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A Poloidal Field Measurement Technique: Pitch Angle Measurements via Injected He⁺ Ions

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Abstract

The poloidal field of a tokamak can be determined by observing the light emitted by He⁺ ions injected into the plasma by a perpendicular He⁰ beam. These ions will orbit in small circles located where the neutral atom became ionized, and they will remain there for a few microseconds. During this time, some of these ions will also emit light at various spectral lines. The observed spectrum of any of these lines will have a peculiar and very wide shape, and it will be offset (Doppler shifted) with respect to the natural line location. The location and width of the spectral pattern provide independent information about the components of the poloidal field which are parallel and perpendicular to the beam velocity, and this information is local to the point where the light is emitted. For a horizontal beam, these components are b_x and b_y , respectively. The difference in Doppler shift between two measurement points above one another (at the top and bottom of the beam) is directly proportional to δb_x , which in turn is proportional to the transform on that flux surface. Thus, this technique provides a means to measure directly local values of $q(r)$. Simulation studies indicate that accurate measurements can be made in milliseconds.

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INTRODUCTION

The poloidal field of a tokamak can be determined by observing the light emitted by He^+ ions injected into the plasma by a perpendicular He^0 beam.^{1,2} These ions will orbit in small circles located where the neutral atom became ionized, and they will remain there during the He^+ lifetime (a few microseconds) provided the component of the beam velocity parallel to the magnetic field is reasonably small. During this lifetime, some of these ions will also emit light at various spectral lines. The observed spectrum of any of these lines will have a peculiar and very wide shape, and it will be offset with respect to the natural line location. The location and width of the spectral pattern will provide independent information about the components of the poloidal field which are, respectively, parallel and perpendicular to the beam velocity. This information is local to the point where the light is emitted.

Consider atoms injected perpendicular to the field. If the emitted light is observed from a point normal to the surface of the disk of the orbit, as in Fig. 1, viewpoint 1, all the light will be found within the normal spectral width of the line, because all the ion motion is perpendicular to the line of sight, and there is no Doppler shift. However, if the point of view is moved to view the disk edge on, as at viewpoint 2, the spectrum will be quite different. Depending upon just where the ion is in its orbit, it could have a large velocity along the sight line, resulting in a Doppler shift of many Å. Since the ions spend more of their time with nearly full velocity directed toward (or away from) the point of view than at other velocities, the spectrum will be split into

Fig. 1 An ion orbiting in a magnetic field B , and 3 points of view.

a two peaked affair, with relatively little light emitted in the vicinity of the line center. Figure 2 is a schematic spectrum of this light.

This effect is not small: for a 50 keV He^0 beam, the maximum shift of the He^+ 4686 Å line is ± 24 Å. The width is also dependent upon the view, and is, in fact, proportional to $\sin\alpha$, where α is the angle between the sight line and the normal to the orbit (viewpoint 3, Fig. 1). This angle is dependent upon the pitch of

Fig. 2. Schematic spectrum.

the field lines, as well as the geometry of the point of view. If, for instance, the view point is chosen such that $\sin\alpha \sim 0.1 \rightarrow 0.2$ when there is no poloidal field, then the pitch itself will be comparable to $\sin\alpha$. The full width of the line then would be $5 \rightarrow 10$ Å without field, with a change in width of ~ 5 Å with field. This is not at all a subtle change.

The 4686 Å light is quite bright; for TFTR conditions [$n_e \sim 4 \times 10^{13} \text{ cm}^{-3}$; optical system étendue of 2.5×10^{-5} ($=f/8$); a 50 keV, 40 mA/cm², 10 cm wide He beam;] one would expect to get about 10^6 photons every msec into the optical system. This should yield 1% results each msec. On a 10 msec time scale, one would expect to detect $\sim 10^6$ photoelectrons. The detected signal should then be good to 0.1%, and the accuracy of measurement determined by other considerations. The intensity calculations used are explained in the section on absolute intensity.

