NOTICE

**CERTAIN DATA
CONTAINED IN THIS
DOCUMENT MAY BE
DIFFICULT TO READ
IN MICROFICHE
PRODUCTS.**

Received JTI

JAN 08 1991

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

LA-UR--90-4168

DE91 005895

TITLE: USE OF A GENETIC ALGORITHM TO ANALYZE ROBUST STABILITY PROBLEMS

AUTHOR(S): T. M. Murdock
W. E. Schmitendorf
S. Forrest

SUBMITTED TO: for the Proceedings of the American Automatic Control Conference,
to be held June 91, in Boston, MA.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

MASTER

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Use of a Genetic Algorithm to Analyze Robust Stability Problems

T. M. Murdock, W. E. Schmitendorf

University of California-Irvine

Irvine, CA 92717

and

S. Forrest

University of New Mexico

Albuquerque, NM 87131

Abstract

This note presents a genetic algorithm technique for testing the stability of a characteristic polynomial whose coefficients are functions of unknown but bounded parameters. This technique is fast and can handle a large number of parametric uncertainties. We also use this method to determine robust stability margins for uncertain polynomials. Several benchmark examples are included to illustrate the two uses of the algorithm.

1 Introduction

Stability of polynomials plays an important part in the analysis and design of control systems. A standard approach to robustness analysis of linear dynamic systems is to examine the characteristic polynomial in the presence of parametric uncertainties. The characteristic polynomial is of the form

$$P(s, \mathbf{q}) = \sum_{m=0}^n a_m(\mathbf{q}) s^m \quad (1)$$

where $\mathbf{q} = [q_1, q_2, \dots, q_l]^T$ is a vector of uncertain parameters which are contained in the compact set $\mathbf{Q} = \{\mathbf{q} : q_i \in [q_i^- ; q_i^+] ; i = 1, 2, \dots, l\}$. The coefficient functions $a_m(\mathbf{q})$ can vary linearly, multilinearly or polynomially with $\mathbf{q}$. Although it exceeds the scope of this paper, we believe the algorithm can be applied to almost all functions $a_i(\mathbf{q})$.

For a particular value of $\mathbf{q}$, the polynomial is said to be **stable** if all of its roots lie in the strict left half complex plane. Similarly, in order to be **robustly stable**, the roots of the polynomial must be contained in the strict left half plane $\forall \mathbf{q} \in \mathbf{Q}$. This means that the maximum real root component of the roots (hereafter referred to as the maximum root) of the characteristic polynomial must also lie in the left half plane $\forall \mathbf{q} \in \mathbf{Q}$. In this paper an optimization scheme known as a genetic algorithm (GA) [1] is applied to this problem of finding the maximum root of $P(s, \mathbf{q}) = 0 \forall \mathbf{q} \in \mathbf{Q}$. If this maximum root is positive then the polynomial is unstable, otherwise we conclude the polynomial is stable. The algorithm allows

for interval, linear affine, multilinear, and polynomial coefficient functions in q . The algorithm will also determine the vector q_{max} , which yields the maximum root. Polynomials of the form 1 also arise when analyzing the stability of a system in state space form. If $\dot{x}(t) = A(q)x(t)$, then the characteristic polynomial $P(s, q) = \det[sI - A(q)] = \sum_{m=0}^n a_m(q)s^m$ and this polynomial is used in checking stability.

In addition to the application of testing stability for a specified Q , we present an extension of this technique which is applied to the problem of measuring the robust stability margin of characteristic polynomials with uncertain coefficient functions. The robust stability margin is denoted in this paper by $\bar{r}$, where $\bar{r}$ is the largest value of r such that $P(s, q)$ is stable for all $|q_i| \leq \bar{r}$. It is also commonly thought of as the largest hypercube in parameter space which yields robust stability. The most important aspects of the genetic algorithm are that it presents a numerically fast method of testing robust stability for characteristic polynomials with any type of coefficient function that varies polynomially in q , and it can treat a large number of uncertain parameters.

