EFFECT OF DAMAGE ON THE MODAL PARAMETERS OF A CYLINDRICAL SHELL

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ABSTRACT

The objective of the study was to investigate the feasibility of assessing damage to structural systems by measuring the changes in the dynamic characteristics of a thin circular cylindrical shell with both ends open. Theoretical and experimental modal analyses were performed for the shell. Subsequently, a notch was machined into the shell simulating a small amount of damage. The shell with the notch was again subjected to experimental modal analysis. A comparison of the modal parameters determined from the tests before and after the shell was damaged showed that the natural frequencies were not sensitive to
the crack introduced. However, some of the mode shapes showed significant changes, establishing that the mode shapes were the more sensitive parameters for damage detection.

NOMENCLATURE

- **m** Integer defining the axial wave pattern
- **n** Integer defining the circumferential wave pattern

1. INTRODUCTION

If modal parameters of a structure are sensitive to damage sustained by the structure, it might be possible to develop methods to characterize the damage in terms of the change in modal parameters. The work described here was the first step in exploring the feasibility of such methods. The objective was to determine *experimentally* whether modal parameters of a structure were sensitive to the occurrence of relatively small amount of damage.

When structural systems are subjected to severe loadings and suffer permanent damage, it is reasonable to expect that their stiffness characteristics would change. It is also reasonable to expect that these changes would be reflected by changes in the modal characteristics of the system if it were linear elastic. This gives rise to the question whether the change in modal characteristics could be used as an indicator and measure of damage to the system.

In a recent survey article on the dynamics of cracked rotors, Wauer [1] notes that many researchers have attempted to identify the location and extent of cracks even in non-rotating structures by studying the vibration characteristics before and after cracking. He has noted that finding the most sensitive procedure for detecting a crack in its initial stage remains to be resolved.
Wang and Zhang [2] performed a sensitivity analysis to conclude that while modal characteristics are not sensitive to incipient faults, the transfer or response functions in certain frequency ranges are sensitive to the same fault. This would seem to contradict some of the results of other investigators who performed experiments to show that changes in modal characteristics are sensitive to the existence of flaws. Cawley and Adams [3] performed vibration tests on rectangular plates before and after holes or saw cuts were introduced into them and found that changes in the natural frequencies could be used to detect, locate and quantify the damage introduced. More recently, Silva and Gomes [4] reported on the results of their experiment with cracked free-free beams showing that the flexural natural frequencies decreased due to cracking. Rizos et. al. [5] used an experiment on a cracked cantilever beam to demonstrate that their method can be used to identify cracks in structures by measuring their modal characteristics; however, they noted that the method lacked accuracy for very small cracks. Their results provide a clue to resolve the seeming contradiction between the conclusions of [2] and the results reported in [3,4] by showing that changes in modal parameters might not be sensitive enough to reveal very small flaws. As noted in [4], experimental results on this subject are rather limited. It is clear that a great deal more of experimental work and theoretical analysis are needed before determining whether experimental modal analysis could be used as a non-destructive method for detecting damage in practical situations.

The objective of the present study was to investigate the feasibility of assessing the damage to structural systems from the changes in the dynamic characteristics of the systems. The immediate purpose was to determine experimentally whether modal parameters of a structure that is more complex than a beam or a plate are sensitive to the occurrence of relatively small amount of damage.

A theoretical modal analysis of the test structure was first performed to provide the basis for proper performance of the vibration tests. The subsequent experimental study involved two sets of vibration tests, one on the structure before any damage was introduced and the other after. The data from each set of tests were then subjected to experimental modal analysis. The results of the two sets of analysis were compared to detect changes in the modal parameters.
The selection of the type of system for the experiment was dictated by two considerations. The first was that it had to be more pertinent to nuclear power plant components than the structural systems that have hitherto been tested. The second was that the type of structure had to be a well defined linear elastic system which was amenable to theoretical modal analysis and could be manufactured, and tested within the limitations of available time and resources. As noted above, only beams and plates are reported to have been tested hitherto. Geometric configurations incorporating cylindrical shells are common in nuclear power plants. Reactor vessels, containment shells, large piping, all consist of circular cylindrical shapes in many cases. Although the test structure was not intended to be a scale model of any particular component, it was considered that the dynamic behavior of a thin circular cylindrical shell would be more relevant to power plant components than other simple structures. It was also relatively easy to make and in order to keep the manufacturing simple, the shell was left open at both ends. The shell was to have free kinematic boundaries because these boundary conditions were relatively easier to achieve in the tests and to be idealized in analysis without significant error.

