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Statistical Analyses in the Study of Solar Wind-
Magnetosphere Coupling

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Abstract

Statistical analyses provide a valuable method for establishing initially the existence (or lack of existence) of a relationship between diverse data sets. Statistical methods also allow one to make quantitative assessments of the strengths of observed relationships. This paper reviews the essential techniques and underlying statistical bases for the use of correlative methods in solar wind-magnetosphere coupling studies. Techniques of visual correlation and time-lagged linear cross-correlation analysis are emphasized, but methods of multiple regression, superposed epoch analysis, and linear prediction filtering are also described briefly. The long history of correlation analysis in the area of solar wind-magnetosphere coupling is reviewed with the assessments organized according to data averaging time scales (minutes to years). It is concluded that these statistical methods can be very useful first steps, but that case studies and various advanced analysis methods should be employed to understand fully the average response of the magnetosphere to solar wind input. It is clear that many workers have not always recognized underlying assumptions of statistical methods and thus the significance of correlation results can be in doubt. Long-term averages (≥ 1 hour) can reveal gross relationships, but only when dealing with high-resolution data (1-10 min) can one reach conclusions pertinent to magnetospheric response time scales and substorm onset mechanisms.

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Introduction

It is often said that there are three forms of untruth: lies, damned lies, and statistics. Statistical methods may often be employed to cover up the truth, but they also can be most illuminating and valuable if used properly. Statistical studies have played an especially important role in the study of solar wind-magnetosphere coupling and, thus, it is important to review some of the principal results of such studies.

As noted, for example, by Reiff (1983), statistical analyses are commonly used in space physics both to search initially for patterns of relationship ("exploratory studies") and to verify an assumed theoretical relationship ("confirmatory studies"). In the early years of solar wind-magnetosphere coupling (SMC) studies, the use of statistical methods fell primarily into the category of exploratory research (e.g., Snyder et al. 1963). People continue to search for the "best" empirical SMC parameter(s) today--still using statistical methods--but theoretical ideas also are being constantly evaluated by statistical means (e.g., Vasyliunas et al., 1982; Bargarze et al. 1985). Statistical methods, in this latter case, become the touchstone for deciding between different theoretical approaches.

In the present review, we will deal mostly with correlation analyses, although several other common methods will also be described briefly. In the course of our review, we will attempt to comment critically on the issues of "best" SMC parameters, the value (or weaknesses) of various data averaging time scales, the importance of determining confidence estimates, and the proper role of statistical methods in the overall approach to understanding magnetospheric response to solar wind variability.

Statistical Uncertainties in Observational Results

There are two components of the real world that make statistical analyses desirable and, indeed, crucial to our descriptions of physical relationships. The first of these is that we can not make perfect measurements of any quantity no matter how carefully we try. One may understand this by thinking of all the small, uncontrollable things that get in our way as we try to make a set of otherwise identical measurements. The result is that for any ensemble of observations, we get a range of values whose mean value (barring systematic errors) approaches the "true" value as the number of measurements goes to infinity.

A second feature of the physical world is that in complicated systems (such as the magnetosphere) there may be many parameters that affect a given "output" in a putative cause-and-effect relationship. For example, the relation between the interplanetary electric field and the cross-magnetospheric electric field may depend not only on dayside magnetic merging rates, but also on varying ionospheric conductivities, magnetospheric plasma compositions, previous ring current history, etc. Thus, even though we might feel we have excellent measures of global electric fields, the relationship determined will not be one-to-one because of covariant relationships over which there is little knowledge or control.

In general, measurement errors are well-described by the normal error (or Gaussian) distribution (Bevington, 1969; Worthing and Giffner, 1950). This function is an approximation to the binomial distribution as the number of observations approaches infinity. As relates to the discussion of the previous paragraphs, it is useful because it describes the distribution of random observations in most experimental situations and it provides reasonable

estimations of the parameters of most probability distributions. The Gaussian curve may be expressed as

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp \left(-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2 \right) \quad (1)$$

Here, y is a probability or frequency of occurrence, μ is the mean value of the parent population from which the parameter x is drawn, and σ is the standard deviation of the parent distribution. The curve is a bell-shaped function symmetric about the mean value of x .

