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MASTER

**A STUDY OF THE GENERAL PLASMA CHARACTERISTICS OF
A HIGH POWER MULTIFILAMENT ION SOURCE**

**Kurt F. Schoenberg
(Ph. D. thesis)**

September 1979

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DISCLAIMER

INTRODUCTION

By the late nineteen forties, the knowledge for potentially utilizing nuclear processes was sufficient to recognize that controlled thermonuclear fusion could provide civilization with a reasonably clean and virtually inexhaustible source of energy. The ensuing years of research, however, discovered that problems of plasma stability and confinement were far more complex than initially anticipated. As a consequence, the main fusion effort for the past thirty years has focused on understanding and controlling plasma stability, resulting in the current optimistic prediction that scientific feasibility of fusion power, a milestone in the quest for plasma confinement, will be achieved when the Tokamak Fusion Test Reactor (TFTR) becomes operational at Princeton in 1981 (Fig. 1-2).^{*} Figure 1-1 schematically illustrates the basic parameters which characterize a conceptual magnetic fusion device. Successful energy extraction requires heating a deuterium-tritium plasma to thermonuclear ignition temperatures (50 million degrees centigrade) and maintaining a plasma density-confinement time product of greater than $5 \cdot 10^{13}$ sec/cm³ (The Lawson Criterion). The confinement force is produced by carefully constructed magnetic fields which hold the hot plasma away from surrounding matter thus maintaining system integrity. Energy output is primarily in the form of high energy neutrons (14 MeV) which necessitates external power production via thermal cycles.

Paramount to the study of plasma confinement and fusion power generation are the problems of plasma production and heating (Energy Investment). Although there exist many plasma heating mechanisms

^{*} Figures appearing in the Introduction are selected from the LBL-MFE group files.

- C. The resonant fluctuation frequency is set by the periodic multifilament structure in the source plasma chamber, indicating an interaction between the filaments (operated in the space charge limited mode) and the ion waves.
- D. Fluctuation amplitude levels are insufficient to cause significant plasma ion heating.

The multifilament source is presently the heart of the Lawrence Berkeley Laboratory Neutral Beam program, slated to provide neutral beam heating for several major U. S. Department of Energy fusion experiments. A brief discussion coupling the measured source plasma parameters with overall neutral beam performance is included.

To my wife, Carol, for her patience and encouragement
throughout the duration of this work and to my parents,
Stephanie and Frank, for providing the educational opportunity.

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A STUDY OF THE GENERAL PLASMA CHARACTERISTICS OF A
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ABSTRACT

A general assessment of the steady state and time dependent plasma properties which characterize a high power multifilament ion source is presented. Steady state measurements, obtained via a pulsed electrostatic probe data acquisition system, in part conclude:

- A. The source electron distribution function consists of a bulk component of thermalized electrons ($kT_e \sim 3$ to 5 eV) comprising 90 to 97 percent of the total electron population, plus a non-thermal high energy tail which is a monotonically decreasing function of energy, and contains a small but experimentally significant electron population at energy values 10 to 15 eV above maximum cathode fall.
- B. Overall steady state source performance approximately follows low pressure discharge theory assuming spatially uniform ion production.

Fluctuation measurements, obtained via a broadband digital spectral analysis system, in part conclude:

- A. Source plasma fluctuations are caused by ion acoustic waves, initially excited by a current driven instability at the thermionic cathode.
- B. Fluctuation spectral energy density is highly non thermal and exhibits a resonant profile.

SPECIFICALLY,

$$T_{\text{ION}} > 5 \text{ KEV } (50,000,000 \text{ } ^\circ\text{K})$$

$$n_{\text{T}} > 5 \times 10^{13} \text{ cm}^{-3} \cdot \text{SECONDS}$$

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Figure I-1. A General Fusion Schematic.

relevant to fusion (radio frequency, ohmic, compressional, shock, laser, electron-beam etc.), the currently most successful and thus most popular technique consists of high power neutral beam injection. Table 1 gives a partial list of current or proposed fusion experiments which employ or will employ neutral beam heating developed by groups at Oak Ridge National Laboratory (ORNL) and Lawrence Berkeley Laboratory/Lawrence Livermore Laboratory (LBL/LLL).¹

Table 1: Neutral Beam Utilization

Device	Location	Development Group
ATC	Princeton	LBL/LLL
Doubiet III	Princeton	LBL/LLL
ISX	Oak Ridge	ORNL
MFTF	Livermore	LBL/LLL
Ormak	Oak Ridge	ORNL
PDX	Princeton	ORNL
PLT	Princeton	ORNL
TFTR	Princeton	LBL/LLL
TMX	Livermore	LBL/LLL
2XIIB	Livermore	LBL/LLL

A fairly typical neutral injection system is shown in Figure 1-3; which illustrates a general schematic of the 120 kV TFTR neutral beam line. The heart of the system is the Berkeley Multifilament Ion Source², where deuterium ions are generated by a diffuse, low pressure thermionic arc. Plasma ions are extracted and accelerated to the desired energy by a multi-electrode structure, and

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Figure 1-2.

subsequently neutralized by charge exchange interactions with background deuterium gas in the neutralizer cell. Residual ions are removed from the neutral beam by a sweep magnet in order to prevent severe beam line ablation and damage; caused by containment field deflection of ions into surfaces near the fusion plasma entrance duct. Vacuum pumps provide isolation for the neutralizer cell and fusion plasma so that background pressure in the beam transport tube (the section between the sweep magnet and plasma entrance port in Fig. 1-3) is insufficient to cause significant beam re-ionization. Computer control and diagnostics allow for optimization of beam operating parameters to within design specifications.

The performance parameters of a neutral beam system must satisfy stringent requirements set by engineering and physics considerations of the target plasma. Many of these parameters are extremely dependent on the operating plasma properties of the beam ion source. Pedagogically, the coupling of source parameters to overall beam performance is best approached by careful examination of the beam design requirements. A partial set of performance parameters for a single 120 kV TFTR neutral beam line is listed in Table 2.

Beam Energy. Beam energy requirements are primarily determined by the necessity of depositing high energy neutrals well inside the target plasma, preferably near the center. The spatial attenuation of a neutral beam through the target plasma results from charge exchange collisions with plasma ions and ionizing collisions with plasma ions and electrons. Mathematically, this effect may be modeled by

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Figure 1-3. 120 kV Neutral Beam Injection Line.

 Table 2: Neutral Beam Requirements for TFTR (single source)

Neutral Gas:	Deuterium
Beam Energy:	120 kV
Beam Current:	65 Amps
Beam Current Density:	0.28 A/cm ²
Pulse Duration:	0.5 sec
Accelerator Grid Transparency:	0.6
Source Extraction Area:	400 cm ²
Beam Divergence Angle ($\pm 1/e$):	<1.0°

the relation:

$$\frac{dI}{dX} = -\lambda^{-1} I \quad (I-1)$$

where for beam energy greater than 80 keV:³

$$\lambda^{-1} \text{ (cm}^{-1}\text{)} \approx 1.8 \cdot 10^{-10} A(\text{amu}) Z_{\text{eff}} n(\text{cm}^{-3})/E(\text{keV}) \quad (I-2)$$

and where I is the total neutral beam current, $(A; E)$ is the neutral particle (mass; energy), and Z_{eff} is the effective charge per plasma ion. The scale length λ is essentially the mean free path for beam ionization and thus must be comparable to the target plasma radius for successful injection. Optimum beam energy is therefore a function of target plasma density and ionic composition.

Beam Current. Demands on injected beam current are set by the total input power required to heat and sustain the fusion target plasma. In terms of plasma operating parameters, the required input power may be

approximately defined in the steady state limit as³

$$P(kw) \approx 2.4 \cdot 10^{-13} n(\text{cm}^{-3})(T_i + T_e)(\text{KeV})V(\ell)/\tau(\text{ms}) \quad (\text{I-3})$$

where n is the mean target plasma density, $(T_i; T_e)$ is the (ion; electron) temperature, V is the plasma volume, and τ represents the mean energy containment time. For TFTR, this translates into hundreds of amperes of injected current. The demands on accelerated ion current are even greater due to the low neutralization efficiency of charge exchange reactions at 120 keV (Figures I-4 and I-5). Thus, total current demand coupled with the availability of injection ducts sets individual source current requirements.

Beam Divergence. Limitations on beam divergence basically result from the configurational design of the beam line - target system. Figure I-2 depicts neutral beam access to the TFTR plasma. Individual beam ducts must accommodate megawatts of neutral injection while remaining small enough to fit through field coils and shields. In addition, beam lines are typically several meters long (9 m for TFTR) in order to accommodate the beam line components (Fig. I-3). As a result, maximum beam divergence, defined in terms of the beam gaussian half width, must typically remain less than one degree.

Properties of the ion source plasma essentially set the minimum or lower limit level of beam divergence since by Liouville's Theorem, the phase space volume of ions extracted from the source is incompressible. Therefore, ion temperature transverse to the beam direction, and all source parameters related to it, become coupled to beam performance.

Figure 1-4.

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XBL 757-3554-C

Figure 1-5. Power and Particle flow required to yield 1 Megawatt of 120 keV D^0 neutrals.

(e.g. parameters describing plasma fluctuations possibly responsible for stochastic ion heating).

Equally important are the electrostatic properties of the beam extractor. Figure I-3 schematically illustrates the extractor's multi-electrode structure. The electrode shape, spacing, and potential configuration are all set to optimize the match between the ion source plasma and accelerator, thereby minimizing extractor induced beam divergence. In principle, this requires extracted beam current density exactly match the space charge limited emission of ions from the source plasma. Hence, the relationship between extracted ion current density, electrode potential and electrode spacing can be approximated by the Child-Langmuir relation:

$$j_{\text{Extract}}^i \propto \frac{(\Delta v)^{3/2}}{d^2} \quad (\text{I-4})$$

where Δv is the potential difference between electrodes with spacing d . For fixed beam energy, equation I-4 indicates that the upper limit of extracted ion current density is set by the lower limit of electrode spacing; a condition imposed by the electrical breakdown threshold across the electrodes. For TFTR, this translates into an upper current density limit of close to 0.28 amps/cm^2 . Given the total beam current requirement (65 amps), it is apparent that beam extraction must occur over a fairly large plasma area (Fig. I-6). In fact, the extraction area is too large for a single aperture accelerator and thus, a multiple aperture extractor is employed (i.e. the total beam is composed of a summation of parallel beamlets, each satisfying the condition of

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Figure I-6. Multiple Aperature Extractor
from a 50 A, 20 kV Accelerator.

space charge limited ion flow). Dividing the total beam current by the multiple aperture transparency and extracted current density, yields a total extraction area of roughly 400 cm^2 (Table 2). Maintaining minimum beam divergence over a large extraction area requires that source plasma parameters remain relatively constant across the extractor aperture. Therefore, source parameters such as the electron distribution function and plasma potential, which influence the source plasma profile, are also coupled to beam performance.

Beam Composition. Due to the molecular properties of deuterium or hydrogen gas, the source plasma is composed of three main ion constituents: D^+ , D_2^+ and D_3^+ . A typical species ratio for the multifilament source is 75: 15: 10 percent. The main effect of species composition on beam performance, apart from changes in extracted current density, is the creation of beam energy dispersion. Since all plasma ions are singly charged, their accelerated energy will be identical. However, the relative high probability of molecular dissociation for high energy collisions in the neutralizer cell, will cause the D_2^+ and D_3^+ beam ions to dissociate into half and third energy atomic neutrals. These components do not satisfy the injection energy requirement (Eq. I-1) and therefore at the very least, they lower injection efficiency (Fig. I-5). Unfortunately, plasma species ratio is not a simple function of source performance, but requires a fairly sophisticated model describing steady state operation in terms of power balance and relevant reaction rates; the coefficients of which are highly dependent on the source electron distribution function.

The preceeding discussion has illustrated the coupling between source plasma characteristics and overall beam performance. The motivation for research presented in this dissertation, apart from academic considerations, was therefore two fold. Knowledge of the steady state properties of the source plasma required measuring the spatial dependence of the electron distribution function, electron and ion density, and plasma potential as a function of operating conditions. Questions addressed to stochastic ion heating by source plasma turbulence, required measuring the auto-power and cross-power spectral density of plasma potential fluctuations. In addition, the existence of a collisionless, cold-ion, high density arc plasma can stand on its own merits for warranting an ion wave propagation study.

The proceeding study is divided into two main sections. Section one presents a succinct description of the steady state source plasma parameters, and the physics governing their behavior. A discussion of the experimental technique and measurement apparatus is presented in the Appendix. Section two presents the experimental approach and results of source plasma fluctuation measurements. Both sections are composed of previously, or soon to be published journal reports,^{4,5} and thus are presented in a compendious manner.

Come forth into the light of things,
let nature be your teacher.

- William Wordsworth (1798)

THE 10 AMPERE SOURCE

The 10 ampere source is basically a small version of the ion sources presently employed in the LBL/LLL neutral beam program. A cross sectional schematic and photographs are shown in Figures 1, 2A and 2B respectively. The source produces a plasma via a diffuse, low pressure, high current electrical discharge. Arc ionization is produced by primary electrons originating at the thermionic cathode (filament ring), and energized by their passage through the cathode-plasma sheath. The arc discharge occurs between the filament ring, consisting of 26 hairpin tungsten filaments connected in parallel, and the anode ring. A pulse line composed of iron core inductors and electrolytic capacitors, supplies roughly 10 to 60 kW of arc power for up to 100 msec. All source chamber walls electrically float at potentials such that the net random current due to electron and ion bombardment is nulled. The main volume of the discharge is relatively free of magnetic fields, although both filament heater current and arc discharge current contribute to a field in the vicinity of the filaments. For this reason, the applied filament current is D.C. to minimize A.C. modulation of discharge conditions. To avoid formation of a substantial anode sheath, which increases discharge noise, the anode area is chosen such that the discharge arc current is supplied by the random electron flux striking the anode.

Experimental access to the plasma is via several radial probe ports near the source midplane and floating extractor grid, one axial probe port, and a section of axially symmetric floating wall which can be used as an extended wall probe.

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Figure 1. Cross Sectional Schematic of the
10 Ampere Source.

Figure 2A. The Ten Ampere Source.

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Figure 2B. The mounted source on a high voltage test stand.

SOURCE PLASMA CHARACTERISTICS

The Electron Distribution Function

Preliminary analysis of the electron distribution function in low pressure, hot cathode discharges was performed by Langmuir in 1925.⁶ Employing the technique of electrostatic probes,⁷ current-voltage characteristics were obtained for a variety of operating conditions in low current (10mA), D.C. mercury vapor discharges. These probe characteristics, in many ways similar to those obtained in the Berkeley source (a typical example is illustrated in Figure 3A),⁸ were interpreted by Langmuir as being derivable from an electron distribution function composed of three main constituents:

1. Primary electrons: thermionically emitted electrons energized by their passage through the cathode-plasma sheath.
2. Secondary electrons: an isotropic, thermal constituent with a temperature considerably less than, but dependent on the primary electron energy. In Figure 3A, the electron probe current due to secondary electrons would roughly correspond to probe bias between 0 and 23 volts. Note that the secondary electrons appear thermal, due to the exponential dependence of probe current on probe bias in this range.
3. Ultimate electrons: a cool isotropic, thermal component constituting the bulk of the electron population with a temperature less than the secondaries, but fairly independent of primary electron energy. In Figure 3A, the ultimate's contribution corresponds to probe bias between 23 volts and the plasma potential. Again, note that the ultimate electrons appear thermal.

Arc Power: 16.3 kW

Gas Flow: 6.5 T-l/sec

Source Center

Bulk Electron Temperature: 3.8 ± 0.2 eVElectron Density: $2.0 \pm 0.3 \cdot 10^{12} / \text{cm}^3$ Ion Density: $1.9 \pm 0.2 \cdot 10^{12} / \text{cm}^3$

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Figure 3A.

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Figure 3B.

Figure 3C.

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XBL786-1149

Figure 3D.

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Figure 3E.

