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A RECOMMENDATION OF A TRIAXIAL FAILURE THEORY FOR GRAPHITE

by
P. Y. TANG

Prepared under
Contract DE-AT03-76ET35300
for the San Francisco Operations Office
Department of Energy

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ABSTRACT

An extensive literature search has been conducted on the application of triaxial failure theories to an anisotropic brittle solid. This study leads to the conclusion that the strength tensor theory of Tsai and Wu appears most promising for HTGR structural graphites. The reduction of this theory to model the transversely isotropic nature of HTGR structural graphites is presented. The extensions to the ongoing biaxial stress experimental program, needed to provide the additional data required to verify this theory, are defined.

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1. INTRODUCTION

1.1 OBJECTIVES AND CONTENTS

For a number of years, the design criteria for structural graphite components in HTGRs have been based upon the maximum (normal) stress failure theory [GA,1]. Considering its analytic simplicity and the limited experimental data available, this was a logical choice; however, its adequacy has been questioned by the Franklin Institute in a recent study performed for the NRC [FI,2]. In response to this study, General Atomic has initiated a theoretical and experimental program to review the issue of appropriate failure criteria for structural graphite in HTGRs. The initial phase of the experimental program, a biaxial test program, has been previously discussed [Ho,3]; this report describes the theoretical portion of the program.

The goals of the theoretical study were to:

1. Identify the best available triaxial failure theory for a general anisotropic brittle solid.
2. Simplify the theory to better suit transversely isotropic graphites.
3. Define any changes needed to adapt the existing experimental plan [Ho,3] to provide the data needed to correlate and verify the recommended failure theory.

These objectives have been achieved and are described in this report.

In the next section, several remarks on the triaxial failure criterion for graphite are given so the general flavor can be grasped. They include the definition of a failure surface, and the necessary and the desirable requirements for a graphite failure criterion. These general remarks also set up ground rules for the critical literature review summarized in Chapter 2.

Section 2.1 critically reviews the literature on triaxial failure theories for a brittle solid. Section 2.2 reviews the experimental investigations on graphite fracture, including attempts to correlate fracture data with existing theories. From this information, the conclusion is drawn, in Section 2.3, that the strength tensor theory developed by Tsai and Wu [4] should be pursued for use in HTGR structural graphite design.

Section 3.1 describes the details of Tsai and Wu's theory for a general anisotropic brittle solid. Section 3.2 defines the proposed failure theory for graphite, which is obtained by simplifying Tsai and Wu's general theory to a form suitable for a transversely isotropic brittle solid.

In Section 3.3 a test program to evaluate the proposed failure theory for graphite is developed and formulated. This program recommends a minimum number of tests, which are 4 uniaxial (normal stress) tests, a biaxial test, a triaxial test, and a torsion test. In Section 3.4, a few further remarks about the proposals are given, including their relation to the existing biaxial stress program.

Finally, the conclusions of this study are given in Chapter 4.

1.2 GENERAL REMARKS ON THE TRIAXIAL FAILURE CRITERION FOR GRAPHITE

For a triaxial stress state, a stress (or strain) tensor generally contains 6 non-trivial components. A triaxial failure criterion is a set of equations which, if satisfied by the six stress (or strain) components, imply fracture. Such a criterion can be written as

$$f(\sigma_k) = 1 \quad , \quad k=1,\dots,6 \quad , \quad (1.1)$$

where σ_k is the contracted notation of the stress tensor. For a given stress state, σ_k , f is characterized by

if $f(\sigma_k) < 1$, no failure occurs;

if $f(\sigma_k) = 1$, failure will occur; and (1.2)

if $f(\sigma_k) > 1$, not admissible.

Geometrically, Equation (1.1) defines a failure surface in 6-dimensional stress space and Equation (1.2) states that a stress point can only reside either on or inside the failure surface and that failure occurs only if the stress point is on the surface. Because of this geometric interpretation, the failure criterion (1.1) is also known as a failure surface.

The function f in Equation (1.1) is usually obtained by a mathematical correlation of experimental observations. It must, however, satisfy the following requirements:

R1. The function f must be a scalar function.

R2. The failure surface represented by Equation (1.1) must be closed, surrounding the zero stress state.

The first requirement ensures that the failure criterion is valid for all coordinate systems; the second requirement ensures that the material strength is finite in all directions.

Graphite is a brittle material with a small amount of plastic deformation prior to fracture. Like most brittle materials, it is compressible and has a much higher compressive strength than tensile strength. This material behavior will be referred to as the Bauschinger effect.

Unlike most brittle materials, graphite generally possesses a special material symmetry.* The material properties on any plane perpendicular to the direction of extrusion for extruded graphite or of molding for molded graphite are practically invariant under any rotation of that plane about any axis along the direction perpendicular to the plane. Such a plane and an axis are referred to as the isotropic plane and the axis of rotational symmetry, respectively. The properties on the isotropic plane are, however, quite different from those existing along the axis of rotational symmetry. The material symmetry so described is called transverse isotropy.** Any coordinate system with one axis parallel to the axes of rotational symmetry and the other two axes on the isotropic plane is referred to as a coordinate system of the material axes of the transversely isotropic material.

* The candidate graphites for use at GA are considered to possess such a special material symmetry.

** In view of this and the above footnote, graphite will, unless otherwise indicated, be referred to as transversely isotropic graphite throughout this report.

From the above discussions on the material characterizations of graphite, it should be clear that additional requirements are necessary for a graphite failure criterion. They are:

- R3. The failure criterion must consider the Bauschinger effect.
- R4. The failure criterion must conform to the transversely isotropic material symmetry.
- R5. The failure criterion must be free of the restriction of incompressibility and depend on the hydrostatic state of stress (i.e., mean stress).

Beside meeting the five necessary requirements, R1-R5, the proposed failure criterion would be more acceptable if it also satisfies the following desirable requirements:

- R6. The failure criterion should be capable of predicting the failure conditions for all stress states.
- R7. The failure criterion should be relatively simple for experimental characterization and engineering analysis; the number of required material strength data should be minimized.

2. A CRITICAL LITERATURE REVIEW

2.1 TRIAXIAL FAILURE THEORIES FOR A BRITTLE SOLID

In the literature, numerous triaxial failure theories have been proposed for a brittle solid. To structure the discussion, these theories will be grouped according to the four basic failure criteria to which they are related. The four basic criteria are: (1) the maximum stress (strain) theory, (2) the maximum shear stress theory, (3) the maximum strain energy theory, and (4) the maximum distortion energy theory. These basic criteria were originally developed for an isotropic*solid. Furthermore, the second and the fourth theories are well-known yield criteria for ductile materials. Although plastic yielding and brittle fracture are two completely different failure phenomena, yield criteria can predict the fracture of a brittle material. This is true because such a material experiences negligible plastic deformation before it fractures.