Although this concept is described in terms of a Helium beam, and 4686 Å light, it is not restricted to Helium. In fact, any neutral beam except atomic hydrogen could be used. All that is required is that the ion under observation have at least one electron available in order to emit light. Specifically, molecular hydrogen could possibly be used. This option would allow the measurement to be made when the diagnostic beam is run with H^0 or D^0 for other experiments.

RELATIVE INTENSITY vs λ

Consider a beam of heavy (i.e., not atomic hydrogen or deuterium) neutral particles injected along a major radius of the tokamak. Consider also a Cartesian coordinate system with the x axis along R (pointing toward the beam source), y vertical, and z parallel to B_ϕ , as shown in Fig. 3. [Note that the coordinate system is displaced from the magnetic axis of the plasma.] Then, for a particle with $v \perp B$ and ionized at $t = x = y = z = 0$:

$$v/v_0 = -i\cos(\omega t) + \sin(\omega t)(j\cos\vartheta - k\sin\vartheta).$$

1

This is v for the orbit shown in Fig. 1, but tilted by the pitch angle ϑ of the magnetic field. [$\tan\vartheta \approx B_y/B_\phi$].

Fig. 3
Measurement
coordinate
system, and
magnetic fields.

A viewer at (x, y, z) would see any light emitted by this ion along a direction vector:

$$g \equiv i\xi + j\eta + k\zeta$$

$$\xi \equiv x/r \quad \text{etc.} \quad 2$$

$$r^2 \equiv x^2 + y^2 + z^2.$$

The velocity along the line of sight is:

$$\begin{aligned}
 v_g/v_0 &= \mathbf{g} \cdot \mathbf{v}/v_0 \\
 &= -\xi \cos(\omega t) + \sin(\omega t)(\eta \cos \vartheta - \zeta \sin \vartheta) \\
 &= \sqrt{\xi^2 + (\eta \cos \vartheta - \zeta \sin \vartheta)^2} \cdot \sin(\omega t + \Delta) \\
 &\quad \left[\tan \Delta = \frac{(\eta \cos \vartheta - \zeta \sin \vartheta)}{\xi} \right]
 \end{aligned} \tag{3}$$

This can be expressed in terms of the wavelength shift λ of the light emitted at time t . [In this discussion, λ , λ_m , $\lambda_{\max}$, etc. will refer to Doppler shifts, and λ^* to the unshifted wavelength.] Thus:

$$\lambda = \lambda_m \sin(\omega t + \Delta) \tag{4}$$

where,

$$\lambda_m = \lambda_{\max} \sqrt{\xi^2 + (\eta \cos \vartheta - \zeta \sin \vartheta)^2}.$$

and,

$$\lambda_{\max} = \lambda^* v_0 / c.$$

$\lambda_{\max}$ is the largest Doppler shift possible for ions at v_0 , obtained by viewing the ion orbit edge on. At other viewing angles, λ_m is less than $\lambda_{\max}$, and is a function of the pitch angle ϑ . λ is the wavelength shift of the light emitted by the ion at a specific, but unknown, time t . The probability of emission at any shifted λ is proportional to the time spent at that λ , and so is the intensity. Since:

$$\omega t + \Delta = \sin^{-1}(\lambda/\lambda_m)$$

then:

$$\begin{aligned}
 \frac{dl}{d\lambda} &\sim \frac{d(\omega t)}{d\lambda} \\
 \frac{dl}{d\lambda} &= \frac{I_0}{\pi \sqrt{\lambda_m^2 - \lambda^2}}. \quad [\text{See Fig. 2}]
 \end{aligned} \tag{5}$$

This rather distinctive shape can be used to determine the value of λ_m , and from that the pitch of the field can be deduced.

