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Verification of the SLC Wake Potentials*

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ABSTRACT

The accurate knowledge of the monopole, dipole and quadrupole wake potentials is essential for SLC. These wake potentials were previously computed by the modal method.¹⁻² The time domain code TBCI³⁻⁴ allows independent verification of these results. This comparison shows that the two methods agree to within 10% for bunch lengths down to 1 mm. TBCI results also indicate that rounding the irises gives at least a 10% reduction in the wake potentials.

1 INTRODUCTION

For a bunch of charged particles offset a distance a from and moving at the speed of light c parallel to the axis of a cylindrically symmetric structure, the source current can be represented (in cylindrical coordinates) as

$$J = \frac{I(z/c - t)\delta(r - a)}{\pi a} \sum_{m=0}^{\infty} \frac{\cos m\varphi}{(1 + \delta_{m,0})} \hat{z}, \quad (1)$$

where $I(t)$ is the longitudinal charge distribution times c and m is the azimuthal mode number. Each term of the sum can be thought of as a charged ring of radius a whose charge varies as $\cos m\varphi$ in the azimuthal direction and $I(z/c - t)$ in the longitudinal direction. Both the modal method and the time domain code TBCI give the wake potential due to any of these rings. For SLC it is sufficient to consider only the wake potentials due to the $m = 0, 1, 2$ rings.¹

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The modal method of Refs. (1) and (2) gave the delta function wake potentials $\bar{W}(t)$, i.e. the voltage gain at time t behind a delta function ring. (Let $I(z/c - t) = \delta(z/c - t)$ in Eqn. (1).) TBCI on the other hand solves for the wake potentials due to smooth distributions of charged particles, which we denote here as the integrated wake potentials $\tilde{W}(t)$. Thus here we will compare only $\tilde{W}(t)$ as given by the two methods, taking $I(t)$ to be Gaussian. Note that given the delta function wake potential, the integrated wake potential is given by

$$\tilde{W}(t) = \int_{-\infty}^t \bar{W}(t - \tau) I(\tau) d\tau. \quad (2)$$

Note also that whereas for smooth bunches the total loss given by

$$k_{tot} = \int_{-\infty}^{\infty} W_z(t) I(t) dt \quad (3)$$

is rather insensitive to errors in the high frequency behaviour of the wake potentials, $\tilde{W}(t)$ in general is not.⁵

2 SETTING UP THE TBCI COMPUTATIONS

Suppose we would like to compute $\tilde{W}(t)$ for an exciting ring of small radius a and azimuthal mode number m due to a perfectly conducting, cylindrically symmetric structure of finite extent that is connected to infinitely long side tubes (of equal radii) using TBCI. TBCI allows infinitely long side tubes, but even so, two difficulties can arise for short exciting bunches. One is a physical problem: an extremely long outgoing tube may be necessary for all the wakefield effect to catch up with the bunch. The other is purely numerical: numerical "noise" due to the finite mesh size can creep into the wake potential necessitating another run with the cylindrical tubes only (no cavity), and then the subtraction of the two wake potentials.⁶

To avoid these problems we note that for any $m > 0$, the components of the total delta function wake potentials, when taken from minus to plus infinity, can be written

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$$\begin{aligned}
W_r(t) &= cf(t)ma^{2m-1} \cos m\varphi \\
W_\varphi(t) &= -cf(t)ma^{2m-1} \sin m\varphi \\
W_z(t) &= -f^4(t)a^{2m} \cos m\varphi
\end{aligned}
\tag{4}$$

for a test charge at $r = a$, at azimuthal angle φ and at time t behind the exciting ring. Thus if we know the wake potential for a given $a > 0$, we know it for any other a . Eqs. (4) also show that the components of $\vec{W}(t)$ are related as

$$\begin{aligned}
W_r'(t) &= -\frac{mc}{a} W_z(t) \\
\frac{W_\varphi(t)}{\sin m\varphi} &= -\frac{W_r(t)}{\cos m\varphi}.
\end{aligned}
\tag{5}$$

If we know one component of $\vec{W}(t)$ we know all components. For the case of $m = 0$, $W_r(t)$ and $W_\varphi(t)$ are both identically equal to zero while $W_z(t)$ is independent of φ and a . Substituting Eqs. (4) into Eqn. (2) it is easy to see that the components of $\vec{W}(t)$ also obey the relations of Eqs. (4) and (5) (with $f(t)$ replaced by another function $g(t)$ in Eqs. (4)) since $W_r(t=0) = 0$.

Due to Eqs. (4) and (5) we can avoid both difficulties mentioned at the beginning of this section. We take the ring radius a to equal the radius of the side tubes. Then after any slice of the ring has crossed the gap, it no longer receives any contribution to $W_z(t)$ or $W_\varphi(t)$ since there can be no force along the surface of a perfect conductor. Thus a short outgoing tube will suffice. Also the numerical "noise" mentioned above disappears. Then from Eqs. (4) and (5) we can easily obtain the components of $\vec{W}$ for any other a .