Following the ground rules established earlier, the four basic failure criteria and their further developments are critically reviewed in the next four subsections.

2.1.1 Theories Related to the Maximum Normal Stress (Strain) Criterion

The maximum normal stress criterion is the simplest and most commonly used failure criterion. It postulates that fracture will occur when the maximum tensile stress reaches the material's ultimate strength. This criterion is one of the oldest theories that can easily incorporate the Bauschinger effect. For an isotropic brittle solid this is done by assigning two

* Isotropy means that the material has no orientation preference.

different values for the ultimate strength, one when the maximum stress is in tension and the other when it is in compression.

For an anisotropic material separate values of the tensile and compressive ultimate strengths are observed for each material axis. To apply the maximum stress criterion to an anisotropic material, these strengths are used to limit the maximum stresses separately for each material axis. However, such an extension is not meaningful when the principal stresses do not align with the material axes. Moreover, for graphite under a biaxial tension, where the principal stress directions do coincide with the material axes, this criterion tends to overestimate the graphite strength.

Obviously, the dual of the maximum normal stress criterion is the maximum normal strain criterion, which postulates that a limiting tensile strain controls failure. This criterion would imply substantially higher strengths in biaxial tension than uniaxial tension, which is not true for graphite [Price,5].

Instead of being used directly as a macroscopic failure criterion, the maximum (tensile) stress criterion has been applied microscopically in the flaw mechanics to develop different macroscopic failure criteria, such as Griffith's biaxial criterion [6] and its three-dimensional extensions by Paul [7] and by Paul and Mirandy [8]. In these developments, it is assumed that a brittle material is microscopically inhomogeneous due to the presence of randomly orientated flaws, which are sufficiently separated to be considered as isolated cracks. Further, it is postulated that brittle fracture occurs as a consequence of the highly localized tensile stress concentration produced at the tip of the longest crack of the most dangerous orientation.

Clearly, the theories developed by Griffith, Paul, and Paul and Mirandy can explain the failure mechanism. In addition to this, they can predict the angle of fracture. These features are certainly desirable.

Unfortunately, these theories were originally developed for an isotropic material and all have the following deficiencies for graphite applications:

- D1. A state of infinite biaxial/triaxial compression is theoretically possible. (This violates the necessary requirement R2 for a failure surface.)
- D2. The criterion implies that a ratio of eight between the compressive and tensile strength is maintained, which is rarely observed for real materials. (This essentially violates the necessary requirement R3 for a graphite failure criterion.)*
- D3. The criterion is expressed in terms of the three principal stress components rather than the six general stress components. Since the material axes and the principal stress axes generally are not coincident for a transversely isotropic material, the criterion, like the maximum stress criterion, is not very useful for graphite. (This essentially violates the necessary requirement R4.)
- D4. The criterion represents a multi-segmented failure surface rather than a single smooth one. It seems to be extremely difficult to generalize such a multi-segmented criterion to include the effect of all stress components for a transversely isotropic material. Furthermore, the use of a multi-segmented failure surface would impose additional complexities in strength characterization and computational effort. (This violates the desirable requirement R7.)

* This deficiency is not observed in Paul and Mirandy's criterion because their criterion has properly taken into account the Bauschinger effect by considering the effects of Poisson's ratio and the flaw geometry.

2.1.2 Theories Related to the Maximum Shear Stress Criterion

Tresca [9] postulated that the plastic yielding of a ductile material will occur across the plane of the maximum shear stress when the shear stress reaches its critical value, the shear strength of the material. Coulomb [10] and Mohr [11] applied the concept of shear failure to brittle materials such as soils and rocks. They postulated that the shear strength of the material depends on two parameters – the cohesive strength of the material and the coefficient of the internal friction, μ , on the plane of fracture, which is different from the plane of the maximum shear stress by $\frac{1}{2} \tan^{-1}(\mu)$. Alternatively, Coulomb-Mohr's criterion can be expressed by a linear equation of the major and minor principal stresses which are normalized by two different parameters, namely, the tensile and the compressive ultimate strengths. This implies that the Bauschinger effect has been taken into account and, therefore, explains why the Coulomb-Mohr criterion is better than the Tresca criterion in their correlations with fracture data and fracture angles of brittle materials.

In spite of its wide acceptance, Coulomb-Mohr's theory has two unrealistic assumptions and implications:

1. The failure criterion is independent of the intermediate principal stress. (This basically violates the requirement R5.)
2. It fails to predict the failure condition where maximum tensile stress reaches a critical value before shear stress does. (This violates the requirement R6.)

Aiming at these deficiencies, Paul [12, 13, 14] proposed a three-parameter generalized pyramidal failure criterion for an isotropic material. Geometrically, his criterion approximates a non-linear curved fracture surface in the three-dimensional principal stress space by a multi-segmented linear surface, which can be made up by adjoining sets of hexagonal pyramids. Like those failure criteria developed in the flaw mechanics and presented

in the last subsection, Paul's theory contains the deficiencies D3 and D4. Hence, it is not recommended for graphite applications either.

2.1.3 Theories Related to the Maximum Strain Energy Criterion

The maximum strain energy criterion states that brittle fracture occurs when the elastic strain energy reaches a critical value. This criterion was modified by Ely [15] for a transversely isotropic material under a biaxial stress state, when he interpreted his biaxial data of graphite. In his modification, more than one independent strength parameter was allowed, and each of the two principal stresses contained in the expression of the elastic strain energy for the biaxial loading was normalized by the appropriate ultimate tensile or compressive strength for the same principal stress direction. Moreover, the Poisson's ratio in Ely's equation was not restricted to a fixed value of $\frac{1}{2}$, which was usually assigned for an incompressible solid.

Ely's criterion satisfies all the necessary requirements, R1-R5, for a failure criterion of graphite under a biaxial loading. However, this criterion may not become a desirable triaxial failure criterion for graphite under general loadings because it is a multi-segmented fracture surface in the principal stress plane and, hence, it has also the deficiencies D3 and D4.

2.1.4 Theories Related to the Maximum Distortion Energy Criterion

Von Mises' maximum distortion energy criterion [16] has been used with considerable success to predict the plastic yielding of those ductile materials which are isotropic, incompressible, and free of the Bauschinger effect. It is hardly surprising that most of the failure criteria for a brittle solid are generalizations of von Mises' theory with different degree of considerations of the Bauschinger effect, anisotropy and the dependence of the hydrostatic state of stress.