The secondary and ultimate electrons compose what is often called a two temperature or Bimaxwellian distribution function. However, the physical origin of this distribution function is difficult to resolve. For high energy electrons comprising a majority of the secondary population, the collisional mean free path is many times greater than the plasma's spatial dimension and hence, collisional relaxation is inadequate to account for thermalization. Several explanations of this apparent conundrum, oftentimes called "The Langmuir Paradox,"⁸ have been proposed involving collective effects such as plasma-sheath oscillations. In recent years, subsequent experimental investigations of low current discharges have provided evidence that questions pertaining to the resolution of Langmuir's Paradox are possibly moot. Work by Rayment and Twiddy,⁹ involving direct distribution function measurement via the harmonic double differentiation technique in mercury vapor discharges, have shown that the electron distribution function is non-thermal, despite the thermal nature indicated by probe characteristic measurements.

With regard to the Berkeley source, distribution function measurements performed under relatively high discharge current conditions, confirms the non-thermal nature of the distribution function tail. Figure 3B illustrates a logarithmic plot of the electron energy distribution function $f_e(\epsilon)$ obtained by both real time and harmonic double differentiation of the probe characteristic shown in Figure 3A. Figures 3C and 3D plot the total electron energy distribution function $F_e(\epsilon)$, including phase space weighting (i.e. $F_e(\epsilon) \propto f_e(\epsilon) \sqrt{\epsilon}$ where $F_e(\epsilon) d\epsilon$ gives the relative number of electrons with energy between ϵ and $\epsilon + d\epsilon$). The distribution function, in contrast to Langmuir's interpretation,

consists of two main constituents:

1. Thermal bulk electrons which constitute the majority of the total electron population (roughly 96% for the function shown in Figure 3C).
2. A non-thermal high energy tail, which comprises the monotonically decreasing high energy component of the distribution function (Figure 3D)¹⁰.

The break between the bulk electron component and non-thermal tail occurs at an energy roughly corresponding to the potential difference between the plasma potential and the source wall floating potential. Measurements made with cylindrical probes oriented in the axial and radial directions, and with spherical probes confirm the near isotropy of the distribution function; which is primarily due to the symmetry of the source chamber.

Qualitatively, distribution function structure is best understood by considering the source chamber as an electrostatic potential well. Excluding the anode, the chamber consists of several electrically isolated sections of conducting wall; each section floating negative with respect to the plasma potential such that the net random current due to electron and ion bombardment is nulled. Source walls therefore form a potential well for electrons; the containment condition being that the projected energy of an electron normal to a wall surface ($\epsilon_{\perp}$), is less than the potential difference between the plasma potential and the wall floating potential ($\Delta\epsilon_{\text{WELL}}$). This containment condition can be employed to conceptually dichotomize the distribution function into the two constituents described above.

1. Electrons whose total energy is less than $\Delta\epsilon_{\text{WELL}}$, and which remain trapped until encountering the anode. Electrons in this group can be shown to have sufficient time to thermalize before escaping,¹⁰ and hence constitute the bulk thermal electrons.
2. Electrons whose total energy is greater than $\Delta\epsilon_{\text{WELL}}$ (the high energy tail). Within this group, electrons where $\epsilon_{\perp} < \Delta\epsilon_{\text{WELL}}$ will remain trapped until collection by the anode, an inelastic encounter with a background neutral, or scattering into the "loss cone" defined by the condition $\epsilon_{\perp} > \Delta\epsilon_{\text{WELL}}$. The containment period, which for typical source operating conditions is .03 μsec , is insufficient to allow thermalization,¹⁰ but evidently sufficient to insure no overall depletion of the tail.

Spatially, the energy distribution of bulk electrons remains fairly constant over the majority of the source volume. Axial and radial scans of the bulk electron temperature (Figures 5A and 5B) show a fair degree of spatial uniformity. The high energy tail, however, exhibits a gradual population enhancement with increasing radial position from source center. This effect can be partially attributed to the high energy containment condition ($\epsilon_{\perp} < \Delta\epsilon_{\text{WELL}}$) which, for the cylindrical geometry of the source chamber, favors energetic electrons with small radial velocity components that can elastically scatter off the plasma-wall sheath.

Figure 3E illustrates the distribution function tail near the filaments at a radial position 6 cm off axis. The tail comprises $\sim 10\%$ of the total electron population as opposed to $\sim 4\%$ for Figure 3D obtained under the same operating conditions on axis. In terms of electron

population capable of ionization, this relates to an off axis density of $1.6 \pm .2 \cdot 10^{11}/\text{cm}^3$ as compared to $1.2 \pm .2 \cdot 10^{11}/\text{cm}^3$ on axis; even though the total electron density maximizes at source center (see Figure 5). Hence, plasma ion production is essentially uniform throughout the source volume.

In conclusion, when computing bulk plasma properties such as electron Debye length, which sets the spatial scale determining sheath structure, the distribution function may be considered thermal at the bulk electron temperature. However, when computing dissociation and ionization rates, the high energy tail has profound influence. For example, the population of ionizing electrons in deuterium is typically one hundred per cent larger than expected from a Maxwellian at the bulk electron temperature. The enhancement of ionization rates is even greater due to cross section weighting. The spatial uniformity of ionizing electrons results in uniform plasma ion production, and performs the beneficial function of flattening the plasma potential and density profile; a necessary condition for large area ion extraction.

The High Energy Tail Beyond Cathode Fall

The existence of electrons with energy values exceeding the cathode fall in electrical discharges have been documented since the early days of vacuum tubes.¹¹ An extensive study of this effect in low current, hot cathode discharges was initially performed by Langmuir.⁶ Using probe characteristic measurements, substantial electron currents were measured for probe bias values up to -20 volts with respect to cathode. Langmuir attributed this effect to primary electron scattering, and proposed an algorithm for determining the temperature by hypothesizing an isotropic Maxwellian, displaced in energy by cathode fall acceleration:

$$f_p(\epsilon) = n_e \left(\frac{m_e}{2kT_p} \right)^{3/2} e^{-(\sqrt{\epsilon} - \sqrt{eV_p})^2 / kT_p} \quad (1)$$

where V_p = Plasma Potential w.r.t. cathode

T_p = Primary Temperature

as the primary electron distribution function. For a 5mA low pressure mercury vapor discharge with a cathode fall of 50 volts, a primary temperature of .12 eV was sufficient to account for the experimental measurements.

Similar measurements performed on the Berkeley source, involving high resolution probe characteristics, generally concur with Langmuir's experimental results. Figure 4A is a computer plot of a probe characteristic obtained under conditions similar to those of Figure 3. Measurable electron current extends to probe bias values 20 volts negative of (-) filament. (Zero bias marks the negative leg of the cathode.) However, as previously mentioned, a more precise determination of the high

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Figure 4A.

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Figure 4B.

energy tail at energy values exceeding cathode fall entails direct distribution function measurement. Measurements performed in low current discharges by Rayment and Twiddy, reveal a monotonically decreasing, non-thermal function over a range 15 to 20 eV greater than cathode fall. The dominant mechanism for this primary scattering is due to a resonant interaction between the primary and bulk plasma electrons. The distance traveled by primaries before being strongly scattered is roughly one tenth the collisional mean free path for electron-electron scattering, and corresponds to a plasma interaction time of approximately three plasma periods; indicating a strong beam-plasma effect.⁹

Analogous distribution function measurements on the Berkeley source, employing a section of axially symmetric floating wall as an extended wall probe, generally concur. Figure 4B illustrates an axially projected distribution function obtained for a background plasma density of $1.3 \cdot 10^{12}/\text{cm}^3$ (Discharge arc power = 10.3 kW). The results parallel those of Rayment and Twiddy, and consist of a monotonically decreasing function out to 20 eV above cathode fall. Models describing this structure for energy values greater than cathode fall can possibly be inferred from similar measurements in low current discharges where beam-plasma interactions dominate. However, at present, no direct experimental confirmation of this mechanism in the Berkeley source exists.

The Spatial Dependence of Source Plasma Parameters

The first comprehensive steady state theory of a low pressure, magnetic field free discharge was formulated by Tonks and Langmuir in 1929.¹² The theory involved solution of Poisson's equation under conditions where:

1. electrons are assumed to have a Maxwellian distribution function; their local density satisfying the Boltzmann relation
 $n_e(X) \propto \exp(e\phi(X)/kT_e)$ where $\phi(X)$ is the local plasma potential.
2. ions are assumed created at rest throughout the plasma volume according to a specified generation function, and subsequently free fall without collisions in the local ambipolar electric field to the walls, where recombination with electrons occur. For the case of planar, cylindrical or spherically symmetric discharges ($\beta = 0, 1, 2$ respectively), the ion continuity equation under the free fall and steady state assumptions, yields the local ion density function:

$$n_i(X) = \left(\frac{M_i}{2e}\right)^{1/2} \frac{1}{X^\beta} \int_0^X \frac{G(Y)Y^\beta dY}{[\phi(Y) - \phi(X)]^{1/2}} \quad (2)$$

where $G(X) \equiv$ ion generation rate per unit volume

$\phi(X) \equiv$ Local plasma potential (a monotonically decreasing function of X , with $\phi(0) = 0$).

$X \equiv$ Variable characterizing the spatial position in the discharge measured with respect to the center of symmetry (i.e., $X = Z$; r for planar; cylindrical or spherical discharges, respectively).

Thus Poisson's equation becomes:

$$\begin{aligned} \nabla^2 \phi(X) &= -4\pi e(n_i(X) - n_e(X)) \\ &= -4\pi e \left[\left(\frac{M_i}{2e}\right)^{1/2} \frac{1}{X^\beta} \int_0^X \frac{G(Y)Y^\beta dY}{[\phi(Y) - \phi(X)]^{1/2}} - n_{e0} e^{(e\phi(X)/kT_e)} \right] \end{aligned} \quad (3)$$

Langmuir termed this result "The complete plasma-sheath equation" and proposed solutions for two simplified but physically interesting cases:

1. The plasma approximation, where quasi-neutrality requires $\nabla^2 \phi(X) = 0$.
2. The thin sheath approximation where volume ion production in the sheath region can be ignored.

The proposed solutions were in power series form, and were asymptotically correct in the limit of small Debye length.

Exact solutions of equation 3 for planar discharges; and asymptotic, thin sheath solutions of equation 3 for cylindrically and spherically symmetric discharges, were formulated by Self assuming plasma ion production spatially uniform, proportional to the electron density, and proportional to the square of the electron density.^{13,14} Since many relevant discharge parameters are calculable once the potential distribution is known, these solutions provide valuable input for predicting ion source performance. However, for most experimental discharge chambers, a two-dimensional generalization of the Self solutions is necessary in order to obtain a complete and accurate spatial mapping of the plasma potential.

An alternative approach to steady-state low pressure discharge theory proposed by Kino and Shaw,^{15,16} retains the Maxwellian assumption for electrons, but treats ion motion via transport equations obtained from moments of the Boltzmann equation. Hence, whereas the Tonks-Langmuir or Self method deals with an exact description of ion motion in terms of the ion distribution function and related local ion density (equation 2), the moment method employs a macroscopic fluid description of ion kinetics. The advantage of the moment equation method, is that it leads to differential equations which are easier to generalize and compute than

the exact plasma sheath equation. The two-dimensional solutions conclude that for cylindrically symmetric discharges, spatial plasma profiles are dependent on R/L_z , the discharge chamber radius to depth ratio. However, for the Berkeley source where $R/L_z \sim 1$, the one-dimensional solutions suffice for estimation purposes across the bulk of the discharge chamber.

Figures 5A and 5B illustrate axial and radial profile data for operating conditions previously illustrated in Figure 3. The ion free fall profile for plasma potential represents solutions to equation 3 for the case of spatially uniform ion production; as concluded from electron distribution function measurements. Ion free fall curves for plasma density are related to the plasma potential profile via the Boltzmann factor at the bulk electron temperature. Saturated ion current densities are defined in terms of the normalized value:

$$j_{i\text{SAT}} = \frac{1}{4} n_i e \sqrt{\frac{8kT_e}{\pi M_i}} \quad (4)$$

The experimental agreement with the one-dimensional solutions, although approximate, remain sufficient to conclude that overall source performance follows steady state low pressure discharge theory. In particular:

1. Plasma potential measurements exhibit the general ambipolar profile predicted by solutions of the collisionless plasma sheath equation (3) for spatially uniform ion production.
2. Local plasma density is related to local potential via the Boltzmann factor at the bulk electron temperature.

3. Extractable ion current density (which is equivalent to the outward ion flux crossing the Tonks-Langmuir Sheath Boundary) follows low pressure discharge theory predictions, and hence is essentially determined by the electron temperature and density at source center.

Admittedly, an exact determination of the effects of multiple ion species,¹⁷ finite ion temperature*, and ion collisions on observable plasma parameters, requires numerical solutions of the appropriate moment equations. However, to within experimental uncertainties, measurements indicate that these effects are minor.

*Ion temperature is presumed to fall between lower limit estimates inferred from background gas temperature measurements ($\sim .3\text{eV}$ for D^0 , $.1\text{eV}$ for D_2^0), and upper limit estimates set by extracted beam divergence ($\sim 1\text{eV}$).¹⁸

Plasma parameters vs. axial position

Arc power: 16.3 kW gas flow: 6 T- ℓ /sec

▬ Axial probe data ▲ Radial probe data

Figure 5A.

XBL787-1379

Figure 5B.

SOURCE PLASMA FLUCTUATIONS - THE EXPERIMENTAL APPROACH

Introduction

Temporal signal processing techniques are widely employed in plasma physics to investigate plasma waves, instabilities and turbulence. For example, experimental knowledge of the spatial and temporal dependence of plasma fluctuations is necessary in order to study their relationship with anomalous transport or non-linear phenomena.¹⁹⁻³⁰ Historically, early temporal techniques were primarily limited to analog measurement of the two-time correlation function. Although correlation techniques were experimentally effective, they required measurement over a narrow bandwidth.^{19,21,29} Wideband fluctuation analysis was therefore a fairly laborious process involving extensive analog filtering and measurement iteration.

An equivalent, but oftentimes more transparent method of fluctuation analysis involves power spectrum measurement. The power spectrum is essentially a Fourier transform of the correlation function, and is equivalent to the correlation function in information content. However, it provides an additional advantage that fluctuation phenomena are described in the frequency domain, which for many applications is advantageous over the time domain correlation format (e.g., the study of plasma dispersion characteristics or the spectral index and mode coupling properties of turbulent plasmas). In addition, recent advances in digital electronics, and the development of a fast Fourier transform (FFT) algorithm³¹⁻³⁵ have greatly facilitated both the speed and accuracy of power spectral acquisition and analysis; allowing broadband measurement with minimal analog filtering. This section presents the digital spec-

tral analysis system used to study ion acoustic fluctuations in the LBL 10 ampere neutral beam ion source.

Theory of Measurement

Let $\phi_A(t)$ represent a general time dependent signal describing some intensive property of a physical system. For example, $\phi_A(t)$ may represent the fluctuating electrostatic potential in a plasma. Apart from the mean averaged signal power, the simplest nontrivial function describing the temporal characteristics of $\phi_A(t)$ is the two-time correlation function:*

$$C_{AA}^{\phi}(\tau) = \langle \phi_A(t) \phi_A(t-\tau) \rangle_t = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \phi_A(t) \phi_A(t-\tau) dt \quad (5)$$

where T represents the total signal observation time. Mathematically, the correlation function is a real and even function of τ ; conditions which result from constraints of reality and the time invariant nature of the physical processes which drive $\phi_A(t)$. Despite its adjudged simplicity, the correlation function contains an appreciable amount of information about the statistical properties of $\phi_A(t)$. It is, in essence, a measure of the dependence between the physical processes driving $\phi_A(t)$ at different times (i.e., a measure of the systems temporal memory) and in this sense, describes its time variation.**

An alternate but equivalent description of the correlation function in the frequency domain is given by the auto power spectral density function:

*Statistically defined as the first moment of the two-time probability distribution function.