There are a number of interesting derivations and justifications for the normal error curve (e.g., Worthing and Geffner, 1950), but one may also simply appeal to plausibility arguments and to its empirically replicated form (Bevington, 1969). An interesting mechanical construction that illustrates the normal error curve is Galton's quincunx (Worthing and Geffner, 1950). As shown in Fig. 1, the apparatus consists of an arrangement of small pegs above a horizontal set of collection bins. Small beads or shot fall through a hole in a hopper at the top and receive a series of small deflections from the pegs as they fall downward. The sidewise deflections occur with equal likelihood to the right and the left, and are the counterpart to small random errors discussed above. The quincunx device, when set in motion, thereby produces a distribution of shot in the collection bins which approximate the normal error curve. The machine thus may be thought of as a mechanical analog of many other systems where small errors contribute to a final measurement uncertainty.

In addition to the Gaussian distribution, sometimes the distribution of possible data values will be described by the Poisson distribution (Bevington, 1969). The Poisson curve is an approximation to the binomial distribution when the mean probability of a given measurement is much smaller than the total

possible number (i.e., the probability is $\ll 1$ that any given value will occur). The Poisson distribution is given by

$$y = \frac{\mu^x}{x!} e^{-\mu} \quad (2)$$

where the parameters have the same meaning as in Eq. (1). This kind of distribution is generally appropriate in a counting experiment where data represent the number of counts per unit time.

In doing statistical analyses (e.g., least-squares regressions), one generally must take into account measurement uncertainties in the data. Either Gaussian or Poisson distributions of uncertainties will ordinarily be appropriate depending on the nature of the measurements.

Visual Correlation Analyses

Very often researchers will notice a seeming relationship between two or more parameters which happen to be plotted together in some way. For example, two variables may be plotted as a function of time and they may vary in a quite similar fashion or they may change in nearly opposite fashions. The human eye and brain are rather good at surveying such plotted data and spotting patterns of relationship. We term this ability to see a related behavior in parameters as "visual cross-correlation." This sort of analysis often leads to serendipitous discovery of physical relationships.

It is quite common to use visual correlation techniques in SMC studies. The time series representing various solar wind parameters (V , B , B_z , etc.) are often plotted on the same time scale as a measure of geomagnetic activity (K_p , AE , etc.) and the reader is asked to notice the similarity of behavior (perhaps with a time delay) between the time series. In this usage, visual cross-correlation is cited as evidence (or proof) of a physical relation being advocated by an author.

An example of the use of the visual correlation method is shown in Fig. 2 taken from Perreault and Akasofu (1978). In the upper part of the figure are shown the values of B_z , VB_z and hourly sums of the southward B_z component (ΣB_{st}) measured in the upstream solar wind. In the lower portion of the top panel are magnetospheric indices (c.f., Baumjohann, 1985, this volume): AE , AL , Dst , and $ASYM$. The lower panel of Fig. 2 shows the value of estimated solar wind input energy ($\epsilon(t)$) and the estimated magnetospheric dissipated energy ($U(t)$) on the same scales. The authors of the paper note the generally similar behavior of $\epsilon(t)$ and $U(t)$ and conclude thereby that a "good" and meaningful relationship exists between ϵ and U .

Many other studies dealing with solar wind-magnetosphere coupling have used the visual correlation technique in order to assess the issue of cause-and-effect relationship, magnetospheric response time scales, and so forth. Among such studies are those of Fairfield and Cahill (1966), Tsurutani and Meng (1972), Akasofu and Chao (1979), Akasofu (1980), and Akasofu et al. (1982).

In addition to the concurrent plotting of several parameter time series, visual correlation analyses may also be presented in the form of scatter plots of one parameter versus another. Without a linear or nonlinear regression fit, the reader is then asked to judge the degree of correlative behavior (or lack of it).

It may be argued that visual cross-correlation is the most primitive form of statistical analysis, but it is in some sense also the most "honest" since the viewer is allowed to judge the quality of relationships. The distinct disadvantage of the method is that there is no quantitative or absolute determination of the strength of correlative behavior. Without such quantification, the reader is not able to make any truly informed decision on the merits of two or more roughly equivalent apparent relationships in raw data sets.

Statistical Cross-Correlation Analyses

It is beyond the scope of this review to present an exhaustive discussion of the method of least-squares fitting using linear or polynomial functions. Furthermore only a brief description of the procedure and concept of linear cross-correlation will be given here. Readers are urged to read texts on these topics (e.g., Worthing and Geffner, 1950; Bevington, 1969); also, an interesting distillation of the essential methods and some amusing examples of what can go wrong are presented by Reiff (1983).

