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OCTOBER 1981

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FLUCTUATIONS ON ION-CYCLOTRON WAVES

BY

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**PLASMA PHYSICS
LABORATORY**

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Effect of Low Frequency Density Fluctuations
on Ion Cyclotron Waves*

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ABSTRACT

Scattering of waves in the ion cyclotron range of frequencies by low-frequency fluctuations is analyzed in the weak turbulence approximation. Finite-ion-Larmor radius, multi-ion-species, ion-drift-motion, and electromagnetic terms are included in the wave kinetic equation. Conditions for significant scattering have been identified for the fast wave as well as for externally launched ion Bernstein waves. Implications for ICRF heating in tokamak plasmas are discussed.

DISCLAIMER

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I. INTRODUCTION

It is now well known that low-frequency fluctuations exist at a significant level $\{\delta n/n = \mathcal{O}(1-10\%)\}$ in tokamak plasmas.¹⁻³ During radiofrequency heating of tokamak plasmas, the heating wave must traverse the region of low-frequency fluctuations in order to reach the plasma core. Mode-mode coupling of the incident wave with such fluctuations can induce scattering of the incident wave trajectory and, therefore, can cause significant deviation from the linear ray-tracing predictions of the wave power flow and deposition profiles. Recent investigations of the scattering of lower-hybrid waves (electron plasma waves),⁴⁻⁸ for example, suggest several important implications for lower-hybrid wave heating. In some cases^{6,8} the scattering can completely alter predictions based on linear wave theory.

In the present paper, using a wave kinetic equation which includes the ion kinetic and electromagnetic terms, we examine the effect of low-frequency density fluctuations on ion cyclotron waves. In Sec. II, we derive a general wave kinetic equation applicable to waves in the ion cyclotron range of frequency. Using an experimentally observed Gaussian fluctuation wave number spectrum, analytic solutions useful for $k_{\perp}/\ell_0 < 1$ as well as $k_{\perp}/\ell_0 > 1$ are given.

In Sec. III, we consider the fast-wave heating case where we identify a non-negligible scattering region near the ion-ion hybrid mode-conversion layer.⁹⁻¹¹ Such scattering can affect the subsequent history of the mode-converted ion Bernstein wave and can strongly affect the electron heating efficiencies based on linear wave theory. In Sec. IV, we consider the scattering of an externally launched ion Bernstein wave. Since the linear ray-tracing calculations^{12,13} predict very favorable penetration and power deposition profiles, any scattering of the ion Bernstein wave is likely to cause some reduction in the wave penetration and overall heating efficiencies. Using PLT tokamak plasma parameters, we present the numerically calculated inverse scattering length as a function of radial position. We then give a simple formula for estimating the scattering probability which can be scaled to other tokamak-type devices. In general, ion Bernstein waves can be expected to reach the plasma core without significant scattering except in a turbulent ($\delta n/n \approx 30\%$), low temperature ($T_{i0} \ll 1$ keV) high field ($B_0 > 2.5$ kG) plasma. The results are summarized in Sec. V.

II. WAVE KINETIC EQUATION

In analyzing the ion cyclotron waves in a thermonuclear plasma, it is often necessary to include various effects such as finite-Larmor-radius, electromagnetic terms, and multi-ion species. Therefore, we shall first derive the wave kinetic equation including these effects and will later consider various limiting cases where the wave kinetic equation can be considerably simplified. For waves in the ion cyclotron frequency range, the wave equation can be well represented by a 3×3 matrix in rectangular coordinates as¹⁴

$$\begin{bmatrix} K_{xx} - n^2 & K_{xy} & n_{\perp} n_{\parallel} \\ -K_{xy} & K_{xx} + K_{yy} - n_{\perp}^2 & 0 \\ n_{\perp} n_{\parallel} & 0 & K_{zz} - n_{\parallel}^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (1)$$

In a non-drifting Maxwellian plasma, the above dielectric tensor elements can be expressed as

$$K_{xx} = 1 + \frac{\omega_p^2}{\omega^2} + \sum_i \frac{\omega_{pi}^2}{\omega k_{\parallel} v_i} \frac{e^{-\lambda_i}}{\lambda_i} \sum_{n=1}^{\infty} n^2 I_n (Z_n + Z_{-n})$$

$$K_{xy} = -i \frac{\omega_p^2}{\omega \Omega_e} + i \sum_i \frac{\omega_{pi}^2}{\omega k_{\parallel} v_i} e^{-\lambda_i} \sum_{n=1}^{\infty} n (I_n - I_n') (Z_n - Z_{-n})$$

$$K_{yy} = \sum_i \frac{\omega_{pi}^2}{\omega k_{\parallel} V_i} 2\lambda_i e^{-\lambda_i} [(I_0 - I_0') Z_0 + \sum_{n=1}^{\infty} (I_n - I_n') (Z_n + Z_{-n})]$$

$$K_{zz} = 1 + \frac{2 \omega_{pe}^2}{k_{\parallel}^2 V_e^2} [(1 + \xi_{e0} Z_0)],$$

where the summation i is over ion species, n is the harmonic number, Z_n is the plasma dispersion function of argument $\xi_{0n} \equiv (\omega + n\Omega_0)/k_{\parallel} V_0$, and I_n is the modified Bessel function of argument $\lambda_i \equiv k_{\perp}^2 T_i / m_i \Omega_i^2$. The off-diagonal terms K_{xz} and K_{yz} can be neglected here.