**For example, in the case of Brownian motion, particle momentum appears as a random temporal variable on a time scale much longer than the particle driving force correlation time.

$$S_{AA}^{\phi}(\omega) = \lim_{T \rightarrow \infty} \left[\frac{\Phi_A^*(\omega) \Phi_A(\omega)}{T} \right] = \int_{-\infty}^{\infty} C_{AA}^{\phi}(\tau) e^{i\omega\tau} d\tau \quad (6)$$

where:

$$\Phi_A(\omega) = \int_{-\infty}^{\infty} \phi_A(\tau) e^{i\omega\tau} d\tau \quad (7)$$

$$\phi_A(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_A(\omega) e^{-i\omega t} d\omega$$

and where $S_{AA}^{\phi}(\omega)$ is a real, even and non-negative function of ω . The inverse transform of equation (6) returns the correlation function:

$$C_{AA}^{\phi}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{AA}^{\phi}(\omega) e^{-i\omega\tau} d\omega \quad (8)$$

Equations (6) and (8) are formally recognized as the Weiner-Khinchine relations which illustrate the equivalence between a correlation and power spectral density description of fluctuation phenomena. Combining equations (5) and (8) for the case of $\tau = 0$ yields the power balance condition:

$$\langle (\phi_A(t))^2 \rangle_t = C_{AA}(0) = \frac{1}{\pi} \int_0^{\infty} S_{AA}^{\phi}(\omega) d\omega \quad (9)$$

which physically states that the mean squared signal power must equal the total integrated power spectral density. Equation (9) thus provides a convenient and reliable method of checking the accuracy and validity of experimental power spectrum measurements.

The generalization from one to two signal sample points is rather straightforward. Given two signals $\phi_A(\vec{x}_A, t)$ and $\phi_B(\vec{x}_B, t)$, again possibly representing the electrostatic potential fluctuation at two spatial locations in a plasma, the two-point, two-time cross correlation function is defined by:

$$C_{AB}^{\phi}(\vec{R}, \tau) = \lim_{\substack{T \rightarrow \infty \\ V \rightarrow \infty}} \frac{1}{2TV} \int_{-T}^T \int_V \phi_A(\vec{x}_A, t) \phi_B(\vec{x}_A - \vec{R}, t - \tau) d\vec{R} d\tau \quad (10)$$

where:

$$\vec{R} = \vec{x}_A - \vec{x}_B$$

The corresponding cross power spectrum is therefore:

$$\begin{aligned} S_{AB}^{\phi}(\vec{k}, \omega) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C_{AB}^{\phi}(\vec{R}, \tau) e^{i(\omega\tau - \vec{k} \cdot \vec{R})} d\tau d\vec{R} \\ &= \lim_{\substack{T \rightarrow \infty \\ V \rightarrow \infty}} \left\{ \frac{\phi_A^*(\vec{k}, \omega) \phi_B(\vec{k}, \omega)}{TV} \right\} \end{aligned} \quad (11)$$

Experimentally, $S(\vec{k}, \omega)$ contains a reasonably complete description of the spatial and temporal characteristics of fluctuations within the physical system. However, it is often unnecessary to measure the full spatial content of $S(\vec{k}, \omega)$ in order to draw relevant physical conclusions from the analyzed data. For example, if plasma potential fluctuations result from the excitation of a unidirectionally propagating normal plasma mode, then within the limitations imposed by linear theory they can be described in terms of a superposition of monochromatic plane waves:

$$\phi(\vec{x}, t) = \int_{-\infty}^{\infty} \alpha(\omega) e^{-i(\omega t - \vec{k}(\omega) \cdot \vec{x})} d\omega \quad (12)$$

where $\vec{k}(\omega)$ is a solution of the linear mode dispersion relation and $\alpha(\omega)$ describes the frequency dependent wave amplitude function. For this case, a complete description of the cross power spectrum (equation 11) is obtainable by measuring the spectral function:

$$\begin{aligned} S_{AB}^{\phi}(\vec{R}, \omega) &= \int_{-\infty}^{\infty} C_{AB}^{\phi}(\vec{R}, \tau) e^{i\omega\tau} d\tau \\ &= \lim_{T \rightarrow \infty} \left[\frac{\phi_A^*(\vec{x}_A, \omega) \phi_B(\vec{x}_B, \omega)}{T} \right] \end{aligned} \quad (13)$$

which is experimentally expeditious since analysis does not require the computation of a spatial Fourier transform and hence, in principle, only two spatial samples are necessary. Fourier transforming equation (12) and substituting into (13) yields:

$$S_{AB}^{\phi}(\vec{R}, \omega) = |S_{AB}^{\phi}(\omega)| e^{i\Theta_{AB}(\vec{R}, \omega)} \quad (14)$$

where

$$|S_{AB}^{\phi}(\omega)| = \lim_{T \rightarrow \infty} \left[(1/T) |\alpha_A^*(\omega) \alpha_B(\omega)| \right] \quad (15)$$

and

$$\Theta_{AB}(\vec{R}, \omega) = [\Theta_B(\vec{x}_B, \omega) - \Theta_A(\vec{x}_A, \omega)] = \vec{k}(\omega) \cdot \vec{R} \quad (16)$$

The cross power spectral density, $|S_{AB}^{\phi}(\omega)|$, is a physical measure of the coherence between temporal fluctuations occurring at sample points A and B. In analogy with optics, a normalized version of $|S_{AB}^{\phi}(\omega)|$

called the spectral coherence function, is defined by:

$$\gamma_{AB}^{\phi}(\omega) = \frac{|S_{AB}^{\phi}(\omega)|}{[S_{AA}^{\phi}(\omega)S_{BB}^{\phi}(\omega)]^{1/2}} \quad (17)$$

where $S_{AA}^{\phi}(\omega)$ and $S_{BB}^{\phi}(\omega)$ are the auto power spectral density at points A and B respectively. In essence, $\gamma_{AB}^{\phi}(\omega)$ is the normalized cross power spectral density ($0 \leq \gamma_{AB}^{\phi}(\omega) \leq 1$) where low (high) values within a particular bandwidth imply incoherence (coherence) between fluctuations at the sample points. The cross power phase spectrum, $\Theta_{AB}(\vec{R}, \omega)$, is defined as the relative spectral phase difference between fluctuations at the sample points and is therefore a measure of the plasma dispersion relation ($k(\omega)$).

Equations 13 through 17 illustrate that under the assumption of normal mode excitation (equation 12), the general cross power spectrum can be obtained by two fixed sample points with arbitrary non-zero separation. The inclusion of wave damping adds the additional constraint that sample point separation must remain less than the wave spatial coherence length. Mathematically, this simplified description results from the well-defined relationship between ω and $\vec{k}$ as defined by the linear dispersion relation, and can be illustrated by spatially transforming equation (14) to obtain the general cross power spectrum:

$$S_{AB}^{\phi}(\vec{k}, \omega) = \int_{-\infty}^{\infty} S_{AB}^{\phi}(\vec{R}, \omega) e^{-i\vec{k} \cdot \vec{R}} d\vec{R} = |S_{AB}^{\phi}(\omega)| \delta(\vec{k} - \vec{k}(\omega)) \quad (18)$$

Thus, in frequency bands where equation (18) is valid, the cross power spectrum will yield a high degree of coherence (equation 17) with the dispersion relation defined by equation 16. For cases where equation

(18) is not valid,* the full cross power spectrum (equation 11) must be measured.

Apparatus

Figure 6 schematically illustrates the digital spectral analysis system. Potential fluctuations are detected by two matched and shielded electrostatic probes, which are mounted on vernier drivers enabling probe separation adjustment while maintaining vacuum integrity. A set of external probe access ports allow axial, radial and azimuthal alignment for wave propagation measurements. The probe signals are processed through a matched pair of pass band filters, amplified and digitized by a Nicolet transient recorder; which is capable of 8 bit/50 nsec per point analog to digital conversion. Filter pass bands are set by requirements to suppress transient turn on and power supply noise (low frequency cut off) and to suppress aliasing effects (high frequency cut off). The digitized data is finally read by a Modcomp Systems computer over a direct I/O link, which subsequently executes a spectral analysis algorithm providing on-line analysis via a Tektronix 4014 graphics terminal. Disk and magnetic tape storage supply the capability for multiple data set averaging which is necessary in order to obtain low noise broadband power spectrum estimates. Hardcopy alphanumeric and graphical output is provided by a Versatek printer/plotter.

The spectral analysis system is capable of power spectrum measurement over a 10MHz bandwidth with a spectral resolution of 2.5kHz, and fluctuation power levels down to $10^{-11} \text{V}^2/\text{kHz}$. Minimum power level

*When measuring a strongly excited spectrum, the delta function dependence of k in equation 18 may be broadened by non-linear coupling between modes.³⁶

Figure 6. Digital Spectral Analysis Schematic.

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resolution is primarily set by input amplifier and digitization noise since shielding minimizes external RF pickup. System performance is periodically checked by using the power balance condition (equation 9) on a calibrated signal. Measured integrated power spectral density typically agrees with RMS signal power to within ten per cent.

Equation 16 demonstrates the importance of matching data acquisition channels in the detection apparatus. Since the cross power phase spectrum is dependent only on the relative phase between channels, proper matching insures minimal systematic measurement error. Measured phase error is typically less than .01 (0.1) radians over a 1(10)MHz bandwidth respectively.

Accuracy Analysis

The accuracy of the spectral analysis system is dependent on the response characteristics of the probe shielded cables, band pass filters and input amplifiers (Fig. 6); and the acquisition and numerical accuracy of the signal digitizer and spectral analysis algorithm. Pedagogically, these effects may be approximately modeled within the formalism of linear network theory where signal modification by detection system characteristics are described in terms of a frequency response function $H_{\text{sys}}(\omega)$. That is, given a time dependent signal $\phi(t)$ with spectral density $\phi(\omega)$, the detected signal is:

$$\phi_{\text{DET}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_{\text{sys}}(\omega) \phi(\omega) e^{-i\omega t} d\omega \quad (19)$$

or equivalently:

$$\hat{\phi}_{DET}(\omega) = H_{sys}(\omega)\phi(\omega) \quad (20)$$

In general, $H_{sys}(\omega)$ is a complex function whose real and imaginary components, describing detection system amplitude and phase response, are related via the Kramers-Kronig relations.³⁷

In application to spectral analysis, the detected cross power spectral density (equation 13-15) of two signals ($\phi_A(\vec{x}_A, t)$, $\phi_B(\vec{x}_B, t)$) measured by two channels of a system with response ($H_A(\omega)$, $H_B(\omega)$) is:

$$S_{AB}^{\hat{\phi}}(\vec{R}, \omega)|_{DET} = \lim_{T \rightarrow \infty} \left\{ \frac{1}{T} |\phi_A^*(\omega) H_A^*(\omega) \phi_B(\omega) H_B(\omega)| e^{i[\Theta_{AB}(\vec{R}, \omega) + \Theta_H(\omega)]} \right\} \quad (21)$$

where $\Theta_H(\omega)$ defines the system induced phase shift. If

$$H_A(\omega) = H_B(\omega) = H(\omega):$$

$$S_{AB}^{\hat{\phi}}(\vec{R}, \omega)|_{DET} = |S_{AB}(\vec{R}, \omega)| |H^2(\omega)| e^{i\Theta_{AB}(\vec{R}, \omega)} \quad (22)$$

In theory, equation (21) or (22) can be digitally inverted to obtain the actual cross power spectrum if an accurate form of $H(\omega)$ is available.³⁸⁻⁴⁰ However, it is experimentally wise to judiciously design the detection apparatus to minimize system induced effects over the measurement bandwidth. This requires signal channels be effectively matched in order to minimize phase error; and the modulus of the response function, $|H(\omega)|$, should remain reasonably flat across the desired cross power spectral bandwidth. Semi-quantitative knowledge of $H(\omega)$ is therefore important in order to provide a basis for reasonable experimental procedure and design.

Probe-Plasma and Analog Circuit Effects

Signal response of the spectral analysis system (Figure 6) can be effectively modeled by the schematic equivalent illustrated in Figure 7 (one channel). The schematic is composed of three conceptually distinct sections; the plasma-probe interface (generally dependent on probe geometry and plasma operating conditions), the analog detection circuit (shielded cable, bandpass filters, input amplifier), and the digital processing circuit which accounts for digital effects on data acquisition and analysis via the transfer function $H_D(\omega)$. The plasma-probe interface is essentially a signal driver with voltage source $\phi(t)$ (representing probe floating potential fluctuations) and output impedance $Z_p(v_0, \omega)$ (the plasma-probe sheath impedance where v_0 is the static sheath potential drop). Assuming constancy of electron temperature over a signal acquisition period, $\phi(t)$ is equivalent to local plasma potential fluctuations. The analog circuit is characterized by an effective input or driving point impedance $Z_i(\omega)$ and potential transfer function $H_c(\omega)$. From equation 19, the net analog detected signal is therefore:

$$\phi_{DET}^A(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) \phi(\omega) e^{-i\omega t} d\omega \quad (23)$$

where the composite analog transfer function is:

$$H(\omega) = \left[\frac{Z_i(\omega)}{Z_p(v_0, \omega) + Z_i(\omega)} \right] H_c(\omega) \quad (24)$$

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Figure 7. Response schematic of the spectral analysis system.

$Z_i(\omega)$, $H_c(\omega)$ and in principle $Z_p(v_0, \omega)$ are all experimentally measurable quantities. For transient plasmas where direct measurement of $Z_p(v_0, \omega)$ is possibly untenable, a semi-quantitative expression is derivable from basic plasma considerations as follows.

The total plasma current density drawn by an electrostatic probe as a function of probe bias is:

$$J_T(v) = J_p(v) + \frac{1}{4\pi} \frac{\partial}{\partial t} E_p(v) \quad (25)$$

where $v(t)$ represents probe bias with respect to the plasma potential (i.e., the probe sheath potential drop), $J_p(v)$ and $E_p(v)$ represent the collected current density and electric field at the probe surface as a function of bias respectively, and $\frac{1}{4\pi} \frac{\partial}{\partial t} E_p(v)$ defines the displacement current contribution due to temporal space charge changes in the probe sheath. The sheath potential drop may be further dichotomized into a static and fluctuating component:

$$v(t) = v_0 + \phi(t) \quad (26)$$

yielding a Taylor series expansion of $J_T(v)$ about v_0 :

$$J_T(v) = J_p(v_0) + j_T(\phi) \quad (27)$$

where:

$$j_T(\phi) = \left. \frac{\partial J_T(v)}{\partial v} \right|_{v_0} \phi(t) = \left[\left. \frac{\partial J_p(v)}{\partial v} \right|_{v_0} + \frac{1}{4\pi} \frac{\partial}{\partial t} \left. \frac{\partial E_p(v)}{\partial v} \right|_{v_0} \right] \phi(t) \quad (28)$$

Fourier Transforming (27) yields:

$$J_T(v_0, \omega) = J_p(v_0) + j_T(\omega) \quad (29)$$

where

$$j_T(\omega) = \left. \frac{\partial J_p(v)}{\partial v} \right|_{v_0} \left[1 + \frac{i\omega}{4\pi} \left. \frac{\partial E_p(v)}{\partial J_p(v)} \right|_{v_0} \right] \phi(\omega) \quad (30)$$

For an ion attracting probe, the probe-plasma sheath is primarily dominated by positive space charge and hence, the dependence of collected current density on sheath electric field (Equation 30) is essentially determined by space charge limited ion flow. For the case of a planar probe, or cylindrical or spherical probe operating in the thin sheath limit (Appendix), collected ion current density is approximately given by the Child-Langmuir relation:

$$J_i(v) = \frac{4}{9} \left(\frac{2q_i}{M_i} \right)^{1/2} \frac{v^{3/2}}{4\pi d^2} \quad (31)$$

where d is the probe sheath thickness. Defining $E_p(v) \sim v/d$, setting v_0 at the probe floating potential, and employing equation (31) yields:

$$\left. \frac{\partial E_p(v)}{\partial J_p(v)} \right|_{v_0} \approx \frac{4\pi d}{C_s} \left[\ln \left(\frac{M_i}{m_e} \right) \right]^{-1/2} \approx 4\pi \tau_i \quad (32)$$

where C_s is the ion sound speed ($C_s = \sqrt{T_e/M_i}$) and τ_i is essentially the mean ion transit time through the probe-plasma sheath. Although equation (32) assumes a floating probe, it is approximately correct for most of the ion attracting probe operation regime where $ev_0 \gg kT_e$

since it is relatively insensitive to probe bias. By employing equations (30) and (32), equation (29) may be cast in the form of an admittance relation:

$$I_T(v_0, \omega) = I_p(v_0) + Y_p(v_0, \omega) \phi(\omega) \quad (33)$$

where $I_p(v_0)$ is the D.C. collected probe current at bias v_0 (zero for a floating probe); $Y_p(v_0, \omega)$, the dynamic probe sheath admittance, is defined by

$$Y_p(v_0, \omega) = Y_p(v_0) [1 + i\omega\tau_1] \quad (34)$$

and $Y_p(v_0)$, the static probe sheath admittance, is:

$$Y_p(v_0) = \left. \frac{\partial I_p(v)}{\partial v} \right|_{v_0} = \left[Z_p(v_0) \right]^{-1} \quad (35)$$

For frequencies well below the ion plasma frequency, probe sheath reactance is negligible and hence sheath admittance is determined by measuring the slope of the probe current-voltage characteristic about the static operating point. Probe sheath impedance, defined as the inverse of admittance, is therefore an easily determined quantity. Since $I_p(v)$ is a nonlinear function of sheath potential, $Z_p(v_0)$ will in general behave like a nonlinear circuit element. Deleterious nonlinear effects in the analog detection circuit may, however, be avoided by minimizing the ratio of probe sheath to analog input impedance over the measurement bandwidth ($|Z_p(v_0)| / |Z_1(\omega)| \ll 1$). This condition must be mitigated by considerations of finite probe size since probe sheath impedance is inversely proportional to probe area (Eq. 35). A probe of

scale length L_p will effectively act as a low pass spatial filter with cut off at $k \approx 2\pi L_p^{-1}$. Accurate spectral measurement therefore requires that probe size remain much smaller than the fluctuation wavelength. In any case, the determination and utilization of the complete analog response function (Eq. 24) is rather straightforward.