As described above, measurement error and/or unknown covariant factors can make the observed relation between two parameters X and Y very scattered and substantially less than one-to-one. Thus, even if we suspect on theoretical grounds that X and Y should be linearly related, nonetheless the measured data may be highly variable. Linear correlation analysis allows one to assess the strength of a correlation (if any) between X and Y, and it allows one to estimate whether the observed correlation is significant. We emphasize that correlation does not necessarily imply causality: all it says is that two parameters vary together and there may very well exist an unknown underlying factor causing the observed correlation.

In order to illustrate the point, suppose we have a set of measurements of two parameters X and Y. Since we are dealing with solar wind-magnetosphere coupling, let us assume X is a measure of some solar wind parameter and let us assume that Y is an hypothesized magnetospheric response of some sort. Generally, then, we may consider X to be a function of time which is called the input time series, I(t), and Y is also a function of time which is called the output time series, O(t). Very often there is a significant delay in the response of the magnetosphere to a solar wind variation and so it is appropriate to compare the value of I(t) with O(t) at a latter time. The lag time of O relative to I for optimum correlation may be represented by τ_L and one then assesses the correlation between I(t) and O(t + τ_L).

As noted previously, when making a limited set of measurements, we are sampling the infinite assumed "parent population" which has a mean ($\mu \equiv \lim_{n \rightarrow \infty} (\frac{1}{n} \sum X_i)$) and a variance ($\sigma^2 \equiv \lim_{n \rightarrow \infty} (\frac{1}{n} \sum (X_i - \mu)^2)$). In practice we only have available our limited set of measurements to estimate the experimental mean ($\bar{X} \equiv \frac{1}{n} \sum X_i$) and variance ($S^2 \equiv \frac{1}{n-1} \sum (X_i - \bar{X})^2$). When X and Y are correlated (but not related one-to-one), we can only speak of a given X having a most-probable value of Y

around which other value are distributed. The closer that the observed values are to the most probable value, the more definite is the relationship of Y to X. It is this underlying assumption which is the basis for most measures of the degree of correlation.

Assuming then that the measured set of data is the most probable set, the most probable value of Y is given by least-squares fitting the values of Y versus X. This is called a regression analysis of Y on X. Conceptually, one can envision a change of variables by removal of the means: $x = (X - \bar{X})$ and $y = (Y - \bar{Y})$. Then, one has a simplified least-squares linear relation (e.g., Worthing and Geffner, 1950)

$$y_c = \left(\frac{\sum xy}{\sum x^2} \right) x = bx \quad (3)$$

for the regression line. A measure of the scatter, or dispersion, of the observed values of y around the regression line is then given by

$$S_y = \sqrt{\frac{\sum (y - y_c)^2}{n}} \quad (4)$$

and is called the standard error of estimate. As may be seen, it is the square root of the average squared deviations from the least-squares linear fit through the data.

From this conceptual framework, one can define the correlation coefficient, r, as

$$r = \sqrt{1 - \left(\frac{S_y}{\sigma_y} \right)^2} \quad (5)$$

As is clear, r is a dimensionless measure of the degree to which data points vary from the best fit line. As noted by Reiff (1983), the value of r can be thought of as the square root of the fraction of the variance (of y) that can be explained by the linear fit with x . Thus, $(1-r^2)$ is the fraction of the variance of y which is not explained by the linear correlation. Many correlations in SMC are in the 0.7-0.8 range. For such values of r , $(1-r^2)$ is in the range 0.4-0.5; hence, even with these high correlations, nearly half of the variance is not explained by the assumed relationship.

In practice, rather than using Eq. (4), it is more convenient to go back to the raw measured data and define the correlation coefficient as

$$r = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{\sqrt{(n\sum X_i^2 - (\sum X_i)^2)(n\sum Y_i^2 - (\sum Y_i)^2)}} \quad (6)$$

We note that for simplicity we have suppressed the weighting of the data by which uncertainties of the X_i 's and Y_i 's should be taken into account explicitly (Bevington, 1969). We also note that Bevington includes very convenient FORTRAN programs to do the correlation analysis. Notice that Eq. (6) is quite useful in practice since only five moments are necessary to calculate r : $\sum X_i$, $\sum Y_i$, $\sum X_i Y_i$, $\sum X_i^2$, and $\sum Y_i^2$.