Bauschinger's effect was considered in the theories generalized by Grassi and Cornet [17], Fisher [18], Coffin [19], and Stassi [20] for an incompressible and isotropic brittle solid. The first three theories were

developed specifically for cast iron composed of a matrix of carbon steel with interspersed graphite. The Bauschinger effect is incorporated into the theories through a factor of stress concentration caused by the graphite flakes. The resulting failure criteria are all multi-segmented. The failure criterion developed by Stassi, on the other hand, is a single smooth surface. However, all of them are biaxial criteria expressed in terms of two principal stresses. This limits their use for general applications to graphite.

The effect of anisotropy was considered in the theories developed by Griffith and Baldwin [21], Forman [22], and Hill [23, 24]. The first two theories proposed distortion-energy failure criteria for orthotropic* materials such as wood by assuming that the materials behave linearly up to fracture. This assumption certainly limits their usefulness. Moreover, these criteria contain only one independent strength parameter, which is not sufficient to describe the fracture behavior of an anisotropic brittle solid. Such shortcomings do not occur in Hill's theory. Nevertheless, Hill's failure criterion fails to account for the Bauschinger effect and the dependence of the hydrostatic state of stress. This is understood because Hill's criterion was originally derived for the yielding of anisotropic metals.

Hill's equation, which consists of quadratic terms of the stress components, has been modified by Marin [25] and Hoffman [26] to include linear terms of the normal stress components so that the Bauschinger effect can be taken into account. Marin's failure criterion is expressed in terms of the principal stress components and, consequently, is not very useful for an anisotropic material under general loadings. Hoffman's criterion contains effects of all 6 stress components. But, its use for graphite structural design is still limited because it is independent of the hydrostatic state of stress.

* Orthotropic means that the material has three mutually perpendicular axes of anisotropy about which property variations are symmetric and along which the properties are different from those that exist along the other two axes.

Norris [27] also derived a failure theory which accounts for the Baushinger effect in an orthotropic material. He assumed that an orthotropic material such as wood could be considered as a material composed of walls of an isotropic material surrounding voids in the shapes of equal rectangular prisms. The walls of isotropic material enclosing these voids were parallel to the three planes of material symmetry. The application of the von Mises criterion to the three isotropic walls resulted in three failure equations in terms of the local stresses in the isotropic material. The local stresses were assumed to be proportional to the gross stresses applied to the orthotropic material. The ultimate strength parameter associated with the normal gross stress in the resulting equations was assumed to take its tensile or compressive value depending on whether the gross stress is in tension or in compression. Thus, Norris' criterion is a three-segment continuous ellipsoid. Moreover, its derivation does not seem to be rigorously correct.

All the failure criteria reviewed so far contain no higher order terms of the stress components than the quadratic ones, but they all have the following shortcomings:

1. They are limited to the case in which the reference axes are coincident with the material axes. The transformations of the strength criteria for a general coordinate system from the material axes are not known. These criteria can only be applied by transforming the external stresses to the material axes.
2. They do not include all the interaction effects of the 6 stress components. Such an inclusion would guarantee the dependence of the failure criterion on the hydrostatic state of stress.

A single quadratic failure criterion, which does not suffer these shortcomings has been developed by Tsai and Wu [4]. Their quadratic tensor polynomial failure theory is valid for any anisotropic brittle solid exhibiting the Bauschinger effect. Simplifications for material symmetries, such as orthotropy, transverse isotropy and isotropy, can be systematically obtained through the use of the strength tensors. These tensors are further restricted by certain stability conditions that ensure the closedness of the proposed failure surface in 6-dimensional stress space.

Zakharov [28] was, perhaps, the first one to use the strength tensors to express a failure criterion in a tensorially invariant form. However, his tensor polynomial criterion consists of only the normal stress components. Col'denblat and Kopnov [29] proposed a more general form for the failure criterion consisting of polynomial terms of all the stress components with empirical powers. Their theory is more general than Tsai and Wu's, but the mathematical expression given by Tsai and Wu is more practical and simpler from the operational point of view. Priddy [30] generalized Tsai and Wu's theory by adding some cubic terms of stress to their equation. However, the addition of extra terms was highly arbitrary and does not seem to be justified for a wide range of materials. Merkle [31] proposed an ellipsoidal yield criterion for a compressible transversely isotropic graphite. His criterion is essentially a special case of the theory proposed by Tsai and Wu because it contains no linear terms of the normal stress components.

2.1.5 The Best Theory Available for Graphite Applications

From the critical literature review described above, particularly the last two paragraphs, it should be clear that the quadratic strength tensor theory developed by Tsai and Wu seems to be the best available failure theory for graphite applications. It should also be clear that this theory satisfies all the necessary requirements R1-R5* for a graphite failure criterion. Besides, since it is a second order theory, the

*Requirements defined in Section 1.2.

criterion is relatively simple to characterize experimentally; thus, the desirable requirement R7 can be satisfied. The satisfaction of the remaining requirement, namely R6, can be verified only by correlation with the fracture data of graphite. This is addressed in the next section where the results of such correlations are reviewed.

2.2 CORRELATIONS OF THEORIES WITH FRACTURE DATA OF GRAPHITE

Several experimental investigations on the fracture of graphite materials are described in the literature. Most of the experiments were conducted under uniaxial and biaxial states of stress at room temperature. The biaxial data were usually obtained by testing thin-walled tube specimens to failure under combinations of internal, external, and axial pressures. The principal stress axes were coincident with the material axes in all tests. Usually, these biaxial data together with uniaxial data were used to define failure surfaces in a biaxial principal stress plane, which were then compared to the predictions of various failure theories.

In the compression-compression quadrant of the biaxial stress plane, it has generally been observed that the measured fracture strengths are considerably lower than the values predicted by the failure theories. A thorough study conducted by Weng [32] confirmed the general belief that this discrepancy is primarily due to the instability and the arch action of the thin-walled tube specimens under biaxial compressive stresses. It can be concluded that, due to the premature tube buckling, the existing biaxial compressive stress data should not be used to verify failure theories.

In this section, the existing experimental studies on graphite fracture will be outlined and the theories used for correlations in these studies will be ranked. In view of the discussions in the last paragraph, the ranking will be based on the uniaxial and triaxial data and the biaxial data in stress quadrants other than the compression-compression quadrant. For convenience, the outline of the experimental studies and

the ranking of the theories will be discussed separately for isotropic graphite and transversely isotropic graphite.

