semiempirical Migdal force. Therefore, there may be improvements in the comparison once this effect and possibly other corrections like pion exchange currents, centre of mass corrections, etc., are included.

With respect to further data taking we can ask where more data are needed in order to improve our knowledge of the transition density most efficiently. Because of the wavelength of these oscillations only the cross sections for $q \geq 2.2 \text{ fm}^{-1}$ begin to determine the structure in ρ_{tr} . There has been considerable

progress on the experimental side. New hydrogen target cooling systems are now available making much larger beam currents possible which together with a resolution of 3×10^{-4} allow to measure cross sections down a factor 100 below the smallest presently measured cross section on this level. Assuming that all the data at momentum transfer $q \geq 2.2 \text{ fm}^{-1}$ can be retaken with the similar kind of improvement we will be in a position to reconstruct the transition charge density in r space with much greater precision.

Fig. 2 shows, however, that all the nuclear structure calculations differ already strongly at small and intermediate momentum transfer which is $q \leq 2.5 \text{ fm}^{-1}$. Therefore, obviously the present data are a sufficient check for the quality of agreement for present theories. We also see that the uncertainties in the nuclear structure calculations are much too big as to draw any conclusions on correction effects such as neutron form factor, exchange currents or centre of mass corrections. We, therefore, hope that from the spatial reconstruction for which the high- q -data are needed we have better access to the deficiencies of the theory and that it may help us in the interpretation of other kind of experiments.

REFERENCES

1. Proceedings of the Cracow Conference on radial shape of nuclei, Institute of Nuclear Physics, Cracow 1976.
2. J. Friar, J. W. Negele and J. Heisenberg - this workshop.
3. P. Ring and J. Speth, Nuclear Physics, A235 (1974) 315.
4. J. P. Blaizot and D. Grogny (1976) Nuclear Physics, A284 (1977) 429, and to be published.
5. I. Hamamoto, Physics Letters, 10C (1974) 64, and Physics Letters, 66B (1977) 410.
6. J. H. Heisenberg and I. Sick, Physics Letters, 32B (1970) 249.
7. N. Voegler, Master thesis, University of Mainz, 1975.
8. H. Rothaas, et al., Physics Letters, 51B (1974) 23.
9. J. Friar and J. W. Negele, Nuclear Physics, A212 (1973) 93.
10. B. Dreher, et al., Nuclear Physics, A235 (1974) 219.
11. B. Frois, et al., Physics Review Letters, 38 (1977) 153.

FIGURE CAPTIONS

- Fig. 1 Transition charge density for the octupole vibration in ^{208}Pb . Shaded area is experimental uncertainty, solid line is RPA-prediction with semiempirical Migdal force, broken line is RPA-prediction with Gogny D1-force.
- Fig. 2 Comparison of experimental data for $\sigma/\sigma_{\text{Mott}}$ to theoretical predictions for the 3^- state in ^{208}Pb .

FIGURE 1

³⁶²A

Momentum transfer q (fm^{-1})

FIGURE 2

7.6 (a) Electroexcitations of High Spin States

W. Knüpfer and M. G. Huber

Institute for Theoretical Physics

University Erlangen-Nürnberg, Erlangen, Germany

I. Introduction

The experimental investigation of low lying high spin states has been quite popular in low energy nuclear physics during the last few years. Those efforts lead to the discovery of various interesting phenomena such as Yrast lines and back bending effects^{1,2)}. Those results have stirred up many theoretical investigations³⁾ in the frame of collective and microscopic models, respectively. It is generally believed that a proper description of those rotational bands and their "interactions" will lead to an improved understanding of the nuclear dynamics at low excitation energies. So far most of those investigations were performed with hadron induced processes: for example with $(\alpha, xn)^{2)}$ - or (heavy ion, $xn)^{4)}$ - type of reactions.

It is the purpose of this contribution to point out that some of those nuclear excitations can best be studied by inelastic scattering of high energy electrons.

II. Example: ^{208}Pb

Such kind of an experiment has recently been performed by the Mainz-group on $^{208}\text{Pb}^{5)}$. They studied systematically the electroexcitation in the region between 4 and 5 MeV. Surprisingly enough they found there strongly collective excitations of high angular momentum: states with $J^\pi = 2^+, 4^+, 6^+, 8^+$ and 10^+ could be identified; the corresponding transition exhibit two remarkable features: (i) the $B(E\lambda)$ -values were generally of the order of 20 single particle units (except for the $0^+ \rightarrow 2^+$ transition); (ii) the energies followed rather closely an $I(I + 1)$ -law.

In a consistent RPA treatment using a spherical shell model basis (including $(1p1h)$ excitation up to $4\hbar\omega$) such an excitation spectrum had been found by Knüpfer et al⁶⁾; it turned out that the calculated energies and formfactors do agree rather nicely with the experimental data (see Fig. 1). The interpretation of the experiment looks at first surprising; everybody associates with such a rotational band a large deformation; on the other hand ^{208}Pb is believed to be one of the

Fig. 1: a, b) The measured 2^+ and 4^+ formfactors at 4.09 and 4.32 MeV, respectively, are compared with the calculated DWBA results at 120 MeV and 289 MeV, respectively (the theoretical excitation energies are given in parenthesis). Experimental points correspond to an electron energy of 120 (○), 200 (Δ) and 290 MeV(+), respectively. c) Comparison of the calculated 4^+ (4.311 MeV) DWBA formfactor with the corresponding PWBA-formfactor. d, e, f) The experimental 6^+ , 8^+ , 10^+ formfactors are compared with the PWBA-results (the calculated excitation energies are given in parenthesis).

best spherical shell model nuclei (if not the best one). It turns out, however, that those "rotational" features can be reproduced quite satisfactorily in a spherical shell model calculation even without introducing effective charges. This may eventually shed some light on the detailed understanding of collective coordinates, such as "deformations", "rotation angles" or "intrinsic coordinates".

III. Extension to other nuclei

From hadron induced reactions we know a number of nuclei which exhibit nice Yrast bands and backbending effects²⁾, particularly in the rare earth region. Such excitations obviously should be investigated by (e, e') . Furthermore, in spherical nuclei low lying high spin states are predicted, too, from a RPA treatment. Some of them might even exhibit a rotational type spectrum. In Fig. 2 such predictions are shown for a number of nuclei which are accessible for a rigorous microscopic treatment.

Fig. 2: Candidates of a quasisrotational band in the spherical nuclei ^{90}Zr , ^{140}Ce and ^{203}Pb with the main components of the wavefunction. The 2^+ states in ^{90}Zr and ^{140}Ce , respectively, contain large admixtures of more complicated configurations and therefore lie outside (closed circles: experimentally known levels; open circles: predicted levels).

IV. Conclusion

From the experimental analysis of Friedrich et al it can be concluded that it is indeed possible to investigate the electro-excitation of low lying high spin states provided three experimental prerequisites are fulfilled:

- (i) sufficiently high end point energy so that the transition density can be measured up to "large" q-values; as a guide the following empirical relation seems to hold for ^{208}Pb :

$$q_I \cdot \bar{\rho} = I$$

with $\bar{\rho} = 5 \text{ fm}$

(here q_I denotes that q value where the $(0^+ \rightarrow I^+)$ transition formfactor exhibits its first maximum) using this result as a guide one arrives at the following estimate for a nucleus of mass number A:

$$q_I(A) = 0.85 \cdot I \cdot A^{1/3} \text{ (fm}^{-1}\text{)}$$

- (ii) sufficiently good energy-resolution:
in the case of ^{208}Pb the level spacing corresponded to that of a strongly deformed rotator (see Fig. 2):

$$E_x(I) = E_0 + aI(I + 1)$$

with

$$E_0 = 4.01 \text{ MeV}$$

$$a = 9.1 \text{ keV}$$

The energy resolution which presently is available at the new electron accelerators should be good enough to investigate quite a number of transitions, particularly in light and medium light nuclei;

- (iii) good background rejection:
this is indeed needed since the formfactors for high spin transitions are rather small (see Fig. 1).

It is suggested that high q inelastic electron scattering be used as a tool to complement the investigations of Yrast bands and backbending effects¹⁾. If those experiments are feasible then they exhibit an obvious advantage over the heavy ion induced reactions previously mentioned: they yield in a rather direct way the corresponding transition densities⁷⁾ and the $B(E\lambda)$ -values; those informations represent a stringent test for the various models and thus can help us to understand better the physics that is responsible for such low lying high spin excitations in complex nuclei.

References

- 1) M. Bertozzi, Intern. School on Electro- and Photonuclear Reactions, ed. S. Costa and C. Schaerf, Lect. Notes in Physics 62, Springer 1977
- 2) C. U. Mayer-Böricke, in Proc. of the Intern. Symposium on Nuclear Structure, Balatonfüred 1975
- 3) A. Faessler, in Proc. of the Intern. Symposium on Nuclear Structure, Balatonfüred 1975
- 4) K. Nagatani et al, Phys. Rev. C 14 (1976) 2133
- 5) J. Friedrich, K. Voegler, H. Euteneuer, Phys. Lett. 64 B (1976) 269
- 6) E. Weber, W. Knüpfer, E. Grecksch, M. G. Huber, Phys. Lett. 65 B (1976) 189
- 7) W. Knüpfer, M. G. Huber, B. Frois, Internal Report, Saclay (to be published)

V.6(b) Electroexcitation of Collective Magnetic Dipole Resonances

W. Knüpfer and M. G. Huber

Institute for Theoretical Physics

University Erlangen-Nürnberg, Erlangen, Germany

I. Introduction

A detailed investigation of the magnetic excitations of complex nuclei is needed for a better understanding of the nuclear dynamics both in finite nuclei and in infinite nuclear matter.