2 Background

Techniques based on necessary and sufficient conditions exist for stability determination of characteristic polynomials whose coefficient functions are interval polynomials or linear affine functions in q . In the case of interval polynomial coefficients, Kharitonov's theorem is necessary and sufficient [2]. It allows us to determine the stability of the entire Q -box by checking the stability of only four polynomials. The Edge Theorem is a necessary and sufficient test for problems with $a_i(q)$ linear affine functions (as well as the interval polynomial case). The Edge Theorem states that we can test stability of the entire Q -box simply by testing the stability of the exposed edges of the Q -box [3].

Tests for cases where the coefficient functions are multilinear or polynomial in q are also available. One such test is the concept of zero inclusion [4]. This occurs when the value set of a polynomial, which is obtained by mapping the Q -box into the complex plane for a given complex point $\hat{s}$, contains the origin $0 + 0i$. The zero inclusion test itself is necessary and sufficient for all classes of polynomials mentioned above; however the implementations of a zero inclusion test, such as the H-function test of Barmish [5], are not. Another test which is necessary and sufficient for any form of coefficient function is the robust root locus. The robust root locus is a plot of the root space of the characteristic polynomial $\forall q \in Q$. It can be generated using a zero inclusion test [6], or by brute force gridding of the Q -box. The zero inclusion and robust root locus tests are necessary and sufficient for coefficient functions which are interval, linear affine, multilinear or polynomial in q . The value set, however, may be difficult to determine (i.e. may require gridding). Use of the zero inclusion test simplifies in the case of interval and linear affine polynomials, since the value set is equivalent to the convex hull of the vertices of the Q -box mapped to the complex plane. Thus, the problem of checking for zero inclusion in these cases reduces to checking whether zero is included in the convex hull (a much simpler check). For the multilinear case, the value set is contained within the convex hull and thus may yield conservative results (i.e. may conclude unstable when stable). For the polynomial case, these simplifications do not hold and use of the convex hull may be completely in error (i.e. may conclude stable when unstable).

All of the techniques previously mentioned work well for interval and linear affine functions in q , but they are at best conservative tests for multilinear functions and can completely fail when the coefficient functions are polynomial. Domain splitting techniques [7] can be used to reduce the conservatism for multilinear cases, and can help increase accuracy for polynomial cases as well. Additional stability tests which eliminate conservatism for polynomials with multilinear and polynomial coefficient functions are also available [8,9]. The computer-aided method presented in [8] is based on the parameter space approach developed in [10]. It is promising in that it can handle polynomial coefficient functions although it still may suffer from extensive computation time if a large number of uncertain parameters are present or dense gridding is required. This method is extensively evaluated in [10,11]. Another method of computer-aided stability analysis uses a value set approach which requires mapping the $\mathbf{Q}$ -box into the complex plane for selected frequencies and checking for zero inclusion. This method is attractive for polynomials with polynomial coefficient functions, but it will work only if the polynomial in question admits to a tree structured decomposition [9]. Note that it is not always possible to determine a tree structured decomposition.

In general, for polynomials with large numbers of parametric uncertainties [12], most stability tests become computationally intractable. Large numbers of perturbations can influence the effectiveness of a test by creating a "combinatoric explosion" in the number of tests to be performed. Barnish points out in [13] that when using the edge test on linear affine polynomials, a combinatoric explosion in the number of exposed edges as a function of the number of perturbation parameters l can occur. That is, for an l -dimensional $\mathbf{Q}$ -box, $l2^{l-1}$ edges must be checked for stability. Combinatoric explosions in calculations are more evident when using the brute force approach of gridding the $\mathbf{Q}$ -box and finding the roots at every point. Calculation times for using this technique can grow large even for a small number of parameters when a dense grid is being used. This type of test is used for comparison in cases where no necessary and sufficient condition test is appropriate. Here we show that a GA based stability test will solve problems for which only conservative results could be obtained using existing tests. In addition, this technique is faster than existing techniques. It avoids combinatoric explosions in calculations caused by the presence of a large number of uncertainties. Lastly, it solves benchmark problems which challenge the most current analytical stability tests.