Note that the discussion above is for the case in which the particle is initially traveling along the x-axis, and the magnetic field is perpendicular to that axis; that is, for particles on the midplane of the tokamak. For particles injected above or below the midplane, the magnetic field would not be exactly perpendicular, but would have a small component parallel to the x-axis (and to the particle velocity). Then the equation for the velocity along the sightline (from Appendix A) would be :

$$\frac{\mathbf{v} \cdot \mathbf{g}}{|\mathbf{v}_0|} = \frac{\lambda}{\lambda_{\max}} = \sin(\omega t + \Delta) \sqrt{\rho^2 \sin^2(\alpha - \theta) + \left(\xi - \frac{b_x}{b_z} \rho \cos(\alpha - \theta) \right)^2} - \frac{b_x}{b_z} \left(\rho \cos(\alpha - \theta) + \xi \frac{b_x}{b_z} \right), \quad 6$$

where η and ζ have been replaced with:

$$\eta = \rho \sin(\alpha), \quad \zeta = \rho \cos(\alpha).$$

α is the angle between the horizontal midplane and the plane including the beam (which is also in the midplane) and the detector; b_x , b_y , and b_z are components of the unit vector along B.

Equation 6 has both an oscillating term, a function of $v_{\perp}$, and a streaming term [the second term on the right], which is proportional to $v_{\parallel}$. For neutrals injected along the x axis, $v_{\parallel} \approx v_0 b_x$. The effect of the streaming is to shift the spectral pattern by a small but easily measurable amount, λ_c . The half-width λ_m is essentially the same as Eq. 4, with a very small correction:

$$\lambda_m = \lambda_{\max} \sqrt{\rho^2 \sin^2(\alpha - \theta) + [\xi - b_x \rho \cos(\alpha - \theta)]^2}, \quad 6a$$

$$\lambda_c = -\lambda_{\max} b_x [\rho \cos(\alpha - \theta) + \xi b_x]. \quad 6b$$

The width of the pattern is (mostly) a function of the vertical field b_y , but the position of the pattern is (mostly) proportional to b_x . Therefore, the position of the pattern can be used as a diagnostic of the poloidal field in and of itself. A particle moving along the x-axis would encounter $b_x = 0$, and so would have no streaming component when ionized, whereas particles at the top or bottom of the beam would have streaming, because $b_x \neq 0$. Thus there would be a shift of the spectral pattern as a function of height within the beam.

A LOCAL q MEASUREMENT SCHEME

Consider a pair of observation points, vertically above each other, one near the top of the beam, the other near the bottom, as depicted schematically in Fig. 4. These measurements would be on the same flux surface. If we denote $\delta\lambda_c$ as the shift in λ_c between the top and bottom patterns [see Fig. 5], then [neglecting the second order term]:

$$\frac{\delta\lambda_c}{\lambda_{\max}} \approx \delta b_x \rho c \gamma s(\alpha - \delta)$$

$$\approx \delta b_x.$$

For circular flux surfaces, δb_x

Fig. 4 Schematic of q measurement geometry.

Fig. 5 Simulated spectra from a pair of points in TFTR separated vertically by 0.1 m and located at $R = 2.75$ m. The plasma was considered to have $q(0) = 0.85$ and $\Lambda \equiv \beta_p + \frac{q}{2} = 2.5$.

is directly proportional to the local transform $\iota \equiv 1/q$ on that surface:

$$b_x = \iota \frac{y}{R_0}, \quad \text{and} \quad \delta b_x = \iota \frac{\Delta y}{R_0}.$$

where R_0 is the magnetic axis of the flux surface being sampled, and y is the height, and Δy the difference in heights, of the sample point(s) above (and below) the axis [see Fig. 4]. Therefore:

$$\iota = \delta b_x \frac{R_0}{\Delta y} = \delta b_x \frac{R-x}{\Delta y} = \delta b_x \frac{R}{\Delta y} + b_y.$$

Here R is the major radius of the measurement points, and x the horizontal distance from there to the magnetic axis of the flux surface R_0 . Equations (6a) and (6b) can be solved to give b_x and b_y in terms of spectroscopically measured quantities. Thus ι (and q) can be determined exactly:

$$\iota(R) = \frac{\delta \lambda_c}{\sqrt{\lambda_{\max}^2 + 0.25(\delta \lambda_c)^2 - 0.5(\lambda_m^2 + 2\lambda_m^2)}} \cdot \frac{R}{\Delta y} + \tan(\vartheta). \quad 7$$