2.2.1 Isotropic Graphite

Jortner [33] measured fracture strengths and strains of Poco AXF-5Q, a fine-grained isotropic graphite, in several biaxial tension-tension and tension-compression stress states at 70°F, 3000°F, and 4000°F. He found that both the maximum stress and the maximum strain criteria did not really represent the fracture behavior of the graphite tubes tested at room temperature. For the higher testing temperatures, he found that the failure surfaces were similar in shape to the room-temperature surfaces but were displaced toward higher stresses.

Taylor, et al. [34] extensively studied the mechanical properties of three types of isotropic reactor graphite with and without fast neutron irradiation at 150°C in DIDO. They used cylindrical rod specimens to measure the following fracture data: ordinary uniaxial tensile and compressive strengths, shear strengths (of simple double shear tests), and triaxial strengths under simultaneous applications of hydrostatic pressure and an axial compressive load. They found that both tensile and shear strengths increased with neutron irradiation and that the tensile/shear strength ratio was roughly constant at a value slightly less than 0.5. They also found that their triaxial results for unirradiated graphite were described better by Mohr's criterion than Griffith's.

Broutman et al. [35] tested tube specimens of Poco AXM-5Q isotropic graphite in tension-tension and tension-compression stress states. They found that their test results did not agree well with the maximum stress, Mohr's, or Ely's theories. However, they did note that, in the tension-compression stress quadrant, the strength data lay between the Mohr's and Ely's theories; the Mohr's theory was the best approximation. In the tension-tension quadrant, Ely's theory was considered to be better than the other two.

Babcock et al. [36] studied the biaxial fracture behavior of ATJ-S graphite at elevated temperatures. Tubular specimens were tested under the compression-compression loadings as well as the tension-tension and compression-tension loadings. They found that the ATJ-S graphite under their study showed very low anisotropy; its fracture stress data at room temperature were in a good agreement with either Mohr's or Norris' criterion; and its fracture strain data showed no systematic failure surface.

The correlation results reviewed so far can be summarized as follows: For predicting the fracture behavior of isotropic graphites, Ely's theory is as good as either Norris' or Mohr's theory and all are better than Griffith's, the maximum stress, or the maximum strain theory. For an easy reference, the last statement is replaced by the following expression:

$$\text{Ely's} = \text{Norris'} = \text{Mohr's} \] \ \text{Griffith's, Maximum Stress, Maximum Strain,} \quad (2.1)$$

where the symbols "=" and "]" stand for "as good as" and "better than," respectively.

2.2.2 Transversely Isotropic Graphite

Ely [37,38,39] conducted extensive biaxial tests on Graph-1-Tite graphite, which is transversely isotropic. He used tubular specimens subjected simultaneously to an axial load and internal fluid pressure. The results of correlating his biaxial data with the existing theories can be summarized by the following expression:

$$\text{Ely's} \] \ \text{Stassi's} \] \ \text{Mohr's} \] \ \text{Max. Shear, Max. Distortion Energy, Max. Stress.} \quad (2.2)$$

Similar biaxial tests on Grade ATJ and JTA graphites and a limited number of triaxial compressive tests on ATJ graphite and shear strength test on JTA graphite have been conducted by Weng [40,32,41,42]. In the

biaxial tension, tension-compression and compression-tension stress quadrants, his analysis on the test results of the two transversely isotropic graphites led to

$$\text{Hoffman's }] \text{ Ely's }] \text{ Norris' } , \text{ Stassi's }] \text{ Hill's } . \quad (2.3)$$

This equation can be readily combined with Equation (2.2) into

$$\begin{array}{l} \text{Hoffman's }] \text{ Ely's }] \text{ Norris', Stassi's } \left\langle \begin{array}{l}] \text{ Mohr's }] \text{ Max Shear, Max Distortion} \\ \text{Energy, Max Stress.} \end{array} \right. \\ \phantom{\text{Hoffman's }] \text{ Ely's }] \text{ Norris', Stassi's } }] \text{ Hill's.} \end{array} \quad (2.4)$$

The relative ranks of Ely's, Norris' and Mohr's theories in Equation (2.4) are different from those in Equation (2.1), which resulted from a comparison based on isotropic graphite data. This is because Ely and Norris treat the transversely isotropic material symmetry more extensively than Mohr, and Ely treat the compressibility better than Norris. Recognizing this, Equations (2.4) and (2.1) can be further combined for transversely isotropic graphites as

$$\begin{array}{l} \text{Hoffman's }] \text{ Ely's }] \text{ Norris', Stassi's } \left\langle \begin{array}{l}] \text{ Mohr's }] \text{ Max Shear, Max Distortion} \\ \text{Energy, Griffith's, Max} \\ \text{Stress, Max Strain.} \end{array} \right. \\ \phantom{\text{Hoffman's }] \text{ Ely's }] \text{ Norris', Stassi's } }] \text{ Hill's.} \end{array} \quad (2.5)$$

Greenstreet, et al. [43] obtained biaxial data on tubular specimens of RAXM support element graphite, which is also transversely isotropic. They found that their data were in good agreement with either the maximum stress or Ely's criterion and in marked disagreement with the maximum shear stress criterion. On the surface, their first finding seems to contradict to Equation (2.5). But, actually it does not because their limited data

have only the biaxial stress ratios of 1:1, 2:1, and 1:-1 and in this range of stress ratios Ely's theory can only predict slightly better than the maximum stress theory.

Greenstreet, et al. [44] reported some uniaxial data for EGCR-Type AGOT graphite. Jortner [45,46] obtained biaxial fracture strength data of ATJ-S graphite* at room, as well as elevated temperature. He also obtained some triaxial stress test data of the same graphite at room temperature. Cobb, Parry, and Engle [47], and Price and Cobb [48] reported some uniaxial and biaxial fracture data for the H327 graphite. When the data on all these transversely isotropic graphites were reported, no attempt was made to compare them with the existing failure theories.

Chang and Weng [49] extensively compared the strength tensor theory of Tsai and Wu with fracture data of graphite, including most of those reported in the last paragraph. More specifically, the data they used were: (1) Weng's data [32,41] on grade ATJ graphite, (2) Weng's [40,42] on JTA, (3) Babcock's [36] and Jortner's [45,46] on ATJ-S, and (4) Cobb, Parry and Engle's [47] and Price and Cobb's [48] on H327. Chang and Weng claimed that very good correlations were obtained. This is shown in Figures 1-4 for the biaxial failure surfaces of four grades of graphite they used.