The problem of parametric perturbations and their effect on stability margin has also been widely studied. For the class of interval matrices, sufficient conditions for stability are given in [14,15], where Lyapunov approaches are used, and also in [16] where the Gershgorin theorem is applied. Lyapunov based approaches can also be used on linearly structured matrix perturbations to derive stability spheres in parameter space [17]. When the uncertain parameters occur polynomially in matrix entries, references [18] and [19] offer a complete solution to checking eigenvalue location in the presence of one or two uncertain parameters. However, [18] states that when the number of uncertain parameters exceeds two, this technique becomes cumbersome from a computational burden point of view. More recently, an algorithm for determining stability perturbation bounds for systems with uncertainties entering the state equation matrix nonlinearly has been presented [20]. This algorithm transforms the stability margin nonlinear problem into a modified signomial programming problem [21] which offers consistent convergence speed improvements and can handle larger numbers of parametric uncertainties. However, it is still slow compared to the GA and, we believe, more complicated to use.

Using the GA, we show that the robust stability margin can be determined for any of the previously mentioned classes of coefficient functions. The GA technique is also much faster and more generalized than current methods. This implies that stability margins can be calculated for characteristic polynomials with large numbers of uncertain parameters as well. It also seems reasonable that this technique can be extended to determine a robust stability margin based on a specified region, Γ , in the complex plane (Γ -stability).

3 Genetic Algorithm

Genetic algorithms were initially developed by Holland [22] and comprehensive descriptions of GAs are found in [1,23]. A GA is a search algorithm based on the principles of Darwinian evolution. GAs work by selecting a random sample of string structures ($\mathbf{q}$), evaluating their worth (determined by the user in a fitness function), and stochastically selecting a new sample based on the information provided by the previous iteration. The algorithm combines the principle of natural selection (by eliminating the least fit structures from the sample) with "genetic" operators such as mutation and cross-over which choose the new sample points at each iteration. We used the GENESIS implementation [24,25] of the GA for the reported work.

The GA is used in stability determination as an objective function minimization routine. That is, the algorithm determines the structure (vector $\mathbf{q}$) that corresponds to the minimum fitness evaluation. The user supplies the fitness function as well as the bounds and precision on the domain of structures (e.g., uncertain parameters q_i). The uncertain parameters are converted to bit strings (structures) for processing by the GA. The conversion process involves several steps: (1) mapping the bounds and precision of each parameter into discrete intervals, (2) producing the binary encoding of the interval corresponding to each q_i in the vector, (3) converting each binary encoding to its Grey code equivalent, and (4) concatenating the resulting Grey codes to produce one bit string. The precision of the uncertain parameters is controlled by the length of the bit string (i.e., the longer the bit string, the more precise the parameters are). In our application, the fitness function consists of the uncertain coefficients (in $\mathbf{q}$) of the characteristic polynomial, a root finding algorithm, and a root sorting scheme to determine the maximum root (root furthest to the right). The structures are sent to the fitness function, where the maximum root is determined and retained for the set of roots corresponding to each $\mathbf{q}$. The maximization problem is converted to a minimization problem by multiplying the maximum root by negative unity. Thus, by finding the vector $\mathbf{q}_{max}$ that corresponds to the best (lowest) fitness, we are actually finding the vector that produces the maximum root.