The second term on the right is less than about 0.1 and negative if R is outside the magnetic axis of the plasma. Near the axis it is very small. ϑ can be determined from the width of either pattern (or the average of the two) and the position along the beam, represented by ξ . x , the distance between R and R_0 , can be determined from the pitch b_y , and thus also both R_0 and $\Delta(r)$, which is the Shafranov shift of the flux surface measured.

$$\tan \vartheta \equiv b_y = \iota \frac{x}{R_0}.$$

This can be rewritten as:

$$x = \Delta y \tan(\vartheta) \cdot \frac{\sqrt{\lambda_{\max}^2 + 0.25(\delta \lambda_c)^2 - 0.5(\lambda_m^2 + 2\lambda_m^2)}}{\delta \lambda_c} \quad 8$$

and

$$\Delta = R - R_m - x,$$

where R_m is the magnetic axis of the outer flux surface [= $R_0(a)$ (or $R_0(b)$, depending upon the tokamak control system)].

The quantities λ_c and λ_m^2 can be determined simply from the spectroscopic data, since they are the first and second moments with respect to λ (normalized to I_0 , the integral over the pattern). λ_m is determined in part by a large position dependent term, so that no general estimate can be given for it: it ranges from 2 to 14 Å. On the other hand, $\delta\lambda_c$ can be estimated:

$$\delta\lambda_c = \lambda \frac{\lambda_{\max} \Delta y}{R_0}.$$

For the TFTR Diagnostic Neutral Beam (DNB) at 50 kV, $\lambda_{\max} = 25\text{Å}$, and $\Delta y \approx 0.1\text{ m}$. Since $R_0 = 2.5\text{ m}$ is a typical value for the magnetic axis in TFTR, then:

$$\delta\lambda_c(\text{Å}) \approx \lambda(r) = 1/q(r).$$

LOCATION OF THE MEASUREMENT POINTS ALONG THE BEAM

The transform λ in the preceding section is a function of some spectroscopic quantities times the major radius R of the measurement point. Therefore, the accuracy with which R is known determines the accuracy of λ . If, for instance, measurements are made by imaging the beam upon a set of fiber optics, the question then is what R is seen by each fiber. This can be determined during a machine opening by backlighting the fiber, and measuring the location of the projected image at the beam. However, if the camera is moved or bumped or installed after the opening, then the fibers are no longer (or not at all) calibrated. Fortunately, R can also be measured spectroscopically.

If the spectrometer is set to look at the light emitted by neutral helium atoms in the injected beam, the observed light will be Doppler shifted, by an amount which is determined by the geometry of the observation point. Specifically:

$$\delta\lambda = \xi \cdot n\lambda_{\max}, \quad 9$$

where ξ , as elsewhere in this paper, is the horizontal component of the direction unit vector $\mathbf{g}$, and $n\lambda_{\max}$ is the maximum shift for the neutral light, analogous to $\lambda_{\max}$ for the ions. ξ by itself is needed in the solution of Eq. (5a), but it is also proportional to distance along the beam, measured from the point R_t at which the viewing window is tangent to the beam. If R_p is the major radius of the viewing window,

and B the toroidal angle between the beam location and the window, then:

$$R_t = R_p \cos(B), \quad 10$$

$$\begin{aligned} x_d &= R_p \sin(B) \sec(\alpha) \frac{\xi}{\sqrt{1-\xi^2}} \\ &= R_p \sin(B) \sec(\alpha) \frac{\delta\lambda}{\sqrt{n\lambda_{\max}^2 - \delta\lambda^2}} \end{aligned}$$

$$\left[R_p \sin(B) \sec(\alpha) = \sqrt{R_p^2 \sin^2(B) + y_p^2} \right].$$

$$R = R_t - x_d.$$

The three quantities R_p , B , and y_p (the height of the port above the midplane) can be determined to within 10^{-3} by surveying the outside of the machine. Therefore, the accuracy with which R is known would be limited by $\delta\lambda$. Typically, $\delta\lambda \sim 1 \text{ \AA}$ for $x_p = 10 \text{ cm}$, so that x_d and R could be determined to $\sim 2.5 \text{ cm}$. This would be 1% accuracy for R (and λ). If the pattern of all the measurement points had been measured during a machine opening, or on a mock-up, then the location of the array of all these points could certainly be determined to within 1 cm. Measuring the pattern would be essential for determining Δy at any rate.