Huang [50] also compared Tsai and Wu's theory with strength data of graphite. He used Ely's data [39] of Graph-I-Tite graphite and Weng's biaxial data [32] of ATJ graphite. He found that the agreement between the theory and the experimental results was good except in the compression-compression stress quadrant (see Figures 5 and 6). In this stress quadrant, a wide scatter exists in the strength data of both grades of graphite. Huang believed that this was partially due to buckling of the tube specimens being tested and that the compression-compression strength of the material obtained from other types of test specimens might be different from that obtained from the tubular specimens.

* The degree of transverse isotropy of ATJ-S graphite may vary somewhat from location to location within a single billet. This may explain why the ATJ-S graphite used by Babcock [36] showed very low transverse isotropy while the same grade of graphite used by Jotner exhibited high transverse isotropy.

Fig. 2. Biaxial failure surface for JTA graphite [49].

Fig. 3. Biaxial failure surface for ATJ-S graphite [49] .

Fig. 4. Biaxial failure surface for H-327 graphite [49].

Fig. 5. Biaxial failure surface for graph-I-tite graphite [50].

Fig. 6. Biaxial failure surface for ATJ graphite [50].

2.2.3 Conclusions

From the above literature review of the experimental work, the following can be concluded:

1. Due to tube buckling, the existing compression-compression strength data of graphite should not be used to verify existing failure theories.
2. All the failure theories, except Tsai and Wu's theory, that had been correlated with fracture data of graphite were ranked by Equation (2.5). This result is consistent with that obtained in the last section from a critical literature review of the theoretical work.
3. Tsai and Wu's theory correlated well with the existing fracture data of graphite, except in the biaxial compression stress quadrant. This exception may be explained by the first conclusion until more data are available.

2.3 A RECOMMENDATION OF A TRIAXIAL FAILURE CRITERION FOR GRAPHITE

The conclusions of the critical literature review of the theoretical and experimental work on triaxial failure theories for a brittle solid were given in Subsection 2.1.5 and the last Subsection. These conclusions collectively show that the quadratic strength tensor theory of Tsai and Wu satisfies all the necessary and desirable requirements given in Section 1.2 for a graphite failure criterion. Thus, this theory is recommended for use in the design of HTGR graphite structures.

3. PROPOSALS

3.1 THE STRENGTH TENSOR THEORY OF TSAI AND WU FOR A GENERAL ANISOTROPIC BRITTLE SOLID

Tsai and Wu's strength tensor theory [4] for a general anisotropic brittle solid can be written as*

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad , \quad (i,j=1,\dots,6) \quad , \quad (3.1)$$

where σ_i is the stress tensor of rank two, F_i and F_{ij} are the strength tensors of rank two and four, respectively. To ensure that the material strength is finite in all directions, F_{ij} must be positive definite, that is,

$$F_{ij} \sigma_i \sigma_j > 0 \quad , \quad (3.2)$$

at all points σ_i in 6-dimensional stress space. Geometrically, Equation (3.2) is a necessary and sufficient condition to ensure that the failure surface represented by the quadratic polynomial Equation (3.1) is closed and ellipsoidal. It should be noted that the condition given by Equation (2) in [Tsai and Wu, 4],**

$$F_{ii} F_{jj} - F_{ij}^2 \geq 0 \quad (i \neq j) \quad , \quad (3.3)$$

* Unless otherwise indicated, the usual summation convention over a repeated index is used throughout this report.

** In Equation (3.3) repeated indices do not indicate summation.

is necessary but not sufficient to ensure the closedness of the failure surface.

Being in tensor form, the criterion (3.1) is valid not only for the material axes but also for any different reference coordinate systems. The transformations of σ_i , F_i and F_{ij} under the coordinate transformations follow the well-known tensorial laws [Sokolnikoff, 51] for the second rank and the fourth rank tensors, respectively. For example, if x'_i ($i=1,2,3$) denotes the reference coordinate system, x_i ($i=1,2,3$) designates the material axis system, and a_{ij} ($ij=1,2,3$) represents the associated coordinate transformation defined by

$$x'_i = a_{ij} x_j \quad , \quad a_{ij} a_{kj} = 1 \quad , \quad (i,j,k=1,2,3) \quad , \quad (3.4)$$

then, a second rank tensor $f'_{\ell m}$ and a fourth rank tensor $f'_{\ell mnp}$ referred to the x'_i system are related to f_{np} and f_{qrst} referred to the x_i system by

$$f'_{\ell m} = a_{\ell n} a_{mp} f_{np} \quad , \quad (3.5)$$

and

$$f'_{\ell mnp} = a_{\ell q} a_{mr} a_{ns} a_{pt} f_{qrst} \quad , \quad (3.6)$$

respectively. All the Latin indices in the last two equations take the values 1, 2, 3.

The major features of Tsai and Wu's theory are as follows:

1. Equation (3.1) is in tensor form. Hence, it is valid for all coordinate systems when it is valid for one coordinate system. This allows transformations of the strength criterion from one coordinate system to another through the well-known tensorial transformation laws.
2. The stability condition imposed on the strength tensors assures the closedness of the failure surface.
3. The criterion is operationally simple because it represents a single smooth surface rather than a multi-segmented surface and it contains no terms of the stress components higher than the quadratic ones.
4. The criterion has been obtained without using any constitutive equation of a brittle solid. More specifically, the linearly elastic stress-strain law has not been employed in the derivation of Equation (3.1). Thus, if a non-linear constitutive law is used to evaluate the stresses in Equation (3.1), there will be no inconsistency.
5. It is possible to derive a complete set of independent strength components for materials with different material symmetries. The graphites of interest in both reactor core and core support design at GA can be modelled as transversely isotropic. This simplification is easily derived from the general theory for anisotropic materials.
6. Equation (3.1) contains linear terms, which take into account the Bauschinger effect. That is, the effect of the difference in tensile and compressive strengths.

7. The effect of the hydrostatic state of stress on fracture can be considered since no restriction of incompressibility is contained in the theory.