Whereas Refs. (1) and (2) gave the wake potentials per cell for an infinitely repeating structure, TBCI cannot solve for such a structure. But the more cells we take between the infinite side tubes (and then divide the total wake potentials by the number of cells), the better approximation we have to the wake potentials per cell of the infinitely

repeating structure. To further minimise the effect of the end conditions we have here taken the wake potentials for a finite number of cavities and then subtracted the wake potentials for the case with one less cavity.

3 RESULTS

The TBCI computations were done for $m = 0, 1, 2$ with Gaussian $I(t)$. To best compare the methods, the same structure was used as approximation to the SLAC structure as in Refs. (1) and (2), i. e. the infinitely repeating structure of which Fig. (1a) gives one cell. (Fig. (1a) shows the "average" SLAC cell, #45,⁸ with the rounded irises replaced by rectangular ones.)

Fig. (2) shows how the integrated wake potentials approach a limiting function going from a structure of 1 cell with infinite side tubes to the "2 minus 1" cell configuration (the wake potential for 2 cells with infinite side tubes minus that for 1 cell with infinite side tubes) to the "3 minus 2" cell configuration for $\sigma_z = 1$ mm. As described above, all plots give the wake potentials offset to the radius of the side tubes (1.163 cm) and at $\varphi = 0$, due to charge rings with radius a set to the side tube radius and of given m . $I(t)$ is given at the bottom of each plot to show the relative positions of $\bar{W}(t)$ and $I(t)$. The convergence to a limiting function already is quite good for the "3 minus 2" cell configuration, the poorest being the $m = 0$ wake potential. Even in this case, however, there is a maximum difference of only 10% between the "3 minus 2" cell and the "2 minus 1" cell configurations, for $-2 \leq ct/\sigma_z \leq 2$.

Figs. (3)(4) and (5) give the final results for the $m = 0, 1, 2$ wake potentials respectively, for bunch length values of $\sigma_z = 1, 2, 5$ and 15 mm. Because of computer resource limitations only the "3 minus 2" cell configuration is given for $\sigma_z = 1$ and 15 mm whereas the better "4 minus 3" cell configuration was used for the other bunch lengths. The comparison between the modal method results and those of TBCI is quite good. In the range of $-2 \leq ct/\sigma_z \leq 2$ the agreement is to within 10%, 8%, 10% of the maximum value for the wake potentials for the $m = 0, 1, 2$ cases respectively.

Since TBCI can also solve for structures with irises having rounded edges, calculations were done for $\sigma_z = 1$ mm using the basic cell shown in Fig. (1b). This is a much better approximation to SLAC cell #45. Fig. (6) shows the comparison of the TBCI results for the basic cell of Fig. (1a) versus for that of Fig. (1b), for the "3 minus 2" cell

configuration. For $-2 \leq ct/\sigma_z \leq 2$ the improved model gives a maximum decrease of the wake potentials of 8%, 13%, 20% respectively for $m = 0, 1, 2$ as compared to the model with square disk edges. For $m = 1$ and 2 the difference for $ct/\sigma_z > 2$ becomes even greater.

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(b)

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Fig. (1): 1a (top) is the model to the "average" SLAC cell used in comparing the two methods. 1b (bottom) is the improved model with rounded irises. All dimensions are given in centimeters.

Fig. (8): TBCI results for 1 cell with infinite side tubes (dotdashes), "2 minus 1" cells (dashes), and "3 minus 2" cells (solid lines). $\sigma_z = 1$ mm; $m = 0, 1, 2$. $I(t)$ is given below. Note that $t < 0$ is in front of the bunch center.

Fig. (9): Comparison of W_2 as given by the two methods for $m = 0$ and various values of σ_z . Solid lines are TBCI results, dashed lines are modal results. The $\sigma_z = 1, 15 \text{ mm}$ TBCI results are in the "3 minus 2" configuration whereas those of $\sigma_z = 2, 5 \text{ mm}$ are in the "4 minus 3" configuration.

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Fig. (4): Comparison of W_r as given by the two methods for $m = 1$ and various values of σ_2 . Solid lines are TBCI results, dashed lines are modal results. The $\sigma_2 = 1, 15$ mm TBCI results are in the "3 minus 2" configuration whereas those of $\sigma_2 = 2, 5$ mm are in the "4 minus 3" configuration.

Fig. (5): Comparison of W_r as given by the two methods for $m = 2$ and various values of σ_z . Solid lines are TBCI results, dashed lines are modal results. The $\sigma_z = 1, 15$ mm TBCI results are in the "3 minus 2" configuration whereas those of $\sigma_z = 2, 5$ mm are in the "4 minus 3" configuration.

Fig. (8): TBCI results for the improved model to the SLAC "average" cavity (Fig. (1b)) for $\sigma_z = 1$ mm, $m = 0, 1, 2$. Dashed lines are TBCI results for the simpler model (Fig. (1a)).

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