With Helium light, a very bright HeI line at $3889 \text{ \AA}$ would be appropriate for calibrating the location of the fibers, and a dim line at $4713 \text{ \AA}$, which could be measured simultaneously with the $4686 \text{ \AA}$ ion light, would be useful for checking up on the calibration. Any other beam (deuterium, say) could also be used.

ABSOLUTE INTENSITY

At any one point along the He^0 beam path, the beam can be ionized by a variety of processes to form He^+ ions. The rate of formation of these ions is:

$$\dot{n}^+ = n_b \sum_{i=1} n_i \langle \sigma v \rangle_i = n_e n_b \sum_{i=1} d_i \langle \sigma v \rangle_i \quad d_i \equiv \frac{n_i}{n_e} \quad 11$$

Here, the subscript i refers to all the processes involved in ionizing the neutral particles. The symbol $0 \rightarrow 1$ refers to the same thing. It is not the summation limit. The density ratios, d_i , are generally of order unity; for electron processes, $d_i = d_e = 1$.

The rate of loss is expressed by a similar equation:

$$n^* = n_e n^* \sum_{1 \rightarrow 2} d_j \langle \sigma v \rangle_j + n_e n^* \sum_{1 \rightarrow 0} d_k \langle \sigma v \rangle_k \quad 12$$

The double ionization processes $1 \rightarrow 2$, which are indicated by the subscript j , are similar to the corresponding processes in Eq. (11), although the σ 's are smaller. However, the relative density terms are the same; term for term, $d_i = d_j$. There are two neutralization processes: charge exchange on either neutral beam particles or on neutralized plasma ions. These processes are indicated by the subscript k . The relative densities are quite small, $d_k < 10^{-3}$, so that the $1 \rightarrow 0$ sum is much smaller than the $1 \rightarrow 2$ sum.

In equilibrium, the source and loss rates are equal. Thus Eqs. (11) and (12) can be solved for n^* .

$$n^* = n_b \frac{\sum_{0 \rightarrow 1} d_i \langle \sigma v \rangle_i}{\sum_{1 \rightarrow 2} d_j \langle \sigma v \rangle_j + \sum_{1 \rightarrow 0} d_k \langle \sigma v \rangle_k} \quad 13$$

The ion lifetime can also be extracted from Eqs. (11) and (12):

$$n^* = \dot{n}^* \tau$$

$$\tau = \frac{1}{n_e \left[\sum_{1 \rightarrow 2} d_j \langle \sigma v \rangle_j + \sum_{1 \rightarrow 0} d_k \langle \sigma v \rangle_k \right]} \quad 14$$

These processes have been modeled using a Gryzinski³ calculation for the electron and ion impact ionization cross sections, and *ad hoc* formulae for the $0 \rightarrow 1$ and $1 \rightarrow 2$ charge exchange cross sections.^{4,5} [The neutralization ($1 \rightarrow 0$) effects were not included in the calculations shown here.] For ions other than hydrogen, the H^+ impact

ionization and charge exchange cross sections were used, scaled by Z_i^2 , and with E/A_i substituted for the relative energy E .