From Eq. (1) one can obtain a wave dispersion relation in the following form:

$$\epsilon = K_{xx} - \frac{K_{xy}^2}{n_{\perp}^2 - K_{xx} - K_{yy}} + \frac{n_{\parallel}^2 K_{zz}}{(n_{\perp}^2 - K_{zz})} = 0, \quad (2)$$

where ϵ is the wave dielectric constant.

In calculating the scattering, we let (ω_k^+, k) correspond to the incident wave, $(\omega_{k'}^-, \vec{k}')$ the scattered wave and $(\omega_{\vec{k}-\vec{k}'}^+, \vec{k}-\vec{k}')$ the low frequency fluctuations. For the ion cyclotron range of frequency $f \approx 50-300$ MHz, one can neglect the frequency shift $f-f'$ due to the scattering since the frequency of the fluctuation ($f-f' \leq 100$ KHz) is still quite small.¹⁻³ The parallel wave number can be also considered to be conserved here since $k_{\parallel}$ for the wave (typically $\sim 0.1 \text{ cm}^{-1}$)

should be large compared to $k_{\parallel}$ for the fluctuations $k_{\parallel} = \mathcal{O}(1/Rq)$ ($\leq 0.01 \text{ cm}^{-1}$).¹⁵ For the nearly perpendicularly propagating component for which $k_{\parallel}$ is comparable to that of the fluctuations, it can be easily shown that the parallel coupling is negligibly small. Moreover, the perpendicular wave numbers of ion cyclotron waves such as the ion Bernstein wave and the fast wave are usually conserved through the wave dispersion relation [Eq. (2)], together with the small shifts in frequency and in $k_{\parallel}$. Therefore, the main effect of the scattering can be well represented in this situation by the rotation of the wave vector $\vec{k}_{\perp}$.

We define the scattering angle β such that $\vec{k}_{\perp} \cdot \vec{k}'_{\perp} = |\vec{k}_{\perp}|^2 \cos \beta$. The wave electric field is defined as

$$\vec{E}(\vec{k}) = E_{\perp} \hat{k}_{\perp} + E_{\theta} \hat{k}_{\parallel} \times \hat{k}_{\perp} + E_{\parallel} \hat{k}_{\parallel}, \quad (3)$$

where $\hat{k}_{\parallel}$ is the unit vector parallel to the local magnetic field direction. The wave electric-field components can be determined through the dielectric tensor [Eq. (11)] to be

$$E_{\theta} = -E_{\perp} K_{xy} / (n_{\perp}^2 - K_{xx} - K_{yy})$$

(4)

and

$$E_{\parallel} = E_{\perp} n_{\perp} n_{\parallel} / (n_{\perp}^2 - K_{zz}).$$

The wave velocity distribution function for particles of type σ is defined.

$$f_{\sigma}(\omega, \vec{k}) = f_{\sigma}^{(1)}(\omega, \vec{k}) + f_{\sigma}^{(2)}(\omega, \vec{k}), \quad (5)$$

where $f_{\sigma}^{(1)}$ is the first-order (linear) and $f_{\sigma}^{(2)}$ the second-order (non-linear) wave velocity distribution functions. The $f_{\sigma}^{(2)}$ arises from the coupling between the incident wave and the low-frequency fluctuation.

Poisson's equation can now be written as

$$\vec{\nabla} \cdot \vec{E}(\vec{k}) = 4\pi e [\sum_i n_i^{(1)} - n_e^{(1)}] + 4\pi e [\sum_i n_i^{(2)} - n_e^{(2)}] , \quad (6)$$

where the perturbed densities $n_{\sigma}^{(1)}$ and $n_{\sigma}^{(2)}$ are obtained from the continuity equations,

$$n_{\sigma}^{(1)}(\omega, \vec{k}) = \int d^3v f_{\sigma}^{(1)}(\omega, \vec{k}) = n_{\sigma 0} [\vec{k} \cdot \vec{v}_{\sigma}^{(1)}(\omega, \vec{k}) / \omega] , \quad (7)$$

$$n_{\sigma}^{(2)}(\omega, \vec{k}) = \int d^3v f_{\sigma}^{(2)}(\omega, \vec{k}) = \tilde{n}_{\sigma}(0, \vec{k} - \vec{k}') [\vec{k} \cdot \vec{v}_{\sigma}^{(1)}(\omega, \vec{k}') / \omega] , \quad (8)$$

and where $\tilde{n}_{\sigma}$ is the Fourier amplitude of the low-frequency fluctuation.

The first order velocity $\vec{v}_{\sigma}^{(1)}$ (see Appendix A) can be obtained using the mobility tensor $\vec{M}_{\sigma}$,

$$\vec{v}_{\sigma}^{(1)}(\omega, \vec{k}) \equiv \int d^3v \vec{v} f_{\sigma}^{(1)}(\omega, \vec{k}) = \frac{c}{B_0} \vec{M}_{\sigma} \cdot \vec{E} . \quad (9)$$