In common usage for SMC studies, X has had values of V_{SW} , B , B_z , B_x , VB_z , VB_s , $V^2 B_s$, ϵ , etc. in the solar wind. On the other hand, in such studies, Y has been K_p , AE , AL , AU , Dst , U_T , etc., and has usually been derived from magnetospheric indices or measurements. Combinations of parameters have generally been formed in order to linearize the assumed relationship before correlation analyses are performed.

We remind readers of the underlying assumptions of these derivations. First, it is assumed that X and Y are distributed normally about their mean (or least-squared) values. Secondly, it is assumed that the X-Y relation is linear. There should be good theoretical (or other) grounds for this second assumption. In particular, if the X-Y relationship is not linear, the meaning of r quickly becomes very unclear. Finally, it is normally assumed that the precision of X is much higher than that of Y; often in practice uncertainties in X can, in fact, be vested in Y.

As noted above, uncertainties in data should definitely be included in the computation of r . Many nonlinear relations (e.g. $y = ab^x$) can be linearized rather simply ($\log y = \log a + (\log b)x$). Attention should then be paid to whether the distributions of the data are log-normal or normal (in the raw, linear sense).

Let us finally consider the issue of confidence levels for a given computed value of cross correlation. Generally one can ask the question: Given a certain computed r , what is the probability that this is only spurious and the data are, in fact, uncorrelated? An answer to this question is shown in Fig. 3 (from Bevington, 1969). It is a plot of r versus the number of independent observations made. The several curves P show the probabilities at the 0.1, 0.05, etc., level that variables are not correlated. For example, from the figure one can determine that with 50 observations and a value of $r \sim 0.6$, then the probability is < 0.001 that the parameters are uncorrelated. Worded differently, the probability is $> 99.9\%$ in such a case that a true correlation exists.

a. Time Scales for Correlation Analyses

Solar wind-magnetosphere coupling studies reported in the literature have utilized data averages ranging from years down to minutes. Figure 4 shows a quasilogarithmic time line marking some of the primary (and convenient) averaging intervals commonly employed in correlative analyses. Time scales greater than a solar cycle could in principle be employed but at such large times, it is not clear that one is learning much about the behavior of solar wind influence on magnetosphere behavior: Such time scales are beyond meaningful averaging ranges for these studies. At the other extreme, for data sampling at < 1 minute, one is constrained by information transit times (e.g., Alfvén time scales) in the magnetosphere. Thus, uses of averages that are smaller than ~ 1 min also are probably not meaningful.

As noted in the figure, it is our opinion that data averages ≥ 1 hr make accessible information about gross coupling relationships between the solar wind and the magnetosphere, but only by using data in the $1 \sim 10$ min range can one probe the detailed coupling relationships. In the one- to multi-hour range, parcels of solar wind plasma have moved from the dayside of the magnetosphere to well beyond the explored reaches of the distant tail and one is, therefore, looking at interactions in the broadest sense. Similarly, time averages of data much less than $1 \sim 5$ min are at such high values in the frequency domain that one is probably largely studying "noise" in the input and/or output time series.

It is noted in Fig. 4 that geomagnetic storm time scales are in the many-hour to daily range. Substorm time scales, on the other hand, are in the several minute range. Since many studies of SMC are concerned with substorm processes, we would argue that correlative analyses should use relatively high resolution data. More will be said about this later.

It is a legitimate question to ask if long term solar wind coupling behavior is a superposition of a lot of short-term effects, or if different physical processes come into play at the longer time scales. If the former is true, then much information is lost using long averages, while in the latter case the long averaging is entirely appropriate. It is probably true that many features (ring current growth and decay, radiation belt diffusion, etc.) occur on slow enough time scales that longer averages are justified and, indeed, proper. On the other hand, for substorm studies one is constrained to use high time resolution data.

b. Correlation Studies on $\geq$ Annual Time Scales

There are many studies in the literature describing semiannual and annual average measurements of solar wind velocities, densities, temperatures, etc. (e.g., Gosling et al., 1971, 1976). These variations have been considered in terms of the 11-year sunspot activity cycle and, upon this basis, the relationship between solar activity and solar wind conditions at 1 AU have been well-documented (e.g., Sheeley et al., 1976).