The cylindrical shell was machined from a 12 in. diameter Schedule 40 carbon steel seamless pipe. The dimensions of the finished shell were as follows: length: 27.016 in (686.2 mm), outer diameter: 12.343 in (313.5 mm), and inner diameter: 12.141 in (308.4 mm).

Damage in structures could take different forms depending on the material and loading. Stiffness degradation, plastic deformation, discrete cracks, clusters of microvoids, or relaxation of boundary restraints are all examples of damage. The controlled damage to be introduced into the shell had to be quantifiable and easy to introduce. Therefore a saw cut of known dimensions was selected to simulate a discrete crack in a metal cylinder. The basis for determining the dimensions and location of the saw cut is discussed in a later section.

2. FINITE ELEMENT MODAL ANALYSIS
The purpose of the pretest analysis was to estimate the approximate modal characteristics of the system so that the test parameters could be properly selected. As the modes of predominantly radial translation are relatively the easiest to excite and measure in the tests, the analysis was focused on determining these modes only.

The ANSYS (Version 4.4) program was used to perform a modal analysis of the cylindrical shell. None of the symmetry properties of the shell was exploited in devising the finite-element model because of the need for comparison with results of possible future analyses involving damage that eliminates any symmetry. The shell was discretized with a total of 544 nodes, with 17 equally spaced nodes along the length of each of the 32 generator lines that were distributed at equal distance along the circumference. Thus the shell consisted of 512 elastic quadrilateral shell elements that were all identical in size and shape.

With six degrees of freedom (DOF) at each node, the total DOF was equal to 3264. Modal analysis process in ANSYS requires the selection of master or dynamic degrees of freedom in the structure from among these total DOF to reduce the complexity of the analysis. As the current interest focused on predominantly radial translational modes, the radial DOF at each of the 544 nodes was included among the master set. The total number of master DOF was set at 600 allowing the program to automatically select an additional 56 DOF to be included in the set of master DOF. From the available set of 600 DOF, only the first 100 mode shapes were calculated. The first six of these are rigid-body modes and were hence ignored.

The lowest deformational natural frequency of the shell was computed to be 70.25 Hz with axial wave pattern given by \( m = 1 \), and circumferential wave pattern given by \( n = 2 \). (See Figure 1). The 94th deformational mode had a frequency of 2358.7 Hz with a mode shape defined by \( m = 7 \) and \( n = 5 \). ANSYS-determined frequencies for modes with \( 1 \leq m \leq 2, \ 2 \leq n \leq 10 \) were compared with those calculated with formulas for the Rayleigh \( (m = 1) \) and Love \( (m = 2) \) modes [6], and the frequencies agreed very well,
adding confidence to the finite-element results. The modal characteristics determined in the finite-element analysis formed the basis for determining the test parameters for the undamaged shell.

3. EXPERIMENTAL MODAL ANALYSIS

3.1 Impulse Testing

The shell was suspended, with its axis vertical, from a relatively rigid frame by means of three steel springs of relatively low stiffness. As the measurements proved subsequently, the rigid body modes of the shell were well below the lowest deformational mode. Impulsive excitation with hammer impacts was the method of choice to excite all the modes of interest. The instrumented hammer used was the PCB Model 086B03 with an integral quartz force sensor. In the present case the pre-test finite element analysis showed that there were 52 radial translational modes of vibration in the frequency range of 0 to 1600 Hz. It was expected that the 'damage' to be introduced to the shell would cause at least some of these modes to change. Following a few trial impacts on the shell with tips of different stiffness, a hard plastic tip was found to produce impulses that were nearly uniform in the above frequency range.

Were it not for the symmetry, only one accelerometer would have been mounted at a single reference location and the shell would have been successively impacted at each of the response locations including the reference location. In a circular cylindrical shell there are two radial translational modes at each radial translational natural frequency due to symmetry. With only one accelerometer mounted at a single reference location it might not be possible to identify both of these modes at every one of these natural frequencies. Consequently at least two reference locations were required. The two reference locations selected were 135 degrees apart along the circumference. Along the axis of the shell, they were located at a distance of one eighth of the length of the shell from the middle circle, one on either side of it.
The two reference accelerations were measured with ENDEVCO Model 22 microminiature, piezoelectric accelerometers, with negligible mass loading. The accelerometers were attached to the inside surface of the shell, positioned at each location to measure the component of the acceleration at that location in the radial direction.