The $\mathbf{Q}$ -box is specified by the bounds on the individual components of the $\mathbf{q}$ vector. Several random points in the $\mathbf{Q}$ -box are chosen as an initial population of structures, and the maximum root is determined for each vector $\mathbf{q}$. The position of the maximum root determines the fitness of the vector $\mathbf{q}$. The GA searches the $\mathbf{Q}$ -box for the global optimum by increasing the chance of reproducing fit structures and reducing the chance of reproducing unfit structures. The genetic operators "cross-over" and "mutation" are used during reproduction of each new generation. The cross-over operator exchanges characteristics (contiguous bits) between two fit structures in the same manner that cross-over among chromosomes exchanges genetic information during meiosis. The cross-over operator allows

the search to avoid converging on local optima by making structured long jumps in the search space. Mutation, on the other hand, selects bits randomly and flips their values. This serves as a randomization procedure and prevents the search from becoming frozen at certain values (i.e., if all the structures in a generation have bit position 6 set to 0, then without mutation, bit position 6 would be set to 0 in all successive generations). The rates at which cross-over and mutation are applied are user-specified, although the mutation rate is typically much lower than the cross-over rate. The GA is a surprisingly fast and accurate technique for searching the $\mathbf{Q}$ -box to obtain the vector $\mathbf{q}$ which yields the maximum root.

Since the GA is a heuristic sampling algorithm designed to escape local optima, it does not suffer from the problems associated with techniques based on derivatives or line searches. By combining the cross-over and mutation operators, the GA has a much better chance of eventually finding a global optimum. For example, assume the GA is searching a two-dimensional $\mathbf{Q}$ -box. If it begins converging on a local optimum (by reproducing the most fit individuals), the randomness inherent in the cross-over and mutation operators will cause the GA to randomly search elsewhere in the $\mathbf{Q}$ -box to avoid overlooking other optima. While this gives us a high degree of confidence that the global optima is eventually found, it should be noted that there is no guarantee that the GA will not become trapped in a local optimum.

4 Stability Analysis

Robust stability analysis of polynomials with interval and linear affine coefficient functions using the GA was verified against known results (using Kharitonov's theorem and the Edge test). The GA was able to correctly determine whether or not a polynomial was stable. The results for multilinear and polynomial coefficient functions are much more interesting because no analytic results are available for checking stability. In this section, we present our results for several such examples.

Example 1: A Multilinear Polynomial

A classic example of a multilinear polynomial which is difficult to solve using known methods is that of the unstable enclave [10,11]. The challenge of this example is that an unstable ball is contained in the $\mathbf{Q}$ -box, while the edges of the $\mathbf{Q}$ -box remain stable. The enclave can be made arbitrarily small and avoids detection by edge tests and brute force gridding.

Consider the third order characteristic polynomial with l parametric uncertainties q_i

$$P(s, \mathbf{q}) = a_0(\mathbf{q}) + a_1(\mathbf{q})s + a_2\mathbf{q}s^2 + s^3 \quad (2)$$

where

$$\begin{aligned} a_0(\mathbf{q}) &= l(l-1) + r^2 + 2(l+1) \sum_{i=1}^l q_i + 2 \sum_{i < j}^{1, \dots, l} q_i q_j \\ a_1(\mathbf{q}) &= l + \sum_{i=1}^l q_i \\ a_2(\mathbf{q}) &= a_1(\mathbf{q}) \end{aligned} \quad (3)$$

The Routh criteria of $a_1 a_2 - a_0 > 0$ yields the stability constraint $\sum_{i=1}^l (q_i - 1)^2 > r^2$, thus r is the radius of a ball of unstable points in $\mathbf{Q}$ that is centered at $\mathbf{q} = [1, 1, \dots, 1]^T$. Note that if $r = 0$, then a single unstable point exists at $\mathbf{q} = [1, 1, \dots, 1]^T$, and if r^2 is replaced by any negative number the polynomial becomes stable $\forall \mathbf{q} \in \mathbf{Q}$. This example is interesting because it allows us to vary the radius of the unstable ball r as well as the number of uncertain parameters l . The accuracy of the GA can be evaluated by shrinking the enclave to be arbitrarily small. In addition, its ability to handle a large number of parameters can be tested by increasing l .