Substituting Eqs. (7)-(9) into Eq. (6) and assuming that the time evolution of $E_{\perp}(\vec{k})$ due to the scattering is slow compared

to the wave oscillation time scale one obtains the following mode-coupling equation^{7,16,17}:

$$i \left(\frac{\partial \varepsilon}{\partial \omega} \right)_{\omega_k} \frac{\partial E_{\perp}(\vec{k})}{\partial t} = \hat{n}_e (\vec{k}-\vec{k}') V E_{\perp}(\vec{k}') \exp[-i(\omega_k - \omega_{k'})t], \quad (10)$$

with

$$V = -i \cos \theta \left(\sum_i \frac{\omega_{pi}^2}{\omega \Omega_i} A_{i\perp} n_{ie} - \frac{\omega_{pe}^2}{\omega \Omega_e} A_{e\perp} \right) + i \frac{\omega_{pe}^2}{\omega \Omega_e} A_{e\parallel} \frac{k_{\parallel}}{k_{\perp}}, \quad (11)$$

$$- i \sin \theta \left(\sum_i \frac{\omega_{pi}^2}{\omega \Omega_i} A_{i\theta} n_{ie} - \frac{\omega_{pe}^2}{\omega \Omega_e} A_{e\theta} \right),$$

where from Appendix A

$$A_{i\perp} = \left(\frac{\Omega_i}{k_{\parallel} v_i} \right) e^{-\lambda_i} i \sum_{n=1}^{\infty} \left[- \frac{i n^2}{\lambda_i} I_n (Z_n + Z_{-n}) - \frac{n K_{xy} (I_n - I_n') (Z_n - Z_{-n})}{(n_{\perp}^2 - K_{xx} - K_{yy})} \right],$$

$$A_{e\perp} = - \frac{K_{xy}}{(n_{\perp}^2 - K_{xx} - K_{yy})} + i \frac{\omega}{\Omega_e}, \quad (12)$$

$$A_{i\theta} = - \left(\frac{\Omega_i}{k_{\parallel} v_i} \right) e^{-\lambda_i} i \sum_{n=1}^{\infty} \left[n (I_n - I_n') (Z_n - Z_{-n}) - \frac{n^2}{\lambda_i} I_n (Z_n + Z_{-n}) \frac{i K_{xy}}{(n_{\perp}^2 - K_{xx} - K_{yy})} \right],$$

$$A_{e\theta} = - \left[1 + i \frac{\omega}{\Omega_e} \frac{K_{xy}}{(n_{\perp}^2 - K_{xx} - K_{yy})} \right],$$

$$A_{e\parallel} = - i \frac{n_{\perp} n_{\parallel}}{(n_{\perp}^2 - K_{zz})} \left(\frac{\omega \Omega_e}{k_{\parallel}^2 v_e^2} \right) Z_0,$$

and where ϵ is the wave dielectric constant defined in Eq. (2), $\hat{n}_e$ is the normalized Fourier amplitude of the low frequency electron density fluctuation $\tilde{n}_e(\vec{k}-\vec{k}')/n_{eo}$ and $n_{ie} \equiv \hat{n}_i(\vec{k}-\vec{k}')/\hat{n}_e(\vec{k}-\vec{k}')$. Equation (11) can be written in the following form:

$$V = b_1 + b_2 \sin^2 \beta/2 + b_3 \sin \beta/2 \cos \beta/2 ,$$

where

$$b_1 = 1 + \frac{n_n^2}{n_i^2 - K_{zz}} - \delta_1 ,$$

$$b_2 = -2 \left[1 + \frac{n_n^2 K_{zz}}{n_i^2 - K_{zz}} - \delta_1 \right] ,$$

$$b_3 = 2 \left[K_{xy} + \frac{K_{xy}(K_{xx}-1)}{(n_i^2 - K_{xx} - K_{yy})} - \delta_\theta \right] ,$$

$$\delta_1 = i \sum_i \frac{\omega_{pi}^2}{\omega \Omega_i} (n_{ie} - 1) A_{i1} ,$$

and

$$\delta_\theta = \sum_i \frac{\omega_{pi}^2}{\omega \Omega_i} (n_{ie} - 1) A_{i\theta} .$$

Since it is reasonable to assume that the waves and the low frequency turbulence have no phase correlation,

one can invoke the well-known random phase-approximation^{16,17} in Eq. (10) and obtain the following time-evolution equation for the wave action density N_k due to the scattering:

$$\left(\frac{dN_k}{dt}\right)_s = \sum_{\vec{k}-\vec{k}'} \frac{2\pi|v|^2}{\left(\frac{\partial \epsilon}{\partial \omega}\right)^2} |\hat{n}_e(\vec{k}-\vec{k}')|^2 (N_k - N_{k'}) \delta(\omega_k - \omega_{k'}) , \quad (13)$$

where $N_k \equiv |E_k|^2 (\partial \epsilon / \partial \omega)$. The summation in Eq. (13) can now be expressed in the following integral form:

$$\left(\frac{dF}{dt}\right)_s = 2\pi \int d^2(\vec{k}_1 - \vec{k}_1') \delta(\omega_k - \omega_{k'}) \frac{|v|^2}{\left(\frac{\partial \epsilon}{\partial \omega}\right)^2} S_e(\vec{k} - \vec{k}') [F(\vec{k}') - F(\vec{k})] , \quad (14)$$

where $S_e \equiv \lim_{L \rightarrow \infty} |\hat{n}_e|^2 (n/L)^{-2}$ and $F(k) \equiv \omega_k N_k$ is the wave energy density. By noting that $|\vec{k}_1 - \vec{k}_1'| = 2k_1 \sin \beta/2 \equiv l$ and $\delta(\omega_k - \omega_{k'}) = \delta(\omega - 2k_1 \sin \beta/2) (\sin \beta/2) / (\partial \omega / \partial k_1)$, Eq. (14) then yields