Digital Effects

The preceding discussion has addressed the topic of spectral analysis from the standpoint of continuum mathematics. That is, all spectral variables were continuous functions of time or frequency. Digital data processing, however, requires all continuous functions be replaced by their discrete counterparts; since digitized quantities are only defined at a finite number of discrete sample points. This process is conceptually illustrated in Figure 8. The signal digitizer essentially multiplies the detected analog waveform by a sampling comb consisting of N samples with sampling frequency $f_s = 1/\Delta t$ and total duration $T = N\Delta t$. In addition, the signal amplitude at each sample point is quantized to a fixed number of discrete levels set by the capacity of the analog to digital converter (e.g. an eight bit ADC will allow 2^8 or 256 quantized amplitude levels). The resulting output signal is a digitized representation of the analog input waveform whose accuracy is strongly dependent on the following considerations.

Bandwidth Limitation. One consequence of the Whittaker-Shannon sampling theorem is that an infinite, but discrete time series will completely describe a continuous waveform, provided the waveform is bandwidth limited to half the discrete sampling frequency (also known

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Figure 8. Conceptual illustration of signal digitization.

as the Nyquist frequency).^{41,42} In effect, the theorem formalizes a well-known concept in information theory that the maximum rate of information transfer is proportional to the transmission bandwidth. Input signal components above the Nyquist frequency will, when sampled, be falsely interpreted as lower frequency information; an effect known as aliasing.⁴³ Aliasing is a straightforward consequence of sampling at equally spaced intervals and cannot be removed from the digitized data once sampling has occurred. Therefore, bandwidth limitation of the analog input is a necessity which entails passing the signal through a suitable low pass or band pass filter before digitization occurs. The effect of filter response on overall system performance is then included in the analog transfer function $H(\omega)$, previously described.

Data Window. The spectral composition of an input bandlimited signal ($|\omega| \leq 2\pi f_s$) may be expressed as an infinite Fourier series over the bandwidth interval:

$$\phi_{DET}^A(\omega) = \sum_{n=-\infty}^{\infty} \phi_{DET}^A(t_n) e^{i\omega t_n} \quad (36)$$

where

$$\phi_{DET}^A(t_n) = \int_t \phi_{DET}^A(t) \delta(t-t_n) dt \quad (37)$$

$$= \frac{1}{2\pi f_s} \int_{-\pi f_s}^{\pi f_s} \phi_{DET}^A(\omega) e^{-i\omega t_n} d\omega \quad (38)$$

and: $\phi_{DET}^A(t)$ represents the continuous analog input to the digital system in the notation of Figure 7.

$t_n = n\Delta t$ represents the discrete sample points where Δt is the intra-sample period and is equivalent to the inverse of the sample frequency f_s .

The act of experimental measurement, however, necessitates a finite data record and consequently, equation 36 must be modified accordingly. The digitally measured signal may be expressed in terms of the band limited input as:

$$\phi_{DET}(t_n) = \phi_{DET}^A(t_n) w_R(t) \quad (39)$$

where $w_R(t)$ defines the rectangular data window:

$$w_R(t) = \begin{cases} 1 & 0 \leq t \leq t_{N-1} \\ 0 & \text{otherwise} \end{cases} \quad (40)$$

Hence, by employing equations 36 through 40, the spectral composition of the digitized signal is given by:

$$\phi_{DET}(\omega_m) = \sum_{n=0}^{N-1} \phi_{DET}(t_n) e^{i\omega_m t_n} \quad (41)$$

where $\phi_{DET}(\omega_m)$ defines the discrete spectral transform of $\phi_{DET}(t_n)$ at frequencies $\omega_m = 2\pi m/N\Delta t$. Note that equation 41 is periodic with period $2\pi f_s$.

The inverse transform relation may be obtained by utilizing the orthogonal property of the complex exponentials over a set of discrete, equally spaced points covering the period:

$$\sum_{m=0}^{N-1} e^{i\omega_m(t_n - t_j)} = N\delta_{nj} \quad (42)$$

Thus, multiplying both sides of equation (41) by $e^{-i\omega_m t_j}$ and summing over m yields:

$$\begin{aligned} \sum_{m=0}^{N-1} \phi_{\text{DET}}(\omega_m) e^{-i\omega_m t_j} &= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \phi_{\text{DET}}(t_n) e^{i\omega_m(t_n - t_j)} \\ &= N \sum_{n=0}^{N-1} \phi_{\text{DET}}(t_n) \delta_{nj} \\ &= N\phi_{\text{DET}}(t_j) \end{aligned} \quad (43)$$

Hence:

$$\phi_{\text{DET}}(t_n) = \frac{1}{N} \sum_{m=0}^{N-1} \phi_{\text{DET}}(\omega_m) e^{-i\omega_m t_n} \quad (44)$$

Again, note that equation 44 is periodic with period T (the total sample duration). Equations 41 and 44 form the discrete counterparts of the continuous spectral transforms defined by equation 7. The spectral relationship between the discrete and continuous transform relations (equations 7 and 41) may be obtained by Fourier transforming equation 39, yielding the periodic convolution:

$$\phi_{\text{DET}}(\omega_m) = \frac{1}{2\pi f_s} \int_{-\pi f_s}^{\pi f_s} \phi_{\text{DET}}(\omega) W_R(\omega_m - \omega) d\omega \quad (45)$$

where $W_R(\omega)$ represents the Fourier transform of the rectangular data window $w_R(\tau)$:

$$W_R(\omega) \approx T \frac{\sin(\omega T/2)}{T/2} e^{i\omega T/2} \quad (46)$$

The transform $W_R(\omega)$ (Figure 9A) effectively acts like a digital transfer function ($H_D(\omega)$) relating analog to digital spectral content. Accurate spectral measurement therefore requires the main lobe width of the window function be considerably less than the inverse coherence time of the detected signal. In addition, the large sidelobe content of $W_R(\omega)$, resulting from the sharp temporal discontinuity in $w_R(t)$, effectively decreases the transform frequency resolution to something greater than the main lobe width due to the leakage of spectral information across a moderately wide frequency range. The usual approach of reducing sidelobe leakage entails utilizing a temporal data window with a less severe discontinuity and hence lower sidelobe content. One example is the Hanning window defined by:

$$w_H(t) = \frac{1}{2} \left[1 - \cos\left(\frac{2\pi t}{T}\right) \right] w_R(t) \quad (47)$$

Figure 9B illustrates the spectral transform of equation 47 ($W_H(\omega)$). The main lobe is twice as wide as the corresponding rectangular window however, side lobe level and associated leakage is greatly reduced. The digital transform is again related to its continuous counterpart by the periodic convolution (equation 45):

$$\Phi_{DET}(\omega_m) = \Phi_{DET}^A(\omega) * H_n(\omega) \Big|_{\omega=\omega_m} \quad (48)$$

where the digital transfer function is given by:

Figure 9. Window Transforms

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$$H_D(\omega) = W_H(\omega) \quad (49)$$

Hence, combining equation 48 with the analog response described by equation 20 yields the total system detected spectral density (Figure 7):

$$\Phi_{DET}(\omega_m) = \Phi(\omega)H(\omega) * H_D(\omega) \bigg|_{\omega=\omega_m} \quad (50)$$

where $\Phi(\omega)$ represents the spectral density of the input plasma potential fluctuations $\phi(t)$, and $H(\omega)$ represents the composite analog response function defined by equation 24.

Cross Power Spectral Estimation: Substituting equation 50 into equation 13, yields the detected cross power spectrum assuming two matched channels (A and B):

$$S_{AB}^{\phi}(\vec{R}, \omega_m) |_{DET} \approx S_{AB}^{\phi}(\vec{R}, \omega) |H^2(\omega)| * |H_D^2(\omega)| \bigg|_{\omega=\omega_m} \quad (51)$$

The measured cross power spectrum is, in actuality, a statistical estimate of the true cross power spectrum since for a finite number of data samples, statistical noise is significant. The effects of statistical noise on spectral measurement may be reduced by ensemble averaging the measurements over a series of statistically independent data sets. That is, letting $\text{Re}[S_{AB}^{\phi}|_{DET}]_j$ and $\text{Im}[S_{AB}^{\phi}|_{DET}]_j$ represent the real and imaginary components of the detected cross power spectrum for data set j , the ensemble averaged cross power spectral quantities are given by:⁴⁴

$$\langle |S_{AB}^{\phi}(\omega_m)| \rangle = \frac{1}{M} \left[\left(\sum_{j=1}^M \operatorname{Re}[S_{AB}^{\phi}|_{DET}]_j \right)^2 + \left(\sum_{j=1}^M \operatorname{Im}[S_{AB}^{\phi}|_{DET}]_j \right)^2 \right]^{1/2} \quad (52)$$

$$\langle \Theta_{AB}(\vec{R}, \omega_m) \rangle = \operatorname{ARCTAN} \left[- \frac{\sum_{j=1}^M \operatorname{Im}[S_{AB}^{\phi}|_{DET}]_j}{\sum_{j=1}^M \operatorname{Re}[S_{AB}^{\phi}|_{DET}]_j} \right] \quad (53)$$

$$\langle \gamma_{AB}^{\phi}(\omega_m) \rangle = \frac{\langle |S_{AB}^{\phi}(\omega_m)| \rangle}{[\langle S_{AA}^{\phi}(\omega_m) \rangle \langle S_{BB}^{\phi}(\omega_m) \rangle]^{1/2}} \quad (54)$$

where M denotes the total number of data sets in the averaging ensemble. Note that the ensemble averages are performed for each of the N discrete frequencies at which the spectral quantities are defined. The statistical variance of the averaged cross power spectral quantities are given by:⁴⁴

$$\operatorname{VAR} \langle |S_{AB}^{\phi}(\omega_m)| \rangle \approx \frac{1}{2M} \langle |S_{AB}^{\phi}(\omega_m)| \rangle^2 \left[1 + \frac{1}{\langle \gamma_{AB}^{\phi}(\omega_m) \rangle^2} \right] \quad (55)$$

$$\operatorname{VAR} \langle \Theta_{AB}(\vec{R}, \omega_m) \rangle \approx \frac{1}{2M} \left[\frac{1}{\langle \gamma_{AB}^{\phi}(\omega_m) \rangle^2} - 1 \right] \quad (56)$$

$$\operatorname{VAR} \langle \gamma_{AB}^{\phi}(\omega_m) \rangle \approx \frac{1}{2M} \left[1 - \langle \gamma_{AB}^{\phi}(\omega_m) \rangle^2 \right]^2 \quad (57)$$

The statistical signal to noise ratio follows the familiar Gaussian result and increases proportional to the square root of the number of independent data sets averaged. In addition, both the cross power phase and coherence are strongly dependent on the averaged coherence spectrum.

One standard method of minimizing statistical noise entails dividing a single data record of N samples into M subsets of N/M samples each. After the application of a suitable data window to each subset, the subsets are transformed and averaged yielding reduced statistical noise at the expense of reduced transform resolution. For many applications, however, this trade off is acceptable as long as transform resolution remains significantly greater than the signal inverse coherence time.

Quantization. In addition to a finite data record, the digitized signal amplitude, represented by equation 39, is quantized to a fixed number of discrete levels by the analog to digital converter. The effects of signal quantization on spectral analysis are conceptually equivalent to the deleterious effects of coefficient quantization on the algorithm used to compute the spectral transform. Although the nature of coefficient quantization is inherently nonstatistical, a rough statistical analysis of quantization noise is possible by replacing each discrete transform coefficient (e.g. equation 41) by its true value plus a white noise source called jitter. The resulting signal to noise ratio for an FFT algorithm is approximately:⁴⁵

$$\left(\frac{\text{Signal}}{\text{Noise}}\right)_{\text{Quantization}} \approx 6 \left[\frac{2^{2b}}{\log_2 N} \right] \quad (58)$$

where b represents the number of bits available for amplitude storage in the ADC and N is the total number of points transformed. The key result of equation 58 is that the signal to noise ratio is a weak function of the total number of transformed points, but has a strong dependence on the number of quantized amplitude levels available in the ADC.

Conclusion

The degree to which a detected signal approximates reality will, to a large extent, depend on a judicious choice of the analog and digital components which comprise the measurement apparatus. The preceding discussion has briefly presented the general considerations behind successful digital spectral measurement. However, in the final analysis, the actual choice of specific system components will directly depend on properties of the spectrum under consideration.

SOURCE PLASMA FLUCTUATIONS - THE EXPERIMENTAL RESULTS

Introduction

A uniform, magnetic field free plasma can support two characteristic electrostatic fluctuations: the high frequency Langmuir wave and the lower frequency ion acoustic wave. In low temperature plasmas, Langmuir waves are basically constant frequency oscillations at the electron plasma frequency and, as previously described, are most likely responsible for strong primary electron scattering at the source filaments via an electron two-stream instability. Langmuir fluctuations indirectly effect source performance through their influence on electron distribution function structure (Section I). In contrast, ion acoustic fluctuations are of greater direct interest to overall source performance since they directly couple to ion motion and thus provide a reasonably effective mechanism for low energy ion heating.³⁶ Ion waves comprise a direct analog to sound waves in a compressible medium where wave motion is sustained by fluid inertia (ions) and pressure (electrons).