Therefore it is suggested to measure the distribution of the magnetic multipole transition strength of various nuclei, particularly with inelastic electron scattering.¹⁾ From an analysis of such data^{2,3,4)} one might obtain information on:

- (i) the strength of the spin- and spin-isospin dependent components of the effective residual interactions and on the properties of the corresponding magnetic eigenmodes of the nuclear many body system⁵⁾;
- (ii) the properties of the virtual pion field inside of the nucleus;
- (iii) the virtual admixtures of isobar configurations (such as of the Δ (33) resonance) in low lying nuclear excitations⁶⁾.

Such information is of current interest in connection with critical phenomena such as pion condensation⁷⁾ or the coupling of external and internal degrees of freedom of an individual bound nucleon.

II. Giant Magnetic Dipole Resonances

In the frame of a conventional shell model the existence of a collective Giant Magnetic Dipole Resonance is predicted^{8,9)}; in closed shell nuclei it should be localized at an excitation energy of roughly $E_{\text{exc}} \approx 100 A^{-1/3}$ (MeV); it is a coherent

superposition of (1p1h) configurations of $2\hbar\omega$. Such eigenmodes cannot be seen in photoabsorption but in (e,e') since the corresponding transition operators do exhibit a different radial behaviour:

for photons: $M_\gamma \propto \langle j_{\text{particle}} | g_e \underline{\ell} + g_s \underline{s} | j_{\text{hole}} \rangle$

for (e,e'): $M_{ee'}(q) \propto \langle j_{\text{particle}} | e^{iqr} (g_e \underline{\ell} + g_s \underline{s}) | j_{\text{hole}} \rangle$

We performed (1p1h)-RPA-calculations in the framework of the MSI-model for the nuclei ^{12}C , ^{28}Si and ^{40}Ca including all 1^+ configurations up to $2\hbar\omega$. In the specific example of ^{12}C we included also all 2p2h states of $2\hbar\omega$ to study the splitting effect of the collectivity which has become important for the Giant Quadrupole Resonance¹⁰⁾. Details are given in ref. 9.

In fig. 1 the formfactors for the M1 ($\Delta T = 1$) excitations in ^{12}C are shown as a function of the excitation energy for

Fig. 1: (a)-(c) The M1 transition form factor as a function of the excitation energy for various values of the momentum transfer q . The full and dashed lines correspond to the results without and with 2p2h admixtures, respectively. (d) The formfactors for the inelastic excitation of the M1 resonance calculated within the full configuration space.

various fixed values of the momentum transfer q . Those giant M1 resonances are localized between 35 and 45 MeV, characterized by a respectable formfactor, which - particularly for $q > 1 \text{ fm}^{-1}$ - clearly dominates the low lying 1^+ state at 15.1 MeV. The effect of the (2p2h) admixtures leads to a splitting of this collective M1 excitation in two parts: one between 30 and 35 MeV and the other between 40 and 45 MeV; this feature is clearly visible for $q > 1.4 \text{ f}^{-1}$.

In fig. 2 a similar calculation for the M1 states in ^{28}Si is shown. In fact the high lying M1 states between 15 and 20 MeV and between 25 and 35 MeV which have vanishing $B(M1)$ values exhibit magnetic formfactors with a pronounced collective behaviour. In ^{40}Ca we predict a number of collective M1-states between 25 and 40 MeV.

Fig. 2: The M1 transition formfactor as a function of the excitation energy for various fixed values of the momentum transfer in the nuclei ^{28}Si and ^{40}Ca , respectively.

All those calculations were performed using a conventional effective residual interaction¹¹⁾. The calculated distribution of the excitation strength depends sensitively on the strength constants of the spin- and spin-isospin-parts of this interaction. All exotic effects - such as the influence of mesonic corrections have so far been ignored.

III. Conclusions

In the conventional π -model collective Giant Magnetic Dipole Resonances are predicted which cannot be seen in photoabsorption, but which should show up in (e, e') . The properties of such excitations do reflect rather directly the spindependent components of the effective nucleon-nucleon interaction which is particularly interesting for the onset of pion condensation. It seems highly interesting to investigate the distribution of the magnetic excitation strength in order to find out whether collective magnetic eigenmodes do exist and if so what their properties look like. Since such excitations carry the quantum number of the pionic field and of the internal Δ -excitation of a nucleon those eigenmodes may also reflect the properties of the mesonic field inside of a nucleus.

References

- 1) W. Bertozzi, in Photonuclear Reactions II, Internal School on Electro and Photonuclear Reactions, ed. S. Costa and C. Schaerf, Lecture Notes in Physics 62
- 2) H. Zarek et al, Phys. Rev. Lett. 38 (1977) 750
- 3) T. Donnelly, J. D. Walecka, G. E. Walker, I. Sick, Phys. Lett. 32 B (1970) 545
- 4) T. Donnelly, in Proc. of Electron Scattering at Intermediate Energies, Saclay 1975
- 5) W. Knüpfer, M. G. Huber, B. Frois, Internal Report, Saclay (to be published)
- 6) E. Grecksch et al (to be published)
- 7) G. E. Brown et al (to be published)
- 8) J. Speth et al, Phys. Lett. 63 B (1976) 257
- 9) E. Grecksch et al, Phys. Rev. C 15 (1977) 837
- 10) W. Knüpfer et al, Z. f. Physik A 276 (1976) 99
- 11) W. Knüpfer, M. G. Huber, Phys. Rev. C 14 (1976) 2254

ON ELECTROMAGNETIC FORM FACTORS OF
ODD-EVEN DEFORMED NUCLEI^{†*}

E. Moya de Guerra

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

and

A.E.L. Dieperink
Kernfysisch Versnellar Instituut
Groningen, The Netherlands

* Contribution to the M.I.T. Workshop, June '77

[†] This work is supported in part through funds provided by
ERDA under Contract EY-76-C-02-3069.*000.

July 1977

CTP #656

I. INTRODUCTION

Electron scattering is a powerful tool for studying electromagnetic charge and current density distributions in nuclei. Experiments using high momentum transfer ($q \sim 2-3 \text{ fm}^{-1}$) and good energy resolution make it possible for the first time to investigate current density distributions in deformed nuclei. Such a study can be expected to give valuable information on the problem how deformed nuclei actually rotate. Up to now little is known about electromagnetic currents in deformed nuclei. For example, our main source of information on the core contribution to the convection current is the value of the gyromagnetic ratio g_R ; the analysis of M1 moments and transition rates indicates that its value is close to but somewhat less than $\frac{Z}{A}$, which corresponds to the value for a rigid body rotation. From electron scattering one expects to obtain additional information on, for example, the momentum transfer dependence of g_R , and also on higher multipole current distributions ($\lambda > 1$). (The aim of this note - that was motivated by electron scattering experiments, currently taking place at Bates, on deformed nuclei ^{25}Mg , ^{169}Er , ^{175}Lu and ^{181}Ta - is to discuss some aspects of transverse electron scattering on odd-even nuclei in terms of various models.

The simplest description of an odd-even nucleus would be in terms of a Hamiltonian that contains no coupling between rotational and intrinsic motion in which case the nuclear wave function factorizes¹; as a model for the intrinsic wave function ϕ_K the Nilsson

5 ref

model could be used. This model could be considered as a lowest order approximation to a systematic expansion of an angular momentum projected Hartree-Fock wave function, proposed by Villars².

The calculation of electromagnetic form factors in the projected HF approximation will be discussed in Section II; some numerical predictions for the simple Nilsson scheme are given in Section III. Problems related to a possible violation of current conservation are discussed in Section IV.

II. PROJECTED HARTREE-FOCK APPROXIMATION

At present, the best approximation to the intrinsic ground state of an axially deformed nucleus is obtained in the Hartree-Fock approach. Since this approach does not yield a solution that is an eigenstate of angular momentum the procedure adopted here consists of projecting eigenstates of the angular momentum operator from an intrinsic axially symmetric Hartree-Fock wave function ϕ_K .