The accuracy of the uncertain parameters in this example was set to twelve bits of resolution per q_i which divides each user-specified parameter range into 2^{12} (4096) points. Results of applying the GA to this example show that, for two parameters, an enclave of radius $r = 1 \times 10^{-5}$ can be detected. When the radius of the enclave was set to zero, the GA verified that the system was unstable and the vector corresponding to the maximum root was determined to be $\mathbf{q} \approx [1, 1]^T$. Conversely, when the term r^2 was replaced with -1×10^{-4} , the GA verified that the polynomial was stable. Figure 1 shows the evolution of the best fitness (maximum root) and the average fitness (average of the maximum roots of the entire population) over 300 generations for the two parameter case with the enclave set to zero.

Next, the problem was considered with a large number of parameters. For 10 uncertain parameters, an enclave of radius $r = 1 \times 10^{-2}$ was detected (Figure 2). More importantly, using the GA with the same enclave radius, we were able to detect instability for up to 50 uncertain parameters! This proves that the GA can handle a larger number of parametric uncertainties than any other stability test to date.

The size of the minimum radius that was detected for the two parameter case does not represent limitations of the GA. Instead, the minimum radius is used to determine a grid size necessary to detect the enclave using a brute force gridding technique. The GA is compared to gridding because the domain of the fitness function is effectively gridded by setting the length of the bit strings representing the q_i 's. It accomplishes its savings over gridding by sampling points in a more intelligent fashion, thus adding some of the innovative flair of a human search. By comparing the GA with brute force gridding, we can illustrate that the GA avoids a combinatoric explosion in calculations. In the two parameter case with the enclave radius $r = 1 \times 10^{-5}$, gridding the intervals $q_1 = [0.0; 2.2]$, $q_2 = [0.0; 3.0]$ would require a grid of 6.6×10^{10} gridpoints. On an Apollo 3500, assuming 3ms for coefficient and root calculations, the computational effort would take two years. The calculation time needed for the GA in the two parameter case was under two minutes on the Apollo 3500. In the case of ten parametric uncertainties, the GA required approximately six minutes to run. For fifty parameters, the unstable enclave was detected in thirty minutes on the same Apollo 3500 machine. Granted that other methods previously mentioned are superior to brute force gridding in terms of calculation time, they may still yield conservative results for a polynomial with multilinear coefficient functions in q . Certainly, none of the methods mentioned earlier are capable of handling a multilinear problem with 50 uncertain parameters. The GA, on the other hand, is both fast (50 uncertain parameters in thirty minutes) and applicable to a more general class of problems.

Example 2: A Polynomial Polynomial

Using another example from Ackermann [8], consider the following ninth order poly-

mial with the coefficient functions

$$\begin{aligned}
a_9(\mathbf{q}) &= 1 + 0.02(q_1^2 q_2 q_3^2 q_4^2) \\
a_8(\mathbf{q}) &= 51(1 + 0.04(q_1^2 q_2 q_3 q_4)) \\
a_7(\mathbf{q}) &= 1156(1 + 0.05(q_1^2 + q_2 + q_3)q_4) \\
a_6(\mathbf{q}) &= 15260(1 + 0.05(q_1^2 + q_2 + q_4)q_3) \\
a_5(\mathbf{q}) &= 128963(1 + 0.05(q_1^2 q_4 + q_2 q_3)) \\
a_4(\mathbf{q}) &= 721505(1 + 0.05(q_1^2 q_3 + q_2 q_4)) \\
a_3(\mathbf{q}) &= 2664608(1 + 0.05(q_1 + q_2 + q_3)q_4) \\
a_2(\mathbf{q}) &= 6253816(1 + 0.05(q_1 + q_2 + q_4)q_3) \\
a_1(\mathbf{q}) &= 8445840(1 + 0.1(q_1 q_4 + q_2 q_3)) \\
a_0(\mathbf{q}) &= 4934800(1 + 0.1(q_1 + q_2 + q_3 + q_4))
\end{aligned} \tag{4}$$