Ion acoustic phenomena have received considerable experimental and theoretical attention since the early days of plasma physics.^{46,47} Many experimental investigations have utilized low density positive columns where discharge parameters were chosen for optimum wave behavior. Also, the strong unidirectional plasma current in a tubular positive column generally justifies one-dimensional wave propagation studies. In comparison, the dimensions and operating characteristics of the multi-filament source are constrained to adequately couple source performance to other beamline components and consequently, a certain amount of experimental latitude must be sacrificed. Nevertheless, the source

provides a suitable high density medium for ion wave propagation for the following reasons:*

1. The plasma is fairly uniform and magnetic field free across the bulk of the discharge chamber.
2. The electron temperature exceeds the effective ion temperature over all source operating conditions ($T_e \geq 10T_i$).
3. The plasma is essentially "collisionless". That is, the ion neutral collision frequency is approximately equal to the inverse of the mean ion lifetime (due to ion free fall) and typically is less than the fluctuation frequencies of interest.

This section presents the experimental results and discussion of fluctuation measurements in the multifilament source plasma.

The Source Plasma Dispersion Relation

The linear response of a plasma constituent ($\rho_s(\vec{r}, t)$) to a spatially and temporally fluctuating potential ($\phi(\vec{r}, t)$) is described by the general relation:

$$\rho_s(\vec{r}, t) = \int_V d\vec{r}' \int_{-\infty}^t dt' G_s(\vec{r}, \vec{r}'; t, t') \phi(\vec{r}', t') \quad (59)$$

where $G_s(\vec{r}, \vec{r}'; t, t')$ defines the species response function which is solely determined by properties of the unperturbed plasma. Assuming the unperturbed plasma is translationally invariant in both space and time,

* A high density medium with respect to low power positive column discharges.

equation 59 becomes:

$$\rho_s(\vec{r}, t) = \int_V d\vec{R} \int_0^\infty d\tau G_s(\vec{R}, \tau) \phi(\vec{r}-\vec{R}, t-\tau) \quad (60)$$

where: $\vec{R} = \vec{r} - \vec{r}'$
 $\tau = t - t'$

The assumption of translational invariance requires the characteristic scale lengths of temporal and spatial inhomogeneities in the unperturbed plasma be substantially greater than those of the potential fluctuations under consideration.*

Potential fluctuations are also coupled to plasma space charge by Poisson's equation:

$$\nabla^2 \phi(\vec{r}, t) = 4\pi \left[\sum_s \rho_s(\vec{r}, t) + \rho_{\text{ext}}(\vec{r}, t) \right] \quad (61)$$

where $\rho_s(\vec{r}, t)$ represents the charge density contribution from each plasma constituent and $\rho_{\text{ext}}(\vec{r}, t)$ describes the charge density distribution introduced by external, non plasma sources. That is,

$$\nabla^2 \phi_{\text{ext}}(\vec{r}, t) = 4\pi \rho_{\text{ext}}(\vec{r}, t) \quad (62)$$

where $\phi_{\text{ext}}(\vec{r}, t)$ represents an external, non plasma driving potential.

Fourier transforming equation 60, 61, and 62 yields:

$$\rho_s(\vec{k}, \omega) = G_s(\vec{k}, \omega) \phi(\vec{k}, \omega) \quad (63)$$

* A more stringent condition on translational invariance is set by the act of spectral measurement, which requires the unperturbed plasma to remain stationary in time over the detection period.

and:

$$\bar{\Phi}(\vec{k}, \omega) = \sum_s \frac{4\pi}{k^2} G_s(\vec{k}, \omega) \bar{\Phi}(\vec{k}, \omega) + \bar{\Phi}_{\text{ext}}(\vec{k}, \omega) \quad (64)$$

Hence:

$$\left[1 + \sum_s \chi_s(\vec{k}, \omega) \right] \bar{\Phi}(\vec{k}, \omega) = \epsilon(\vec{k}, \omega) \bar{\Phi}(\vec{k}, \omega) = \bar{\Phi}_{\text{ext}}(\vec{k}, \omega) \quad (65)$$

where:

$$\chi_s(\vec{k}, \omega) = - \frac{4\pi}{k^2} G_s(\vec{k}, \omega) \quad (66)$$

defines the plasma susceptibility for species s , and

$$\epsilon(\vec{k}, \omega) = \left[1 + \sum_s \chi_s(\vec{k}, \omega) \right] \quad (67)$$

defines the longitudinal plasma dielectric function which essentially describes the complete electrostatic properties of the plasma system. From equation 65, the normal mode or non driven plasma fluctuations are obtained by setting $\bar{\Phi}_{\text{ext}}(\vec{k}, \omega)$ and therefore $\epsilon(\vec{k}, \omega)$ equal to zero.

In general, ω and $\epsilon(\vec{k}, \omega)$ are complex quantities whose real and imaginary parts describe the propagation and temporal growth or decay of plasma waves. For the case of weak wave growth or damping, ω_i and $\text{Im}[\epsilon(\vec{k}, \omega)]$ are small compared to their real counterparts and therefore, the complex dielectric function may be expressed in a Taylor series expansion about the real normal mode frequency ($\omega_r(\vec{k}) \equiv \omega_k$):

$$\epsilon(\vec{k}, \omega) = \epsilon(\vec{k}, \omega_k^r + i\omega_i) \approx \epsilon(\vec{k}, \omega_k^r) + i\omega_i \left. \frac{\partial \epsilon}{\partial \omega} \right|_{\omega_k^r} = 0 \quad (68)$$

where:

$$\epsilon(\vec{k}, \omega) = \text{Re} [\epsilon(\vec{k}, \omega)] + i \text{Im} [\epsilon(\vec{k}, \omega)] \quad (69)$$

To first order in small quantities:

$$\text{Re} [\epsilon(\vec{k}, \omega_k^*)] = 0 \quad (70)$$

$$\omega_i(\vec{k}) \approx - \frac{\text{Im} [\epsilon(\vec{k}, \omega_k^*)]}{\frac{\partial}{\partial \omega} \text{Re} [\epsilon(\vec{k}, \omega)] \big|_{\omega_k^*}} \quad (71)$$

The accuracy with which equations 70 and 71 describe plasma fluctuations depends entirely on an adequate formulation of plasma dynamics. For example, if $T_i \ll T_e$, a fluid model obtained from moments of the Boltzmann equation may sufficiently describe ion wave propagation in the long wave length limit ($k\lambda_D \ll 1$). As T_i/T_e or $k\lambda_D$ approach unity, the fluid model is no longer applicable due to the effects of ion or electron Landau damping, and thus a more accurate kinetic description is necessary (the Vlasov equation). The addition of ion-neutral or electron-neutral collisions may require the inclusion of an appropriate collision operator in the kinetic equations (the Boltzmann equation).

Over all source operating conditions, the fluid model adequately describes ion wave propagation apart from wave growth or collisionless damping. For a stationary, collisionless, magnetic field free plasma; the linearized fluid equations, governing the dynamics of a particular plasma constituent, are given by:

$$\text{Continuity:} \quad \frac{\partial}{\partial t} \rho_s(\vec{r}, t) + \rho_s^0 \nabla \cdot \vec{u}_s(\vec{r}, t) = 0 \quad (72)$$

Momentum

Conservation:
$$M_s n_s^0 \frac{\partial \vec{u}_s(\vec{r}, t)}{\partial t} = - Z_s e n_s^0 \nabla \phi(\vec{r}, t) - \nabla P_s(\vec{r}, t) \quad (73)$$

Equation
of State:

$$\begin{aligned} \nabla P_s(\vec{r}, t) &= n_s^0 T_s [(n_s^0 + n_s(\vec{r}, t)) / n_s^0]^\gamma \\ &= \gamma T_s \nabla n_s(\vec{r}, t) \end{aligned} \quad (74)$$

where ρ_s^0 , n_s^0 represent the unperturbed plasma charge density, particle density respectively; $Z_s e$ denotes the particle charge; and $\vec{u}_s(\vec{r}, t)$ is the perturbed fluid velocity field for plasma species s . Fourier transforming equations 72, 73, and 74 and solving for $\rho_s(\vec{k}, \omega)$ yields:

$$\rho_s(\vec{k}, \omega) = \frac{k^2}{4\pi} [\omega_{ps}^2 \phi(\vec{k}, \omega)] [\omega^2 - \gamma k^2 v_s^2]^{-1} \quad (75)$$

where ω_{ps} defines the constituent plasma frequency:

$$\omega_{ps} = [4\pi n_s^0 (Z_s e)^2 / M_s]^{1/2} \quad (76)$$

and $v_s = [T_s / M_s]^{1/2}$ defines the species thermal velocity.

From equations 63, 66, and 75, the constituent susceptibility is found to be:

$$\chi_s(\vec{k}, \omega) = - \frac{4\pi \rho_s(\vec{k}, \omega)}{k^2 \phi(\vec{k}, \omega)} = - \frac{\omega_{ps}^2}{(\omega^2 - \gamma k^2 v_s^2)} \quad (77)$$

The source plasma consists of four main constituents: D^+ , D_2^+ , D_3^+ and electrons. Ion thermal effects are primarily dependent on the D^+ ions, and thus the molecular ion susceptibilities may be realistically modeled by the zero temperature limit of equation 77. At ion acoustic

frequencies of interest, the electrons may be treated as a massless, isothermal fluid. Hence, electron susceptibility is given by the zero frequency limit of equation 77:

$$\chi_e(k, \omega) \xrightarrow{\omega=0} \frac{1}{(k\lambda_D)^2} \quad (78)$$

where λ_D is the electron Debye length:

$$\lambda_D = \left[\frac{T_e}{4\pi n_e e^2} \right]^{1/2} \quad (79)$$

The approximate ion wave dispersion relation in the long wavelength limit is therefore:*

$$\frac{\omega_k^+}{k} = C_s = \left[\frac{T_e + \gamma_i T_{D+}}{\bar{M}_i} \right]^{1/2} \quad (k\lambda_D \ll 1) \quad (80)$$

where $\bar{M}_i$ defines the effective ion mass:

$$\bar{M}_i = n_e \left[\frac{n_{D+}}{M_{D+}} + \frac{n_{D_2^+}}{M_{D_2^+}} + \frac{n_{D_3^+}}{M_{D_3^+}} \right]^{-1} \quad (81)$$

For an approximate species ratio of $(D^+, D_2^+, D_3^+ : .75, .15, .10)$ and $T_{D+} \approx .4$ ev, the predicted sound speed is $C_s \approx 1.4 \cdot 10^6$ cm/sec which, due to the offsetting effects of effective ion mass and finite ion temperature, is reasonably close to the simple Tonks - Langmuir prediction of $C_s = \sqrt{T_e/M_{D+}}$.

* The adiabatic coefficient $\gamma_i \approx 3$ for one dimensional, longitudinal propagation.

Thermal Background

The absolute lower limit of ion acoustic spectral energy in any physical system is set by thermal equilibrium considerations. Consequently, ion wave thermal background provides a convenient reference for comparing experimentally measured spectral quantities. From electrodynamic theory, the momentum space energy density of an electrostatic fluctuation may be expressed in terms of the plasma dielectric function by the relation: ⁴⁸

$$\epsilon_{\vec{k}} = \frac{\partial}{\partial \omega} (\omega \epsilon) \left\langle \frac{E^2(\vec{k})}{8\pi} \right\rangle \quad (82)$$

where $\langle E^2(\vec{k}) \rangle$ represents the statistically averaged electrostatic field. Under conditions of weak damping, equation 82 becomes:

$$\epsilon_{\vec{k}} = \omega_{\vec{k}}^+ \frac{\partial}{\partial \omega} \operatorname{Re} [\epsilon(\vec{k}, \omega)] \bigg|_{\vec{k}} \left\langle \frac{E^2(\vec{k})}{8\pi} \right\rangle \quad (83)$$

For estimation purposes, the dielectric function may be modeled by the low frequency, cold ion result:

$$\epsilon(\vec{k}, \omega) \approx 1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{1}{(k\lambda_D)^2} \quad (84)$$

Hence:

$$\epsilon_{\vec{k}} = \frac{1}{4\pi} \frac{\omega_{pi}^2}{\omega^2} \langle E^2(\vec{k}) \rangle \quad (85)$$

For isotropic systems, ϵ_k^+ may be more conveniently expressed in terms of the electrostatic spectral density:

$$\epsilon_k^+ \xrightarrow{\text{Isotropy}} \frac{1}{16\pi^2 k^2} \left(\frac{\omega_{pi}^2}{\omega^2} \right) \frac{\partial \omega_k}{\partial k} \langle E^2(\omega) \rangle \quad (86)$$

In thermal equilibrium, the total energy contained per k mode is given by the equipartition theorem:

$$\epsilon_k^+ = T_e \quad (87)$$

Therefore, from equation 86, the auto power spectral background of thermal potential fluctuations is found to be:

$$\langle \phi^2(\omega) \rangle = \frac{\langle E^2(\omega) \rangle}{k^2} = \frac{16\pi^2 T_e}{C_s} \left(\frac{\omega^2}{\omega_{pi}^2} \right) \quad (88)$$

For typical source operation conditions, $\langle \phi^2(\omega) \rangle = 10^{-12} \text{ (volt)}^2 / \text{kHz}$ at an observation frequency of one megahertz. For comparison, the Johnson noise in the detection circuit schematically illustrated in Figure 6 is roughly:

$$\langle \phi^2(\omega) \rangle_{\text{Johnson}} = \frac{T_{\text{amb}} Z_i}{\pi} \approx 10^{-12} \text{ (volt)}^2 / \text{kHz} \quad (89)$$

where T_{amb} represents the ambient laboratory temperature and Z_i is the detection circuit input impedance. Both thermal noise figures are comparable and are about one order of magnitude less than the maximum resolution of the digital detection system.

Experimental Results

Multifilament source operation is experimentally controlled by three external parameters:

1. The flow of deuterium gas through the source chamber which determines the background neutral density in the source plasma.*
2. The arc pulse line voltage (APLV) which essentially controls arc power.
3. The filament heater voltage which determines filament temperature and consequently the thermionic emission characteristics of the cathode.

For a given value of these operating parameters, within the confines of stable arc operation, the source plasma will self consistently adjust its internal properties to maintain steady state operation. Although plasma properties such as electron distribution function, potential profile and plasma density are complicated functions of the external operating parameters, overall source performance may be characteristically dichotomized into two distinct operational modes.

The source plasma is driven by a diffuse arc originating at a thermionic cathode consisting of 26 hairpin filaments connected in parallel. Tungsten is chosen as the thermionic emitter since its boiling point (3683 °K) is much higher than its emission temperature (2370 °K), and hence activation is not required. The maximum current density emitted by a thermionic filament is approximately given by the Richardson - Dushman

* An estimate of the relation between input gas flow and background neutral density, based on steady state balance between input and output flow rates, concludes that an input flow of 1 T-l/sec corresponds to a neutral density of $2 \times 10^{15} / \text{cm}^3$. This estimate concurs with the calculated estimates of Chan for typical operating conditions. ¹⁷

equation:

$$J_e \Big|_{\text{max}} = A_0 T_f^2 \exp \{ - e \phi_w / k_b T_f \} \quad (90)$$

where A_0 is Dushman's constant ($1.2 \cdot 10^6$ Amp/ $\text{m}^2 \text{ } ^\circ\text{K}$), T_f is the filament temperature in degrees Kelvin, ϕ_w is the filament work function in volts and k_b is Boltzmann's constant. At low emission current densities, equation 90 will approximately describe the dependence of arc current on filament temperature. Hence, while holding gas flow and APLV constant, arc current should be a monotonically increasing function of filament voltage. An arc with this operating characteristic is said to run emission limited.

As filament voltage (temperature) continues to increase, arc current will remain emission limited until emission current density approaches an upper limit imposed by space charge shielding in the sheath surrounding the filament. Beyond this point, emission current density is independent of filament voltage and is given by the Child - Langmuir equation:

$$J_e = \frac{1}{4\pi} \frac{4}{9} \left(\frac{2e}{m_e} \right)^{1/2} \frac{V}{d^2} \beta^{-2} \quad (91)$$

where V is the cathode fall potential, d gives the space charge sheath thickness and β is a geometrical correction factor which is dependent on filament radius. Thus, an arc operating independent of filament temperature is said to run space charge limited.

