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DISCONTINUITY EFFECTS ON RADIAL CAVITY TRANSMISSION LINES

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ABSTRACT

Pulse propagation in radial cavity transmission lines such as those found on a radial line accelerator is considered. Specifically, the effects of discontinuities along the line are examined in detail. It is found that previous analyses of such effects have been incorrect, and here two alternate solution techniques are presented. Depending upon the parameters of such a radial line, the discontinuity effects considered here may or may not be significant; however, if they are significant, it is recommended that the alternate solution techniques presented here be used.

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Introduction

In the development of radial line accelerators, an understanding of electromagnetic pulse propagation in radial cavity transmission lines such as that depicted in Fig. 1 has become important. In a recent paper by Eccleshall and Temperley¹ considerable attention has been devoted to this subject. They show that pulse propagation can be approximated using principal-mode theory in which the fields are approximated only in terms of the lowest order propagating mode, which is the low frequency, TEM transmission line mode. In addition, to treat the effects of the discontinuity between regions 1 and 2 more accurately (see Fig. 1), an equivalent circuit model developed by Whinnery, et al.²⁻⁴ for the treatment of such discontinuities was used. It will be shown here that the equivalent circuit parameters obtained by Whinnery, et al. are not correct. In fact, the method used to obtain them was not well-posed, and consequently this equivalent circuit model is simply not appropriate for this case. Depending upon the parameters of the radial transmission line, the effects of this discontinuity may be negligible. However, if instead these effects are important, alternate techniques of solution should be used. Two such alternate methods will be shown here.

Fig. 1. Radial cavity transmission line.

Examination of the Equivalent Circuit Model

To demonstrate the difficulties of using the equivalent circuit model, consider the simplest discontinuity which preserves the essential features. That discontinuity is the parallel plate structure shown in Fig. 2a, along with the equivalent circuit (Fig. 2b) obtained for it in Ref. 2.

To obtain this equivalent circuit model, Whinnery, et al. represented the fields in each of the three regions (see Fig. 2a) in an eigenmodal expansion appropriate to each region with unknown coefficients $\{A_n\}$, $\{B_n\}$, and $\{C_n\}$ as follows:

$$E_x \Big|_A \Big|_{z=0} = \sum_{n=0}^{\infty} A_n \cos \frac{n\pi x}{a} \quad (1)$$

$$H_y \Big|_A \Big|_{z=0} = \sum_{n=0}^{\infty} Y_{A_n} A_n \cos \frac{n\pi x}{a} \quad (2)$$

$$Y_{A_n} = -i\omega\epsilon a / (n\pi K_{A_n}) , \quad n \neq 0 \quad (3)$$

$$K_{A_n}^2 = 1 - (2a/n\lambda)^2 \quad (4)$$

with an assumed $e^{i\omega t}$ time dependence. For regions B and C, replace A by B or C, a by b or c, and change the sign of (3).

Excitation is provided by a TEM transmission line mode incident from the left in regions B and/or C. Thus B_0 and C_0 include the incident wave amplitude, i.e.,

$$B_0 = B^i + B^r \quad (5)$$

Fig. 2. Simplified parallel plate geometry (a) and its associated equivalent circuit model (b).

where B^i and B^r are the incident and reflected TEM mode amplitudes, respectively. Then the TEM wave admittance becomes

$$Y_{B_0} = \sqrt{\frac{\epsilon}{\mu}} (B^i - B^r)/B_0 \quad (6)$$

and thus is dependent upon the excitation. Now, by requiring continuity of the tangential fields (1), (2) at the region interfaces, and then making use of the orthogonality of the eigenmodes, one obtains ²

$$aA_0 = bB_0 + cC_0 \quad (7)$$

$$A_m = \frac{2B_0}{m\pi} \sin \frac{m\pi b}{a} \left[1 + \left(\frac{mb}{a} \right)^2 \sum_n \frac{(-1)^n B_n}{n^2 B_0 \{ (mb/na)^2 - 1 \}} \right] - \frac{2C_0}{m\pi} \sin \frac{m\pi b}{a} \left[1 + \left(\frac{mc}{a} \right)^2 \sum_n \frac{C_n}{n^2 C_0 \{ (mc/na)^2 - 1 \}} \right] \quad (8)$$