3.2 THE PROPOSED TRIAXIAL FAILURE THEORY FOR GRAPHITE

For a general anisotropic material, there are 6 independent strength components for F_i and 21 for F_{ij} . If a material has symmetry, these numbers can usually be reduced substantially. For example, they reduce to 3 and 9, 2 and 5, and 1 and 2, respectively, for orthotropic, transversely isotropic, and isotropic brittle materials. Thus, if the material principal axes $x_i (i=1,2,3)$ are taken as the reference axes, with the $x_1 x_2$ coordinate plane being coincident with the isotropic plane for the case of transversely isotropic material, one may write F_i and F_{ij} for an orthotropic material as

$$F_i = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ & F_{22} & F_{23} & 0 & 0 & 0 \\ & & F_{33} & 0 & 0 & 0 \\ & & & F_{44} & 0 & 0 \\ \text{Symmetric} & & & & F_{55} & 0 \\ & & & & & F_{66} \end{bmatrix}, \quad (3.7)$$

for a transversely isotropic material as

$$F_i = \begin{bmatrix} F_1 \\ F_1 \\ F_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ & F_{11} & F_{13} & 0 & 0 & 0 \\ & & F_{33} & 0 & 0 & 0 \\ & & & F_{44} & 0 & 0 \\ & \text{Symmetric} & & & F_{44} & 0 \\ & & & & & 2(F_{11}-F_{12}) \end{bmatrix}, \quad (3.8)$$

or for an isotropic material as

$$F_i = \begin{bmatrix} F_1 \\ F_1 \\ F_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} F_{11} & F_{12} & F_{12} & 0 & 0 & 0 \\ & F_{11} & F_{12} & 0 & 0 & 0 \\ & & F_{11} & 0 & 0 & 0 \\ & & & 2(F_{11}-F_{12}) & 0 & 0 \\ & \text{Symmetric} & & & 2(F_{11}-F_{12}) & 0 \\ & & & & & 2(F_{11}-F_{12}) \end{bmatrix}. \quad (3.9)$$

In Equations (3.7)-(3.9), the first three values of the indices i and j are associated with the normal stresses in the x_1, x_2, x_3 directions, while the last three values are associated with the shear stresses on the $x_1 x_3$, $x_2 x_3$, and $x_1 x_2$ coordinate planes.

Let us confine our attention to the transversely isotropic material from now on. For this material, Equation (3.1) can be combined with Equation (3.8) as

$$\begin{aligned}
 & F_1(\sigma_1 + \sigma_2) + F_3\sigma_3 + F_{11}(\sigma_1^2 + \sigma_2^2 + 2\sigma_6^2) + \\
 & + F_{33}\sigma_3^2 + F_{44}(\sigma_4^2 + \sigma_5^2) + 2F_{12}(\sigma_1\sigma_2 - \sigma_6^2) + 2F_{13}(\sigma_1\sigma_3 + \sigma_2\sigma_3) = 1 .
 \end{aligned}
 \tag{3.10}$$

This equation was written with reference to the material axes. When the chosen reference axes are different from the material axes, one can still use Equation (3.10) by either transforming the stress tensor from the reference axes to the material axes by using Equations (3.5) and (3.4)₂, or transforming the strength tensors from the material axes to the reference axes by using Equations (3.5) and (3.6).

In Equation (3.10), the strength constants (F_1, F_3) and ($F_{11}, F_{33}, F_{44}, F_{12}, F_{13}$), respectively, are the 2 and 5 independent strength tensor components mentioned earlier for a transversely isotropic material. These strength constants can be determined readily by the simple strength tests proposed in the next section. However, they are not free material parameters because they are restricted by the stability condition (3.2). For a transversely isotropic material, this condition is equivalent to

$$\begin{aligned}
 & F_{11} > 0 , \\
 & F_{33} > 0 , \\
 & F_{44} > 0 , \\
 & F_{11}^2 - F_{12}^2 > 0 , \\
 & F_{11} F_{33} - F_{13}^2 > 0 , \\
 & F_{33}(F_{11}^2 - F_{12}^2) - 2F_{13}^2(F_{11} - F_{12}) > 0 .
 \end{aligned}
 \tag{3.11}$$

The quadratic tensor polynomial Equation (3.10) with the strength constants subjected to the stability condition given by Equation (3.11) is the proposed triaxial failure criterion for graphite.

3.3 THE PROPOSED TEST PROGRAM

A minimum number of tests are proposed here to completely characterize the seven strength constants ($F_1, F_3, F_{11}, F_{33}, F_{44}, F_{12}, F_{13}$) in the failure criterion (3.10). The tests consist of four uniaxial (normal stress) tests, a biaxial test, a triaxial test, and a torsion test. A description of these experiments and the analytical relations between the strength constants and the strengths measured are given in the following three subsections.

3.3.1 Uniaxial Tests

Because of the transverse isotropy of graphite, two types of specimens should be prepared to determine the uniaxial tensile and compressive strengths. In one, the loading axis of the specimen should be parallel to the x_3 axis (i.e., the axis of rotational symmetry) of graphite and, in the other, the loading axis should be perpendicular to this direction. Let X_t and Z_t denote the uniaxial strengths of graphite along the x_1 and x_3 directions and let X_c and Z_c be the uniaxial compressive strengths along these two directions. Then, Equation (3.10) gives

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c} ,$$

$$F_3 = \frac{1}{Z_t} - \frac{1}{Z_c} ,$$

(3.12)

$$F_{11} = \frac{1}{X_t X_c} ,$$

$$F_{33} = \frac{1}{Z_t Z_c} .$$

3.3.2 Biaxial Test

Thin-walled cylindrical specimens are used to determine the constant F_{13} . The specimen is prepared in such a way that the geometric axis of the cylinder coincides with the x_3 axis of graphite. The cylinder is simultaneously subjected to internal pressure and axial force. The non-vanishing stresses in the cylinder are

$$\sigma_3 = \sigma_B \quad (3.13)$$

$$\sigma_1 = \alpha \sigma_B ,$$

where α is a proportional constant, and both σ_1 and σ_3 are not simultaneously negative to avoid tube buckling under biaxial compressions. Let B denote the value for σ_B at failure. Then, Equation (3.10) yields

$$F_{13} = \frac{1}{2\alpha B^2} [1 - B(\alpha F_1 + F_3) - B^2(\alpha^2 F_{11} + F_{33})] , \quad (3.14)$$

where the constants F_1 , F_3 , F_{11} , and F_{33} have been determined by Equation (3.12).

3.3.3 Triaxial Test

Cylindrical rods with their geometric axes parallel to the x_3 axis can be used to determine F_{12} . The rod is laterally confined by a constant hydrostatic pressure P , and axially subjected to a monotonically increasing tensile force σ_T . Thus, the stress state in the rod is

$$\sigma_3 = \sigma_T , \quad (3.15)$$

$$\sigma_1 = \sigma_2 = -P .$$

Let T denote the value of σ_T at failure. Then, Equation (3.10) gives

$$F_{12} = \frac{1}{2P^2} (1 + 2F_1 P - F_3 T) - (F_{11} - \frac{2T}{P} F_{13} + \frac{T^2}{2P^2} F_{33}) \quad , \quad (3.16)$$

in which $(F_1, F_3, F_{11}, \text{ and } F_{33})$ and F_{13} have been found from Equations (3.12) and (3.14), respectively.