The plasma was modeled with the usual $(1 - \rho^2)^m$ formula for n_e and T_e , where m is an arbitrary value, and $\rho = r/a$. Different values of m were used for n_e and T_e ; T_i and T_e had different values, but the same shape (same m). The density input was actually a presumed n_{e-l} ; the program then calculated central n_{e0} . The ions were considered to be D^+ and (optionally) C^{+6} . If the Carbon option was chosen, the plasma was presumed to have a base density of C^{+6} ($1.0 \times 10^{19} \text{ m}^{-2}$ for TFTR); any density in excess of this was presumed to be D^+ . Z_{eff} and the various $d_i (= dp, dc)$ were calculated accordingly. The density and temperature functions for a typical calculation are shown in Fig. 6, along with the beam attenuation (meaning remaining beam, actually). The plasma parameters used are shown listed on the right side of the figure. Included is the neutral beam current density, which is calculated according to a $V^{3/2}$ law, with also a model for the neutralization efficiency.⁶

Fig. 6. Beam attenuation (remaining beam) for a beam entering from the right, along with n_e and T_e profiles for the beam energy and plasma conditions listed. A C^{+6} population with a line density $n_{i-l} = 1 \times 10^{19} \text{ m}^{-2}$ was presumed for this calculation.

Fig. 7. He^+ density and lifetimes for the conditions of Fig. 6

The ion densities and lifetimes are shown in Fig. 7. The lifetimes can be seen to be on the order of a few microseconds. If the beam is indeed perpendicular to the magnetic field, then the only parallel velocity component of the He^+ ions would come from the transverse velocity of the ion accelerator and ion source. If the beam spreading is $\sim 1.5^\circ$ ($= 1/40$ radian), then $v_\perp \sim 3\text{--}4 \cdot 10^6$ cm/s. Thus the ions would travel ~ 10 cm in a lifetime. This is comparable to the width of the beam itself. Although this travel would dilute the ion density at the beam location itself, a sight line close to the horizontal midplane ($\alpha \leq 20\text{--}30^\circ$) would pass through regions about this length on either side of the beam region, and would pick up compensating ion light coming from other parts of the beam area.

With ion density in hand, the emission intensity can then be calculated:

$$\dot{n}_{\text{hv}} = n_e n_+ \sum_{\text{exc}} d_i \langle \sigma v \rangle_i \quad \frac{\text{photons}}{\text{cc} \cdot 4\pi \text{ steradian} \cdot \text{sec}} \quad 15$$

Here the index l includes electron and ion impact excitation; these are all variations of one cross section, which is calculated by the Gryzinski model. Prompt emission has been ignored; the prompt cross section is $\sim 1/10$ of this cross section, and relevant density, n_p is also $\sim n^*/10$, so the effect should be relatively small. There are no charge exchange effects. The relative density terms d_i are the same as the corresponding ionization terms d_i .

Fig. 8. Emission intensity for the conditions of Fig. 6, and the intensity at the input of an optical instrument with an étendue of $5 \times 10^{-5} \text{ cm}^2\text{-ster}$ (the beam is presumed to be 10 cm wide).

The intensities shown in Fig. 8 were also calculated using the Gryzinski model of the excitation cross section, along with the plasma model shown in Fig. 6 and the ion densities shown in Fig. 7. The scale on the left is the volume emission intensity, in photons/cm³-ster-sec; the scale on the right gives the intensity in photons/s at the input of an optical device with an étendue of $5 \times 10^{-5} \text{ cm}^2\text{-ster}$ (f/6, 600 μ diameter

fiber). The beam width is presumed to be 10 cm. It should be noticed that many more than 10^6 photons enter the optical system each msec over much of the plasma diameter. This should allow good time resolution with $\sim 1\%$ statistics in the photoelectrons.

Although not shown here, a scan of plasma conditions shows that the intensity is not very sensitive to variations in plasma or beam conditions. Decreasing T_e and T_i to ~ 3 keV produces the most effect, increasing the attenuation, and decreasing the light intensity, but not below 10^6 photons/msec at the plasma center.

SUMMARY

The light emitted by DNB-injected Helium ions can provide a direct measurement of both components of the poloidal magnetic field, b_x and b_y . This light has a peculiar two peaked spectrum. The shift of the spectrum with respect to the natural line location is directly proportional to b_x (for a horizontal DNB), and the width is a function of b_y ; the location and width are determined directly by the first and second moments of the spectrum with respect to λ . For a measurement above the magnetic axis, b_x is proportional to $1 \cdot y$. With a differential measurement between two points above one another, spaced by Δy , $\delta b_x \sim \delta \lambda_c - 1 \cdot \Delta y$, where $\delta \lambda_c$ is the difference in first moments between the top and bottom measurements. λ_c (and more importantly, $\delta \lambda_c$) is thus a direct measurement of the local transform of the flux surface being viewed. The equations for λ_m and $\delta \lambda_c$ can be solved directly to give both the transform 1 and the Shafranov shift Δ .