For reflection-symmetric nuclei the projected Hartree-Fock wave function for $K \neq 0$ can be written as:

$$\psi_{MK}^{I\pi}(x) = \frac{1}{N_K^I} \int d\Omega \frac{\pi^{1/2}}{2} \left(\mathcal{D}_{KM}^{+I}(\Omega) R(\Omega) \phi_K^\pi(x) + (-1)^{I-K} \mathcal{D}_{-KM}^{+I}(\Omega) R(\Omega) \phi_{-K}^\pi(x) \right), \quad (2.1)$$

with $\pi = \pm 1$ being the parity of the intrinsic nuclear wave function and N_K^I a normalization factor. The conventions for $\mathcal{D}$ functions and rotation operators are as in reference [2]. This projected H-F wave function has the normal behavior under time-reversal:

$$T \psi_{MK}^{I\pi} = (-1)^{I-M} \psi_{-M K}^{I\pi}.$$

Using standard techniques² the reduced matrix element of any tensor operator $T_\mu^\lambda(x)$ between states of angular momentum I, I' in the same rotational band K can be written as:

$$\langle I' K \| T^{\lambda} \| I K \rangle = \frac{1}{N_K^I N_K^{I'}} \sum_{\nu} \langle I K - \nu \mid \nu \mid I' K \rangle \frac{1}{2 I'} \times$$

$$\int_0^{\pi/2} \sin \beta d\beta \left\{ d_{K-\nu, K}^I(\beta) \operatorname{Re} \langle \phi_K | T_{\nu}^{\lambda} e^{-i\beta J_y} | \phi_K \rangle + \right. \\ \left. + (-1)^{I-K} d_{K-\nu, -K}^I(\beta) \operatorname{Re} \langle \phi_K | T_{\nu}^{\lambda} e^{-i\beta J_y} | \phi_{-K} \rangle \right\}. \quad (2.2)$$

For strongly deformed nuclei the integral over β can be carried out leading to an expansion in powers of $\frac{1}{\langle J_1^2 \rangle}$. Using many-body techniques² one obtains

$$\langle \phi_K | T_{\nu}^{\lambda} e^{-i\beta J_y} | \phi_K \rangle = \\ = \langle \phi_K | T_{\nu}^{\lambda} e^{-i\beta J_y} | \phi_K \rangle_L \exp. (\langle \phi_K | e^{-i\beta J_y} | \phi_K \rangle_C) \simeq \\ \simeq e^{-\beta^2/\beta_0^2} \langle \phi_K | T_{\nu}^{\lambda} e^{-i\beta J_y} | \phi_K \rangle_L \quad (2.3)$$

with

$$\beta_0^2 = \frac{4}{\langle J_1^2 \rangle} = \frac{2}{\langle J_y^2 \rangle}, \quad (2.4)$$

$\langle J_1^2 \rangle \gtrsim 100$ for strongly deformed nuclei. By carrying out an expansion in powers of β of the linked matrix elements in eq. (2.3) and of the $d_{m, \nu}^I$ functions, equation (2.2) takes the form:

$$\langle I' K \| T^{\lambda} \| I K \rangle \simeq \hat{I} \left\{ \langle I K \mid \lambda 0 \mid I' K \rangle \langle \phi_K | T_0^{\lambda} | \phi_K \rangle_L \frac{1}{2} (1 + (-1)^{I+P}) + \right. \\ \left. + (-1)^{I-K} \langle I - K \mid \lambda 2K \mid I' K \rangle \operatorname{Re} \langle \phi_K | T_{2K}^{\lambda} | \phi_{-K} \rangle_L \frac{1}{2} (1 + (-1)^{2K+P}) + \right.$$

$$\begin{aligned}
& + \frac{1}{\langle J_1^2 \rangle} \left\{ \langle I \ K-1 \ \lambda 1 | I' K \rangle \sqrt{(I+K)(I-K+1)} R_e \langle \phi_K | T_1^1 J_- | \phi_K \rangle_L + \right. \\
& \quad + \langle I \ K+1 \ \lambda -1 | I' K \rangle \sqrt{(I-K)(I+K+1)} R_e \langle \phi_K | T_{-1}^1 J_+ | \phi_K \rangle_L - \\
& \quad \left. - \langle I \ K \ \lambda 0 | I' K \rangle R_e \langle \phi_K | T_0^1 2 J_y | \phi_K \rangle_L \right\} \quad (2.5)
\end{aligned}$$

where $p=0$ and 1 for Coulomb and transverse multipoles respectively (see for example reference [3]), and only the leading terms up to first order in $\frac{1}{\langle J_1^2 \rangle}$ have been kept.

The zeroth order terms in equation (2.5) reproduce the result that one gets in the factorization approximation of Bohr and Mottelsson¹. In the "pair filling" approximation in which the even-even core of the odd-even nucleus is time reversal invariant, only the last particle contributes to the zeroth order terms of transverse electric and magnetic transitions. The collective contributions to these transitions are given by the first order terms in eq. (2.5). Below we summarize the selection rules for longitudinal and transverse transitions in odd-even nuclei based on equation (2.5) and pair filling approximation. The Clebsch-Gordan coefficients give different weight to the different multipoles in any particular transition.

Coulomb multipoles: C_λ , $\lambda=\text{even}$, $|I'-I| \leq \lambda \leq I'+I$, any I' .

In zeroth order only the first term contributes, this contribution is a collective one: all protons contribute to the m.e. $\langle \phi_K | C_O^\lambda | \phi_K \rangle$. In particular for elastic scattering the C_0 transition will dominate, a well known experimental fact. The first order terms

give a small correction ($\sim \frac{1}{\langle J_1^2 \rangle}$) to the zeroth order term in this case.

Transverse magnetic: $M\lambda$, $\lambda=\text{odd}$, $|I'-I| \leq \lambda \leq I'+I$, any I' .

Both zeroth order terms contribute to M but only the odd nucleon contributes to the m.e.'s $\langle \phi_K | M_O^\lambda | \phi_K \rangle$ and $\langle \phi_K | M_{2K}^\lambda | \phi_{-K} \rangle$. These s.p. contributions will be studied in more detail in section III.

The two first terms in $\frac{1}{\langle J_1^2 \rangle}$ give the core contributions to magnetic transitions for any multipolarity $\lambda \geq 1$. The third term in $\frac{1}{\langle J_1^2 \rangle} - \langle \phi_K | M_O^\lambda J_Y^2 | \phi_K \rangle_L$ - gives a small correction to the zeroth order terms and can be neglected in first approximation.

For $M1$ moments and transitions the collective factor g_R in this approximation is given by:

$$g_R = \frac{\langle \phi_K | M'_x J_x + M'_y J_y | \phi_K \rangle}{\langle \phi_K | J_1^2 | \phi_K \rangle} \quad (2.6)$$

and therefore (in the limit $q \rightarrow 0$) $g_R \approx \frac{Z}{A}$.

Equation (2.6) represents an average of the self-consistent cranking formula⁴ for the collective g factor.

Transverse electric: $E\lambda$, $\lambda=\text{even}$, $|I'-I| \leq \lambda \leq I'+I$, $I' \neq I$.

There is no zeroth order contribution to $E(\lambda)$ multipoles except for the single particle contribution $\langle \phi_K | E_{2K}^\lambda | \phi_{-K} \rangle$ to inelastic transitions with $\lambda \geq 2K+1$.

For $\lambda < 2K+1$ the only contribution to $E\lambda$ multipoles comes from the first order terms in eq. (2.5) which are of collective

type. The collective contribution being given by:

$$E(\lambda)_{col.} = (-1)^{I+K} \frac{\hat{I} \hat{I}'}{\langle \hat{I}_\perp^2 \rangle} \operatorname{Re} \langle \phi_k | E_1^\dagger J_- | \phi_{+k} \rangle \times$$

$$\left\{ \sqrt{V(I+K)(I-K+1)} \begin{pmatrix} I & \lambda & I' \\ K-1 & 1 & -K \end{pmatrix} - \right.$$

$$\left. - \sqrt{V(I'+K)(I'-K+1)} \begin{pmatrix} I' & \lambda & I \\ K-1 & 1 & -K \end{pmatrix} \right\}$$

(2.7)

III. NILSSON APPROXIMATION

In order to obtain some idea about the behavior of transverse form factors for axially deformed nuclei it is instructive to consider the leading order term in eq. (2.5) in some detail, using a simple Nilsson model for the intrinsic state ϕ_K . Assuming the wave functions ψ_{MK}^I can be factorized as ($K \neq 0$)

$$\psi_{MK}^I = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \mathcal{D}_{KM}^{+I}(\Omega) \phi_K(x') + (-1)^{I-K} \mathcal{D}_{-KM}^{+I}(\Omega) \phi_{-K}(x') \right\} \quad (3.1)$$

The reduced matrix elements of intrinsic tensor operators T^λ that do not depend on the orientation Ω of the body-fixed frame, i.e., that transform as

$$T_{\mu}^{\lambda}(\text{lab.}) = \sum_{\nu} T_{\nu}^{\lambda}(\text{int.}) \mathcal{D}_{\nu\mu}^{+\lambda}, \quad (3.2)$$

can be written as (see eq. 2.5)

$$\begin{aligned} \langle I'K || T^{\lambda} || IK \rangle = \sqrt{2I+1} \frac{1}{2} \bigg\{ & \langle IK \lambda 0 | I'K \rangle \langle K | T_0^{\lambda} | K \rangle (1 + (-1)^{\lambda+p}) + \\ & + (-1)^{I-K} \langle I-K \lambda 2K | I'K \rangle \langle K | T_{2K}^{\lambda} | -K \rangle (1 + (-1)^{2K+p}) \bigg\} \end{aligned} \quad (3.3)$$

We note that the second term in (3.3) only contributes for $\lambda \geq 2K$. For odd-even nuclei with a high ground state spin $I=K$ this has the interesting consequence that the intermediate magnetic multipoles ($1 < \lambda < 2K$) are rather strongly suppressed compared to their intrinsic values, since the value of the Clebsch-Gordan Coefficient

$\langle II\lambda 0 | I'I \rangle$ becomes very small, whereas the highest multipoles ($\lambda \geq 2I$) which receive a contribution from the second term, are of the order of a single-particle contribution.