$$\left(\frac{dF(\phi)}{dt}\right)_s = \frac{2\pi k_1}{\left(\frac{\partial \omega}{\partial k_1}\right) \left(\frac{\partial \epsilon}{\partial \omega}\right)^2} \int_{-\pi}^{\pi} |v|^2 S_e(2k_1 \sin \beta/2) [F(\phi + \beta) - F(\phi)] d\beta , \quad (15)$$

where S_e the density fluctuation wave number spectrum is related to $|\hat{n}_e|^2$ by $|\hat{n}_e|^2 = \int_0^\infty S_e(l) 2\pi l dl$ where $l = |\vec{k}_1 - \vec{k}_1'|$, and ϕ is the polar angle so that $k_r = k_1 \cos \phi$. By noting b_3 to be an imaginary quantity, one can write $|v|^2$ in the form

$$|V|^2 = a_1 + a_2 \sin^2 \beta/2 + a_3 \sin^4 \beta/2, \quad (16)$$

where $a_1 = b_1$, $a_2 = 2b_1b_2 - b_3^2$ and $a_3 = b_2^2 + b_3^2$.

We shall now adopt a definition of the scattering length L_s suggested in Ref. 7 which does not depend on the approximation $\xi_0^2 \ll k_0^2$,

$$L_s = (k_1 \partial \epsilon / \partial k_1)^2 (4\pi k_1^3)^{-1} \left[\int_{-\pi}^{\pi} |V|^2 S(2k_0 \sin \beta/2) \sin^2(\beta/2) d\beta \right]^{-1}. \quad (17)$$

The present scattering definition is based on the damping rate of the lowest non-vanishing eigenfunction, $\cos \phi$, without invoking the small β approximation.⁷ In the limit of small β , Eq. (17) reduces to the usual diffusion form, $L_s = (\partial \omega / \partial k_1) / D_{\phi\phi}$ where $D_{\phi\phi} \equiv (dF/dt)_s / (\partial^2 F / \partial \phi^2)$. For the density fluctuation spectrum S , we shall use the experimentally observed Gaussian spectrum²

$$S(\xi) = [\pi \xi_0^2]^{-1} |\hat{n}_e|^2 \exp(-\xi^2 / \xi_0^2).$$

Equation (17) can now be readily calculated by numerical means. One can also perform the integral semi-analytically by expressing $\exp(-x) = \sum_{n=0}^{\infty} (-1)^n x^n / n!$ which then yields

$$\begin{aligned} L_s &= (k_1 \frac{\partial \epsilon}{\partial k_1})^2 \left(\frac{\xi_0^2}{8k_1^3} \right) |\hat{n}_e|^2 \left[\sum_{n=1}^{\infty} C_n \int_0^{\pi} \sin^{2n}(\beta/2) d\beta \right]^{-1} \\ &= (k_1 \frac{\partial \epsilon}{\partial k_1})^2 \frac{\pi \xi_0^2}{4k_1^3} |\hat{n}_e|^{-2} \left[\sum_{n=1}^{\infty} C_n \frac{(2n)!}{(n!)^2 2^{2n}} \right]^{-1}, \end{aligned} \quad (19)$$

where C_n can be computed from Eqs. (12), (16), (17), and (18) to be

$$C_1 = a_1 ,$$

$$C_2 = - a_1 \left(\frac{2k_\perp}{\xi_0} \right)^2 + a_2 ,$$

$$C_3 = \frac{a_1}{2} \left(\frac{2k_\perp}{\xi_0} \right)^4 - a_2 \left(\frac{2k_\perp}{\xi_0} \right)^2 + a_3 ,$$

$$C_4 = - \frac{a_1}{6} \left(\frac{2k_\perp}{\xi_0} \right)^6 + \frac{a_2}{2} \left(\frac{2k_\perp}{\xi_0} \right)^4 - a_3 \left(\frac{2k_\perp}{\xi_0} \right)^2 ,$$

$\vdots$

$$C_n = (-1)^{n-1} \left[\frac{a_1}{(n-1)!} \left(\frac{2k_\perp}{\xi_0} \right)^{2n-2} - \frac{a_2}{(n-2)!} \left(\frac{2k_\perp}{\xi_0} \right)^{2n-4} + \frac{a_3}{(n-3)!} \left(\frac{2k_\perp}{\xi_0} \right)^{2n-6} \right] ,$$

where a_1 , a_2 , and a_3 are given in Eq. (16). Although Eq. (19) is valid for all values of $k_\perp/\xi_0$, it is particularly useful for $k_\perp^2 \lesssim \xi_0^2$ where the series converges reasonably rapidly. For $k_\perp^2 \gg \xi_0^2$, the series convergence becomes tediously slow so that Eq. (19) may be awkward for analytical use. In that case, since $\xi_0/k_\perp = 2 \sin \beta/2 \ll 1$, β is small and, thus, one can go back to Eq. (17) and perform the integral using a small- β expansion, $\sin \beta = \beta - \beta^3/6$, $\cos \beta = 1 - \beta^2/2$, etc. Changing the variable in the integration to $k_\perp/\xi_0 \beta$, because of the exponential term, one can let the limits of integration go to $\pm \infty$. Equation (17) may then be easily integrated to give

$$L_s = \frac{2(k_1 \partial \epsilon / \partial k_1)^2}{\sqrt{\pi} \xi_0 |\hat{n}_e|^2} \left[a_1 + \left(\frac{\xi_0}{k_1} \right)^2 \frac{3}{8} \left(a_2 - \frac{a_1}{3} \right) + \left(\frac{\xi_0}{k_1} \right)^4 \frac{15}{64} \left(a_3 - \frac{2}{3} a_2 + \frac{2}{45} a_1 \right) \right]^{-1}. \quad (20)$$

Using Eqs. (17), (19), and (20) we shall now consider various cases for the relevant waves in the ion cyclotron range of frequency.