Eighty ‘response’ locations on the outer surface of the shell were marked at each of which the hammer impacts were made. Seventy two of these were at the points of intersection of eight generator lines lying on the lateral surface, equally spaced at 45° along the circumference, and nine circles along the length, equally spaced at one eighth the length of the shell, as shown in Fig. 2. The additional eight locations were marked on the topmost circle such that there were a total of sixteen equally spaced points on this circle as may be seen in Fig. 2.

There were two reasons for not exploiting the symmetry of the shell in selecting the above locations. The first was the need to define the mode shapes as fully as possible in order to characterize the changes in them. It was for this reason that the top circle had 16 points rather that the eight that the other circles had. With 16 points, it would be possible to identify modes that have up to eight circumferential waves. The second reason was that, in the general case, the damage to the shell would destroy all the symmetries and it would anyway be necessary to instrument the entire shell.

The highest axial wave pattern occurring at frequencies below 1600 Hz is characterized by a value of 5 for m. The 9 measurement locations on each of the eight lines assure a clear definition of mode shapes with such axial wave patterns, as might be seen from Fig.1.

The data acquisition and processing equipment for the dynamic testing consisted of a HP-3565 signal processor system with four input channels. The system also included a HP 300 series computer that was running the data acquisition software VISTA. VISTA performs analog to digital conversion and Fast Fourier Transform (FFT) calculations and gives transfer functions as its result. An exponential response
window with a decay constant of 500 ms was applied in the time domain. The force window was also exponential (with the same decay constant as the response window) for the time duration from zero to 200 ms. For time greater than 200 ms, the force window had zero value.

The shell was subjected to impulse testing twice, i.e. before and after introducing the ‘damage’ in the form of a notch. Each test consisted of two runs, with the first run covering 0 - 800 Hz, and the second covering 800 - 1600 Hz, with a frequency resolution of 0.25 Hz.

To reduce the effect of random errors, ten impacts were made at each response location and the final transfer function, or the frequency response function (FRF), was obtained as an equally-weighted average of the ten FRFs. The final product of the test was a set of 160 FRFs. These formed the input to the modal parameter identification procedure.

3.2 Determination of Modal Parameters

The modal analysis was performed with the TDAS module of I-DEAS (Level 4.0) software, licensed from SDRC [7]. The present case called for a multiple-degree-of-freedom identification technique because of the occurrence of two modes at each natural frequency. The Polyreference algorithm of TDAS was selected as the best method. The main advantage of this technique is its ability to use all the available FRFs to give a single global estimate for the modal parameters.

The Polyreference technique uses a curve-fitting algorithm in time domain and makes use of multiple response functions for more than one reference location. It is an extension of the complex exponential method in which a polynomial fit is assumed for the response function at a point due to a unit impulse at a single reference location (obtained as the inverse Fourier transform of the FRF for that point) and the roots of the polynomial are determined by a least-square technique [8]. Details of the theoretical development of the Polyreference method are given by Vold and co-workers [9, 10].
In any curve fitting technique, it is usually necessary to provide an initial estimate of the number of modes contributing to the FRFs included. The peaks of the FRF themselves usually provide this initial estimate, provided each peak is known to represent one mode only. The number of radial modes was estimated from the pretest analysis. A visual examination of any given FRF was not sufficient to reveal whether all of these modes were actually excited in the test runs. The TDAS software provides an option to compute a mode indicator function. For a single reference, the mode indicator function is the ratio of two sums. The numerator is the sum of the products of the real parts of the FRFs and their magnitudes. The denominator is the sum of the squares of the FRF magnitudes. Localized or lower amplitude modes are apparent in this function since the maximum amplitude is normalized to unity. For multiple reference test data, as in the present situation, the Multivariate Mode Indicator function calculation was performed. The number of mode indicator functions is equal to the number of the references used in the calculation. Therefore, with two references, two mode indicator functions, primary and secondary, were generated. Repeated roots were indicated at frequencies at which both the primary and the secondary functions showed a local minimum. These functions made it possible to arrive at a very good estimate of the modes excited.