For simplicity we approximate the intrinsic state ϕ_K by a Nilsson single-particle state $[N n_z \Lambda \Sigma]$. For given deformation parameter this state can be expanded in terms of basis states $|N\lambda\Lambda\Sigma\rangle$ (expansion coefficients $a_{\lambda\Lambda}^K$), which in turn can be expressed in terms of spherical states $|lj\Omega\rangle$. In this way the intrinsic matrix elements are given by

$$\langle K' | T_{\mu=K'-K}^{\lambda} | K \rangle = \sum_{\substack{\ell\ell'\Lambda\Lambda' \\ j j' \Omega \Omega'}} a_{\ell\Lambda}^K a_{\ell'\Lambda'}^{K'} \langle \ell\Lambda \frac{1}{2} \Sigma | j K \rangle \times \quad (3.1)$$

$$\langle \ell' \Lambda' \frac{1}{2} \Sigma' | j' K' \rangle \frac{1}{\sqrt{2j'+1}} \langle j K \lambda \mu | j' K' \rangle \langle \ell' j' || T^{\lambda} || \ell j \rangle_{s.p.}$$

By substituting eq. (3.4) in eq. (3.3) one obtains the desired expression.

As an illustration we consider the matrix elements of the magnetic multipole operators. The simplest case occurs if the ground state with $I=K$ is described by a "stretched" s.p. wave function $[N n_z=0 \Lambda=N \Sigma=K-\Lambda]$ in which case the expansion coefficients $a_{\ell\Lambda}^K$ in eq. (3.4) are unity (neglecting $\Delta N=2$ mixing). This is the case for ^{25}Mg , with the Nilsson particle in the $d_{5/2}$ shell. Neglecting the core contribution the matrix elements of the magnetic multipole operator in the rotational model are given by

$$\langle I' K=5/2 \| T^\lambda(q) \| I=5/2 K=5/2 \rangle = \langle d_{5/2} \| T^\lambda(q) \| d_{5/2} \rangle_{s.p.}$$

$$\times \left(\langle \frac{5}{2} \frac{5}{2} \lambda 0 | I' \frac{5}{2} \rangle \langle \frac{5}{2} \frac{5}{2} \lambda 0 | \frac{5}{2} \frac{5}{2} \rangle + \langle \frac{5}{2} \frac{-5}{2} \lambda 5 | I' \frac{5}{2} \rangle \langle \frac{5}{2} \frac{-5}{2} \lambda 5 | \frac{5}{2} \frac{5}{2} \rangle \right)$$

For the elastic scattering the rotational model thus predicts a q -independent quenching of the $\lambda=5,3$ magnetic multipoles compared to the independent particle model by factors 0.55 and 0.12, these numbers are in qualitative agreement with both results of shell model calculations and a recent Bates experiment⁵. As for M1 multipole, the present model cannot be expected to give reliable results, since it is well known that the magnetic dipole operator receives a non-negligible collective core contribution (see eq. 2.6). Similar predictions can be made for the inelastic transitions within the $K=\frac{5}{2}$ band. Experimentally it could be of interest to look for a possible M7 contribution in the $\frac{5}{2}^+ \rightarrow \frac{9}{2}^+$ transition. The observation of such a transition (forbidden in the present mode) could be an indication of $\Delta N=2$ mixing in the Nilsson model.

As a second example we consider ^{181}Ta , in the Nilsson approximation described by a $[404\downarrow]$ proton intrinsic state^(*). One easily sees that the matrix elements of the magnetic multipole operators can be expressed as a linear combination of the matrix elements in the spherical basis $\langle q_{7/2} \| T^\lambda \| q_{7/2} \rangle$, $\langle q_{7/2} \| T^\lambda \| q_{7/2} \rangle$, $\langle q_{7/2} \| T^\lambda \| q_{7/2} \rangle$. For convenience the radial wave functions are approximated by harmonic oscillator

(*) For $\delta = .3$, $[404\downarrow] = .976 |444 \frac{1}{2}\rangle - .218 |443 \frac{1}{2}\rangle$

wave functions. The calculated square of the elastic transverse form factor is shown in Fig. 1 together with the prediction of the extreme s.p. model. One sees that compared to the independent particle model the strong coupling model predicts an appreciable depression of the cross section for intermediate q -values due to the large reduction of the M3 and M5 multipoles. (Experimentally this effect enables one to study the M7 multipole distribution over a larger q region than is possible in the spherical nuclei). It would be interesting to compare this result with both the predictions using projected Hartree-Fock techniques (section II) and the results from the Bates experiments, currently being analyzed. In reality the dip could be less pronounced due to effects of Coulomb distortions and possibly a slower fall of the M1-contribution. In the present case with (large) $I=K$ the core contribution $\sim g_R^2 (I - \frac{K^2}{I+1})$ is rather unimportant.

In addition to the elastic scattering inelastic excitations of the $K=\frac{7}{2}$ band in ^{181}Ta have been observed up to $I^\pi = \frac{21}{2}^+$. As an example we have calculated the transverse form factor for the $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ transition. In the Nilsson model one predicts in this case nonvanishing M3, M5, M7, E8 and M9 transitions. By inserting the proper Clebsch-Gordan coefficients one finds that the M5 multipole is highly retarded, whereas the M3 and M7 multipoles are quenched by about a factor 4 compared to the single particle values. The E8 and M9 transitions arise from $\ell=4$ $j=9/2$

components in the Nilsson wave function. However, these components are rather small ($\sim 1\%$), and moreover give rise to a q -dependence very similar to that of the dominant $\langle g_{7/2} || M^7 || g_{7/2} \rangle$ matrix element. Therefore they appear to only slightly modify the high q side of the transverse form factor (see Fig 2). The absence of a $M1$ matrix element and the suppression of the $M5$ multipole lead - on the low q side - to a rather characteristic shape of the cross section.

It is obvious that a systematic investigation (within the ground state rotational band) of several inelastic transverse form factors, which can be expressed as linear combinations, of a limited number of multipoles with different weighting for each transition, would be a powerful tool for mapping out the nuclear current distribution.

VI. DISCUSSION AND SUMMARY

The advantage of the projected Hartree-Fock approximation is that it allows to compute charge and current densities in the same framework. The collective contributions to the current densities appear in a straightforward way. To the extent that these collective contributions are described in a good approximation it also gives current conservation to this very same approximation. How good the approximation is can only be judged by numerical computation (now under way).

Its main limitations come from the fact that the projected H-F states do not minimize the energy. To obtain a more rigorous determination of the collective contribution to the current density and to satisfy in a better approximation the continuity equation the variation after projection Hartree-Fock approximation should be used.

We now discuss briefly the problem of current conservation. Since the projected Hartree-Fock states are not eigenstates of the Hamiltonian, H , the equation of continuity,

$$\begin{aligned} \vec{\nabla} \langle \psi_f | \vec{j} | \psi_i \rangle &\equiv -i \langle \psi_f | [H, \rho] | \psi_i \rangle = \\ &= -i (E_f - E_i) \langle \psi_f | \rho | \psi_i \rangle \end{aligned} \quad (4.1)$$

is not exactly satisfied in this approximation. Let us consider for simplicity a transition in an even-even nucleus from the ground state 0^+ to an excited state I^+ ($I=\text{even}$).

The left-hand side of equation (4.1) gives to lowest order:

$$\begin{aligned} \langle \Psi_{M,0}^I | [H, C_M^I] | \Psi_{0,0}^0 \rangle &= \frac{1}{N_0 N_I} \frac{1}{\hat{I}} \int_0^{\pi/2} \sin \beta d\beta \langle \phi_0 | [H, C_0^I] e^{-i\beta J_y} | \phi_0 \rangle \\ &\approx \frac{2 I(I+1)}{\langle J_z^2 \rangle^2} \langle \phi_0 | C_0^I H J_y^2 | \phi_0 \rangle_L. \end{aligned} \quad (4.2)$$

The right-hand side of equation (4.1) gives to lowest order:

$$(E_I - E_0) \langle \Psi_{M,0}^I | C_M^I | \Psi_{0,0}^0 \rangle \approx \frac{I(I+1)}{2 \mathcal{Y}} \langle \phi_0 | C_0^I | \phi_0 \rangle \quad (4.3)$$

the moment of inertia $\mathcal{Y}$ is in this formalism given by²:

$$\frac{1}{2 \mathcal{Y}} = \frac{2 \langle \phi_0 | H J_y^2 | \phi_0 \rangle_L}{\langle J_z^2 \rangle^2} \quad (4.4)$$

Therefore the difference between the left and right hand sides of equation (4.1) is proportional to

$$\begin{aligned} &= \langle \phi_0 | C_0^I H J_y^2 | \phi_0 \rangle_L - \langle \phi_0 | C_0^I | \phi_0 \rangle \langle \phi_0 | H J_y^2 | \phi_0 \rangle_L = \\ &= \sum_{\mu, \sigma} \langle \mu | C_0^I | \sigma \rangle \langle \sigma | H J_y^2 | \mu \rangle \end{aligned} \quad (4.5)$$

with μ and σ representing occupied and unoccupied states, respectively. Hence the continuity equation is only satisfied to the extent that this difference may be small as compared to

$$\langle \phi_0 | C_0^I | \phi_0 \rangle \langle \phi_0 | H J_y^2 | \phi_0 \rangle_L.$$

One of the interesting aspects of the projected H.F. approach is that it predicts a collective core contribution to the convection current, without the complications imposed by the factorization approach (3.1) of having to carry a transformation into rotational and intrinsic coordinates as well as to take into account the band mixing due to the Coriolis coupling^{1,4}.