III. FAST-WAVE HEATING

We shall first show that the fast wave usually would not suffer scattering due to the low-frequency density fluctuations. Here we can assume that $k_{\perp}^2 < \xi_0^2$ and $k_{\perp} \rho_j < 1$. In a thermonuclear plasma, it is also appropriate to assume $|K_{zz}| \approx \omega_{pe}^2/\omega^2 \gg n_{\perp}^2 \approx \omega_{pi}^2/\Omega_i^2$. The dispersion relation can then be written from Eq. (2) as

$$\epsilon \approx K_{xx} - \frac{K_{xy}^2}{n_{\perp}^2 - K_{xx}} - n_{\perp}^2 = 0, \quad (21)$$

where $n_{\perp}^2$ is usually small compared to $K_{xx} = \mathcal{O}(\omega_{pi}^2/\omega^2) \approx 10^3 - 10^4$. From Eq. (19), one can then compute the scattering length to leading order in $(k_{\perp}/\xi_0)^2$, yielding

$$L_s \approx 4\pi \left(\frac{\Omega_i}{\omega}\right)^2 \left(\frac{\xi_0}{k_{\perp}}\right)^2 k_{\perp}^{-1} |\hat{n}_e|^{-2}. \quad (22)$$

Since $\xi_0/k_{\perp} \sim \mathcal{O}(10)$, $k_{\perp} \approx \mathcal{O}(1)$, and $\Omega_i/\omega \approx \mathcal{O}(1)$, Eq. (22) yields $L_s \approx 10^3 |\hat{n}_e|^{-2} \text{ cm}$ which is much larger than the expected radial travel distance $a \approx 200 \text{ cm}$.

The situation, however, changes significantly near a wave resonance such as the two-ion hybrid resonance $\omega \approx \omega_{TH}^{18,19}$ where linear mode-conversion into ion Bernstein waves can take place.⁹⁻¹¹ At a mode conversion layer, the wave perpendicular group velocity vanishes for a finite $k_{\perp}$ causing the scattering rate to diverge. We shall therefore assess the degree of importance

of the scattering near the mode-conversion layer using the wave kinetic equation. In the vicinity of the mode-conversion layer, one can express the wave dispersion relation to first order in $k_{\perp}^2 \rho_i^2$ as

$$\varepsilon = -g_1 \frac{x_c + x}{R} + k_{\perp}^2 g_2 + \frac{1}{k_{\perp}^2} g_3 - g_4 = 0, \quad (23)$$

where R is the major radius, $x_c + x$ is the distance from the ion-ion hybrid resonance layer, and x_c is the distance of the turning point from the resonance layer. The coefficient g_i 's as obtained from Eq. (11) are

$$g_1 \approx \frac{2 \omega_{p1}^2}{(\Omega_1^2 - \omega^2)} \left[\frac{\omega^2 (\Omega_1^2 - \Omega_2^2)}{(\Omega_1^2 - \omega^2) (\omega^2 - \Omega_2^2)} \right],$$

$$g_2 \approx 3 \frac{\omega_{p1}^2}{(\Omega_1^2 - \omega^2)} \left[-\frac{T_1/m_1}{(4\Omega_1^2 - \omega^2)} + \frac{T_2/m_2}{(4\Omega_2^2 - \omega^2)} \right],$$

$$g_3 \approx \frac{\omega^2}{c^2} \left\{ \frac{\omega_{p1}^2}{(\Omega_1^2 - \omega^2)} \left[\frac{\omega}{\Omega_2} - \frac{\omega}{\Omega_1} \right] \right\}^2,$$

$$g_4 \approx \left[-1 - \frac{\omega_{pe}^2}{\Omega_e^2} + n_{\parallel}^2 \right],$$

where $\omega \approx \omega_{IH}$, $x/R \ll 1$, $\Omega_1 > \Omega_2$, and a $1/R$ magnetic variation is assumed. We shall now compute the scattering probability P_s between positions x_1 and x_2 defined as

$$P_S = \int_{x_1}^{x_2} L_S^{-1} dx. \quad (24)$$

We will let the limit of integration x_2 approach as close as possible to the turning point without violating the W.K.B. approximation, $(\partial k_1 / \partial x) k_1^{-2} < 1$. Using Eq. (19) in Eq. (24) we obtain

$$P_S = \int_{x_1}^{x_2} dx \frac{k_1^3(x) |\hat{n}_e|^2}{(g_2 k_1^2(x) - g_3/k_1^2(x))^2 (\pi \xi_0^2)} \left[\sum_{n=1}^{\infty} C_n \frac{(2n)!}{(n!)^2 2^{2n}} \right], \quad (25)$$

where $k_1(x)$ is determined from Eq. (23). The integration in (25) can be performed by changing the integration variable from x to k_1 yielding

$$P_S = \frac{2}{\pi} \frac{R}{g_1 g_2} \frac{|\hat{n}_e|^2}{\xi_0^2} \int_{k_{11}}^{k_{12}} dk_1 \frac{k_1^4}{(g_3/g_2 - k_1^4)^2} \left[\sum_{n=1}^{\infty} C_n \frac{(2n)!}{(n!)^2 2^{2n}} \right]. \quad (26)$$