3.3.4 Torsion Test

Finally, F_{44} can be determined from a torsion test of thin-walled cylinders with the same configuration as those used in the biaxial test. Let M denote the value of the applied end torque at failure, a the average radius of the cylinder, and t the wall thickness. Then, Equation (3.10) leads to

$$F_{44} = \frac{4\pi^2 a^4 t^2}{M^2} \quad . \quad (3.17)$$

3.4 REMARKS ON THE PROPOSALS

A few further remarks about the proposed failure criterion and test program will be given in the following subsections.

3.4.1 Chang and Weng's Correlations

In correlating Tsai and Wu's theory with the existing fracture data of four different grades of graphite, Chang and Weng [49] computed the values of the strength constants, which are summarized in Table 1. They further checked these values with the stability condition given by Equation (3.3). For a transversely isotropic material, this equation is equivalent to

$$F_{11}^2 - F_{12}^2 > 0 \quad , \quad (3.18)$$

$$F_{11} F_{33} - F_{13}^2 > 0 \quad .$$

Table 1

Computed Strength Constants (1 psi = 6.895×10^3 Pa) [49.]

Constant Graphite	F_1 10^{-3} (psi) $^{-1}$	F_3 10^{-3} (psi) $^{-1}$	F_{11} 10^{-7} (psi) $^{-2}$	F_{33} 10^{-7} (psi) $^{-2}$	F_{13} 10^{-7} (psi) $^{-2}$	F_{12} 10^{-7} (psi) $^{-2}$	F_{44} 10^{-7} (psi) $^{-2}$
ATJ	0.1602	0.2775	0.2945	0.4758	-0.1603	0.9476*	1.3620
JTA	0.0949	0.2281	0.0719	0.1370	-0.0425	--	0.3367
ATJ-S	0.1382	0.1163	0.2233	0.3891	-0.1300	0.0493	0.4927
H-327	0.3750	0.3000	1.5620	1.000	-0.5578	--	--

* F_{12} was computed from the available triaxial test data. However, the value so determined violates the stability condition given by Equation (3.3).

Clearly, Equation (3.18) is only a subset of the proposed stability condition (3.11). Obviously, more restrictions have to be satisfied in Equation (3.11) than Equation (3.18). Therefore, it is interesting to further check the strength constants for the four grades of graphite with Equation (3.11). Before this can be done, however, two points in Chang and Weng's computation of the strength constants need to be clarified.

First, there is an error in their application of Jortner's triaxial data [45] to compute F_{12} for Grade ATJ-S Graphite. The tensile strength for the axial loading of Jortner's triaxial tests was misused as the compressive strength. The correct triaxial strength data that should have been used by Chang and Weng are

$$\begin{aligned} T &= +3590 \text{ psi} , \\ P &= 3830 \text{ psi} . \end{aligned} \tag{3.19}$$

Using this and the strength constants (F_1 , F_3 , F_{11} , F_{33} , and F_{13}) given in Table 1, the correct value for F_{12} of ATJ-S Graphite can be computed from Equation (3.16) as

$$F_{12} = - 0.0785 \times 10^{-7} (\text{psi})^{-2} . \tag{3.20}$$

Second, there is an undesirable footnote for Table 1, which casts a cloud on the applicability of the proposed theory to Grade ATJ graphite. The footnote stated that a violation of the stability condition (3.18), had occurred in ATJ graphite for the F_{12} value computed from the available triaxial data.

The triaxial strength data used by Chang and Weng were

$$\begin{aligned} \sigma_3 &= - 12980 \text{ psi} , \\ \sigma_1 &= \sigma_2 = - 2190 \text{ psi} , \end{aligned} \tag{3.21}$$

which were obtained by subjecting specimens to a constant hydrostatic pressure and a varying axial end pressure [Weng, 32]. Only two specimens were run to generate such data. In view of this and the wide scatter of graphite data for other tests, the triaxial test data recorded in Equation (3.21) may not be valid to compute F_{12} . Moreover, it is of vital importance to have the stability requirement fulfilled. Therefore, it is desirable to have F_{12} determined by (and, hence, automatically satisfied) the stability condition (3.18)₁ and see if the prediction of the triaxial strength agrees with experiment to within random experimental error.

Toward this end, Equation (3.10) is solved for σ_3 of a triaxial test as

$$\sigma_3 = \frac{1}{2F_{33}} \left\{ -(4F_{13}\sigma_1 + F_3) \pm \sqrt{(4F_{13}\sigma_1 + F_3)^2 - 4F_{33}[2(F_{11} + F_{12})\sigma_1^2 + 2F_1\sigma_1 - 1]} \right\}. \quad (3.22)$$

To satisfy Equation (3.18)₁, the values for F_{12} must be ranged by

$$F_{11} > F_{12} > -F_{11}. \quad (3.23)$$

When the values of F_{33} , F_{13} , F_3 , F_1 , F_{11} and σ_1 are given, Equations (3.22) and (3.23) reveal that the maximum value of the axial compressive load, $(-\sigma_3)_{\max}$, occurs at

$$F_{12} \approx -F_{11}. \quad (3.24)$$

Using Table 1, Equations (3.24) and (3.21)₂, F_{12} and $(-\sigma_3)_{\max}$ can be calculated from Equation (3.22) as

$$F_{12} = -0.2940 \times 10^{-7} \text{ (psi)}^{-2}, \quad (3.25)$$

and

$$(-\sigma_3)_{\max} = -11810 \text{ psi}, \quad (3.26)$$

respectively. About 9% difference in the axial compressive strength is found between the value calculated here and the experimental data given by Equation (3.21)₁. Thus, if F_{12} is determined by satisfying the stability condition (3.18), 9% of the calculated error results for Weng's triaxial tests on ATJ graphite.