Taking long-term averages of solar wind properties as input, Crooker et al. (1977) examined the very long time scale response of the magnetosphere to such gross solar wind variations. The planetary index A_p averaged over yearly time scales was compared to various solar wind parameters (and combinations thereof). Figure 5 shows some of the primary results of Crooker et al. In the upper portion of the figure, the average solar wind speed ($\bar{V}$) is compared with $\bar{A}_p$ for the period 1962 through 1975. The authors found a relation $\bar{A}_p = 10^{-5} \bar{V}^{2.3}$ with a correlation coefficient of 0.81. An improvement in the relation was found by doing a linear regression between $\bar{V}^2$ and A_p (see the lower part of

Fig. 5). This analysis yielded $\bar{A}_p = 7 \times 10^{-5} \bar{V}^2 - 1.8$ and a correlation coefficient of 0.86.

Such high correlation coefficients are not uncommon in very long-term studies (see papers in the review by Feynman, 1983). As noted above, results such as those of Crooker et al. (1977) are of considerable interest for understanding gross solar wind-magnetosphere relationships, but they have little to say about magnetospheric responses on substorm time scales.

c. Correlation Studies on 3-24 Hour Time Scales

Interplanetary magnetic field (IMF) parameters change significantly on time scales of minutes and even seconds. On the other hand, solar wind plasma parameters typically change significantly on time scales of 10's of minutes to hours. It has been quite common for workers to average solar wind, IMF, and planetary activity indices for periods of many hours to study coupling problems.

Contributing significantly to the prevalence of studies in this class has been the common availability of both interplanetary and geomagnetic activity data sets averaged over periods of 3, 6, 12 and 24 hours. For example, one of the most commonly used indices, K_p , is presented on 3- and 24-hour time scales. Furthermore, many spacecraft data sets are conveniently summarized on such time scales (e.g., ~100 min for low-altitude orbits, or daily for the geosynchronous orbit).

An example of the use medium scale data for SMC studies is shown in Fig. 6 (from Le1 et al. (1981)). The solar wind electric field y-component ($E_y = VB_z$) for average periods of 3 to 6 hours is plotted versus the magnetospheric electric field (E_M) inferred from the magnetopause location as determined by GEOS-1. Such a measurement is taken by the authors to provide a direct measure

of the efficiency of dayside magnetic reconnection. As shown in Fig. 6, the result was a relation given by $E_M = 0.13 E_y + 0.09$. This suggests a merging "efficiency" on the dayside of 13% and the high-degree of linear correlation in this analysis ($r = 0.97$) argues for a close coupling relation between E_y and E_M .

Again, such relatively long-term averages of solar wind and magnetospheric properties are telling about coupling on storm-like scales (but probably not on substorm scales). A great strength of such studies is the wide variety of data on planetary-wide basis that are available going back in time for decades, or more. As illustrated by the example above, such data have been quite commonly used and very quantitative relationships have been developed in many instances (e.g., Snyder et al., 1963; Olbert, 1968; Ballif et al., 1969; Bobrov, 1973; Svalgaard, 1977; Murayama et al., 1980; Gallagher and D'Angelo, 1981; Holzer and Slavin, 1982).

d. Correlation Studies on ~1-Hour Time Scales

Some of the truly seminal work on solar wind-magnetosphere coupling has been based upon excellent data sets of hourly-averaged quantities. With such data, one is beginning to approach (c.f., Fig. 4) the time scales of substorm development.

An example of a SMC correlation study using hourly data is the classic work of Arnoldy (1971). Figure 7 shows several of the lagged cross-correlation results obtained by Arnoldy using various parameters upstream (note the different plotting symbols) compared with the hourly-averaged AE index. As seen in the upper part of Fig. 7, solar wind and IMF parameters such as density (n), velocity (V), and total field strength (B) gave relatively unimpressive correlation results. In contrast, the sum of the southward (or negative) part

of the IMF B_z for the 1-hour interval preceding the AE averaging period (denoted by ΣB_{gT}) gave an impressively high correlation (~ 0.8) with AE. This result, borne out by many subsequent studies, suggested that substorm activity (as measured by AE) is directly and closely related to the amount of southward B_z in the interplanetary medium in the hour or so before a substorm expansive phase.

The lower part of Fig. 7 shows further results from the Arnoldy study. Convolving ΣB_{gT} with V gave nearly identical results whereas AE was only mildly correlated with solar wind pressure. These results show the exploratory and confirmatory power of the lagged cross-correlation analysis in choosing between various theoretical possibilities.