By noting $C_1 = 1 \ll C_2 \approx -C_3 \approx -4K_{xy}^2 \approx \omega_{pi}^4/\omega^4$, to leading order in k_{12}^2/ξ_0^2 , Eq. (26) reduces to

$$P_S = \frac{1}{2} \frac{R}{g_1 g_2} \frac{|\hat{n}_e|^2}{\pi \xi_0^2 \omega^2} \frac{c^2 g_3^{5/4}}{\omega^{5/4}} \int_{\gamma_1}^{\gamma_2} d\gamma \left[\frac{\gamma^4}{1-\gamma^4} \right], \quad (27)$$

where $\gamma \equiv k_1/k_c$, and $k_{1c} = (g_3/g_2)^{1/4}$ is the value of k_1 at the mode-conversion point. The integral can be easily performed to yield:

$$\int_{\gamma_1}^{\gamma_2} d\gamma \frac{\gamma^4}{1-\gamma^4} = \left[-\gamma + \frac{1}{4} \ln \left(\frac{1+\gamma}{1-\gamma} \right) + \frac{1}{2} \tan^{-1} \gamma \right] \Big|_{\gamma_1}^{\gamma_2} . \quad (28)$$

One can see that the integral in Eq. (27) diverges logarithmically as $\gamma \rightarrow 1$. By limiting it to only the W.K.B. region one can show that the integral assumes a value of order unity. One can also show that the mode-converted wave suffers scattering of a similar magnitude as it propagates out of the conversion layer. Finally, after some algebra, Eq. (26) reduces to

$$P_s = \frac{R |\hat{n}_e|^2}{100 (\xi_0 \rho_i)^2} \left(\frac{\omega_{pi} \omega}{c v_i} \right)^{1/2} Q , \quad (29)$$

where Q is a coefficient of order of unity. For a D-T thermonuclear plasma with equal ion concentrations $(\omega_{pi} \omega / c v_i)^{1/2} \approx 2$ and $R \approx 600$ cm, Eq. (28) yields $P_s \sim 10 |\hat{n}_e|^2 / (\xi_0 \rho_i)^2$, or for $\hat{n}_e \approx 5\%$ and $\xi_0 \rho_i \approx 0.2$, $P_s \approx 1$ which is quite large. Since such scattering can strongly affect the electron Landau damping characteristics of the mode-converted ion Bernstein wave through the change in wave n_z due to magnetic shear, the scattering can influence the electron heating efficiency of ICRF heating near the ion-ion hybrid resonance.

IV. ION BERNSTEIN WAVE HEATING

In ion Bernstein wave heating, or finite-Larmor-radius ICRF (FLR-ICRF), an externally launched ion Bernstein wave propagates from the low density, low field region where $k_{\perp} \rho_i < 1$ to the high density higher field plasma core where $k_{\perp} \rho_i > 1$. Since throughout its propagation path, the wave assumes a wide range of $k_{\perp} \rho_i \approx 0.1-10.0$, it is necessary to consider the effect of scattering accordingly.

In the low density edge region, $\omega_{pi} < \omega$, the wave is an electron plasma wave. In this region $k_{\perp} \rho_i \ll 1$, and one can use Eq. (19) to obtain

$$L_S = 2\pi \frac{\xi_0^2}{k_{\perp}^2} |\hat{n}_e|^{-2} \left[1 + \frac{1}{8} \frac{\omega_{pi}^2}{\omega^2 \Omega_i^2} + \mathcal{O}\left(\frac{k_{\perp}^2}{\xi_0^2}\right) \right]^{-1}. \quad (30)$$

The second term in the bracket is the coupling term due to the E×B drift. One recalls that for lower-hybrid heating, $(\omega_{pi}^2/\omega\Omega_i)^2 \sim 10^2 - 10^3$ causing the E×B term to dominate the polarization term (the first term). However for $\omega \approx 2\Omega_i$, $\omega_{pi}^2/\omega\Omega_i^2 \lesssim 1$, and the E×B term is relatively small. Since $k_{\perp} < \xi_0 \sim \mathcal{O}(1 \text{ cm}^{-1})$, Eq. (22) gives $L_S \gtrsim 2\pi |\hat{n}_e|^{-2}$ or for $\hat{n}_e \sim 10\%$, $L_S > 600 \text{ cm}$ which is much larger than the expected thickness of the electron plasma region $\approx 3(1 \text{ cm})$. Therefore the externally launched electron plasma wave in this frequency range should easily reach the mode-transformation region ($\omega = \omega_{pi}$ layer) without significant scattering for $\hat{n} = \mathcal{O}(10\%)$.

In the medium to high density region, the wave transforms into an ion Bernstein wave.²⁰ In this region, $\omega_{pi}^2 \gg \omega^2$ and the $E \times B$ term dominates the coupling. From Eq. (17), one can express L_s as

$$L_s = \frac{1}{64} \frac{\xi_0^2}{k_0^3} |\hat{n}_e|^{-2} \frac{(k_\perp \partial \epsilon / \partial k_\perp)^2}{|K_{xy}|^2} \left[\int_0^{\pi/2} \sin^4 \theta \cos^2 \theta \exp \left[-4 \left(\frac{k_\perp}{\xi_0} \right)^2 \sin^2 \theta \right] d\theta \right]^{-1}$$

$$= \frac{\sqrt{\eta_i}}{B_0} |\hat{n}_e|^{-2} f\left(\frac{\omega}{\Omega_i}, \xi_0 \rho_i\right), \quad (31)$$

where the function f is determined by ω/Ω_i and $\xi_0 \rho_i$. It should be noted that for a given $\xi_0 \rho_i$, $L_s \propto \sqrt{\eta_i}/(B_0 |\hat{n}_e|^2)$ and is independent of the plasma density due to the cancellation of ω_{pi}^2 terms in ϵ and K_{xy} . Since the function f is a complicated function involving the modified Bessel function and the integral, it was necessary to determine it by numerical means. As an example, we show here a calculation done using PLT-type tokamak parameters.