It is an interesting question whether the simple arguments-based upon a rigid rotator model - that lead to the expression $g_R = \frac{Z}{A}$ - can be generalized to finite q , and higher multipole densities. On the basis of a rigid rotator model the collective contributions to the magnetic multipole operators $M_\mu^\lambda(q)$ would be of the form

$$M_\mu^\lambda(q) \sim \vec{\omega} \cdot \int d\vec{r} \left(r^2 - \frac{(\vec{r} \cdot \vec{\omega})^2}{\omega^2} \right) \rho(\vec{r}) \vec{\nabla} j_\lambda(qr) Y_\mu^\lambda(\hat{r})$$

where $\rho(\vec{r})$ represents the intrinsic charge density distribution of the nucleus and $\vec{\omega}$ the angular velocity.

Qualitatively one expects from this model a core contribution to $M^\lambda(q \rightarrow 0)$ proportional to $\int \rho(\vec{r}) r^{\lambda+1} Y_{\lambda-1}^\lambda(\hat{r}) d\vec{r}$, i.e. proportional to the charge multipole distribution of order $2^{\lambda-1}$. As a consequence, since more core nucleons contribute than in the extreme s.p. model, the collective transition density would be located more to the inside of the nucleus. This behavior would be in agreement with the observation that magnetic scattering experiments performed at 180° and at low q yield in many cases a range parameter smaller than the one corresponding to the extreme single-particle model.

Finally it is interesting to see what one would obtain for the behavior of magnetic multipoles in a weak coupling scheme.

Assuming for simplicity a ground state ($I=j$) wave function

$$\psi(I) = \alpha_0 \phi_0([R=0, j]^I) + \sqrt{1-\alpha_0^2} \phi_2([R=2, j]^I)$$

one obtains for the matrix elements of the higher magnetic multipoles ($\lambda \geq 5$)

$$\langle I || T^\lambda || I \rangle = (\alpha_0^2 + (1-\alpha_0^2) \left\{ \begin{matrix} I & I & 2 \\ I & I & \lambda \end{matrix} \right\} (2I+1)) \langle j || T^\lambda || j \rangle_{s.p.}$$

For the highest multipole $\lambda=2I$, $(2I+1) \left\{ \begin{matrix} I & I & 2 \\ I & I & 2I \end{matrix} \right\} \sim \frac{1}{2}$, for $I \geq \frac{7}{2}$,

leading to a small quenching by $\frac{1+\alpha_0^2}{2}$ compared to the single particle value. On the other hand for the next higher multipole

($\lambda=2I-2$) one has $(2I+1) \left\{ \begin{matrix} I & I & 2 \\ I & I & 2I-2 \end{matrix} \right\} \sim -\frac{1}{2}$, leading to a quenching which is more substantial than for $\lambda=2I$. This suggests that the weak and strong coupling schemes might in practice lead to rather similar total form factors.

REFERENCES

1. A. Bohr and B. R. Mottelsson "Nuclear Structure" Vol. II, W. A. Benjamin Inc., 1975.
2. F. Villars, Rendiconti della Scuola Internazionale di Fisica "E. Fermi" - XXXVI corso, p. 13. F. Villars and N. Schmeing-Rogerson, Ann. Phys. 63 (1971) 443.
3. T. De Forest, and J. D. Walecka, Adv. in Phys., 15, (1966)1.
4. F. Villars and G. Cooper, Ann. Phys. 56 (1970) 224.
5. G. Peterson et al., Phys. Rev., to be published.

FIGURE CAPTIONS

Figure (1): Elastic transverse form factor squared for ^{181}Ta calculated for a [404+] Nilsson orbit (full line) and the $g_{7/2}$ single proton orbit (dotted line). The dashed curves represent the various single-proton multipole contributions. In PWBA.

Figure (2): Transverse form factor squared calculated in the Nilsson model for the $|\frac{7}{2} \frac{7}{2}\rangle \rightarrow |\frac{11}{2} \frac{7}{2}\rangle$ transition in ^{181}Ta . The dashed curve represents the sum of the M3, M5, M7 multipoles, the full curve includes the contributions of the E8 and Mg multipoles also. In PWBA.

FIGURE 1

FIGURE 2

21 ref

WHAT CAN ONE LEARN WITH POLARIZED TARGETS IN (e, e') ?

BATES 1977 SUMMER WORKSHOP REPORT BY G. RAWITSCHER
PHYSICS DEPARTMENT, UNIVERSITY OF CONNECTICUT
STORRS, CONNECTICUT 06268

In short, the answer is that to some extent one can separate from each other the contributions from the various electromagnetic multipoles which lead from the initial to the final state of the nucleus. The question of the precision and uniqueness with which the separation can be effected is a quantitative one, the answer to which will not be attempted here. The purpose of the present discussion is to review the general properties of the formulae which have been derived in the literature. The study of parity non-conserving terms by means of longitudinally polarized electrons is discussed by Sauder in a different workshop¹ and will not be included.

The most used reference for the present discussion is Uberall's book² which gives in Section 4.4 the general first order Born Approximation result for polarized electrons scattering on polarized targets. Polarized target nucleons have been used in the study of photo-pion production from nucleons by Althoff et al³ at Bonn. The scattering of polarized nucleons from nuclear targets is being studied by several laboratories including Saclay and Berkeley, a good review article being that by Glashauser.⁴ The interaction of polarized deuterons with nuclei has also been explored, particularly by Haeberli at Wisconsin who has given an excellent discussion of the meaning of vector and tensor polarization.⁵ Ron Johnson at Surrey, England has also studied extensively those properties of the deuteron-nucleus force which can be

learned from tensor polarizations.⁵ Several theoretical studies of electron scattering from tensor polarized deuterons exist in the literature.⁶ A recent review of the monopole and quadrupole deuteron form factors plus corrections due to charge exchange effects is given by Haftel and Kloeet.⁷ Saylor⁸ has a proposal for measuring the tensor polarization of recoiling deuterons in elastic electron scattering. Polarization effects involving photonuclear reactions $(\gamma, \vec{n})$ or $(\vec{n}, \gamma)$ have been reviewed by Glarish recently.⁹

The discussion becomes particularly easy if one uses the spherical tensor decomposition of the density matrix which characterizes the target polarization, as is described in Appendix A. The rank of each polarization tensor is K , where K is not larger than twice the spin of the nucleus' ground state. When $K = 0, 1, 2$, etc., one has an unpolarized, vector polarized tensor polarized etc., component for the target. (For example, the spin $1/2$ nucleus can have at most a vector polarization.) In the literature the odd K polarizations are called "polarizations" while the even K polarizations are called "alignments". We will generally use the word polarization for either case.

For a polarized target the cross section not only depends on θ , the angle between the initial and final electron momenta k_1 and k_2 , but also on the angles γ and ϕ which describe the direction of the target's polarization vector relative to the scattering plane. These functions of γ and ϕ are spherical harmonics $Y_{KQ}(\gamma, \phi)$ and the corresponding coefficients $T_{KQ}(\theta)$ contain interference terms between the various transition matrix elements $C\lambda$, $E\lambda'$ and $M\lambda''$ which connect the initial to the final nuclear state. Here λ is the multipolarity of the

transition, C stands for the nuclear transition amplitudes which are due to the Coulomb interaction and E and M are the transverse electric and transverse magnetic nuclear transition amplitudes. These quantities are real but not necessarily positive.

In first order Born approximation, if the incident electrons are unpolarized, one finds that K has to be even.² Thus, for example, if the target is a deuteron, then the polarization must contain a tensor component if one is to see a deviation of the cross section from the unpolarized target case. In the elastic electron deuteron scattering case, the deviation from the unpolarized cross sections will contain interference-between C_0-C_2 , C_2-C_2 , M_1-M_1 and M_1-C_2 . In the inelastic case the $E\lambda$ amplitudes also enter. If first order Born approximation is replaced by distorted wave Born approximation, where the distortion is caused by the monopole part C_0 of the Coulomb interaction, then the odd values of K also enter. The respective coefficients contain the imaginary parts of the products of two scattering amplitudes (which in first order plane Born approximation would only have real parts). Such terms might lend themselves for the study of dispersion corrections, and might be more suitable for this purpose than the comparison of electron scattering with positron scattering. All of the above points will be amplified in the discussion which follows.