Since the highly successful work of Arnoldy, a vast literature has grown up in this category of 1-hour analysis studies (e.g., Garrett et al., 1974; Murayama and Hakamada, 1975; Maezawa, 1979; Holzer and Slavin, 1978, 1979; Murayama, 1979; Akasofu, 1979; Reiff et al., 1981; Meloni et al., 1982). In all these instances, linear correlation analyses have been used as the discriminator between "good" and "poor" solar wind drivers of geomagnetic activity. These studies have all constituted extremely valuable guides in solar wind-magnetosphere coupling studies and have sprung from the widely accessible interplanetary and geomagnetic data sets that are available (e.g., King, 1975).

We conclude from our examination of this literature set that hourly (and greater) time averaging is basically a low-pass filtering operation. Many magnetospheric processes and phenomena are of a relatively high frequency and, thus, long-term averaging eliminates these high frequency components. Therefore, this averaging procedure artificially increases correlation and masks magnetospheric response time scales. Hence, despite the convenience and

value of the hourly-average studies, they still are not the best way to address substorm processes and mechanisms.

c. High Time Resolution Correlation Studies

Because of the potential problems noted above, viz., the possible masking of the details of solar wind-magnetosphere coupling mechanisms, many researchers have gone from the medium- and coarse-resolution data sets (≥ 1 hr) to high time-resolution (1-10 min) studies. Correlation analyses employing such data hold the possibility of addressing details of substorm relationships.

Early studies of this class tended to preselect intervals for statistical analysis. For example, Meng et al. (1973) chose ~ 30 days on which B_z (IMF) was southward to do their analysis, while other authors (e.g., Burton et al., 1975) have examined only storm intervals. Studies examining rather clear, isolated substorm activity have found that cross-correlations between VB_z (or related parameters in the solar wind) and the AE index show a maximum correlation coefficient between 0.6 and 0.7 at a lag time of ~ 50 min (c.f., Rostoker et al., 1972).

In recent high-resolution correlation studies, Baker et al. (1981, 1983) have emphasized evaluations of the VB_B , V^2B_B , and ϵ parameters. In the 1981 study they found that VB_B and V^2B_B were virtually identical in their correlative behavior with AE. Furthermore, in this nonselective study using 2.5-min time resolution data, Baker et al. found VB_B both qualitatively and quantitatively superior to ϵ in the prediction of substorms (as measured by AE). This result is illustrated in the upper panel of Fig. 8.

This work was extended by Baker et al. (1983) to include continuous solar wind measurements by ISEE-3 at 1-min time resolution. The lower two panels of Fig. 8 show results separated according to general levels of geomagnetic

activity. The overall result found by Baker et al. (1983) was that VB_s , V^2B_s , and ϵ had virtually identical correlation properties with AE during quiet intervals, whereas ϵ was substantially less well correlated with AE than VB_s during disturbed intervals. Generally speaking, all correlation levels were lowered during disturbed conditions presumably due to higher frequency, uncorrelated components both in the solar wind (input) and magnetospheric (output) time series.

Nearly all workers using high resolution data for cross-correlation analyses have interpreted the lag times between solar wind and magnetospheric parameters in terms of a "response" time of the magnetosphere. This interpretation is probably justified to an extent, but due caution should be applied. Figure 9 is taken from Meng et al. (1973) and shows the result of a simulation analysis of an idealized input and output time series comparison through cross-correlation. Since cross-correlation is, in effect, a convolution of the input and output functions, Meng et al. showed that the detailed strength and shapes of the lagged cross-correlation profile depends on the specific shape and other characteristics of the original time series (see, also, Hirschberg and Holzer, 1975). Thus, one should be cautioned, in the absence of other data and analysis techniques, against interpreting peak lag times too literally as the response time of the magnetospheric systems.

We conclude, however, that analysis involving high time resolution data have revealed a great deal about substorm coupling mechanisms including magnetospheric response times. It is found that correlation coefficients are generally reduced from those found in longer-term average studies indicating the presence of high-frequency, internal magnetospheric instability processes. Generally, parameters involving V and B_z in the solar wind are highly

"successful" in correlations with AE, AL, and other indices, but levels of correlation vary substantially with general levels of geomagnetic activity.

f. Other Cross-Correlation Methods

Two other methods deserve mention here: multiple regression analyses and linear prediction filtering. Both of these methods rely essentially on techniques closely related to linear cross-correlation. Multiple regression is a generalization of linear correlation analysis allowing simultaneous multivariate studies (e.g., Meloni et al., 1982). The method allows nonpolynomial functions and makes specific provision for parameter optimization. It is particularly useful for indicating which parameters should be included in a complete theoretical description. The method, however, greatly increases the complexity of analyses and substantial physical insight needs to be applied in order to understand results.