$$Y_{B_0} B_0 = Y_{A_0} A_0 + \frac{a}{\pi b} \sum_m \frac{Y_{A_m} A_m}{m} \sin \frac{m\pi b}{a} \quad (9)$$

$$Y_{C_0} C_0 = Y_{A_0} A_0 - \frac{a}{\pi c} \sum_m \frac{Y_{A_m} A_m}{m} \sin \frac{m\pi b}{a} \quad (10)$$

$$Y_{B_n} B_n = \frac{2b(-1)^n}{n^2 \pi a} \sum_m \frac{mY_{A_m} A_m \sin(m\pi b/a)}{\{ (mb/na)^2 - 1 \}} \quad (11)$$

$$Y_{C_n} C_n = -\frac{2c}{n^2 \pi a} \sum_m \frac{mY_{A_m} A_m \sin(m\pi b/a)}{\{ (mc/na)^2 - 1 \}} \quad (12)$$

Next, (7) and (8) are substituted into (9-12) which then become an infinite set of equations for the modal coefficients in regions B and C. Now in order to describe the problem using the equivalent circuit in

Fig. 2b, (9) and (10) must be equated with the equivalent circuit's two nodal current equations

$$J_{o_B} = J_{o_A} + i\omega C'_B V_{o_B} + i\omega C'_A V_{o_A} \quad (13)$$

$$J_{o_C} = J_{o_A} + i\omega C'_C V_{o_C} + i\omega C'_A V_{o_A} \quad (14)$$

It is easy to show that Y_{D_o} is the TEM mode current J_{o_D} , where D is A, B, or C. Similarly, $-d D_o$ is the voltage V_{o_D} across each region at the interface where d is a, b, or c. Since by (7), $V_{o_C} = V_{o_B} + V_{o_C}$, it is simply a matter of algebra to obtain²

$$C'_A = -\frac{2a^2\epsilon}{\pi^3 bc} \sum_m \frac{\sin^2(m\pi b/a)}{m^3 K_{A_m}} \left[1 + \left(\frac{mc}{a}\right)^2 \sum_n \frac{C_n}{n^2 C_o [(mc/na)^2 - 1]} \right] \quad (15)$$

$$C'_B = \frac{2a^2\epsilon}{\pi^3 b^2} \sum_m \frac{\sin^2(m\pi b/a)}{m^3 K_{A_m}} \left[1 + \left(\frac{mb}{a}\right)^2 \sum_n \frac{(-1)^n B_n}{n^2 B_o [(mb/na)^2 - 1]} \right] - C'_A \quad (16)$$

$$C'_C = \frac{2a^2\epsilon}{\pi^3 c^2} \sum_m \frac{\sin^2(m\pi b/a)}{m^3 K_{A_m}} \left[1 + \left(\frac{mc}{a}\right)^2 \sum_n \frac{C_n}{n^2 C_o [(mc/na)^2 - 1]} \right] - C'_A \quad (17)$$

It is at this point in the analysis of Whinnery, et al. that an inconsistency arises. By comparing (9) to (13), (15) and (16) were obtained. By comparing (10) to (14), (17) was obtained. However, the latter comparison also generates the following:

$$C_A = -\frac{2a^2 \epsilon}{\pi^3 bc} \sum_m \frac{\sin^2(m\pi b/a)}{m^3 K_{A_m}} \left[1 + \left(\frac{mb}{a}\right)^2 \sum_n \frac{(-1)^n B_n}{n^2 B_0 \{ (mb/na)^2 - 1 \}} \right] \quad (18)$$

[In (17) and (18) the prime on C'_A has been deleted to denote the two different expressions (15) and (18).] Clearly (15) and (19) represent two distinct expressions for the same quantity. Unless these two expressions are equivalent, the discontinuity on Fig. 2a cannot be modelled by the circuit in Fig. 2b. If the inner sums are neglected in C'_A [from (15)] and in C_A [from (18)], then the expressions are equivalent and are equal to C_{A_0} given by

$$C_{A_0} = -\frac{2a^2 \epsilon}{\pi^3 bc} \sum_m \frac{\sin^2(m\pi b/a)}{m^3 K_{A_m}}. \quad (19)$$

This, in fact, is the approximation plotted in Fig. 17 of Ref. 2.

Being unable to demonstrate the equivalence of (15) and (18) analytically, we test for equivalence numerically. To do that, (9-12) were solved for a finite number of modal coefficients B_n and C_n by truncating the infinite set of equations. To insure that the solution obtained was correct, the results were checked against an independent solution to be discussed later.