A Gryzinski calculation of the expected intensity of the emitted light -- for the very bright 4686 Å ion line -- shows that for a wide variety of plasma conditions, the light is indeed intense enough so that the measurement time could be milliseconds.

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APPENDIX A GENERAL EQUATIONS.

The complete general formula for the ion velocity is:

$$\begin{aligned}
 \mathbf{v} = & i\{b_x v_{||} + \cos\omega t[v_{x0}(b_y^2 + b_z^2) - b_x(v_{y0}b_y + v_{z0}b_z)] \\
 & + \sin\omega t(v_{y0}b_z - v_{z0}b_y)\} \\
 & + j\{b_y v_{||} + \cos\omega t[v_{y0}(b_x^2 + b_z^2) - b_y(v_{x0}b_x + v_{z0}b_z)] \\
 & + \sin\omega t(v_{z0}b_x - v_{x0}b_z)\} \\
 & + k\{b_z v_{||} + \cos\omega t[v_{z0}(b_x^2 + b_y^2) - b_z(v_{x0}b_x + v_{y0}b_y)] \\
 & + \sin\omega t(v_{x0}b_y - v_{y0}b_x)\}
 \end{aligned} \tag{A1}$$

where,

$\mathbf{b} = ib_x + jb_y + kb_z$ is a unit vector parallel to $\mathbf{B}$, and

$$v_{||} = \mathbf{b}(\mathbf{b} \cdot \mathbf{v}_0) = b(b_x v_{x0} + b_y v_{y0} + b_z v_{z0})$$

For the case $v_0 = -v_{x0}$, $v_{y0} = v_{z0} = 0$, and, as before, with the unit vector $\mathbf{g} (= i\xi + j\eta + k\zeta)$ pointing from the ion to the observation point:

$$\begin{aligned}
 \frac{\mathbf{v} \cdot \mathbf{g}}{|\mathbf{v}_0|} = & -\cos\omega t[\xi(b_y^2 + b_z^2) - b_x(\eta b_y + \zeta b_z)] + \sin\omega t(\eta b_z - \zeta b_y) \\
 & - b_x(\xi b_x + \eta b_y + \zeta b_z)
 \end{aligned} \tag{A2}$$

$$\begin{aligned}
 = & \sin(\omega t + \Delta) \sqrt{(\eta b_z - \zeta b_y)^2 + [\xi(1 - b_x^2) - b_x(\eta b_y + \zeta b_z)]^2} \\
 & - b_x[\xi b_x + (\eta b_y + \zeta b_z)]
 \end{aligned}$$

$$= \lambda / \lambda_{\max}.$$

This can also be written as:

$$\frac{\mathbf{v} \cdot \mathbf{g}}{|\mathbf{v}_0|} = \sin(\omega t + \Delta) \sqrt{\rho^2 \sin^2(\alpha - \gamma) + \left(\xi - \frac{b_x}{b_z} \rho \cos(\alpha - \gamma) \right)^2} \\ - \frac{b_x}{b_z} \left(\rho \cos(\alpha - \gamma) + \xi \frac{b_x}{b_z} \right) \quad \text{A3}$$

using the pitch angle γ and the detector angle α as used in the preceding sections. [Note: the approximation $b_z = 1 = \sec \gamma$ was used to derive Eq. (A3) from Eq. (A2).]

This equation differs from the equivalent equation used earlier, mainly in that there is a streaming term [the second term on the right], which is proportional to b_x . A particle moving along the x -axis would encounter $b_x = 0$, and so would have no streaming component when ionized. However, particles at the top or bottom of the beam would encounter $b_x \neq 0$, and would have streaming.