Figure 10 illustrates measurements of arc current vs. filament voltage for fixed gas flow and APLV. The emission and space charge limited modes are clearly defined. Emission limited operation (0-600

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Figure 10.

relative units) is characterized by a strong arc dependence on filament voltage (temperature) as described by equation 90. Space charge limited operation is essentially independent of filament voltage as predicted.*

The effects of emission or space charge limited operation on plasma oscillations are illustrated in Figure 11, which plots the auto power spectral density of potential fluctuations over a 5 MHz bandwidth for several values of filament voltage. Under emission limited conditions, plasma fluctuations consist of low level, broadband noise with no distinguishing characteristics (Fig. 11A). As filament voltage is increased, the first vestiges of a resonant spectrum appear when the arc discharge begins a transition between emission and space charge limited operation (Fig. 11B). The resonant fluctuation profile continues to develop with increasing filament voltage until a fully resonant spectrum emerges under full space charge limited operation (Figures 11C and 11D). A complete plot of peak spectral density vs. filament voltage is illustrated in Figure 12. Peak spectral density grows exponentially with increasing filament voltage over three orders of magnitude until saturation.

The exact nature of the resonant fluctuations may be examined in detail by measuring their cross power spectrum. Figure 13 shows computer generated plots of the cross power spectral functions, defined by equations 15 through 17, under space charge limited operation.

* The small dependence (monotonically decreasing) of arc current on filament voltage during space charge limited operation is presumably due to the effect of increasing magnetic field strength around the filaments as heater voltage, and consequently heater current is increased. These fields can cause a slight reduction in electron transport to the source chamber walls immediately adjacent to the filament ring (Fig. 1), thus modifying source operation.

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Figure 11. Auto Power Spectral Density
as a function of operational mode.

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Figure 12.

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Figure 13A.

XBL 799-11693

Figure 13B.

XBL 799-11694

Figure 13C.

The measurements were performed with two shielded, unidirectional disk probes positioned on axis, with a relative separation of .4 cm and facing towards the source gas inlet (Figure 1). The cross power spectral density, coherence and phase measurements indicate the fluctuations are reasonably coherent and possess a fairly linear dispersion relation (constant slope) across the resonant spectrum (800 - 2000 kHz); consistent with the properties of ion acoustic fluctuations. However, wave propagation characteristics which, in principle, are obtained by measuring the slope of the phase spectrum, are inconclusive due to the uncertainty in propagation direction and the possibility of phase mixing at the probes due to a spatially broad region of wave excitation.

Discussion

The spontaneous excitation of ion acoustic waves in a partially ionized plasma is governed by competition between wave growth due to inverse Landau damping, and wave destruction due to ion Landau and collisional damping. Consequently, the existence of ion waves in the source plasma can be expected on the basis of two important considerations: the plasma's ability to support ion waves with minimal damping, and the probable existence of a cathode-originated current-driven instability.

The Ion Acoustic Instability

Ion acoustic generation may be mathematically demonstrated within the linear formalism described by equations 59 through 71, assuming a kinetic description of plasma susceptibility. Utilizing the Vlasov equation, the susceptibility of a particular plasma constituent is given by:⁴⁹

$$\chi_s(\vec{k}, \omega) = -\frac{\omega_s^2}{k^2} \left[\mathcal{P} \int du \frac{\frac{\partial}{\partial u} f_s(u, \hat{k})}{(u - V_\phi)} + i\pi \frac{\partial}{\partial u} f_s(u, \hat{k}) \Big|_{u=V_\phi} \right] \quad (92)$$

where $\mathcal{P}$ denotes the Cauchy Principle Value and $f_s(u, \hat{k})$ defines the projected velocity distribution function of species s in direction $\hat{k}$:

$$f_s(u, \hat{k}) = \int d\vec{v} f_s(\vec{v}) \delta(\hat{k} \cdot \vec{v} - u) \quad (93)$$

where $V_\phi = \omega/k$ is the wave phase velocity. For a two component plasma consisting of drifting electrons and stationary ions with distribution functions:

$$f_e(u, k) = 1/[2\pi v_e^2]^{1/2} \exp[-(u - u_d)^2 / 2 v_e^2] \quad (94)$$

where $u_d = \vec{v}_d \cdot \hat{k}$ and $\vec{v}_d$ represents the electron drift velocity in the rest frame of the ions, and:

$$f_i(u) = 1/[2\pi v_i^2]^{1/2} \exp[-u^2 / 2 v_i^2] \quad (95)$$

where v_e, v_i represent the electron and ion thermal velocities respectively; the kinetically determined longitudinal dielectric function (equation 67), in the ion acoustic limit, is found to be:

$$\epsilon(k, \omega) = 1 + \frac{1}{k^2 \lambda_D^2} - \frac{\omega_{pi}^2}{\omega^2} \left[1 + 3 \frac{v_i^2}{v_e^2} \right] + \quad (96)$$

$$i \left(\frac{\pi}{2} \right)^{1/2} \left\{ \frac{\omega_{pe}^2}{k^2} \left(\frac{v_i - u_d}{v_e} \right) \exp \left[-\frac{(v_i - u_d)^2}{2 v_e^2} \right] + \frac{\omega_{pi}^2}{k^2} \left(\frac{v_i}{v_i^3} \right) \exp \left[-\frac{1}{2} \left(\frac{v_i}{v_i} \right)^2 \right] \right\}$$

Hence, from equation 71, the imaginary wave frequency in the long wavelength and weak damping limit is given by:

$$\omega_i(k) = -k \sqrt{\frac{\pi}{8}} \left(\frac{m_e}{M_i} \right)^{1/2} \left\{ (C_s - u_d) \exp \left[-\frac{1}{2} \left[\frac{(C_s - u_d)^2}{v_e} \right] \right] + C_s \left[\frac{M_i}{m_e} \frac{T_e^3}{T_i^3} \right] \exp \left[-\frac{T_e}{2T_i} \right] \right\} \quad (97)$$

When $T_e \gg T_i$, the second term in equation 97 describing ion Landau damping becomes negligible. Consequently, equation 97 demonstrates that ion acoustic waves become unstable when electron drift velocity exceeds the ion sound speed.

In the multifilament source, primary electrons are continuously injected into the plasma at the thermionic filaments. Although exact computation of the primary electron drift velocity is unfortunately impossible due to the effects of local magnetic fields resulting from filament heater and arc discharge current,⁵⁰ and electrostatic oscillations resulting from possible electron two stream instabilities⁹, it is highly likely that a reasonable population of electrons satisfy the growth criterion for ion waves in the general vicinity of the filaments.

The Source Plasma Resonance

Once an ion wave instability is established, the fluctuation spectrum temporally grows at a rate governed by an appropriate form of equation 97. The operational region of instability defined by the condition that $\omega_i(k) > 0$ and the spectral profile determined by the wave number dependence of $\omega_i(k)$, are highly dependent on the electron to ion temperature ratio and the electron drift velocity.^{51,52} If ion

collisions are important, then the ratio of ion plasma to ion collision frequency enters the theory.⁵³ The fluctuation spectrum will continue to grow until saturated by a non-linear mechanism such as electron trapping in the fluctuating wave potential.^{54,55,26} When the electron drift velocity greatly exceeds the ion sound speed, the plasma can become turbulent from non-linear ion-wave interactions.^{36,56,57} For the case of strong turbulence, measurements indicate that the ion wave spectrum peaks about the ion collision frequency. 58-61

In contrast, the spectral characteristics of the multifilament source fluctuations do not seem dependent on either plasma operational state described above. The source resonant frequency (~1 MHz) is far below the ion plasma frequency (~100 MHz) which excludes its self determination from linear wave growth alone. Turbulent effects are difficult to reconcile due to the low absolute values of the fluctuating potentials, and the experimental inability to precisely determine electron drift velocity and ion collision frequency which characterize the theory.

Based on spectral measurements similar to those illustrated in Figure 13, the multifilament source effectively functions as a resonant cavity oscillator, whose spectral profile is solely determined by the multifilament spacing around the perimeter of the source chamber (Fig. 1). Within the source, the temporal behavior of potential fluctuations may be described by damped harmonic oscillations:

$$\begin{aligned}\phi(t) &= \phi_0 e^{-\delta t} e^{-i\omega_0 t} & t > 0 \\ &= 0 & t < 0\end{aligned}\quad (98)$$

where ω_0 represents the resonant cavity frequency and δ denotes the energy loss properties of the cavity. That is:

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Figure 14.

$$\delta = \frac{\omega_0}{2Q} \quad (99)$$

where Q , the cavity quality factor, is defined in its standard notation as:

$$Q = 2\pi \frac{\text{Energy Stored in Cavity}}{\text{Power Loss from Cavity}} \quad (100)$$

A continuous excitation of the resonant oscillations within the source chamber will lead to a Lorentzian spectral profile, obtained from the Fourier transform of equation 98:

$$I^2(\omega) = \frac{\dot{\omega}_0^2}{[(\omega - \omega_0)^2 + \delta^2]} \quad (101)$$

Figure 14 illustrates a computer generated least squares Lorentzian fit to the cross power spectral data of Figure 13. The Lorentzian profile is mathematically dependent on three fit parameters: the fluctuation amplitude $\dot{\omega}_0$, the resonant frequency ω_0 , and the resonance half width at half maximum δ . Physically, each parameter lends insight into characterizing the fluctuations.

1.6.5.500 Frequency. Figure 15 shows a pyrometric photograph of an emitting hairpin filament similar to those employed in the multifilament source.⁵⁰ The filament's thermal distribution clearly maximizes near its negative leg. This distribution results from a combination of three competing thermal effects: resistive heating by the filament current, thermal cooling by conduction along the filament to the water cooled

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Figure 15.

filament chucks (Figure 1), and evaporative cooling by electron emission. Since thermionic emission is highly dependent on filament temperature (equation 90), most electron emission is spatially localized near the negative leg of the emitting filaments. Thus, as previously noted, the source plasma is driven by 26 periodically spaced electron injectors.

The injection periodicity is hypothesized to fix the resonant wavelength at the interfilament spacing due to several considerations:

1. The emission properties of the filaments are intimately connected with the resonant phenomena as indicated by Figure 12.
2. The filaments are connected in parallel (electrically shorted together), and thus only waves satisfying the condition of integral wavelength to filament spacing can participate in the feedback mechanism responsible for the resonant behavior; others being damped due to phase randomization.

Assuming ion waves, equation 80 predicts:

$$\omega_0 = \frac{C_s}{\Delta X_{\text{filament}}} \cong \left[\frac{1}{\Delta X_{\text{filament}}} \sqrt{\frac{T_e}{M_D}} \right] \quad (102)$$

Consequently, the resonant frequency should be inversely proportional to filament spacing and directly proportional to the square root of the plasma electron temperature.

In the source plasma, electron temperature is a monotonically increasing function of decreasing gas flow and increasing arc power. Experimental measurements of resonant frequency versus gas flow and arc power, under space charge limited conditions, are illustrated in Figures 16A and 16B. The solid curve plots the calculated results of equation 102

Figure 16A.

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Figure 16B.

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Figure 17.

from independently determined electron temperature measurements.

Figure 17 shows a computer generated auto power spectral plot for the case of half filament operation under conditions similar to those of Figures 13 and 14. Doubling the interfilament spacing almost exactly halves the resonant frequency as predicted by equation 102. The excellent experimental confirmation of equation 102 serves to validate both the resonant cavity hypothesis and the identification of ion waves in the source plasma.

The Resonance Halfwidth. As previously discussed, the spatial dependence of source plasma parameters follows the "ion free fall" model, where ions are uniformly created throughout the source volume and subsequently accelerate towards the walls in the ambipolar potential (Figure 5). Although ion current entering the sheath edge bounding the plasma is a function of operating conditions, it must enter at essentially the ion sound speed to maintain sheath stability (the Bohm condition).⁶² Ion waves in the source plasma may therefore be conceptualized as propagating in a flowing medium which goes supersonic at the plasma-sheath boundary. Hence, the source walls are excellent absorbers of ion waves,⁶³⁻⁶⁵ and consequently the wave power lost from the source plasma is effectively equal to the wave energy stored in the source divided by the mean ion lifetime (τ_{ion}).

Estimates of the mean ion lifetime may be obtained from general physical considerations of source operation. For steady state operation, ion generation by electron-neutral collisions must equal ion loss to the walls:

$$\int_{\text{Source Volume}} G \, dV = \int_{\text{Plasma-Sheath Boundary}} n_i(x) \vec{v}_i(x) \cdot d\vec{A} \quad (103)$$

where G denotes the ion generation rate per unit volume (approximately constant), and $n_i(x)$; $v_i(x)$ represent the spatially dependent ion density and ion velocity respectively. Under free fall conditions, Gauss's Theorem yields:

$$G V_{\text{source}} = (n_{is} v_{is}) A_{\text{source}} \cong n_{is} C_s A_{\text{source}} \quad (104)$$

where n_{is} ; v_{is} represent the ion density; ion velocity evaluated at the sheath edge. The mean ion lifetime may be effectively defined as:

$$\tau_{\text{ion}} = \frac{\bar{n}_i}{G} \quad (105)$$

where $\bar{n}_i$, the mean ion density, is given by:

$$\bar{n}_i = \frac{1}{V} \int_V n_i(x) dV \quad (106)$$

Therefore:

$$\tau_{\text{ion}} = \left[\frac{V}{A} \right]_{\text{source}} \cdot \frac{1}{C_s} \left[\frac{\bar{n}_i}{n_{is}} \right] \quad (107)$$

Under typical operating conditions: $V/A \approx 1.7$, $\bar{n}_i/n_{is} \approx 2$, $C_s \approx 1.4$ to $1.5 \cdot 10^6$ cm/sec, and thus $\tau_{\text{ion}} \approx 2.3$ to 2.4 μsec .

Combining equations 99, 100 and 102; the ratio of resonance half width to resonant center frequency may be expressed as:

$$\frac{\delta}{\omega_0} = \frac{1}{2\omega_0} \left[\frac{\text{Wave Power Lost}}{\text{Wave Energy Stored}} \right] \approx \frac{1}{2\omega_0 \tau_{\text{ion}}} = \frac{1}{2} \Delta X_{\text{fil}} \left(\frac{A}{V} \right) \left[\frac{n_{is}}{\bar{n}} \right] \quad (108)$$

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Figure 15.

Figure 18 illustrates experimental measurements of δ/ω_0 versus arc power under space charge limited conditions. The solid line plots the calculated predictions of equation 108. Since $n_{IS}/\bar{n}$ is a weak function of source operation, δ/ω_0 should remain relatively independent of changing arc parameters during resonant operation, as confirmed by the data. The reasonable agreement between calculation and measurement serves to confirm the contention that finite ion lifetime is primarily responsible for ion wave damping in the source plasma. This does not preclude ion-Landau or ion-collisional damping processes from being present. It does, however, agree with the previously stated conclusions of the ion free fall model: that to within experimental uncertainties, the effects of finite ion temperature and ion-neutral collisions are minor.

Fluctuation Amplitude and Resonant Excitation. Resonant excitation in the source plasma is highly dependent on space charge limited operation as illustrated by Figures 10 through 12. Qualitatively, this operational dependence may reside in the distinct wave reflection characteristics of the filaments when operated in the space charge versus emission limited mode.

The emission properties of a space charge limited filament is dependent on its cathode fall potential (equation 91). A small potential increase (reduction) in the cathode fall due to a local potential fluctuation (eg. an ion wave) will result in an increase (decrease) in electron emission which tries to reduce (increase) the potential back to steady state. In essence, this emission current dependence acts as a negative feedback mechanism which attempts to clamp the cathode fall at its unperturbed value. Consequently, the filaments become nodal points for wave reflection. The periodic multifilament structure then tends to

preferentially scatter waves from the background noise which satisfy the condition of integral wavelength to filament spacing; the others being damped due to phase randomization. This scattering characteristic is non-existent in the emission limited mode since, in that case, filament emission is independent of potential.