in [8], the parameter intervals for q_3 and q_4 are fixed at $[-1.0; 1.0]$ and it is shown that the polynomial is stable for $q_1 \in [-0.8; 1.8]$, $q_2 \in [-0.8; 1.2]$. Using these intervals and the GA algorithm, the maximum real component of the root was found to be -1.239×10^{-1} , indicating stability (Figure 3), and occurred for $\mathbf{q} = [-0.79492, -0.79902, -0.99707, 0.99601]^T$. By changing the q_1 interval to $q_1 \in [-1.1; 1.8]$, the polynomial became unstable (Figure 4) and the maximum real root component, 2.675×10^{-2} occurred at $\mathbf{q} = [-1.1, -0.8, -1.0, 1.0]^T$. Thus, the GA can effectively analyze this polynomial with polynomial coefficients.

The computation time required for this example was approximately fifteen minutes on the Apollo 3500 for both the stable and unstable cases. The computer-aided method presented in [8] takes two hours to run on the Apollo 3500 for the same example, but also provides more information for stability bounds when only two of the parameters are allowed to vary, but the other two are fixed.

While the GA provides less information in the two parameter variation case, it does allow us to vary all four parameters simultaneously instead of fixing q_3 and q_4 and examining the remaining parameters. This gives us the added benefit of being able to find the largest hypercube for which the above polynomial is stable ($\bar{r} = 0.955$). (The computation of robust stability margin is discussed more fully in the next section.) This versatility can ultimately give us more information about the limiting stability values for each individual parameter.

5 Robust Stability Margin

From the previous examples, we see that this technique can be extended to determine the robust stability margin for a system (i.e. the largest hypercube in parameter space for which the characteristic polynomial is stable). Using the GA, the stability margin for a given characteristic polynomial can be established through simple iteration. The hypercube is enlarged proportionally in increments until the characteristic polynomial becomes unstable. The hypercube is then reduced in smaller increments until the polynomial becomes stable again. The accuracy of the stability margin is controlled by the number of iterations the user makes. Calculation times vary, depending on the order of the polynomial more than the number of parameters since the calculation of the roots of a high order polynomial can

be time consuming. Two examples taken from [7,20,27], illustrate the effectiveness of using the GA to compute robust stability margin.

Example 3: A Multilinear Coefficient Polynomial

Consider the following coefficient functions for a fourth order monic polynomial.

$$\begin{aligned} a_0(\mathbf{p}) &= 2p_1 \\ a_1(\mathbf{p}) &= 10p_2p_3 + p_1 \\ a_2(\mathbf{p}) &= p_2p_3 + 10p_2 + 10p_3 \\ a_3(\mathbf{p}) &= 10 + p_2 + p_3 \end{aligned} \tag{5}$$

with

$$\begin{aligned} p_1 &= 800 + 800q_1 \\ p_2 &= 4 + 2q_2 \\ p_3 &= 6 + 3q_3 \end{aligned} \tag{6}$$

This example arose in the determination of the l_∞ stability margin for a closed loop system [27]. We would like to determine the largest $\bar{r}$ where $|q_i| \leq \bar{r}$; $i = 1, 2, \dots, l$ for which the fourth order polynomial. $P(s, \mathbf{p})$ is stable. By iteration, we find that the polynomial is unstable for $\bar{r} = 0.3418$ and stable for $\bar{r} = 0.3416$. One could iterate even further for a more exact measure of stability margin. This compares well to the results of [20], where $\bar{r}$ was found to be 0.3417. The computational time for this example cannot be compared to that of the algorithm presented in [20], as the GA was run on a different system.