There is furthermore the general requirement valid in first order (either plane or distorted wave) Born approximation that the multipolarities λ and λ' of the two interfering amplitudes, together with the rank K of the target polarization tensor, must form a triangle. Thus, from this rule a vector polarized target cannot produce inter-

ference between C0 and C2, but C2 and M1 could interfere. This triangular condition is demonstrated in detail in Appendix B, which together with Appendix C illustrates the elements of a distorted wave Born calculation.* The main argument hinges on the fact that if

$|j_0 m_0\rangle$ is the initial state of the target nucleus and $|jm\rangle$ is the final state, where j represents the spin and m the projection along a Z axis, then the corresponding scattering amplitude $f_{jmm_0}^{(\lambda)}(\theta, \phi)$ caused by the λ -multipole part of the electron nucleus interaction is proportional to the Clebsch-Gordan coefficient $\langle jm | j_0 \lambda m_0 \rangle$. The scattering amplitude starting from a different initial state $|j_0 m'_0\rangle$ to the same final state $|jm\rangle$ contains an analogous factor $\langle jm | j_0 \lambda' m'_0 \rangle$, and the interference between these two amplitudes is modulated by the density matrix of states m_0 and m'_0 , which represents the polarization of the target and which itself is described by a Clebsch-Gordan coefficient $\langle j_0 m_0 j_0 -m'_0 | KQ \rangle$. Thus we arrive at a product of three C-G's, which when summed over m_0, m'_0 and m (the latter if the state of polarization of the final nucleus is not observed) gives rise to a 6-j symbol

$$\left\{ \begin{matrix} \lambda & \lambda' & K \\ j_0 & j_0 & j \end{matrix} \right\}$$

which implies the triangular relation $\Delta(\lambda \lambda' K)$ mentioned above.

Returning to first plane Born approximation, and assuming now

* Appendices B and C are available from the author upon request.

that the electrons are polarized, then that part of the elastic or inelastic cross section which depends on the direction of the initial electron polarization vector $\hat{\epsilon}_1$ will only be sensitive to target polarization with odd K. For example, if the target is unpolarized then the cross section does not depend on the polarization of the electron. This is at first surprising since one might expect that for a magnetic transition M should depend on the direction of polarization of the electron; even for an unpolarized target. Presumably this is the case but only the imaginary parts of the products of two scattering amplitudes enter, which vanish in 1st order plane wave Born approximation. If the target is vector polarized, in addition to the incident electron beam being polarized, then one gets interference terms between transverse electric and magnetic as well as transverse and Coulomb matrix elements but not between Coulomb-Coulomb elements. For the case of elastic electron deuteron scattering, since the transverse electric multipole transition amplitudes vanish, one only gets interference terms $M_1 \cdot C_0$ and $M_1 \cdot C_2$ which enter in the combination $M_1(\langle 1010 | 11 \rangle C_0 + \langle 1210 | 11 \rangle C_2)$. One does not get $C_0 \cdot C_2$ interferences. The latter only appear with a tensor polarized target, where they enter in the combination $C_2(\langle 2000 | 20 \rangle C_0 + \langle 2200 | 20 \rangle C_2)$.

It is interesting to note that to get an effect, it is not necessary for the direction of the polarization vector of the incident electrons to have a component perpendicular to the scattering plane. Hence if in the experiments on parity violations¹ polarized targets are present, they may give rise to undesirable background effects.

The above first order plane wave Born approximation results are

based on Sections 4.2-4.4 in Uberall's book,² where K is denoted by I , where C is denoted as M^{if} and where $E\lambda$ and $M\lambda$ are denoted as $\overline{C}_\lambda^{if}$, with $\sigma = 0$ and 1 , respectively. A complete first order Born approximation expression for the cross section for polarized electrons incident on polarized targets is presented in this treatise, according to a paper by Weigert and Rose¹⁰ corrected for misprints in various Clebsch-Gordan coefficients.

If the polarization $\vec{\zeta}$ of the electrons after the scattering is also observed there are three effects to be considered¹⁰ (all in 1st order plane wave Born approximation)

- a) The incident electron is polarized and the target is unpolarized. In this case one learns nothing more than what one already learns from conventional unpolarized electrons scattering on unpolarized targets.
- b) The incident electron is unpolarized. In this case the target has to be polarized with odd K polarization. One learns the same type of information as that from polarized electrons scattering on polarized targets (without observing the final polarization of the electron). One has interferences $E\lambda - E\lambda'$, $M\lambda - M\lambda'$, $M\lambda - E\lambda'$; and $C\lambda - E\lambda'$, $C\lambda - M\lambda'$, but not $C\lambda - C\lambda'$.
- c) The incident electron is polarized and the target is polarized. This case involves only even target polarizations. One gets interferences $C\lambda - C\lambda'$; $E\lambda - E\lambda'$, $M\lambda - M\lambda'$, $E\lambda - M\lambda'$; and $C\lambda - E\lambda'$, $C\lambda - M\lambda'$.

The only way one can obtain interferences between Coulomb multipoles $C\lambda - C\lambda'$ is to have a tensor (or K even) polarized target, regardless of the polarization of the incident or outgoing electron. Coming back to the case of the elastic electron deuteron scattering, if one cannot, for experimental reasons, have a tensor polarized target available, then a vector polarized target together with polarized incident electrons will still be very useful since one will get $C0-M1$ and $C2-M1$ interferences. The first will give one consistency checks on $C0$ and $M1$ and the latter will give information on $C2$.

Apart from the case of polarized holmium, no electron scattering experiments seem to exist using polarized nuclei. Theoretical calculations have been done however. One by Langworthy and Uberall¹¹ will be discussed in some detail. It illustrates the sensitivity of the elastic cross section on a polarized B^{10} target to the quadrupole

FIG. 1. Sensitivity of the charge form factor $F_{CH}(q)$ to variations in the quadrupole moment Q for 200 MeV electrons incident on polarized ^{10}B , with orientation type L described below from Langworthy and Uberall, Ref. 11, with permission, where it is Fig. 3.

moment. The target nucleus is in the state of maximum polarization $m_0=j=3$ (all other m 's are unpopulated) and the direction of polarization $\hat{N}$ is along the incident electron momentum. Figure 1 illustrates the sensitivity to the sign and magnitude of the quadrupole moment. The form factor represents the charge form factor, i.e., the part which does not contain the factor $(\frac{1}{2} + \tan^2 \theta/2)$. In calculations for this figure, only $C0$ and $C2$ amplitudes were included. If all the transition amplitudes allowed by parity considerations $C0, C2, C4, C6, M1, M3$ and $M5$ were included, then there would in principle be enough measurable quantities (1 for $K=0$ and 3 each for $K=2, 4$ and 6) to over-determine the 7 form factors. Langworthy and Uberall also discuss the inelastic transition from the ground state ($j=3^+$) to the 4^+ level at 6.02 MeV in ^{10}B . In this case ten transition matrix elements would be allowed in principle ($C2, C4, C6, E2, E4, E6, M1, M3, M5, M7$) and there would be just enough measurable quantities to give 10 equations bilinear in the 10 transition matrix elements. Since these equations are not linear, a unique determination of the 10 form factors at a single momentum transfer is in this case not possible in principle.

In their calculation¹¹ the authors include only $C2$ and $C4$ transition matrix elements. The inelastic charge transition form factor for this case is reproduced in Figure 2. R denotes the result for the random orientation of the nucleus, L is when the incident electron direction $\vec{k}_1$ is along the axis $\vec{N}$ of polarization of the target nucleus, and T_i , $i=1, \dots, 3$ corresponds to an angle of 90° between incident electron and target polarization, for 3 directions between the scattering plane and N .

FIG. 2. Form factor $F_{in}(q)$ for the $3^+ \rightarrow 4^+$ transition in ^{10}B at $k_1 = 200$ MeV: (a) random orientation with experimental data of Ref. 8, (b) orientations as described in the text. From Langworthy and Uberall, Ref. 11, where it is Fig. 6.

Another interesting calculation is that of Ravenhall and Mercer on oriented holmium.¹² These authors use both a distorted wave as well as a coupled channel formulation, but include only Coulomb transition matrix elements C0 and C2. Since experimentally the inelastic transitions from the ground state ($7/2^+$) to the two excited states $9/2$ and $11/2$ were not resolved from the elastic transition, these two inelastic cross sections were also calculated for the polarized target and then added to the elastic cross section. Unfortunately the authors do not show the sensitivity of the cross sections to the degree of polarization of the target or to the size of the assumed quadrupole deformations parameter. However, they do show the difference between aligned (a) and unaligned cross sections (u), for the inelastic transitions.

$$\Delta = (\sigma_a - \sigma_u) / \sigma_u$$

Here the target is polarized along the normal to the scattering plane (the Y case) with a mix of scalar, vector, tensor, etc., polarizations as given by experiment (35% for $K=0$, 41% for $K=1$, 24% for $K=2$, 9% for $K=3$, 2% for $K=4$). The result is shown in Figure 3. One sees that the

FIG. 3. Orientation effects for energy-resolved scattering of 200 MeV electrons by holmium nuclei oriented in the Y direction. The full curve is the result of the coupled-channel calculation, the dashed curve from the DWBA method. From Ravenhall and Mercer, Ref. 9, with permission where it is Fig. 12.

sign of Δ is different for the two inelastic transitions, indicating the feasibility of using target polarization in order to determine the quantum numbers of the excited state.