In Fig. 1(a), the plasma density and temperature profiles used in the calculation are shown. The density and temperature at the center are $T_0 = 1$ keV and $n_0 = 3 \times 10^{13} \text{ cm}^{-3}$ which are assumed to fall to 1/100 of the central values at the plasma edge. In Fig. 1(b), the magnetic field profiles are shown. A standard q -profile (the safety factor) with $q(\text{edge}) = 4$ and $q(\text{center}) = 1$ has been assumed. Using Eq. (17), we have

calculated the inverse scattering length L_s^{-1} for a 1% density fluctuation as a function of radial position in a hydrogen plasma. In Fig. 2, our result is shown for various values of $\xi_0 \rho_i$ (as labeled). In general, the scattering is strong when $k_\perp \approx \xi_0$ and decreases for both $k_\perp \ll \xi_0$ and $k_\perp \gg \xi_0$ as can be expected from Eqs. (19) and (20). We also observe that the ion Bernstein wave suffers most scattering when $\xi_0 \rho_i = 1$. One can apply the present result to other similar aspect ratio devices by using the scaling $L_s \propto \sqrt{T_i} / (B_0 |\hat{n}_e|^2)$. The total scattering probability defined here as $P_s = \int_0^a L_s^{-1} dr$ is typically 10^{-3} for the case shown in Fig. 2. If one assumes a uniform density fluctuation to exist in the outer half of the plasma, then one obtains an expression for the scattering probability,

$$P_s \lesssim 5 \times 10^{-6} T_{i0}^{-1/2} (\text{keV}) |\hat{n}|^2 (\%) a (\text{cm}) B_0 (\text{T}), \quad (32)$$

where a is a minor radius of a device. For PLT-parameters, $T_{i0} = 1 \text{ keV}$, $|\hat{n}| (\%) = 10\%$, $a = 40 \text{ cm}$, and $B_0 = 2.5 \text{ T}$, then $P_s = 0.05$. For reactor-parameters, $T_{i0} = 10 \text{ keV}$, $|\hat{n}| (\%) = 5\%$, $a = 200 \text{ cm}$, and $B_0 = 5 \text{ T}$, and $P_s = 0.04$. In either case, the scattering is relatively small. It should also be noted that in the Princeton ACT-1 device,²⁰ coherent ion Bernstein wave excitation and propagation have been observed for $T_i \approx 1 \text{ eV}$, $|\hat{n}| \approx 7\%$ (everywhere), $B_0 \approx 0.5 \text{ T}$, and $a \approx 8 \text{ cm}$ making $P_s \approx 0.05$. Therefore in a thermonuclear plasma, the scattering of the externally launched ion Bernstein wave can be expected to be small. However, the scattering problem can still

exist for a low temperature $T_{i0} \ll 1$ keV plasma with a large density fluctuation. For example, for $T_{i0} = 1$ eV, $|\hat{n}| = 30\%$, $B_0 = 3$ T, the scattering length is only about 2 cm.

V. CONCLUSIONS AND DISCUSSION

Scattering of waves in the ion cyclotron range of frequency by low frequency density fluctuations is analyzed in the weak-turbulence approximation. A general wave kinetic equation including the ion kinetics and the electromagnetic terms is derived for the ion cyclotron range of frequency. Analytic expressions are given for $k_{\perp}/\xi_0 < 1$ and $k_{\perp}/\xi_0 > 1$ calculated for a Gaussian fluctuation spectrum. Using this wave kinetic equation, we have examined two ICRF heating cases, one for the fast wave and another for the externally launched ion Bernstein wave (FLR-ICRF).

For the fast-wave case, the only region susceptible to this scattering appears to be near the mode-conversion layer where the scattering probability can diverge logarithmically. By limiting our analysis only to the WKB region, we could show that the scattering probability can approach $\mathcal{O}(1)$ for $\delta n/n = 5\%$. The resulting scattering probability appears to be large enough to warrant further investigation of the scattering process as well as the possible consequences on plasma heating via the mode-conversion process.

One of the obvious implications of the scattering is in the electron heating efficiency.^{21,22} The mode-converted ion Bernstein wave, if not electron Landau damped would presumably travel to the nearest ion cyclotron (harmonic) resonance layer where it can be completely absorbed by the ions.

Often in a tokamak geometry, such ion cyclotron absorption layers lie near the plasma edge (or even outside the plasma) which then could cause undesirable edge heating. However, if significant scattering can take place near the layer, giving the converted ion Bernstein wave a large enough poloidal wave-number, then, due to the magnetic shear effect, a good fraction of the wave can assume large enough $n_{\parallel}$ values to be absorbed efficiently via electron Landau damping near the conversion layer. Therefore, a moderate amount of scattering can enhance the electron heating near the mode-conversion layer and thus, is likely to play a beneficial role in increasing the ICRF electron heating efficiency.