Example 4: A Polynomial Coefficient Polynomial

In [7], a fourth order monic polynomial is studied with the coefficient functions

$$\begin{aligned} a_0(\mathbf{p}) &= p_3^3 \\ a_1(\mathbf{p}) &= p_1 p_2^3 p_3^2 \\ a_2(\mathbf{p}) &= p_1^2 p_2^2 p_3 \\ a_3(\mathbf{p}) &= p_1^3 p_2 \end{aligned} \tag{7}$$

where

$$\begin{aligned} p_1 &= 1.4 + 0.25q_1 \\ p_2 &= 1.5 + 0.2q_2 \\ p_3 &= 0.8 + 0.2q_3 \end{aligned} \tag{8}$$

Using the GA and iterating to find the largest hypercube yields a stability margin of $\bar{r} = 1.09$. Once again, the computing time depends on the accuracy desired by the user and is difficult to compare with the algorithm in [20]; however, we believe the GA is simpler conceptually and can accommodate larger numbers of parametric uncertainties.

6 Conclusions

In this paper we have addressed the problem of robust stability analysis of linear dynamic systems. We used a genetic algorithm to find the maximum root of the system's characteristic polynomial in the presence of uncertain parameters. We believe that it is superior to present stability tests for the following reasons: (1) The coefficient functions may be any of the four classes of polynomial functions (interval, linear affine, multilinear, polynomial) mentioned previously. (2) The coefficient functions can contain a large number of parameters without producing a combinatoric explosion in calculations. (3) The GA is an intelligent search which produces results faster than existing techniques.

This algorithm can be used to determine robust stability for polynomials which previously presented difficulties due to a combinatoric explosion in the number of calculations involved, or to the lack of an appropriate necessary and sufficient condition test. In addition, the GA minimization scheme was extended to determine the robust stability margin of characteristic polynomials with polynomial coefficient functions in q . It proved to function as well as complicated numerical techniques [7,20,27], which are not nearly as versatile in the types of problems they can handle. The GA can be extended to Γ -stability problems and shows promise of being applicable to any controls problem which lends itself to a minimization scheme. While we have not shown that the GA is the best algorithm, our success on many examples shows that it merits consideration in analyzing robust stability problems.

7 Acknowledgement

This research was supported by Los Alamos National Laboratory under the INCOR Program. The authors thank Dr. David K. Campbell for his support under this program.

References

- [1] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, Mass., 1989.
- [2] V. L. Kharitonov, "Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations," *Differential'nye Uravneniya*, Vol. 14, No. 11, pp.1483-1485, 1978.
- [3] A. C. Bartlett, C. V. Hollot, and L. Lin, "Root Locations of an Entire Polytope of Polynomials: It Suffices to Check the Edges," *Mathematics of Control, Signals, and Systems*, Vol. 1, pp. 61-71, 1987.
- [4] L. A. Zadeh and C. A. Desoer, *Linear System Theory*, McGraw Hill, 1963, Section 9.17.
- [5] B. R. Barmish, "New Tools for Robustness Analysis," *Proc. IEEE Conf. Decision and Control*, Austin, Texas, pp. 1-6, Dec. 1988.
- [6] B. R. Barmish and R. Tempo, "The Robust Root Locus," *Proc. IEEE Conf. Decision and Control*, Austin, pp.1386-1391, Dec. 1988.