Linear prediction filtering uses autocorrelation and cross-correlation methods (see Clauer, 1985, this volume) to determine the most general linear relationship between an input and an output time series. It has proven to be an excellent method for understanding magnetospheric temporal response to solar wind variations (Iyemori et al., 1979; Clauer et al., 1981; Bargatze et al., 1985). The method, described in detail by Clauer, thus allows a very general and quantitative assessment of prediction of output based upon an assumed input.

Selection-Based Methods

Available space does not allow a detailed treatment of the many other statistical methods that have been employed in SMC studies. Two methods deserve special mention however. The first of these we term "transfer-interval analyses" (Holzer and Slavin, 1982). In this method, the AL index is taken as a measure of the rate at which magnetic flux is returned from the tail by substorm processes to the dayside magnetopause. Solar wind parameters such as VB_s or ϵ are taken as measures of the rate of erosion of flux from the dayside by magnetic reconnection. The time integrals of AL and VB_s (or ϵ , V^2B_s) are then compared quantitatively to assess the issue the "best" predictor of geomagnetic activity. Intervals of disturbed conditions bracketed at beginning and end by quiet conditions are chosen and may range in length from hours to days.

This method, involving 1-hr data resolution and quiet-to-quiet interval selection, is assessed by cross-correlation analysis. It is a careful and highly successful method (see, also, Holzer and Slavin, 1979, 1981a,b). Although criticized (Kamide, 1983), the technique is a very valuable technique of understanding the energy transfer process from the solar wind to the magnetosphere (Holzer and Slavin, 1983).

A second additional statistical method deserving special mention is superposed epoch analyses (see, e.g., Reiff, 1983). The method has been employed very profitably for many interplanetary, magnetospheric, and SMC problems (e.g., Foster et al., 1971; Caan et al., 1978; Hakamada, 1980; Bleber et al., 1982). As demonstrated by these studies, the method is very valuable in showing the average behavior of the system, e.g., the average AE response to a southward turning of the IMF. Given proper physical insight in the choice of

parameters and in the selection of zero-epoch time, the method can be a very valuable adjunct to cross-correlation studies.

Summary

We conclude that a very large number of solar wind-magnetosphere studies have employed statistical methods. A partial listing of these is shown in Table 1 (Baker et al., 1984; Akasofu, 1981). In many cases the confidence levels of obtained correlation coefficients is not given nor are the underlying assumptions concerning normality of distribution, etc., carefully assessed or specified. Thus, it is difficult at times to be sure of how much to trust the statistical results of many studies.

In the final analysis, we suggest in the strongest terms that researchers make every effort to familiarize themselves in great detail with the methods and bases of statistical analysis (e.g., Bevington, 1969) before blindly proceeding with correlation (or other) studies.

We consider that SMC studies using hourly (or greater) averaging time scales have often obtained large correlation coefficients, but this is due to the low pass filtering effects of the temporal averaging method. This greatly downplays important internal, high frequency magnetospheric phenomena.

High time resolution (1-10 min) data sets have revealed a great deal about magnetospheric response times to solar wind variations. Further, the issue of "best" solar wind predictors has been illuminated by high resolution studies. It is clear that magnetospheric response character appears different as general levels of activity increase. We suggest that correlation studies be considered as merely part of a much broader set of methods (case studies, prediction filtering, superposed epoch analysis, etc.) in order to characterize completely the complex issue of solar wind-magnetosphere coupling.