The higher order effects (coupled channel as compared to the DWBA) also show up more strongly in the inelastic transitions, as is seen by the comparison of the dashed and solid lines. This is very likely due to the fact that the most important correction to the first order inelastic transition is the reorientation correction

(used mainly in nonrelativistic Coulomb excitation experiments), which is only one order higher in the quadrupole deformation parameter β_2 than the DWBA amplitude. This is to be contrasted with the elastic case, where the first non-vanishing correction is of second order in the deformation parameter, and hence much smaller. These reorientation corrections for the inelastic scattering cross sections also occur for unoriented target nuclei. They can be understood as rescattering from an oriented excited state, the orientation of the latter being produced by the inelastic excitation itself. The reorientation amplitude is of second order in the inelastic transition matrix element for the case of an electron incident on a ground state unpolarized nucleus. This is of the same order as the two step processes which consist of a transition from the ground to one or several high lying excited states, such as a dipole giant excitation, and from there to the inelastic state in question. These latter processes could be of the same size as the reorientation ones, and have so far been included only once in the calculation of elastic as well as inelastic cross sections by Onley.¹³ If it could be shown that the two step processes, when summed over all inelastic states, cancel each other to a large extent then the reorientation term would predominate and the scattering cross section would be sensitive to the quadrupole moment form factor in the excited state. In this case the comparison between electron and positron inelastic scattering could help to isolate the reorientation effect. For electrons the reorientation amplitude would add coherently with the first order inelastic amplitude, and for positrons the same amplitudes would subtract coherently, both giving

interference terms of opposite sign. There actually is some evidence that the two step transitions between different excited states have a tendency to cancel each other.¹⁴ More theoretical work is desirable.

While on the topic of dispersion corrections, it is interesting to note that it is likely that they should make their presence felt more strongly in the scattering from polarized nuclei, than in the unpolarized case. The reason is that second order processes give rise to a correction to the scattering amplitude which is mainly imaginary, while the 1st order part is mainly real, with the exception of the interference minima, where the imaginary part of the distorted wave dominates over the real part. (The interference of the dispersion correction or second order part with the first order part in elastic scattering thus occurs mainly near the interference minima for unpolarized target nuclei.) But for polarized targets with a polarization of odd tensor rank the dependence of the cross section on the direction of the target polarization depends on the imaginary part of the product of the two scattering amplitudes for example the product of two C2 scattering amplitudes, or of M1 times C0. The second order dispersion amplitude when added to the C2 or M1 amplitudes in the above examples being mainly imaginary will thus interfere with the real first order amplitude of C2 or C0, respectively at all angles where the latter is real and large, for instance near the maxima of the scattering cross section. By contrast, it is expected that the ordinary Coulomb distortion (2^d and higher order effects in which the target remains in the ground state at all times) give only a small background correction to the cross section for unpolarized electrons scattering off vector

polarized targets. In first order plane wave Born approximation the correction vanishes entirely, and in the similar case in which the target is unpolarized and the electrons are polarized, the Coulomb distortion effects are found to be very small¹⁵ other than at far backward angles because of the helicity stiffness of the high energy electrons. It would be very interesting to examine theoretically the dispersion correction to the polarization function $S(\theta)$ for polarized electrons scattering off unpolarized targets. It could well be that virtual excitation of magnetic resonances (or other transitions involving transverse multipole operators) permit a change of the spin direction of the electrons, and hence affect $S(\theta)$ in a measurable way. $S(\theta)$ is the function which is the coefficient of the dot product of the normal to the scattering plane and the polarization of the incident electron.

Particularly for the case of inelastic scattering from unpolarized targets with unpolarized electrons the effect of dispersion corrections on the polarization of the final state could be non negligible. These are the so called sub-state population cross sections, which can be obtained by simultaneously detecting the de-excitation γ ray in coincidence with the inelastically scattered electron. These experiments could become feasible with 100% duty cycle accelerators. In this case the degree of population of the various m states of the final nucleus is not dependent on the helicity stiffness of the electron (it also occurs for spinless particles) and depends sensitively on the sign of the deformation of the final state,¹⁶ (the reorientation effect gets enhanced) as well as on the presence of imaginary terms in the transition potential (due to dispersion corrections).¹³

Measurements of the type described above, i.e., measurements of the angular distribution of the de-excitation radiation emitted by the excited target nucleus as it settles back to the ground state provide a powerful tool for the determination of the spin and parity of the excited state. This feature has been emphasized by O'Connell in a separate workshop report,¹⁷ (calculations for electron scattering already exist in the literature)¹⁸ and for the case of incident α particles as well as heavy ions the method has already been put to practical use. See for example Steadman et al¹⁹ for the heavy ion case.

In summary, scattering from polarized targets as well as angular correlations of the inelastically scattered electrons with the radiation emitted during the de-excitation process will help to determine the various transition matrix elements which enter in the electron-nucleus interaction in a model independent way. In particular the scattering of electrons from polarized deuterons at large momentum transfers (near and beyond the first zero of C_0) should be very useful for separating the C_2 from the C_0 amplitudes and thus will help to clarify the properties of the two nucleon system and the role played by the exchange currents.⁷ Scattering from polarized targets may also help to clarify the effect of dispersion corrections.

The author acknowledges very stimulating discussions at the workshop, in particular with J. Heisenberg who asked the initial questions and B. Bertozzi who helped to clarify the simplicity of the use of the momentum transfer $\vec{q}$ as the Z axis for polarization considerations.

References

1. P. Sauder, p. 26, Report on the Bates Midwinter Workshop, 1/3/77.
2. H. Uberall, "Electron Scattering from Complex Nuclei", Academic Press, N.Y. (1971).
3. K. H. Althoff et al., Nucl. Phys. B116, 253 (1976).
4. C. Glashausser and J. Thirion, Adv. Nucl. Phys. 2, 79 (1969) and C. Glashausser in Nuclear Spectroscopy and Reactions, Part B, ed. J. Cerny, p. 195 (Academic Press, 1974).
5. W. Haeberli in "Nuclear Spectroscopy and Reactions" Part A, ed. J. Cerny, p. 51 (Academic Press, N.Y., 1974); W. Haeberli in "Polarization Phenomena in Nuclear Reactions", Ed. H. H. Barschall and W. Haeberli (University of Wisconsin Press, (1971), p. 235; R. C. Johnson, *ibid*, p. 143.
6. M. J. Moravcsik and P. Gosh, Phys. Rev. Lett. 32, 321 (1974); T. J. Brady, E. L. Tomusiak and J. S. Levinger, Can. J. Phys. 52, 1322 (1974); F. Coester and A. Ostebee, Phys. Rev. C11, 1836 (1975).
7. M. I. Haftel and W. M. Kloet, Phys. Rev. C15, 404 (1977); E. L. Lomon, E. Hadjimichael and W. Turchinets, report in the present summer study, and E. L. Lomon, to be published.
8. D. Saylor, p. 16, Report on the Bates Midwinter Workshop, 1/3/77.
9. H. F. Glarish "Polarization effects in the Giant dipole resonance", in Proc. Intl. Conf. on Photonuclear reactions and applications, ed. by B. L. Berman, Asilomar (1973), p. 255.
10. L. J. Weigert and M. E. Rose, Nucl. Phys., 51, 529 (1964).
11. J. Langworthy and H. Uberall, Phys. Rev. C2, 911 (1970).

12. Ravenhall and Mercer, Phys. Rev. C13, 2324 (1976).
13. D. S. Onley, Nucl. Phys. A118, 436 (1968).
14. J. L. Friar and M. Rosen, Ann. Phys., 87, 289 (1975).
15. P. Uginicius, H. Uberall and G. H. Rawitscher, Nucl. Phys. A158, 418 (1970).
16. P. K. Glanz and G. H. Rawitscher, Nucl. Phys. A217, 299 (1973);
W. Eyrich, A. Hofman, U. Schreib, S. Schneider, F. Vogler and
H. Rebel, Phys. Lett. 63B, 406 (1976) and to be published.
17. J. O'Connell, report to the present workshop.

18. D. Drechsel and H. Uberall, Phys. Rev. 181, 1383 (1969).
19. S. G. Steadman et al, Phys. Rev. Lett. 33, 498 (1974).
20. U. Fano (1951) National Bureau of Standards, Report #214, Phys.
Rev. 90, 577 (1953), Rev. Mod. Phys. 29, 76 (1957); U. Fano and
G. Racah, "Irreducible Tensorial Sets" (Academic Press Inc.,
N. Y., 1959).
21. D. M. Brink and G. R. Satchler, "Angular Momentum" (Clarendon
Press, Oxford, 1968), 2nd ed., Section 6.4.1).

APPENDIX A

The purpose of this Appendix is to give a simple review of the use of the density matrix in the description of polarization effects, and also to remind the reader of the significance of the spherical tensor decomposition of polarization phenomena.

The state of polarization is described by a bilinear (or quadratic) function of the amplitude components of the wave function. If a nuclear state has total angular momentum j and projection m on a Z axis, then the amplitude components are d_m , and the state $|\psi\rangle$ is described by the superposition

$$|\psi\rangle = \sum_m d_m |jm\rangle \quad (\text{A.1})$$

If, for a given direction of the Z axis, the target nucleus is initially in a well-defined initial state $|jm_0\rangle$ then for a rotated Z axis the initial state will again appear as a coherent superposition of m states. Hence (A.1) is the general description of the initial state. Now, for a spinless projectile incident on a given nuclear state $|jm_0\rangle$, there may be a spin flip during the scattering process and for each projection m there can be an outgoing scattering amplitude $f_{mm_0}(\theta, \phi)$

$$|\psi\rangle \rightarrow e^{ikz} |jm_0\rangle + \frac{e^{ikr}}{r} \sum_m f_{mm_0} |jm\rangle, \quad (\text{A.2})$$

but for the general case given by Eq. (1), one has

$$| \psi \rangle \rightarrow \left(\sum_{m_0} d_{m_0} | j m_0 \rangle \right) e^{i k z} + \left(e^{i k r} / r \right) \sum_m \left(\sum_{m_0} d_{m_0} f_{m m_0} \right) | j m \rangle \quad (\text{A.3})$$

The cross section, being an observable, contains the absolute value squared of the coefficient of $e^{i k r} / r$, and since all the $| j m \rangle$'s are orthogonal to each other, one is left with the sum

$$\sum_{m m_0 m'_0} d_{m_0}^* d_{m'_0} f_{m m_0}^* f_{m m'_0} \quad (\text{A.4})$$

The above is the answer for a "pure" state.