Figure 3 shows such a comparison of (15), (18), and (19) with $b/a = .5$ and an incident wave in region C. Note that C_A is unequal to C'_A and that neither approach C_{A_0} as $a/\lambda \rightarrow 0$ (λ is wavelength). Also, note that where C_{A_0} is real, C_A and C'_A are in general complex.

Figure 4 shows another comparison with $b/a = .25$. In this case C_A and C'_A are each computed twice, once with the incident wave in

Fig. 3. Comparison of C_{A_0} , C'_A , and C_A ($b/a = .5$).

Fig. 4. Comparison of C_{Ao} , C'_A , C_A for two different excitations ($b/a = .25$).

region B and again with the incident wave in region C. We see that in addition to the fact that C_A is again unequal to C'_A , they are not even independent of excitation, which is a crucial requirement for the equivalent circuit model. It should be noted that these difficulties are in addition to the limitation that (as pointed out by Whinnery, et al.) the capacitance is independent of frequency only when the transverse dimensions of the transmission line are small compared to wavelength λ . This means that the circuit model is not even a valid low-frequency approximation.

Alternate Field Solution

In order to verify the previously obtained solution of (9-12) for $\{B_n\}$ and $\{C_n\}$, and hence the values computed for C'_A , C_A , and C_{A_0} from (15), (18), and (19), a solution for the fields by an alternate technique was obtained. Briefly, this method employs standard Green's function techniques⁵ in the three regions of Fig. 2a. Here we add a shorting plane in region A at $z = l$ (see Fig. 2a), which is accounted for in the previously discussed solution of Whinnery, et al.,² by changing the region A mode impedance (3) to

$$Y_{A_n} = - (i\omega\epsilon a/n\pi K_{A_n}) \coth(n\pi K_{A_m} l/a) . \quad (20)$$

Then, using Green's theorem and enforcing continuity of the tangential fields at the region interfaces, the following two coupled integral equations are obtained:

$$\int_0^c E_x(x' + b) [g_1(x - b, x', c) + g_2(x, x' + b)] dx' + \int_0^b E_x(x') g_2(x, x') dx' = \frac{\lambda}{\pi i} E_x^{\text{inc}}(x) \Big|_{z=0}, \quad b < x < a \quad (21)$$

$$\int_0^b E_x(x') [g_1(x, x', b) + g_2(x, x')] dx' + \int_0^c E_x(x' + b) g_2(x, x' + b) dx' = \frac{\lambda}{\pi i} E_x^{\text{inc}}(x) \Big|_{z=0}, \quad 0 < x < b \quad (22)$$

where

$$g_1(x, x', \alpha) = \frac{1}{\alpha} \sum_{n=0}^{\infty} \frac{\varepsilon_n}{v_\alpha} \cos \frac{n\pi x}{\alpha} \cos \frac{n\pi x'}{\alpha} \quad (23)$$

$$g_2(x, x') = \frac{1}{a} \sum_{n=0}^{\infty} \frac{\varepsilon_n}{v_a} \coth v_a \ell \cos \frac{n\pi x}{a} \cos \frac{n\pi x'}{a} \quad (24)$$

$$v_\alpha^2 = \left(\frac{n\pi}{\alpha} \right)^2 - \left(\frac{2\pi}{\lambda} \right)^2, \quad \text{Im}(v_\alpha) \geq 0$$

$$\varepsilon_n = \begin{cases} 1, & n = 0 \\ 2, & n \neq 0 \end{cases}$$

Equations (21) and (22) can be solved numerically for the unknown interface electric field $E_x(x)$ by employing moment methods⁶ to reduce them to coupled matrix equations. Specifically, the unknown electric field (E_x) is approximated by a finite number of subsectional rectangular pulses of unknown amplitude (pulse expansion). The integral equations

are then enforced at a finite number of values of x corresponding to the center of each rectangular pulse (collocational δ -function testing). If a sufficient number of pulses are used, the solution can be shown to converge.

Figure 5 shows a comparison of the electric field as a function of x at $z = 0$ as computed by both techniques. For the integral equation method, 20 expansion pulses were used for each interface. For the mode-matching method of Whinnery,² 15 modal coefficients were computed for both regions B and C. We see that the two solutions are in good agreement except for the oscillation in the mode-match solution which is obviously due to the truncation of the modal expansion. This agreement verifies that the previously given numerical computations of C_A , C_A' , and C_{AO} are indeed accurate.