A local potential disturbance can also cause a modification of the space charge distribution in a cathode fall sheath on a time scale greater than an ion sheath transit time.^{66,67} For frequencies much less than the ion plasma frequency, a space charge disturbance will oscillate with the potential fluctuation and may excite or enhance an ion wave.⁶⁸⁻⁷⁰ The excitation mechanism is fairly inefficient since the wave must be excited in the pre-sheath region external to the plasma-sheath boundary, in order to avoid absorption due to supersonic ion flow into the cathode. However, the mechanism may prove adequate to account for the low spectral power densities observed in the source plasma. The peak spectral power density would then be determined by a steady state balance between wave creation by enhanced scattering and wave destruction by wall absorption and phase randomization.

Conclusion

From a practical standpoint, source plasma fluctuations do not contribute to ion heating. For typical ion lifetimes, a plasma ion will at most experience several potential fluctuations in the millivolt range before it is lost. This interaction time and amplitude scale is insufficient for any substantial wave - particle energy transfer, which is consistent with the contention that ion wave energy is absorbed by the source chamber walls.

From a plasma physics standpoint, the fluctuation phenomena proved extremely rich. The preceeding discussion has addressed only the basic fluctuation properties observed. A more thorough treatment, however, would probably require increased experimental control of and access to the parameters which characterize the source plasma.

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APPENDIX - ELECTROSTATIC PROBE DIAGNOSTICS

Theory of Measurement

The Berkeley source operates in a regime where cylindrical or spherical probe operating conditions are adequately described by the collisionless thin sheath approximation, i.e. $\bar{\lambda} \gg r_p \gg \lambda_D$

where:

$$\bar{\lambda} \equiv \text{Collisional mean free path} = \frac{18\pi n_e \lambda_D^4}{\ln(12\pi n_e \lambda_D^3)}$$

$$\lambda_D \equiv \text{Electron Debye length} = \sqrt{kT_e / 4\pi n_e e^2}$$

$$r_p \equiv \text{Probe radius.}$$

A schematic typical of a probe current voltage characteristic in this regime is shown in Fig. A-1. The experimentally important quantities are:

1. Accurate determination of the plasma potential V_p .
2. Accurate determination of the probe floating potential V_f .
3. Accurate measurement of the ion saturation region (A).
4. Accurate measurement of the electron transition region (B).

For the case where the electron temperature far exceeds the ion temperature, collected ion current density is quite insensitive to ion temperature and is expressed as:

$$j_i(V_\phi) = \left[\frac{1}{4} n_i Z_i e \sqrt{\frac{8kT_e}{\pi m_i}} \right] i_+(V_\phi; T_e, T_i, r_p, \lambda_D)$$

where i_+ is an ion current correction factor computed by the theory of Laframboise,¹ and V_ϕ is the probe bias voltage measured with respect

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Figure A-1. Probe current-voltage characteristic.

to the plasma potential, i.e. $V_\phi = V_p - V$.

The electron current density in the transition region can be expressed in terms of the isotropic electron velocity distribution function

$f_e(v)$, as

$$j_e(V_\phi) = n_e e \langle v \rangle(V_\phi) = e \int_{\sqrt{\frac{2eV_\phi}{m_e}}}^{\infty} v^3 f_e(v) dv \int_0^1 \frac{1}{v} \sqrt{\frac{2eV_\phi}{m_e}} \cos\theta d(\cos\theta) \int_0^{2\pi} d\phi \quad (A2)$$

Performing the angle integration yields

$$j_e(V_\phi) = \pi e \int_{\sqrt{\frac{2eV_\phi}{m_e}}}^{\infty} v^3 f_e(v) \left(1 - \frac{2eV_\phi}{m_e v^2}\right) dv \quad (A3)$$

Considering that electron current density is experimentally measured

as a function of bias potential, a more convenient description of

$j_e(V_\phi)$ is obtained by expressing it as a function of $f_e(\epsilon)$, the isotropic electron energy distribution function. Defining $f_e(\epsilon)$ as

$$f_e(\epsilon) = \int_0^{\infty} f_e(v) \delta\left(\epsilon - \frac{1}{2} m_e v^2\right) dv \quad (A4)$$

equation (A3) becomes

$$j_e(V_\phi) = \frac{2\pi e}{m_e} \int_{V_\phi}^{\infty} \epsilon f_e(\epsilon) \left(1 - \frac{eV_\phi}{\epsilon}\right) d\epsilon \quad (A5)$$

For the special case of a Maxwellian Plasma,

$$f_e(\epsilon) = n_e \left(\frac{m_e}{2\pi kT_e}\right)^{3/2} e^{-\epsilon/kT_e} \quad (A6)$$

which when substituted into equation (A5), yields the familiar result

$$j_e(V_\phi) = \frac{1}{4} n_e e \sqrt{\frac{8kT_e}{\pi m_e}} e^{-eV_\phi/kT_e} \quad (A7)$$

where the electron temperature is given by the inverse slope of $\ln j_e(V_\phi)$ vs V_ϕ . For the general case where $f_e(\epsilon)$ deviates from a Maxwellian, the electron current density will behave according to equation (A5). The fact that $j_e(V_\phi)$ is related to $f_e(\epsilon)$ through an integral equation, coupled with experimental measurement uncertainties, makes it quite insensitive to all but gross distribution function structure. Finer grained resolution is possible by performing the first and second derivatives of $j_e(V_\phi)$. Taking the second derivative with respect to bias voltage of equation (A5) yields

$$\frac{d^2 j_e(V_\phi)}{dV_\phi^2} = \frac{d}{dV_\phi} \left(\frac{2\pi e}{m_e} \int_{eV_\phi}^{\infty} \frac{\partial}{\partial V_\phi} \epsilon f_e(\epsilon) \left(1 - \frac{eV_\phi}{\epsilon}\right) d\epsilon \right) = \frac{2\pi e^3}{m_e} f_e(V_\phi) \quad (A8)$$

Hence,

$$f_e(\epsilon) = \frac{m_e^2}{2\pi e^3} \left. \frac{d^2 j_e(V_\phi)}{dV_\phi^2} \right|_{eV_\phi = \epsilon} \quad (A9)$$

which relates the electron energy distribution function to the second derivative of the electron probe current density.^{2,3*}

A further generalization for anisotropic distribution functions is possible by defining $f_e(u, \hat{n})$, the projected electron velocity distribution function in the spatial direction $\hat{n}$, as

*The above treatment required $f_e(\epsilon)$ to be isotropic. However, the applicability of equation (A9) to anisotropic distribution functions, may also be quasi-valid under certain restrictive conditions for spherical probes.^{3,4}

$$f_e(u, \hat{n}) = \int_{\text{all } \vec{v}} d\vec{v} f_e(\vec{v}) \delta(\hat{n} \cdot \vec{v} - u) \quad (\text{A10})$$

where $f_e(\vec{v})$ is the general electron velocity distribution function.

In terms of $f_e(u, \hat{n})$, equation (A2) reduces to

$$j_e(V_\phi) = e \int_{\sqrt{\frac{2eV_\phi}{m_e}}}^{\infty} f_e(u, \hat{n}) u du \quad (\text{A11})$$

where $\hat{n}$ now refers to the spatial direction normal to the probe surface.

Again, defining $f_e(\epsilon, \hat{n})$, the projected electron energy distribution function as

$$f_e(\epsilon, \hat{n}) = \int_0^{\infty} f_e(u, \hat{n}) \delta(\epsilon - \frac{1}{2} m_e u^2) du \quad (\text{A12})$$

equation (A11) becomes

$$j_e(V_\phi) = \frac{e}{m_e} \int_{eV_\phi}^{\infty} f_e(\epsilon, \hat{n}) d\epsilon \quad (\text{A13})$$

Performing the first derivative with respect to bias voltage yields

$$\frac{dj_e(V_\phi)}{dV_\phi} = -\frac{e}{m_e} f_e(V_\phi, \hat{n}) \quad (\text{A14})$$

or

$$f_e(\epsilon, \hat{n}) = -\frac{m_e}{e} \left. \frac{dj_e(V_\phi)}{dV_\phi} \right|_{eV_\phi = \epsilon} \quad (\text{A15})$$

When used with a judiciously designed plane or wall probe, equation (A15) affords a convenient method of measuring the projected distribution

function in any spatial direction.

Experimental Apparatus

All probe characteristic measurements and the vast majority of all electron distribution function measurements were made using a pulsed data acquisition system, which digitally records the probe characteristic and its first and second derivatives. Several distribution function measurements employed the second harmonic or intermodulation technique which uses the nonlinearity of the probe current-voltage characteristic to obtain its second derivative. This technique was primarily employed as an auxiliary check on the reliability and accuracy of the pulsed acquisition system. A complete description and accuracy analysis of the two techniques are given below. A photograph of the overall apparatus is shown in Figure A-2.

1. Pulsed Probe Data Acquisition System

The motivation for a pulsed detection system, in addition to the plasma's transient nature, is readily apparent from its low frequency noise spectrum (Fig. A-3). The large noise increase below 1 kHz is presumably due to power supply effects. Data acquisition in a time less than 1 ms is necessary to minimize this noise influence.

Figure A-4 schematically illustrates the probe driver/detection circuit. An initial pulse, obtained from the source logic, operates the timing of various source inputs (e.g. application of arc power, filament power, gas injection, etc.). The pulse is also applied to a variable delay gate (enabling data acquisition at any subsequent

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Figure A-2. Experiment 1 Apparatus.

Figure A-3. Low frequency plasma noise.

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Figure A-4. Probe driver- detector schematic.

moment during source operation), amplified, and then used to trigger the probe driver and data recording system. The probe driver initially applies a large positive bias to the probe to insure a clean collection surface, followed by a linearly decreasing voltage ramp which sweeps the probe bias over its entire operating range. Sweep speeds of $.1 \text{ V}/\mu\text{sec}$ to $1 \text{ V}/\mu\text{sec}$ over a total range of 100 volts are typical. The probe current is differentially detected across a standard resistance and then processed by the differentiation network, which outputs the probe current and its first and second derivatives. These signals are digitally recorded by a Nicolet transient digitizer, which simultaneously samples the processed current signal and its corresponding bias voltage. The stored data is accessible both graphically and as a digitized set of data points; each point consisting of two, twelve bit words (e.g. (V,i) for probe characteristic measurements) with a maximum of 2048 points per data set. The digitized data is finally read by a Modcomp Systems computer via a direct I/O link, which subsequently executes an analysis algorithm providing on line data analysis.

Over all conditions encountered in the Berkeley source, a single data set is sufficient to accurately determine all important experimental quantities obtainable from the probe characteristic or its first derivative. Typical times for complete data acquisition range from .1 ms to 1 ms. However, the comparatively large effective noise bandwidth inherent in the operation of a differentiation network, oftentimes requires an ensemble type average over multiple data sets for second derivative measurements, in order to minimize ambient noise effects and increase measurement resolution. For transient plasmas, this technique requires

a good shot to shot reproducibility in the plasma's operating conditions. The Berkeley source is found to satisfy this requirement.

In general, accurate temporal differentiation by an electronic network, requires the network frequency response function, $D(\omega)$, be directly proportional to ω . This is easily seen by noting that the Fourier Transform of the time derivative operator (d/dt) , is $(-j\omega)$. Using the standard definition of dB voltage gain per octave,

$$G_{\text{oct}} = \left(\begin{array}{c} \text{Voltage gain} \\ \text{per octave in} \\ \text{dB} \end{array} \right) = 20 \log_{10} \frac{D(2\omega)}{D(\omega)} \quad (\text{A16})$$

a time derivative network will have a gain per octave of

$$G_{\text{oct}} = 20 \log_{10}(2) \approx 6 \text{ dB} \quad (\text{A17})$$

The differentiation network illustrated in Fig. A-5 consists of a series of ganged stages, each stage tailored to a particular frequency response which minimizes overall network noise and instability, while maintaining differentiation accuracy over a 100 kHz bandwidth. All stages utilize compensated AD 507 wideband, low noise operational amplifiers, which have proven quite cost effective. Components for the differentiation stages (Fig. A-6) were chosen to insure a 6 dB/octave gain increase over a 100 kHz bandwidth and stability over all operating conditions. Buffer stages are low pass butterworth filters with flat pass bands from 0-100 kHz.

II. Accuracy Analysis of the Pulsed Acquisition System

The accuracy of the data acquisition process is dependent on the response characteristic of the probe driver circuit, the acquisition accuracy of the probe current detection network, and the temporal

Figure A-5. Differentiation network block diagram.

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Figure A-6. Differentiation stage with compensation.

linearity of the probe bias voltage ($V_\phi \propto t$); which enables a direct connection between the time derivatives performed by the differentiation network, and the derivatives with respect to voltage shown proportional to distribution functions (Equations A9 and A15).

A circuit equivalent model of the probe/driver is illustrated in Fig. A-7. A discrete circuit model should remain a reasonable approximation for response times slow compared to an ion plasma period.

The temporal response of the circuit model is roughly

$$\tau_{\text{system}} = \frac{R_D R_S (C_D + C_S)}{(R_D + R_S)} \quad (\text{A18})$$

where C_D (R_D) are the effective driver circuit capacitance (resistance) and C_S (R_S) are the effective probe-plasma sheath capacitance (resistance) respectively. For realistic experimental systems, $C_S \ll C_D^5$ and R_D can usually be made much smaller than R_S through judicious driver circuit and probe design. Therefore, the probe driver electronics completely determines the temporal response of the system. The measured frequency response of the driver section depicted in Fig. A-7, was linear over a 100-kHz bandwidth, with a loaded slew rate of 10 V/ μ sec.

Since the probe current detection network operates on time dependent probe current, network acquisition accuracy is strongly dependent on its temporal response. The spectral density of the probe current density $j(t)$, is given by its Fourier Transform

$$J(\omega) = \int_{-\infty}^{\infty} j(t) e^{i\omega t} dt \quad (\text{A19})$$

Given $D(\omega)$, the network response function of the detection system, the detected current density is*

$$j_{\text{DET}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} D(\omega) J(\omega) e^{-i\omega t} d\omega = \int_{-\infty}^t d(t-\tau) j(\tau) d\tau \quad (\text{A20})$$

where $d(t-\tau)$, the Network Transfer Function, is given by

$$d(t-\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} D(\omega) e^{-i\omega(t-\tau)} d\omega \quad (\text{A21})$$

A rather exact form of $D(\omega)$ is obtainable from amplitude and phase response measurements for each of the network functions of interest, although an analytic solution of equation (A21) is practically unobtainable. Numerical solutions of equations (A20) and (A21) are possible, albeit somewhat tedious.^{6,7} For estimation purposes, a qualitative solution of equation (A21) is possible by noting that since all network functions perform effectively over a 100-kHz bandwidth, the analysis of network accuracy reduces to finding the effect of a finite bandwidth response on the detected signal. This amplitude response can be modeled by the step function

$$\begin{aligned} |D(\omega)| &= 1 & 0 \leq |\omega| \leq \omega_c \\ &= 0 & |\omega| > \omega_c \end{aligned} \quad (\text{A22})$$

where ω_c is the angular high frequency cut-off. Substituting in equation (A21) yields

$$d(t-\tau) = \frac{\omega_c}{\pi} \text{Sinc}[\omega_c(t-\tau)] \quad (\text{A23})$$

*In general, the causal behavior of equation (A20) requires $D(\omega)$ be a complex function whose real and imaginary parts, describing network amplitude and phase response, are related via the Kramers-Kronig relations.

Figure A-7. Probe/driver equivalent circuit.

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Figure A-8. The Sine transfer function.

From equation (A20), the approximate detected signal is therefore*

$$\begin{bmatrix} j(t) \\ j'(t) \\ j''(t) \end{bmatrix}_{\text{DET}} = \frac{\omega_c}{\pi} \int_{-\infty}^{\infty} \text{sinc}[\omega_c(t-\tau)] \begin{bmatrix} j(\tau) \\ j'(\tau) \\ j''(\tau) \end{bmatrix} d\tau \quad (\text{A24})$$

Figure A-8 illustrates the Sinc transfer function, with a temporal resolution of roughly $1/2f_c$. For $f_c = 100$ kHz, the temporal resolution is approximately 5 μsec , which corresponds to a voltage resolution of 1/2 volt for a constant sweep speed of .1 V/ μsec .