For a mixture of many states one replaces the averaged value of $d_{m_0}^* d_{m'_0}$ by the density matrix ρ

$$d_{m_0}^* d_{m'_0} \rightarrow \rho_{m'_0 m_0}.$$

The density matrix has $(2j+1) \times (2j+1)$ components, and for a pure state it can be thought of as a matrix which is the direct product of the vectors

$$(d_{m'_0}) \otimes \overline{(d_{m_0}^*)}$$

The usefulness of decomposing this matrix into a sum of spherical tensors of rank K , with $2K+1$ elements each, stems from various reasons. One is the fact that, because of the Wigner Eckart theorem, each $f_{m m_0}$ is also proportional to a spherical tensor $(f_{m m_0} \propto \langle j' m' | j \lambda m_0 \nu \rangle)$. Hence the sum over the m 's in Eq. (A.4) can be done by taking advantage of the vector addition properties of the

spherical tensors. Another advantage stems from the ease with which the density matrix can be expressed in terms of a new rotated coordinate system, if it is expressed in spherical tensorial form. This decomposition has been introduced by Fano²⁰ and extensive discussions exist in the literature. Nevertheless, a few simple remarks are made below, based on the book by Brink and Satchler.²¹ Following the notation of Ref. 21, $\rho_{\mu\mu'}$ above is expressed as $\rho_{jm, j'm'}$. The fact that the amplitudes d_{μ} in Eq. (1) refer to a well-defined angular momentum state is thus clearly brought out. Under rotation of the coordinate system, since each $|j\mu\rangle$ transforms according to a linear combination of rotation functions $D_{\mu M}(\theta)$ of the Euler angles $\theta \equiv (\alpha\beta\gamma)$ of the rotation, one has for ρ' in the new coordinate system

$$\rho'_{jm, j'm'} = \sum_{\mu\mu'} D_{\mu m}^j(\theta)^* \rho_{jm, j'\mu'} D_{\mu' m'}^j(\theta) \quad (\text{A.5})$$

Now, using $D_{\mu m}^j(\theta)^* = (-)^{j-m} D_{- \mu -m}^j(\theta)$ together with the property

$$D_{aa'}^A(\theta) D_{bb'}^B(\theta) = \sum_C (2C+1) \begin{pmatrix} A & B & C \\ a & b & c \end{pmatrix} \begin{pmatrix} A & B & C \\ a' & b' & c' \end{pmatrix} D_{\delta\delta'}^C(\theta)^*$$

one obtains

$$\begin{aligned} \rho'_{jm, j'm'} &= \sum_K \sum_{\mu\mu'} (2K+1) \begin{pmatrix} j & j' & K \\ -\mu & \mu' & -k \end{pmatrix} \rho_{jm, j'\mu'} (-)^{\mu} \\ &\times (-)^m \begin{pmatrix} j & j' & K \\ -m & m' & -k \end{pmatrix} D_{-k -k'}^K(\theta)^* \end{aligned} \quad (\text{A.6})$$

The sum over μ and μ' in the first line of Eq. (A.6) can be recognized as the projection of ρ on a tensor of rank K . This suggests the following definition of the spherical density matrix com-

ponents $\rho_{Kk}(jj')$:

$$\begin{aligned} \rho_{Kk}(jj') &= \sum_{\mu\mu'} \rho_{j\mu, j'\mu'} (-)^{K-j'-\mu} \langle jj'-\mu\mu' | Kk \rangle \\ &= \sum_{\mu\mu'} \rho_{j\mu, j'\mu'} (-)^{K+j+\mu+k} (2K+1)^{1/2} \begin{pmatrix} j & j' & K \\ -\mu & \mu' & -k \end{pmatrix} \quad (A.7) \end{aligned}$$

which can be inverted as

$$\begin{aligned} \rho_{jm, j'm'} &= \sum_{Kk} \rho_{Kk}(jj') \langle jj'-m m' | Kk \rangle (-)^{K-j-m} \\ &= \sum_{Kk} \rho_{Kk}(jj') (-)^{K-j'-k-m} \hat{K} \begin{pmatrix} j & j' & K \\ -m & m' & -k \end{pmatrix} \quad (A.8) \end{aligned}$$

where $\hat{K}$ means $(2K+1)^{1/2}$. As a result of (A.7) or (A.8), (A.5)

becomes identical to Eq. (A.8) transcribed for the primed quantities

$\rho'_{jm, j'm'}$ and $\rho'_{Kk}(jj')$ with

$$\rho'_{Kk}(jj') = \sum_k \rho_{Kk} D_{kk'}^K(\theta) \quad (A.9)$$

In other words, the bilinear quantities $\rho_{mm'}$ can be decomposed into sums of tensor quantities ρ_{Kk} according to Eq. (8), each of which is indeed a spherical tensor since under rotation it transforms by means of a D function in the usual way, Eq. (A.9). A general normalization property of the ρ 's is that

$$\sum_{Kq} [\rho_{Kq}(jj')]^2 = \sum_{mm'} [\rho_{jm, j'm'}]^2 \quad (A.10)$$

Examples of Eqs. (7) and (8) will now be given.

1. The system is in a pure state $|jm\rangle$ with maximum polarization, i.e., d_m in Eq. (1) is δ_{jm} . Then

$$S_{mm'} = d_m d_{m'}^* = \delta_{mm'} \delta_{mj}$$

and, according to Eq. (7)

$$S_{Kk} = (-)^{K-2j} \langle jj-jj | K0 \rangle \delta_{k0} \quad (\text{A.11})$$

This means that tensor polarization of all ranks $0 \leq K \leq 2j$ are simultaneously present, each with a weight given by Eq. (A.11). For example for $K=0,1,2,\dots$, the corresponding values of ρ_{K0} are $(2j+1)^{-1/2}$, $[3j/(j+1)(2j+1)]^{1/2}$, $[5j(2j-1)/(j+1)(2j+3)(2j+1)]^{1/2}, \dots$. This example was used for the case of electron scattering on B^{10} , discussed in the text. For deuterons one has $j=1$ and the three values of ρ_K are $1/\sqrt{3}$, $1/\sqrt{2}$ and $1/\sqrt{6}$. The sum of the squares of these three numbers is equal to unity, consistent with Eq. (A.10).

2. The polarization of the target nucleus has cylindrical symmetry. This is the usual case where the polarization is along some well-defined magnetic or electric field, all directions $\perp$ to this field being equivalent. In this case, choosing the Z axis along the polarizing field one has

$$S_{mm'} = P_m \delta_{mm'} \quad (\text{A.12})$$

where the P_m 's are the (positive definite) population probabilities.

As a result of the diagonal nature of the density matrix the projection

$k \neq 0$ of each spherical density of rank K is zero, i.e.,

$$S_{K0} = \sum_m P_m (-)^{K-j-m} \langle jj -m m | K0 \rangle \quad (A.13)$$

It has been assumed in all the examples that $j=j'$, where j represents the spin of the target nucleus.

If all the P_m 's are equal to each other, i.e., $P_m = P$, one finds that $\rho_{K0} = \delta_{K0} \hat{j} P$, i.e., only the $K=0$ tensor survives, and the sample is unpolarized. Here $\hat{j} = (2j+1)^{1/2}$.

If the state is aligned, i.e., $P_m = P_{-m}$ then the properties of the C-G coefficients imply that $\rho_{K0} = (-)^K \rho_{K0}$, i.e., all the odd K polarizations vanish.

For a general distribution P_m , $m=-j$ to $+j$, one determines the ρ_K 's from Eq. (A.13). For example, for the case of a deuteron, $j=1$, the P_m 's and the ρ_K 's obey the relations

$$\begin{pmatrix} P_1 \\ P_0 \\ P_{-1} \end{pmatrix} = \frac{S_{00}}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{S_{10}}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \frac{S_{20}}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (A.14)$$

The general expression for ρ_{mm}^K , which is due to a K polarization along any direction whose polar angles relative to the frame of reference are θ_p and ϕ_p is given by

$$S_{mm}^K = S_{K0} \langle K\gamma | jj' -m m' \rangle (-)^{K-j-m} \times \left[4\pi / (2K+1) \right]^{1/2} Y_{K\gamma}(\theta_p, \phi_p) \quad (A.15)$$

This result follows by inserting $\rho_{Kk} \delta_{k0}$ into Eq. (A.9) and (A.8) and remembering that the Euler angles Θ in that expression rotate the axis of polarization into the frame of reference axes. Then using

$$D_{Om}^j(\alpha, \beta, \gamma) = D_{m'0}^{j*}(-\gamma, -\beta, -\alpha) = (4\pi/2K+1)^{1/2} Y_{K\gamma}(-\beta, -\alpha) = (4\pi/2K+1)^{1/2} Y_{Kp}(\Theta, \phi)$$

p p

one obtains the above result.