Time Domain Results

Using Laplace transform techniques, Eccleshall and Temperley¹ have incorporated the equivalent circuit in Fig. 2b to obtain the transmitted voltage pulse in region B due to a time step excitation in region C. Their analysis is readily extended to obtain the reflected voltage pulse in region C and also to a more general excitation as well.

Similarly, using the results of the integral equation solution, voltage and reflection coefficients R and T can be obtained using

$$R = \frac{1}{cE_x} \int_0^c E_x(x' + b) dx' - 1 \quad (25)$$

and

$$T = \frac{1}{bE_x} \int_0^b E_x(x') dx' \quad (26)$$

Fig. 5. Comparison of real and imaginary parts of interface electric field.

Then, for any given incident waveform, the time response can be obtained by numerical inverse Fourier transformation. For results shown here, Filon's method⁷ is used to perform this transform.

Figure 6 shows a comparison between this transformed integral equation solution and Eccleshall and Temperley's solution using the equivalent circuit model from Ref. 2 for a step input waveform. The dimensions of the structure are comparable to those of the RADLAC accelerator: $b = c = 30$ cm, $l = 15$ cm. Based upon the results of Whinnery, et al.,² it can be shown¹ that the response using the equivalent circuit model should be valid for $t > t_{\min}$ where $t_{\min} = a/2v$ and v is the phase velocity of the electromagnetic wave. We see that this is not the case, which reflects the other difficulties associated with the equivalent circuit model discussed previously. (For this structure $t_{\min} = 1$ ns.)

Figure 7 shows the same comparison except the incident voltage pulse now has a 10-90 risetime of 6 ns. In this case, the equivalent circuit result is more accurate; however, it is essentially unchanged from the incident pulse. Consequently, although the equivalent circuit is still wrong, its effect is in fact negligible, and it would be sufficient to ignore the discontinuity altogether.

An additional method of treating this discontinuity has also been used by Tucker.⁸ The distributed transmission line circuit is approximated by a lumped element circuit. The capacitance of each element is found from the electrostatic field solution (in this case obtained numerically using the computer code JASON⁹). The corresponding inductances are found from the analogous magnetostatic fields, which have

Fig. 6a. Voltage transmission for step input.

Fig. 6b. Voltage reflection for step input.

Fig. 7a. Voltage transmission for 6 ns risetime input.

Fig. 7b. Voltage reflection for 6 ns risetime input.

the trivial solution of constant transverse B-field. The resulting lumped circuit is then solved numerically using the computer code SCEPTRE.²⁰

Figures 8 and 9 show a comparison of the total voltages across the interfaces between regions A and B (V_E) and between regions A and C (V_C) for a step input and a 6 ns risetime input, both with maximum voltage V_0 . For this case, each L-C element pair corresponds to about 3.25 cm of line. The agreement is quite good between the waveforms, even for the step response (Fig. 8). It should be noted that the resonant mode of each L-C element pair in the lumped circuit approximation has a period of roughly 0.6 ns. The excitation of these "artificial" resonances are seen to be the primary source of deviation between the two solutions. Also note that although in this case we have assumed an ideal, matched voltage source and a matched output line, the lumped circuit approach is capable of treating multiple reflection due to line terminations.

Conclusions

It has been shown that the equivalent circuit model² is not correct. Resulting errors become important for relatively fast risetime pulses when the effects of the line's end discontinuity become significant. In this event one should consider a more accurate approach such as the integral equation technique discussed here. Another alternative would be to use the lumped circuit approach (JASON-SCEPTRE), keeping in mind the effect of the resonant frequency of the elemental L-C circuits. By increasing the number of circuit elements used in the approximation, the accuracy could thus be improved. Also, this approach has the attractive capability of

Fig. 8a. V_C for step input.

Fig. 8b. V_B for step input.

Fig. 9a. V_c for 6 ns risetime input.

Fig. 9b. V_B for 6 ns risetime input.

being able to model complicated line terminations which couple to the discontinuity. Finally, it should be noted that although the discontinuity effects may be negligible for a given set of radial line parameters, any combination of faster risetime pulses, larger physical dimensions, or slower medium propagation velocity will serve to increase the effects of such a discontinuity.

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