Next, we need to know the sample standard deviation of the mean for a triaxial test. Apparently, we can not directly get this for ATJ graphite because there are only two samples available for this graphite. This piece of information has to be borrowed from other graphites. The three types of DIDO isotropic reactor graphites studied by Taylor, et al., [34] are probably the only graphites which have been tested extensively by triaxial tests. Four to eight specimens were run per increment of hydrostatic pressure in these tests. The sample standard deviation of the mean for these tests can be estimated from Fig. 6 of [34]. It ranged from 5% to 12.5%, with an average value of about 9%. Thus, should the sample standard deviation of the mean for a triaxial test be the same for ATJ graphite and the graphite studied by Taylor, et al., the calculated error by the proposed theory agrees with test on ATJ graphite to within random experimental error.

With the values of F_{12} being adjusted by Equations (3.25) and (3.20) for ATJ and ATJ-S graphites, respectively, we are now ready to use Table 1 to check the complete set of the stability Equation (3.11). Straightforward calculations show that no violation of this equation can be found for all of the four grades of graphite under investigation. For completeness, the correct values for F_{12} are combined with Table 1 into Table 2.

3.4.2 Effective Stress Criterion

In predicting failure of a ductile metal under a combined stress state, the effective stress approach is well-known. In this approach, a triaxial stress state is represented by a single stress quantity called the effective stress, and the failure of the material will not occur if such a quantity is less than the uniaxial ultimate strength. It is quite natural to ask what is the analogue for graphite to such an approach for metal.

Table 2

Corrected Strength Constants* (1 psi = 6.895×10^3 Pa)

Constant Graphite	F_1 10^{-3} (psi) $^{-1}$	F_3 10^{-3} (psi) $^{-1}$	F_{11} 10^{-7} (psi) $^{-2}$	F_{33} 10^{-7} (psi) $^{-2}$	F_{13} 10^{-7} (psi) $^{-2}$	F_{12} 10^{-7} (psi) $^{-2}$	F_{44} 10^{-7} (psi) $^{-2}$
ATJ	0.1602	0.2775	0.2945	0.4758	-0.1603	-0.2940	1.3620
JTA	0.0949	0.2281	0.0719	0.1370	-0.0425	--	0.3367
ATJ-S	0.1382	0.1163	0.2233	0.3891	-0.1300	-0.0786	0.4927
H-327	0.3750	0.3000	1.5620	1.000	-0.5578	--	--

* Table 2 is the same as Table 1 except the values for F_{12} in Table 1 are replaced by those in Equations (3.25) and (3.20).

Aiming at this analogue, we note that F_{33} is positive in view of Equation (3.11)₂. Hence, F_{33} can be used to divide Equation (3.10). The resulting equation, after rearranging terms, is

$$\begin{aligned} \sigma_3^2 + \frac{F_{11}}{F_{33}} (\sigma_1^2 + \sigma_2^2) + \frac{2F_{13}}{F_{33}} (\sigma_1 \sigma_3 + \sigma_2 \sigma_3) + \frac{2F_{12}}{F_{33}} \sigma_1 \sigma_2 + \frac{F_{44}}{F_{33}} (\sigma_4^2 + \sigma_5^2) \\ + \frac{2(F_{11} - F_{12})}{F_{33}} \sigma_6^2 + \frac{F_3}{F_{33}} \sigma_3 + \frac{F_1}{F_{33}} (\sigma_1 + \sigma_2) = \frac{1}{F_{33}} \end{aligned} \quad (3.27)$$

Since the right-hand side of Equation (3.27) is positive, the left-hand side of Equation (3.27) can be replaced by the square of a single quantity, σ_{eff} , called the "effective stress":

$$\begin{aligned} (\sigma_{\text{eff}})^2 \equiv \sigma_3^2 + \frac{F_{11}}{F_{33}} (\sigma_1^2 + \sigma_2^2) + \frac{2F_{13}}{F_{33}} (\sigma_1 \sigma_3 + \sigma_2 \sigma_3) + \frac{2F_{12}}{F_{33}} \sigma_1 \sigma_2 + \frac{F_{44}}{F_{33}} (\sigma_4^2 + \sigma_5^2) \\ + \frac{2(F_{11} - F_{12})}{F_{33}} \sigma_6^2 + \frac{F_3}{F_{33}} \sigma_3 + \frac{F_1}{F_{33}} (\sigma_1 + \sigma_2) \end{aligned} \quad (3.28)$$

Substituting Equation (3.28) for σ_{eff} and Equation (3.12)₄ for F_{33} into Equation (3.27), the failure condition becomes

$$\sigma_{\text{eff}}^2 = Z_t Z_c \quad (3.29)$$

Thus, a triaxial stress state in graphite can be represented by a single stress quantity and it will not fail if such an quantity is less than the product of axial tensile and compressive strenghts. Clearly, Equation (3.29) with the effective stress defined by Equation (3.28) is the analogue we sought for graphite. Hence, the proposed criterion (3.10) can also be referred to as an effective stress criterion.

3.4.3 Relation to the Existing Experimental Program

As mentioned in the first section of this report, GA has been carrying out a biaxial stress test program [Ho, 3] for the candidate graphite materials for the HTGR core support structure. The immediate objective of this program is to obtain enough data to adequately describe the biaxial failure surface in a plane perpendicular to the isotropic plane of these materials. The biaxial data will be used to correlate with various failure theories to establish the one which will best represent the fracture behavior of graphite.

Mathematically, the biaxial stress test can be described by

$$\sigma_1 \neq 0, \sigma_3 \neq 0 \text{ while, } \sigma_2 = \sigma_4 = \sigma_5 = \sigma_6 = 0. \quad (3.30)$$

When Equation (3.30) is substituted into Equation (3.10), one obtains

$$F_1 \sigma_1 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{33} \sigma_3^2 + 2 F_{13} \sigma_1 \sigma_3 = 1. \quad (3.31)$$

From the last equation, it is clear that the biaxial tests can at most determine five strength constants (i.e., F_1 , F_3 , F_{11} , F_{33} , and F_{13}) leaving the two constants, F_{44} and F_{12} , undetermined. In view of this, the triaxial and torsion tests recommended in the last section are necessary additions to the existing biaxial program to obtain all the strength constants needed.

Thus, it is concluded that the existing biaxial stress program can be used to verify the proposed failure theory for graphite by additionally including the triaxial and the torsion tests.

4. CONCLUSIONS

The critical literature review of both the theoretical and the experimental results led to the following conclusions:

1. The strength tensor theory developed by Tsai and Wu appears to be the best practical failure theory for graphite.
2. Biaxial compression tests on the thin-walled tube specimens should be avoided in any verification of the proposed theory in view of the consistent reports of premature failures caused by tube buckling.

To verify the proposed failure theory, both the triaxial and the torsion tests are recommended to supplement the on-going biaxial stress program.

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