A direct connection between time and voltage response of the probe current detection network assumes the bias voltage driving the probe, $V_\phi(t)$, is a linear function of time. Nonlinearities in $V_\phi(t)$ have minimal impact on measurement of the probe characteristic, since the signal digitizer simultaneously samples both the probe bias and its corresponding probe current. The measured probe characteristic is therefore independent of the exact form of $V_\phi(t)$, within the constraints imposed by driver response and finite bandwidth effects mentioned above. Derivative measurements, however, are strongly dependent on $V_\phi(t)$. Straightforward differentiation yields the relation between derivatives with respect to time and derivatives with respect to bias voltages:

$$\frac{d}{dt} = \left[\frac{dV_\phi(t)}{dt} \right] \frac{\partial}{\partial V_\phi} \quad (\text{A25})$$

$$\frac{d^2}{dt^2} = \left[\frac{d^2V_\phi(t)}{dt^2} \right] \frac{\partial}{\partial V_\phi} + \left[\frac{dV_\phi(t)}{dt} \right]^2 \frac{\partial^2}{\partial V_\phi^2} \quad (\text{A26})$$

* The non-causal behavior of equation (A24) is due to the exclusion of phase response from equation (A22).

For $V_{\phi}(t) \propto t$, time and voltage derivatives are directly proportional.

A rough estimate of the errors incurred by a nonlinear $V_{\phi}(t)$ is possible by including a small second order nonlinearity in the bias voltage

$$V_{\phi}(t) = A_0 + \alpha t + \beta t^2 \quad (A27)$$

A nonlinearity of 0.5% (i.e. $\beta t / \alpha \leq .005$ for all t during the pulse) will incur an error of $\leq 1\%$ in directly relating d/dt to d/dV , and an error of $\leq 2\%$ in directly relating d^2/dt^2 to d^2/dV^2 .

Measured nonlinearity in the driver section depicted in Fig. A-7, was less than 0.5% over the entire range of bias voltage. The resulting errors incurred in derivative measurements are therefore insignificant compared to other systematic uncertainties.

Ion Current Effects

Equations (A9) and (A15) relate the electron distribution function to derivatives of the electron probe current. However, since the differentiation network operates on the total probe current, it is necessary to examine the effect of ion current on distribution function measurements.

Under the operating conditions encountered in the Berkeley source, an analytic expression for the ion current as a function of probe bias does not exist. This is primarily due to electron thermal effects which, for an ion attracting probe, allow an electric potential of approximately kT_e to exist in the quasi-neutral plasma region exterior to the plasma-probe sheath. Thus, for the case where $T_i < T_e$, ion probe current is quite insensitive to ion temperature and mainly depends

on the complex relation between plasma sheath growth and probe operating parameters. To obtain the exact form of this relation requires a numerical solution of the equations which govern the behavior of ion attracting probes in a collisionless plasma.

One numerical calculation which is particularly well suited for the plasma conditions prevalent in the Berkeley source, is given by Laframboise.¹ Recall that within the Laframboise theory, ion probe current density as a function of probe bias is described by equation (A1). Several approximate analytic fits to the numerical results of Laframboise have been made for a wide range of probe-plasma operating conditions.⁸ For the Berkeley source, a typical analytic fit to i_+ , the ion current correction factor, is given by

$$i_+(V_\phi) = \begin{cases} 0.9 \chi^{.469} & \text{for } 0 \leq \chi < 2 \text{ within } 8\% \\ 1.09 \chi^{.182} & \text{for } \chi \geq 2 \text{ within } 1\% \end{cases} \quad (\text{A28})$$

where $\chi = \frac{eV_\phi}{kT_e}$. Defining R as the ratio of $j_i(V_\phi)$ to $j_e^*(V_\phi)$, and utilizing the results of equations (A1), (A7) and (A28) yields

$$R = \frac{\left(\frac{d^2 j_i(V_\phi)}{dV_\phi^2} \right)}{\left(\frac{d^2 j_e(V_\phi)}{dV_\phi^2} \right)} = \sqrt{\frac{M_e}{M_i}} (kT_e)^2 e^\chi \left(\frac{d^2 i_+(V_\phi)}{dV_\phi^2} \right) = \begin{cases} 3.7 \cdot 10^{-3} \left(\frac{e^\chi}{\chi^{1.53}} \right) & \chi < 2 \\ 2.7 \cdot 10^{-3} \left(\frac{e^\chi}{\chi^{1.82}} \right) & \chi \geq 2 \end{cases} \quad (\text{A29})$$

For $R < 1$, electron effects dominate, which implies from equation (A29) that probe bias remain in the approximate range $.1 \text{ kTe} \leq eV_\phi \leq 10 \text{ kTe}$. In practice, uncertainties in distribution function measurement tend to limit results to well within this range.

Other Effects

A compendium of experimental complications associated with probe measurements is presented in most standard probe references.^{2,3,8,9} Important effects like probe surface contamination and probe area variation can usually be minimized by careful probe/driver design. For example; as previously noted, the Probe Driver (Fig. A-4) initially applies a large positive bias (~ 70 volts) to the probe in order to insure a clean collection surface by electron bombardment. Probe area variation is minimized by the use of a shadowed cavity probe (Fig. A-9), which prevents an electrical contact from developing between the probe surface and deposited metallic materials on adjacent insulators. This technique is especially important in high current, hot cathode arcs; where tungsten plating due to ablation of the filaments is large.

Probe perturbation of the plasma is, however, unavoidable. The degree to which the perturbation effects the probe measurement is a function of probe-plasma operating conditions, and for many systems becomes appreciable only when probe bias approaches the plasma potential, where the collected electron current is large. Regarding the probe current-voltage characteristic, this effect tends to round off the ideally sharp break occurring between the electron transition region and the electron saturation region (Fig. A-1). A reasonable approximation of the plasma potential and associated electron saturation current, is customarily achieved by linearly extrapolating the two regions in the neighborhood of the break and obtaining their intersection (Fig. A-1).

In distribution function measurements, the perturbation's effect

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Figure A-9. Shadowed cavity cylindrical probe.

appears as a depletion in the number of electrons with energy roughly less than or equal to kT_e . This effect is mitigated by the condition that for most applications, distribution function structure in this energy region can be inferred from probe characteristic measurements.

III. Electron Distribution Function Measurement by Harmonic Analysis

If a small A.C. potential at frequency ω_0 is added to the D.C. probe bias, that is

$$V_{\phi}(t) = V_{\phi}^0 + v \cos(\omega_0 t) \quad (\text{A30})$$

where

$V_{\phi}^0 \equiv$ D.C. probe bias

$v \equiv$ A.C. probe bias amplitude

then the corresponding probe current density as a function of probe bias can be expressed in terms of its Taylor series expansion about V_{ϕ}^0 :

$$\begin{aligned} j(V_{\phi}) = j(V_{\phi}^0) + v \cos(\omega_0 t) \left. \frac{dj(V_{\phi})}{dV_{\phi}} \right|_{V_{\phi}=V_{\phi}^0} + \frac{v^2}{2} \cos^2(\omega_0 t) \left. \frac{d^2j(V_{\phi})}{dV_{\phi}^2} \right|_{V_{\phi}^0} \\ + \frac{v^3}{6} \cos^3(\omega_0 t) \left. \frac{d^3j(V_{\phi})}{dV_{\phi}^3} \right|_{V_{\phi}=V_{\phi}^0} + \dots \quad (\text{A31}) \end{aligned}$$

Noting the trigonometric identities:

$$\cos^2(\omega_0 t) = 1/2 \cos(2\omega_0 t) + 1$$

$$\cos^3(\omega_0 t) = 1/4 \cos(3\omega_0 t) + 3 \cos(\omega_0 t)$$

$$\cos^4(\omega_0 t) = 1/8 \cos(4\omega_0 t) + 4 \cos(2\omega_0 t) + 3$$

and substituting into equation (A31) yields:

$$j^{2\omega}(V_{\phi}^0) = \frac{v^2}{4} \left. \frac{d^2 j(V_{\phi})}{dV_{\phi}^2} \right|_{V_{\phi}=V_{\phi}^0} + \frac{v^4}{48} \left. \frac{d^4 j(V_{\phi})}{dV_{\phi}^4} \right|_{V_{\phi}=V_{\phi}^0} + \dots \quad (A32)$$

where $j^{2\omega}(V_{\phi}^0)$ is the spectral component of the probe current density at frequency $2\omega_0$ and D.C. bias V_{ϕ}^0 (i.e. $j(V_{\phi}^0, 2\omega_0) = j^{2\omega}(V_{\phi}^0) \cos(2\omega_0 t)$). Neglecting higher order terms in equation (A32), and substituting into equation (A9),

$$j_e^{2\omega}(V_{\phi}^0) = \frac{\pi v^2 e^3}{2m_e} f_e(\epsilon) \bigg|_{eV_{\phi} = \epsilon} \quad (A33)$$

which relates the electron energy distribution function to the second harmonic component of the electron probe current density.

Figure (A-10) schematically illustrates the harmonic detection network. The network consists of four main sections; the probe driver (previously described), the driving signal network which supplies A.C. probe bias, the reference frequency generator or frequency doubler, and a phase sensitive lock-in detector. In the driving signal network (Fig. A-11) the output of a sine wave generator is filtered by a tuned amplifier (Q-40) composed of μA 741 op amps, which limits the total harmonic distortion of the driving signal to less than .04% at output. Isolation amps, composed of μA 741 voltage followers, provide buffering to avoid resonant interaction among stages.

Outputs of the driving signal network are applied to the sum point of the probe driver op-amp, where a variable D.C. bias is added; and to the frequency doubling network (Fig. A-12) which provides a

Figure A-10. Second harmonic detection network.

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Figure A-11. Driving signal network.

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Figure A-12. Frequency doubling network.

reference signal at the second harmonic. The probe network supplies a constant amplitude A.C. bias over the entire range of D.C. bias values. Nonlinearities in the probe current-voltage characteristic generate harmonic distortion of the fundamental driving frequency, which is differentially detected by a lock-in amplifier. The filtered harmonic content of the probe current is mixed with the reference frequency, time integrated to minimize noise effects, and recorded by a storage scope; the recorded signal being directly proportional to the second harmonic content of the probe current.

IV. Accuracy Analysis of the Harmonic Technique

Several of the effects previously mentioned in the accuracy analysis of the pulsed probe detection network apply to the harmonic technique as well. In particular, analysis of probe/plasma response (for $\omega_0 < \omega_{pi}$), ion current effects, shot to shot reproducibility, probe perturbation of the plasma etc. are practically identical for the two techniques. In addition, accuracy of the harmonic technique depends on nonideal circuit effects in the harmonic detection network, and the effect of higher order derivatives on the probe current second harmonic content (equation A32).

Since the harmonic technique employs the non-linear property of the probe characteristic, nonlinearities in the detection network (Fig. A10) will induce systematic errors (circuit related harmonic distortion) in distribution function measurements. Also, the non-ideal behavior (due to finite bandwidth effects) of tuned amplifiers in the driving signal network and lock-in amplifier, can cause systematic

error due to non zero mixing of fundamental frequency components.

If necessary, this effect can be minimized by designing the apparatus to detect intermodulation rather than harmonic distortion.¹⁰

In general, the simple relation between the electron energy distribution function and the probe current second harmonic content (Eq. A33), may not be completely valid if higher order derivatives in Eq. A32 are significant. A generalized form of Eq. A33 is obtainable by recalling the previously derived relation describing electron probe current as a function of bias:

$$j_e(V_\phi) = \frac{2\pi e}{m_e} \int_{eV_\phi}^{\infty} \epsilon f_e(\epsilon) \left(1 - \frac{eV_\phi}{\epsilon}\right) d\epsilon \quad (A5)$$

where for the apparatus described in Fig. A10,

$$V_\phi = V_\phi^0 + V \cos(\omega_0 t) \quad (A30)$$

Letting $\cos(\omega_0 t) = \chi$, $j_e(V_\phi)$ can be expanded in a Chebyshev power series about V_ϕ^0 :

$$j_e(V_\phi) = 1/2 j_e^0(V_\phi^0) + \sum_{K=1}^{\infty} j_e^{K\omega}(V_\phi^0) T_K(\chi) \quad (A34)$$

where

$$j_e^{K\omega}(V_\phi^0) = \frac{2}{\pi} \int_{-1}^1 j_e(V_\phi) \frac{T_K(\chi)}{\sqrt{1-\chi^2}} d\chi \quad (A35)$$

is the spectral component of the electron probe current density at frequency $K\omega_0$ and D.C. bias V_ϕ^0 , and $T_K(\chi)$ is a Chebyshev Polynomial of degree K. Thus, the second harmonic component of $j_e(V_\phi)$ is

$$j_e^{2\omega}(V_\phi^0) = \frac{2}{\pi} \int_{-1}^1 j_e(V_\phi) \frac{T_2(\chi)}{\sqrt{1-\chi^2}} d\chi \quad (A36)$$

Substituting for $j_e(V_\phi^0)$ from Equation A5, and V_ϕ from Equation A30

$$j_e^{2\omega}(V_\phi^0) = \frac{4e^2 v}{m_e} \int_{-1}^1 d\chi \frac{T_2(\chi)}{\sqrt{1-\chi^2}} \int_{e(V_\phi^0 + v\chi)}^{\infty} (\alpha(\epsilon) - \chi) f_e(\epsilon) d\epsilon \quad (A37)$$

where:

$$\alpha(\epsilon) = \left(\frac{\epsilon - eV_\phi^0}{e v} \right) \quad (A38)$$

Interchanging the order of integration yields:

$$j_e^{2\omega}(V_\phi^0) = \frac{4e^2 v}{m_e} \left[\int_{e(V_\phi^0 - v) \geq 0}^{e(V_\phi^0 + v)} f_e(\epsilon) d\epsilon \int_{-1}^{\alpha(\epsilon)} (\alpha(\epsilon) - \chi) \frac{T_2(\chi)}{\sqrt{1-\chi^2}} d\chi \right. \\ \left. + \int_{e(V_\phi^0 + v)}^{\infty} f_e(\epsilon) d\epsilon \int_{-1}^1 (\alpha(\epsilon) - \chi) \frac{T_2(\chi)}{\sqrt{1-\chi^2}} d\chi \right] \quad (A39)$$

Integrating over χ , and defining the function:

$$\kappa(V_\phi^0, \epsilon) = e v \left(1 - \left(\frac{\epsilon - eV_\phi^0}{e v} \right)^2 \right)^{3/2} \quad (-e v \leq \epsilon - eV_\phi^0 \leq e v) \\ = 0 \quad (|\epsilon - eV_\phi^0| \geq e v) \quad (A40)$$

yields:

$$j_e^{2\omega}(V_\phi^0) = \frac{4e}{3\pi^2} \int_0^\infty \kappa(V_\phi^0, \epsilon) f_e(\epsilon) d\epsilon \quad (A41)$$

Equation A41 is the generalized form of Equation A33, including higher order effects. The function $\kappa(V_\phi^0, \epsilon)$ (Figure A-13) acts like a network transfer or instrument function analogous to Equation A24. Qualitatively, $\kappa(V_\phi^0, \epsilon)$ tends to smear out distribution function structure on a scale smaller than v , the A.C. bias amplitude. Typical experimental values of v chosen to maximize resolution and signal to noise effects range from .3 to .8 volts.

Equation A41, like equation A24, can be solved to yield an exact form of $f_e(\epsilon)$ via the technique of numerical deconvolution.^{6,7} However, experimentally judicious circuit design and/or minimal A.C. bias will yield instrument functions whose half widths are considerably less than the energy spread characterizing distribution function structure (e.g. for a Maxwellian distribution function, the instrument function half width must remain much less than the electron temperature). In this way, measurement errors incurred by not deconvoluting equations A24 or A41 are minimal with respect to other experimental considerations.

Figure A-13. Harmonic transfer function.

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