This streaming would result in a shift of the spectral pattern as a function of height within the beam, and, since the effect is not small, can be used as a diagnostic of the poloidal field in and of itself.

APPENDIX B MAGNETIZED THERMAL IONS

In the unmagnetized thermal case, the ion velocity $\perp$ to the sight line is ignorable, and to get the relative intensity at λ ($\rightarrow u$), one integrates over the two ignorable velocity coordinates v_y and v_z . Thus:

$$\frac{dI}{du} = \frac{I_0}{\pi^{3/2} v_{th}^3} e^{-(u/v_{th})^2} \int_{-\infty}^{\infty} dv_y e^{-(v_y/v_{th})^2} \int_{-\infty}^{\infty} dv_z e^{-(v_z/v_{th})^2} \quad \text{B1} \\ = \frac{I_0}{\sqrt{\pi} v_{th}} e^{-(u/v_{th})^2},$$

where here, $u = v_x$. Thus, the perceived distribution function dI/du is the component of the real distribution function along the sight line.

In the magnetized thermal case, if the sight line is $\perp$ to $\mathbf{B}$, then an ion with speed $v > |u|$ has a velocity u along the sight line for only a small fraction of each orbit. However, ions with other speeds will also have velocity u for some fraction of time. In order to calculate what the perceived distribution function is, one must integrate the magnetized intensity function over all three velocity components (v_x, v_y, v_z) with u as a parameter. In doing this we note that one of the components ($v_z \parallel \mathbf{B}$) is ignorable, but that v_y feeds u just as much as v_x , and this intensity function must be inside both integrals.

In terms of velocity u , the intensity function [Eq. (5)] of a single gyrating ion with orbital speed v ($v \geq u$) is:

$$\frac{dl}{du} = \frac{l_0}{\pi \sqrt{v^2 - u^2}} \quad \text{where } \lambda_m \rightarrow v$$

$$\text{and } \lambda \rightarrow u.$$

To get the results we want, we must integrate this over a Maxwellian distribution function.

$$\int (dl, du) = \int (l_0, \pi^2 \sqrt{\pi} v \lambda_0(t_h, j)) \int (-\infty, \infty, dv_z e^{-\frac{(v_z/v_{th})^2}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dv_x e^{-(v_x/v_{th})^2} dv_y e^{-(v_y/v_{th})^2}}{\sqrt{v^2 - u^2}} \quad B2$$

$$v^2 = v_x^2 + v_y^2$$

Converting the v_x and v_y integrals to cylindrical coordinates, v and ϕ , and integrating over both ϕ and v_z :

$$\frac{dl}{du} = \frac{l_0}{\pi v_{th}^2} \int_u^\infty \frac{dv^2 e^{-(v/v_{th})^2}}{\sqrt{v^2 - u^2}}$$

$$= \frac{l_0 u}{\pi v_{th}^2} \int_1^\infty \frac{dx^2 e^{-x^2(u/v_{th})^2}}{\sqrt{x^2 - 1}} \quad x \equiv \frac{v}{u}$$

$$= \frac{I_0 u}{\sqrt{\pi} v_{th}} \int_1^{\infty} \frac{dt e^{-st}}{\Gamma(\frac{1}{2}) \sqrt{t-1}} \quad t \equiv x^2 \quad B3$$

$$s \equiv (u/v_{th})^2$$

The last integral is the LaPlace Transform of $\frac{e^{-s}}{\sqrt{s}}$. Thus:

$$\frac{dl}{du} = \frac{I_0}{\sqrt{\pi} v_{th}} e^{-(u/v_{th})^2}. \quad B4$$

Because the LaPlace transform returned a Gaussian function from our input Maxwellian distribution function, the results are the same as the unmagnetized case. This is fortuitous. If the distribution function were not Maxwellian, the integral would return a function of u which would not have the same functional form as the distribution function itself. This is a very different situation from the unmagnetized case, where one integrates over two ignorable coordinates, and retains the part of the distribution function along the sight line, whatever its functional form.

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