- [7] A. Sideras and R. S. Pena, "Fast Computation of the Multivariable Stability Margin for Real Interrelated Uncertain Parameters," *IEEE Trans. Auto. Contr.*, Vol. 34, pp. 1272-1276, 1989.
- [8] J. E. Ackermann and W. Sienel, "Computer-aided Robustness Analysis for Characteristic Polynomials with Polynomial Parameter Dependence," *Proc. IEEE Conf. Decision and Control*, Tampa, Fla., pp.1934-1935, Dec. 1989.
- [9] B. R. Barinich, J. Ackermann, and H. Z. Hu, "The Tree-Structured Decomposition, A New Approach to Robust Stability Analysis," DLR-Report 515-89-10, 1989.
- [10] J. E. Ackermann, D. Kaesbauer and R. Muench, "Robust Γ -stability Analysis in a Plant Parameter Space," DLR-Report 515-89-12, 1989, to appear in *Automatica*.
- [11] J. E. Ackermann, H. Z. Hu, and D. Kaesbauer, "Robustness Analysis: A Case Study," *Proc. IEEE Conf. Decision and Control*, Austin, Texas, pp. 123- 128, Dec. 1989.
- [12] J. E. Ackermann and W. Sienel, "What is a "Large" Number of Parameters in Robust Systems?," *Proc. IEEE Conf. Decision and Control*, Honolulu, Hawaii, Dec. 1990.
- [13] B. R. Barmish, "A Generalization of Kharitonov's Four Polynomial Concept for Robust Stability Problems with Linearly Dependent Coefficient Perturbations," *Proc. Amer. Contr. Conf.*, Atlanta, Ga.,1988.
- [14] R. K. Yedavalli, "Perturbation Bounds for Robust Stability in Linear State Space Models," *Int. J. Contr.*, Vol. 42, pp.1507-1517, 1985.
- [15] K. Zhou and P. Khargonekar, "Stability Robustness Bounds for Linear State- Space Models with Structured Uncertainty," *IEEE Trans. Automat. Contr.*, Vol. AC-32, pp. 621-623, 1987.
- [16] M. B. Argoun, "On Sufficient Conditions for the Stability of Interval Matrices," *Int. J. Contr.*, Vol. 44, pp. 1245-1250, 1986.
- [17] L. H. Keel, S. P. Bhattacharyya, and J. W. Howze, "Robust Control with Structured Perturbations," *IEEE Trans. Automat. Contr.*, Vol. 33, pp. 68-78, Jan. 1988.
- [18] A. Tesi and A. Vincino, "Robust Stability of State-Space Models with Structured Uncertainties," *IEEE Trans. Automat. Contr.*, Vol. 35, Feb. 1990.
- [19] R. Genesio and A. Tesi, "Results on the Stability Robustness of Systems with State Space Perturbations," *Syst. Contr. Lett.*, Vol. 11, pp. 39-47, 1988.
- [20] A. Vicino, A. Tesi, and M. Milanese, "Computation of Nonconservative Stability Perturbation Bounds for Systems with Nonlinearly Correlated Uncertainties," *IEEE Trans. Automat. Contr.*, Vol. 35, No. 7, pp. 835-841, July 1990.
- [21] J.E. Falk, "Global Solutions of Signomial Programs," George Washington Univ., Washington, DC, Tech. Rep. T-274, 1973.

- [22] J. H. Holland, *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, MI, 1975.
- [23] K. Kirshnakumar and D. E. Goldberg, "Genetic Algorithms in Control System Optimization," *AIAA Guidance, Navigation and Control Conf.*, pp.1568-1577, Portland, OR, Aug.1990.
- [24] J. J. Grefenstette, "GENESIS: A system for using genetic search procedures," *Proc. of the 1984 Conf. on Intelligent Systems and Machines*, pp. 161-165.
- [25] J. J. Grefenstette, "A user's guide to GENESIS," (Technical Report No. CS-84-11), Nashville; Vanderbilt University, Department of Computer Science.
- [26] J. E. Ackermann, "Does it Suffice to Check a Subset of Multilinear Parameters in Robustness Analysis?," Department of Electrical and Computer Engineering, University of California, Irvine, CA., 1989.
- [27] R. R. E. de Gaston and M. G. Safonov, "Exact Calculation of the Multiloop Stability Margin," *IEEE Trans. Automat. Contr.*, vol.33, pp. 156-171, 1988.

List of Figures

- Figure 1. Fitness Evolution for 2 Parameters, Enclave Radius = 0
- Figure 2. Fitness Evolution for 10 Parameters, Enclave Radius = 0.01
- Figure 3. Fitness Evolution for Example 2, Stable Case
- Figure 4. Fitness Evolution for Example 2, Unstable Case

END

DATE FILMED

05 / 08 / 91