The Polyreference technique identifies a number of computational modes in addition to the physical modes. A computational mode is generated to account for unwanted effects such as noise, filter characteristics, leakage, residual flexibility and inertia, and nonlinearities reflected in the FRFs. To distinguish between physical and computational modes, TDAS provides the option to compute the Modal Confidence Factor (MCF) for each identified mode. The MCF exploits the redundant phase relationships that are satisfied only by the physical modes. It takes on a value close to unity for physical modes and a value less than one for computational modes. The degree to which the MCF can distinguish between physical and computational modes is data dependant since poor data quality results in a poor estimation and low MCF values even for physical modes. In the tests on the cylindrical shell, almost all the modes identified as physical had an MCF value of 1.0 or 0.99.
A means of global validation of the identified modal parameters is provided by the Modal Assurance Criterion (MAC) matrix [11]. Each element of the MAC matrix is a scalar value between 0 and 1 representing the correlation between any two mode shapes. It is the normalized inner product of the two mode shape vectors. A MAC value close to one indicates a high degree of correlation or consistency between two mode shapes. While MAC is not a true orthogonality check since the mass or stiffness matrices have not been included in its calculation, it can be used as an approximation of an orthogonality check. If such possibilities as nonstationarity or nonlinearity of the system, insufficiency of the number of degrees of freedom, presence of noise, and the occurrence of unmeasured forces on the system could be ruled out, the MAC value would be zero for linearly independent mode shapes and unity for mode shapes that represent the same motion differing only by a scalar.

In the present series of tests care was taken to rule out or minimize the sources of error noted in the previous paragraph and therefore the MAC matrix was used for validating the mode shapes derived in each test run.

4. RESULTS AND DISCUSSION

4.1 Modal Characteristics of Undamaged Shell

The natural frequencies of the undamaged shell, computed with ANSYS and identified from the tests are given in columns 4 and 5 of Table 1. The natural frequencies identified from the tests were smaller than the calculated values only by about 3%. Considering that the calculations were based on estimated, and not measured, material properties, this difference is negligible.

The two modes for each combination of m and n do not occur at the same frequency because of deviation from perfect biaxial symmetry introduced by unintended defects of manufacture in the case of tests, and inherent numerical errors in the case of analysis. The results show that all the targeted modes below 1600
Hz were excited in the tests, the notable exceptions being modes 10 and 12. Both these modes have a circumferential wave number of 4. The companion modes for these, the modes 9 and 11 respectively, were well excited. The circumferential nodes for the missing modes might occur at intervals of integer multiples of 45° (see Fig. 1). The two reference locations were 135° apart and that might explain why one or the other of the accelerometers coincided with a nodal location for the missing modes. The other modes with n=4 have been identified in pairs perhaps because the frequency separation at which each pair occurred was larger than that for the above modes.

The damping identified (Cols. 6 and 8) is seen to be very low for almost all the modes, with most values below 0.1%. Mode shapes were determined for each mode and examples of these are given by Figs. 3 and 4.

4.2 Selection of Damage

A number of considerations influenced the size and location of the crack that represents damage in this case. Petroski and Glazik [12] had shown, using finite-element analysis, that non-penetrating longitudinal cracks, of lengths equal to the full length of the shell and of various depths, lower the frequencies of bending modes and that the mode shapes with n>3 or 4 reveal the existence of the cracks. It seemed to the present authors that even a shorter longitudinal crack would affect these mode shapes in an obvious manner and that it would be more interesting to consider circumferential cracks which have not been studied before.

From the point of view of relevance to practical situations, the crack had to extend over only a small part of the circumference and it would have to be non-penetrating too. Otherwise there might be easier and more obvious indicators of such cracks. Also the technique would be impractical if measurement points had to be spaced as closely as the length of the crack to be detected. Therefore the length of the crack had to be no more than an eighth of the circumference in this case. As to the depth of the crack, even
though it was known from previous experimental work by others on beams etc. that cracks of relatively small depths do not make a measurable difference in modal frequencies, there were no theoretical basis to determine the right depth. So the depth was selected to be about one half the shell thickness. The width of the crack was dictated by the machining process.

As to the location of the crack, there was only one important factor that influenced the choice. The location had to be such that at least for some specific modes, the presence of the crack would not make any difference to the frequency of the mode. This was necessary as a control measure to eliminate the possibility of extraneous influences affecting the natural frequencies. For example, Adams et al. [13] found that changes in the temperature of the test specimens affected the natural frequencies. This would mean that the crack should be located along a circle known to be a nodal circle for a specific mode. From Fig. 1 it is obvious that the most suitable location for this purpose would be the middle circle of the shell.

The cylindrical shell with the machined crack is shown in Fig. 5. The center of the crack coincided with one of the response locations.

4.3 Changes in Modal Parameters due to Damage

First of all we note that the changes in the natural frequency of modes due to damage (col. 9 of Table 1) are negligible. While for most modes the change is less than a tenth of a percent, even the largest change is less than 1 percent. More importantly, the change in most cases is comparable to the frequency resolution of the test data, i.e. 0.25 Hz. This means that the changes are attributable more to the numerical processing of the curve fitting algorithm than to a physical frequency shift. It is clear that the modal frequencies are practically not sensitive to the crack in the shell.