For plasma heating by the externally launched ion Bernstein wave, we have numerically calculated the scattering integral since the wave assumes a large range of $k_{\perp}\rho_i$, $k_{\perp}\rho_i < 1$ at the plasma edge and $k_{\perp}\rho_i > 1$ in the plasma core. The wave excited near the low density edge (electron plasma wave) is shown to reach the mode-transformation layer ($\omega \approx \omega_{pi}$) without significant scattering. Using PLT-type tokamak plasma parameters, the inverse scattering length is numerically calculated as a function of radial position for various values of $\xi_0\rho_i$. The most scattering occurs for $\xi_0\rho_i \approx 0.5-1.0$ as expected since the ion Bernstein wave on the average assumes $k_{\perp}\rho_i \approx 1$ which satisfies $k_{\perp} \approx \xi_0$. We then obtain a simple formula for the scattering probability which can be scaled to other

tokamak parameters. For a reasonable range of device, plasma and fluctuation parameters, we find that the externally launched ion Bernstein wave can reach the plasma core without suffering significant scattering, $P_s < 0.1$. Such a result is consistent with the observation in the ACT-1 device where coherent ion Bernstein wave excitation and propagation have been experimentally observed.²⁰

The relatively small scattering probability for the externally launched ion Bernstein wave suggests that the modification of the linear ray-tracing calculation by the scattering process should be small. Also, a ray-tracing calculation¹³ including moderate amounts of scattering does not show significant changes in the wave trajectory and power deposition profiles. Coherent wave propagation is also important for the proposed method of ion temperature measurement using externally-launched ion Bernstein waves detected via laser or microwave scatterings.²³

Finally, it should be noted that the currently available data on low-frequency density fluctuations in tokamak plasmas still contain relatively large uncertainties in the wavenumber spectrum, spatial localization and the absolute fluctuation level. These fluctuation characteristics are also observed to vary from device to device. In our analysis, we have allowed for these uncertainties by treating these quantities as variables. In computing the scattering probability, we may have overestimated the effect of scattering by assuming a

uniform $\delta n/n$ and $\xi_0 \rho_i$. When more precise information on the fluctuation spectra becomes available, further refinement in the calculation can be easily made, and accordingly, one should be able to predict the degree of scattering more precisely.

APPENDIX A

The mobility tensor $\vec{M}_\sigma$ is defined as¹⁴

$$\vec{v}_\sigma^{(1)} = \frac{c}{B_0} \vec{M}_\sigma \cdot \vec{E} = \int d^3v f_\sigma^{(1)} \vec{v}.$$

Then the relevant components of the first order velocity for the ion cyclotron range of frequency can be expressed in the following form:

$$v_{i\parallel}^{(1)} = E_\perp \frac{c}{B_0} \left(\frac{\Omega_i}{k_\parallel v_i} \right) e^{-\lambda_i} \sum_{n=1}^{\infty} \left[-\frac{in^2}{\lambda_i} I_n(Z_n + Z_{-n}) - \frac{nK_{xy}(I_n - I_n')(Z_n - Z_{-n})}{(n^2 - K_{xx} - K_{yy})} \right],$$

$$v_{e\parallel}^{(1)} = -E_\perp \frac{c}{B_0} \left[-i \frac{\omega}{\Omega_e} + \frac{K_{xy}}{(n^2 - K_{xx} - K_{yy})} \right],$$

$$v_{i\theta}^{(1)} = -E_\perp \frac{c}{B_0} \left(\frac{\Omega_i}{k_\parallel v_i} \right) e^{-\lambda_i} \sum_{n=1}^{\infty} \left[n(I_n - I_n')(Z_n - Z_{-n}) - \frac{n^2}{\lambda_i} I_n(Z_n + Z_{-n}) - \frac{iK_{xy}}{(n^2 - K_{xx} - K_{yy})} \right],$$

$$v_{e\theta}^{(1)} = -E_\perp \frac{c}{B_0} \left[1 + i \frac{\omega}{\Omega_e} - \frac{K_{xy}}{(n^2 - K_{xx} - K_{yy})} \right],$$

$$v_{e''}^{(1)} = -E_\perp \frac{c}{B_0} \left(\frac{n_\perp n_\parallel}{n^2 - K_{zz}} \right) \left(i \frac{\omega \Omega_e}{k_\parallel v_e^2} Z_0' \right).$$

Equation (4) has been used to eliminate E_θ and $E_\parallel$, and the ion motion along the field line $v_{i''}^{(1)} \ll v_{e''}^{(1)}$ has been neglected.

We can now define a vector $\vec{A}_\sigma$ so that

$$\vec{v}_\sigma^{(1)} = \frac{c}{B_0} \mathbf{E}_\perp \cdot \vec{A}_\sigma .$$

ACKNOWLEDGMENT

The author would like to thank Drs. P. L. Colestock, D. Q. Hwang, T. H. Stix, J. Wilson, and K. L. Wong for fruitful discussions and suggestions. He also wishes to thank Drs. E. Mazzucato, C. M. Surko, and G. Rewoldt for useful information concerning drift waves in tokamak plasmas.

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FIGURE CAPTIONS

Fig. 1. Radial profiles. (a) Plasma density and temperature profiles. (b) Toroidal and poloidal magnetic field profiles.

Fig. 2. Inverse scattering length as a function of the radial position in a hydrogen plasma. The values of $\xi_0 \rho_i$ are as labeled. $f = 55$ MHz, $B_0 = 2.4$ T, $\delta n/n = 1\%$, $T_{i0} = 1$ keV, $n_0 = 3 \times 10^{13} \text{ cm}^{-3}$, $R = 130$ cm, $a = 40$ cm.

Fig. 1

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Fig. 2