Table 1. LIST OF THE CORRELATION STUDIES BETWEEN GEOMAGNETIC INDICES AND SOLAR WIND PARAMETERS

Author(s)	Geomagnetic Index (Time resolution)	Relation with solar wind parameters
Snyder et al. (1963)	ΣKp (24 hr)	$\Sigma Kp = (V - 330)/8.44$
Olbert (1968)	ΣKp (24 hr)	$\Sigma Kp = (V - 252)/6.3$
Ballif et al. (1969)	Kp (3 hr)	$Kp = 9 \left[-\exp \left(\frac{-0.8T_N - 0.35}{7.70} \right) \right]$
Arnoldy (1971)	AE (1 hr)	$AE = -0.26(\Sigma B_s \tau)_0 - 0.91(\Sigma B_s \tau),$ $= -0.33(\Sigma B_s \tau)^2 + 0.12P_0$
Rostoker et al. (1972)	AE (5.5 min)	B_z
Meng et al. (1973)	AE (5.5 min)	B_z
Bobrov (1973)	Kp (3 hr)	$Kp = f(V, B_z, \Delta B_z)$
Garrett et al. (1974)	ap, AE (1 hr)	$ap, AE = C_1 + C_2 V(B_s) + C_3 Vd$
Murayama and Hakamada (1975)	AE (1 hr)	$AE = CB_s V^2$
Burton et al. (1975)	Dst (2.5 min)	$\frac{d}{dt} Dst_0 = F(E) - a Dst_0$
Crooker et al. (1977)	Ap	$Ap = 3.5 \times 10^{-5} B_z V^2 - 1.9$
Svalgaard (1977)	am (3 hr)	$am = 6.6q(f, \alpha) \frac{BV_0}{21} \left[\frac{nV_0^2}{105} \right]^{1/3}$ $\times \frac{1.157}{(1+3 \cos^2 \psi)^{2/3}}$
Maezawa (1979)	Al, AU, am (1 hr)	$Al \propto B^{0.85} V^{2.08} (\sin \theta)^{0.54}$ $AU \propto B^{0.67} V^{1.15} (\sin \theta)^{0.34}$ $am \propto B^{1.03} V^{2.34} (\sin \theta)^{0.37} n^{0.2}$
Holzer and Slavin (1977)	AL (1 hr)	Rate of erosion $\frac{d\phi_E}{dt} = 0.2(1.9 \times 10^{10} \text{ cm}) B_z V_{sw} (\text{km s}^{-1})$ Rate of return $\frac{d\phi_r}{dt} = 1.8 \times 10^{10} AL(\gamma)$
Murayama (1979)	AL (1 hr)	$AL = 60(B_s + 0.5) V^2 F(x, B_y) n^{0.13}$
Murayama et al. (1980)	AL (1 hr)	$AL/V^2 = K(B_s + 0.5)$
Baker et al. (1981)	AE (2.5 min)	$VB_z, V^2 B, B^2 V, \text{etc.}$
Lei et al. (1981)	Correlation between the interplanetary and magnetospheric electric fields	
Reiff et al. (1981)	Correlation between ϕ_{PC} and B_z, c	
Holzer and Slavin (1982)	AL	$BV^2, B_s V, B_s^2 V$
Iijima and Potemra (1982)	Correlation between field-aligned current density and $VB_z, V^2 B, B^2 V, \text{etc.}$	
Meloni et al. (1982)	AE	B_z, VB_z, c
Baker et al. (1983)	AE, U_T (1 min)	$VB_s, V^2 B_s, c$

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Figure Captions

- Fig. 1. Galton's quincunx, a device for illustrating the physical basis of the normal frequency distribution law (from Worthing and Geffner (1950); used by permission of the publishers).
- Fig. 2. Various interplanetary and geomagnetic data sets plotted on similar time scales for period in February 1967. These plots illustrate the visual correlation method discussed in the text (from Perreault and Akasofu, 1973).

- Fig. 3. The linear correlation coefficient r versus the number of observations N . The solid curves give the probability P that variables exhibiting a given correlation coefficient are not correlated (from Bevington (1969); used by permission of the publishers).
- Fig. 4. A quasilogarithmic time scale illustrating commonly used data averaging intervals in solar wind magnetosphere coupling studies.
- Fig. 5. Comparisons of yearly averaged values of A_p , V , V^2 showing the gross relationships between solar wind variations and geomagnetic response (from Crooker et al., 1977).
- Fig. 6. A scatter plot and regression fit between the cross-magnetospheric electric field (E_M) inferred from measurements from GEOS-1 and the interplanetary electric field (E_y) inferred using VB_z (from Lei et al., 1981).
- Fig. 7. Lagged cross-correlation analysis results showing the relationship between hourly averages of the AE index and various interplanetary parameters as discussed in the text (from Arnoldy, 1971).
- Fig. 8. High time resolution, lagged cross-correlation analysis results comparing the efficacy of ϵ , VB_s , and V^2B_s as predictors of the AE index. (a) Results from an analysis using IMP-8 data (Baker et al., 1981). (b,c) Analyses using ISEE-3 solar wind data during both quiet and disturbed intervals (Baker et al., 1983).
- Fig. 9. Cross-correlation coefficients for two idealized sets of IMF B_z and AE variations (from Meng et al., 1973).

Fig. 1

Fig. 2

Fig. 3

CORRELATIVE ANALYSIS TIME SCALES

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 8

Fig. 9