Second, the damping ratios show a change for many modes. However these changes could not be related to the introduction of the crack for some important reasons. The first is that among the modal
parameters estimated, the damping ratios have the highest uncertainty. This became evident during the curve fitting process during which changing the number of roots of the polynomial resulted in significant change in the damping values even when the modal frequencies changed little. Secondly, the damping ratios are of the order of 1% or less in this case and depend not only on the material damping in the shell itself, but also on the ambient air which helps dissipate the energy through radiation. Finally the viscous modeling of the damping is only a mathematical artifice and is not as physically meaningful as the mass and stiffness representations. Therefore the change in damping is not a quantitative measure of the cracking of the shell.

4.4 Change in Mode Shape as Indicator of Cracking

The other modal parameter that might be an indicator of damage is the mode shape. The mode shapes are more easily compared by plotting the deformed shapes than by comparing the mode shape vectors numerically. Such a comparison was made for each mode and it was found that while many mode shapes do not show the effect of the crack some do indeed show differences. To give examples of mode shapes showing obvious differences, Figs. 6 and 7 are given. Figures 3 and 6 give the mode shape for mode 29 before and after the crack was introduced. Figures 4 and 7 similarly show the shapes for mode 30. Comparison of the two sets show that the mode shapes have noticeably changed as a result of cracking of the shell. This is in spite of the fact that the natural frequencies for modes 29 and 30 have changed only by 0.05 and 0.07% respectively!

A single quantity representing the magnitude of change in each mode shape would be very useful as an indicator of damage. The diagonal terms of the MAC matrix obtained by correlating the mode shapes of the undamaged and cracked shells provide this quantitative measure. Using TDAS, these values are easily obtained and are given in column 10 of Table 1. A MAC value of 1 would show that the two mode shapes are perfectly correlated or, in this case, identical. The greater the deviation from unity, the less the two modes are correlated. The last column of Table 1 gives the deviation of MAC from unity. It is seen that
at least 10 out of the 52 modes show a deviation greater than 25%. The change in natural frequency for these same modes is negligible. Thus it is clear that certain mode shapes are much more sensitive to damage in the shell.

From Table 1, it is seen that most of the mode shapes that show a significant change have an odd number for the value of \( m \). This is consistent with the fact that these modes have their maximum deformation at the middle of the shell where the crack is present.

The above results show partial agreement with the conclusions of Wang and Zhang [2] in that natural frequencies are not sensitive to the introduction of damage. These authors have also stated that the mode shape vectors are not sensitive unless the frequencies are very close. The present results show that this aspect needs further investigation. For the cylindrical shell every mode involves two closely spaced frequencies. Yet only some of the modes show mode shape sensitivity to damage.

5. CONCLUSIONS

The results of the experimental modal analyses of a cylindrical shell, before and after a crack was machined into the shell, have shown that mode shapes are more sensitive to the presence of damage than the modal frequencies. The sensitivity of at least some of the mode shapes is sufficient to indicate the existence of cracks that might not be detected by other more obvious indicators such as a leak. The development of a method that uses this sensitivity as a practical non-destructive examination method requires further research.

The observations reported above need to be confirmed through additional tests and finite-element analyses before proceeding with the development of methods to define the damage on the basis of change in the mode shapes. First, tests on additional cylindrical shells with different dimensions should be performed to assure the general validity of the results. Alternatively, finite-element simulations might
be performed to verify the phenomenon. The sensitivity of mode shapes to damage needs to be analytically investigated. The more difficult inverse problem of relating the change in modal parameters could be addressed only after the solution to these direct problems are determined.

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References


Table 1 Comparison of Mod/F Parameters of Undamaged and Cracked Shells

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Wave Pattern Integer</th>
<th>AnsYS determined Freq., Hz</th>
<th>Identified from test on Undamaged Shell Freq., Hz</th>
<th>Identified from test on Cracked Shell Freq., Hz</th>
<th>Change in Freq. due to damage in %</th>
<th>MAC %</th>
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<td>1 2</td>
<td>70.25</td>
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... (Continued)
Figure 1. Axial and Circumferential Wave Patterns in Shell Vibration Modes
Figure 2. Shell Model for Experimental Modal Analysis

Figure 3. Mode Shape for Mode 29: Undamaged Shell

Figure 4. Mode Shape for Mode 30: Undamaged Shell

Figure 5. Shell with Machined-in Crack

Figure 6. Mode Shape for Mode 29: Shell with Crack

Figure 7. Mode Shape for Mode 30: Shell with Crack